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Theory

2.1 Indices

```
def indices(N, d):
    list = [0 for x in range(d)]
    if d != 0:
        indices_recurse(N, d, list, 1)

def indices_recurse(N, d, list, index):
    if index == len(list):
        for i in range(N + 1):
            print(list)  # print a unit of output
            list[index-1] += 1
        list[index-1] = 0
    else:
        for i in range(N + 1):
            indices_recurse(N, d, list, index+1)
            list[index-1] += 1
        list[index-1] = 0
```

Q: "If we say an output of (0, 0, … , 0) is one unit of output, how many units of output in terms of N and d are there?"

A: $(N+1)^d$

Q: "Now dropping the assumption that $n_1, n_2, \dots, n_d = N$, how many units of output in terms of $n_1, n_2, \dots, n_d$ and d are there?"

A: $(n_1+1)(n_2+1) \dots (n_d+1)$

2.2 Loops

Prove that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \forall n \in \mathbb{N}, n \geq 1.$$

Base Case:

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1$$

Induction hypothesis:

Assume n = k is correct, ∀k ∈ **N,** k >= 1

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

Induction step:

Prove that k+1 works

$$\sum_{i=1}^{k+1} i = \frac{(k+1)\big((k+1)+1\big)}{2}$$

$$\sum_{i=1}^{k} i + (k+1) = \frac{(k+1)\big((k+1)+1\big)}{2}$$

Substitute using the induction hypothesis

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)\big((k+1)+1\big)}{2}$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)\big((k+1)+1\big)}{2}$$

$$\frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)\big((k+1)+1\big)}{2}$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)\big((k+1)+1\big)}{2}$$

$$\frac{(k+1)\big((k+1)+1\big)}{2} = \frac{(k+1)\big((k+1)+1\big)}{2} \blacksquare$$

2.3 Dishonest Professors

1. Make a last in first out stack.
2. Push two professors into the stack.
3. Ask the top two professors about their opinion of the other.
4. If either of the profs say dishonest, pop both profs out of the stack. If both say honest, keep both in the stack.
5. Push one new professor into the stack.
6. Repeat steps 3-5 until there are no more profs left.
7. The professor on the bottom will be an honest prof.
8. Use this honest prof to discern the honesty of all 99 other profs using the 99 remaining questions.