Environmental Remote Sensing
GEOG 2021

Lecture 6

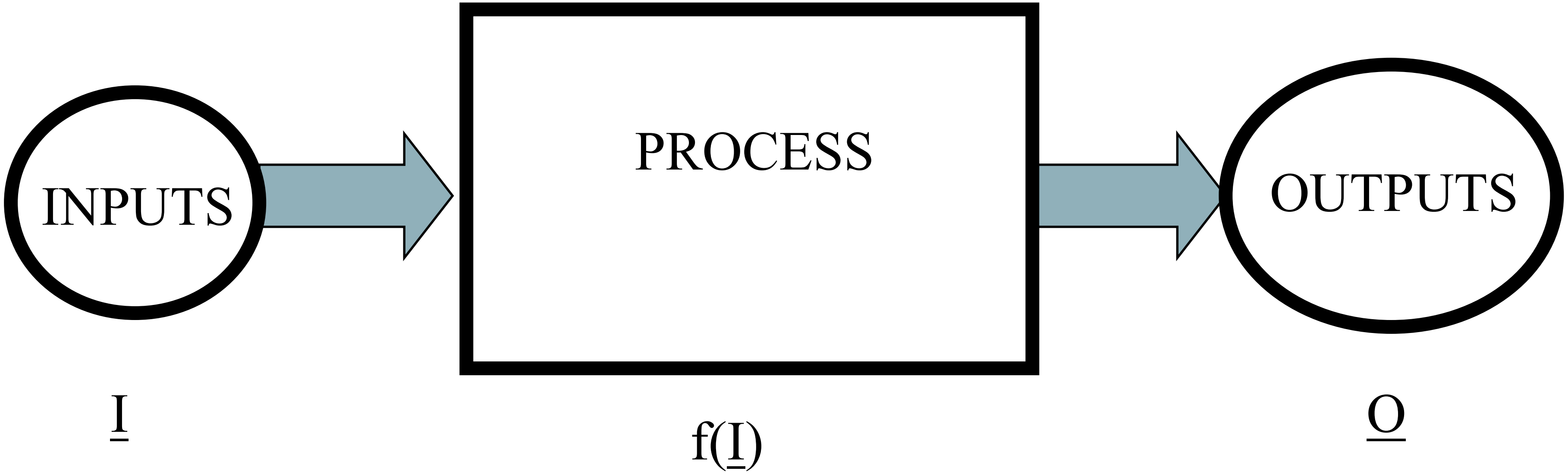
Mathematical Modelling in Geography I

Mathematical Modelling

- **What is a model?**
  - an abstracted representation of reality
- **What is a mathematical model?**
  - A model built with the ‘tools’ of mathematics
- **What is a mathematical model in Geography?**
  - Use models to simulate effect of actual or hypothetical set of processes
  - to forecast one or more possible outcomes
  - Consider spatial/temporal processes

Mathematical Modelling

Functional model representation



O=f(I)

Type of Mathematical Model

Main choice:

- **Statistical and/or empirical**
  - Use statistical description of a system rather than exact
  - Or look for empirical (expereimental/evidence-based) relationships to describe system
- **Physically-based**
  - model physics of interactions
  - in Geography, also used to include many empirical models, if it includes some aspect of physics
    - e.g. conservation of mass/energy - e.g. USLE (universal soil loss equation)
  - similar concepts:
    - Theoretical model
    - Mechanistic model

Type of Mathematical Model

- May chose (or be limited to) combination in any particular situation
- Definitions / use varies

Type of Mathematical Model

Other options:

- **deterministic**
  - relationship **a**=f(**b**) is always same
    - no matter when, where calculate it
- **stochastic**
  - exists element of randomness in relationship
    - repeated calculation gives different results

Type of Mathematical Model

2 modes of operation in modelling:

- **forward model**
  - **a**=f(**b**)
  - measure **b**, use model to predict **a**
- **inverse model**
  - **b**=f^{-1}(**a**)
  - measure **a**, use model to predict **b**
  - THIS is what we nearly always want from a model – invert model against observations to give us estimates of model parameters….

Type of Mathematical Model: e.g. Beer's Law

E.g.:

- **forward model**

$$backscatter = a + be^{-c*biomass}$$

- **inverse model**

$$biomass = -\frac{1}{c}\ln\left(\frac{basckscatter - a}{b}\right)$$

- Model analytical in this case – not usually…..

Type of Mathematical Model

Practically, always need to consider:

- **uncertainty**
  - – in measured inputs
  - – in model
  - – and so **should have distribution of outputs**
- **scale**
  - – different relationships over different scales
    - principally consider over **time / space**

Why Mathematical Modelling?

1. Improve process / system understanding

– by attempting to describe important aspects of process/system mathematically

e.g.

– measure and model planetary geology / geomorphology to apply understanding to Earth

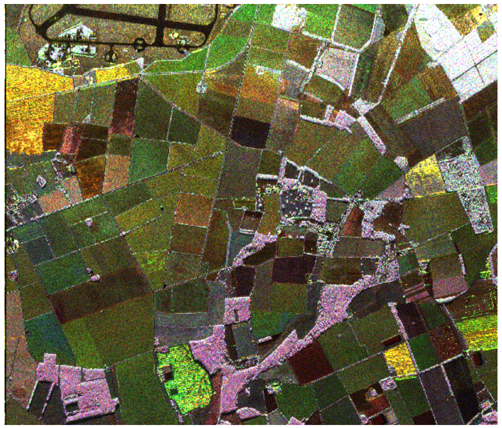
– build statistical model to understand main factors influencing system

Why Mathematical Modelling?

2. Derive / map information through surrogates

e.g.:

REQUIRE	**spatial distribution of biomass**
DATA	spatial observations of microwave backscatter
MODEL	**model relating backscatter to biomass**



Crop biomass map ??

Soil moisture map ??

Why Mathematical Modelling?

3. Make past / future predictions from current observations (extrapolation)

tend to use 'physically-based' models

e.g.:

short term:

weather forecasting, economic models

longer term:

climate modelling

Why Mathematical Modelling?

4. Interpolation based on limited sample of observations

- use statistical or physically-based models

e.g.:

- – vegetation / soil surveys
- – political surveys

How useful are these models?

- Model is based on a set of assumptions

  ‘As long as assumptions hold’, should be valid

- When developing model
  – Important to define & understand assumptions and to state these explicitly
- When using model
  – important to understand assumptions/limitations
  – make sure model is relevant

How do we know how ‘good’ a model is?

- Ideally, **‘validate’** over wide range of conditions

For environmental models, typically:

- – characterise / measure system
- – compare model predictions with measurements of ‘outputs’
  - noting error & uncertainty

‘Validation’: essentially - how well does model predict outputs when driven by measurements?

How do we know how 'good' a model is?

For environmental models, often difficult to achieve

- can't make (sufficient) measurements
  - highly variable environmental conditions
    - 'noisy' measurements
  - prohibitive timescale or spatial sampling required
- systems generally 'open'
  - no control over all interactions with surrounding areas and atmosphere
- use:
  - 'partial validations'
  - sensitivity analyses

How do we know how ‘good’ a model is?

‘partial validation’

- compare model with other models
- analyses sub-components of system
  - e.g. with lab experiments

sensitivity analyses

- vary each model parameter to see how sensitive output is to variations in input
  - build understanding of:
    - model behaviour
    - response to key parameters
    - parameter coupling

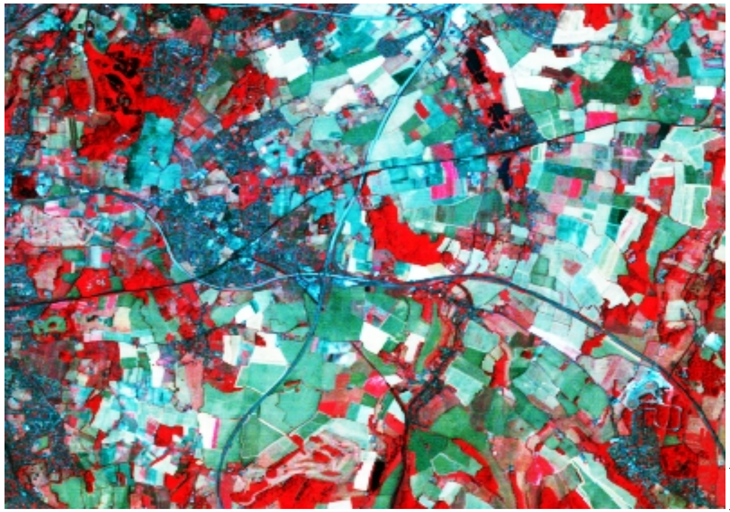
Statistical / empirical models

- **Basis:** simple theoretical analysis or empirical investigation gives evidence for relationship between variables
  - Basis is generally simplistic or unknown, but general trend seems predictable
- Using this, a statistical relationship is proposed

Statistical / empirical models

E.g.:

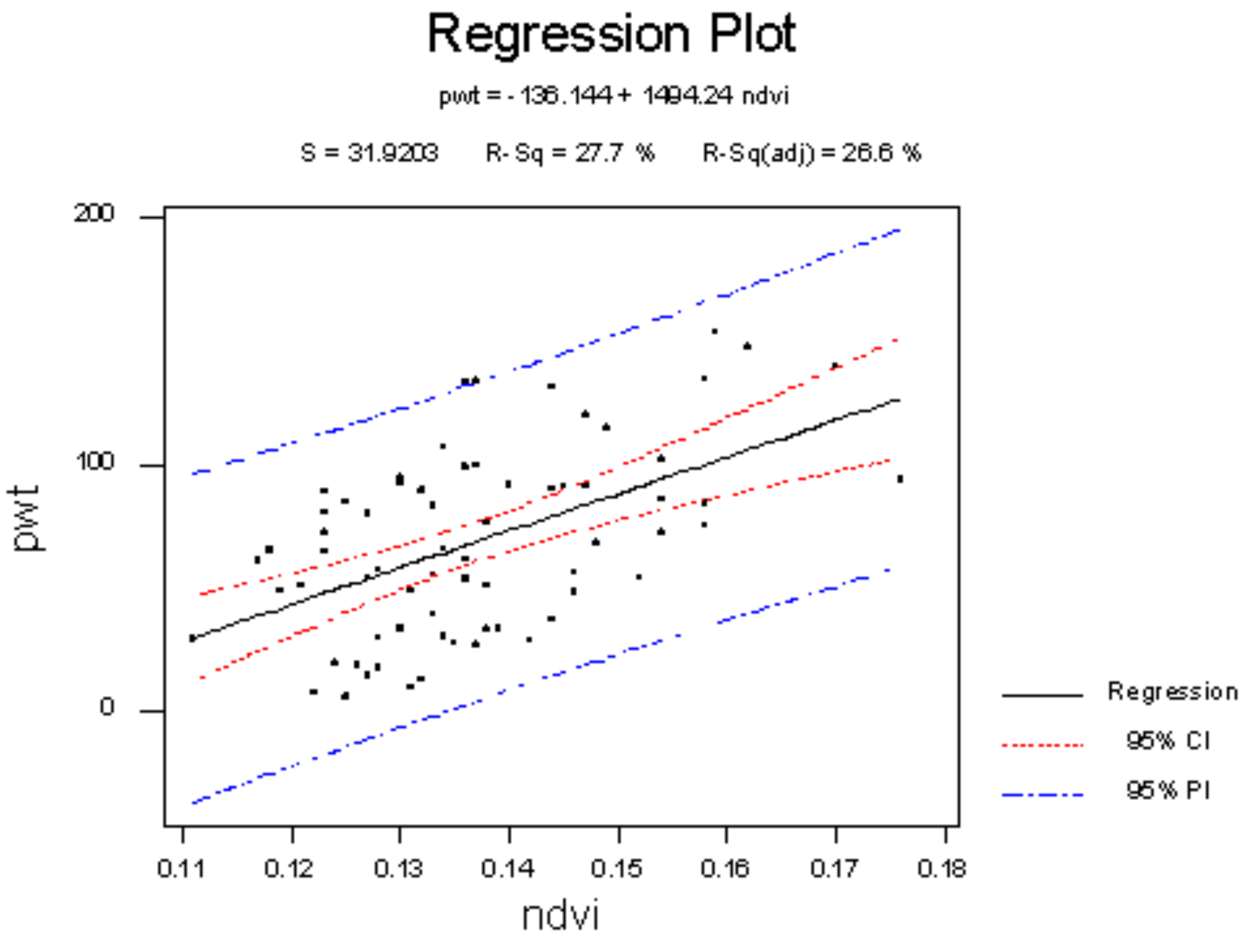
- From observation & basic theory, we observe:
  - vegetation has high NIR reflectance & low red reflectance
  - different for non-vegetated



FCC

NDVI

Biomass vs NDVI: Sevilleta, NM, USA



http://sevilleta.unm.edu/research/local/plant/tmsvinpp/documents/sevsymp2001/sevsymp2001_files/v3_document.htm

Statistical / empirical models

- Propose **linear** relationship between vegetation amount (biomass) and NDVI
  - Model fit ‘reasonable’, $r^2 = 0.27$ (hmmm….)
- Calibrate model coefficients (slope, intercept)
- **Biomass/ (g/m^2) = -136.14 + 1494.2*NDVI**
  - biomass changes by 15 g/m^2 for each 0.01 NDVI
  - X-intercept (biomass = 0) around 0.10
    - value typical for non-vegetated surface

http://sevilleta.unm.edu/research/local/plant/tmsvinpp/documents/sevsymp2001/sevsymp2001_files/v3_document.htm

Statistical / empirical models

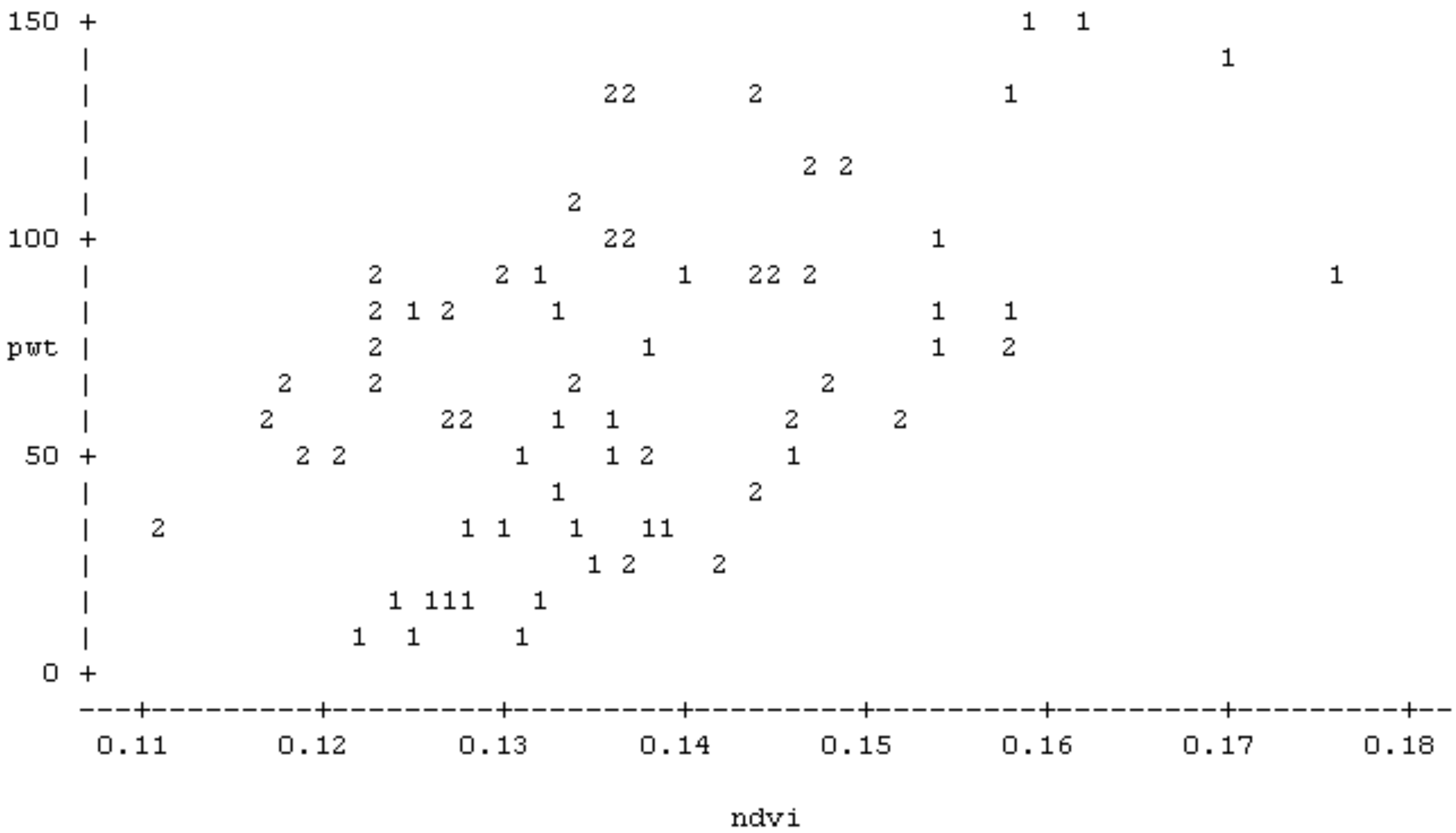
Dangers:

- changing environmental conditions (or location)
  - i.e. lack of generality
- surrogacy
  - apparent relationship with X through relationship of X with Y
- Don’t have account for all important variables
  - tend to treat as ‘uncertainty’
  - But we may miss important relationships

Include season during which measurements made...

- Biomass versus NDVI & Season

SAS Plot of pwt*ndvi. Symbol is value of season.



NOTE: 1 obs hidden.

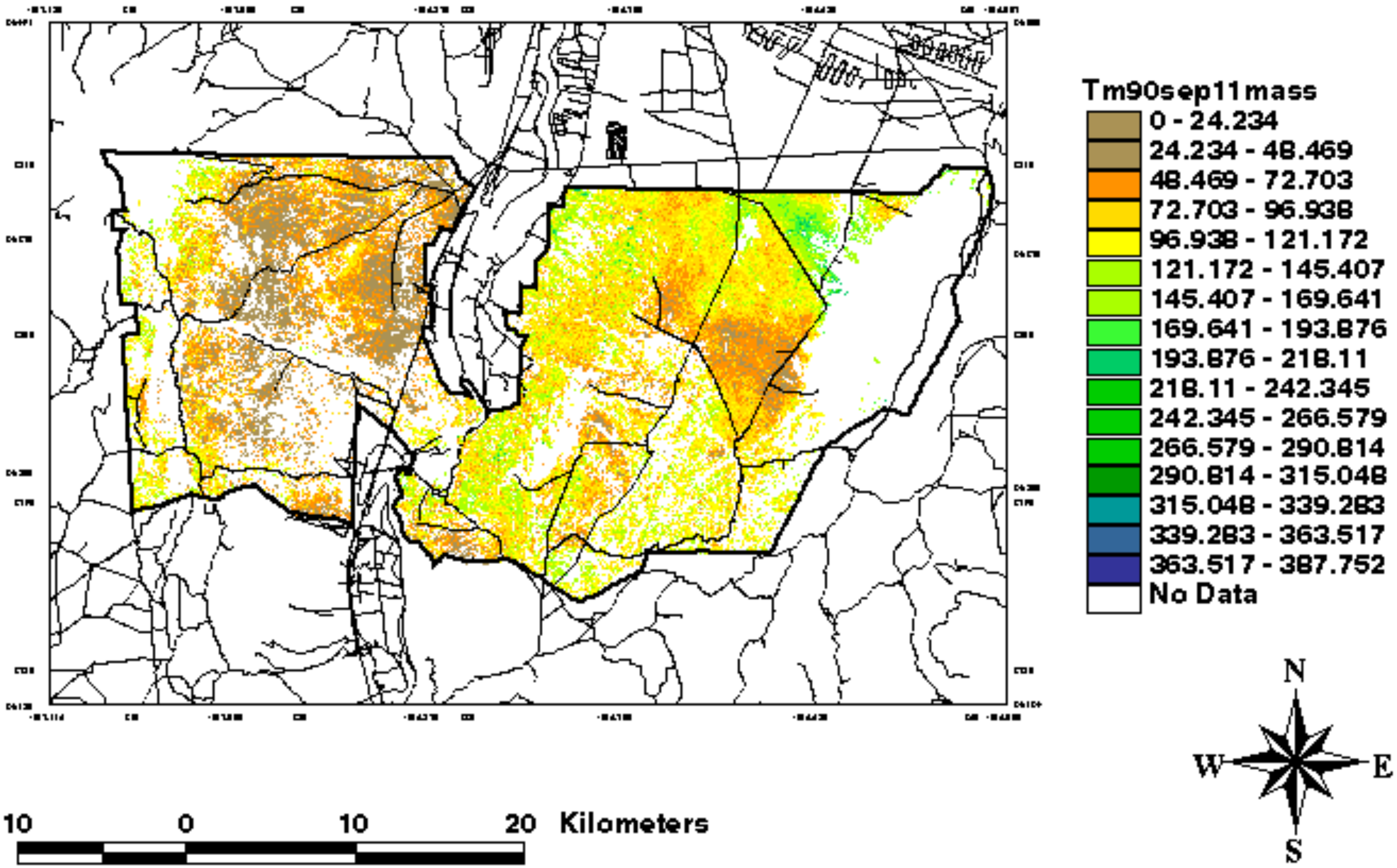
http://sevilleta.unm.edu/research/local/plant/tmsvinpp/documents/sevsymp2001/sevsymp2001_files/v3_document.htm

Statistical / empirical models

- Model fit improved, R^2 value increased to 38.9%
  - Biomass = -200.1 + 1683*NDVI + 25.3*Season
    - biomass changes by 17 g/m^2 for each 0.01 NDVI
    - X-intercept is 0.104 for Spring and 0.89 for summer

http://sevilleta.unm.edu/research/local/plant/tmsvinpp/documents/sevsymp2001/sevsymp2001_files/v3_document.htm

Estimated Live Plant Biomass: 1989 Sep 11



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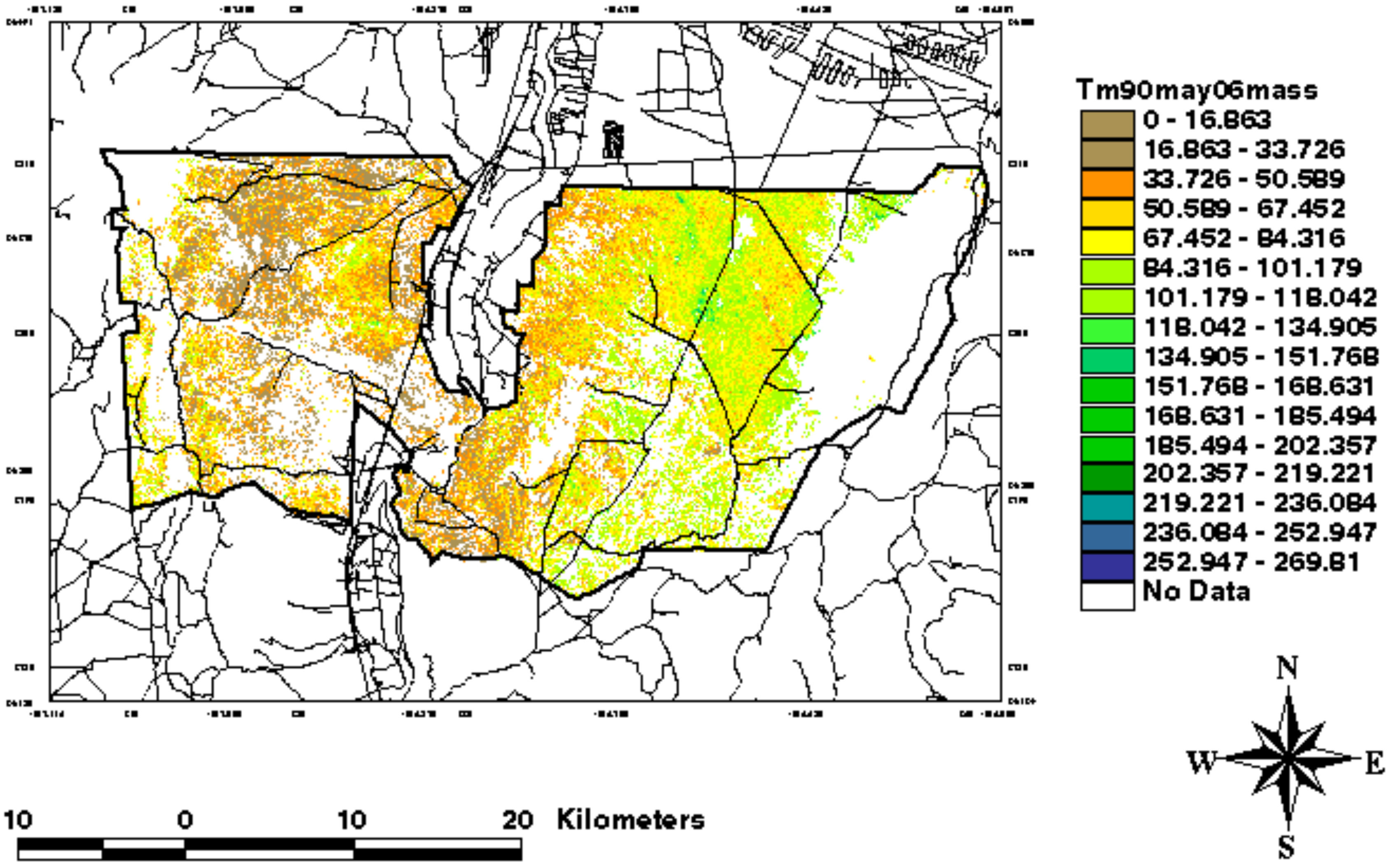
Estimated Live Plant Biomass: 1990 May 6



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Statistical / empirical models

- Model ‘validation’
  - should obtain biomass/NDVI measurements over wide range of conditions
  - R^2 quoted relates only to conditions under which model was developed
    - i.e. no information on NDVI values outside of range measured (0.11 to 0.18 in e.g. shown)

Summary of part I

- Model types
  - Empirical, statistical, physically-based
  - Requirements for models, why we do it
  - Spatial/temporal considerations….

Computerised Environmetal Modelling: A Practical Introduction Using Excel, Jack Hardisty, D. M. Taylor, S. E. Metcalfe, 1993 (Wiley)

Computer Simulation in Physical Geography, M. J. Kirkby, P. S. Naden, T. P. Burt, D. P. Butcher, 1993 (Wiley)