$$
\begin{aligned}
\operatorname{ray}(t) &= \mathbf{o} + \mathbf{d}t \\
0 &= (x - \mathbf{c}_x)^2 + (z - \mathbf{c}_z)^2 - r^2 \\
&\Downarrow \\
0 &= (\mathbf{o}_x + \mathbf{d}_x t - \mathbf{c}_x)^2 + (\mathbf{o}_z + \mathbf{d}_z t - \mathbf{c}_z)^2 - r^2 \\
&= ((\mathbf{o}_x - \mathbf{c}_x) + \mathbf{d}_x t)^2 + ((\mathbf{o}_z - \mathbf{c}_z) + \mathbf{d}_z t)^2 - r^2 \\
&= (\mathbf{o}_x - \mathbf{c}_x)^2 + 2(\mathbf{o}_x - \mathbf{c}_x)\mathbf{d}_x t + \mathbf{d}_x^2 t^2 + (\mathbf{o}_z - \mathbf{c}_z)^2 + 2(\mathbf{o}_z - \mathbf{c}_z)\mathbf{d}_z t + \mathbf{d}_z^2 t^2 - r^2 \\
&= \mathbf{d}_x^2 t^2 + \mathbf{d}_z^2 t^2 + 2(\mathbf{o}_x - \mathbf{c}_x)\mathbf{d}_x t + 2(\mathbf{o}_z - \mathbf{c}_z)\mathbf{d}_z t + (\mathbf{o}_x - \mathbf{c}_x)^2 + (\mathbf{o}_z - \mathbf{c}_z)^2 - r^2 \\
&= \left(\mathbf{d}_x^2 + \mathbf{d}_z^2\right) t^2 + 2\left((\mathbf{o}_x - \mathbf{c}_x)\mathbf{d}_x + (\mathbf{o}_z - \mathbf{c}_z)\mathbf{d}_z\right) t + (\mathbf{o}_x - \mathbf{c}_x)^2 + (\mathbf{o}_z - \mathbf{c}_z)^2 - r^2 \\
&= \left\| \begin{bmatrix} \mathbf{d}_x \\ \mathbf{d}_z \end{bmatrix} \right\|^2 t^2 + 2 \begin{bmatrix} \mathbf{o}_x - \mathbf{c}_x \\ \mathbf{o}_z - \mathbf{c}_z \end{bmatrix} \cdot \begin{bmatrix} \mathbf{d}_x \\ \mathbf{d}_z \end{bmatrix} t + \left\| \begin{bmatrix} \mathbf{o}_x - \mathbf{c}_x \\ \mathbf{o}_z - \mathbf{c}_z \end{bmatrix} \right\|^2 - r^2 \\
&= \underbrace{\|\mathbf{d}_{xz}\|^2}_{A} t^2 + \underbrace{2(\mathbf{o}_{xz} - \mathbf{c}_{xz}) \cdot \mathbf{d}_{xz}}_{B} t + \underbrace{\|\mathbf{o}_{xz} - \mathbf{c}_{xz}\|^2 - r^2}_{C} \\
&\Downarrow \\
t_{1,2} &= \frac{-B \pm \sqrt{D}}{2A} \\
D &= B^2 - 4AC
\end{aligned}
$$