a)

Linear system of equations for the finite difference approximation AT=b:

$a = -2(\frac{1}{h_x^2} + \frac{1}{h_y^2})$

$b = \frac{1}{h_x^2}$

$c = \frac{1}{h_y^2}$

$$
\begin{pmatrix}
a & b & & & & & & c & & & & \\
b & a & b & & & & & & c & & & \\
 & b & a & b & & & & & & c & & \\
 & & \ddots & \ddots & \ddots & & & & & & \ddots & \\
 & & & b & a & b & & & & & & \\
 & & & & b & a & & & & & & \\
c & & & & & & a & b & & & & \\
 & c & & & & & b & a & b & & & \\
 & & c & & & & & b & a & b & & \\
 & & & \ddots & & & & & \ddots & \ddots & \ddots & \\
 & & & & & & & & & b & a & b \\
 & & & & & & & & & & b & a
\end{pmatrix}
\begin{pmatrix}
T_{1,1} \\
T_{2,1} \\
T_{3,1} \\
\vdots \\
T_{N_x-1,1} \\
T_{N_x,1} \\
T_{1,2} \\
T_{2,2} \\
T_{2,3} \\
\vdots \\
T_{N_x-1,N_y} \\
T_{N_x,N_y}
\end{pmatrix}
$$

$$
= -2\pi^2
\begin{pmatrix}
sin(\pi h_x)sin(\pi h_y) \\
sin(2\pi h_x)sin(\pi h_y) \\
sin(3\pi h_x)sin(\pi h_y) \\
\vdots \\
sin((N_x-1)\pi h_x)sin(\pi h_y) \\
sin(N_x\pi h_x)sin(\pi h_y) \\
sin(\pi h_x)sin(2\pi h_y) \\
sin(2\pi h_x)sin(2\pi h_y) \\
sin(3\pi h_x)sin(2\pi h_y) \\
\vdots \\
sin((N_x-1)\pi h_x)sin(N_y\pi h_y) \\
sin(N_x\pi h_x)sin(N_y\pi h_y)
\end{pmatrix}
$$