Matrices and Vector Spaces

Daniel Long

December 29, 2020

Contents

1 Vector Spaces

1.1 Conditions for a Vector Space

A set of vectors **a**,**b**,**c**,... form a linear vector space if:

i. Set is closed under commutative and associative addition

ii. Set is closed under distributative and associative multiplication by a scalar

iii. Set contains a null vector

iv. Set contains a unity vector

v. A corresponding negative exists for all vectors

1.2 Vector Space Definitions

Span of a Set of Vectors
The set of all vectors that can be expressed as a linear sum of the original set

Linear Dependence
A set of vectors are linearly independent if there exists a linear combination equal to the null vector

Basis Vectors
In an N-dimensional vector space V any set of N linearly independent vectors form a set of basis vectors for V

2 Inner Product

The inner product is a generalisation of the dot or scalar product, it has the following properties:

i. $\langle \mathbf{a}|\mathbf{b}\rangle = \langle \mathbf{b}|\mathbf{a}\rangle^*$

ii. $\langle \mathbf{a}|\lambda\mathbf{b} + \mu\mathbf{c}\rangle = \lambda\langle \mathbf{a}|\mathbf{b}\rangle + \mu\langle \mathbf{a}|\mathbf{c}\rangle$

2.1 Inner Product Definitions

Orthogonality
Vectors are orthogonal if $\langle \mathbf{a}|\mathbf{b}\rangle = 0$

Vector Norm
The norm of a vector is given by $|\mathbf{a}| = \langle \mathbf{a}|\mathbf{a}\rangle^{\frac{1}{2}}$

Orthonormality
A basis is orthonormal is $\langle \mathbf{e}_i|\mathbf{e}_j\rangle = \delta_{i,j}$