Linear Optics Quantum Computation: an Overview

C. R. Myers and R. Laflamme

▼ Quantum Information Processing with Linear Optics

Quantum Optics

Classical Electromagnetism

EM waves emerge from the source free Maxwell equations

$$\nabla^2 \mathbf{E}(\mathbf{r},t) = \frac{1}{c}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t)$$

This has the positive and negative field solutions

$$\mathbf{E}(\mathbf{r},t) = i\sum_k \left(\frac{\hbar\omega_k}{2}\right)^{1/2}\left[a_k \mathbf{u}_k(\mathbf{r})e^{-i\omega_k t} + a_k^* \mathbf{u}_k^*(\mathbf{r})e^{i\omega_k t}\right]$$

The energy associated with these fields is given by

$$H = \frac{1}{2}\int_V (\mathbf{E}^2 + \mathbf{B})\,\mathrm{d}\mathbf{r} = \sum_k \left(\frac{\hbar\omega_k}{2}\right) a_k a_k^*$$

Quantisation

To quantise the field $\mathbf{E}(\mathbf{r},t)$ we convert the coefficients a_k into mode operators (aka annihilation and creation operators), which satisfy the canonical commutation relation

$$\left[\hat{a}_i, \hat{a}_j^\dagger\right] = \delta_{ij}$$

This results in the energy defined as

$$\hat{H} = \sum_k \hbar\omega_k \left(\hat{a}_k^\dagger \hat{a}_k + 1/2\right)$$

which resembles a harmonic oscillator.

The eigenstates of the quantised field Hamiltonian are Fock states $|n\rangle$, corresponding to a mode of the field. We can then define the number operator as

$$\hat{n}|n\rangle = \hat{a}^\dagger \hat{a}|n\rangle = n|n\rangle$$

Given the canonical commutation relation we can define the action of the mode operators on the Fock states as

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

We can express the annihilation operator as a linear combination of two Hermitian operators

$$\hat{a} = \frac{\hat{Q}_1 + i\hat{Q}_2}{2}$$

where $\hat{Q}_1$ is the in-phase component and $\hat{Q}_2$ is the out of phase component. These operators then satisfy the following relations relation

$$\left[\hat{Q}_1, \hat{Q}_2\right] = 2i$$
$$\Delta\hat{Q}_1 \Delta\hat{Q}_2 \geq 1$$

where the second relation corresponds to the Heisenberg uncertainty principle.

For a general Fock state $|n\rangle$ we have

$$\langle Q_1|n\rangle = \Psi_n(Q_1) = \frac{H_n(\frac{Q_1}{\sqrt{2}})}{\sqrt{2^n n!\sqrt{\pi}}} \exp\left(-\frac{Q_1^2}{4}\right)$$

were $H_n(x)$ are the Hermite polynomials.

Coherent States

Linear Optics Quantum Computation

LOQC and Quantum Error Correction