

mvdcmp: Multivariate Decomposition for Nonlinear Response Models

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Abstract. We have developed a general-purpose multivariate decomposition module for nonlinear response models that incorporates a number of recent contributions to overcome various problems dealing with path dependence and identification. This work extends existing Stata modules in important ways by allowing weights and model offsets. A companion routine handles covariate grouping to facilitate summarization of results.

Keywords: st0001, `mvdcmp`, multivariate decomposition, Oaxaca-Blinder decomposition

Introduction

Multivariate decomposition is widely used in social research to quantify the contributions to group differences in average predictions from multivariate models. The technique utilizes the output from regression models to parcel out components of a group difference in a statistic, such as a mean or proportion, which can be attributed to compositional differences between groups (i.e., differences in characteristics or endowments) and to differences in the effects of characteristics (i.e., differences in the returns, coefficients, or behavioral responses). These techniques are equally applicable for partitioning change over time into components attributable to changing effects and changing composition.

This paper introduces a new Stata module `mvdcmp` for carrying out multivariate decomposition for a variety of models, including: the classical linear model, probit, logit, complementary log-log, Poisson and negative-binomial regression. This program is comparable to several existing Stata modules including: `oaxaca` (Jann 2008), `gdecomp` (Bartus 2006), `fairlie` (Jann 2006), and `nldecompose` (Sinning, Hahn, and Bauer 2008). One feature of `mvdcmp` is that it provides the detailed decomposition for both the characteristics and coefficients components for various models and also provides

standard errors of these components.¹

`mvdcmp` is primarily intended for use in nonlinear decomposition and is based on a number of recent contributions which include convenient methods to handle path dependency (Yun 2004), computing asymptotic standard errors (Yun 2005a), and overcoming the identification problem associated with the choice of a reference category when dummy variables are included among the predictors (Yun 2005b; 2008). This paper is organized as follows: Section 1 describes our general approach to aggregate and detailed decomposition, Section 2 provides several illustrative examples, and Section 3 outlines some possible extensions and future plans for this project.

1 Multivariate Decomposition

Decomposition techniques for linear regression models have been used for many decades. This heterogeneous collection of techniques is more generally referred to as regression standardization (Althauser and Wigler 1972, Duncan 1969, Duncan, Featherman and Duncan 1968, Coleman and Blum 1971, Coleman, Berry, and Blum 1971, Winsborough and Dickinson 1971). Oaxaca (1973) and Blinder (1973) are usually credited with introducing regression decomposition in the econometric literature in the early 1970's. Although their methods are formally identical to those developed in sociology and demography, the technique has become more commonly known as Oaxaca-Blinder, Oaxaca, or Blinder-Oaxaca decomposition.

Regression decomposition has been extended to nonlinear models including: probit (Gomulka and Stern 1990; Even and Macpherson 1993; Pritchett and Yun 2009), logit (Fairlie 2005, Nielson 1998, Bowblis and Yun 2010), count models (see e.g., Bauer et al. 2007; Park and Lohr 2010), and hazard rate models (Powers and Yun 2009). For linear regression, logit, and count models, the observed difference in group means, proportions, or counts (i.e., a difference in the “first moment”) is additively decomposed into a characteristics (or endowments) component and a coefficient (or effects) component. It should be noted that in any given application a researcher may be interested in one or the other of these components, such as in the portion of the total differential that could be attributed to compositional differences between groups, or to the change in characteristics over time for a single group (see e.g., Even and Macpherson 1993 and Nielsen 1998).

1. The `fairlie` package decomposes a difference in proportions based on logit or probit models into the characteristics portion only, whereas `gdecomp` provides both components and extends to models for count data, but with a different decomposition scheme from that implemented in this module. The `nldecompose` module handles a variety of nonlinear models, but does not carry out a detailed decomposition. `oaxaca` decomposes differences in means using results from the classical linear model and is the only module with the option to provide a normalized solution for dummy variables, covariate grouping, weighting, and survey design adjustments. None of the aforementioned modules for nonlinear decomposition provide options for model weights and offset terms as provided by `mvdcmp`.

Table 1: Mapping of X to Y for `mvdcmp` Models.

	Linear	Logit	Probit [†]	Poisson	Neg. Bin. [‡]	C. log-log
$F(X\beta)$	$X\beta$	$\frac{e^{X\beta}}{1 + e^{X\beta}}$	$\Phi(X\beta)$	$e^{X\beta}$	$e^{X\beta}$	$1 - e^{-e^{X\beta}}$

[†] $\Phi(\cdot)$ denotes the cumulative normal distribution function.

[‡] includes a gamma-distributed random effect to account for extra-Poisson variation (i.e., overdispersion).

1.1 Overall Decomposition

We begin with the standard problem of decomposing a difference in first moments in which the dependent variable is a function of a linear combination of predictors and regression coefficients

$$Y = F(X\beta),$$

where Y denotes the $N \times 1$ dependent variable vector, X is an $N \times K$ matrix of independent variables, and β is a $K \times 1$ vector of coefficients. $F(\cdot)$ is any once-differentiable function mapping a linear combination of X ($X\beta$) to Y . The mean difference in Y between groups A and B can be decomposed as

$$\begin{aligned} \bar{Y}_A - \bar{Y}_B &= \overline{F(X_A\beta_A)} - \overline{F(X_B\beta_B)} \\ &= \underbrace{\left[\overline{F(X_A\beta_A)} - \overline{F(X_B\beta_A)} \right]}_E + \underbrace{\left[\overline{F(X_B\beta_A)} - \overline{F(X_B\beta_B)} \right]}_C. \end{aligned} \quad (1)$$

The component labeled E refers to the part of the differential owing to differences in *endowments* or characteristics, usually called the explained component or characteristics effects. The C component refers to that part of the differential attributable to differences in *coefficients* or effects, usually called the unexplained component or coefficients effects. In Eq. 1 we have chosen group A as the comparison group and group B as the reference group. Thus, E reflects a counterfactual comparison of the difference in outcomes from group A 's perspective (i.e., the expected difference if group A were given group B 's distribution of covariates). C reflects a counterfactual comparison of outcomes from group B 's perspective (i.e., the expected difference if group B experienced group A 's behavioral responses to X).²

The same differential (with a change in sign) can be obtained from an alternative decomposition that switches the roles of the reference and comparison groups. This is referred to as the “indexing” problem (Neumark 1988, Oaxaca and Ransom 1988; 1994). By fixing the coefficients in the composition component to group A levels, we assess the

2. Only a standard two-way decomposition is available in `mvdcmp`. Alternative decomposition methods parcel out the EC interaction as a third component.

contribution to the differential that would have occurred if the behavioral responses to the characteristics were fixed to the values in group *A*. By fixing characteristics to group *B* levels in the coefficient component, we assess the contribution to the differential that is due to the difference in effects. An equivalent decomposition would reverse this procedure. That is, we could perform a different decomposition by weighting the composition component by group *B*'s coefficients while using the observed characteristics of group *A* as weights in the coefficient component. Sometimes the average of the results of the two specifications is reported.

The mapping function $F()$ differs between models as shown in Table 1. For the linear, logit, and Poisson regression models it is the case that $\overline{F(X\beta)} = \overline{Y}$. For these models, the maximum likelihood estimates satisfy the estimating equations $X'Y = X'\hat{\mu}$, where $\hat{\mu}$ is a vector of predicted responses, and therefore $\sum Y = \sum \hat{\mu}$ and $\overline{Y} = \overline{\hat{\mu}}$. Thus, for the linear, logit, and Poisson regression models, *mvdcmp* will exactly decompose the difference in the average observed outcomes (Agresti 2002; Greene 2008). However, though very close, the equality above does not hold for the probit, negative binomial, and complementary log-log regression models. In this case, *mvdcmp* decomposes the difference in average *predicted* outcomes.

1.2 Detailed Decomposition

The decomposition thus far has been described at the aggregate level. To understand the unique contribution of each predictor to each component of the difference requires a detailed decomposition. That is, we wish to partition E and C into portions, E_k and C_k ($k = 1, \dots, K$) that represent the unique contribution of the k th covariate to E and C , respectively. One may attempt to compute E_k (C_k) by sequentially substituting one group's covariates (coefficients) with other group's. However, unlike the decomposition for a linear model, a nonlinear decomposition is sensitive to the order in which the independent variables enter the decomposition. This problem is referred to as "path dependence" (see e.g., Yun 2004). Solutions to this problem, involving a strategy of sequential covariate replacement and randomization of ordering of replacement, have been proposed by Fairlie (2005) and are implemented in the Stata module *fairlie* (Jann 2006).

Even and Macpherson (1993), Nielsen (1998), and Yun (2004) suggested simpler methods using weights. Yun (2004) obtained weights from a 1st order Taylor linearization of Eq. 1 around $\overline{X}_A\beta_A$ and $\overline{X}_B\beta_B$. The detailed decompositions obtained in this way are invariant to the order that variables enter the decomposition, thus providing a convenient solution to path dependency. After linearization, the weight component for E is

$$W_{\Delta_{X_k}} = \frac{\beta_{A_k}(\overline{X}_{A_k} - \overline{X}_{B_k})}{\sum_{k=1}^K \beta_{A_k}(\overline{X}_{A_k} - \overline{X}_{B_k})} \quad (2)$$

and the k th weight component for C is,

$$W_{\Delta\beta_k} = \frac{\bar{X}_{A_k}(\beta_{A_k} - \beta_{B_k})}{\sum_{k=1}^K \bar{X}_{A_k}(\beta_{A_k} - \beta_{B_k})} \quad (3)$$

where

$$\sum_k W_{\Delta X_k} = \sum_k W_{\Delta\beta_k} = 1.0.$$

Thus, the composition weights $W_{\Delta X_k}$ reflect the contribution of the k th covariate to the linearization of E as determined by the magnitude of the group difference in means weighted by the reference group's effect. Similarly, the coefficient weights $W_{\Delta\beta_k}$ reflect covariate k 's contribution to the linearization of C as determined by the magnitude of the group difference in the effects weighted by the comparison group's mean. Thus, the weights are proportional to the contributions to the decomposition of the linear predictor, in which the relative sizes of the contributions to the explained or unexplained portions of the outcome differential are equal to the relative contributions to the decomposition of the linear predictor. The weights are invariant to change in the scale of the covariates. The raw difference can now be expressed in terms of the overall components as a sum of weighted sums of the unique contributions.

$$\bar{Y}_A - \bar{Y}_B = E + C = \sum_{k=1}^K W_{\Delta X_k} E + \sum_{k=1}^K W_{\Delta\beta_k} C = \sum_{k=1}^K E_k + \sum_{k=1}^K C_k. \quad (4)$$

1.3 Variability in Decomposition Estimates

The characteristics and effects components do not provide information about the precision of the contributions to group differences *per se*. For this reason, it is important to gauge the sampling variability (asymptotic variance) of E and C , as well as the detailed components, in substantive applications. Because the components used in the decomposition are functions of maximum likelihood estimates, the delta method described by Rao (1973, Pp. 321-323) can be used to derive asymptotic standard errors of the detailed contributions. Interval estimation and significance testing can be done in the usual way (see, e.g., Yun 2005a). This approach utilizes expressions for the gradients of the detailed components with respect to the estimates, in addition to the variance covariance matrix of the estimates from each group, as we show next.

The endowment component is obtained as a weighted sum of the individual contributions, E_k ,

$$E = \sum_{k=1}^K E_k = \sum_{k=1}^K W_{\Delta X_k} \{ \overline{F(X_A \beta_A)} - \overline{F(X_B \beta_A)} \}. \quad (5)$$

Interval estimation and statistical hypothesis testing of the components of the detailed decomposition require computation of the asymptotic variances of the E_k and C_k components appearing in the decomposition equation. First we compute the gradient for

E_k , i.e., $\partial E_k / \partial \beta'_A$ which is a $1 \times K$ vector, the l th element of which is

$$\frac{\partial E_k}{\partial \beta_{A_l}} = W_{\Delta X_k} \left\{ \frac{\overline{\partial F(X_A \beta_A)}}{\partial \beta_{A_l}} - \frac{\overline{\partial F(X_B \beta_A)}}{\partial \beta_{A_l}} \right\} + \frac{\partial W_{\Delta X_k}}{\partial \beta_{A_l}} \left\{ \overline{F(X_A \beta_A)} - \overline{F(X_B \beta_A)} \right\} \quad (6)$$

where

$$\frac{\partial W_{\Delta X_k}}{\partial \beta_{A_l}} = I(k=l) \left[\frac{\overline{X_{A_k}} - \overline{X_{B_k}}}{\sum_k \beta_{A_k} (\overline{X_{A_k}} - \overline{X_{B_k}})} \right] - \frac{\beta_{A_k} (\overline{X_{A_k}} - \overline{X_{B_k}}) (\overline{X_{A_l}} - \overline{X_{B_l}})}{\left\{ \sum_k \beta_{A_k} (\overline{X_{A_k}} - \overline{X_{B_k}}) \right\}^2}, \quad (7)$$

and where $I(\cdot)$ is the indicator function. For nonlinear models considered by **mvdcmp**,

$$\frac{\partial F(X_j \beta_j)}{\partial \beta_l} = f(X_j \beta_j) X_{jl} \quad j \in \{A, B\}.$$

Letting $\mathbf{E} = (E_1, \dots, E_K)$ denote the $K \times 1$ detailed characteristics effect vector and Σ_{β_A} denote the variance/covariance matrix of β_A , the asymptotic covariance matrix of the detailed characteristics component is

$$\Sigma_{\mathbf{E}} = \mathbf{G}_E \Sigma_{\beta_A} \mathbf{G}'_E,$$

where

$$\mathbf{G}_E = \left(\frac{\partial E_1}{\partial \beta'_A}, \frac{\partial E_2}{\partial \beta'_A}, \dots, \frac{\partial E_K}{\partial \beta'_A} \right)$$

is the $K \times K$ gradient matrix.

Following the same logic, the coefficient component can be written as the sum of individual contributions as,

$$C = \sum_{k=1}^K C_k = \sum_{k=1}^K W_{\Delta \beta_k} \left\{ \overline{F(X_B \beta_A)} - \overline{F(X_B \beta_B)} \right\}. \quad (8)$$

Each covariate's contribution to the overall coefficient component depends on the parameter vectors, β_A and β_B . The l th elements of the gradient for C_k are

$$\frac{\partial C_k}{\partial \beta_{A_l}} = W_{\Delta \beta_k} \overline{f(X_B \beta_A) X_{B_l}} + \frac{\partial W_{\Delta \beta_k}}{\partial \beta_{A_l}} \left\{ \overline{F(X_B \beta_A)} - \overline{F(X_B \beta_B)} \right\}$$

and

$$\frac{\partial C_k}{\partial \beta_{B_l}} = \frac{\partial W_{\Delta \beta_k}}{\partial \beta_{B_l}} \left\{ \overline{F(X_B \beta_A)} - \overline{F(X_B \beta_B)} \right\} - W_{\Delta \beta_k} \overline{f(X_B \beta_B) X_{B_l}},$$

where

$$\frac{\partial W_{\Delta \beta_k}}{\partial \beta_{A_l}} = I(k=l) \left[\frac{\overline{X_{B_k}}}{\sum_k \overline{X_{B_k}} (\beta_{A_k} - \beta_{B_k})} \right] - \frac{\overline{X_{B_k}} \overline{X_{B_l}} (\beta_{A_k} - \beta_{B_k})}{\left\{ \sum_k \overline{X_{B_k}} (\beta_{A_k} - \beta_{B_k}) \right\}^2} \quad (9)$$

and

$$\frac{\partial W_{\Delta_{\beta_k}}}{\partial \beta_{Bl}} = \frac{\bar{X}_{B_k} \bar{X}_{Bl} (\beta_{A_k} - \beta_{B_k})}{\{\sum_k \bar{X}_{B_k} (\beta_{A_k} - \beta_{B_k})\}^2} - I(k=l) \left[\frac{\bar{X}_{B_k}}{\sum_k \bar{X}_{B_k} (\beta_{A_k} - \beta_{B_k})} \right], \quad (10)$$

where $I(\cdot)$ is the indicator function.

Let Σ_{β_A} and Σ_{β_B} denote the covariance matrix of the estimates from the group A and B regressions, and let $\mathbf{C} = C_1, \dots, C_K$ denote the $K \times 1$ detailed coefficient effects vector. The asymptotic covariance matrix of the detailed coefficient components is then

$$\Sigma_{\mathbf{C}} = \mathbf{G}_{C_A} \Sigma_{\beta_A} \mathbf{G}_{C_A}' + \mathbf{G}_{C_B} \Sigma_{\beta_B} \mathbf{G}_{C_B}'$$

where

$$\mathbf{G}_{C_j} = \left(\frac{\partial C_1}{\partial \beta_j'}, \frac{\partial C_2}{\partial \beta_j'}, \dots, \frac{\partial C_K}{\partial \beta_j'} \right), \quad j \in A, B$$

is the $K \times K$ gradient matrix and $\frac{\partial C_k}{\partial \beta_j'}$ is the $1 \times K$ gradient vector defined above.

Significance tests on individual components, blocks of components, or for the overall decomposition, can be carried out using Wald tests by defining sub-vectors of \mathbf{E} and \mathbf{C} along with the corresponding sub-matrices of $\Sigma_{\mathbf{E}}$ and $\Sigma_{\mathbf{C}}$. For example, the variance E_k can be found from the k th element of the main diagonal of $\Sigma_{\mathbf{E}}$. The variance estimates derived above assume that the independent variables are fixed and that groups A and B are independent. They will underestimate the true variances if this is not the case.

1.4 Normalization of Dummy Variables

It is well known that the detailed Oaxaca decomposition is not invariant to the choice of the reference category when sets of dummy variables are used (Oaxaca and Ransom 1999).³ Particularly, if a model includes dummy variables, then the sum of the detailed coefficients effects attributed to the dummy variables is not invariant to the choice of the reference, or omitted, category. Suppose that we examine the following regression model containing dummy variables (d 's) representing a factor with I levels:

$$y = a + \sum_{i=2}^I d_i \alpha_i + z\gamma + \varepsilon.$$

The identification problem is that

$$\sum_{i=2}^I \bar{d}_{Bi} (\hat{\alpha}_{Ai} - \hat{\alpha}_{Bi}) \neq \sum_{i=1}^{I-1} \bar{d}_{Bi} (\tilde{\alpha}_{Ai} - \tilde{\alpha}_{Bi}),$$

3. This invariance pertains to the coefficients contribution, C_k .

where $\hat{\alpha}$ and $\tilde{\alpha}$ are estimates when the omitted category is the first and the last category, respectively.

Intuitively, the identification problem can be resolved by averaging the coefficients effects of a set of dummy variables while permuting the reference groups (Yun 2005b). This is equivalent to first computing a normalized equation which can identify the intercept and coefficients of all dummy variables including reference groups by averaging estimates obtained by permuting the reference groups, then using these along with the augmented design matrix to perform the decomposition analysis.

The prototypical normalized equation is

$$y = a^* + \sum_{i=1}^I \alpha_i^* d_i + z\gamma + \varepsilon.$$

mvdcmp offers the option of constructing a normalized decomposition using a practical algorithm outlined by Yun (2008) that transforms the estimates of the usual regression equation. This algorithm, initially developed by Suits (1984), transforms estimates (α) by imposing an anova-type (or centered-effects) restriction, $\sum_{i=1}^I \alpha_i^* = 0$. When the coefficients of the normalized equation are further specified as $\alpha_i^* = \alpha_i + \mu_\alpha$, then the solution for the constraint is

$$\mu_\alpha = -\bar{\alpha} = -\sum_{i=1}^I \alpha_i / I,$$

where the coefficient of the reference group is zero, i.e., $\alpha_1 = 0$. The normalized equation will be

$$y = (a + \bar{\alpha}) + \sum_{i=1}^I (\alpha_i - \bar{\alpha}) d_i + z\gamma + \varepsilon.$$

Following **oaxaca** (Jann 2008), we use the **devcon** module (Jann 2005) to construct the augmented coefficient vectors and covariance matrices which are input to the decomposition routine along with the augmented design matrix.

1.5 Syntax

```
mvdcmp groupvar [ , reverse norm(varlist) scale(#) ] :model depvar indepvars
      [if] [in] [weight]
```

The mandatory *groupvar* denotes the binary grouping variable. The available models and options are summarized in Table 2. The options of particular interest are **offset** and **scale**. **mvdcmp** automatically determines the high-outcome group and uses the low-outcome group as the reference. This can be overridden with the **reverse** option. The **scale** option is particularly useful when decomposing small differences. It simply reports results multiplied by a user-specified value. The **norm** option is based on the same

Table 2: Models and Options Available in `mvdcmp`.

Models	
<code>regress</code>	linear regression model
<code>logit</code>	logit model
<code>probit</code>	probit model
<code>cloglog</code>	complementary log -log model
<code>poisson</code>	Poisson regression model
<code>nbreg</code>	negative binomial regression model
Options	
<code>reverse</code>	reverse the decomposition by switching the comparison group
<code>norm(<i>varlist</i>)</code>	identify dummy variable sets for ANOVA normalization
<code>scale(#)</code>	scale the results [default <code>scale(1)</code>]

option provided in Jann's (2008) `oaxaca` module. Additionally, a companion routine `mvdcmpgroup` can be called after `mvdcmp` to generate a coarser decomposition based on grouping several covariates, such as a collection of socioeconomic variables that may be more informative when considered together as a single component. This is similar to that provided by Jann's (2006) `fairlie` module. The syntax for `mvdcmpgroup` is:

```
mvdcmpgroup (effect_name: indepvars)
```

The first argument in parenthesis *effect_name* is the name of the aggregate effect and *indepvars* is a list of covariates that are to be combined into the aggregate effect.

2 Examples

2.1 Logistic Regression

The first example illustrates the basic syntax of the command for decomposing a difference in proportions using a logit model. Here we decompose the observed black-white difference in the prevalence of first premarital birth using a logit model with data on a sample of non-Hispanic whites and blacks from the 1979 National Longitudinal Survey of Youth (NLSY). We consider a logit model with a set of predictors including: number of family structure changes up to the time of event (`nfamtran`), dummy variables for maternal education (`medu1` for less than 12 years of schooling and `medu3` for more than 12 years of schooling), family income in thousands of dollars (`inc1000`), and mother's age at respondent's birth (`magebir`). We first read the data, construct dummy variables for maternal education, and examine summary statistics to evaluate compositional differences.

```
use pmbNLSY, clear
* illustrate logit decomposition
gen medu1 = 0
```

```

replace medu1 = 1 if medu < 12
gen medu2 = 0
replace medu2 = 1 if medu == 12
gen medu3 = 0
replace medu3 = 1 if medu > 12

```

```
bysort blk: sum devnt nfamtran medu1 medu2 medu3 inc1000 magebir, separator(1000)
```

The summary statistics indicate large black-white differences in the proportion experiencing a first premarital birth (**devnt**). We observe substantial compositional differences in number of family transitions, educational attainment, family income and mother's age at the time of the respondent's birth, with blacks exhibiting lower average income and educational attainment and a greater number of family transitions.

```
-> blk = 0
```

Variable	Obs	Mean	Std. Dev.	Min	Max
devnt	2287	.111937	.3153579	0	1
nfamtran	2287	.4879755	.9509905	0	10
medu1	2287	.2706603	.4443981	0	1
medu2	2287	.5137735	.4999196	0	1
medu3	2287	.2155662	.4113045	0	1
inc1000	2287	.9960198	.6180757	0	4.9497
magebir	2287	25.4769	5.988704	12.25	46.41667

```
-> blk = 1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
devnt	1357	.559322	.4966515	0	1
nfamtran	1357	.6226971	.9228423	0	7
medu1	1357	.5092115	.5000994	0	1
medu2	1357	.3817244	.4859886	0	1
medu3	1357	.1090641	.3118346	0	1
inc1000	1357	.547693	.4173302	0	3.7501
magebir	1357	24.90985	6.769633	12	53.5

Next, we fit models to gauge differences in returns to risk.

```
. logit devnt nfamtran medu1 medu3 inc1000 magebir if blk==0
```

Logistic regression	Number of obs	=	2287
	LR chi2(5)	=	84.73
	Prob > chi2	=	0.0000
Log likelihood = -759.33443	Pseudo R2	=	0.0528

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
nfamtran	.3159368	.058431	5.41	0.000	.2014142	.4304595
medu1	.6543009	.1482065	4.41	0.000	.3638215	.9447804
medu3	-.24361	.2092747	-1.16	0.244	-.6537809	.166561
inc1000	-.3659533	.1419346	-2.58	0.010	-.6441399	-.0877666
magebir	-.006541	.0113161	-0.58	0.563	-.0287202	.0156382
_cons	-1.951179	.3384512	-5.77	0.000	-2.614531	-1.287827

```
. logit devnt nfamtran medu1 medu3 inc1000 magebir if blk==1
```

Logistic regression	Number of obs	=	1357
	LR chi2(5)	=	77.60
	Prob > chi2	=	0.0000
Log likelihood = -892.22964	Pseudo R2	=	0.0417

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
nfamtran	.1761604	.063537	2.77	0.006	.0516301	.3006907
medu1	.0804944	.1226039	0.66	0.511	-.1598048	.3207936
medu3	-.8089405	.2015699	-4.01	0.000	-1.20401	-.4138708
inc1000	-.776516	.1524453	-5.09	0.000	-1.075303	-.4777288
magebir	-.0144374	.0084113	-1.72	0.086	-.0309233	.0020485
_cons	.9614928	.2491499	3.86	0.000	.473168	1.449818

We find larger effects family transitions and low maternal education and smaller effects of high maternal education and family income among whites. Next we carry out the decomposition. The overall and detailed results are presented below.

```
. mvdcmp blk: logit devnt nfamtran medu1 medu3 inc1000 magebir
```

Decomposition Results	Number of obs =	3644
-----------------------	-----------------	------

High outcome group: blk==1 --- Low outcome group: blk==0

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
E	.11008	.013362	8.24	0.000	.083886	.13627	24.604
C	.33731	.024517	13.76	0.000	.28926	.38536	75.396
R	.44738	.020816	21.49	0.000	.40658	.48819	

Due to Difference in Characteristics (E)

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
nfamtran	.0053819	.0019315	2.79	0.005	.0015961	.0091676	1.203
medu1	.0043545	.0066454	0.66	0.512	-.0086705	.017379	.97331
medu3	.019537	.0047194	4.14	0.000	.010287	.028787	4.367
inc1000	.078946	.014059	5.62	0.000	.05139	.1065	17.646
magebir	.0018565	.0010749	1.73	0.084	-.00025026	.0039633	.41497

Due to Difference in Coefficients (C)

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
nfamtran	-.011755	.0073001	-1.61	0.107	-.026063	.002553	-2.6275
medu1	-.026766	.0091023	-2.94	0.003	-.044607	-.0089256	-5.9828
medu3	-.021003	.010891	-1.93	0.054	-.04235	.00034419	-4.6946
inc1000	-.070476	.035804	-1.97	0.049	-.14065	-.00030035	-15.753
magebir	-.034671	.061914	-0.56	0.575	-.15602	.086679	-7.7498
_cons	.50198	.075473	6.65	0.000	.35405	.64991	112.2

We find that differences in effects account for 75% of the observed race differential in the prevalence of premarital birth, with differences in intercepts (baseline logits) accounting for most of this. Equalizing family income (`inc1000`) would be expected to reduce the black-white premarital birth gap by about 18%. A positive E_k coefficient indicates the expected reduction in the black-white premarital birth gap if blacks were equal to whites on the distribution of X_k . In this case, shifting the black distribution on income and higher maternal education to white levels would provide the largest decrease in the black-white differential. A negative C_k coefficient indicates the expected *increase* in the black-white gap if blacks had the same returns to risk, or behavioral responses, as whites. In this case we find that if blacks were penalized by family change to the same extent as whites, the black-white gap would be expected to increase by about 3%. The protective effects of family income are not as strong for whites as they are for blacks. If blacks were “protected” from risk to the same degree as whites, the black-white gap would be expected to increase by about 16%.

2.2 Normalization

In the example above, two categories of maternal education are used—`medu1` (mother’s education < 12 years) and `medu3` (mother’s education > 12 years), with the reference category of exactly 12 years of education. In this case, adopting a different reference category would change both the education effects and the intercept. We overcome this limitation by first defining a dummy variable corresponding to each level of the factor

and including the complete set of dummy variables in the `norm()` option.⁴

```
. mvdcmp blk, norm(medu1 medu2 medu3): logit devnt nfamtran medu1 medu3 inc1000
> magebir
```

Decomposition Results Number of obs = 3644

High outcome group: blk==1 --- Low outcome group: blk==0

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	Pct.
E	.11008	.013362	8.24	0.000	.083886 .13627	24.604
C	.33731	.024517	13.76	0.000	.28926 .38536	75.396
R	.44738	.020816	21.49	0.000	.40658 .48819	

Due to Difference in Characteristics (E)

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	Pct.
nfamtran	.0053819	.0019315	2.79	0.005	.0015961 .0091676	1.203
medu1	.01749	.0047173	3.71	0.000	.008244 .026736	3.9094
medu2	-.0072711	.0026092	-2.79	0.005	-.012385 -.0021571	-1.6252
medu3	.013673	.0029876	4.58	0.000	.0078171 .019529	3.0562
inc1000	.078946	.014059	5.62	0.000	.05139 .1065	17.646
magebir	.0018565	.0010749	1.73	0.084	-.00025026 .0039633	.41497

Due to Difference in Coefficients (C)

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	Pct.
nfamtran	-.011755	.0073001	-1.61	0.107	-.026063 .002553	-2.6275
medu1	-.0090538	.0062777	-1.44	0.149	-.021358 .0032505	-2.0237
medu2	.033622	.011866	2.83	0.005	.010364 .05688	7.5152
medu3	-.006896	.0068572	-1.01	0.315	-.020336 .0065441	-1.5414
inc1000	-.070476	.035804	-1.97	0.049	-.14065 -.00030035	-15.753
magebir	-.034671	.061914	-0.56	0.575	-.15602 .086679	-7.7498
_cons	.43654	.073851	5.91	0.000	.29179 .58129	97.576

4. Though the example includes only one set of dummy variables, `norm()` can handle multiple sets of dummy variables. If multiple sets of dummy variables are included, then normalization for each set should be separated by |. Dummy variables for *all* levels of a factor must be included in the `norm()` statement. For example, `norm(a1-a3 | b1 b2)`, indicates that normalization is to be applied to a 3-category factor denoted by the dummy variables `a1`, `a2`, and `a3` and a 2-category factor denoted by the dummy variables `b1` and `b2`.

The coefficients for maternal education now reflect the results of model fitting using an anova-type normalization in which the coefficients in the logistic regression model sum to zero across levels of maternal education. The model's constant term thus reflects a scaled grand mean of the baseline log odds. The transformed augmented coefficients and covariance matrix are input to the decomposition routine along with the augmented model design matrix. It can be easily verified that aggregate effects and the sum of the characteristics effect of the dummy variables do not change with normalization. However, the coefficients effect of the constant and sum of the coefficients effect of the dummy variables change with normalization.

2.3 Grouping

Researchers are often interested in determining the total contribution to the differential for a subset of covariates. In the example below, we aggregate maternal education and family income into a “socioeconomic component” called SES. The aggregate SES effect is the sum of the relevant elements of \mathbf{E} and \mathbf{C} and the covariance matrix is the sum of the elements of relevant submatrices of $\Sigma_{\mathbf{E}}$ and $\Sigma_{\mathbf{C}}$. We provide a post-estimation command called `mvdcmpgroup` to carry out the necessary aggregation. It works with the returned results from the most recent `mvdcmp` command, and can be called as many times as needed.⁵

```
. mvdcmpgroup (SES: medu1 medu3 inc1000)
```

Decomposition Results					Number of obs =		3644

High outcome group: blk==1 --- Low outcome group: blk==0							

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.

E	.11008	.013362	8.24	0.000	.083886	.13627	24.604
C	.33731	.024517	13.76	0.000	.28926	.38536	75.396

R	.44738	.020816	21.49	0.000	.40658	.48819	

Due to Difference in Characteristics (E)							

devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.

SES	.10284	.013306	7.73	0.000	.076758	.12892	22.986

5. A `nocons` option can be used to show the contributions pertaining to all other model terms excluding the constant. The subgroup aggregation for multiple groups can be done at once rather than calling the command multiple times by separating each group with (). For example, `mvdcmpgroup (SES: medu1-medu3 inc1000)(MEDU: medu1-medu3), nocons`.

Due to Difference in Coefficients (C)							
devnt	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
SES	-.11825	.039915	-2.96	0.003	-.19648	-.040013	-26.43
SES: medu1 medu3 inc1000							

Focusing on composition, we find that equalizing all SES factors across groups would reduce the black-white premarital birth gap by about 23%.

2.4 Negative Binomial Regression

Next we provide an illustration of a count model. This example considers a negative binomial model for the number of abortions occurring to individual women in the NLSY over the period from 1979 to 1997. It is reasonable to expect that there may be dependence between the pregnancy outcomes that comprise the pregnancy history for a given woman. This *unobserved heterogeneity* is referred to as *frailty* in demographic research (see, e.g., Heckman and Singer 1985; Hougaard 1984; Vaupel and Yashin 1985), the effects of which have been recognized for some time (see, e.g., Blumen et al. 1955; Greenwood and Yule 1920; Strehler and Mildvan 1960).⁶ We can build this dependency into the model by specifying a multiplicative factor v that raises or lowers the expected number of abortions for a specific woman in the population. The negative binomial regression model assumes that v follows a gamma distribution normalized to have a mean of 1 with variance α . The resulting conditional distribution of Y is a Poisson-gamma mixture. Integrating out the random effect v yields the unconditional distribution for Y (number of abortions), which follows a negative binomial distribution with mean μ and variance $\mu + \mu^2\alpha$ (see, e.g. Cameron and Trividi 1998; Long 1997; Long and Freese 2006). Following the notation of Long (1997), we express a woman's expected number of abortions under the negative binomial model as

$$\tilde{\mu} = ve^{X\beta},$$

where it follows from the assumptions about v that

$$E(\tilde{\mu}) = E(v)e^{X\beta} = e^{X\beta} = \mu = F(X\beta).$$

The negative binomial and Poisson regression models have the same mean structure, resulting in identical decomposition equations. However, coefficients and standard errors from a negative binomial model will differ from those of a similarly specified Poisson regression model when $\alpha > 0$.

6. The standard Poisson model assumption equates the mean to the variance, i.e., $\mu = E(Y) = \text{var}(Y) = e^{X\beta}$. Including a frailty term introduces a component of variance, thereby permitting the variance in Y to exceed the mean. Thus, the resulting model handles the potential *overdispersion*, or extra-Poisson variation in count data.

An Example using the `offset` Option

We have used the `offset` option in a negative binomial regression model for the example decomposition below. The offset effectively adjusts the model's linear predictor so that covariate effects can be interpreted as changes in the log rate as opposed to changes in the log count.⁷ If the offset is specified, then the expected abortion rate for a woman is

$$\lambda = e^{X\beta},$$

and the expected number of abortions for that woman is $\mu = \lambda R$, where R is the exposure to the risk of abortion (i.e., the number of pregnancies reported by that woman). In this case, $\log R$ is included as an offset in the count model to yield the predicted log abortion rate for that woman.⁸

In the case of a Poisson or negative binomial regression model with an offset term, the decomposition pertains to a difference in aggregate group rates as opposed to a difference in average counts. We define the overall (or *central*) rate in group j in the usual demographic sense as the number of occurrences (total number of abortions) divided by total exposure to risk (total number of pregnancies), or

$$\bar{\lambda}_j = \frac{\sum Y_j}{\sum R_j} = \frac{\bar{Y}_j}{\bar{R}_j} = \frac{\sum F(X_j\beta_j + \log R_j)}{\sum R_j} = \overline{F(X_j\beta_j + \log R_j)} / \bar{R}_j.$$

The decomposition equations for count models—with or without offset terms—can then be expressed in a unified manner as⁹

$$\bar{Y}_A / \bar{R}_A - \bar{Y}_B / \bar{R}_B = \overline{F(X_A\beta_A + \log R_A)} / \bar{R}_A - \overline{F(X_B\beta_B + \log R_B)} / \bar{R}_B. \quad (11)$$

We match each group's offset vector to their respective X matrix in the decomposition. Thus, with group B as the referent, the characteristics component is

$$E = \overline{F(X_A\beta_A + \log R_A)} / \bar{R}_A - \overline{F(X_B\beta_A + \log R_B)} / \bar{R}_B$$

and the coefficients component is

$$C = \overline{F(X_B\beta_A + \log R_B)} / \bar{R}_B - \overline{F(X_B\beta_B + \log R_B)} / \bar{R}_B.$$

It can be shown that the decomposition weights $W_{\Delta_{x_k}}$ and $W_{\Delta_{\beta_k}}$ for count models with offset terms are identical in form to those in Eq. 2 and Eq. 3

As an illustration we utilize a sample of women from the NLSY and decompose the difference in abortion rates for women who were raised in conservative protestant families (`consprot=1`) and those from other religious backgrounds (`consprot=0`), including those with no particular religious upbringing.¹⁰ The empirical abortion rates are 10.6

7. The `offset` option is available only for the Poisson and negative binomial models.

8. The offset must be entered as the logged exposure as `mvdcmp` does not accept the “exposure” option at this time.

9. The offset term is a $N \times 1$ vector of 0's for a model without an offset. That is $\log R = 0$, hence $R = \bar{R} = 1$, and $\bar{\lambda} = \bar{\mu}$. Future versions of `mvdcmp` may offer alternative treatments of the offset term.

10. We have excluded women who were never pregnant, and therefore never at risk for an abortion.

per 100 pregnancies among Conservative Protestants and 14.1 per 100 pregnancies for those from other backgrounds, yield a difference of 3.4 abortions per 100 pregnancies. The decomposition results below are scaled to reflect the impact of differences in characteristics and effects on the abortion rate per 100 pregnancies.

```
. use nabort, clear
. mvdcmp consprot, reverse scale(100) : nbreg nabort medu adjinc south urban profam
> books, offset(lognpreg)
```

Decomposition Results Number of obs = 2807

 High outcome group: consprot==0 --- Low outcome group: consprot==1

nabort	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
-----+-----							
E	1.2755	.62229	2.05	0.040	.05576	2.4952	35.839
C	2.2834	1.0993	2.08	0.038	.12873	4.4381	64.161
-----+-----							
R	3.5588	.94184	3.78	0.000	1.7128	5.4049	

Due to Difference in Characteristics (E)

nabort	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
-----+-----							
medu	1.1296	.28405	3.98	0.000	.57285	1.6863	31.74
adjinc	-.067618	.23457	-0.29	0.773	-.52738	.39215	-1.9
south	.3378	.5004	0.68	0.500	-.64298	1.3186	9.4918
urban	.098114	.038851	2.53	0.012	.021967	.17426	2.7569
profam	.35992	.084695	4.25	0.000	.19392	.52592	10.113
books	-.58235	.2222	-2.62	0.009	-1.0179	-.14683	-16.363

Due to Difference in Coefficients (C)

nabort	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]		Pct.
-----+-----							
medu	.35006	2.7748	0.13	0.900	-5.0886	5.7887	9.8363
adjinc	-.089086	.67844	-0.13	0.896	-1.4188	1.2406	-2.5032
south	2.2287	.96319	2.31	0.021	.34086	4.1165	62.624
urban	-.11297	1.1812	-0.10	0.924	-2.4281	2.2022	-3.1742
profam	1.722	2.7573	0.62	0.532	-3.6824	7.1264	48.388
books	-3.6944	1.6054	-2.30	0.021	-6.8409	-.54784	-103.81
_cons	1.879	4.4598	0.42	0.674	-6.8622	10.62	52.798

We include various measures of socioeconomic background including: family income (*adjinc*), respondent's mother's education (*medu*), and a 0–3 scale of presence of books, magazines, or newspapers (*books*) in the home during adolescence. We also include a pro-family attitude scale (*profam*) constructed as the sum of several NLSY survey items,

as well as dummy variables for urban (`urban`) and southern (`south`) residence. Here we find that 39.6% of the religious background differential in abortion rates can be attributed to differences in characteristics, in particular religious background differences in mother's education, urban residence, "pro-family" attitudes and presence of books magazines or newspapers. The contribution due to the difference in the effects of southern residence (`south`) and reading materials (`books`) is also significant, suggesting a differential salience of regional context and cultural capital on behavioral outcomes.

3 Discussion

Decomposition techniques have a long history in social science research and their popularity is growing partly as a result of the increasing availability of user-friendly computer routines. We have developed a general-purpose decomposition routine that incorporates a number of recent contributions to overcome various problems involving the ordering of variables entered into the decomposition (i.e., the problem of path dependence) and the sensitivity of the results of the *coefficient* portion of the decomposition to the choice of the reference category when regression models include dummy variables (i.e., the identification problem). This work extends existing Stata modules in important ways as a result of these refinements and extensions. The `mvdcmp` module does not provide the full range of models and decomposition strategies provided by `nldecompose` (Sinning, Hahn, and Bauer 2008), however it provides *detailed* decomposition results and standard errors for an important subset of those models. It should also be mentioned that `mvdcmp` provides a single type of decomposition referred to as the *standard* or two-component decomposition. Although it provides for a reverse decomposition, it could be made more flexible by offering alternative options, such as the three-way decomposition described by Daymont and Andrisani (1984).

The programming tasks of the normalization procedure were considerably less daunting due to the availability of Ben Jann's (2005) `devcon` utility. The options for including normalization were inspired by `oaxaca` (Jann 2008) and the `fairlie` (Jann 2006) module provided the motivation to include covariate grouping. Further work remains to be done to include normalized interaction terms as in `oaxaca`. We include the same suite of models that are available in `gdecomp` (Bartus 2006), however we have added options for model weights and offsets in count models. It should be straightforward to include options for robust standard errors and survey adjustments. Future work will consider methods to decompose the estimated unobserved heterogeneity from negative binomial models. We have also extended these methods to other settings and have working versions of modules for the multivariate decomposition of discrete and continuous time hazard models as described by Powers and Yun (2009).

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