Invited review

A survey on problems and methods in generalized assembly line balancing

Christian Becker, Armin Scholl *

Friedrich-Schiller-Universität Jena, Fakultät für Wirtschaftswissenschaften, Carl-Zeiß-Straße 3, D-07743 Jena, Germany

Available online 11 September 2004

Abstract

Assembly lines are traditional and still attractive means of mass and large-scale series production. Since the early times of Henry Ford several developments took place which changed assembly lines from strictly paced and straight single-model lines to more flexible systems including, among others, lines with parallel work stations or tasks, customer-oriented mixed-model and multi-model lines, U-shaped lines as well as unpaced lines with intermediate buffers.

In any case, an important decision problem, called assembly line balancing problem, arises and has to be solved when (re-) configuring an assembly line. It consists of distributing the total workload for manufacturing any unit of the product to be assembled among the work stations along the line.

Assembly line balancing research has traditionally focused on the simple assembly line balancing problem (SALBP) which has some restricting assumptions. Recently, a lot of research work has been done in order to describe and solve more realistic generalized problems (GALBP). In this paper, we survey the developments in GALBP research.
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Keywords: Assembly line balancing; Mass-production; Literature survey; Production process

1. Introduction

Assembly lines are flow oriented production systems which are still typical in the industrial production of high quantity standardized commodities and even gain importance in low volume production of customized products. Among the decision problems which arise in managing such systems, assembly line balancing problems are important tasks in medium-term production planning.

An *assembly line* consists of (*work*) *stations* $k = 1, \ldots, m$ arranged along a conveyor belt or a similar mechanical material handling equipment. The workpieces (jobs) are consecutively launched down the line and are moved from station to station. At each station, certain operations are

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* Corresponding author. Tel.: +49 3641 943171.
   E-mail addresses: c.becker@wiwi.uni-jena.de (C. Becker), a.scholl@wiwi.uni-jena.de (A. Scholl).

repeatedly performed regarding the *cycle time* (maximum or average time available for each workcycle). The decision problem of optimally partitioning (balancing) the assembly work among the stations with respect to some objective is known as the *assembly line balancing problem* (ALBP).

Manufacturing a product on an assembly line requires partitioning the total amount of work into a set of elementary operations named *tasks* $V = \{1, \ldots, n\}$. Performing a task j takes a *task time* t_j and requires certain equipment of machines and/or skills of workers. Due to technological and organizational conditions *precedence constraints* between the tasks have to be observed.

These elements can be summarized and visualized by a *precedence graph*. It contains a node for each task, node weights for the task times and arcs for the precedence constraints. Fig. 1 shows a precedence graph with $n = 10$ tasks having task times between 1 and 10 (time units). The precedence constraints for, e.g., task 5 express that its processing requires the tasks 1 and 4 (*direct predecessors*) and 3 (*indirect predecessor*) be completed. The other way round, task 5 must be completed before its (direct and indirect) *successors* 6, 8, 9, and 10 can be started.

Any type of ALBP consists in finding a feasible *line balance*, i.e., an assignment of each task to a station such that the precedence constraints and further restrictions are fulfilled (see Section 2). The set S_k of tasks assigned to a station k $(=1, \ldots, m)$ constitutes its *station load*, the cumulated task time $t(S_k) = \sum_{j \in S_k} t_j$ is called *station time*. When a fixed common *cycle time* c is given (paced line; cf. Section 2), a line balance is feasible only if the station time of neither station exceeds c. In case of $t(S_k) < c$, the station k has an *idle time* of $c - t(S_k)$ time units in each cycle.
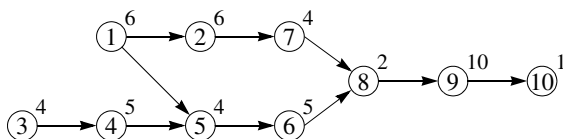
For the example of Fig. 1, a feasible line balance with cycle time $c = 11$ and $m = 5$ stations is given by the station loads $S_1 = \{1, 3\}$, $S_2 = \{2, 4\}$, $S_3 = \{5, 6\}$, $S_4 = \{7, 8\}$, $S_5 = \{9, 10\}$. While no idle time occurs in stations 2 and 5, stations 1, 3, and 4 show idle times of 1, 2, and 5, respectively.

The installation of an assembly line is a long-term decision and usually requires large capital investments. Therefore, it is important that such a system is designed and balanced so that it works as efficiently as possible. Besides balancing a new system, a running one has to be re-balanced periodically or after changes in the production process or the production program have taken place. Because of the long-term effect of balancing decisions, the used *objectives* have to be carefully chosen considering the strategic goals of the enterprise. From an economic point of view *cost and profit related objectives* should be considered (cf. Section 4). However, measuring and predicting the cost of running a line over months or years and the profits achieved by selling the products assembled is rather complicated and error-prone. A usual surrogate objective consists in maximizing the line utilization which is measured by the *line efficiency* as the productive fraction of the line's total operating time and directly depends on the cycle time c and the number of stations m (cf. Section 3).

2. Characteristics of assembly line systems

Because of very different conditions in industrial manufacturing, assembly line systems and corresponding ALBPs are multifaceted. In the following, we shortly characterize the most relevant properties for classifying assembly lines. For more detailed classifications and overviews on balancing issues we refer to, e.g., Buxey et al. (1973), Baybars (1986), Shtub and Dar-El (1989), Ghosh and Gagnon (1989), Erel and Sarin (1998), Scholl (1999, Chapter 1) as well as Rekiek et al. (2002b). Furthermore, see Rekiek and Delchambre (2001).

In case of a *paced assembly line*, the station time of every station is limited to the *cycle time c* as a maximum value for each workpiece. Since tasks are indivisible work elements, *c* can be no smaller

Fig. 1. Precedence graph.

than the largest task time $t_{max} = max\{t_j \mid j = 1, \ldots, n\}$. Due to the cycle time restriction, paced assembly lines have a fixed production rate (reciprocal of the cycle time).

In the absence of a common cycle time, i.e., all stations operate at an individual speed, workpieces may have to wait before they can enter the next station and/or stations may get idle when they have to wait for the next workpiece. These difficulties are partially overcome by buffers between the stations. In this case of an *buffered* (*unpaced*) *assembly line*, the ALBP is accompanied by the additional decision problem of positioning and dimensioning buffers (cf., e.g., Buzacott, 1968; Suhail, 1983; Baker et al., 1990; Hillier and So, 1991; Hillier et al., 1993; Malakooti, 1994; Powell, 1994; Dolgui et al., 2002a).

If only one product is assembled, all workpieces are identical and a *single-model line* is present. If several products (models) are manufactured on the same line, the ALBP is connected to a *sequencing problem* which has to decide on the sequence of assembling the model units (cf. Yano and Bolat, 1989; Sumichrast and Russell, 1990; Sumichrast et al., 1992; Bard et al., 1992; Merengo et al., 1999). The sequence is important with respect to the efficiency of a line, because the task times may differ considerably between the products.

Depending on the type of intermixing the units two variants arise: a *mixed-model line* produces the units of different models in an arbitrarily intermixed sequence (cf. Bukchin et al., 2002), whereas a *multi-model line* produces a sequence of batches (each containing units of only one model or a group of similar models) with intermediate setup operations. Therefore, balancing and sequencing are connected to a lot sizing problem in the latter case (cf., e.g., Burns and Daganzo, 1987; Dobson and Yano, 1994). The different line types are characterized in Fig. 2, where different models are symbolized by different geometric shapes. Depending on these line types, single-model, mixed-model and multi-model versions of ALBP have to be considered and solved (cf. Section 10).

A further important characteristic defining different versions of ALBP is the variability of task times. Whenever the expected variance of task times is sufficiently small, as in case of, e.g., simple tasks or highly reliable automated stations, the task times are considered to be *deterministic* (cf., e.g., Johnson, 1983). Considerable variations, which are mainly due to the instability of humans with respect to work rate, skill and motivation as well as the failure sensitivity of complex processes, require considering *stochastic* task times (cf. Section 9 and Buzacott, 1990; Robinson et al., 1990; Hillier and So, 1991, 1993; Pike and Martin, 1994). Besides stochastic time variations, systematic reductions are possible due to *learning effects* or successive improvements of the production process (cf., e.g., Boucher, 1987; Chakravarty, 1988).

Because of the rigid process orientation, the *layout* of flow-line production systems is partially predetermined by the flow of materials. Nevertheless, some layout possibilities exist. Traditionally, an assembly line is organized as a *serial line*, where single stations are arranged along a (straight) con-

a. single-model line

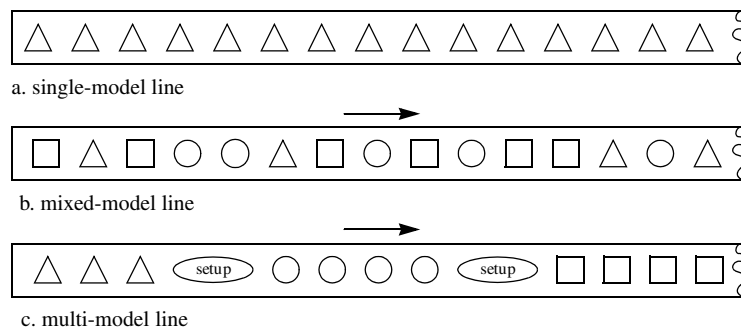b. mixed-model line

c. multi-model line

Fig. 2. Assembly lines for single and multiple products.

veyor belt. Such serial lines are rather inflexible and have other disadvantages which might be overcome by a *U-shaped assembly line* (cf. Section 7). Both ends of the line are closely together forming a rather narrow "U". Stations may work at two segments of the line facing each other simultaneously (*crossover stations*). Besides improvements with respect to job enrichment and enlargement strategies, a U-shaped line design might result in a better balance of station loads due to the larger number of task-station combinations (cf. Miltenburg and Wijngaard, 1994; Monden, 1998; Scholl and Klein, 1999).

Further improvements in flexibility and failure sensitivity of an assembly line system may be achieved by introducing some type of *parallelism* (cf. Section 6): In a multi-model context installing complete *parallel lines* each designed for one product or family of related products often allows better balances and increased productivity. Then the ALBP is accompanied by the additional decision problems concerning the number of lines to be installed and assigning products and work forces to lines (cf. Lehman, 1969; Geoffrion and Graves, 1976; Globerson and Tamir, 1980; Ahmadi et al., 1992). Even with a single line the advantages of parallelization can be utilized by installing *parallel stations*, i.e., the workpieces are distributed among several operators who perform the same tasks. As is the case with parallel lines, the equipment has to be installed several times. Parallel stations allow the reduction of the (global) cycle time of the system if certain tasks have task times longer than the desired cycle time (cf. Freeman and Jucker, 1967; Buxey, 1974; Pinto et al., 1981; Sarker and Shanthikumar, 1983; Bard, 1989; Daganzo and Blumenfeld, 1994). Another possibility of reducing the global cycle time below the largest task time is the concept of *parallel tasks* (cf. Arcus, 1966; Pinto et al., 1975; Inman and Leon, 1994). Respective tasks are assigned to several stations of a serial line which cyclically perform them completely on different workpieces.

Whenever there are process alternatives, i.e., tasks may be performed by different equipment and/or by using different technologies, the balancing problem is connected to an *equipment* or *process selection problem* (cf. Section 5).

For the assembly of heavy workpieces it may be necessary to operate a *two-sided line* which consists of two connected serial lines in parallel. Instead of single stations, pairs of opposite stations on either side of the line (left-hand side and right-hand side stations) work in parallel, i.e., they work simultaneously at opposite sides of the same workpieces (cf. Section 8 and Hautsch et al., 1972; Bartholdi, 1993).

In order to perform a task assigned, the station must be equipped by operators and machines which have the skills and technological capabilities required. Especially in case of complex products it is usually not possible to have all stations equipped equally resulting in *station related assignment restrictions* (cf. Section 8 and Kilbridge and Wester, 1961; Bukchin and Tzur, 2000). Additionally, the assignment of tasks may be restricted by *task related* constraints such as incompatibilities between tasks, minimum or maximum distances (in terms of time or space) between stations performing a pair (or subset) of tasks (cf. Ignall, 1965; Deckro, 1989; Agnetis et al., 1995). Furthermore, *position related* constraints are relevant for workpieces which are heavy, large or fixed at the conveyor belt such that they cannot be turned in any position which is required for performing a task in a certain station (cf. Buxey and Sadjadi, 1976; Wang and Wilson, 1986). Another type of assignment restrictions is *operator related*, because operators have different levels of skill such that only certain task combinations are possible when an operator is assigned to a particular station. Furthermore, aspects of job satisfaction have to be observed (cf. Agrawal, 1985; Iskander and Chou, 1990).

3. Simple versus generalized assembly line balancing

Most of the research in assembly line balancing has been devoted to modelling and solving the *simple assembly line balancing problem* (SALBP) which has the following main characteristics (cf. Baybars, 1986; Scholl, 1999, Chapter 2.2; Scholl and Becker, this issue):

- mass-production of one homogeneous product; given production process;

- paced line with fixed cycle time c;
- deterministic (and integral) operation times t_j;
- no assignment restrictions besides the precedence constraints;
- serial line layout with m one-sided stations;
- all stations are equally equipped with respect to machines and workers;
- maximize the *line efficiency* $E = t_{sum}/(m \cdot c)$ with total task time $t_{sum} = \sum_{j=1}^{n} t_j$.

Several problem versions arise from varying the objective as shown in Table 1.

The versions of SALBP may be complemented by a secondary objective which consists of *smoothing station loads* (*vertical balancing*; cf. Merengo et al., 1999). For example, one may minimize the smoothness index $SX = \sqrt{\sum_{k=1}^{m}(c - t(S_k))^2}$ provided that the combination (m, c) is optimal with respect to line efficiency (see, e.g., Moodie and Young, 1965; Rachamadugu and Talbot, 1991).

Since SALBP-F is an NP-complete feasibility problem, the optimization versions of SALBP are NP-hard (cf. Wee and Magazine, 1982; Scholl, 1999, Chapter 2.2.1.5). All the more, the same is true for any relevant generalized problem.

The assumptions of SALBP are very restricting with respect to real-world assembly line systems. Therefore, researchers have recently intensified their efforts to identify, formulate and solve more realistic problems which consider the characteristics described in Section 2. Following a classification scheme of Baybars (1986) such problems are embraced by the term *generalized assembly line balancing problem* (GALBP). Because most of the models and procedures are directly based on their SALBP counterparts, we refer to Scholl and Becker (this issue) for state-of-the-art solution procedures for SALBP and concentrate here on surveying the way of generalizing the problem rather than describing all methodical aspects in solving these problems.

4. Cost- and profit-oriented objectives

As stated in Section 1, the installation of an assembly line requires large (long-term) *capital investments*. Furthermore, operating the line causes short-term *operating costs* such as wages, material, set-up, inventory and incompletion costs (cf. Scholl, 1999, p. 20). In case of a non-fixed production rate and different levels of production quality, these costs have to be contrasted with the *profit* attained by the line (cf. Zäpfel, 1975).

The installation and operating costs as well as the profits mainly depend on the cycle time and the number of stations (cf. Deckro, 1989), such that cost-oriented models are strongly related to SALBP-E. The latter problem is usually solved by iterating on SALBP-1 or SALBP-2 instances, respectively (see Scholl and Becker, this issue, Section 4.3). Thus, the same procedures can likewise be used for cost or profit oriented objectives on principle. However, in some situations it is necessary to consider models which incorporate costs and/or profits explicitly. This is especially true when the balancing problem is connected with the decision problem of selecting processing or equipment alternatives (cf. Section 5).

4.1. Cost-oriented models

Rosenberg and Ziegler (1992) assume that the operation of a station k causes a *wage rate* w_k per time unit that is equal to the maximum wage rate of all tasks that are assigned to that station. The background of this assumption is that the most demanding task assigned to a station defines the level of qualification the operator(s) must have (for other definitions of station wage rates see Steffen, 1977). The objective is to minimize the aggregate wage rate over all stations, while the number of stations is a variable. Production costs per product unit are obtained by multiplying that rate with the given cycle time. The considered objective is

Table 1
Versions of SALBP

	Cycle time c	
	Given	Minimize
No. m of stations		
Given	SALBP-F	SALBP-2
Minimize	SALBP-1	SALBP-E

equivalent to minimizing the number of stations, if all tasks have the same wage rate. Hence, the problem is a direct generalization of SALBP-1. Rosenberg and Ziegler describe and evaluate priority rule based heuristics, where some of the rules available for SALBP-1 (Scholl and Becker, this issue, Section 5) are extended to allow for smoothing the wage rates within each station.

Amen (2000a) extends the problem by additionally considering station related *costs of capital*, i.e., each station is assumed to require a constant prespecified investment. Amen (1997, 2000a) presents an exact branch-and-bound procedure which extends respective procedures for SALBP-1 (cf. Scholl and Becker, this issue, Section 3) for this problem which uses a station-oriented construction scheme and a laser search strategy based on a topological task labeling. The enumeration is restricted by means of (global and local) lower bounds extending such for SALBP- 1 and dominance rules, where the maximal load rule which is essential for solving SALBP-1 is shown to be inappropriate for the cost-oriented problem. Therefore, only weaker versions of this rule and some other SALBP-1 based rules are applied (cf. Scholl and Becker, 2003).

For the same problem, Amen (2000b, 2001) develops station-oriented priority rule based procedures with cost-oriented dynamic priority rules and compares them to existing ones using a large set of randomly generated problem instances. The new rule which controls the idle time and the difference of wage rates in a station (''best change of idle cost'') performs best. Further improvements are obtained by approaches which use several priority rules. The best results are reported for a restricted version of the branch-and-bound procedure outlined above which is based on successively solving small problems each representing a feasible subset of remaining tasks. However, the latter procedure takes much longer computation times.

Malakooti (1991, 1994) and Malakooti and Kumar (1996) consider a multi-objective ALBP with capacity- and cost-oriented objectives and propose different solution approaches including generation of efficient alternatives, interactive approaches and goal programming.

4.2. Profit-oriented models

The cost-oriented models may be extended by additionally considering profits. Zäpfel (1975, pp. 31) proposes a model with the objective of maximizing the total contribution margin per shift. It considers operating expenses, idle time costs, material costs, and advertising expenses as well as constant selling prices. The model of Klenke (1977, pp. 30) includes fixed selling prices, material costs as well as wages and equipment costs. A similar model which considers individual fixed costs for the different stations is proposed by Rosenblatt and Carlson (1985). This model is extended by Martin (1994) for the case of unpaced lines with buffers, where inventory related cost components are relevant.

5. Equipment selection and process alternatives

The cost-oriented models described in Section 4 assume that the equipment of the stations is given and that the production process is fixed. However, selecting the equipment should be related to the requirements induced by the tasks assigned to a station. Furthermore, the way of manufacturing a product depends on the equipment (machines, manpower). When these decisions are connected to the balancing problem, the term *assembly line design problem* (ALDP) is frequently used in the literature (cf. Baybars, 1986).

Pinto et al. (1983) consider a model which combines the balancing problem with the decision on *process alternatives*. The model considers a basic process which may be complemented by one or more optional process alternatives each of which reduces some task times or even removes certain tasks completely such that each combination of alternatives defines a precedence graph. These alternatives cause fixed costs per time unit. As further cost category, wage rates per time unit are considered. The total wage depends on the number of stations, each of which is manned by one operator, and the cycle time, which may vary in a given range. Due to a desired production rate, a lower bound on the cycle time is given. Taking a larger cycle time causes a certain percentage of overtime

which has to be paid by higher wage rates. Pinto et al. propose a branch-and-bound procedure which branches by selecting/discarding single alternatives and computes lower and even upper bounds by solving respective SALBP instances (cf. Domschke et al., 1997, Chapter 4.3.6.3).

Graves and Lamar (1983) as well as Graves and Holmes Redfield (1988) consider models for one or several products, where the stations to be installed at an assembly line are chosen from a *set of non-identical station types* with different equipments. However, the balancing problem is simplified by assuming a fixed task sequence (serial precedence graph).

Bukchin and Tzur (2000) consider *equipment alternatives* and minimize the total equipment costs for a given cycle time. Every station is provided with *one* equipment chosen from a set of equipment types. Each type has individual costs and an individual influence on the task times. So two problems arise: (1) A variable number of stations need to be installed and provided with equipment. (2) The tasks have to be assigned to the stations considering station related assignment restrictions because some tasks can only be performed with a subset of the equipment types (cf. Section 8). Bukchin and Tzur present an exact and a heuristic algorithm for solving the problem. The first one is a branch-and-bound procedure which is based on the task-oriented construction scheme and uses the MLB strategy (cf. Scholl and Becker, this issue, Sections 3.2 and 3.6). Accompanying the assignment of the first task to a new station the equipment for this station is selected. Lower bounds are computed by discarding the precedence and integrality constraints and considering aggregate cycle time restrictions. The branch-and-bound procedure is capable for solving problems of moderate size with up to 30 tasks and at most 10 equipment types. Therefore, a heuristic version of the procedure is developed which skips nodes of the branch-and-bound tree controlled by a user-specified parameter.

The same problem is examined by Nicosia et al. (2002) who propose a DP procedure (based on Schrage and Baker, 1978; Kao and Queyranne, 1982; cf. Scholl and Becker, this issue, Section 3.5) and a branch-and-bound procedure (based

on FABLE and OptPack; cf. Scholl and Becker, this issue, Section 3.6). Falkenauer (1997) presents a grouping genetic algorithm (cf. Scholl and Becker, this issue, Section 5.2) for a quite similar problem with resource dependent task times. Rubinovitz and Bukchin (1993) consider a restricted version of the problem, where all equipment types have identical installation costs, i.e., the number of stations is minimized. Pinnoi and Wilhelm (1997, 1998) propose branch and cut procedures for basic and generalized ALDP.

An ALDP where an operating mode defining the task times and equipment costs has to be chosen for each task is considered by Rekiek et al. (2002a).

Bukchin and Rubinovitz (2002) show that the *parallel station problem* (cf. Section 6) is a special case of the above mentioned equipment selection problem (p stations in parallel correspond to an equipment which is p times as fast as the basic equipment in a single station). Therefore, the parallel station problem can be solved with the methods outlined above and can be combined with the equipment selection problem without changing the model.

An extension of the balancing problem to *transfer lines* is described in Dolgui et al. (1999) and refined in Dolgui et al. (2001a). A (synchronized) transfer line consists of a sequence of automated stations which perform blocks of tasks *sequentially* and the tasks within each block in parallel using specialized equipment (spindle heads). Combining tasks to blocks is subject to assignment restrictions (cf. Section 8). The block time is determined by the maximal task time in the block and an additional time for transfer operations. The objective function is to minimize the life cycle line cost per part, which is composed of fixed costs per station and additional costs per block, given the cycle time. The problem is solved via shortest constrained path problems (Dolgui et al., 1999, 2001c, 2003), by a decomposition method (Dolgui et al., 2001a), based on a mixed integer program (Dolgui et al., 2001b,c) and by stochastic heuristic procedures (Dolgui et al., 2002b). Dolgui et al. (this issue) consider the problem variation with the blocks of a station being performed *simultaneously* and adapt the shortest constrained path approach.

The equipment selection problem is equivalent to a *worker selection problem*, where workers with different qualifications in terms of production speed or quality are available and are paid according to their qualifications (cf. Akagi et al., 1983; Wilson, 1986; Lutz et al., 1994).

6. Parallel stations and tasks

Due to the indivisibility of tasks, the maximal task time t_{max} is a lower bound on the cycle time c. If there are one or more tasks with task times greater than the desired cycle time, paralleling of stations can resolve this conflict (cf. Buxey, 1974). In the simplest form of paralleling, the *duplication of stations*, there are two identical stations, that execute the same tasks and are provided with the same equipment. Duplicated stations have a *local cycle time* of twice the regular cycle time and are fed with workpieces and release them alternately. Due to the increased local cycle time, the number of feasible loads is greatly enlarged for parallel stations.

Considering the example in Fig. 1 and the feasible solution in Section 1 with $c = 11$ and $m = 5$, for SALBP-2 there is no way for decreasing c. When parallel stations are available, a feasible solution with $m = 5$ and $c = 10$ can be found (Fig. 3), decreasing the total idle time from 8 to 3 time units and increasing the production rate. Stations 4a and 4b show local cycle times of 20. Their station time is 18 and they are fed with workpieces by station 3 alternately.

Of course, the installation of parallel stations causes additional fixed costs such that cost-oriented objectives are necessary to evaluate a certain line design. Pinto et al. (1981) present an approach, where duplicating of stations is allowed. The objective is to minimize labour costs, which
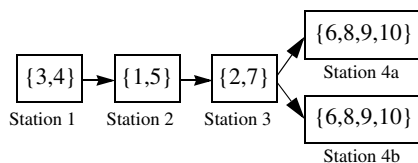
consist of fixed costs for duplicating a station, regular wage costs and overtime costs, which arise if the realized cycle time exceeds a desired cycle time. A branch-and-bound procedure for assigning tasks to stations and deciding whether a station is to be duplicated or not is presented (for details see Domschke et al., 1997, Chapter 4.3.6.1).

Pinto et al. (1975) consider the concept of *parallel tasks*. Long-lasting tasks are assumed to be decomposable into shorter tasks having the same precedence relations as the original one. Now these parallel tasks are assigned to different stations in order to get a feasible balance for the desired cycle time at all and/or to improve the line efficiency. Because of the indivisibility of tasks, however, the original task can be executed only by one station per cycle. This is dealt with by alternately performing this task in each of the respective stations thereby accepting the local cycle times to vary from cycle to cycle. The objective is to minimize total costs, which consist of facility costs and labour costs defined as in the parallel station case. Facility costs arise though in the parallel task case no additional stations are needed, because a transport system has to be installed, that supports temporary violations of the cycle time. Pinto et al. give a mathematical model of the problem and solve it by a branch-and-bound procedure. In their algorithm, parallel tasks must not be assigned to more than two stations and the task time is subdivided equally among the two parts of the task.

Bard (1989) considers parallel tasks and stations as well as *dead time*, which is the time that is needed for transporting workpieces from one station to the next, meanwhile no tasks can be executed. For a serial line, dead time decreases the employable cycle time. If, e.g., c is 10 and the dead time is 1, workers have to be paid for 10 time units, while $t(S_k)$ must not exceed 9 for every serial station k. With parallel stations the unproductive portion of the cycle time can be decreased. If we consider the above example, duplicated stations each have a local cycle time of 20 and the productive time is 19. That is, in the example, the parallel stations use 95% and the serial stations only 90% of the cycle time in a productive manner. Bard proposes a DP procedure, which is based on DP approaches to SALBP-1 (cf. Scholl and Becker,

Fig. 3. Parallel stations.

this issue, Section 3.5), for solving the outlined problem.

Buxey (1974) considers a model with parallel stations which passes on fixed costs for additional equipment and aims at minimizing the total idle time. He proposes a generalized priority rule based procedure to solve the problem. A further heuristic procedure is developed by Sarker and Shanthikumar (1983).

Another approach of duplicating stations is to arrange them *side by side in a serial line*. This can be useful due to space limitations or when a less complex transport system is required. In this case, buffers are needed in front of and behind duplicated stations. Consider an example with four serial stations 1, 2a, 2b, and 3 with 2a and 2b being duplicates of each other. When station 1 finishes its work on a workpiece while the stations 2a and 2b are still busy, the workpiece enters the in-buffer. In the next cycle, 2b is fed with the workpiece from the in-buffer, while 2a is directly delivered by station 1. The output of station 2b is carried to the following station 3 and that of 2a moves through 2b to the out-buffer. Inman and Leon (1994) present an stochastic approach to this problem, where random failures, repair times and processing times are variates. In the stochastic case, duplicated stations often do not start and end their processes at the same time. As long as the first of the stations is busy, the second station cannot be loaded with the next workpiece. Four policies of feeding incoming workpieces to duplicated stations are compared by a simulation.

Another type of paralleling is to assign more than one operator to a station (*multiple manning*). Shtub (1984) considers an objective which minimizes the number of operators given cycle time and number of stations. He describes a heuristic similar to that of Buxey (1974). Wilson (1986) reformulates and reduces the problem and solves it by a standard MIP solver. Chakravarty and Shtub (1986b) consider the case of dynamic change in manning due to learning effects.

Finally, an effect similar to that of paralleling stations consists of combining stations to larger units (aggregate stations) which are operated by *teams of operators*. The aggregate stations have a

multiple of the original cycle time available and operators may rotate increasing the job satisfaction. Respective problems are considered by Johnson (1991), Bukchin et al. (1997) and Bukchin and Masin (2004). Sometimes, the teams are responsible for the complete product such that the "line" has only one aggregate station left. That is, team-oriented assembly systems may remove main characteristics of traditional assembly lines such as strict division of labour and pacing (cf. Bukchin et al., 1997).

7. U-shaped line layout

The *U-line assembly line balancing problem (UALBP)* is introduced and modelled by Miltenburg and Wijngaard (1994). In that problem the assembly line is arranged in an U-shape (cf. Fig. 4). Stations can be arranged so that during the same cycle two workpieces at different positions on the line can be handled. In the example in Fig. 4, station 1 will execute the first tasks on one workpiece, which is starting its production process in that cycle, and the last tasks of another workpiece, that is to be finished at the same cycle. So the difference to SALBP is that a station k can contain not only tasks whose predecessors are assigned to one of the stations $1, \ldots, k$, but also tasks whose predecessors will be finished until the product returns to station k for the second time (cf. Monden, 1998). A station which handles the same workpiece in two different cycles is called a *crossover station* (stations 1 and 5 in Fig. 4).

We can define three problem versions of UALBP regarding to SALBP (cf. Section 3):
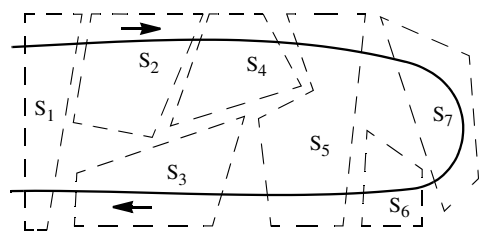

Fig. 4. U-shaped assembly line.

UALBP-1. Given the cycle time c, minimize the number of stations m.

UALBP-2. Given the number of stations m, minimize the cycle time c.

UALBP-E. Maximize the line efficiency E for c and m being variable.

Of course UALBP can undergo the same generalizations as SALBP with respect to cost-oriented objectives, paralleling, equipment selection etc. (see below and the respective sections).

Every solution feasible for SALBP is feasible for UALBP as well, because an U-line does not need to include crossover stations. However, the optimal UALBP solution may have an improved line efficiency compared to the optimal SALBP solution due to the increased possibilities of combining tasks to station loads.

Applying the example of Fig. 1 to UALBP-2 with given $m = 5$, an optimal solution with $c = 10$ can be found, which is shown in Fig. 5. The first station starts executing tasks 3 and 4, but gets back every workpiece at the end of the production process to perform task 10. Hence, the optimal UALBP-2 solution has a better line efficiency than the optimal SALBP-2 solution which requires a cycle time of 11.

Miltenburg and Wijngaard (1994) modify the DO approach of Held et al. (1963) and the ranked positional weight technique of Helgeson and Birnie (1961) to UALBP-1. Ajenblit and Wainwright (1998) use a genetic algorithm to solve UALBP-1. Urban (1998) gives an integer programming formulation for UALBP-1 and solves problem instances with CPLEX. Scholl and Klein (1999) develop the procedure ULINO (U-Line Optimizer) which is an extension of SALOME-1 for SALBP-1 (cf. Scholl and Klein, 1997) and apply it to all versions of UALBP distinguished above. Erel et al. (2001) present a simulated annealing algorithm.
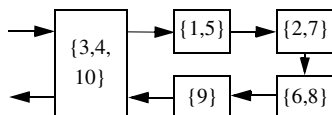
Nakade et al. (1997) develop bounds and approximations for the cycle time when task times are random variables and the number of stations is given (stochastic UALBP-2).

Urban and Chiang (this issue) present a chance-constrained, piecewise linear, integer program for UALBP-1 with stochastic task times which can be solved with CPLEX for small problems. Chiang and Urban (2002) propose a hybrid heuristic composed of priority rule based procedures and an improvement step.

Miltenburg (2000) investigates the effect of the shape of the production line on its effectiveness when breakdowns occur. He points out that an U-line is better or equal to a straight line when buffer inventories are arranged at all contact points between stations. The reason is that crossover stations cause more contact points than regular stations and so more buffer inventories can be installed. Straight lines are to be preferred when there are no buffer inventories. Admittedly, the costs of installing and operating buffer inventories are not considered.

Miltenburg (1998) presents the N-UALBP where there exist a number N of U-lines with the given identical cycle time c. The objective is to assign tasks to stations, while each station can include tasks of at most two adjacent production lines. With a DP algorithm optimal solutions for small problem instances are obtained. Sparling (1998) develops an algorithm to find multiline stations which include tasks of up to three production lines. Furthermore, he considers the general case, where production line locations are not fixed already.

8. Assignment restrictions

As discussed in Section 2, several types of restrictions may curtail the assignment of tasks to stations. *Position related restrictions* induce the need for left hand side and right hand side stations. Especially large workpieces, such as buses or trucks, acquire stations that only execute tasks at one of their sides, because moving around the workpiece would cost to much time. This leads to *task related restrictions* because left hand side

Fig. 5. U-shaped assembly line.

tasks must not be combined with right hand side ones. In general, tasks that cannot be assigned to the same station are called *incompatible*.

Bartholdi (1993) presents *two-sided assembly lines*, where pairs of workstations are located opposite to each other on the left and on the right hand side of the line. Each pair of stations is working on one item at the same time. Tasks are grouped by the side of the vehicle they can be executed at. So there are right hand tasks, e.g. mounting the right wheel, left hand tasks, tasks which can be assigned to either side of the line, e.g., mounting a radio, as well as tasks that have to be executed by both paired stations simultaneously, e.g., installing the rear seat.

Of course, a two-sided line may have unmanned stations. In that case some stations do not have an opposite companion. Bartholdi shows, that for a given c in some cases, depending on the precedence constraints, a two-sided line requires less stations than a traditional one-sided line, but never requires more stations. An important remark is that for a pair of opposite stations the precedence constraints of assigned tasks need to be respected. If we consider a task 3 with its predecessors 1 and 2, task times $t_1 < t_2$, with 1 and 3 being assigned to one station and 2 assigned to the opposite one, then task 3 must not start before the latter station has finished task 2. So idle times may occur even at the beginning or in the middle of a cycle, when one station has to wait with starting a task until the opposite station has finished its predecessors.

Bartholdi implements a modified version of a priority rule based heuristic (cf. Scholl and Becker, this issue, Section 5.1) into a software program, which allows users to fix some tasks to specific stations. This is needed, e.g., when some tasks require special equipment, which is available only at a specific station. Such *station related constraints* can easily be included into models and procedures for SALBP (cf. Johnson, 1983), because they simply prevent from assigning a certain task to a subset of the stations. A DP procedure for a problem with incompatibilities between tasks and some fixed task-station combinations, is presented by Agnetis et al. (1995). The objective is to smooth workload among a given number of stations (vertical balancing; see Section 3).

Kim et al. (2000a) present a genetic algorithm which tries to minimize the number of stations in a two-sided ALBP. Lee et al. (2001) describe two alternative objectives for this problem: the first objective attempts at assigning tasks, which are directly related in the precedence graph, to the same stations. The latter tries to avoid assigning related tasks to opposite stations or, if this is not possible, to maximize the slack time between the finishing time of one task and the starting time of its successor in the opposite station. A priority rule based procedure (cf. Scholl and Becker, this issue, Section 5.1) which is based on grouping related tasks is proposed.

Pastor and Corominas (2000) describe a real-world single-sided line with given number of stations and different types of assignment restrictions. (1) Task related: some tasks are required to be assigned to the same station and, thus, may be combined to a single task. (2) Position related: some tasks can only be performed at the left hand or the right hand side of a large, irremovable workpiece. Furthermore, some tasks need to be executed at the top or at the bottom of the workpiece. Left and right hand side tasks as well as top and bottom tasks must not be assigned to the same stations, respectively. This has to be done without using a two-sided line, i.e., some stations have to be arranged at the left-hand side of the conveyor belt, others at the right-hand side, furthermore, some stations require the worker to operate in the height, others have to allow for working at the bottom. To solve this large real world problem Pastor and Corominas present a mixed integer linear program and a two-phase heuristic procedure (phase 1: truncated DP approach, phase 2: improvement by local search with tabu search). The objective consists of smoothing the station loads such that all station times have a value out of a given target interval.

Bautista et al. (2000) consider SALBP-1 with additional incompatibilities between groups of tasks and the secondary objective of minimizing the cycle time once the minimum number of stations is found. They develop a greedy randomized adaptive search procedure and a genetic algorithm to solve the problem. Park et al. (1997) consider SALBP-2, extended by task incompatibilities and

"range constraints" which allow for flexibility in precedence relations, and develop a local search heuristic.

In a robotic assembly line, certain *tools and parts* are required for performing tasks by the robot cells (stations). Due to limited space to store the parts and tools, restrictions for the joint assignment of tasks to stations are imposed. Kim and Park (1995) propose a mathematical formulation and a cutting plane procedure for this extension of SALBP-1.

Operator related restrictions concerning the levels of qualification are considered by Johnson (1983) who tries to combine tasks with similar levels of complexity in a station. Carnahan et al. (2001) incorporate physical demand into ALBP. Tasks differ in their grip strength demands, while workers differ in their grip strength capacity. In addition, performing a task fatigues a worker, causing a decrease of his capacity. Workers must not be physically overloaded by the tasks that are assigned to their stations. The objective is to minimize a composite score, which consists of the weighted cycle time and a weighted fatigue measure for a given number of stations. The problem is solved by a multi-pass heuristic and two different genetic algorithms.

9. Stochastic task times

In all previous sections, we assume the task times to be deterministic. Though this is regularly the case in highly automated lines (like transfer lines, see Section 5), task times usually vary from cycle to cycle, especially when human operators are engaged (cf. Buzacott, 1990; Scholl, 1999, Chapter 1.3.3). In general, the variance of a task's time increases with its complexity.

Because the literature on stochastic ALBP is wide, we give only an outline of the different lines of research for the case of paced assembly systems (for details, see Scholl, 1999, Chapter 2.3.1).

Moodie and Young (1965) assume the task times to be independent normal variates which is considered to be realistic in most cases of human work. Other distributions are, e.g., examined by

Kao (1976, 1979), Sniedovich (1981) and Nkasu and Leung (1995).

In some cases it is sufficient to use a task time t'_j which contains a *safety factor* such that the realized (stochastic) task time of task j does not exceed this value at a certain probability. Then, the stochastic problem can be (heuristically) transformed into a deterministic one and solved by modified SALBP procedures (for different respective approaches see, e.g., Kottas and Lau, 1976, 1981; Sphicas and Silverman, 1976; Henig, 1986; Carraway, 1989).

Specific *time-oriented objectives* of stochastic models are the minimization of the probability of exceeding the cycle time in any station (cf. Reeve and Thomas, 1973) or the stochastic variations of station times (cf. Raouf and Tsui, 1982). Usually, time-oriented objectives do not consider the consequences of exceeding the cycle time. Since it is generally not possible to use a cycle time large enough to obtain sufficiently small excess probabilities, one has to consider effects of incomplete tasks explicitly:

- The conveyor is *stopped* until the incomplete or missed operations have been performed online. So one station can block the production process of all other stations and the production rate is decreased.
- The process continues, while the incomplete tasks and all their successors in the precedence graph are left out. These tasks are performed at special *off-line stations* after the incomplete unit has left the line or by *mobile workers* while the process continues with a workpiece that has been repaired earlier and waits in a buffer.
- Stations which tend to exceed the cycle time get an additional capacity by employing *additional operators* which increase the work pace but increase the costs per workpiece.

Regardless of how incompletion is dealt with, they cause additional costs. These *incompletion costs* are reduced by decreasing the station utilizations. This can be done by increasing the number of stations or the cycle time. However, this raises the *labour and equipment costs* (cf. Kottas and Lau, 1973, as well as Sections 4 and 5).

Furthermore, the ordering of tasks inside the stations influences the incompletion costs with respect to their value and probability (cf. Kottas and Lau, 1976). The higher the potential incompletion costs of a task are, the more idle time has to be introduced into a station in order to avoid actual incompletion. Therefore, the idle time should be concentrated in early stations of the line, because the incompletion costs depend on the number of affected successor tasks.

Problems with off-line error handling are described by Kottas and Lau (1973, 1976, 1981), Carter and Silverman (1984). Shtub (1984) additionally considers the possibility of dynamically assigning additional operators. Problems with line stoppages in case of task incompletion are examined by Silverman and Carter (1986), Lau and Shtub (1987). In either case, it is proposed to solve the problem in a two-stage manner: (1) Generate a number of feasible line balances. (2) Select the one with minimal cost.

More recently, Sarin and Erel (1990) also consider a problem with off-line error handling and solve it heuristically by a restricted DP approach. Sarin et al. (1999) present an improved heuristic. First, the problem is decomposed into subproblems which are solved by the above mentioned DP procedure. After improving these initial solutions by applying a branch-and-bound procedure they are appended forming the final solution.

Gökcen and Baykoc (1999) compare the off-line repair with the following alternative (see above) by a simulation approach: Every time, a workpiece remains unfinished at a station, the missing tasks are performed by a mobile worker off the line and then the item enters the buffer. On the production line the workpiece is replaced by an item from the buffer which has been repaired earlier. So the objective has to be expanded in order to include buffer costs.

Sotskov et al. (this issue) perform a stability analysis of SALBP-1 solutions, i.e., the effect of (ceteris paribus) variations of task times on the stability of the optimal solution is examined. Considering the sensitivity of such a solution one may decide whether it is necessary to model the problem in a stochastic manner or sufficient to solve some deterministic problem.

A simulated annealing (SA) approach for a stochastic variant of SALBP-1 is proposed by Suresh and Sahu (1994), a genetic algorithm by Suresh et al. (1996). McMullen and Frazier (1998) and McMullen and Tarasewich (2003) develop an SA procedure and an ant algorithm for a GALBP with respect to parallel stations, stochastic task times, mixed-model production and alternative objectives. SALBP-2 with fuzzy task times is considered by Tsujimura et al. (1995).

10. Mixed-model lines

Mixed-model assembly lines manufacture several *models* (versions) of a standardized commodity in an intermixed sequence (cf. Section 2). The models may differ from each other with respect to size, colour, used material, or equipment such that their production requires different tasks, task times and/or precedence relations. As a consequence, finding a line balance whose station loads have the same station time and equipment requirements whatever model is produced is almost impossible. Therefore, the line must be flexible enough with respect to the equipment and the qualification of operators as well as local cycle time violations. In opposite to the (deterministic) single-model case, the cycle time is no longer the maximum time available in each station to perform the tasks on a workpiece but the average time (defined on the basis of a desired production rate).

By analogy with SALBP, the (medium-term) *mixed-model assembly line balancing problem* (MALBP) consists of finding a number of stations and a cycle time as well as a line balance such that a capacity- or even cost-oriented objective is optimized. Corresponding to the problem versions SALBP-1 and SALBP-2, the cycle time or the number of stations may be given, respectively, such that different versions MALBP-1, MALBP-2, and MALBP-E arise (cf. Scholl, 1999, Chapter 3.2.2). However, the problem is more difficult than in the single-model case, because the station times of the different models have to be smoothed for each station (*horizontal balancing*; cf. Merengo et al., 1999) in order to avoid operating inefficiencies like work overload or idle time. Hence, the goals

of single-model problems should be extended to regard this aspect adequately (see below).

The better this horizontal balancing works, the better solutions are possible in the connected short-term *mixed-model sequencing problem* (MSP) which is to find a sequence of model units which meets the demands of all models given by the production program of a short-term planning period and optimizes some objective. Such objectives mostly correspond to inefficiencies arising from variations of the station times of the models (which are set by the line balance). Usually, the demands result from individual orders with certain delivery dates. Therefore, the sequencing problem arises per shift, day or week with particular demands for all models. Surveys of mathematical models and solution procedures for MSP can, e.g., be found in Yano and Bolat (1989), Bard et al. (1992), Scholl et al. (1998), Scholl (1999, Chapters 3.3 and 6.2). Domschke et al. (1996) and Scholl (1999, Chapter 3.4) define a hierarchical planning approach connecting MALBP and MSP at several hierarchical levels.

10.1. Characterization of MALBP

MALBP relies on the same basic assumptions as SALBP (deterministic task times, no assignment restrictions, serial line-layout, fixed rate launching; cf. Section 3). Additionally given are task times t_{jp} of the tasks $j = 1, \ldots, n$ for the models $p = 1, \ldots, P$ (with $t_{jp} = 0$ if j is not required for assembling p) and individual precedence relations for each model which can be combined to a single joint precedence graph. Furthermore, the expected *model mix* with demand portions d_p of the different models $p = 1, \ldots, P$ with $\sum_{p=1}^{P} d_p = 1$ is given (cf. Scholl, 1999, Chapter 3.2). The model mix allows for (partially) anticipating the data of the short-term MSP.

For modelling and solving MALBP two basic approaches can be distinguished: Reduction to single-model problems and horizontal balancing in the multi-model context.

10.2. Reduction to single-model problems

A straightforward transformation of the mixed-model data to some version of SALBP consists in computing *average task times* $\bar{t}_j = \sum_{p=1}^{P} d_p \cdot t_{jp}$ for the tasks $j = 1, \ldots, n$ (cf. Thomopoulos, 1970; Macaskill, 1972; McMullen and Frazier, 1997; Scholl, 1999, Chapter 6.1.2). Solving the resulting (average) SALBP instance guarantees that the cycle time is sufficient to perform all tasks on average. However, even in case of an optimal solution for the average model, considerable inefficiencies may occur when operating the line.

By *relaxing* the assumption that identical tasks have to be assigned to the same station for all models, the mixed-model problem is *decomposed into P independent SALBP instances*. A respective generalization of the shortest-path formulation of SALBP-1 (cf. Scholl and Becker, this issue, Section 3.5) is presented by Roberts and Villa (1970) as well as Rao (1971). However, the assignment of identical tasks to different stations is usually not desired with respect to additional facility requirements, loss of specialization effects, complicated production control, and setup inefficiencies. Only in the case of multi-model production, where batches of models are processed, this relaxation of MALBP may be useful (cf. Section 2).

A rather restrictive problem is obtained by imposing the *cycle time restrictions for every model* (cf. Deutsch, 1971). That is, the station times of all models in all stations must not exceed the (common) cycle time c. This, however, may lead to a poor efficiency of the line, because compensation effects between the models cannot be utilized. Due to these restrictive capacity constraints, no sequencing problem occurs. A MALBP-1 with model-dependent cycle times c_p is modelled by Gökcen and Erel (1998) and solved through a shortest-path based procedure by Erel and Gökcen (1999).

10.3. Horizontal balancing

In any case where the cycle time restriction is formulated on an aggregate/average basis, considerable inefficiencies may occur when operating the line. This is due to the variations in the station times $\tau_{pk} = \sum_{j \in S_k} t_{jp}$ over the models $p = 1, \ldots, P$ considering a certain station k. This imbalance has the following impacts on the performance of a mixed-model line:

- *Work overload*. Whenever the operator of a station is not able to complete the assigned tasks before the workpiece leaves the station (due to a restricted station length or due to the transport system), *work overload* occurs. Work overload may be compensated by the temporary employment of utility workers, stopping the line or another sanction (cf. Section 9). Whatever is selected, work overload is inefficient and expensive and should be minimized. Unfortunately, the amount of work overload which really occurs cannot be computed for a solution of the balancing problem directly, because it depends on the unknown short-term production programs and corresponding production sequences. Therefore, it is essential to obtain balances in which *potential* work overload situations are minimized.
- *Idle time*. As in the single-model case, idle time occurs when a station has completed its work on a workpiece and has to wait for the next one arriving at the station. The idle times per cycle are constant if only one model is produced. If several models are assembled, the idle times differ and depend on the sequence. Similar to work overload, not the realized idle times can be known in advance, but the *potential* idle times should be minimized.

In order to reduce horizontal imbalances, additional *secondary objectives* given the number m of stations and the cycle time c are proposed in the literature.

Thomopoulos (1970) proposes the following objective function which minimizes the sum of absolute differences between the average station time $\bar{\tau}_p = \sum_{j=1}^{n} t_{jp}/m$ of model $p = 1, \ldots, P$ and its realized station times τ_{pk} in all stations $k = 1, \ldots, m$:

$$\text{Minimize} \quad \Psi_1 = \sum_{k=1}^{m} \sum_{p=1}^{P} |\tau_{pk} - \bar{\tau}_p|. \tag{1}$$

Thomopoulos proposes a heuristic procedure for solving some version of MALBP combined with the secondary objective (1) that is similar to the heuristic of Hoffmann (1963) for SALBP-1. Different heuristics for MALBP-1 including the

one of Thomopoulos are compared by Van Zante-de Fokert and de Kok (1997).

The following objective function minimizes the maximal deviation of a station time of any model from the average station time per unit (cf. Decker, 1993):

$$\text{Minimize} \quad \Psi_2 = \max\{|\tau_{pk} - \bar{\tau}_p| \quad \text{with} \\ k = 1, \ldots, m \quad \text{and} \quad p = 1, \ldots, P\}. \tag{2}$$

A third objective function minimizes the sum of cycle time violations of all models in all stations, i.e., it considers only potential work overload (cf. Domschke et al., 1996):

$$\text{Minimize} \quad \Psi_3 = \sum_{k=1}^{m} \sum_{p=1}^{P} \max\{0, \tau_{pk} - c\}. \tag{3}$$

Each of the above objectives may also be used on an aggregate basis by multiplying the single differences with the demand proportions d_p of the models p.

How to solve some version of MALBP with an arbitrary secondary objective by an exact solution procedure for SALBP-1 is explained by Scholl (1999, Chapter 6.1.2).

Theoretical and experimental comparisons of the objectives by Domschke et al. (1996) and Scholl (1999, Chapter 3.2.2.3) shows that the third objective has the best potential in anticipating and avoiding inefficiencies in operating the line. Another comparison of the first objective function with further ones is performed by Bukchin (1998).

Bukchin et al. (2002) propose a three-stage solution approach for MALBP-1 using different secondary objective functions as discussed by Bukchin (1998). The first stage consists of solving SALBP-1 for the average (combined) model determining the number of stations and fixing the tasks, which are common to all models, to stations. The second stage reassigns the tasks of each model which are specific for this model preserving the fixed assignments made before and optimizing the horizontal balancing objective used. The third stage is a local search procedure changing the assignment of the common tasks and applying

stage 2 for completing the solution by assigning specific tasks as described above.

Merengo et al. (1999) define two horizontal balancing functions, which are based on weighted differences between the maximal station time (caused by any model) and the station times of all other models. In their station-oriented heuristic (similar to Hoffmann, 1963), Merengo et al. use the horizontal balancing as primary goal while the minimization of the number of stations (MALBP-1) is taken as secondary goal unlike most other approaches mentioned above.

10.4. Generalized problems

Pastor et al. (2002) consider MALBP-2 with an additional objective that tries to increase the uniformity of tasks at the stations. The problem is solved using priority rule based and tabu search heuristics.

Vilarinho and Simaria (2002) consider and model MALBP-1 with additional assignment restrictions and parallel stations. As secondary objective, terms for measuring vertical and horizontal imbalances are minimized. As solution procedure a two-stage simulated annealing approach is proposed.

Karabati and Sayin (2003) consider an MALBP-2 with a given production sequence containing several copies of each model. In every cycle, the maximum station time (considering the models being currently performed in all stations) defines the realized cycle time. The objective is to minimize the sum of realized cycle times for the production sequence. The problem is modelled as a binary linear program and solved by a modified priority rule based procedure. The special case of a dominating main model is solved as a SALBP-2.

Kim et al. (2000b) consider a combined mixed-model balancing and sequencing problem which is based on avoiding utility work and solved by different genetic algorithms. Bock et al. (this issue) also consider a combined balancing and sequencing problem incorporating different problem extensions which is solved by a parallel computing approach.

Askin and Zhou (1997) present a nonlinear integer program for a cost-oriented extension of MALBP with parallel stations and equipment selection. For solving the problem a priority rule based procedure using the average model is proposed. Chakravarty and Shtub (1985, 1986a) describe priority based and shortest path heuristics for another cost-oriented version of MALBP.

Sparling and Miltenburg (1998) describe and analyse a mixed-model U-line balancing problem and provide a branch-and-bound based solution procedure. A combined balancing and sequencing problem is formulated and solved with a genetic algorithm by Miltenburg (2002). A further genetic algorithm for a similar combined problem is proposed by Kim et al. (this issue).

11. Conclusions and further research

The survey shows that assembly line balancing research which traditionally was focused upon simple problems (SALBP) has recently evolved towards formulating and solving generalized problems (GALBP) with different additional characteristics such as cost functions, equipment selection, paralleling, U-shaped line layout and mixed-model production. While a lot of relevant problems have been identified and modelled, however, the development of sophisticated solution procedures has just begun. Thus, additional research is necessary to adopt state-of-the-art solution concepts like metaheuristics and highly developed enumeration and bounding schemes to GALBP. Furthermore, standardized and realistic test beds are required for testing and comparing methodical enhancements. Because research has produced a large variety of problem definitions without a clear direction (one might say by arbitrarily combining problem characteristics), it seems to be necessary to provide a classification which facilitates distinguishing and referencing those problem types.

Going a step further, it is required to develop user-friendly computer software that is flexible enough to be applied to real-world problems and contains state-of-the-art solution procedures. A first attempt which tries to combine these needs has been made by the softwarelab "Optimal Design" (www.optimaldesign.com) by developing

the MS Windows software *Optiline*. This software combines user-friendly input of data, interactive and automatic optimization tools based on genetic algorithms and tools for analysis of results and reporting. Another system which is based on state-of-the-art SALBP procedures has been developed by Fidan et al. (2003). Most other software products available for managing assembly systems do not contain powerful procedures for assembly line balancing but concentrate on data management. To conclude, there is a considerable demand for further developments.

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