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Metaheuristic Approach to Assembly Line Balancing

SUPAPORN SUWANNARONGSRI^{1*} and DEACHA PUANGDOWNREONG²

¹ Department of Industrial Engineering, Faculty of Engineering

South-East Asia University

19/1 Petchkasem Road, Nongkham District,

Bangkok, 10160, THAILAND

² Department of Electrical Engineering, Faculty of Engineering

South-East Asia University

19/1 Petchkasem Road, Nongkham District,

Bangkok, 10160, THAILAND

* corresponding author: supaporn-eng@sau.ac.th

<http://www.sau.ac.th>

Abstract: - In this paper, the metaheuristic method consisting of the adaptive tabu search (ATS) and the practicing heuristic (PH) technique is proposed to provide optimal solutions of the assembly line balancing (ALB) problems. With the multi-objective approach, the ATS is used to address the number of tasks assigned for each workstation, while the PH is conducted to assign the sequence of tasks for each workstation according to precedence constraints. The proposed approach is tested against six benchmark ALB problems suggested by Scholl and one real-world ALB problem. Comparisons of optimal results obtained by the proposed method with those obtained by the single objective approach are elaborated.

Key-Words: - assembly line balancing, metaheuristic approach, adaptive tabu search, multi-objective function

1 Introduction

Over six decades, the assembly line balancing (ALB) problem has been one of the most interesting topics among industrial researchers. Considered as the class of NP-hard combinatorial optimization problems [1], the ALB problem is one of the classic problems in industrial engineering. By literatures, several methods to provide the optimal solutions of the ALB problems were launched, for example, heuristic approaches [2-4], artificial intelligent (AI) search techniques such as the genetic algorithm (GA) [5] and the tabu search (TS) [6], and hybrid AI methods [7-10]. Based on the optimization context, the multi-objective optimizations can probably give better solutions than the single objective approach. With this idea, many researches have moved to use multi-objective approach to solve the ALB problems [11,12].

In 1989, Glover introduced the tabu search (TS) method to solve the combinatorial optimization problems [13,14]. Based on the neighborhood search approach, the TS method consists of two main strategies namely intensification and diversification [15,16]. In 2004, the modified version of the TS method named the adaptive tabu search (ATS) method was launched [17]. The ATS possesses two distinctive mechanisms denoted as back-tracking (BT) regarded as one of the diversification strategies

and adaptive radius (AR) considered as one of the intensification strategies. The ATS can be regarded as one of the most powerful AI search techniques. Convergence proof and performance evaluation of the ATS have been reported [17,18]. The ATS has been widely applied to various real-world engineering problems, e.g. power system protection [19], dynamic system identification [20,21], control system design [22,23] and audio signal processing [24]. In 2008, the ATS associated with the partial random permutation (PRP) technique was developed to solve the ALB problems [25]. As previous results, it was found that such the approach could provide satisfy solutions. However, it spent amount of search time, when applied to solve the ALB problems containing a lot of tasks.

In this paper, the metaheuristic approach consisting of the ATS and the practicing heuristic (PH) technique are proposed to provide optimal solutions of the ALB problems. The ATS is used to address the number of tasks assigned for each workstation, while the proposed practicing heuristic (PH) technique is conducted to arrange the sequence of tasks according to the precedent constraints. The workload variance, the idle time and the line efficiency are performed together as the multi-objective functions. To perform its effectiveness, the proposed approach is tested against six benchmark

ALB problems suggested by Scholl [26] and one real-world ALB problem from a survey of literature [27]. Obtained results will be compared with those obtained by the single objective approach. This paper consists of five sections. The assembly line balancing problem formulation is described in Section 2. The proposed metaheuristic approach consisting of the ATS method and the PH technique is illustrated in Section 3. Results and discussions are provided in Section 4, while conclusions are given in Section 5.

2 ALB Problem Formulation

The assembly line balancing (ALB) problem is considered as one of the classic industrial engineering problems. An assembly line is a sequence of workstations connected together by a material handling system. It is used to assemble components or tasks into a final product. The fundamental of the line balancing problems is to assign the tasks to an ordered sequence of workstations that minimize the amount of the idle time of the line, whereas satisfying two particular constraints. The first constraint is that the total processing time assigned to each workstation should be less than or equal to the cycle time. The second is that the task assignments should follow the sequential processing order of the tasks or the precedence constraints.

The assembly lines are traditionally represented by the precedent diagram as shown in Figure 1, for example, where the numbers surrounded by circles present the tasks or work-element and the node weights stand for the task time in time units. The precedent diagram in Figure 1 is of the Buxey problem, one of the benchmark ALB problems suggested by Scholl [26].

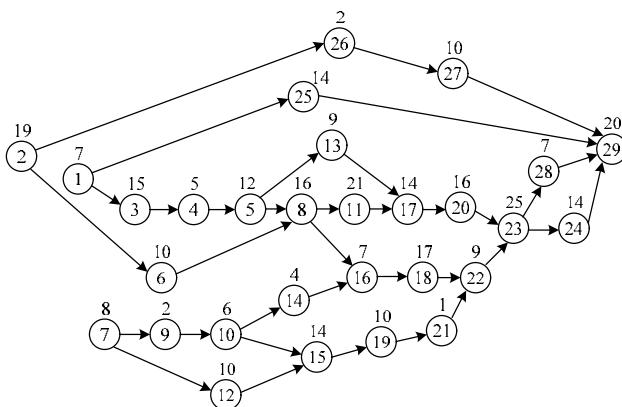


Fig. 1 The precedent diagram of Buxey problem.

In this paper, the single-model assembly line balancing problem is considered. Balancing of the lines can be measured by the idle time (T_{id}), the workload variance (w_v) and the line efficiency (E). Therefore, the goals of balancing assembly lines are to minimize the idle time, to minimize the workload variance and to maximize the line efficiency. Analytical formulations for the ALB problems are stated in (1)-(4), where m is the number of workstations, W is the total processing time, c is the cycle time and T_i is the processing time of the i^{th} workstation.

$$m = W / c \quad (1)$$

$$T_{id} = \sum_{i=1}^m (c - T_i) \quad (2)$$

$$w_v = \sum_{i=1}^m [T_i - (W / m)]^2 / m \quad (3)$$

$$E = \sum_{i=1}^m T_i / (mc) \quad (4)$$

3 The Metaheuristic Approach

The ATS and PH technique are developed together to provide optimal solutions of the single-model ALB problems. The algorithms of the ATS and the PH are briefly given as follows.

3.1 Adaptive Tabu Search (ATS)

With its neighborhood search approach, the ATS [17,18] begins the search with some random initial solutions belonging to a neighborhood search space. All solutions in neighborhood search space will be evaluated via the objective function. The solution giving the minimum objective cost is set as a new starting point of next search round and kept in the tabu list (TL). Figure 2 illustrates some movements of the ATS. With the back-tracking (BT) mechanism regarded as a diversification strategy, the search can escape from an entrapment caused by a local solution. The BT mechanism looks up the TL and selects one of the visited solutions as a new starting point. A new search could begin in a new direction. Hence, an entrapment by a local solution can be released. When the search approaches the local or the global solution, the adaptive radius (AR) mechanism regarded as an intensification strategy is invoked to speed up the search process. The search

radius is subsequently reduced to provide finer and finer solutions within a short search time consumed. The ATS algorithm is summarized step-by-step as follows.

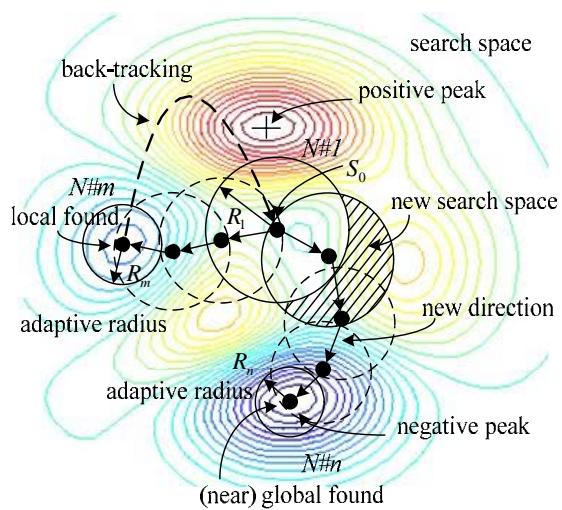


Fig. 2 Movement of the ATS.

- Step 1** Initialize a search space (Ω), $TL = \emptyset$, search radius (R), $count$, and $count_{max}$.
- Step 2** Randomly select an initial solution S_0 from a certain search space Ω . Let S_0 be a current local minimum.
- Step 3** Randomly generate N solutions around S_0 within a search radius R . Store the N solutions, called neighborhood, in a set X.
- Step 4** Evaluate the objective value of each member in X via objective functions. Set S_1 as a member giving the minimum cost.
- Step 5** If $f(S_1) < f(S_0)$, put S_0 into the TL and set $S_0 = S_1$, otherwise, store S_1 in the TL instead.
- Step 6** Activate the BT mechanism, when a local entrapment occurs.
- Step 7** If the termination criteria: $count = count_{max}$ or desired specification are met, then stop the search process. S_0 is the best solution, otherwise go to Step 8.
- Step 8** Invoke the AR mechanism, once the search approaches the local or the global solution to refine searching accuracy.
- Step 9** Update $count = count + 1$, and go to Step 2.

Following recommendations are provided [17,18] for potential users to apply the ATS effectively.

- The initial search radius, R , should be 7.5-15% of search space radius.

- The number of neighborhood members, N , should be 30-40.
- The number of repetitions of a solution before invoking the BT mechanism should be 5-15.
- The k^{th} backward solution selected by the BT mechanism should be equal or close to the number of repetitions of a solution before invoking the BT mechanism.
- The adaptive search radius should employ 20-25% of radius reduction.
- A well educated guess of the search space that is wide enough to cover the global solution is necessary.

3.2 Practicing Heuristic (PH)

Based on the heuristic logics of practicing engineers, the PH is developed in this work to arrange the sequence of tasks assigned for each workstation according to the precedence constraints. In practice, assigning task will be considered from its processing time, number of succeeding tasks and number of precedent tasks. The proposed PH algorithm is thus described as follows.

- Step 1** Let number of tasks be n .
- Step 2** Initialize the sequence of tasks $\Delta = \emptyset$ and $i = 1$.
- Step 3** If a current task, δ_i , possesses properties:
 - (3.1) no precedent tasks,
 - (3.2) maximum succeeding tasks, and
 - (3.3) maximum processing time,
 put δ_i into Δ respectively, delete δ_i , $n = n - 1$, then go to Step 4, otherwise, update $i = i + 1$, then go back to Step 3.
- Step 4** If $n = 0$, terminate the sequencing process. The sequence of tasks stored in Δ is successfully arranged, otherwise set $i = 1$, go back to Step (3).

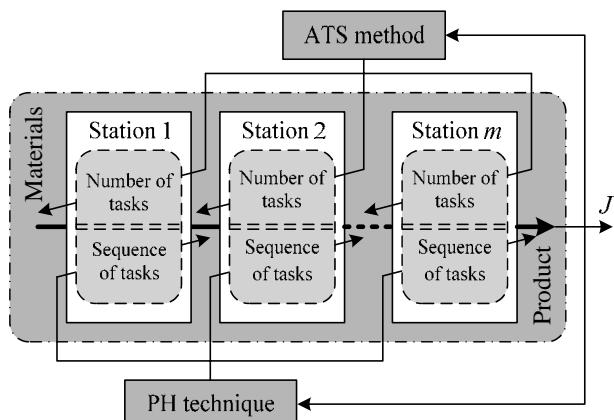


Fig. 3 The metaheuristic approach for ALB.

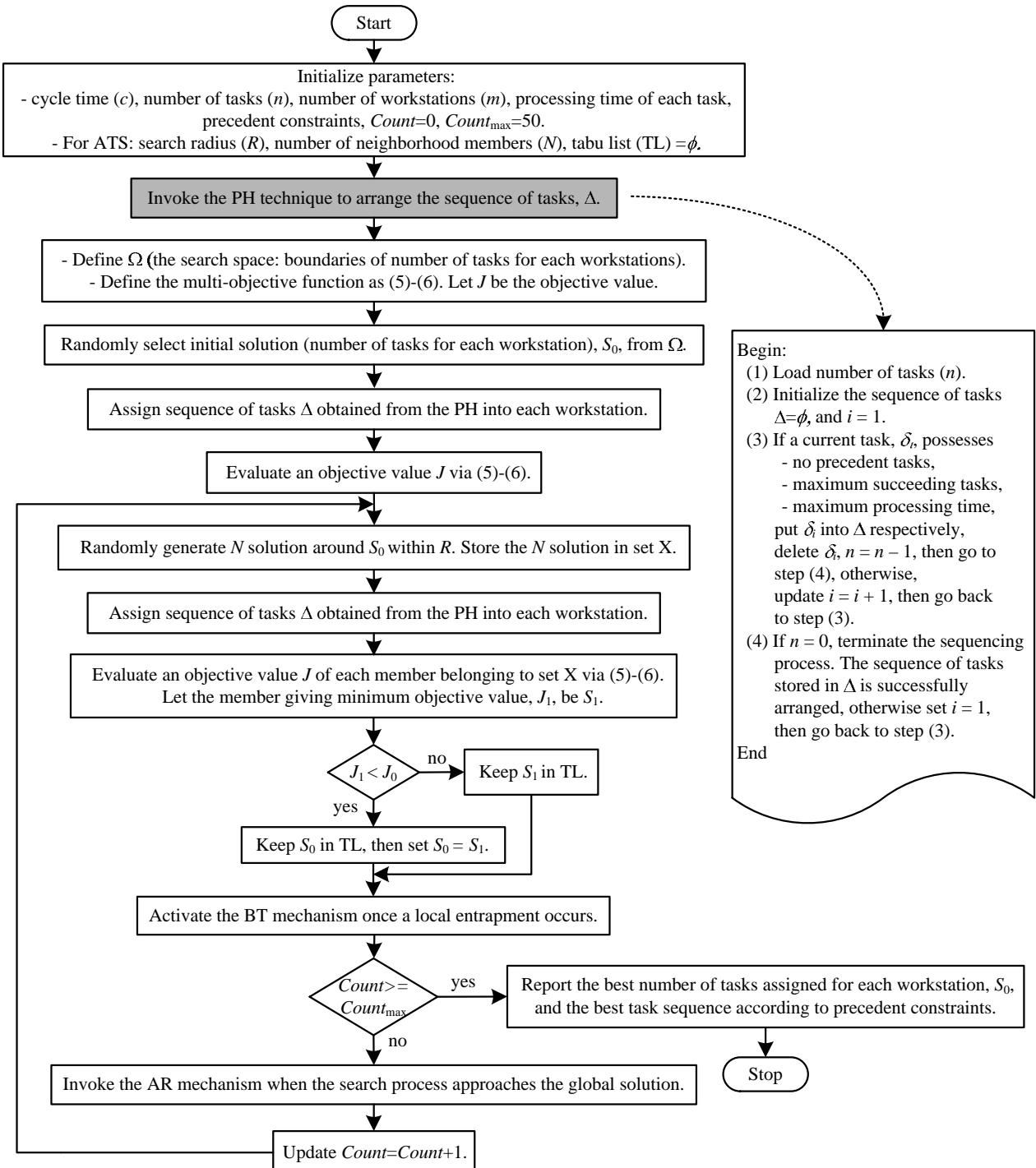


Fig. 4. The flow diagram of the proposed metaheuristic approach for ALB.

Figure 3 represents the proposed metaheuristic approach based on the ATS and the PH associated with the multi-objective function to obtain optimal solutions of the ALB problems. The ATS is employed to address the number of tasks assigned for each workstation, while the PH is used to arrange the sequence of tasks according to the precedent constraints. The multi-objective function (J), including the workload variance (w_v), the idle time (T_{id}) and the line efficiency (E), is performed and

expressed in (5), where γ_1 , γ_2 and γ_3 are weighted factors ($\gamma_1+\gamma_2+\gamma_3=1.0$). J will be fed back to the ATS and the PH boxes to be minimized as stated in (6). The maximum number of search round of the ATS ($count_{max}$) is set as the termination criterion. The flow diagram in Figure 4 reveals the search process of the proposed approach for solving the ALB problems. The flow diagram gives a clear view for the readers to follow.

$$J = (\gamma_1 \cdot w_v) + (\gamma_2 \cdot T_{id}) + (\gamma_3 \cdot E^{-1}) \quad (5)$$

$$\begin{aligned} & \min J \\ & \text{subject to } T_i \leq c \text{ and} \\ & \text{precedent constraints} \end{aligned} \quad (6)$$

4 Results and Discussions

To perform its effectiveness, the proposed metaheuristic approach is tested against six benchmark single-model ALB problems suggested by Scholl [26] and one real-world ALB problem, the motorcycle assembly line balancing of Kawasaki motor enterprise Co. Ltd. [27]. In this work, the number of workstations, m , is prior given. The ATS and the PH are coded by MATLAB. For all tests, the parameter settings of the ATS are as follows: $N = 100$ and $R = 20\%$ of search spaces. $count_{max} = 50$ is set as the termination criteria. In each ALB problems, 20 trials of simulation are carried out to find average values of the multi-objective (MO) function J (w_v , T_{id} and E) and average times consumed. Each trial starts with a random initial solution generated by MATLAB. For the weighted factors, $\gamma_1 = 0.4$, and $\gamma_2 = \gamma_3 = 0.3$ are arbitrarily set. Obtained results will be compared with those obtained by the single objective (SO) approach by setting $\gamma_1 = 1.0$, and $\gamma_2 = \gamma_3 = 0.0$.

Table 1 Details of the tested ALB problems.

| Entry | Name | n | W | c | m |
|-------|------------|-----|------------|----------|-----|
| 1 | Buxey | 29 | 324 min. | 50 min. | 7 |
| 2 | Roszieg | 25 | 125 min. | 20 min. | 7 |
| 3 | Sawyer | 30 | 324 min. | 40 min. | 9 |
| 4 | Tonge | 70 | 3,510 min. | 325 min. | 11 |
| 5 | Kilbridge | 45 | 552 min. | 80 min. | 7 |
| 6 | Warnecke | 58 | 1,548 min. | 160 min. | 10 |
| 7 | Motorcycle | 60 | 2,475 sec. | 360 sec. | 7 |

Referring to Table 1, seven single-model ALB problems are listed in the table with their corresponding details. These ALB problems are Buxey (entry 1), Roszieg (entry 2), Sawyer (entry 3),

Tonge (entry 4), Kilbridge (entry 5), Warnecke (entry 6) and Motorcycle (entry 7), respectively. The readers can find more details of first six ALB problems from [26] and those of entry-7 ALB problem from [27]. When the proposed metaheuristic approach is applied, the boundaries of number of tasks for each workstation must be set to perform the search spaces. Table 2 gives the boundaries of number of tasks for each workstation to be searched. Once the search process stopped, results obtained by the proposed metaheuristic approach with MO and SO functions are summarized in Table 3.

Table 3 Results obtained by the metaheuristic approach with MO and SO functions.

| Entry 1 (Buxey) | | | | |
|----------------------|-------|------------|--------|-------------|
| Apps. | w_v | T_{id} | E | Time |
| SO | 11.06 | 26.00 min. | 94.46% | 265.24 sec. |
| MO | 1.35 | 26.00 min. | 96.42% | 266.82 sec. |
| Entry 2 (Roszieg) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 2.59 | 15.00 min. | 93.98% | 251.76 sec. |
| MO | 0.69 | 15.00 min. | 93.98% | 255.23 sec. |
| Entry 3 (Sawyer) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 6.00 | 36.00 min. | 90.00% | 278.92 sec. |
| MO | 1.56 | 36.00 min. | 97.30% | 281.35 sec. |
| Entry 4 (Tonge) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 11.54 | 65.00 min. | 98.18% | 612.37 sec. |
| MO | 1.17 | 65.00 min. | 99.40% | 633.85 sec. |
| Entry 5 (Kilbridge) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 4.12 | 8.00 min. | 98.00% | 311.57 sec. |
| MO | 0.12 | 8.00 min. | 99.82% | 328.12 sec. |
| Entry 6 (Warnecke) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 18.56 | 52.00 min. | 97.35% | 486.48 sec. |
| MO | 6.56 | 52.00 min. | 97.35% | 492.33 sec. |
| Entry 7 (Motorcycle) | | | | |
| Apps. | w_v | T_{id} | E | Time |
| SO | 21.96 | 45.00 sec | 98.21% | 547.72 sec. |
| MO | 17.67 | 45.00 sec | 98.49% | 571.18 sec. |

Note: Apps. stands for approaches.

Time stands for average times consumed.

Table 2 List of search spaces.

| Entry | Search spaces (boundaries of number of tasks for each workstation) | | | | | | | | | | |
|-------|--|---------|---------|---------|--------|---------|---------|---------|---------|----------|----------|
| | S_1 | S_2 | S_3 | S_4 | S_5 | S_6 | S_7 | S_8 | S_9 | S_{10} | S_{11} |
| 1 | [2, 7] | [3, 8] | [2, 6] | [3, 7] | [2, 7] | [2, 5] | [2, 5] | - | - | - | - |
| 2 | [2, 5] | [2, 5] | [2, 6] | [3, 7] | [1, 5] | [2, 6] | - | - | - | - | - |
| 3 | [2, 5] | [2, 6] | [3, 6] | [3, 6] | [2, 5] | [1, 4] | [1, 5] | [1, 4] | [2, 5] | - | - |
| 4 | [6, 9] | [5, 8] | [2, 6] | [3, 6] | [5, 7] | [3, 6] | [5, 10] | [6, 12] | [6, 12] | [5, 8] | [5, 8] |
| 5 | [5, 10] | [5, 8] | [4, 8] | [4, 8] | [4, 7] | [4, 7] | [8, 12] | - | - | - | - |
| 6 | [3, 6] | [3, 6] | [4, 8] | [6, 9] | [5, 9] | [3, 6] | [4, 7] | [5, 8] | [5, 8] | [4, 8] | - |
| 7 | [5, 10] | [5, 10] | [7, 10] | [7, 12] | [5, 8] | [5, 10] | [9, 14] | - | - | - | - |

Note: S_i stands for the i^{th} workstation.

From Table 3, it is found that the proposed metaheuristic approach with MO is capable of producing better solutions for all ALB problems. Figure 5 shows the convergent rates of the search processes of the entry-1 ALB problem as an example. The curves in Figure 5 confirm that the proposed approach gives optimal solutions before the search process termination. The search convergence curves of other entries are omitted because they have a similar form to that of the entry-1 ALB problem shown in Figure 5. Figure 6 shows details of results for entry-1 ALB problem. In addition, results in details of each entry ALB example are contained in Table 4 – 10, respectively.

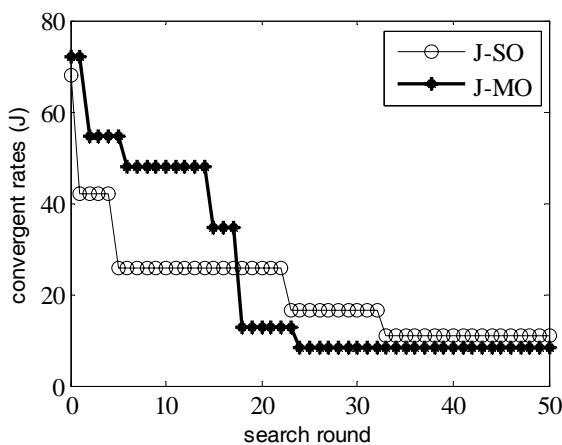


Fig. 5 The convergence rates of Buxey problem.

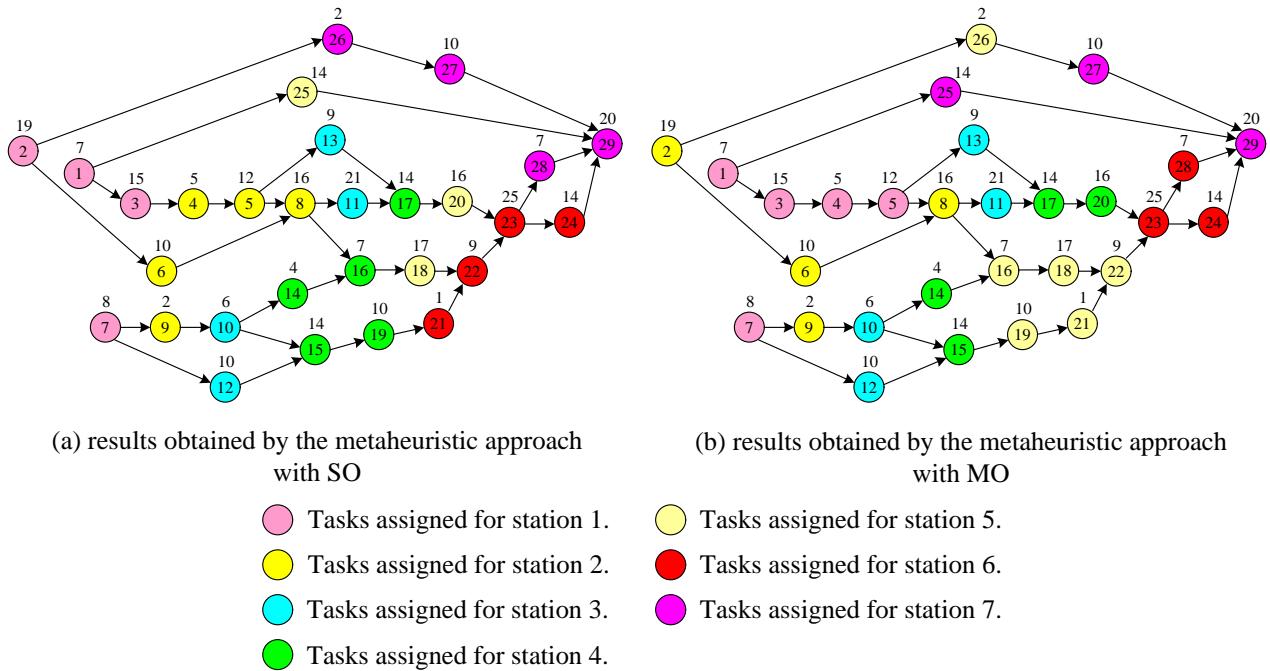


Fig. 6 Results of entry-1 ALB problem (Buxey) in details.

Table 4 Results obtained by the metaheuristic approach for the entry-1 (Buxey).

| Metaheuristic approach with SO | | | |
|--------------------------------|----------------|------------------------|------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,3,7 | 49 | 1 |
| 2 | 4,5,6,8,9 | 45 | 5 |
| 3 | 10,11,12,13 | 46 | 4 |
| 4 | 14,15,16,17,19 | 49 | 1 |
| 5 | 18,20,25 | 47 | 3 |
| 6 | 21,22,23,24 | 49 | 1 |
| 7 | 26,27,28,29 | 39 | 11 |

The total idle time (T_{id}) = 26.00 min.

The workload variance (w_v) = 11.06

The line efficiency (E) = 94.46%

| Metaheuristic approach with MO | | | |
|--------------------------------|-------------------|------------------------|------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,3,4,5,7 | 47 | 3 |
| 2 | 2,6,8,9 | 47 | 3 |
| 3 | 10,11,12,13 | 46 | 4 |
| 4 | 14,15,17,20 | 48 | 2 |
| 5 | 16,18,19,21,22,26 | 46 | 4 |
| 6 | 23,24,28 | 46 | 4 |
| 7 | 25,27,29 | 44 | 6 |

The total idle time (T_{id}) = 26.00 min.

The workload variance (w_v) = 1.35

The line efficiency (E) = 96.42%

Table 5 Results obtained by the metaheuristic approach for the entry-2 (Roszieg).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|---|------------------|---------------------------|---------------------|---|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,3 | 16 | 4 | 1,2,3 | 16 | 4 |
| 2 | 4,8,9,12 | 18 | 2 | 4,5,6 | 18 | 2 |
| 3 | 5,6,10,15 | 19 | 1 | 7,8,11 | 18 | 2 |
| 4 | 7,11,13,14 | 19 | 1 | 9,10,12,13,14,16 | 18 | 2 |
| 5 | 16,17,20 | 19 | 1 | 15,17 | 18 | 2 |
| 6 | 18,19,21,22 | 19 | 1 | 18,20,21,23 | 18 | 2 |
| 7 | 23,24,25 | 15 | 5 | 19,22,24,25 | 19 | 1 |
| The total idle time (T_{id}) = 15.00 min. | | | | The total idle time (T_{id}) = 15.00 min. | | |
| The workload variance (w_v) = 2.59 | | | | The workload variance (w_v) = 0.69 | | |
| The line efficiency (E) = 93.98% | | | | The line efficiency (E) = 93.98% | | |

Table 6 Results obtained by the metaheuristic approach for the entry-3 (Sawyer).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|---|------------------|---------------------------|---------------------|---|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,3,10 | 38 | 2 | 1,3,4 | 37 | 3 |
| 2 | 11,12,13 | 34 | 6 | 2,5,12,16,17 | 36 | 4 |
| 3 | 4,5,16,17,18 | 34 | 6 | 10,13,14,20 | 37 | 3 |
| 4 | 6,7,8,24 | 37 | 3 | 6,7,18,24 | 37 | 3 |
| 5 | 14,15,20 | 37 | 3 | 8,9,15,25 | 37 | 3 |
| 6 | 21,25 | 38 | 2 | 21,22 | 35 | 5 |
| 7 | 9,22,23,26 | 40 | 0 | 19,23 | 34 | 6 |
| 8 | 27,28 | 32 | 8 | 26,27 | 34 | 6 |
| 9 | 19,29,30 | 34 | 6 | 11,28,29,30 | 37 | 3 |
| The total idle time (T_{id}) = 36.00 min. | | | | The total idle time (T_{id}) = 36.00 min. | | |
| The workload variance (w_v) = 6.00 | | | | The workload variance (w_v) = 1.56 | | |
| The line efficiency (E) = 90.00% | | | | The line efficiency (E) = 97.30% | | |

Table 7 Results obtained by the metaheuristic approach for the entry-4 (Tonge).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|---|--------------------------------------|---------------------------|---------------------|---|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,3,5,9,15,41 | 321 | 4 | 1,2,3,5,9,15,24 | 317 | 8 |
| 2 | 4,6,7,8,30,69 | 323 | 2 | 4,6,7,10,41,68 | 319 | 6 |
| 3 | 10,11,16,68 | 317 | 8 | 11,16,18 | 318 | 7 |
| 4 | 12,13,14,70 | 317 | 8 | 8,12,14,30,69 | 318 | 7 |
| 5 | 17,18,19,20,22,24 | 318 | 7 | 13,17,19,20,22,57 | 319 | 6 |
| 6 | 21,23,25,29,57 | 323 | 2 | 21,23,25,27 | 320 | 5 |
| 7 | 26,27,28, 33,34,58 | 316 | 9 | 26,29,31,32, 33,34,58,59 | 320 | 5 |
| 8 | 31,32,35,36, 44,48,49,51 | 325 | 0 | 28,35,36,37,38, 44,45,48,51,70 | 319 | 6 |
| 9 | 37,38,39,40,42,43, 45,46,47,61,62 | 319 | 6 | 39,40,56,61, 62,63,64 | 320 | 5 |
| 10 | 50,52,53, 54,56,59 | 318 | 7 | 42,43,46,47, 49,52,67 | 321 | 4 |
| 11 | 55,60,63,64, 65,66,67 | 313 | 12 | 50,53,54,55, 60,65,66 | 319 | 6 |
| The total idle time (T_{id}) = 65.00 min. | | | | The total idle time (T_{id}) = 65.00 min. | | |
| The workload variance (w_v) = 11.54 | | | | The workload variance (w_v) = 1.17 | | |
| The line efficiency (E) = 98.18% | | | | The line efficiency (E) = 99.40% | | |

Table 8 Results obtained by the metaheuristic approach for the entry-5 (Kilbridge).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|--|-----------------------------------|---------------------------|---------------------|--|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,11,12,39,3,7,8 | 80 | 0 | 1,2,4,6,8,11,12 | 79 | 1 |
| 2 | 4,6,10,13,14,30 | 80 | 0 | 3,5,7,13,14,15 | 79 | 1 |
| 3 | 5,9,15,16,17 | 79 | 1 | 9,10,16,18,19,30,39 | 79 | 1 |
| 4 | 18,19,20,21,31,32 | 80 | 0 | 17,20,21,32,37 | 79 | 1 |
| 5 | 22,23,24,27,29 | 79 | 1 | 22,23,25,27,31 | 79 | 1 |
| 6 | 25,26,28,33,36 | 80 | 0 | 24,26,28,29,33 | 78 | 2 |
| 7 | 34,35,37,38,40, 41,42,43,44,45 | 74 | 6 | 34,35,36,38,40, 41,42,43,44,45 | 79 | 1 |
| The total idle time (T_{id}) = 8.00 min. The workload variance (w_v) = 4.12 The line efficiency (E) = 98.00% | | | | The total idle time (T_{id}) = 8.00 min. The workload variance (w_v) = 0.12 The line efficiency (E) = 99.82% | | |

Table 9 Results obtained by the metaheuristic approach for the entry-6 (Warnecke).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|--|--------------------------|---------------------------|---------------------|---|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (min.) | Idle time (min.) | Assigned Task | Processing time (min.) | Idle time (min.) |
| 1 | 1,2,3,4,9 | 154 | 6 | 1,2,3,4,6 | 157 | 3 |
| 2 | 5,10,11,12 | 154 | 6 | 5,7,8,12,16 | 159 | 1 |
| 3 | 7,13,14,15,17 | 154 | 6 | 28,9,11,13,14,15,24 | 158 | 2 |
| 4 | 8,16,18,19, 20,21,22 | 157 | 3 | 17,18,19,20, 21,22,26 | 154 | 6 |
| 5 | 6,23,24,26, 27,29,30 | 156 | 4 | 23,25,27, 29,30,34 | 154 | 6 |
| 6 | 28,31,32,33,35 | 156 | 4 | 31,33,36,37,40 | 156 | 4 |
| 7 | 34,36,37,39,41,42 | 159 | 1 | 32,35,38,39,41 | 151 | 9 |
| 8 | 25,40,43,44,45,49 | 156 | 4 | 42,43,44,45,46,48 | 154 | 6 |
| 9 | 38,47,51,52,53,54 | 159 | 1 | 47,49,50,51,52,53,54 | 154 | 6 |
| 10 | 46,48,50,55, 56,57,58 | 143 | 17 | 10,55,56, 57,58 | 151 | 9 |
| The total idle time (T_{id}) = 52.00 min. The workload variance (w_v) = 18.56 The line efficiency (E) = 97.35% | | | | The total idle time (T_{id}) = 52.00 min. The workload variance (w_v) = 6.56 The line efficiency (E) = 97.35% | | |

Table 10 Results obtained by the metaheuristic approach for the entry-7 (Motorcycle).

| Metaheuristic approach with SO | | | | Metaheuristic approach with MO | | |
|--|--------------------------------------|---------------------------|---------------------|--|---------------------------|---------------------|
| Station (m) | Assigned Task | Processing time (sec.) | Idle time (sec.) | Assigned Task | Processing time (sec.) | Idle time (sec.) |
| 1 | 1,2,3,4,5,7,49,9 | 360 | 0 | 1,2,4,5,6,49,53 | 353 | 7 |
| 2 | 6,8,10,11, 12,14,15,19 | 348 | 12 | 3,7,8,9,10, 11,13,15,51 | 359 | 1 |
| 3 | 13,16,17,18, 22,24,26,51,52 | 356 | 4 | 12,14,16,17, 18,20,21,22,54 | 348 | 12 |
| 4 | 20,21,28,30,32, 38,40,50,53,55 | 357 | 3 | 24,26,28,29, 30,31,32,52 | 359 | 1 |
| 5 | 23,25,27,29, 42,54,56 | 357 | 3 | 19,23,25, 27,33,55 | 348 | 12 |
| 6 | 31,33,34,35, 37,39,57 | 348 | 12 | 34,35,36,37, 38,39,40,41,43 | 355 | 5 |
| 7 | 36,41,43,44,58, 59,60,45,46,47,48 | 349 | 11 | 42,44,45,56,57,58, 59,60,45,46,47,48 | 353 | 7 |
| The total idle time (T_{id}) = 45.00 sec. The workload variance (w_v) = 21.96 The line efficiency (E) = 98.21% | | | | The total idle time (T_{id}) = 45.00 sec. The workload variance (w_v) = 17.67 The line efficiency (E) = 98.49% | | |

5 Conclusions

The paper has proposed the metaheuristic approach consisting of the adaptive tabu search (ATS) method and the practicing heuristic (PH) technique associated with multi-objective function to obtain optimal solutions of the ALB problems. The ATS is used to address the appropriate number of tasks assigned for each workstation, while the PH is employed to arrange the appropriate sequence of tasks according to the precedent constraints. The proposed approach has been tested against six benchmark ALB problems and one real-world ALB problem. Results confirm that optimal solutions of all ALB problems can be successfully obtained and solutions obtained by the proposed metaheuristic approach are superior to those obtained by the single objective approach.

Acknowledgements

The authors wish to express their sincere thanks to Prof. Dr. S. Sujitjorn, Suranaree University of Technology, Thailand, and his colleagues for launching a powerful AI search technique namely the adaptive tabu search.

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