restrained_shrinkage_fib_MC2010_zadani

October 21, 2023

1 Výpočet odezvy při vázaném smrštění dle fib MC 2010

Vypracované řešení zašlete včetně scriptu nebo pomocných výpočtů na adresu petr.havlasek@cvut.cz

Řešení je možné odezvdat do přespříštího cvičení.

Jako předmět zprávy uveďte **PRPM: BONUS**

1.1 Zadání:

Deska z prostého betonu tloušťky 200 mm se v jednom směru může volně zkracovat a ve druhém směru je neposuvně podepřená. Uvažujte beton třídy C30/37. Deska byla ošetřována po dobu 7 dní a poté byla vystavena prostředí s průměrnou hodnotou relativní vlhkosti 50%. Účinky vlastní tíhy, dalšího nahodilého zatížení a autogenního smrštění zanedbejte. Dále předpokládejte, že vlivem vysychání dochází ke smršťování, které namáhá celý průřez stejnoměrně.

Předpokládejte, že tahová pevnost betonu se vyvíjí dle doporučení MC 2010 a není redukována dlouhodobým zatížením. Střední tahová pevnost betonu ve stáří 28 dnů je pro třídu do C50 aproximována výrazem

$f_{ctm,28} = 0.3\left(f_{ck}\right)^{2/3}$

kde f_{ck} je charakteristická hodnota tlakové pevnosti v tomto stáří. Pro jednoduchost předpokládejte, že vývoj tahové pevnosti se řídí stejnou funkcí jako tlaková pevnost, tj. $f_{ctm}(t) = \beta_{cc}(t) f_{ctm,28}$, kde funkce $\beta_{cc}(t)$ je definována dála v tomto notebooku (buňka pro výpočet modulu pružnosti).

Vaším úkolem je vykreslit vývoj napětí způsobeného smršťováním od vysychání. Do jednoho grafu vykreslete vývoj napětí za předpokladu, že beton dotvaruje a dále pro případ, že je jeho chování idealizováno jako lineárně pružné s tuhostí rovnou sečnovému modulu. Do stejného obrázku nakreslete vývoj tahové pevnosti betonu dané třídy a vývoj předepsaného smrštění.

Stanovte délku vysychání, při které tahové napětí dosáhne tahové pevnosti betonu.

Pro výpočet napětí můžete využít vztah 1.89 ze skripta PRPM. Relaxační funkci vyhodnoťe vždy v polovině časového intervalu.

Pro Vaše řešení můžete využít vykreslení v poslední buňce tohoto notebooku.

Notebook obsahuje potřebné i nepotřebné informace a grafy.

Pozn: vyhodnocení napětí za předpokladu relaxace je poměrně náročná úloha a její výpočet může trvat několik vteřin (v závislosti na časové diskretizaci)

2 fib Model Code 2010: creep and shrinkage

Implementation of expressions from fib bulletin 65: Model Code 2010, Final draft, Volume 1, 2012. Relaxation function computed using equations (5.24)-(5.25) presented in Z.P. Bažant and M. Jirásek: Creep and Hygrothermal Effects in Concrete Structures, 2018

2.1 Range of applicability

- unreinforced concrete moist cured at normal temperature up to 14 days
- age at loading $tt \geq 1$ day
- structural concrete with 20 MPa ≤ f_cm ≤ 130 MPa
- stress level up to 0.4 $f_{cm}(tt)$ for linear creep
- stress level 0.4-0.6 $f_{cm}(tt)$ for nonlinear creep
- mean relative humidity 40-100%
- mean temperature 5-30 C

2.2 Input parameters

2.2.1 Material & Geometry

`fcm28` … mean compressive strength of concrete measured on cylinders at the age of 28 days [MPa]

`h` … notional size [mm] reflecting cross section geometry and surface exposed to drying

`aggregate type` … basalt/quartzite/dense limestone/sandstone

'cement type' … (decision tree = first applicable criterion according to the composition)

- CS = CEM III, CEM II/B, strength class 32.5 N, 42.5 N, more than 65% of GGBS or more than 35% FA
- CN = CEM II, CEM I, strength class 42.5 N, 32.5 R, more than 35% GGBS or more than 20 % FA
- CR = CEM I, strength class 42.5 R, 52.5 N, 52.5 R

`t0` … duration of curing = concrete age at the onset of drying

2.2.2 Loading

`RH` … mean relative humidity of the environment [%]

`tt` … age at loading [day]

`stress` … magnitude of compression [MPa]

2.3 Imports

[14]:
```
%matplotlib inline

import math
import numpy as np

import matplotlib.pyplot as plt
```

2.4 Definition of Global Variables

[43]:
```
# onset of drying (concrete age) [days]
t0 = 7

# concrete age at loading (default value) [days]
tt = 14

# magnitude of loading stress [MPa]
stress = 10

# mean compressive strength on cylinders at the age of 28 days [MPa]
fcm_28 = 38

# relative humidity of the environment [%] (range of validity 20-100%)
RH = 50

# notional size of the member [mm] (h = 2*cross section / perim. of drying␣
↪surface)
h = 200

# default value of parameters related to concrete composition

aggregate_type = "quartzite"
# aggregate_type = "basalt"
# aggregate_type = "dense limestone"
# aggregate_type = "limestone"
# aggregate_type = "sandstone"


# CREEP-SPECIFIC PARAMETERS
# cement_type = "CS"
cement_type = "CN"
# cement_type = "CR"



# ASSIGNING MATERIAL PARAMETERS - DO NOT MODIFY

```

```
if (fcm_28 <= 60): # MPa

    if (cement_type == "CS"):
        s = 0.38

    elif (cement_type == "CN"):
        s = 0.25

    else: # R
        s = 0.20

else:
    s = 0.20

if (cement_type == "CS"):
    alpha = -1.

elif (cement_type == "CN"):
    alpha = 0.

else: # CR
    alpha = 1.

if (cement_type == "CS"):
    alpha_as = 800.
    alpha_ds1 = 3.
    alpha_ds2 = 0.013

elif (cement_type == "CN"):
    alpha_as = 700.
    alpha_ds1 = 4.
    alpha_ds2 = 0.012

else: # CR
    alpha_as = 600.
    alpha_ds1 = 6.
    alpha_ds2 = 0.012


if (aggregate_type == "basalt"):
    alpha_E = 1.2

elif (aggregate_type == "quartzite"):
    alpha_E = 1.0

elif (aggregate_type == "dense limestone"):
    alpha_E = 1.0
```

```
elif (aggregate_type == "limestone"):
    alpha_E = 0.9

else: #(aggregate_type == "sandstone")
    alpha_E = 0.7
```

[44]:
```
# variables - time discretization and plotting

log_scale = True

t_div = 50 # default number of time points used for time integration etc.

t_aux_min = 1.e-2
t_aux_max = 1.e3
```

3 Functions in MC 2010

3.1 Modulus of Elasticity

Described in Section 5.1.7.2

E_{cm} is defined as a tangent modulus of elasticity at the origin of the stress-strain diagram which is approximately equal to the slope of the unloading branch at 40% of the compressive strength

The value of Young's modulus at the age at loading is used to determine the instantaneous deformation while its value at 28 days, $E_{cm}(28)$ or `E_cm_28`, is used to compute the delayed deformation by means of the creep coefficient.

[45]:
```
def compute_modulus_of_elasticity(t):

    E_c0 = 21.5e3

    E_cm_28 = E_c0 * alpha_E * ( fcm_28/10. )**(1./3.)  # 5.1-21

    beta_cc = math.exp( s * (1. - math.sqrt(28./t) ) ) # 5.1-51

    beta_E = math.sqrt(beta_cc) # 5.1-57

    E_cm_t = beta_E * E_cm_28 # 5.1-56

    return E_cm_t
```

3.2 Creep Coefficient

[46]:
```python
def compute_creep_coeff(t,tt):

    phi_bc = compute_creep_coeff_basic(t, tt)
    phi_dc = compute_creep_coeff_drying(t, tt)

    return phi_bc + phi_dc
```

[47]:
```python
def compute_creep_coeff_basic(t, tt):

    beta_bc_fcm = 1.8 * fcm_28**(-0.7) # 5.1-65

    # adjusted age reflecting the cement type but (here) without temperature␣
 ↪effect
    tt_adj = compute_adjusted_age(tt)

    if (t > tt):
        beta_bc_t_tt = math.log( (30./tt_adj + 0.035)**2. * (t-tt) + 1.) # 5.
 ↪1-66
    else:
        beta_bc_t_tt = 0.

    phi_bc = beta_bc_fcm * beta_bc_t_tt # 5.1-64

    return phi_bc
```

[48]:
```python
def compute_creep_coeff_drying(t, tt):

    # adjusted age reflecting the cement type but (here) without temperature␣
 ↪effect
    tt_adj = compute_adjusted_age(tt)
    #--------------------
    beta_dc_fcm = 412. * fcm_28**(-1.4) # 5.1-68
    #--------------------
    beta_RH = (1. - RH/100.) * (0.1 * h/100.)**(-1./3.) # 5.1-69
    #--------------------
    beta_dc_tt = 1. / (0.1 + tt_adj**0.2) # 5.1-70
    #--------------------
    gamma_tt = 1. / ( 2.3 + ( 3.5 / math.sqrt(tt_adj) ) ) # 5.1-71b

    alpha_fcm = math.sqrt(35./fcm_28) # 5.1-71d

    beta_h = min(1.5 * h + 250.*alpha_fcm, 1500 * alpha_fcm) # 5.1-71c

    beta_dc_t_tt = ( (t-tt) / ( beta_h + (t-tt) ) )**gamma_tt # 5.1-71a
```

```
    #--------------------
    phi_dc = beta_dc_fcm * beta_RH * beta_dc_tt * beta_dc_t_tt # 5.1-67

    return phi_dc
```

[49]:
```
def compute_adjusted_age(tt):
    # without temperature effect
    tt_adj = max( tt * ( 9./ ( 2. + tt**1.2 ) + 1. )**alpha, 0.5) # 5.1-73

    return tt_adj
```

3.3 Compliance and Relaxation Functions

[50]:
```
def compute_total_compliance_function(t, tt):

    if (t < tt):
        J = 0.
    else:
        E_cm_t = compute_modulus_of_elasticity(tt)
        E_cm_28 = compute_modulus_of_elasticity(28.)

        # 5.1-63
        # sum of basic and drying creep coefficient
        phi = compute_creep_coeff(t,tt)

        # 5.1.61
        J = 1./E_cm_t + phi/E_cm_28

    return J

def compute_basic_compliance_function(t, tt):

    if (t < tt):
        J = 0.
    else:
        E_cm_t = compute_modulus_of_elasticity(tt)
        E_cm_28 = compute_modulus_of_elasticity(28.)

        # 5.1-63
        phi = compute_creep_coeff_basic(t, tt)

        # 5.1.61
        J = 1./E_cm_t + phi/E_cm_28

    return J
```

```
[51]: # use this function only to evaluate relaxation function at particular time
def compute_total_relaxation_function(t, tt):

    if (t < tt):
        R = 0.

    else:
        R_tot = np.zeros(t_div)

        # create array with loading durations [0., 0., logscale with t_div-2␣
↪entries]
        times = np.logspace( round(math.log10(t_aux_min))-1, ( math.log10(t-tt)␣
↪), num = t_div-2 )
        times = np.insert( times, 0, [0., 0.])

        times += tt # duration of loading changed to age

        # zero loading duration, times[1] = times[0]
        R_tot[1] = 1. / compute_total_compliance_function(times[1], times[0])
        # first non-zero loading duration
        R_tot[2] = 1. / compute_total_compliance_function(times[2], times[0])

        for k in range(2,t_div-1):

            sum = 0.
            for i in range(0,k-1):
                J_kplus1_i = compute_total_compliance_function( times[k+1],␣
↪(times[i+1]+times[i])/2. )
                J_k_i = compute_total_compliance_function( times[k],␣
↪(times[i+1]+times[i])/2. )
                delta_J_k_i = J_kplus1_i - J_k_i
                sum += delta_J_k_i * ( R_tot[i+1] - R_tot[i] )

            J_kplus1_k = compute_total_compliance_function( times[k+1],␣
↪(times[k+1]+times[k])/2. )
            R_tot[k+1] = R_tot[k] - sum / J_kplus1_k

        # return the last value from the array which corresponds to the desired␣
↪time "t"
        R = R_tot[-1]

    return R
```

3.4 Mean Total Shrinkage

5.1.9.4.4: characteristic values corresponding to 5 and 95 percentile are 0.42 eps_cs and 1.58 eps_cs, respectively

[52]:
```python
def compute_mean_total_shrinkage(t):

    eps_cas = compute_mean_auto_shrinkage(t)
    eps_cds = compute_mean_drying_shrinkage(t)
    eps_cs = eps_cas + eps_cds # 5.1-75

    return eps_cs
```

3.4.1 Mean Autogenous Shrinkage

[53]:
```python
def compute_mean_auto_shrinkage(t):

    if (t <= 0.):
        eps_cas = 0.
    else:
        # 5.1-78, notional autogenous shrinkage coefficient
        eps_cas_fcm = -alpha_as * ( fcm_28 / ( 60. + fcm_28 ) )**2.5 * 1.e-6
        # 5.1-79, development of autogenous shrinkage
        beta_as_t = 1. - math.exp( -0.2 * math.sqrt(t) )

        eps_cas = eps_cas_fcm * beta_as_t # 5.1-76

    return eps_cas
```

3.4.2 Mean Drying Shrinkage

[54]:
```python
def compute_mean_drying_shrinkage(t):

    t_dry = t-t0

    if (t_dry <= 0.):
        eps_cds = 0.

    else:
        # 5.1-80, notional drying shrinkage coefficient
        eps_cds_fcm = (220. + 110 * alpha_ds1) * math.exp(-alpha_ds2 * fcm_28)␣
↪* 1.e-6

        # 5.1-83
        beta_s1 = min( (35./fcm_28)**0.1, 1.0 )

        # 5.1-81 effect of relative humidity on drying shrinkage
        if (RH < 99. * beta_s1):
            beta_RH = -1.55 * (1. - (RH/100.)**3 )
        else:
```

```
            beta_RH =  0.25

        # 5.1-82, development of drying shrinkage
        beta_DS_t_dry = math.sqrt( t_dry / (0.035 * h**2  + t_dry) )

        eps_cds = eps_cds_fcm * beta_RH * beta_DS_t_dry # 5.1-77

    return eps_cds
```

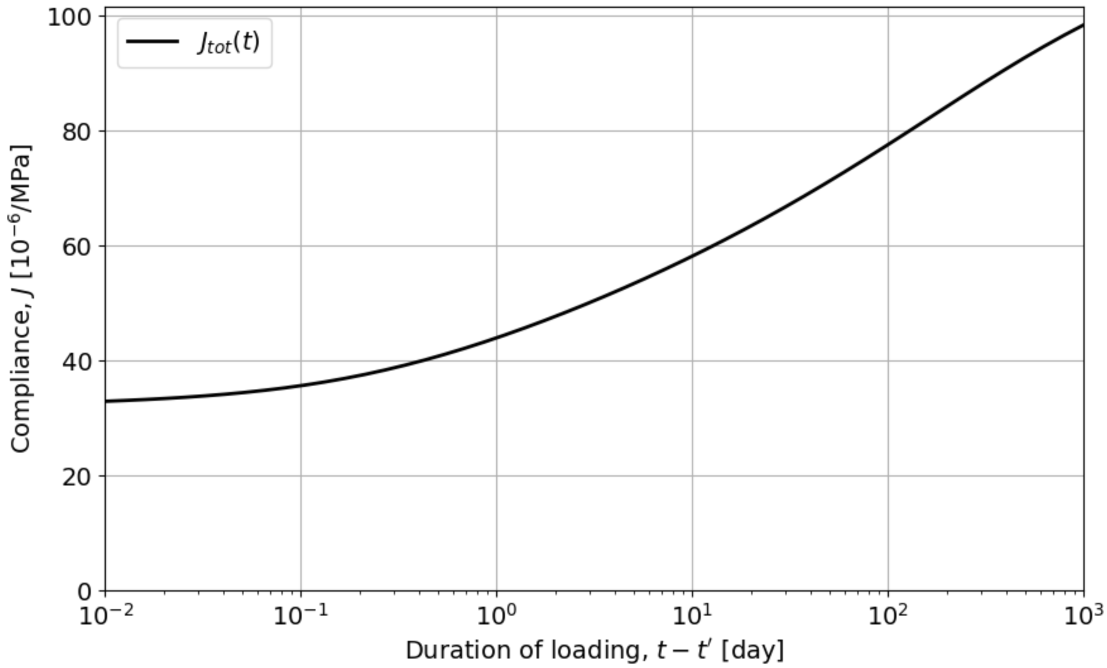
4 Plotting

4.1 Plot of a compliance function

[55]:

```
# prepare data
# times here is the duration of loading
times = np.logspace( round(math.log10(t_aux_min)), round(math.
 ↪log10(t_aux_max)), num = t_div )

J_tot = np.zeros(t_div)

for i in range(t_div):
    # here t is shifted by the onset of loading
    t = times[i] + tt
    J_tot[i] =  compute_total_compliance_function(t, tt)


plt.figure(figsize=(10,6))
plt.rcParams.update({'font.size': 14})

plt.plot(times, J_tot * 1.e6, lw=2., color="black", label=r'$J_{tot}(t)$')


if (log_scale):
    plt.xscale('log')
    plt.xlim([t_aux_min, t_aux_max])
else:
    plt.xlim([0., t_aux_max])

# keep top range unbounded
plt.ylim(bottom = 0)

plt.grid(True)
plt.legend()
plt.xlabel('Duration of loading, $t-t\'$ [day]')
plt.ylabel('Compliance, $J$ [$10^{-6}$/MPa]')
```

```
plt.savefig('fib_MC2010_compliance_function.pdf')

plt.show()
```

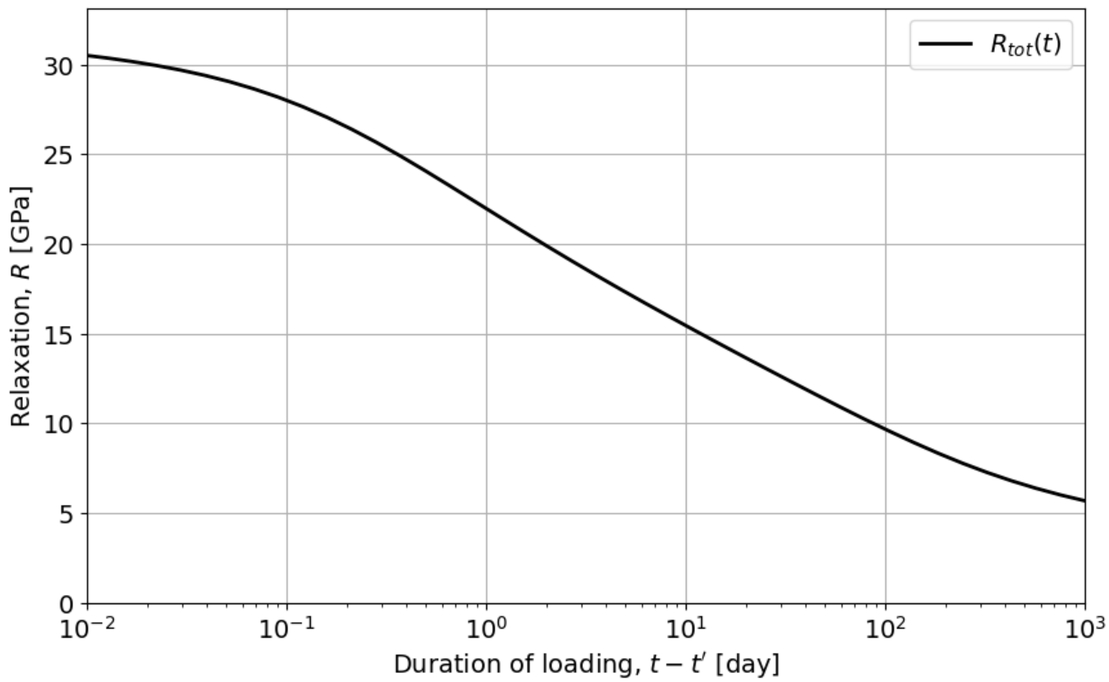


4.2 Plot of a relaxation function

[56]:
```
def evaluate_relaxation_function_at_times(ages):

    aux = ages[0]
    ages = np.insert( ages, 0, [aux, aux])

    R_tot = np.zeros(len(ages))
    # zero loading duration, times[1] = times[0]
    R_tot[1] = 1. / compute_total_compliance_function(ages[1], ages[0])
    # first non-zero loading duration
    R_tot[2] = 1. / compute_total_compliance_function(ages[2], ages[0])

    for k in range(2,len(ages)-1):

        sum = 0.
        for i in range(0,k-1):
            J_kplus1_i = compute_total_compliance_function( ages[k+1],␣
↪(ages[i+1]+ages[i])/2. )
            J_k_i = compute_total_compliance_function( ages[k],␣
↪(ages[i+1]+ages[i])/2. )
```

```
            delta_J_k_i = J_kplus1_i - J_k_i
            sum += delta_J_k_i * ( R_tot[i+1] - R_tot[i] )

        J_kplus1_k = compute_total_compliance_function( ages[k+1],␣
 ↪(ages[k+1]+ages[k])/2. )
        R_tot[k+1] = R_tot[k] - sum / J_kplus1_k

    return R_tot[2:]
```

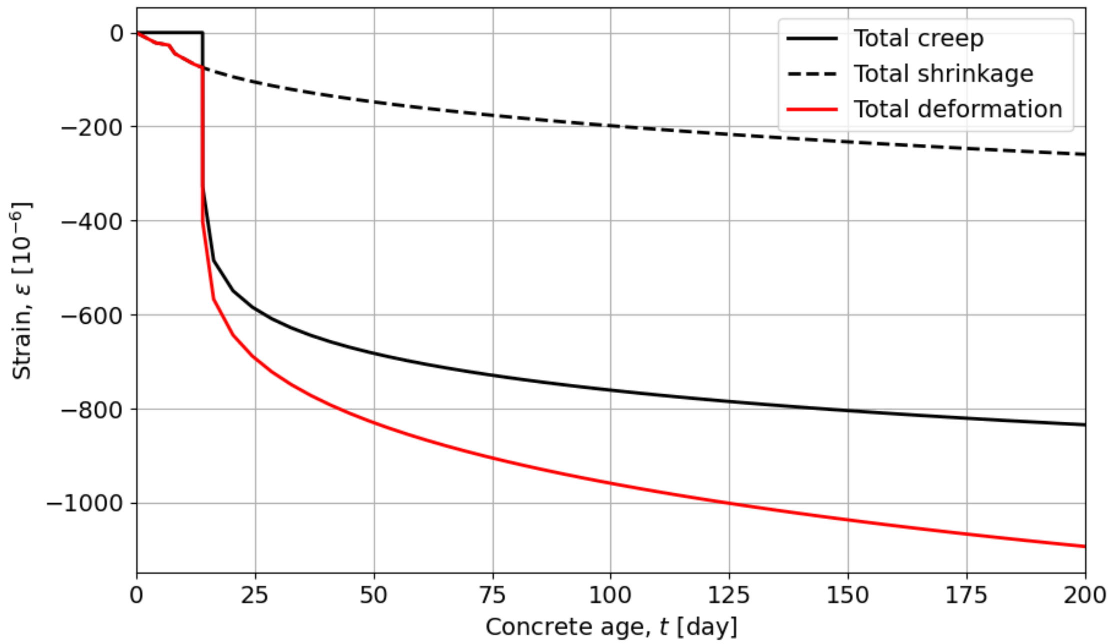
[57]:
```
# create array with loading durations [0., 0., logscale with t_div-2 entries]
times = np.logspace( round(math.log10(t_aux_min)-1), round(math.
 ↪log10(t_aux_max)), num = t_div)
ages = times + tt # duration of loading changed to age

R_tot = evaluate_relaxation_function_at_times(ages)

plt.figure(figsize=(10,6))
plt.rcParams.update({'font.size': 14})

plt.plot(times, R_tot * 1.e-3, lw=2., color="black", label=r'$R_{tot}(t)$')



if (log_scale):
    plt.xscale('log')
    plt.xlim([t_aux_min, t_aux_max])
else:
    plt.xlim([0., t_aux_max])

# keep top range unbounded
plt.ylim(bottom = 0)

plt.grid(True)
plt.legend()
plt.xlabel('Duration of loading, $t-t\'$ [day]')
plt.ylabel('Relaxation, $R$ [GPa]')

plt.savefig('fib_MC2010_relaxation_function.pdf')

plt.show()
```



4.3 Plot of total deformation with strain decomposition

[58]:
```
times = np.linspace(0, 200, num = t_div )

times = np.append(times, [tt-t_aux_min, tt, tt+t_aux_min])
times = np.append(times, [t0+t_aux_min, t0, t0+t_aux_min])
# eliminates redundancy and sorts
times = np.unique(times)

# local variable redifinition only
len_times = len(times)


creep_tot = np.zeros(len_times)

shr_tot = np.zeros(len_times)

def_tot = np.zeros(len_times)


for i in range(len_times):
    # here t is shifted by the onset of loading
    t = times[i]
```

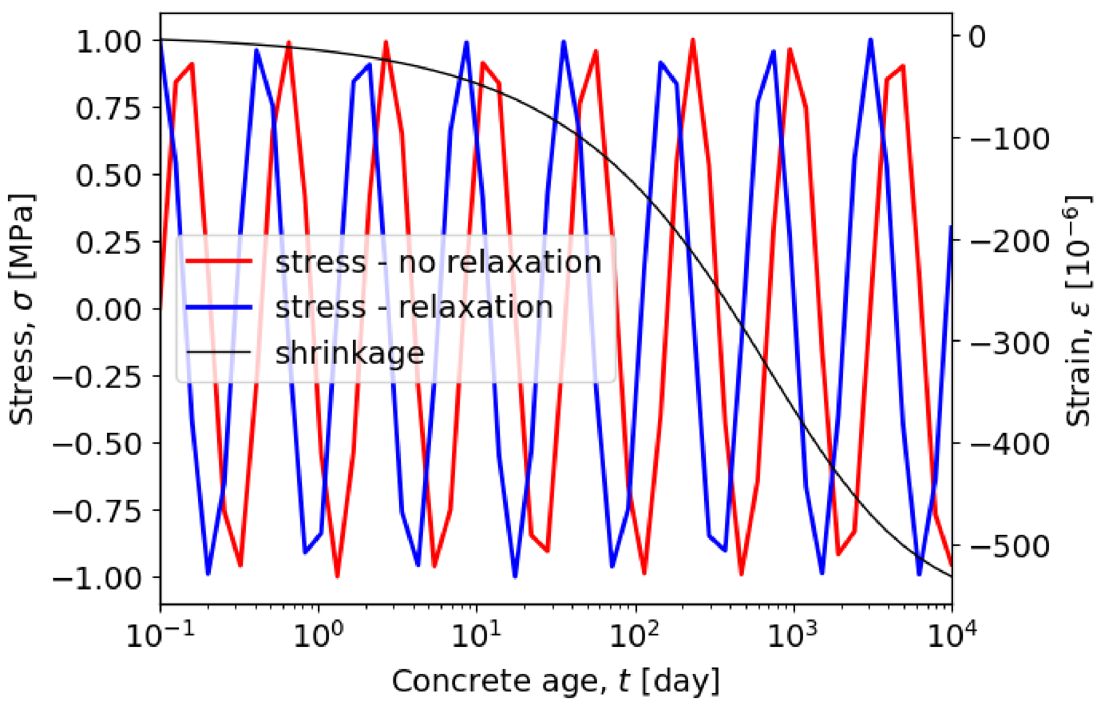
```
    shr_tot[i] = compute_mean_total_shrinkage(t) * 1.e6

    if (t >= tt):

        creep_tot[i] =  stress * compute_total_compliance_function(t, tt) * -1.
 ↪e6

    def_tot[i] = creep_tot[i] + shr_tot[i]


plt.figure(figsize=(10,6))
plt.rcParams.update({'font.size': 14})

plt.plot(times, creep_tot, lw=2., color="black", label=r"Total creep")
plt.plot(times, shr_tot, lw=2., color="black", label=r"Total shrinkage",␣
 ↪linestyle='dashed')
plt.plot(times, def_tot, lw=2., color="red", label=r"Total deformation")

plt.xlim([0., 200])

# keep top range unbounded
#plt.ylim(bottom = 0)

plt.grid(True)
plt.legend()
plt.xlabel('Concrete age, $t$ [day]')
plt.ylabel('Strain, $\epsilon$ [$10^{-6}$]')

plt.savefig('fib_MC2010_evolution_of_total_strain.pdf')

plt.show()
```



4.4 Response to a prescribed shrinkage

[61]:
```
t_div_numer = 50

times_numer = np.logspace(-1, 4, num = t_div_numer )
times_numer += t0

shr = np.zeros(t_div_numer)

for i, t in enumerate(times_numer):
    shr[i] = compute_mean_drying_shrinkage(t)

sig_tot_relax = np.zeros(t_div_numer)
sig_tot_no_relax = np.zeros(t_div_numer)

for i in range(t_div_numer):

    sig_tot_relax[i] = math.cos( i )
    sig_tot_no_relax[i] = math.sin( i )


fig, ax1 = plt.subplots()
plt.rcParams.update({'font.size': 14})

ax2 = ax1.twinx()
```

```
lns1 = ax1.plot(times_numer-t0, sig_tot_no_relax, lw=2., color="red",␣
    ↪label=r"stress - no relaxation")
lns2 = ax1.plot(times_numer-t0, sig_tot_relax, lw=2., color="blue",␣
    ↪label=r"stress - relaxation")
lns3 = ax2.plot(times_numer-t0, shr*1e6, lw=1., color="black",␣
    ↪label=r"shrinkage")

lns = lns1+lns2+lns3
labs = [l.get_label() for l in lns]
ax1.legend(lns, labs, loc=0)

ax1.set_xlim([0.1, 1e4])

ax1.set_xlabel('Concrete age, $t$ [day]')
ax1.set_ylabel('Stress, $\sigma$ [MPa]')
ax2.set_ylabel('Strain, $\epsilon$ [$10^{-6}$]')

plt.xscale('log')

plt.savefig('fib_MC2010_evolution_of_shrinkage-induced_stress.pdf')

plt.show()
```



[ ]:

```

```

[ ]:

```

```