5 Inertial Sensors

Inertial sensors make measurements of the internal state of the vehicle. A major advantage of inertial sensors is that they are non-radiating and non-jammable and may be packaged and sealed from the environment. This makes them potentially robust in harsh environmental conditions. Historically, Inertial Navigation Systems (INS) have been used in aerospace vehicles, military applications such as ships, submarines, missiles, and to a much lesser extent, in land vehicle applications. Only a few years ago the application of inertial sensing was limited to high performance high cost aerospace and military applications. However, motivated by requirements for the automotive industry, a whole variety of low cost inertial systems have now become available in diverse applications such as heading and attitude determination. The most common type of inertial sensors are accelerometers and gyroscopes. Accelerometers measure acceleration with respect to an inertial reference frame. This includes gravitational and rotational acceleration as well as linear acceleration. Gyroscopes measure the rate of rotation independent of the coordinate frame. The most common application of inertial sensors is in the use of a heading gyro. Integration of the gyro rate information provides the orientation of the vehicle. Another application of inertial sensors is the use of accelerometers as inclinometers and to perform vibration analysis. The most sophisticated use of inertial sensor is in the full six degree if freedom navigation systems. In this case at least three gyros are used to track the orientation of the platform and a minimum or three accelerometers are integrated to provide velocity and position. They can also provide 3-D position information and, unlike encoders, have the potential of observing wheel slip. This section presents different type of inertial sensors and the fundamental principles inertial navigation.

5.1 Fundamental Principles of Accelerometers and Gyroscopes

5.1.1 Accelerometers

Accelerometers measure the inertia force generated when a mass is affected by change in velocity. This force may change the tension of a string or cause a deflection of beam or may even change the vibrating frequency of a mass. The accelerometers are composed of three main elements: a mass, a suspension mechanism that positions the mass and a sensing element that returns an observation proportional to the acceleration of the mass. Some devices include an additional servo loop that generates an apposite force to improve the linearity of the sensor. A basic one-degree of freedom accelerometer is shown in Figure 14. This accelerometer is usually referred to as an open loop since the acceleration is indicated by the displacement of the mass.

The accelerometer described in this example can be modelled with a second order equation:

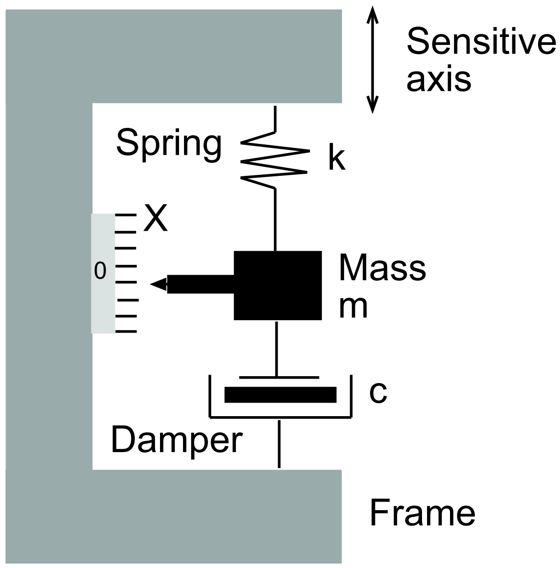

Figure 14: Basic Components of an open loop accelerometer

$$F = m\frac{d^2x}{d^2t} + c\frac{dx}{dt} + kx \tag{88}$$

where F is the applied force to be measured. The damping can be adjusted to obtain fast responses without oscillatory behaviour. Many of the actual commercial accelerometers are based on the pendulum principle. They are built with a proof mass, a spring hinge and a sensing device. These accelerometers are usually constructed with a feedback loop to constrain the movement of the mass and avoid cross coupling accelerations. There are accelerometers that implement variation of this principle such as the Sundstrand "Q-Flex" design and the Analog Device silicon micro-machined accelerometer. One important specification of the accelerometers is the minimum acceleration that can be measured. This is of fundamental importance when working in land vehicle applications, where the acceleration expected is usually in the range of 0.1-0.3 g. Figure 15 shows the acceleration measured when travelling with a car at low speed. A standard accelerometer for such applications is usually capable of measuring acceleration of less than 500 μg. The dependence of the bias with temperature and the linearity of the device are also important specifications.

5.1.2 Gyroscopes

These devices return a signal proportional to the rotational velocity. There is a large variety of gyroscopes that are based on different principles. The price and quality of these

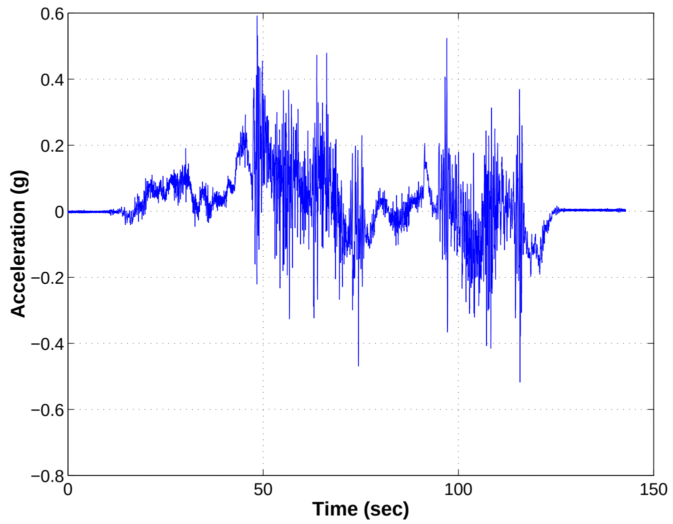

Figure 15: Typical acceleration in land vehicle applications

sensors varies significantly. The following sections present some of the most common types of gyroscopes available for industrial applications.

5.1.3 Vibratory gyroscopes

These types of gyroscopes can be manufactured in very different forms but they are all based on obtaining rotation rate by measuring coriolis acceleration. The device can be modelled by a simple mass-spring system as shown in the Figure 16. The purpose of the gyroscope is to measure the angular velocity of the particle that is supposed to be rotating about the axis OZ.

In this approach the particle is made to vibrate with constant amplitude along the axis OX. This motion is usually referred to as primary motion and is controlled by an embedded circuit that maintains the oscillation at constant amplitude. Under rotation the mass will experience a coriolis inertia force that will be proportional to the applied rate of turn and will have a direction parallel to the OY axis. This motion is referred to as secondary motion and its amplitude is measured to provide information proportional to the angular rotation. Some devices provide an additional feedback mechanism for this secondary motion to increase the linearity of the sensor. The Performance of gyroscopes is mainly characterized by two parameters: scale factor and drift. A gyro with low distortion in the scale factor will be able to accurately sense angular rates at different angular velocities. Various system imperfections and as well as environmental conditions can cause significant variations in the scale factor. Gyro drift is the nominal output of the gyro in steady state.

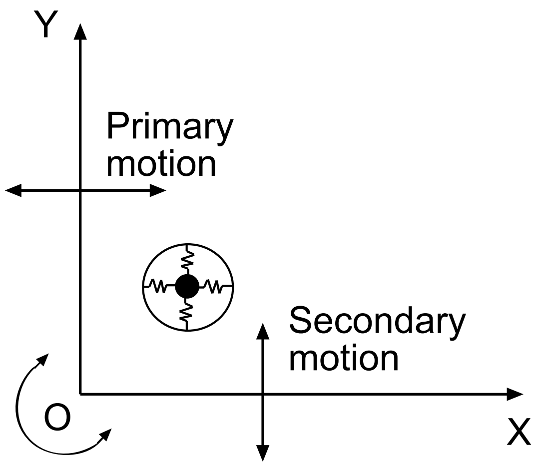

Figure 16: Mass spring model

This non-zero output is usually temperature dependent.

Murata manufactures a gyroscope model Gyrostar ENV05 that is based on this principle. This device presents very good high frequency characteristics but poor low frequency performance. The drift expected is in the order of 1 degree / minute. Noise and significant bias temperature dependence also affect the performance of this particular sensor. Better performance can be expected from the Vibrating Structure Gyroscopes from BAE and the quartz rate sensors build by Systron Donner. These devices have a drift of less that 0.15 degree/minute and the linearity is better than 0.05 % of the full range.

5.1.4 Fiber Optic Gyros

The fiber optic gyros are based on the Sagnac effect discovered by Georges Sagnac in 1913. This effect can be easily explained assuming two waves of light circulating in opposite direction around a path of radius R. If the source is rotating at speed ω, the light traveling in the opposite direction will reach the source sooner than the wave traveling in the same direction, as shown in Figure 17. The wave traveling with the rotation will covers a distance D_1 in a transit time t_1, while the other signal covers a distance t_2 in time D_2

$$\begin{aligned} D_1 &= 2\pi R - R\omega t_1 \\ D_2 &= 2\pi R - R\omega t_2 \end{aligned} \tag{89}$$

Making the waves travel the path N times, the difference in transit time becomes:

$$\Delta t = N(t_2 - t_1) = \frac{4\pi N R^2 \omega}{c^2} \tag{90}$$

It is important to relate this time difference with a phase shift at a particular frequency.

$$\phi = 2\pi \Delta t f \tag{91}$$

For a given rotation ω the phase shift will be

$$\Delta\phi = \frac{8\pi^2 RN}{c}\omega \tag{92}$$

Most low cost implementations of these devices works in an open loop manner. The maximum phase shift that can be evaluated without ambiguities is 90 degrees.

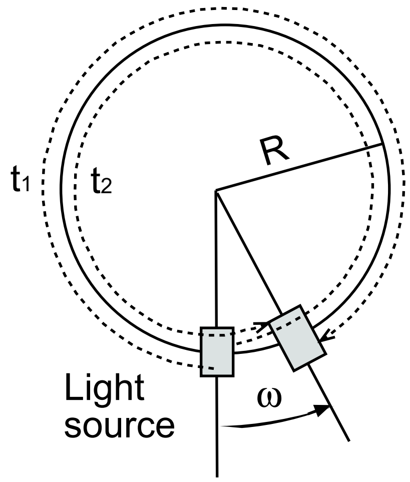

Figure 17: Transmission time difference

There are commercial laser gyros such as the KVH model Ecore 2000 and 1000, which are capable of drifts as low as 0.036 and degree 0.12 per minute respectively. Closed loop optical gyros are also available but they are more expensive

5.2 Sensor Errors

The measurement errors associated with inertial sensors are dependent on the physical operational principle of the sensor itself. The general error equation used for gyroscopic performance along, say, the x direction, is written as

$$\delta\omega_x = b + b_g \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} + s_f\omega_x + m_y\omega_y + m_z\omega_z + \eta, \tag{93}$$

and that of the accelerometers as

$$\delta f_x \;=\; b + s_f a_x + m_y a_y + m_z a_z + \eta, \tag{94}$$

where

- b is the residual biases
- b_g is a 1×3 vector representing the g-dependent bias coefficients
- s_f is the scale factor term
- m represents mounting and internal misalignments and
- η is random noise on the sensor signal

Other terms such as anisoelastic and anisoinertia errors mainly affect mechanical gyros while vibro-pendulous errors affect mechanical accelerometers, and hence will not be considered here.
Apart from the random noise term η, all other error terms are, in principle, predictable thus allowing for some form of compensation.

5.2.1 Evaluation of the Error Components

This section discusses tests which can be performed to gyros in order to systematically remove the errors. Similar tests can be conducted on the accelerometer. It should be noted that temperature and memory effect play a significant role in the stability of the output of low cost inertial sensors. It is for this reason that when one purchases low cost inertial units, not all the values for the error terms are available, and so testing is required based on the application at hand.
If the gyro is left stationary then the only error term encountered is that of the g-independent bias. One of strongest correlations that can be found in inertial sensors is that between the g-independent bias and temperature, also known as in-run bias. Thus by cycling through the temperature that would be encountered in the target application the bias on the gyro can be determined. The better the gyro, the smaller the bias variation over the temperature range. Furthermore, the better the sensor the greater the linearity of this bias variation.
There is also a hysterisis effect encountered with most inertial sensors. Thus in an ensemble of tests, cycling through the temperature in the same manner each time, the variation in the bias between ensembles can be determined. This is known as the switch-on to switch-on bias. Again the better the gyro, the better the switch-on to switch-on bias.
When testing for the remaining error components, the g-independent bias is assumed

known and is removed.
Start with a rate table and place the gyro such that its supposed sensitive axis is orthogonal to input rotation vector, then any output signal will be due to internal misalignment of the sensor's input axis. The purpose of mounting the sensor orthogonal to the input rotation is to deal with a small error about null as opposed to a small error about a large value (as would be encountered if the sensitive axis was parallel to the rotation input).
With the misalignment and g-independent bias available, the gyro is placed on the rate table with its sensitive axis parallel to rotation input. The rate table is cycled from stationary to the maximum rotation measurable by the gyro in steps, holding the rotation rate at particular points. The scale factor and the scale factor linearity can then be determined by comparing the output of the gyro to the rotation rate input. With low cost gyros the temperature also has a part to play with scale factor stability, thus the tests should also be conducted with varying temperature.
Finally the gyro can be placed in a centrifuge machine which applies a rotation rate and acceleration to the gyro. The output reading from the gyro minus the previous terms calibrated, results in the determination of bias due to acceleration or g-dependent bias.
By mathematically integrating Equations 93 and 94, the effect of each error term on the performance of the gyro can be determined. For each error term that is accounted for there is a corresponding increase in performance.
Table 5 compares the error values available for the RLG, FOG, ceramic and silicon VSGs

Characteristic	RLG	FOG	Ceramic VSG	Si VSG
g-independent bias $^o/hr$	0.001-10	0.5-50	360-1800	> 2000
g-dependent bias $^o/hr/g$	0	< 1	36-180	36-180
scale factor non-linearity %	0.2 - 0.3	0.05 - 0.5	5 - 100	5 - 100

Table 5: Comparison of some of the major errors with various gyro implementations

5.2.2 Faults associated with Inertial Sensors

The accelerometers measure the absolute acceleration with respect to an inertial frame. We are interested in the translational acceleration hence the algorithms used should compensate for all other accelerations. For practical purposes it can be considered that gravity is the only other acceleration present. In Figure 15 it can be seen that the average acceleration obtained from a land vehicle is smaller that 0.3 g. This implies that the orientation of the accelerometer has to be known with very good accuracy in order to compensate for gravity without introducing errors comparable to the actual translation acceleration.

This has been the main limitation that has prevented the widespread use of inertial sensors as position sensors in low cost land navigation applications. The orientation of the accelerometer can be tracked using a gyroscope. These sensors provide an output proportional to the rotation speed but are always contaminated by noise and drift. For short periods of time the drift can be approximated by a constant bias. The orientation is evaluated with a single integration of the gyroscope output:

$$\theta_m = \int \dot{\theta} + b + \nu \, dt = \theta + bt + \int \nu \, dt \tag{95}$$

The integration given in equation 95 returns the rotation angle with two additional undesired terms. A random walk due to the noise and another term that grows with time proportional to the gyro bias. As shown in figure 18, the gyro drift will introduce an error

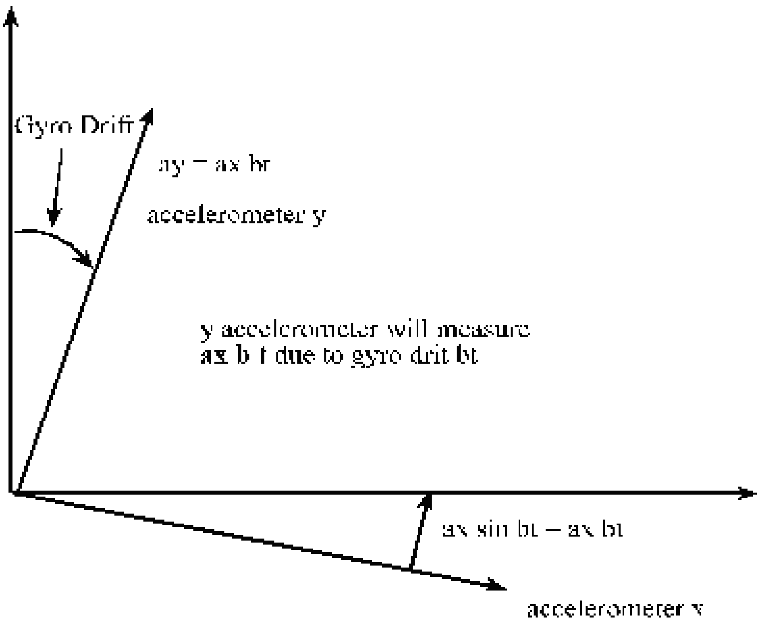

Figure 18: Errors due to drift in the gyro

in acceleration given by

$$a = a_x \sin(bt) \tag{96}$$

For small angle increments the errors in acceleration, velocity and position are given by:

$$a = a_x b t \tag{97}$$
$$v = \frac{1}{2} a_x b t^2 \tag{98}$$
$$p = \frac{1}{6} a_x b t^3 \tag{99}$$

It is important to remark that a constant gyro bias introduces an error in position determination that grows proportional to t^3. For standard low cost, good quality gyros the bias expected is in the order of 1-10 degrees / hour. Without calibration, the bias could introduce an error of approximately 140 meters due to the incorrect compensation of gravity after only 2 minutes of operation. Another aspect that needs to be considered is the resolution of the analog to digital converter used, especially when working with low drift gyroscopes. The original noise standard deviation of the gyro noise could be increased selecting an inappropriate digital to analog converter. A gyro with stability of the order of 1 degree per hour that works in a range of +/-100 deg/sec and will require a 16-bit converter to operate under its specifications. The bias in the accelerometer will also increase the error in position and is proportional to the square of time:

$$v = b_a t \tag{100}$$
$$p = \frac{1}{2} b_a t^2 \tag{101}$$

The error introduced by the accelerometer is important enough to be neglected. Figure 19 presents the bias measured in a set of identical accelerometers corresponding to an inertial unit over a period of six hours whilst they were stationary. Evidently the biases do not remain constant and in fact it can be clearly seen that any one accelerometer is not indicative of the general performance of the other accelerometers. The different biases values indicates that they are physically different low cost inertial sensors. The change in the value occur due to an increase of the temperature of the unit due to ambient temperature variations. The estimation of the biases needs to be done each time the vehicle is stationary to minimize this effect. The calibration procedure must incorporate the identification of gyro and accelerometer biases at the beginning of the navigation task and become active each time the vehicle stops. These identification of the biases can also be done on-line but information from other sensors is required. A second mayor error is due to the integration of random walk ($\int \nu \, dt$) which causes a growing error term known as random walk. Figure 20 presents the effects if integrating a gyro signal on several occasions after the bias was removed and the unit stationary. Although the average error is zero in any particular run the error grow will occur in a random direction. Assuming unity strength white Gaussian noise and letting $\mathbf{x}(k) = \int_0^t \nu(u)\delta(u)$,then the ensemble

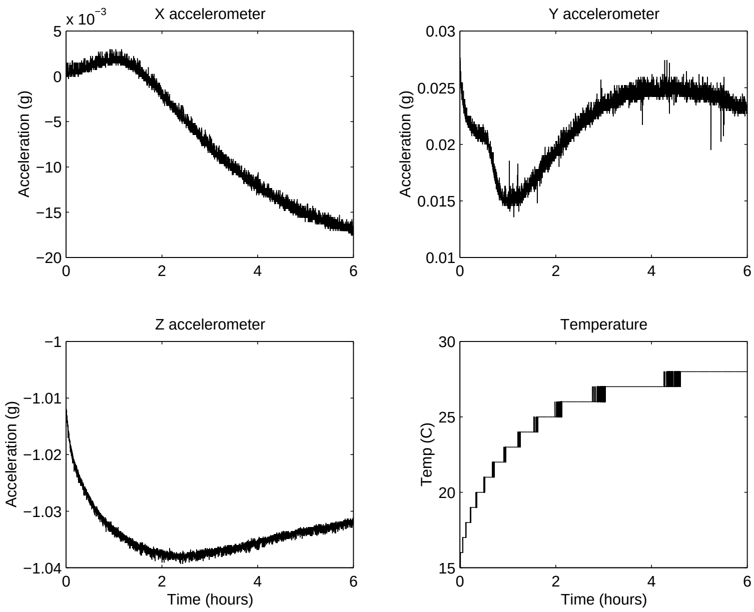

Figure 19: Bias change of the accelerometers over a period of 6 hours.

mean value is:

$$E[\int_0^t \nu(u)\delta u] = \int_0^t E[\nu(u)]\delta u \tag{102}$$

and the variance is

$$E[x^2(t)] = E[\int_0^t \nu(u)\delta u \int_0^t \nu(v)\delta v] = \int_0^t \int_0^t E[\nu(u)\nu(v)]\delta u\delta v \tag{103}$$

$E[\nu(u)\nu(v)]$is the auto-correlation function, which in this case (assuming uncorrelated errors) is simply a Delta Dirac function $\delta(u-v)$. Hence Equation 103 becomes:

$$E[x^2(t)] = \int_0^t \int_0^t \delta(u-v)\delta u\delta v = t \tag{104}$$

The standard deviation of the noise is:

$$\sigma_\nu = \sqrt{t} \tag{105}$$

Therefore without any aiding the white noise will cause an unbounded error growth in the inertial sensors whose value at any particular point will be bounded by $\pm 2\sqrt{t}$ 95 % of the time. For white noise of strength K the standard deviation is $\sigma_\nu = K\sqrt{t}$. The larger the standard deviation of the noise the greater the standard deviation of the error. This is an important specification of the system, since a good gyro can be degraded with

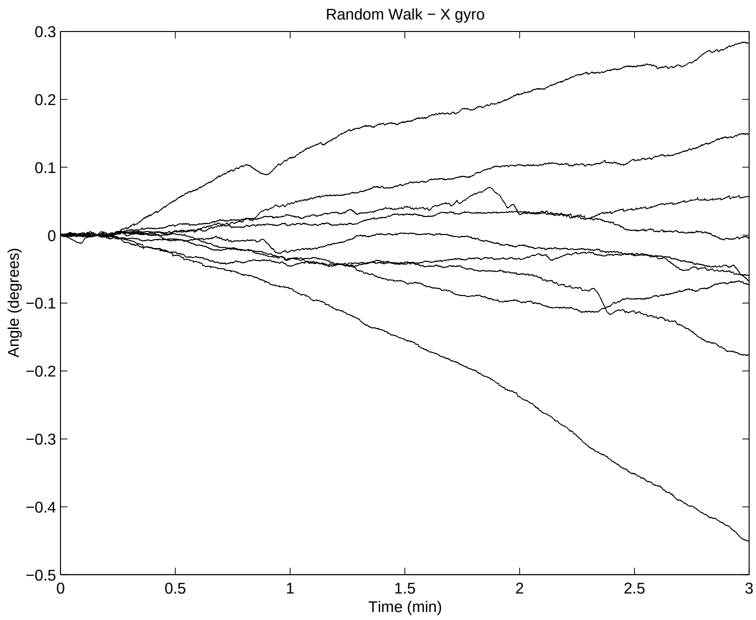

Figure 20: Typical acceleration in land vehicle applications

inappropriate signal conditioning and electronic.

5.3 Application Inertial Sensors

Inclinometers: These sensors are based on accelerometers that resolve the direction of the gravity vector. The gravity vector is a known quantity that is constant in magnitude and direction. Figure 21 presents a basic example of a tilt sensor. These sensors are usually calibrated through a look-up table to determine the offset as a function of temperature. There are tilt sensors available for this purpose that can provide accurate information while the vehicle is stationary. Low cost devices can provide bank and elevation information with an accuracy of up to 0.1 degrees. These sensors can not be used when the platform is moving since they are sensitive to translational and rotational accelerations. Pendulum accelerometers are also used to determine the bank and elevation angles. These devices use the principle that a pendulum with an equivalent radius of the earth will always point to the vertical irrespective of the acceleration. This effect can be approximated with appropriate feedback loop compensation. Although they are able to obtain some rejection of the undesired acceleration due to the movement of the platform they can only be used under very low vehicle accelerations.

Vibration: Vibration Analysis consist of measuring the frequency, strength over a range of frequencies of vibrations. Machinery vibration problems can consume excessive power and impose additional wear on bearings, seals, couplings and foundations. Vibrations

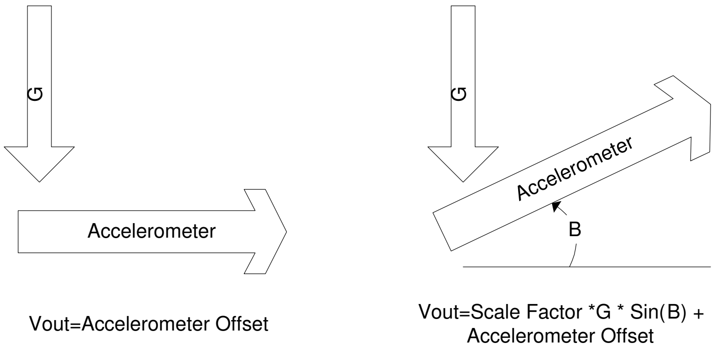

Figure 21: Basic principle of inclinometers

are typically caused by machinery misalignment and unbalance. These problems can be detected by obtaining the frequency response of the vibration of the machine under investigation. This is commonly done via analysis of the FFT of an acceleration signal. There are other application where vibration monitoring may be useful such as the analysis and identification of vibration sources, problems in structures, vibration and shock testing, analysis to ensure products comply with specified vibration required for the application etc.

Heading: Another application of inertial sensors is in the use of a heading gyro. Integration of the gyro rate information provides the orientation of the vehicle. A good quality gyro will have zero or constant bias, and small noise variance. There are many types of gyroscopes but few in the price range that can be afforded in commercial mobile robot applications. Fiber optic and vibratory gyroscopes have been released with satisfactory specifications and reasonable price ranges for many industrial applications

Inertial Measurement Unit (IMU) A full inertial navigation system (INS) consists of at least three (triaxial) accelerometers and three orthogonal gyroscopes that provide measurements of acceleration in three dimensions and rotation rates about three axes. The Physical implementation of inertial sensors can take on two forms:

- Gimballed arrangement: This unit consists of a platform suspended by a gymbal structure that allows three degree of freedom, as shown in Figure 22. This device is usually attached to a vehicle through the outermost gymbal. The vehicle can then perform any trajectory in 3-D while the platform is maintained level with respect to a desired navigation frame using the feedback control loops. These loops use the gyro signal to provide appropriate control singnal to the actuators places in each gymbal. With this approach the platform can be maintained aligned with respect to a coordinate system. The information provided by the accelerometers can then be integrated directly to obtain position and velocities.

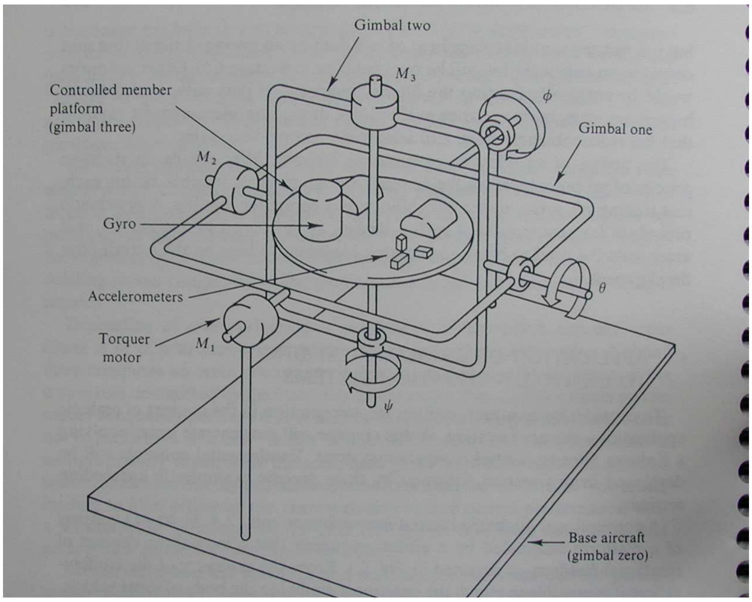

Figure 22: Gymbaled Six Degree of Freedom INS. Each axis has a control loop to maintain the axis at a particular orientation with respect to a coordinate frame. The gyro provides the feedback information to control the actuators of each gymbal

- Strapdown arrangement: The IMU system assembled from low cost solid-state components is almost always constructed in a "strap-down" configuration. This term means that all of the gyros and accelerometers are fixed to a common chassis and are not actively controlled on gimbals to align themselves in a pre-specified direction. As shown in Figure 23, a gyro and accelerometer is placed in each axis. This design has the advantage of eliminating all moving parts. The strapdown construction, however, means that substantially more complex software is required to compute and distinguish true linear acceleration from angular acceleration and body roll or pitch with respect to gravity. Once true linear acceleration has been determined, vehicle position may be obtained, in principle, by double integration of the acceleration. Vehicle orientation and attitude may also, in principle, be determined by integration of the rotation rates of the gyros. In practice this integration leads to unbounded growth in position errors with time due to the noise associated with the measurement and the non-linearity of the sensors.

5.4 Coordinate Frames

Inertial Navigation systems require the transformation of the different measurement quantities between different frames of reference. The accelerometer and gyros measure

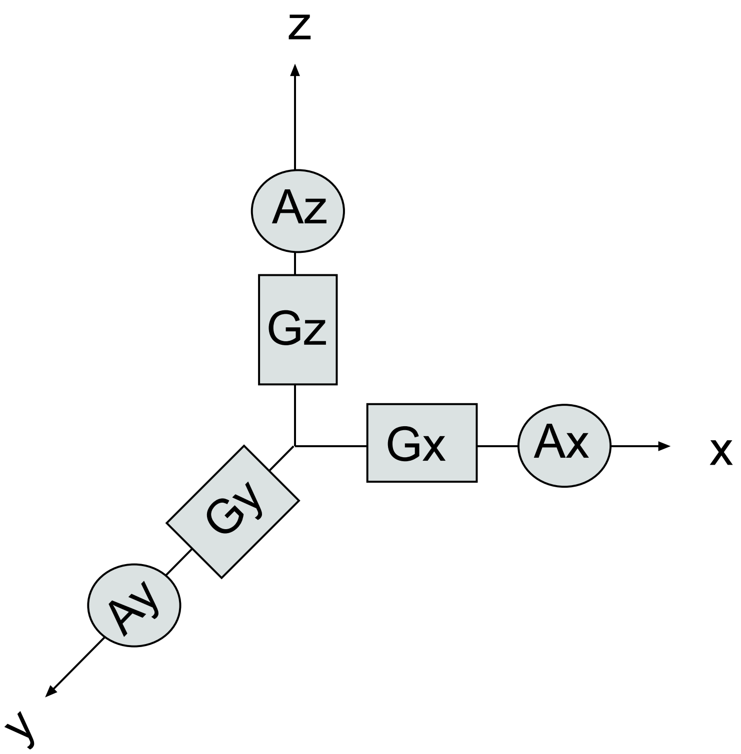

Figure 23: Strapdown Six degree of Freedom INS. Each axis has an accelerometer and a gyro. The gyro signal are used to determine the attitude of the unit and compensate the acceleration information for gravity.

acceleration and rotation rates with respect to an inertial frame. Other sensors such as Global Positioning System (GPS) report information in Earth Centered Earth Fixed (ECEF) coordinate frame. The navigation system needs to transform all the information to a common coordinate frame to provide the best estimate and sometime be able to report the output in various other coordinate frames. This section will introduce the different coordinate system used in this course.

Inertial Frame According to Newtown's law of motion, an inertial frame is a frame that does not rotate or accelerate. It is almost impossible to find a true inertial frame. A good approximation is the one that is inertial with a distant star. For practical purposes we define an inertial frame with origin at the earth center of mass, with x axis pointing towards the vernal equinox at time T_0, the z axis along the earth's spin axis and the y axis completing the rigth handed system.

Earth-centered Earth-Fixed (ECEF) This frame has origin at the mass centre of the Earth, the x axis points to the Greenwich Meridian in the equatorial plane, the y plane at 90 degree east in the same plane and the z axis in the direction of rotation of the reference ellipsoid. This frame is not an inertial frame. It can be approximated to an inertial frame by a negative rotation equivalent to the Greenwich mean Sidereal Time (GMST). The earth's geoid is a surface that minimize the difference with the earth's sea level. This geoid is approximated with an ellipsoid around the earth's minor axis. Different parameter are available for different areas of the world. The most common reference ellipsoid used is the WGS 84. This frame is also referred as Transport Frame.

Local Level Earth Frame This frame, also known as Geographic or Earth frame, is defined locally relative to the earth's geoid. The axis x points to the North's direction, the z axis is perpendicular to the tangent of the ellipsoid pointing towards the interior of the earth, not necessarily to the centre of the earth. Finally the y axis points east to complete a right handed orthogonal system

Wander Frame The Local Level frame presents some numerical problems to integrate the data from inertial system when close to the poles. In this areas significant rotation around the axis z are required to maintain the x axis pointing North. This is true even with small movement in the y direction. This problem can be solved performing all the computation in a frame that does not require to point to North. The x axis then wanders from North at a rate chosen by the user.

Geocentric frame This frame is similar to the Local Level frame with the difference that the z axis points to the centre of the earth.

Local Geodetic This coordinate system is very similar to the Local level Frame. The main difference is that when the system is in motion the tangent plane origin is fixed, while in the local level frame the origin is the projection of the platform origin into the earth's geodic. This is frame is mainly used to perform navigation in local areas relative to a given origin point.

Body Frame Most navigation systems have sensors attached to the vehicle. The body frame constitutes a new frame that is rigidly attached to the vehicle. The origin of this frame can be arbitrarily chosen although some location may simplify the kinematics models.

Summary on frames

Figure 24 shows the coordinates system used in this course. Absolute sensors like GPS provide position in ECEF coordinates, using X, Y, Z or latitude, Longitude and Height Φ, λ, H. Inertial sensors will return information in its body frame, which is in permanent rotation and translation from the navigation frame N, E, D shown in the figure. All the sensory information needs to be converted to the navigation frame prior to perform the data fusion task to obtain the best position estimate. This is one of the tasks of the Navigation system.

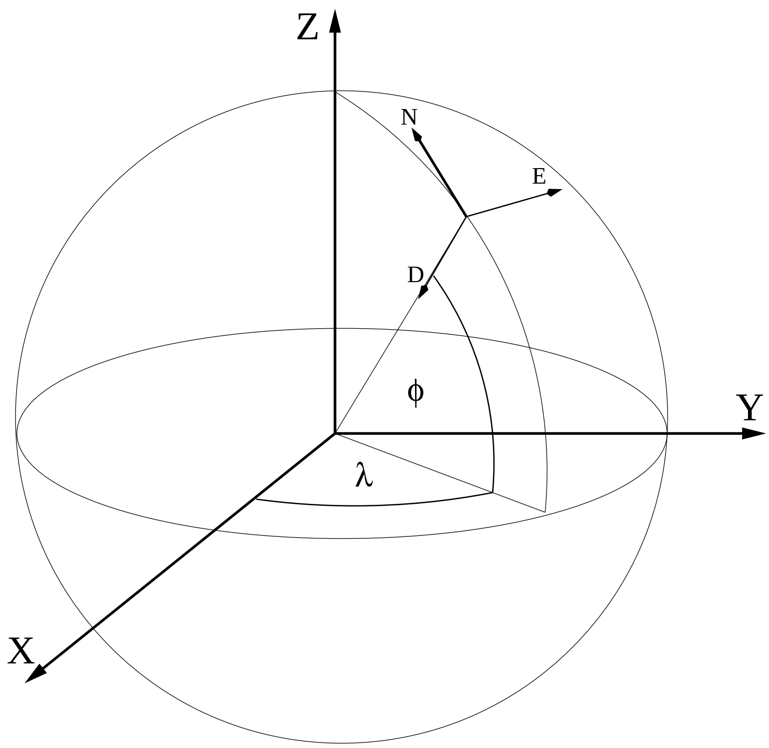

Figure 24: Different Navigation Frames

5.5 Inertial Navigation Equations

This section presents the derivation of the inertial navigation equations for global and local area navigation. For additional information the reader can see [24]

5.5.1 The Coriolis Theorem

Navigation with respect to a rotating frame such as the earth requires the Coriolis theorem. The theorem states that the velocity of a vehicle with respect to a fixed inertial frame $\mathbf{v}_i$,