Homework 9

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1 Volume of Hypersphere

1.1 Question

Find the volume of a hypersphere using a Monte Carlo method.

1.2 Solution

The volume of n-dimensial hypersphere V is given by

$$V = \int_{x_1^2+x_2^2+...+x_n^2 \leq r^2} dx_1 dx_2 ... dx_n = \begin{cases} \frac{(2\pi)^{\frac{n}{2}}}{n!!} r^n & \text{n is even} \\ \frac{2(2\pi)^{\frac{n-1}{2}}}{n!!} r^n & \text{n is odd} \end{cases} \tag{1}$$

In the code ,we use the importance sampling,where the function is gaussian.

1.3 Psudocode

Algorithm 1 Volume of Hypersphere

Input: dimension n, times of repeat

Output: Volume of Hypersphere

```
for i in range time do
    x ~ N(0, σ)
    if x^2 < 1 then
        fi = 1/n dimension gauss distribution
        sum+ = fi
    end if
end for
sum=sum/time
return VolumeofHyperspheresum
```

1.4 Results

We set the repeat time = 100000.When dimension n=2,the simulation result is 3.138,and the exact result is 3.141.When dimension n=3,the simulation result is 4.188,and the exact result is 4.193.When dimension n=4,the simulation result is 4.946,and the exact result is 4.934.When dimension n=5,the simulation result is 5.257,and the exact result is 5.263.

2 Heisenberg Spin Model

2.1 Question

Use the Heisenberg spin model and estimate the ferromagnetic Curie temperature.

2.2 Solution

Using the Metropolis algorithm, we can flip the spins one by one. For computing convenience, we set $k_B = 1$. It should noted that the random vector with magnitude 1 can be generated by generating ϕ from $Unif[0, 2\pi]$, $cos\theta$ from $Unif[-1, 1]$. Thus the random vector is $(sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$.

2.3 Psudocode

Algorithm 2 Heisenbuerg Spin Model

Input: Temperature T,side length n

Output: the spins of the configuration s

function H(x,y,z,s,vector)
    s1=s[(x+1)%n,y,z]   s2=s[(x-1)%n,y,z]
    s3=s[x,(y+1)%n,z]   s4=s[x,(y-1)%n,z]
    s5=s[x,y,(z+1)%n]   s5=s[x,y,(z-1)%n]
return $-vector \cdot (s1 + s2 + s3 + s4 + s5 + s6)$
function Get_Random_Vector
    ϕ=Unif[0,2] θ=arccosUnif[-1,1]
return $(sin\theta cos\phi, sin\theta sin\phi, cos\theta)$
for time in range $1000n^3$ do
    $a, b, c \sim IntUnif[0, n]$
    s'[a,b,c]=Get_Random_Vector()
    $r = e^{-\frac{H'-H}{T}}$
    if r>1 then
        s=s'
    else

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            x ~ Unif[0, 1]
            if x<r then
                s=s'
            end if
        end if
    end for
    return s
```

2.4 Results

Plotting the spin vectors,we can see how spin towards in the space. We get relationship between the magnetization density and the temperature shown as the following plot. From the figure, we can can conclude that the Curie temperature is around 1.00-1.50K/kB.