Homework 5

Chai Shengdu 19307110142

November 2, 2021

1 Richardson extrapolation

1.1 Question

Evaluate the derivative of $f(x) = sinx$ at $x = \frac{\pi}{3}$ with the Richardson extrapolation algorithm,and output the Richardson table.

1.2 Solution

The Richadson extrapolation:

$$D(n,m) = D(n,m-1) + \frac{1}{4^m - 1}[D(n,m-1) - D(n-1,m-1)] \tag{1}$$

where $D(n,0) = \varphi(\frac{h}{2^n})$,and $\varphi(h) = \frac{1}{2h}(f(x+h) - f(x-h))$

1.3 Psudocode

Algorithm 1 RICHARDSON EXTRAPOLATION

Input: initial $h = 1$

Output: estimated derivative with six significant decimal digits and Richardson table

```
initial iteration number itg and initial err = 1e-7
for i in range(0,itg) do
    for j in range(0,i) do
        DM[i, j] = D(i, j, h, x)
        if (abs(DM[i, i] - DM[i-1, i-1]) < err) and (i > 1) then
            sol = DM[i, i]
            itg = i
        end if
    end for
end for
return DM,itg
```

1.4 Results

```
0.4999999999999883 4
[[0.42073549 0.         0.         0.        ]
 [0.47942554 0.49898889 0.         0.        ]
 [0.49480792 0.49993538 0.49999848 0.        ]
 [0.49869893 0.49999594 0.49999998 0.5       ]]
```

Figure 1: the number of rows =4

With 4 iterations can we get the derivative with 6 decimal places.

2 Simpson's rule

2.1 Question

Use Simpson's rule to calculate

$$T(\varphi_0) = 4(\frac{l}{2g})^{\frac{1}{2}} \int_0^{\varphi_0} \frac{1}{(cos\varphi - cos\varphi_0)^{\frac{1}{2}}}\, d\varphi \tag{2}$$

2.2 Solution

Let's introduce a new variable $sin(\varphi/2) = sin(\varphi_0/2)sin\theta$.And we can get

$$f(= 4(\frac{l}{g})^{\frac{1}{2}} \int_0^{\pi/2} \frac{1}{(1 - sin^2(\varphi_0/2)sin^2(\theta))^{\frac{1}{2}}}\, d\theta \tag{3}$$

For convenience ,we can set $4(\frac{l}{g})^{\frac{1}{2}} = 1$.Thus we can evaluate $T(\varphi_0)$ with Simpson's rule:

$$\int_a^b f(x)\, dx = \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)] \tag{4}$$

2.3 Psudocode

Algorithm 2 SIMPSON'S RULE

Input: the initial pendulum angle φ_0

Output: the oscillation period $T(\varphi_0)$

```
f(θ) = 1 / (1 − sin²(φ0/2) sin²(θ))^(1/2)
x = linspace(0, π/2, n),n,I=0
h = π/(2n)
for i in range(int(n/2)) do
    I = I + (h/3)(f(x_{2i}) + 4f(x_{2i+1}) + f(x_{2i+2}))
end for
return I
```

2.4 Results

We can get the relationship with T and φ_0

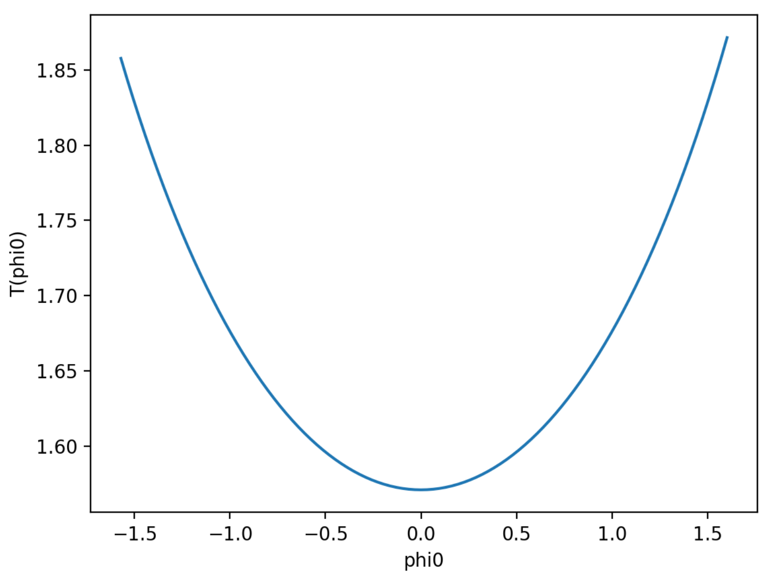

Figure 2: $T(\varphi_0) - \varphi_0$