

Importance Sampling on CTMC Models

Introduction

This project contains Java source and PRISM model files used to study importance sampling methods in Continuous Time Markov Chain (CTMC) models. In past literature, significant work was done on IS for special sub-classes of CTMC models, for instance Mean Time to Fail estimation in regenerative models that always return to their initial state.

One of our goals is to develop novel IS approaches for CTMC models, including non-regenerative models. A collection of “abstract” models is provided using the PRISM language. Each N-state model has the following characteristics:

- States are indexed $1, 2, \dots, N$
- State 1 is the initial state
- State N has no exiting transitions and is considered the “target” or terminal state.
- States $1, \dots, N-1$ all have transitions leading to state N, and are fully connected to each other
- Between any states a, b there is a transition with rate Ra_b , where a is in $1, \dots, N-1$ and b is in $1, \dots, N$.

The abstract models are fully-connected, and can be pruned by setting select edge transition rates to zero. In each model, state N is considered a “target” state, and the IS objective is to estimate the probability and/or mean time to reach state N.

Two-State Model

`models/abstract/2_states/2_state.pm`

The trivial two-state model has only one possible transition which occurs with probability 1. There is a single path.

Three-State Model

`models/abstract/3_states/3_state.pm`

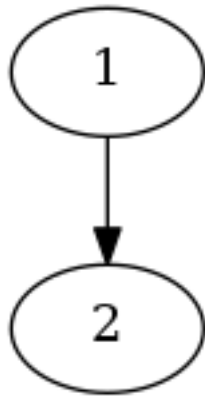


Figure 1: Trivial two-state model.

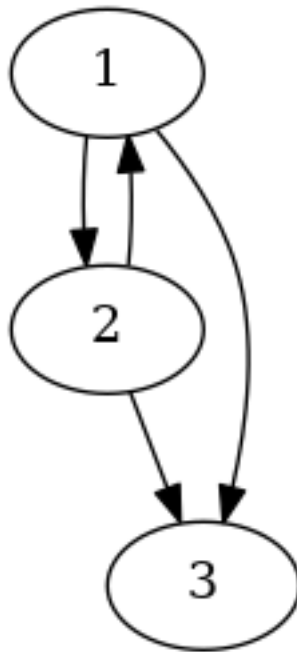


Figure 2: Three-state model.

A general three-state model has two non-regenerative paths that terminate in state 3:

- 1 3
- 1 2 3

There is also a single regenerative path:

- 1 2 1

The probability to reach state 3 before returning to state 1 is

$$\frac{\frac{R_{13}}{R_{12}+R_{13}} + \frac{R_{12}}{R_{12}+R_{13}} \frac{R_{23}}{R_{23}+R_{21}}}{\frac{R_{12}}{R_{12}+R_{13}} \frac{R_{21}}{R_{23}+R_{21}}}$$