

Bottom Up Parsing

TEACHING ASSISTANT: DAVID TRABISH

LR(0) Parsing

- Build the derivation tree from the bottom
- First build the children, then connect to the parent
- Can handle left recursion
 - Which is common in real-world grammars

LR(0) Item

An LR(0) item is of the form:

- $N \rightarrow \alpha.\beta$

The **dot** gives us the current location (a local view).

LR(0) Item

An LR(0) item with the dot at the end is called **reduce** item:

- $N \rightarrow \alpha\beta.$

Otherwise, it's a **shift** item:

- $N \rightarrow .\alpha\beta$
- $N \rightarrow \alpha.\beta$

LR(0) Item Closure Set

The LR(0) closure set of an LR(0) item i is a set S such that:

- $i \in S$
- If $A \rightarrow \alpha.N\beta \in S$ then for each rule $N \rightarrow \gamma$:
 - $N \rightarrow.\gamma \in S$

LR(0) Item Closure Set

For example, given the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = X$
- $E \rightarrow \{ID\}$
- $X \rightarrow INT$

the closure set of the $S \rightarrow .E\$$ contains:

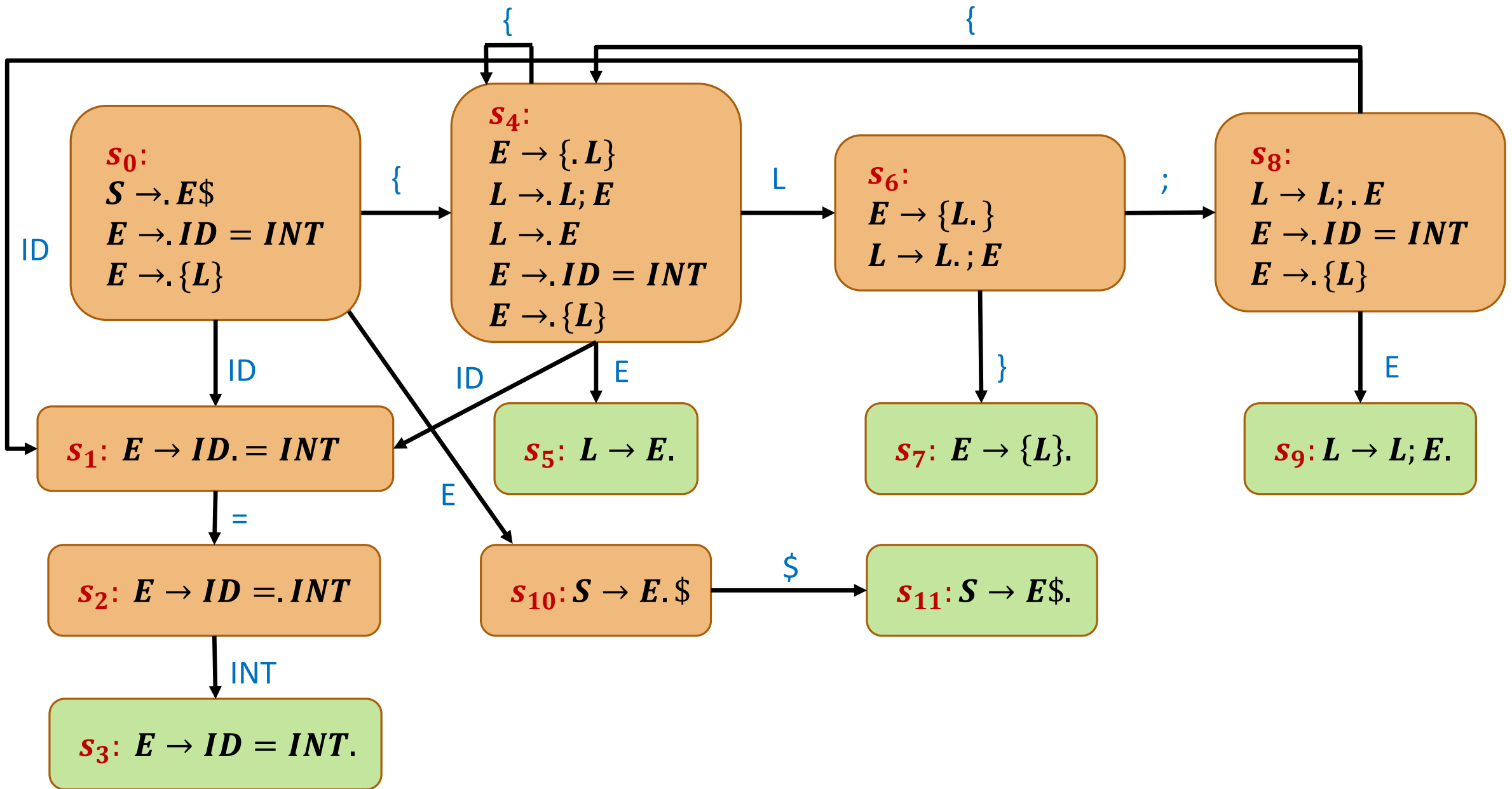
- $S \rightarrow .E\$$
- $E \rightarrow .ID = X$
- $E \rightarrow .\{ID\}$

LR(0) Parsing

Consider the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = INT$
- $E \rightarrow \{L\}$
- $L \rightarrow E$
- $L \rightarrow L; E$

What will be the **transition system** of the LR(0) parser for this CFG?



LR(0) Parser

We start with the initial LR(0) item (that comes from the initial rule):

- $S \rightarrow .E\$$

The initial state is the ϵ -closure of that item, which contains:

$$\begin{aligned} S &\rightarrow E\$ \\ E &\rightarrow ID = INT \\ E &\rightarrow \{L\} \\ L &\rightarrow E \\ L &\rightarrow L; E \end{aligned}$$

LR(0) Parser

We start with the initial LR(0) item (that comes from the initial rule):

- $S \rightarrow .E\$$

The initial state is the ϵ -closure of that item, which contains:

- $S \rightarrow .E\$$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

s_0 :

$S \rightarrow .E\$$

$E \rightarrow .ID = INT$

$E \rightarrow .\{L\}$

LR(0) Parser

From s_0 , if we recognized ID , then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_0 , if we recognized ID , then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

- $E \rightarrow ID. = INT$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

s_0 :

$S \rightarrow \cdot E \$$

$E \rightarrow \cdot ID = INT$

$E \rightarrow \cdot \{L\}$

ID

s_1 : $E \rightarrow ID \cdot = INT$

LR(0) Parser

From s_1 , if we recognized $=$, then the next state will contain:

- $E \rightarrow ID =.INT$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_1 , if we recognized $=$, then the next state will contain:

- $E \rightarrow ID =.INT$

So the next state (the ϵ -closure) contains:

- $E \rightarrow ID =.INT$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

s_0 :

$S \rightarrow .E\$$

$E \rightarrow .ID = INT$

$E \rightarrow .\{L\}$

ID

s_1 : $E \rightarrow ID. = INT$

=

s_2 : $E \rightarrow ID =.INT$

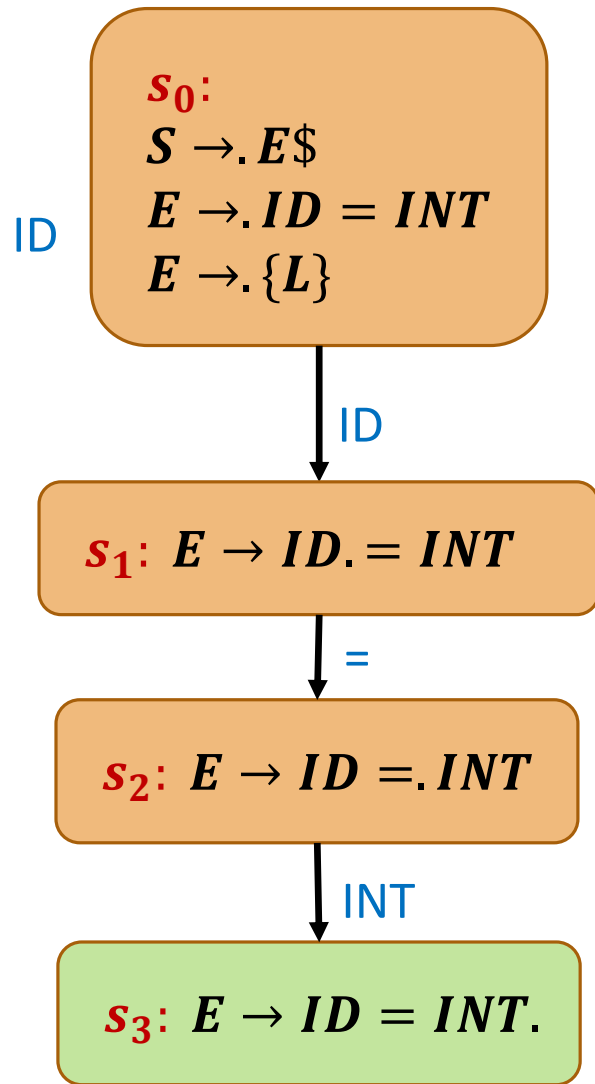
LR(0) Parser

From s_2 , if we recognized INT , then the next state will contain:

- $E \rightarrow ID = INT$.

Which is a reduce state.

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_0 , if we recognized $\{$, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

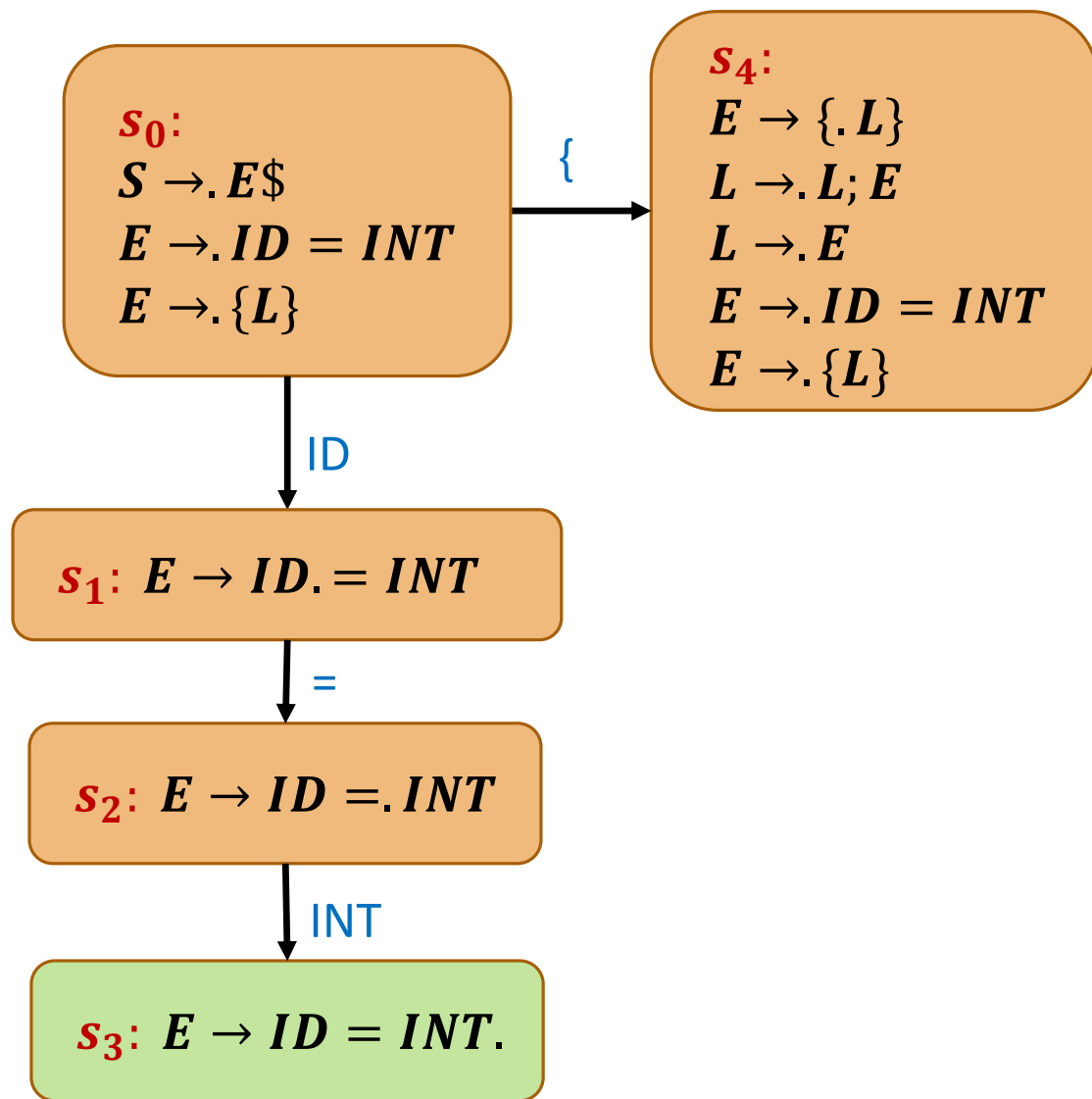
From s_0 , if we recognized $\{$, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L; E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_4 , if we recognized $\{$, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_4 , if we recognized $\{$, then the next state will contain:

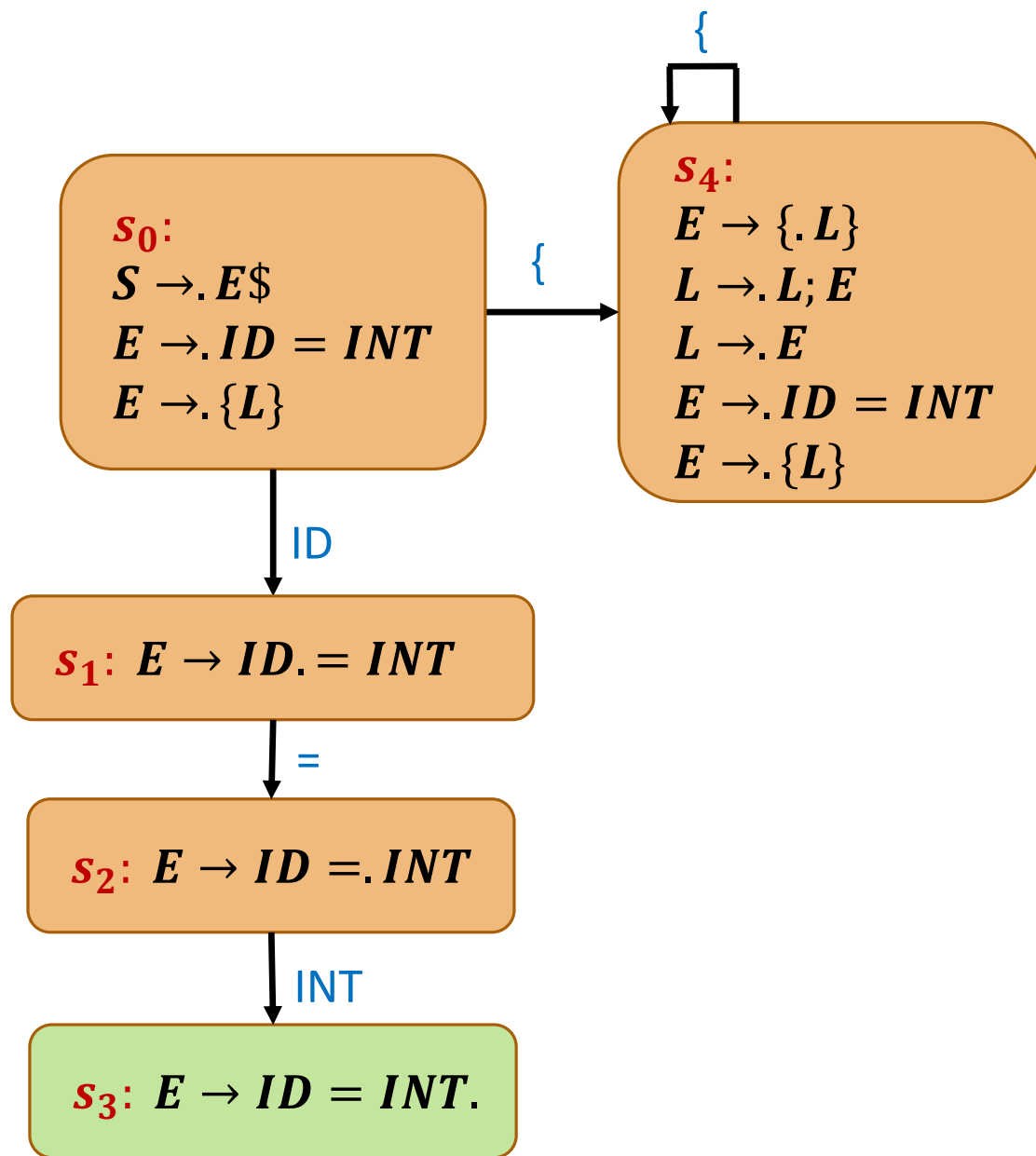
- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L; E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_4 , if we recognized ID , then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_4 , if we recognized ID , then the next state will contain:

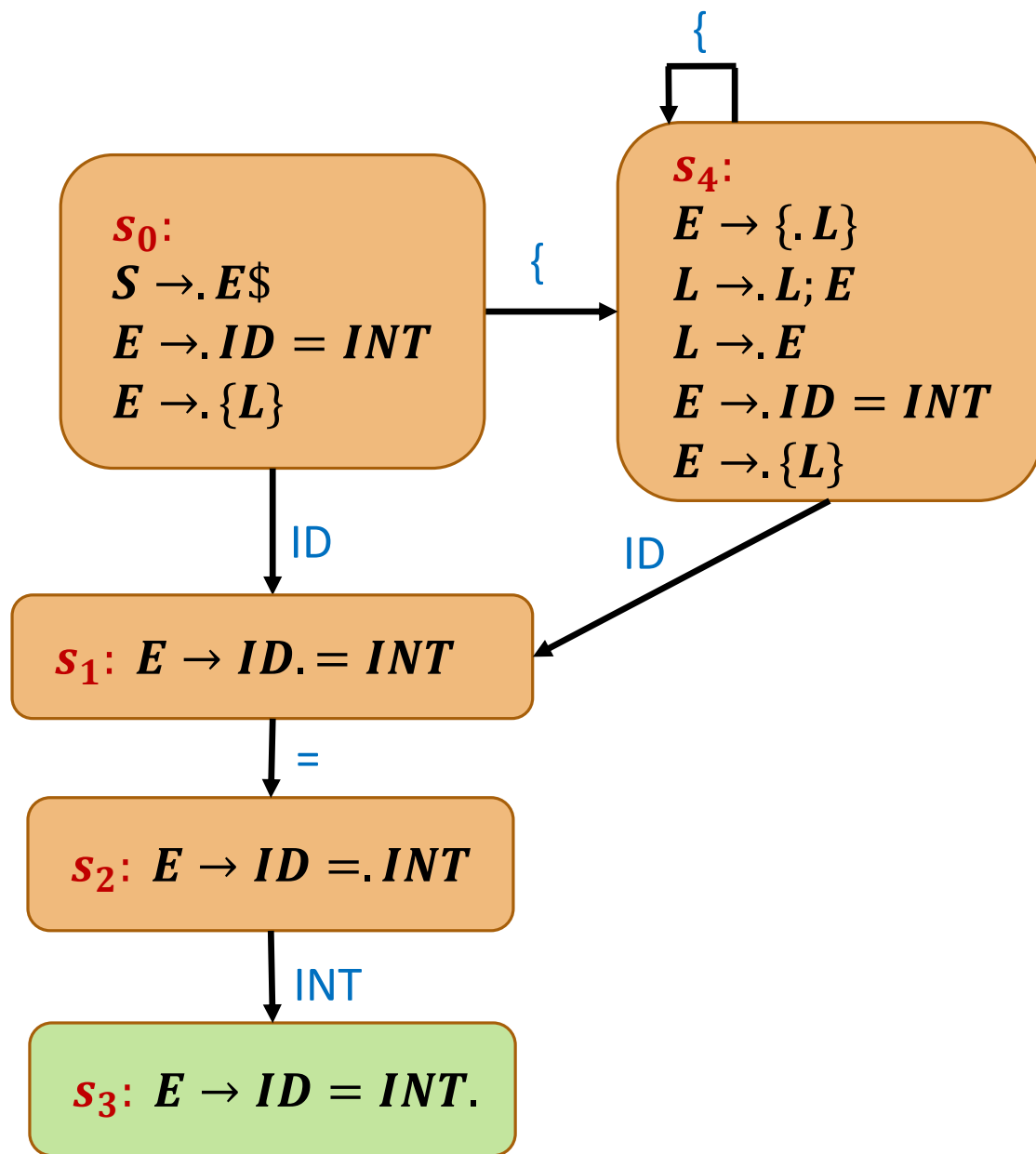
- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

- $E \rightarrow ID. = INT$

which was already computed: s_1

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



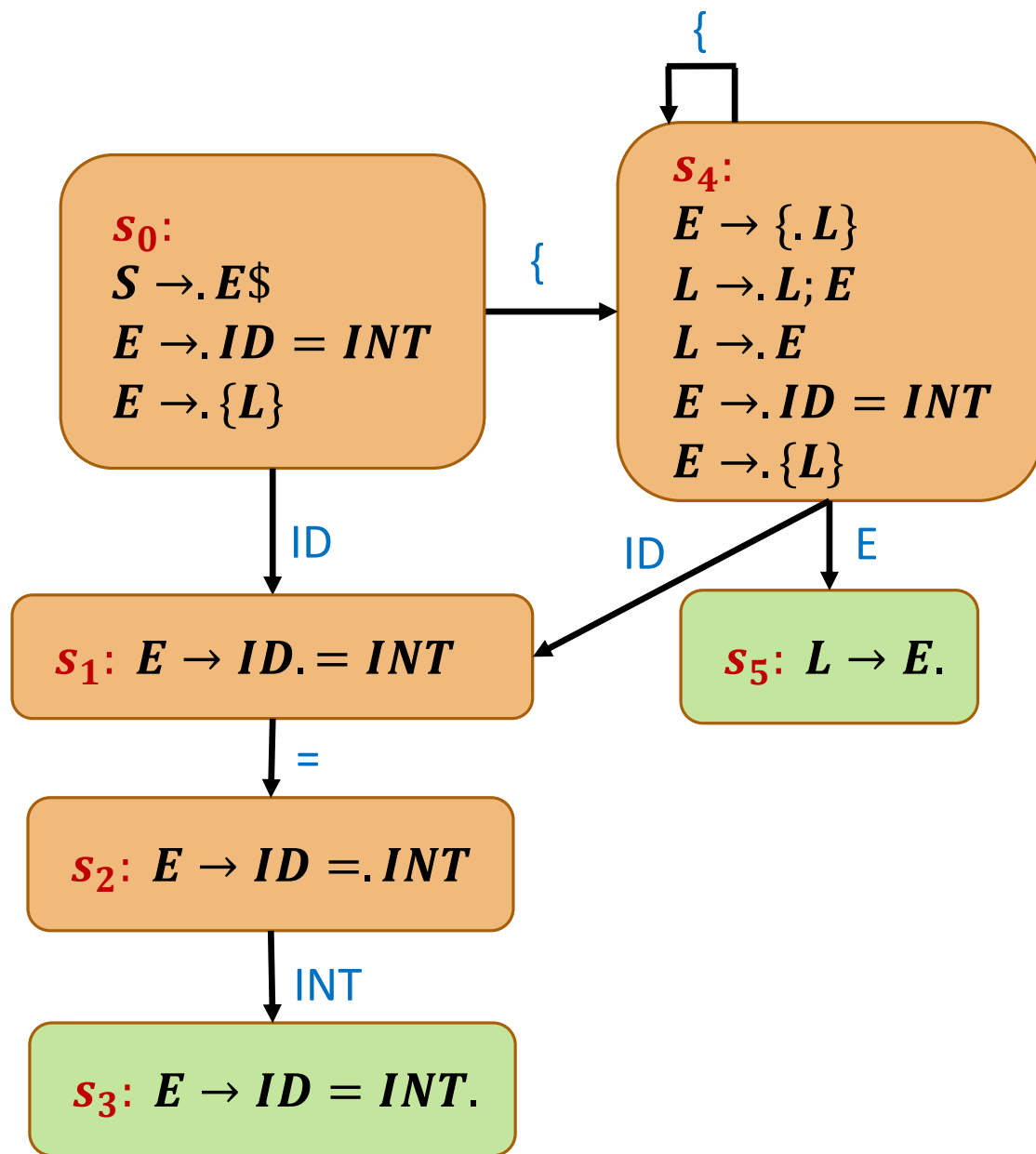
LR(0) Parser

From s_4 , if we recognized E , then the next state will contain:

- $L \rightarrow E$.

which is a reduce state.

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_4 , if we recognized L , then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.; E$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

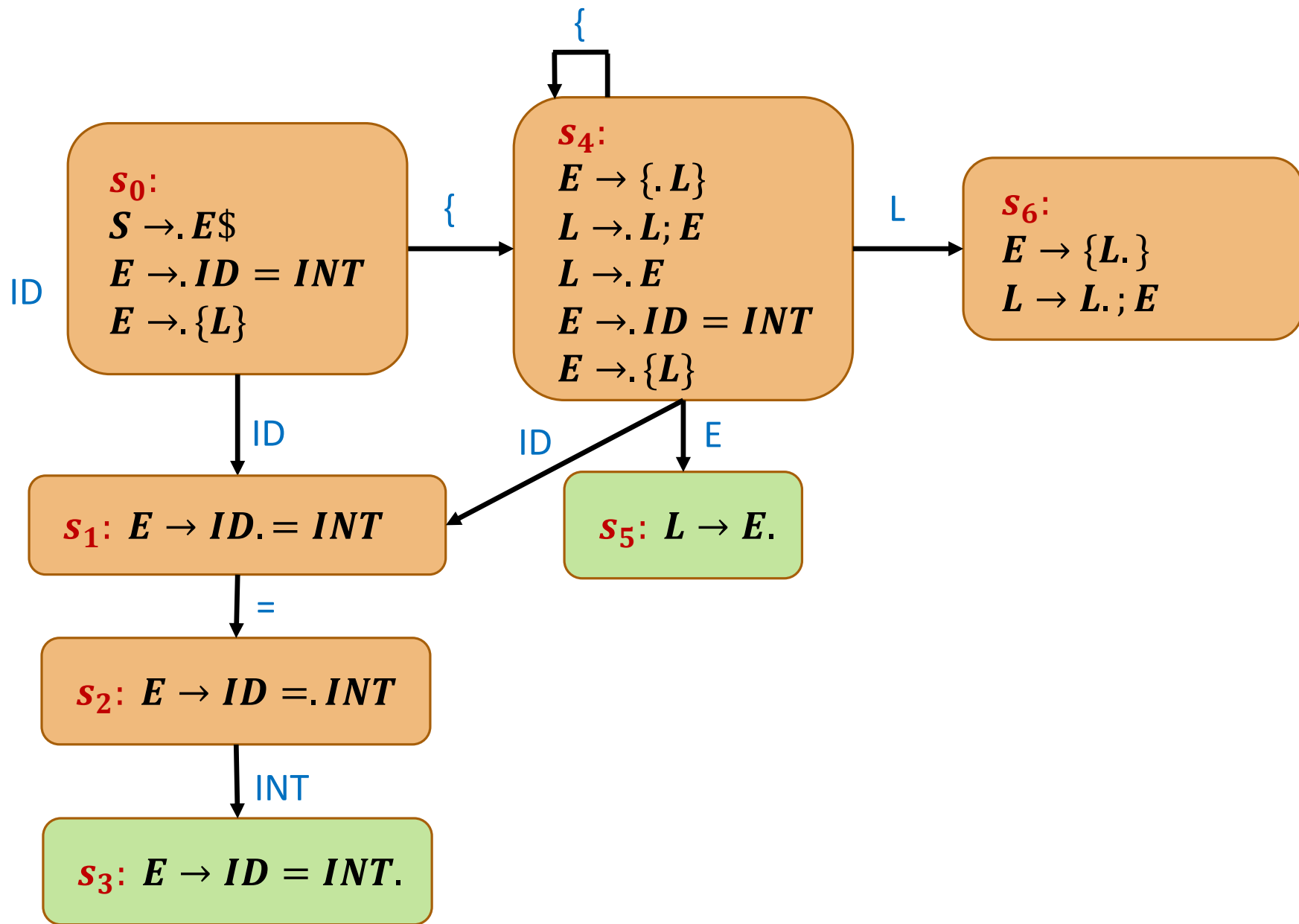
From s_4 , if we recognized L , then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.; E$

So the next state (the ϵ -closure) contains:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.; E$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



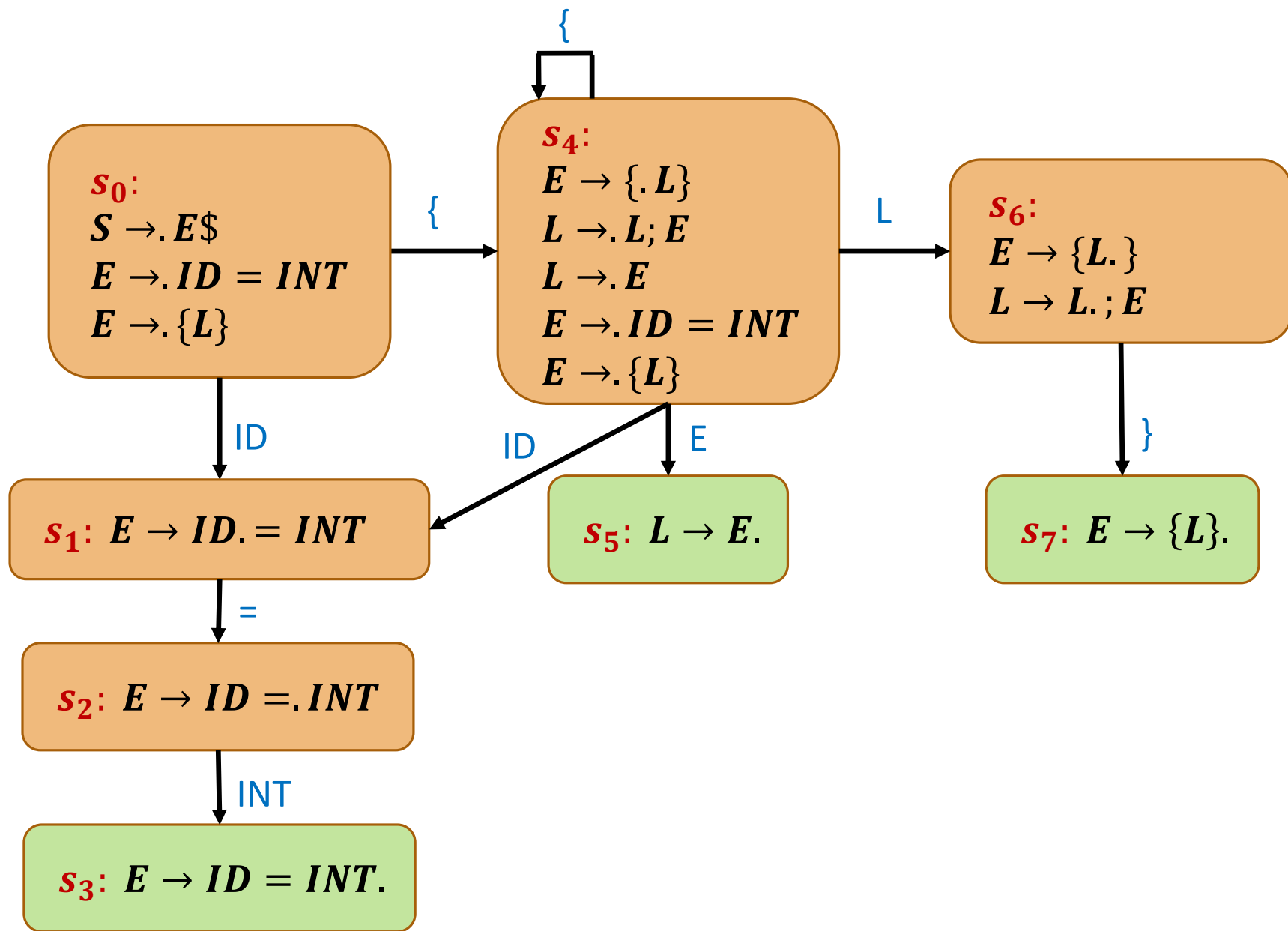
LR(0) Parser

From s_6 , if we recognized $\}$, then the next state will contain:

- $E \rightarrow \{L\}$.

Which is a reduce state.

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_6 , if we recognized $;$, then the next state will contain:

- $L \rightarrow L; . E$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_6 , if we recognized $;$, then the next state will contain:

- $L \rightarrow L; . E$

So the next state (the ϵ -closure) contains:

- $L \rightarrow L; . E$
- $E \rightarrow . ID = INT$
- $E \rightarrow . \{L\}$

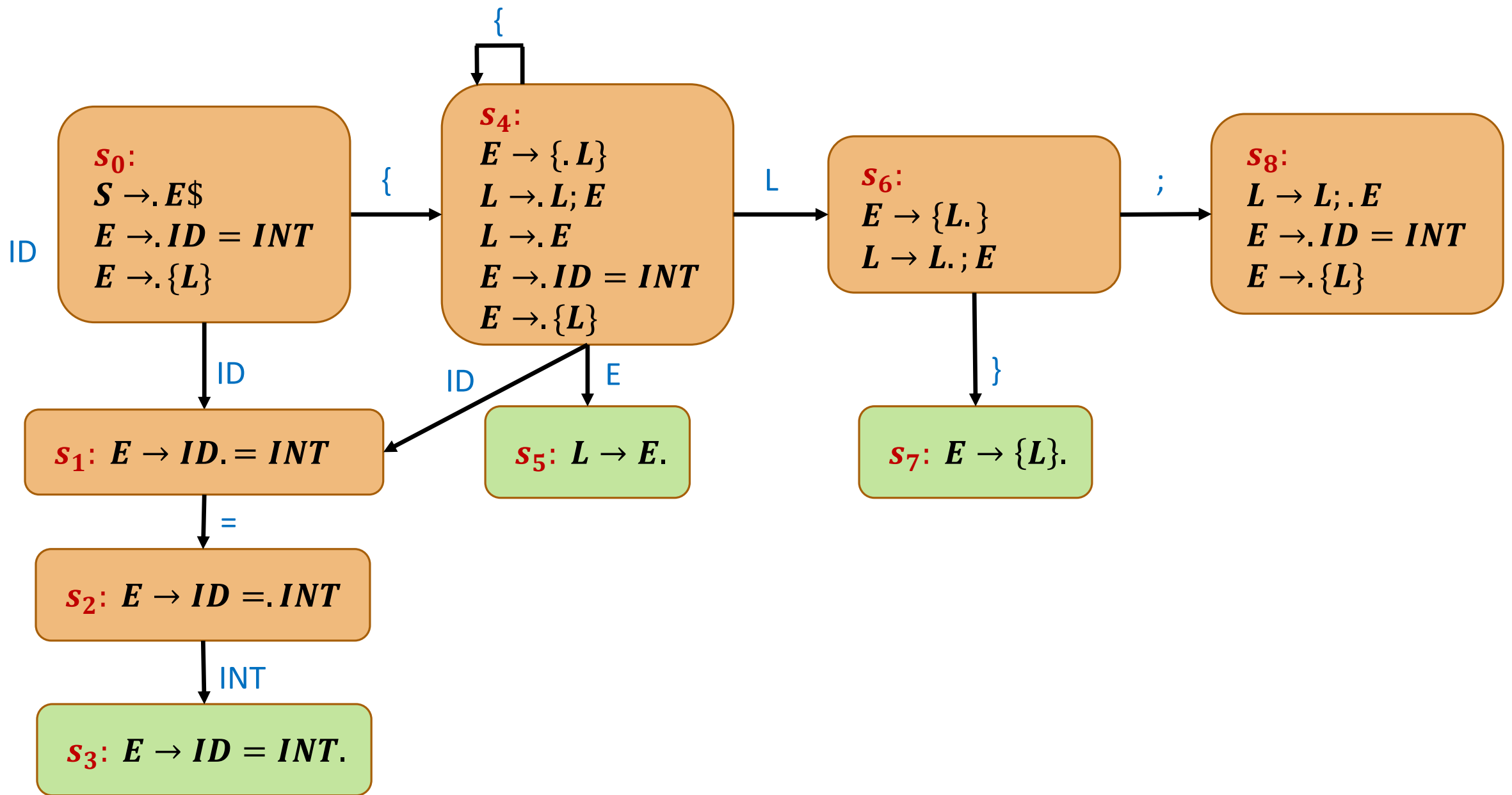
$S \rightarrow E\$$

$E \rightarrow ID = INT$

$E \rightarrow \{L\}$

$L \rightarrow E$

$L \rightarrow L; E$



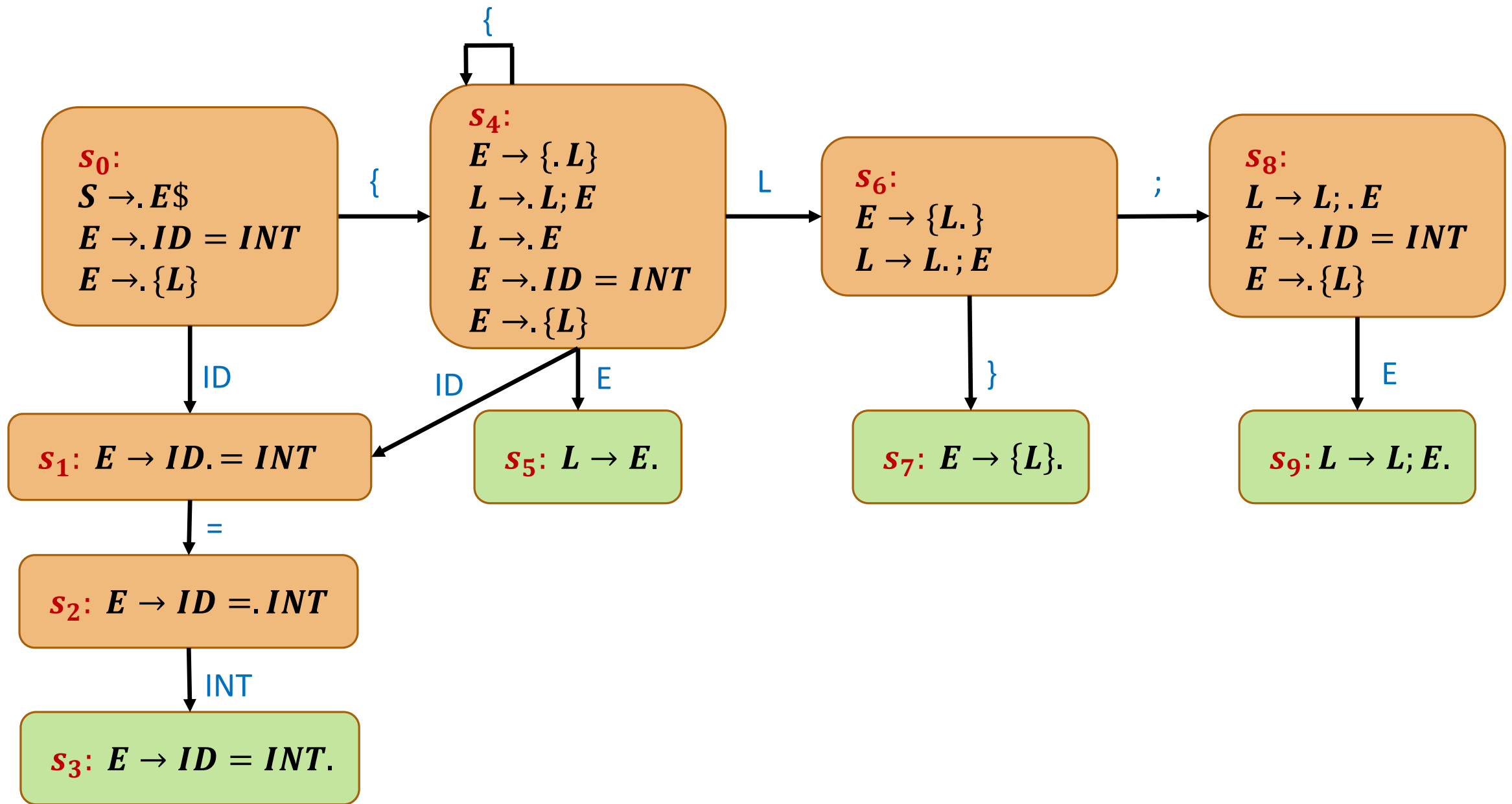
LR(0) Parser

From s_8 , if we recognized E , then the next state will contain:

- $E \rightarrow L; E$.

which is a reduce state.

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_8 , if we recognized $\{$, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_8 , if we recognized $\{$, then the next state will contain:

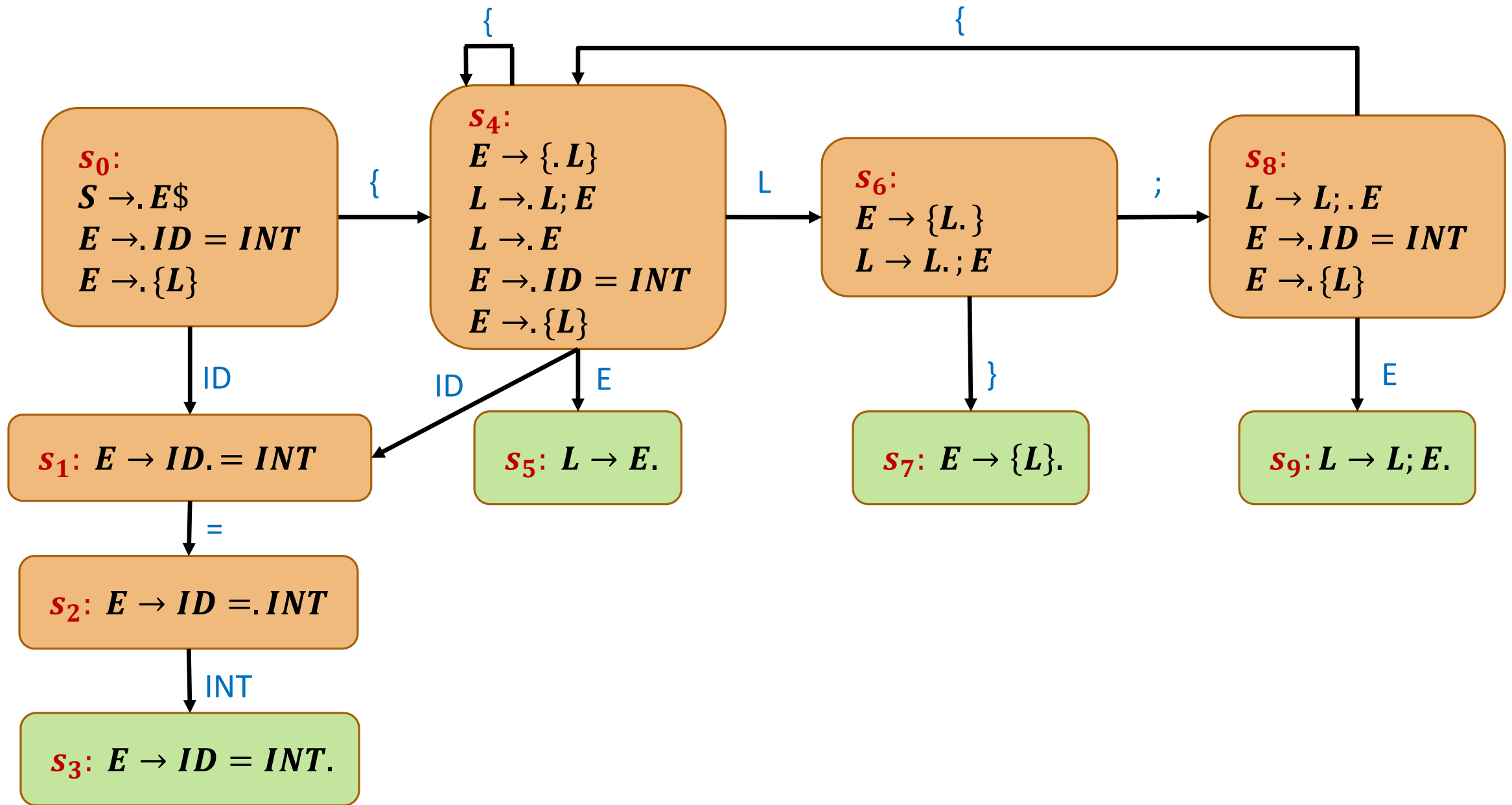
- $E \rightarrow \{.L\}$

So the next state (the ϵ -closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L; E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_8 , if we recognized ID , then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

From s_8 , if we recognized ID , then the next state will contain:

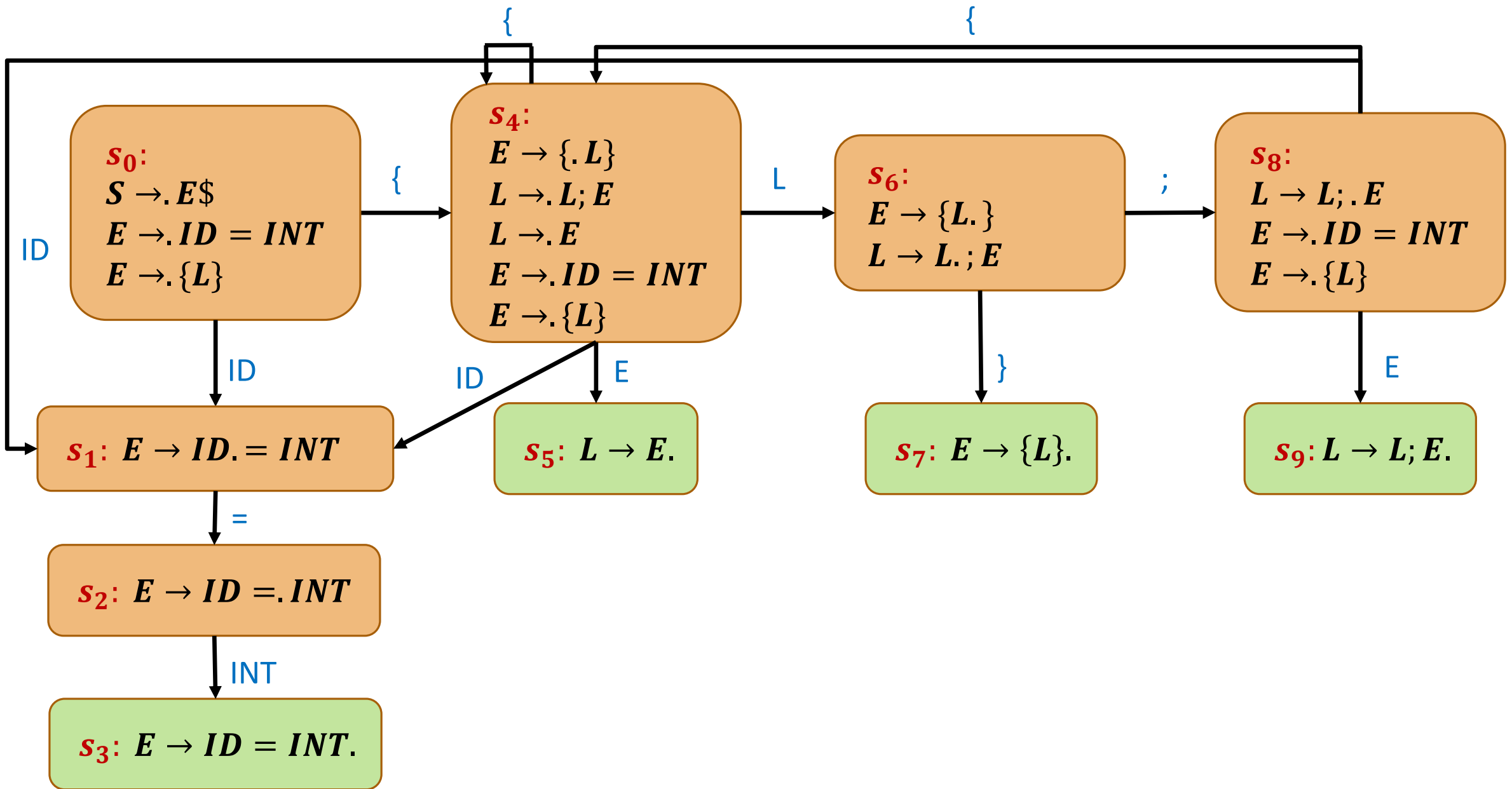
- $E \rightarrow ID. = INT$

So the next state (the ϵ -closure) contains:

- $E \rightarrow ID. = INT$

which was already computed: s_1

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



LR(0) Parser

From s_0 , if we recognized E , then the next state will contain:

- $S \rightarrow E.\$$

So the next state (the ϵ -closure) contains:

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

LR(0) Parser

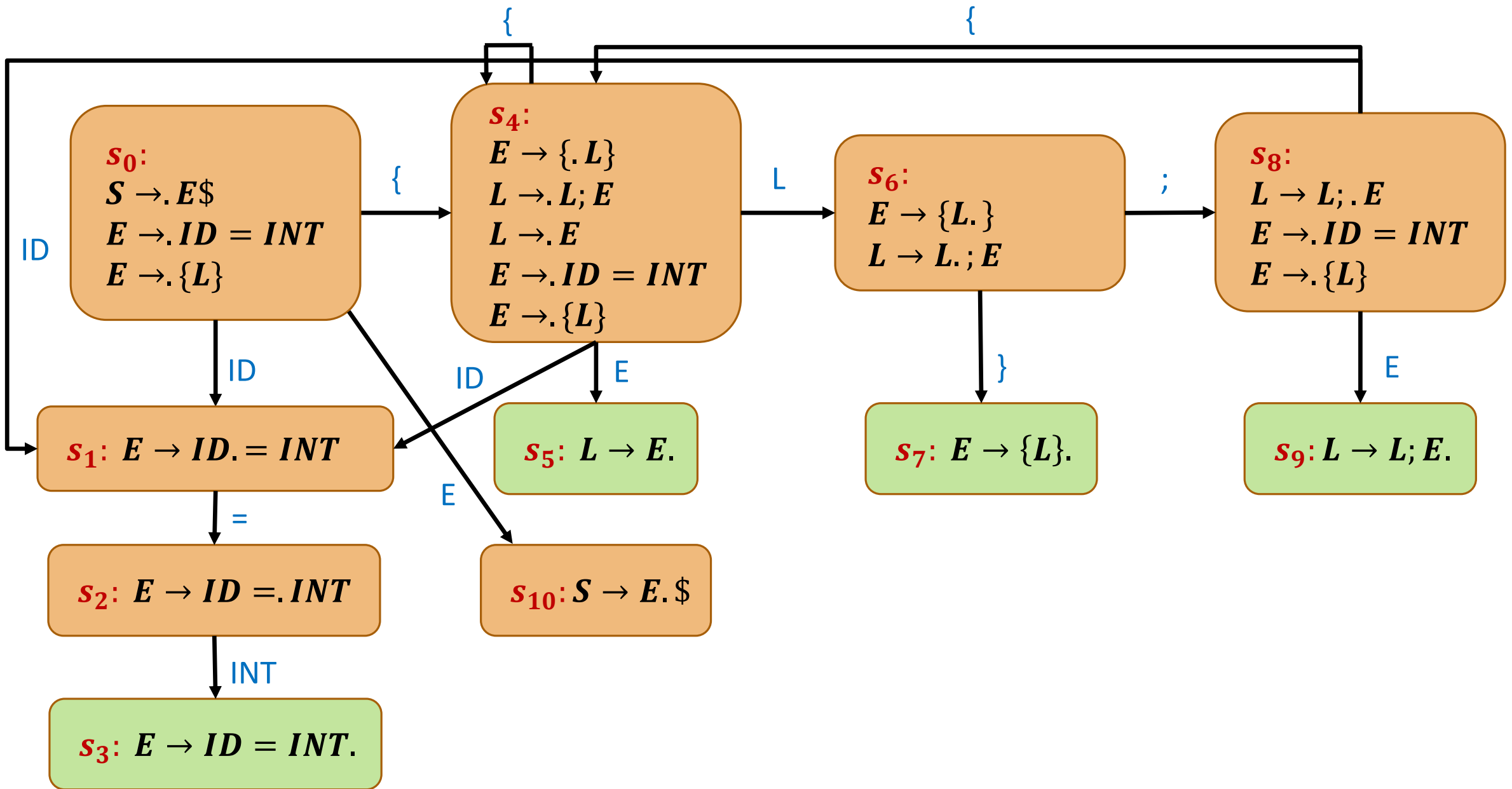
From s_0 , if we recognized E , then the next state will contain:

- $S \rightarrow E.\$$

So the next state (the ϵ -closure) contains:

- $S \rightarrow E.\$$

$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$



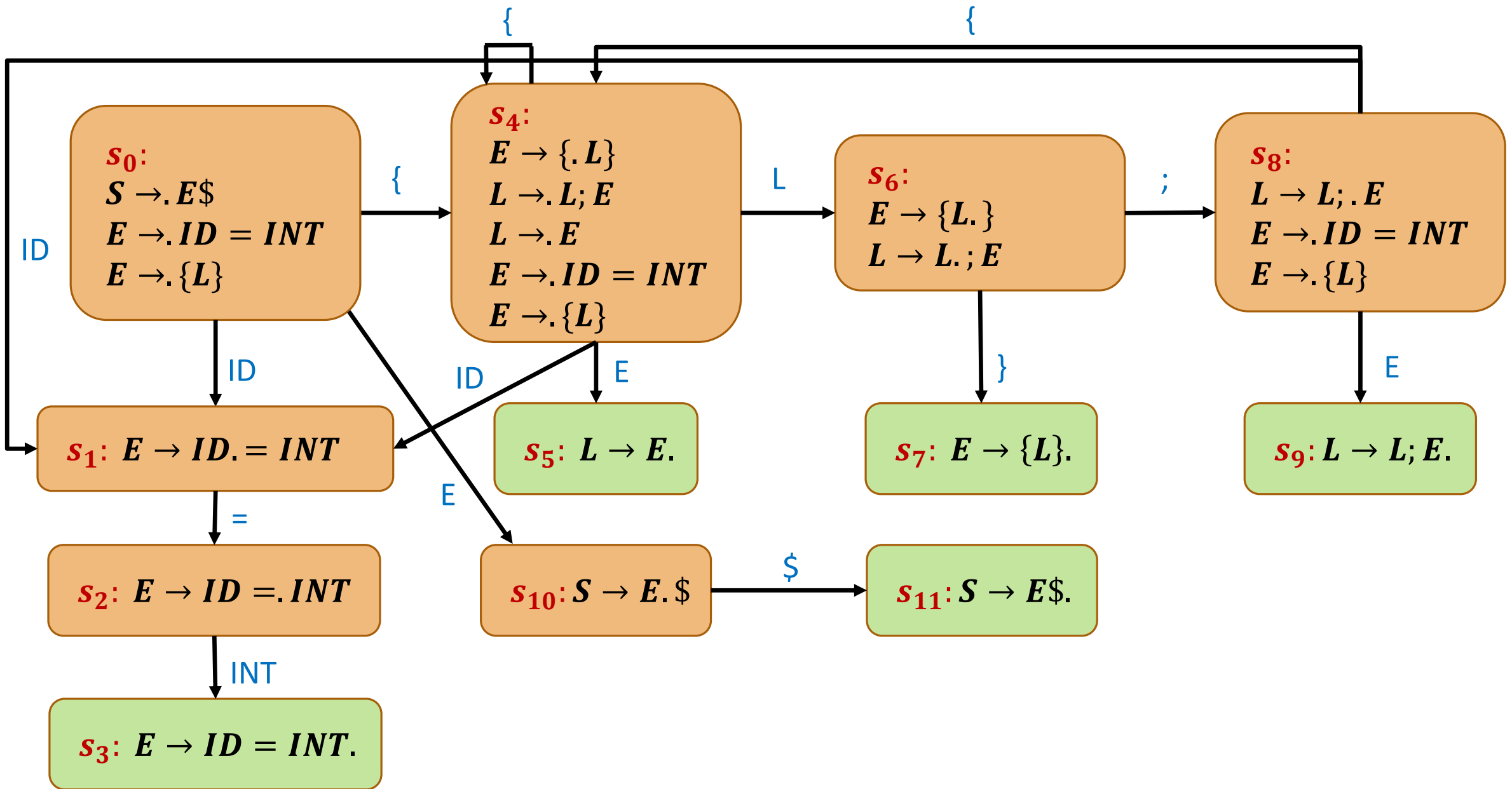
LR(0) Parser

From s_{10} , if we recognized \$, then the next state will contain:

- $S \rightarrow E\$$.

which is a reduce state.

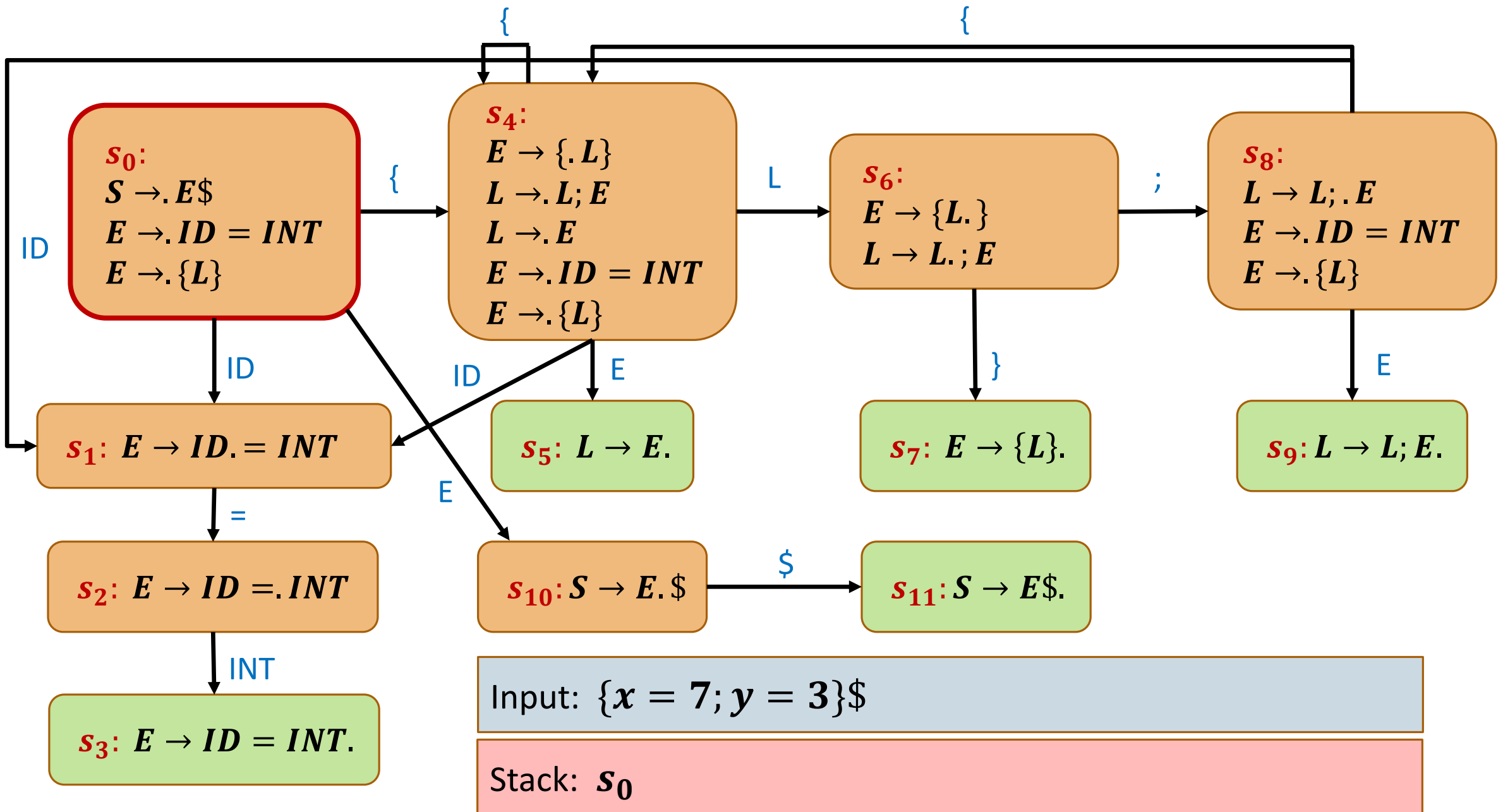
$S \rightarrow E\$$
 $E \rightarrow ID = INT$
 $E \rightarrow \{L\}$
 $L \rightarrow E$
 $L \rightarrow L; E$

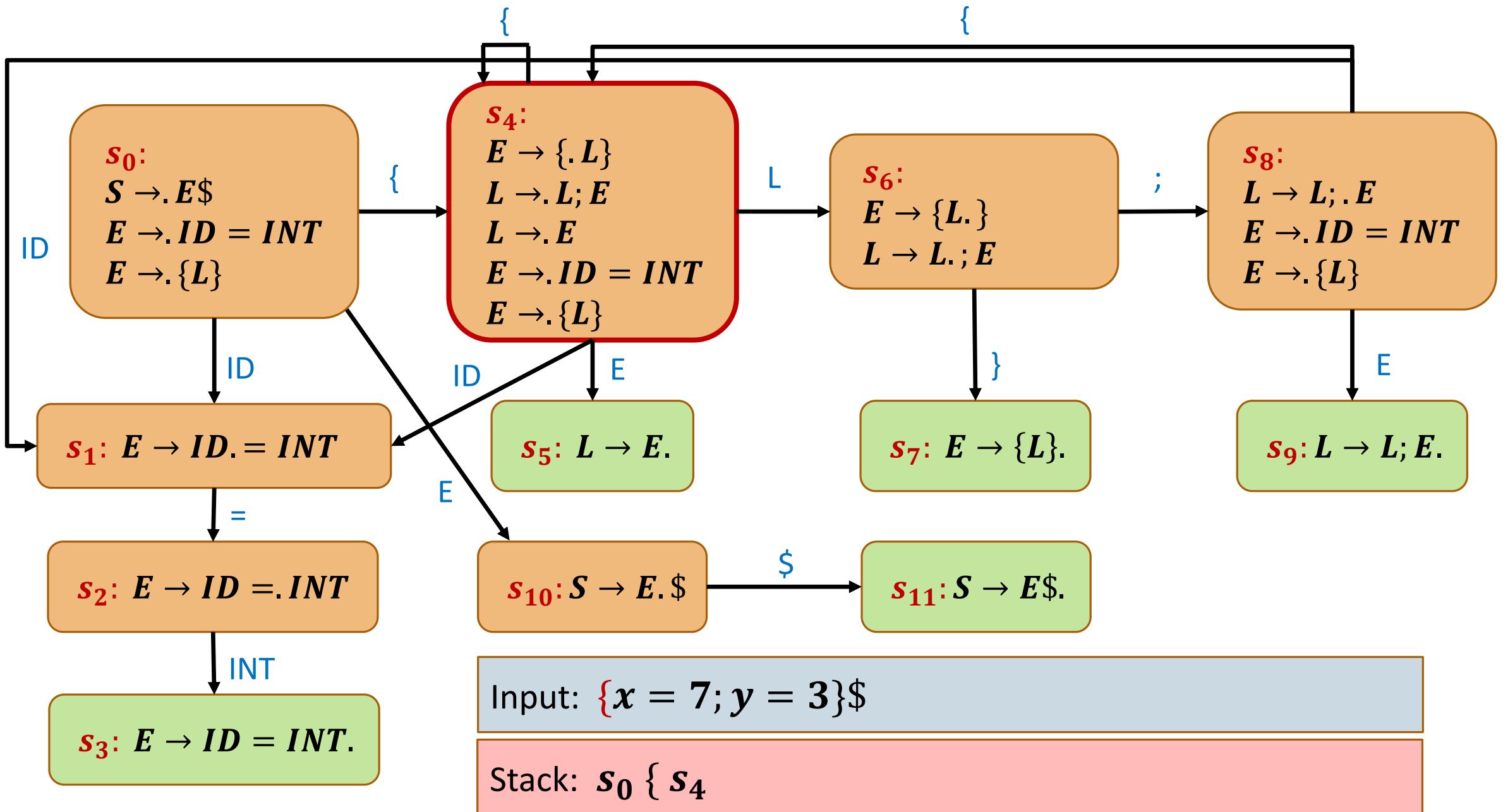


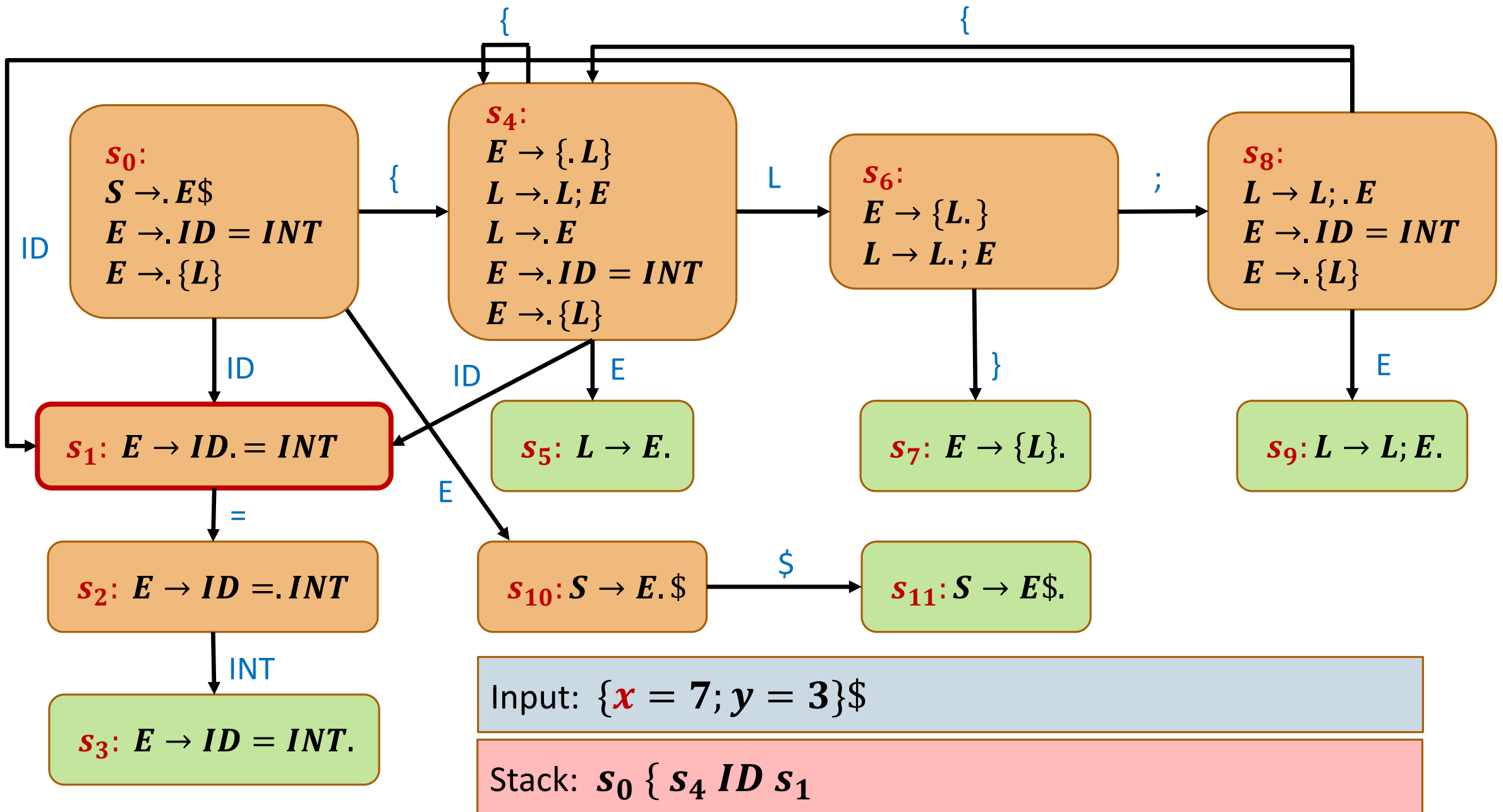
LR(0) Parser: Running Example

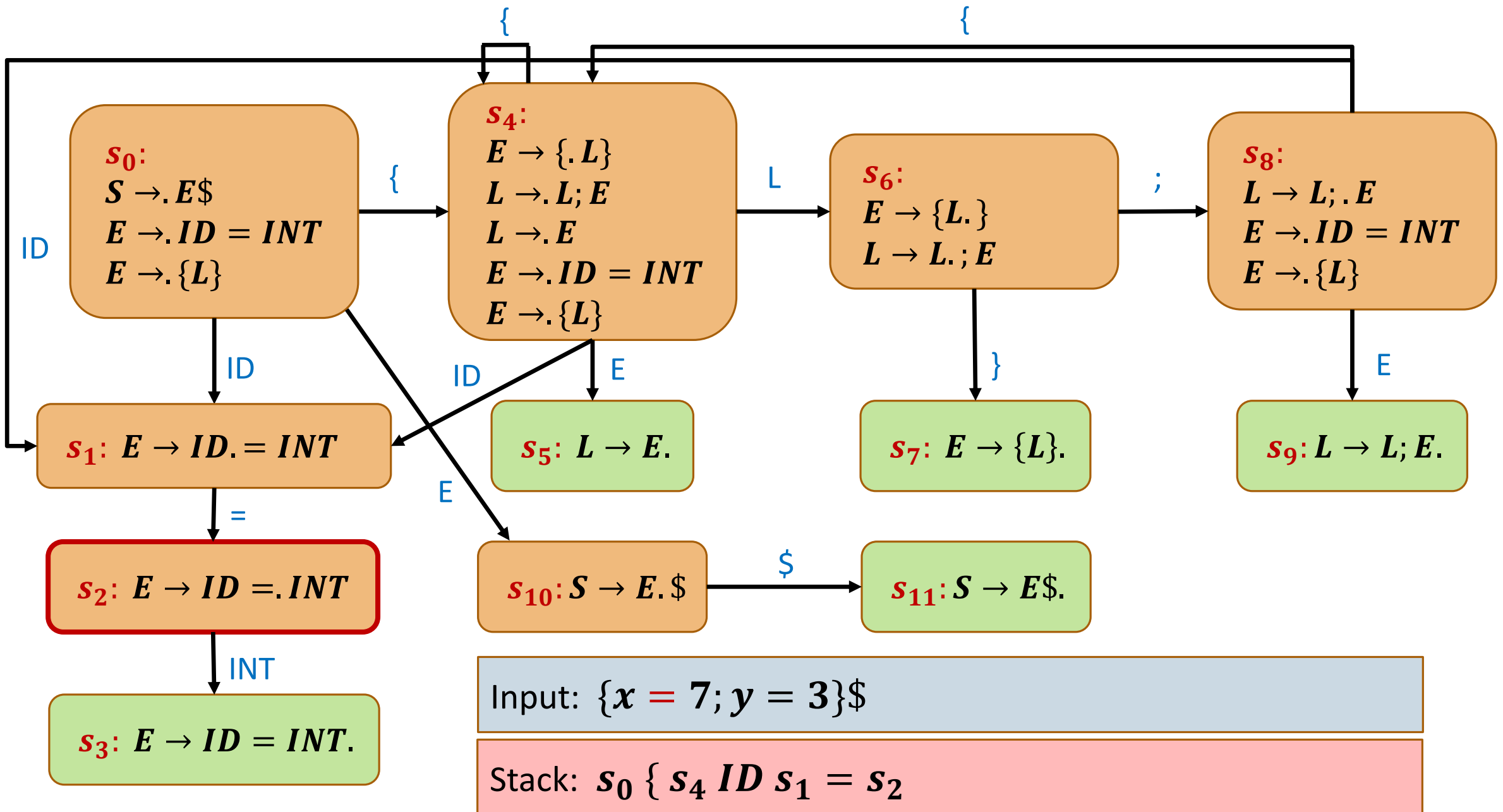
What will happen with the following input:

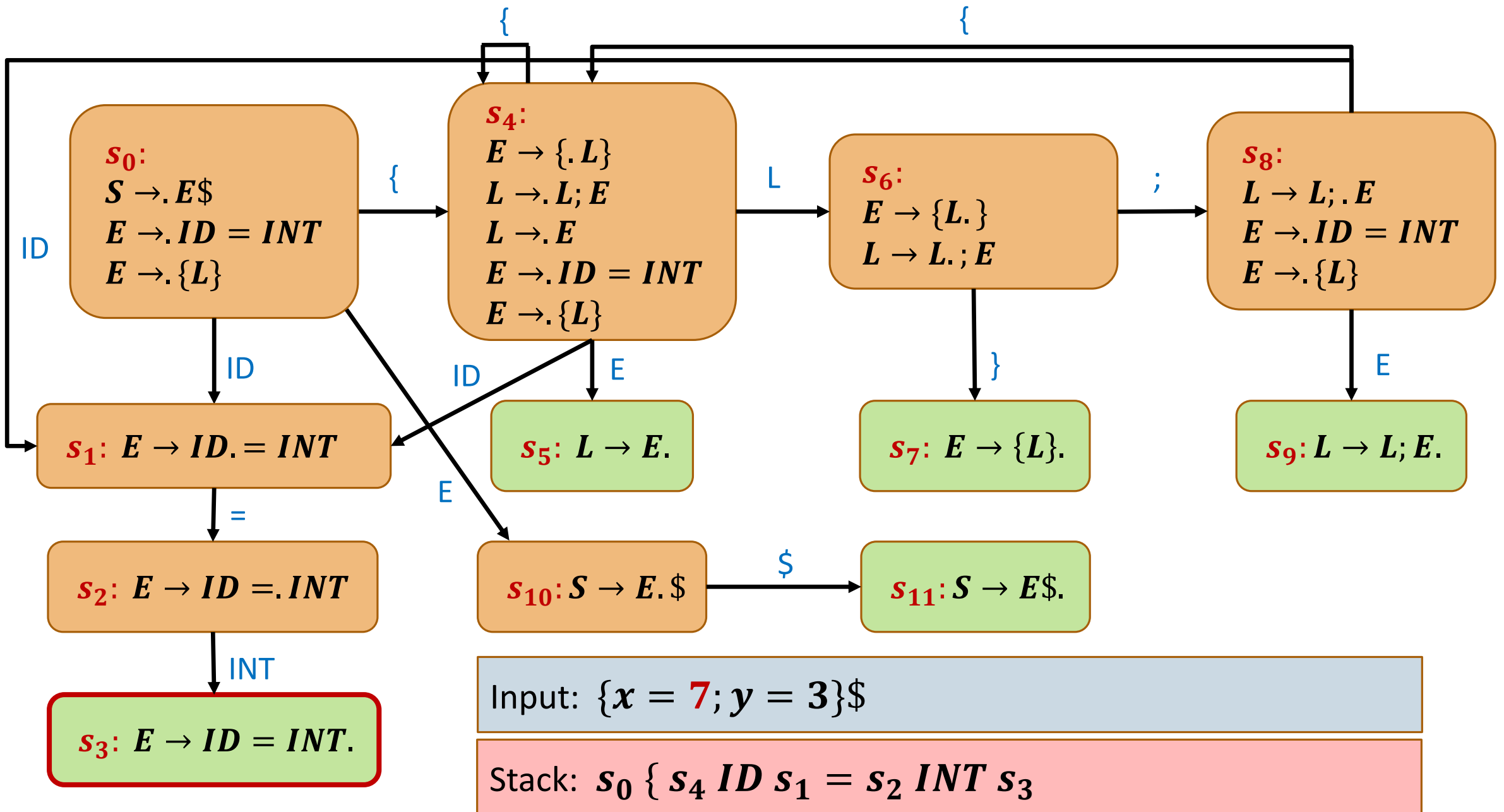
- $\{x = 7; y = 3\}$ \$

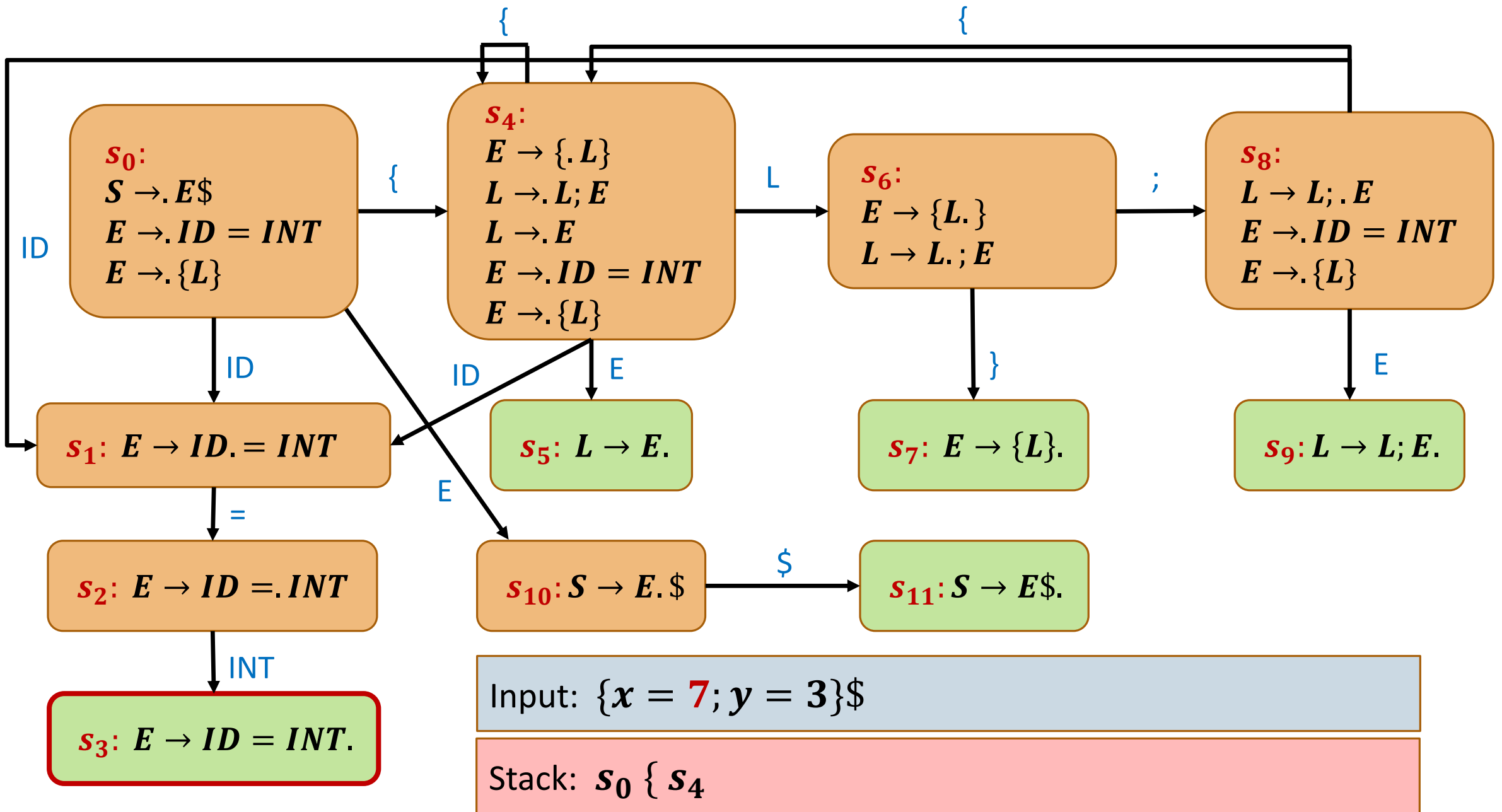


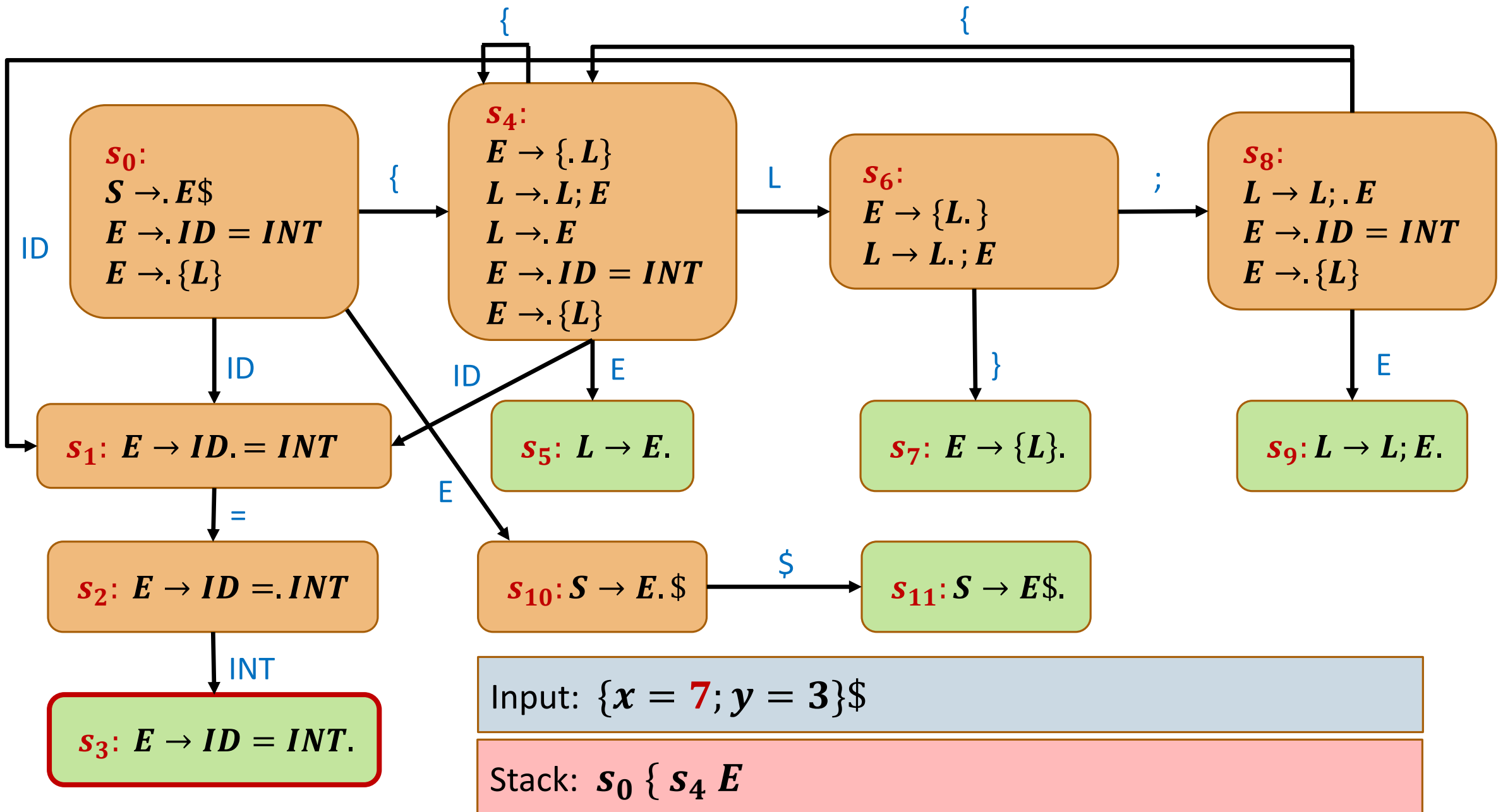


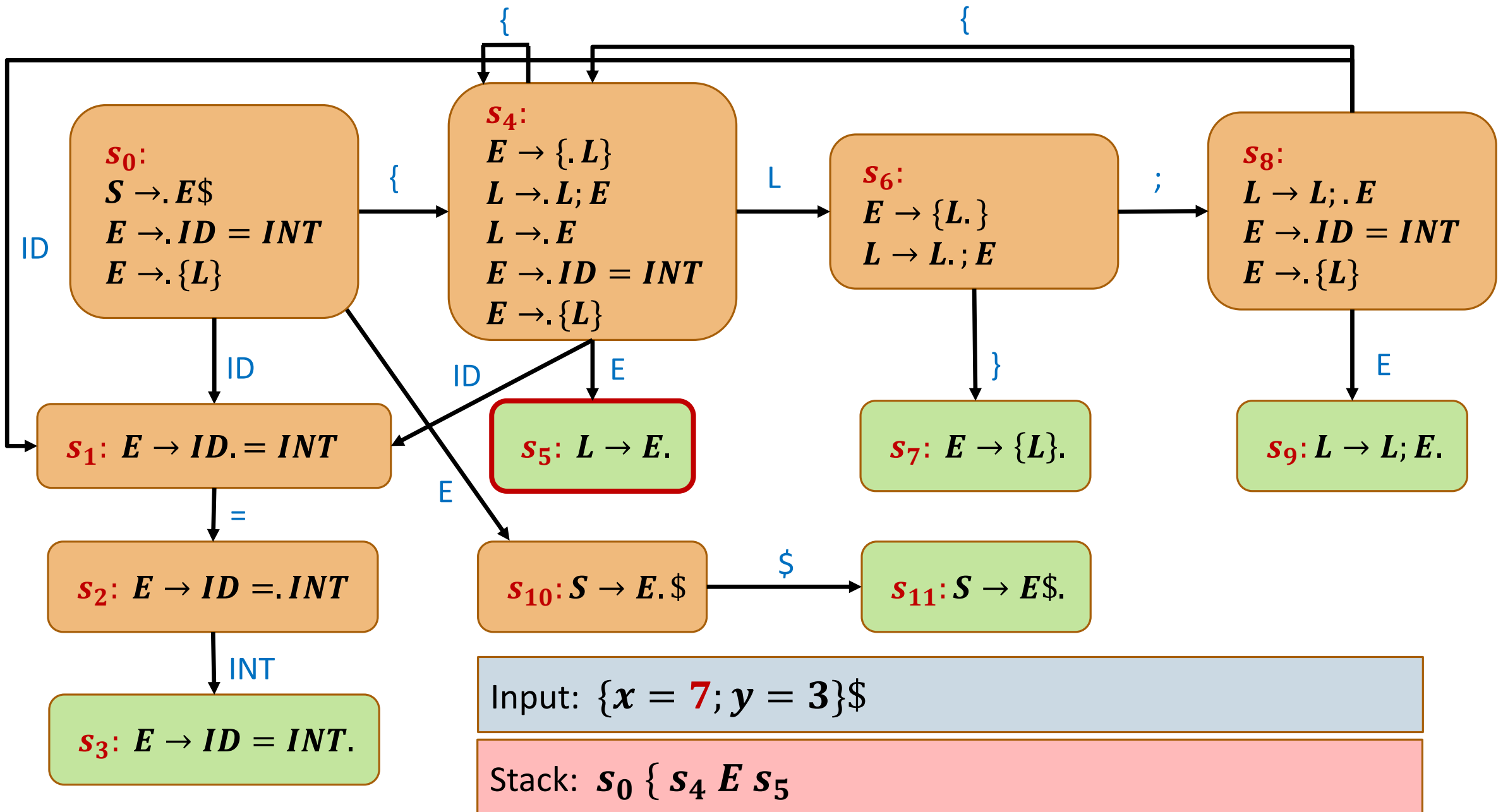


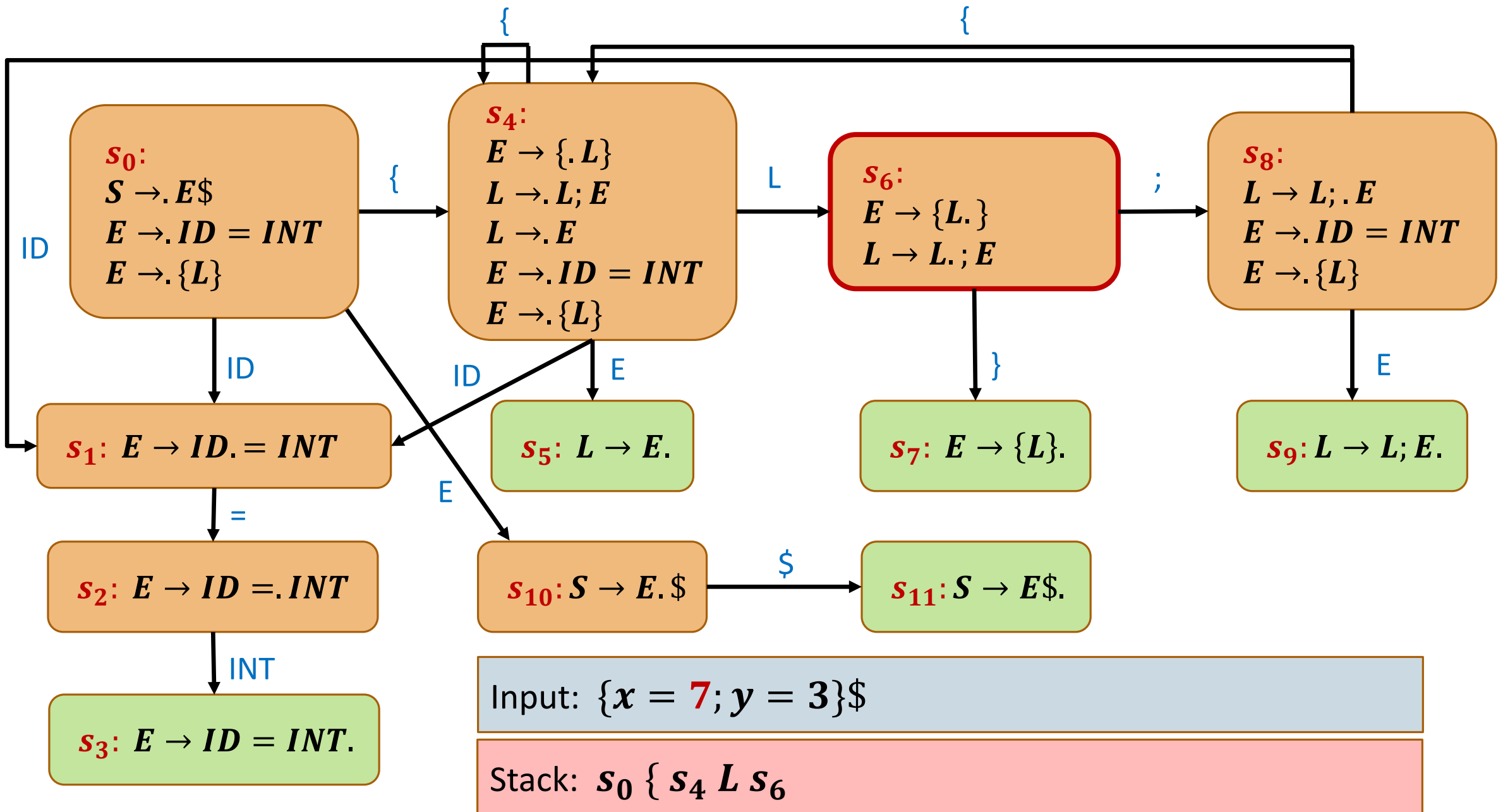


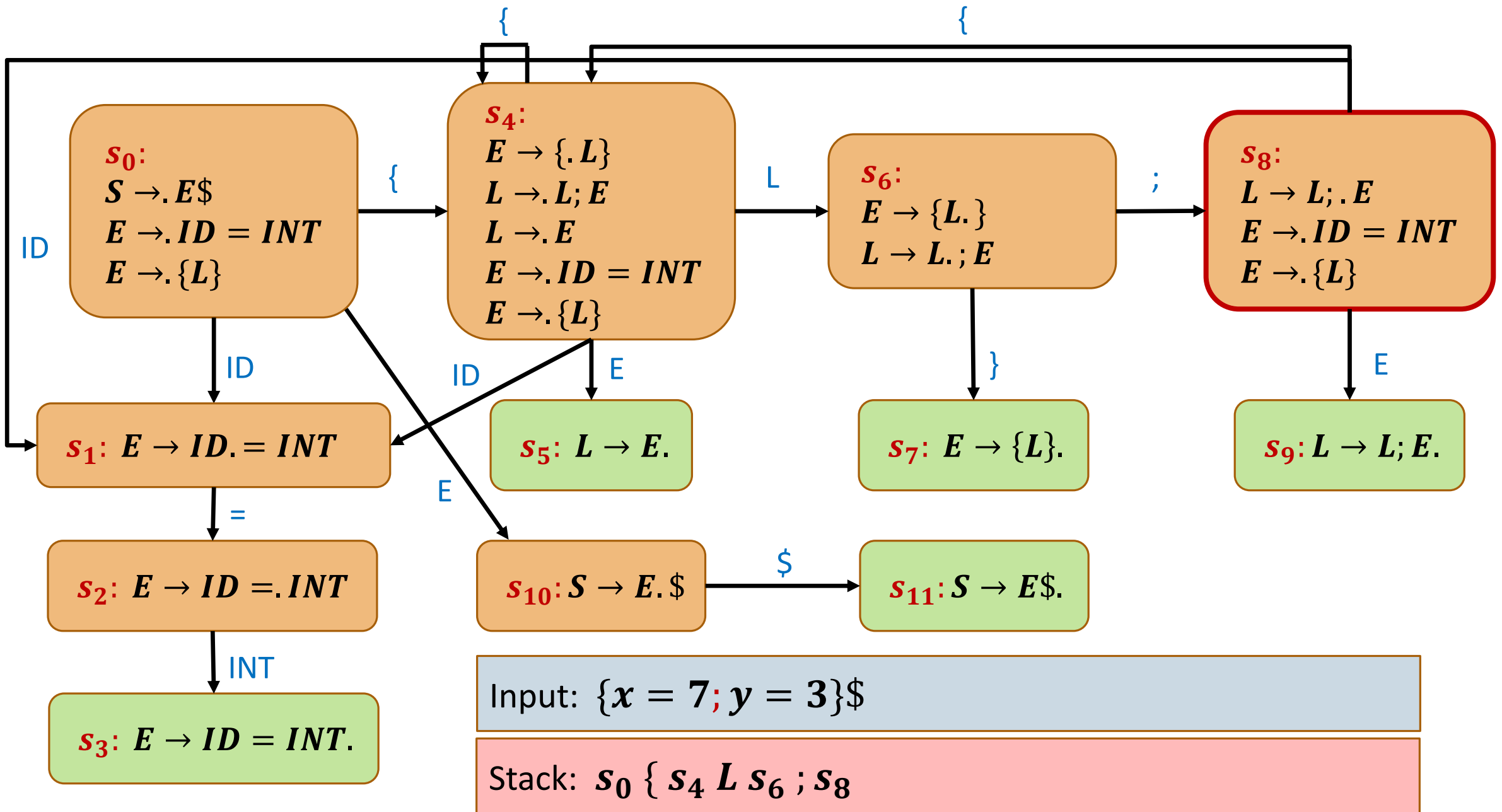


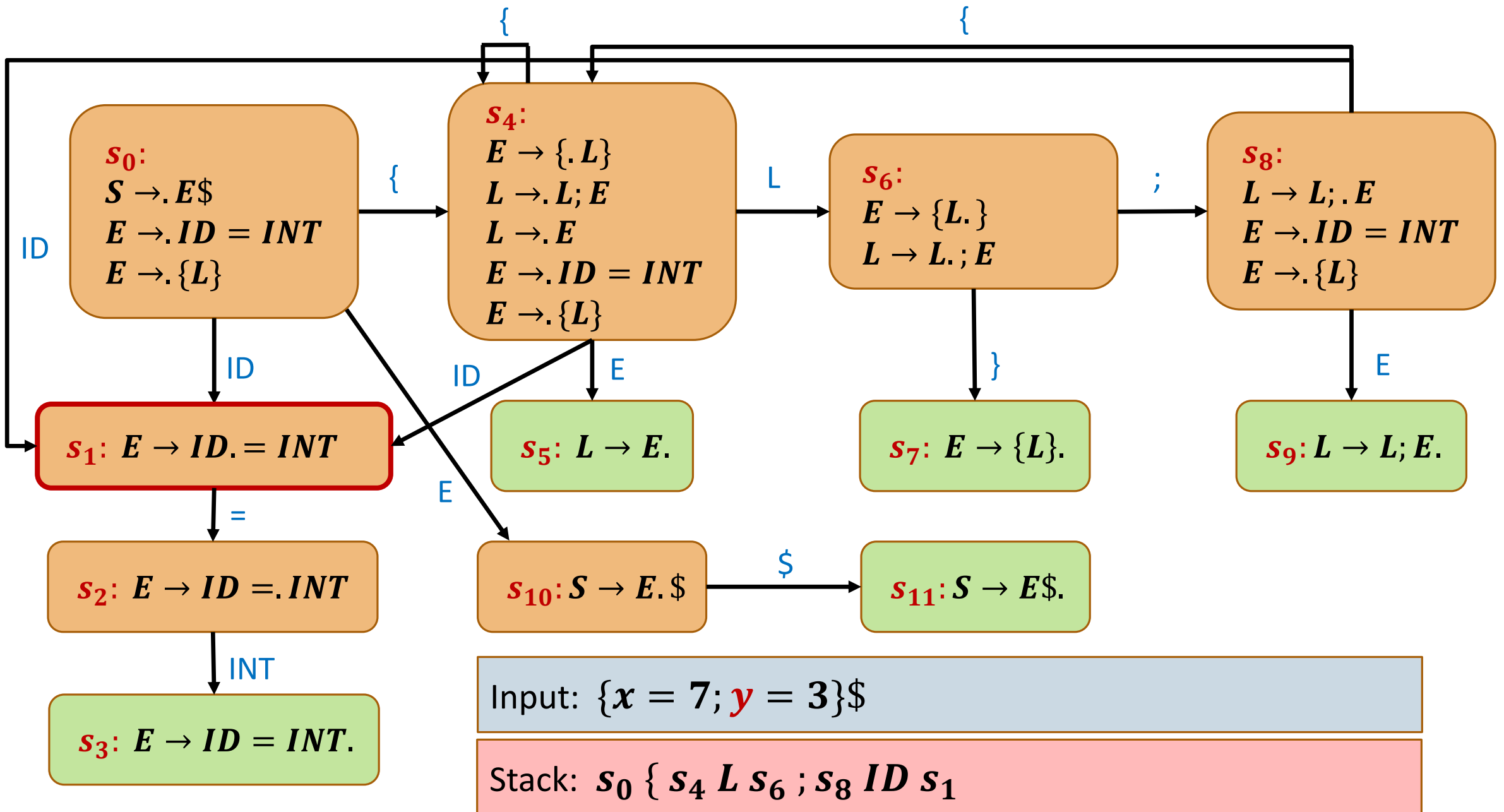


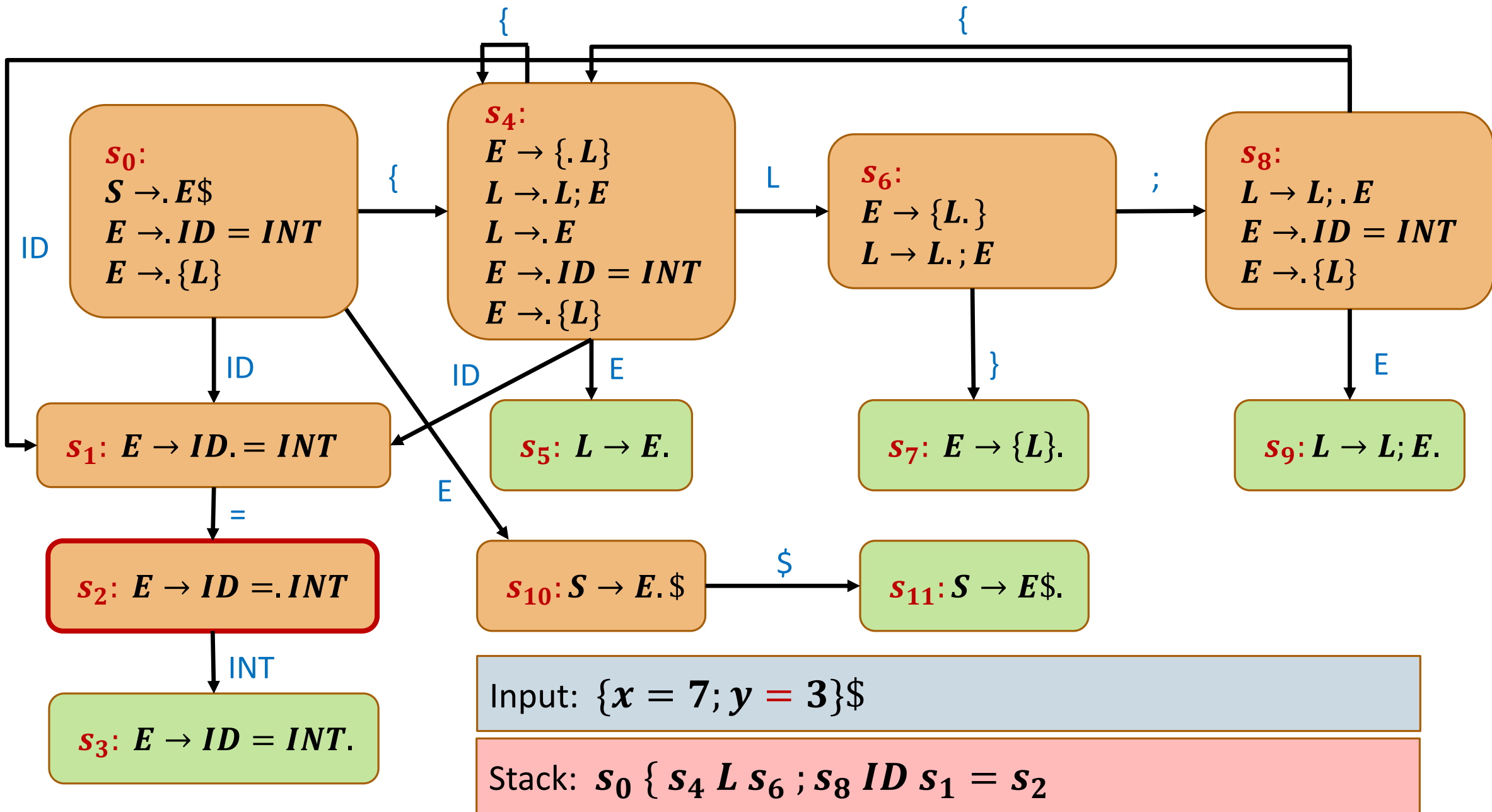


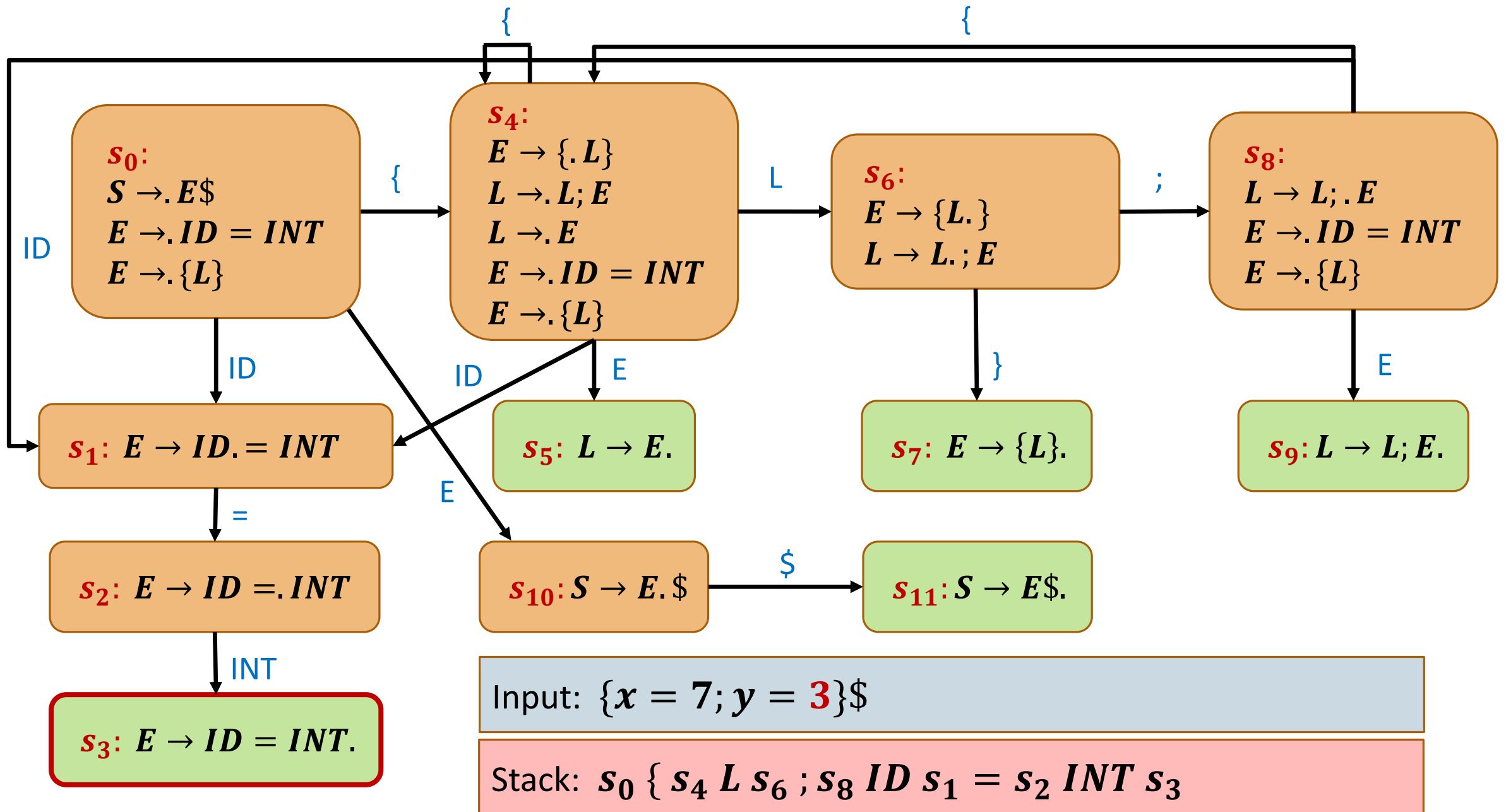


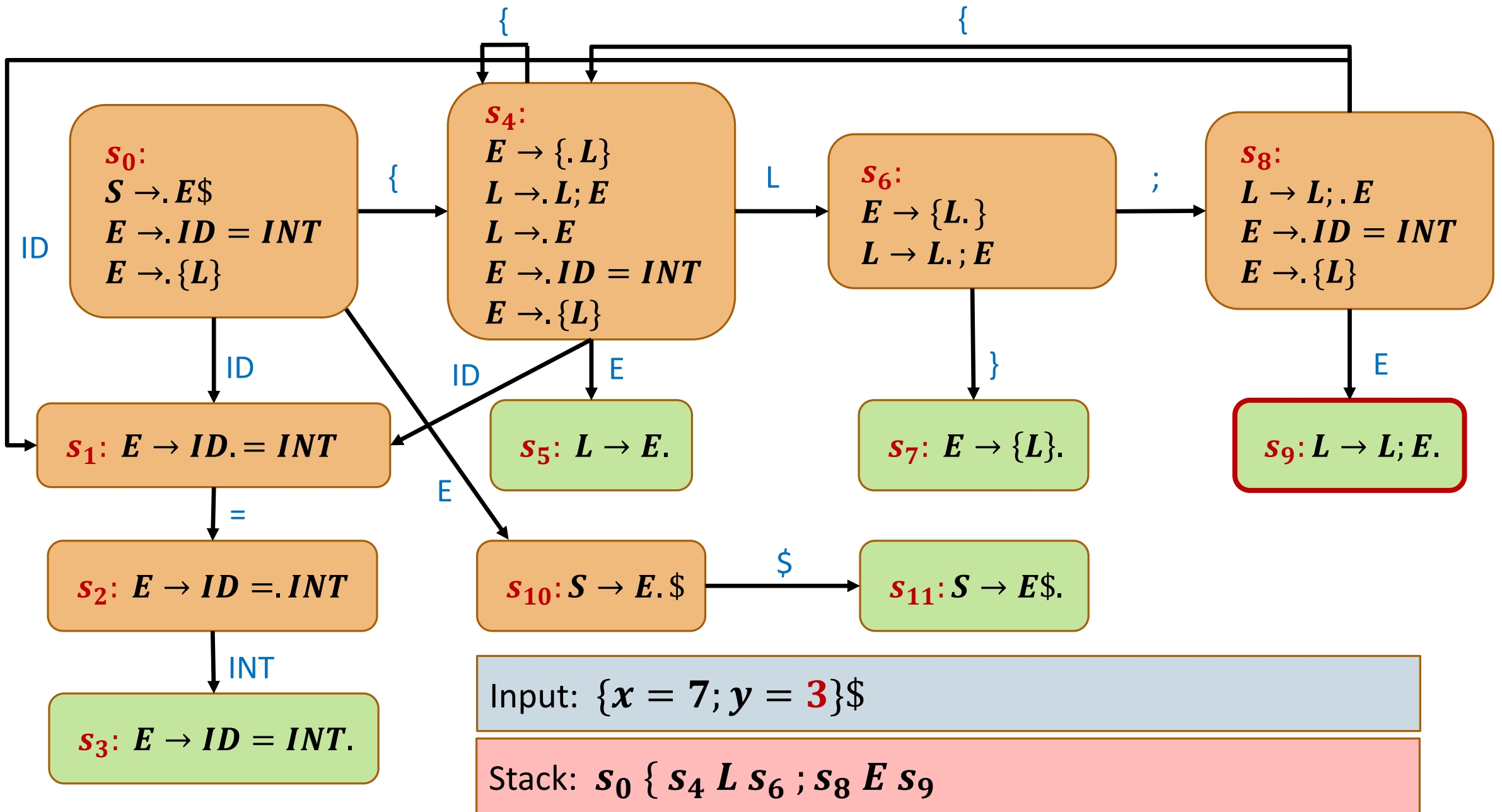


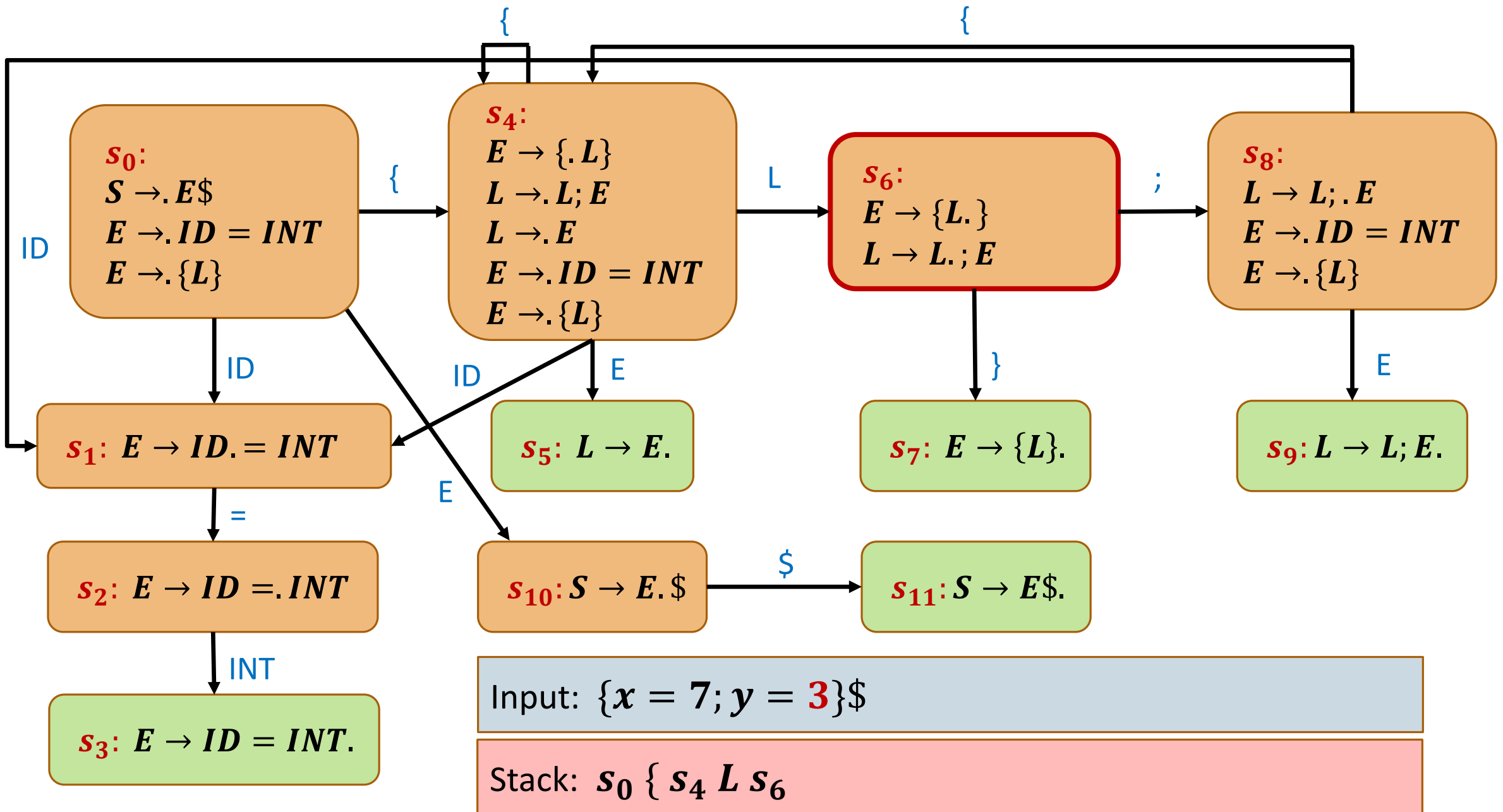


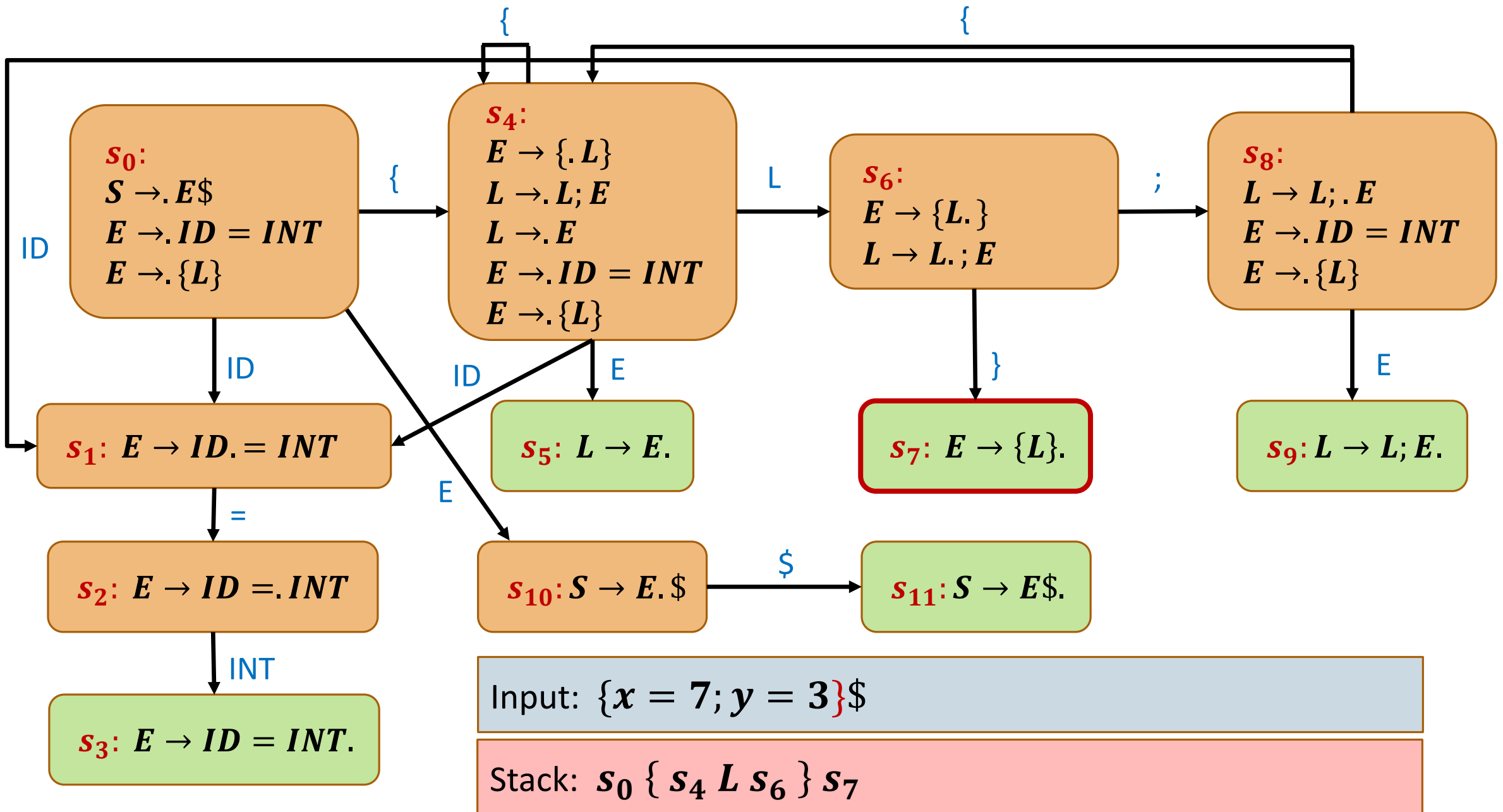


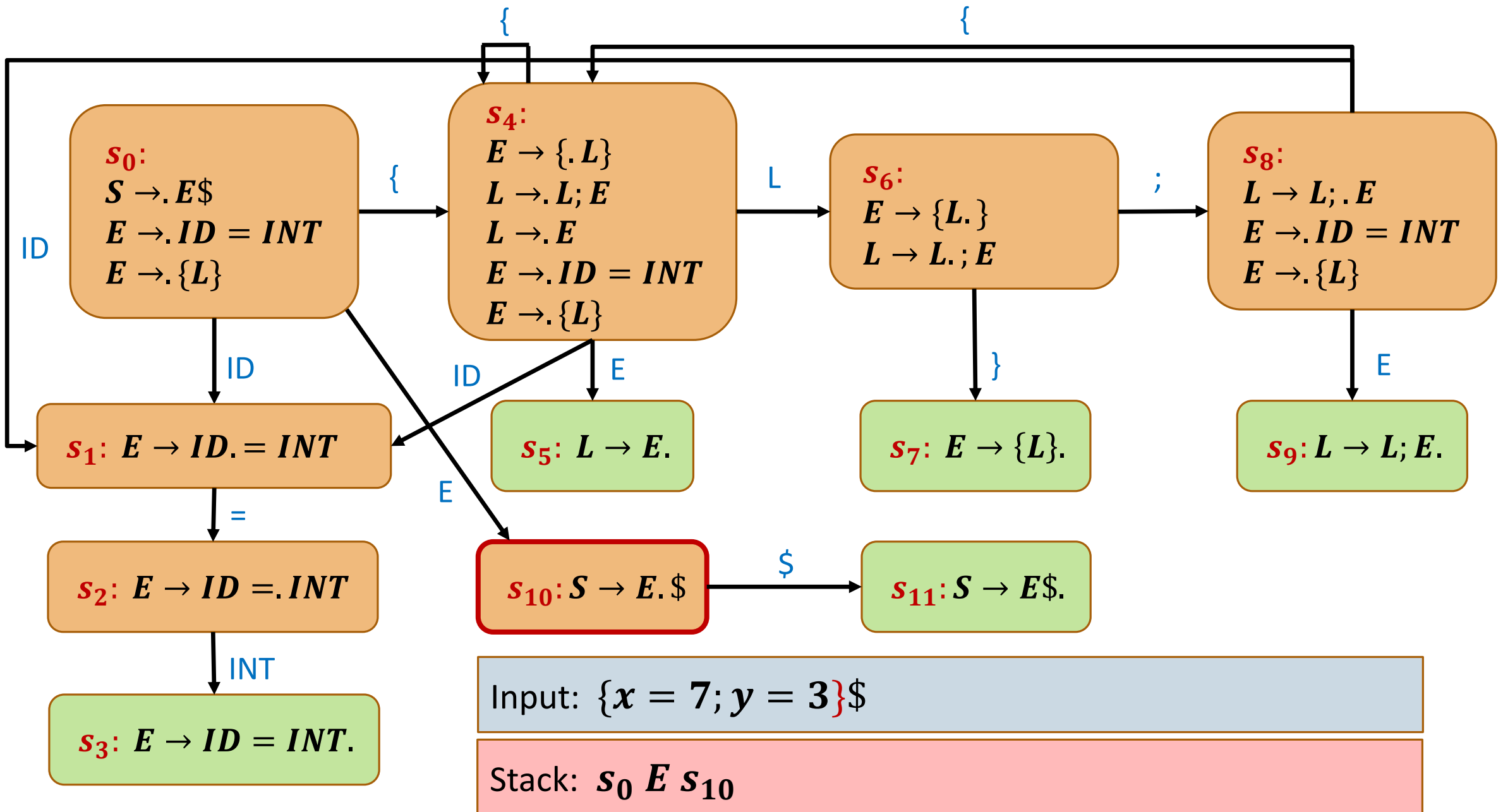


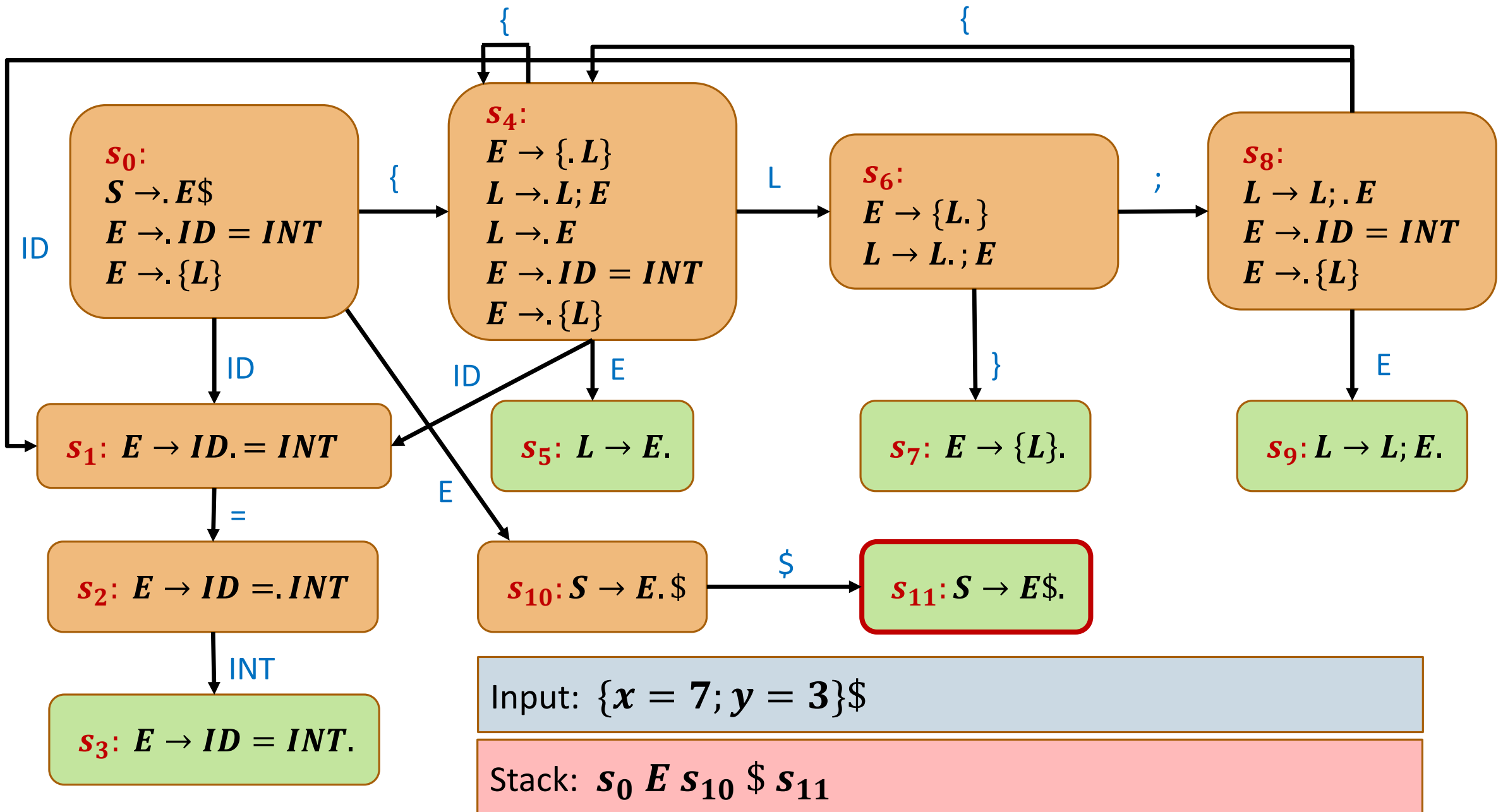


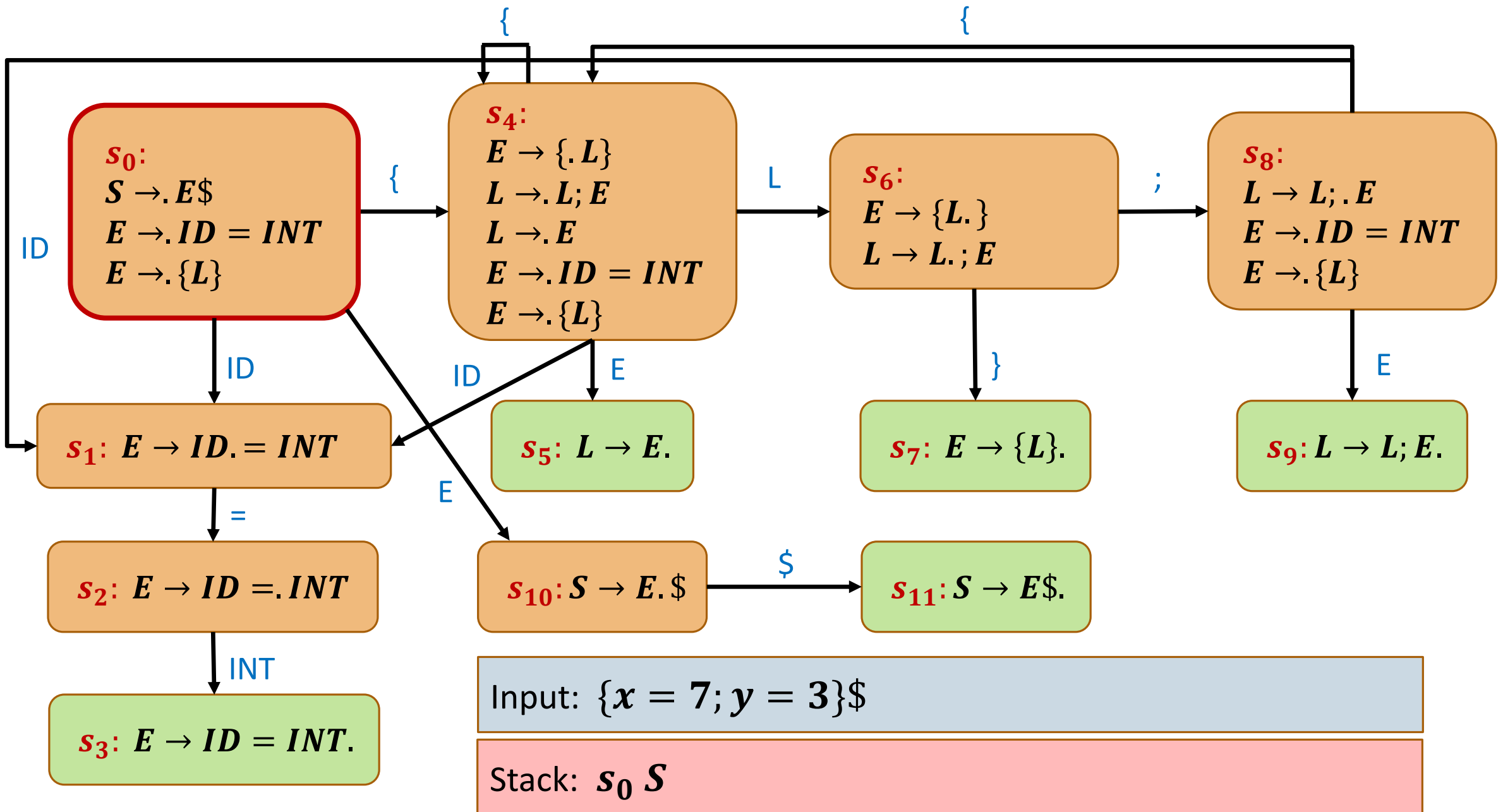










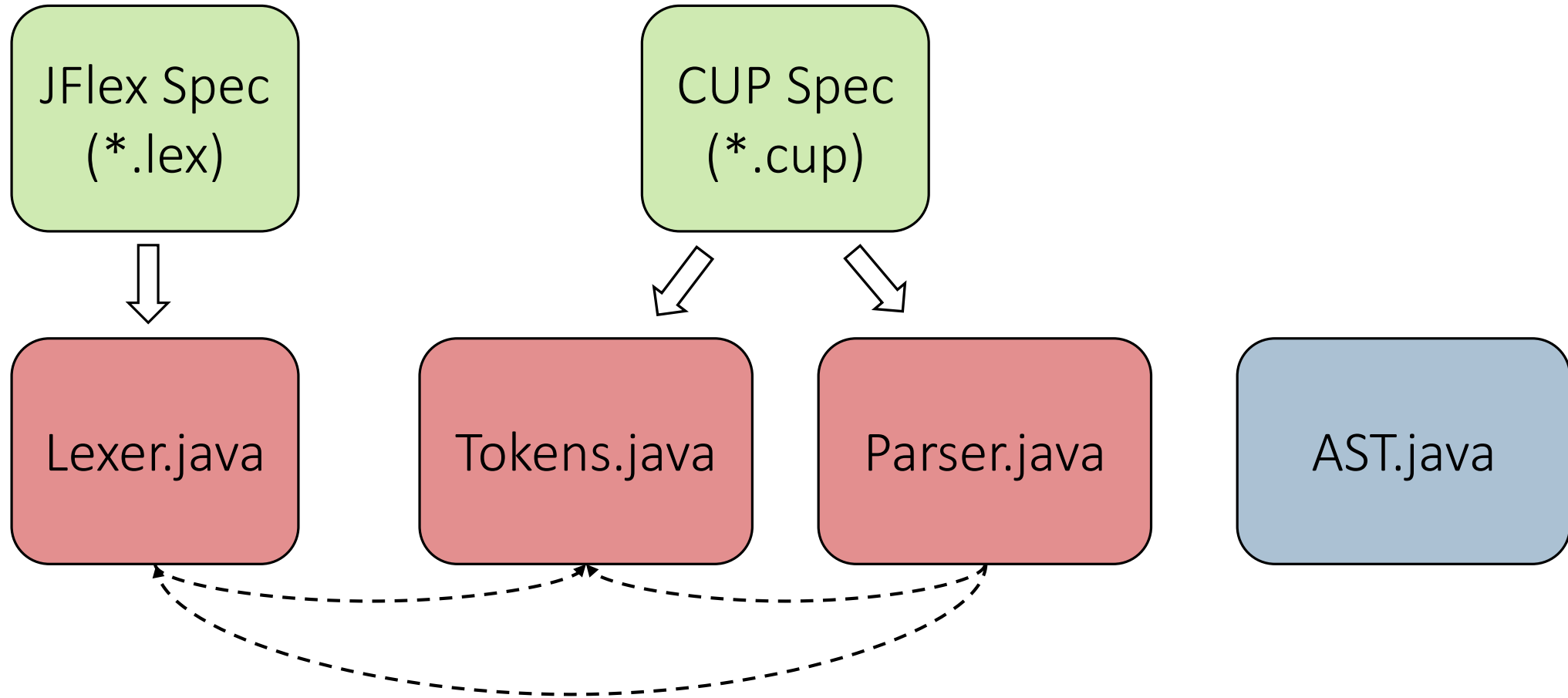


Parsing with CUP

CUP

- Given a user-specified grammar, generates an LALR parser
- Works with JFlex, which provides the parsed tokens
- Other tools:
 - Bison (for C)

CUP/JFlex Workflow



CUP Format

parser setup {

```
parser code {:  
...  
:}
```

lexer setup {

```
scan with {:  
...  
:}
```

grammar {

```
terminal ...  
non terminal ...  
start with ...  
<derivation rules...>
```

CUP Spec: Parser Setup

parser code {:

```
    public Lexer lexer;
```

```
    public Parser(Lexer lexer) {
```

```
        super(lexer);
```

```
        this.lexer = lexer;
```

```
    }
```

```
    public void report_error(String message, Object info) {
```

```
        System.exit(0);
```

```
    }
```

```
:.}
```

CUP Spec: Lexer Setup

scan with {:

Symbol s;

s = lexer.next_token();

// print token...

return s;

:};

CUP Spec: Terminals

terminal T1;

terminal T2;

terminal T3;

terminal T4;

...

CUP Spec: Non-Terminals

non terminal AST_NODE_1 E1;

non terminal AST_NODE_2 E2;

non terminal AST_NODE_3 E3;

...

CUP Spec: Operator Precedence

precedence left OP1;
precedence left OP2;
precedence left OP3;
precedence left OP4;
...

These are token names...

CUP Spec: Grammar

start with **S**;

S ::=

E1:**v1** { : RESULT = new **AST_NODE_CLASS_1**(**v1**); : } ;

S ::=

E1:**v1** **E2**:**v2** { : RESULT = new **AST_NODE_CLASS_2**(**v1**, **v2**); : }

...

CUP Spec: AST Nodes

- We need to **decide** which node types we have in our AST
- We need to **define** the classes for these AST nodes

CUP Example

Consider the following CFG:

- $E \rightarrow INT$
- $E \rightarrow V$
- $E \rightarrow E + E$
- $E \rightarrow E - E$
- $V \rightarrow ID$
- $V \rightarrow V . ID$

CUP Example: Terminals

terminal Integer INT;

terminal String ID;

terminal PLUS;

terminal MINUS;

terminal DOT;

CUP Example: Non-Terminals

non terminal AST_EXP EXP;

non terminal AST_VAR VAR;

CUP Example: Operator Precedence

precedence left PLUS;
precedence left MINUS;

CUP Example: Grammar

start with **EXP**;

EXP ::=

INT:*i* {: RESULT = new **AST_EXP_INT**(*i*); :} |

VAR:*v* {: RESULT = new **AST_EXP_VAR**(*v*); :} |

EXP:*e1* **PLUS** **EXP**:*e2* {: RESULT = new **AST_EXP_BINOP**(*e1*, *e2*, 0); :} |

EXP:*e1* **MINUS** **EXP**:*e2* {: RESULT = new **AST_EXP_BINOP**(*e1*, *e2*, 1); :};

VAR ::=

ID:*name* {: RESULT = new **AST_VAR_SIMPLE**(*name*); :} |

VAR:*v* **DOT** **ID**:*fieldName* {: RESULT = new **AST_VAR_FIELD**(*v*, *fieldName*); :};

CUP Example: AST Nodes

For the non-terminal *VAR*:

```
public abstract class AST_VAR extends AST_Node {  
  
}
```


CUP Example: AST Nodes

For the rule *VAR ::= ID:name*:

```
public class AST_VAR_SIMPLE extends AST_VAR {  
    public String name;  
    public AST_VAR_SIMPLE(String name) {  
        this.name = name;  
    }  
}
```

CUP Example: AST Nodes

For the rule *VAR ::= VAR:v DOT ID:fieldName :*

```
public class AST_VAR_FIELD extends AST_VAR {  
    public AST_VAR var;  
    public String fieldName;  
    public AST_VAR_FIELD(AST_VAR var, String fieldName) {  
        this.var = var;  
        this.fieldName = fieldName;  
    }  
}
```

CUP Example: AST Nodes

For the non-terminal *EXP*:

```
public abstract class AST_EXP extends AST_Node {  
  
}
```

CUP Example: AST Nodes

For the rule *EXP ::= INT:i*:

```
public class AST_EXP_INT extends AST_EXP {  
    public int value;  
    public AST_EXP_INT(int value) {  
        this.value = value;  
    }  
}
```

CUP Example: AST Nodes

For the rule *EXP ::= VAR:v*:

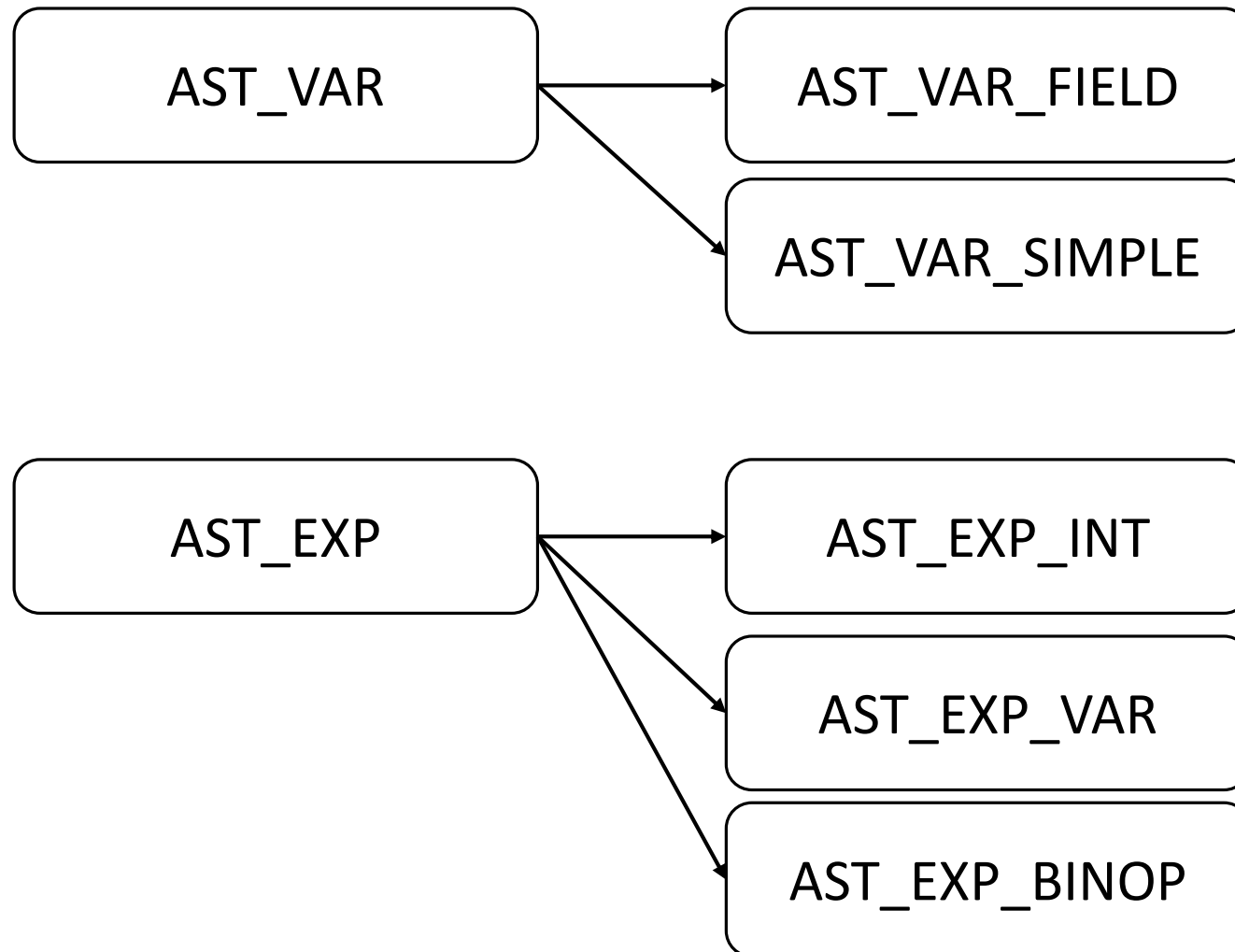
```
public class AST_EXP_VAR extends AST_EXP {  
    public AST_VAR var;  
    public AST_EXP_VAR(AST_VAR var) {  
        this.var = var;  
    }  
}
```

CUP Example: AST Nodes

For the rule $EXP ::= EXP:e1 <OP> EXP:e2 :$

```
public class AST_EXP_BINOP extends AST_EXP {  
    int OP;  
    public AST_EXP left;  
    public AST_EXP right;  
    public AST_EXP_BINOP(AST_EXP left, AST_EXP right, int OP) {  
        this.left = left;  
        this.right = right;  
        this.OP = OP;  
    }  
}
```

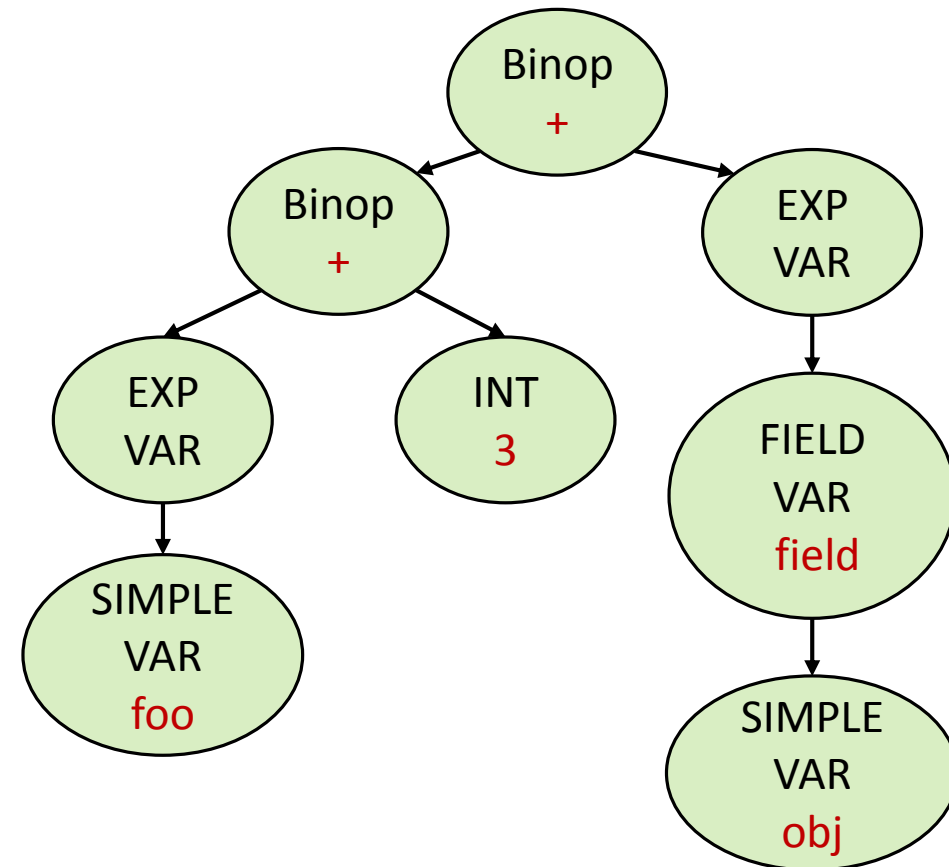
Class Hierarchy (Inheritance)



CUP Example: Debugging

We can generate an image of the AST (using the exercise template)

For the input `foo + 3 + obj.field` we have:



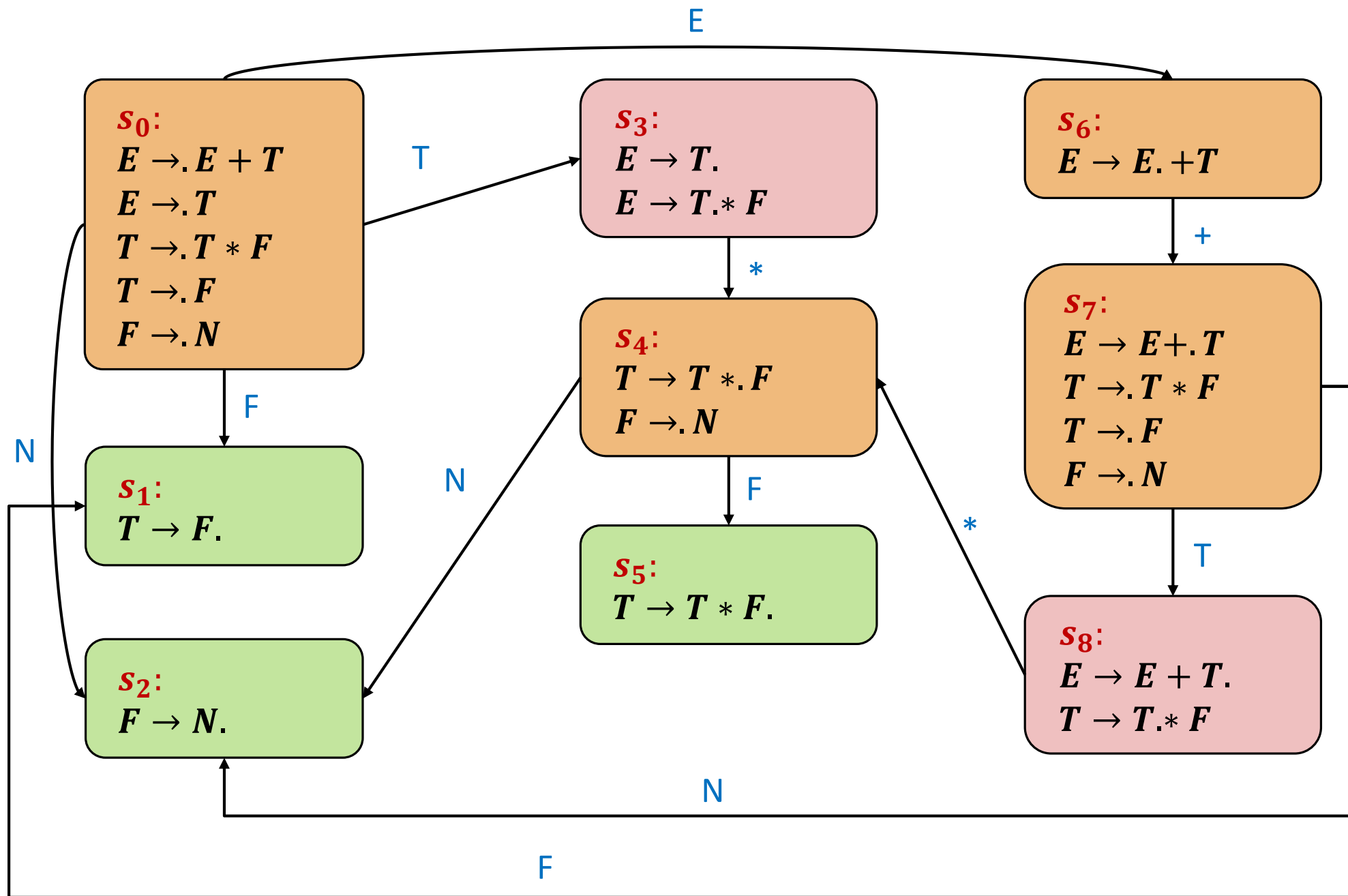
SLR(1), LR(1)

LR(0) Parsing

Consider the following CFG:

- $E \rightarrow E + T$
- $E \rightarrow T$
- $T \rightarrow T * F$
- $T \rightarrow F$
- $F \rightarrow N$

What will be the **transition system** of the LR(0) parser for this CFG?

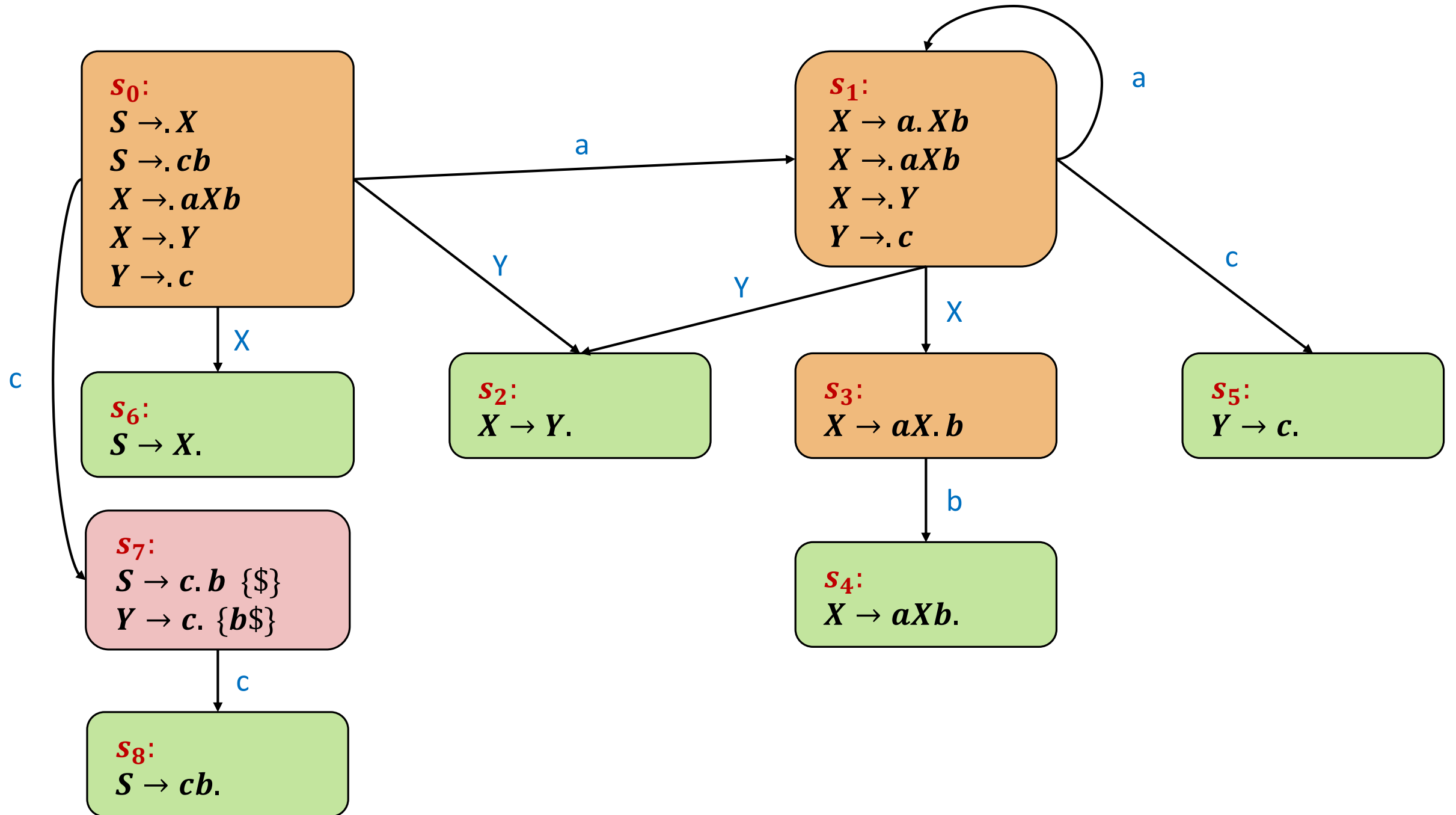


SLR(1) Parsing

Consider the following CFG:

- $S \rightarrow X$
- $S \rightarrow cb$
- $X \rightarrow aXb$
- $X \rightarrow Y$
- $Y \rightarrow c$

What will be the **transition system** of the SLR(1) parser for this CFG?



LR(1) Parsing

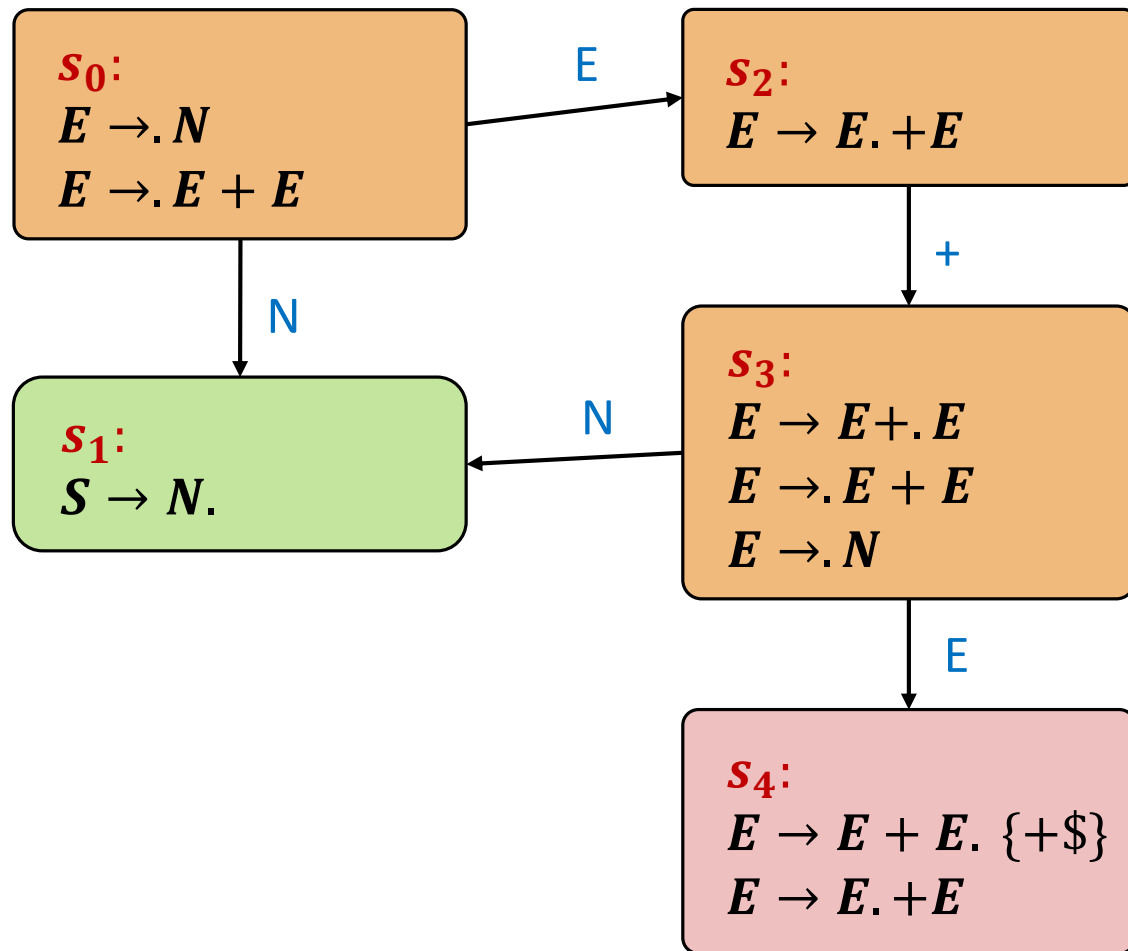
...

Resolving Conflicts

Consider the following CFG:

- $E \rightarrow N$
- $E \rightarrow E + E$

What will be the **transition system** of the SLR(1) parser for this CFG?



Resolving Conflicts

Consider the following CFG:

- $E \rightarrow N$
- $E \rightarrow E + E$
- $E \rightarrow E * E$

What will be the **transition system** of the SLR(1) parser for this CFG?

