Bottom Up Parsing

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LR(0) Parsing

- Build the derivation tree from the bottom
- First build the children, then connect to the parent
- Can handle left recursion
  - Which is common in real-world grammars

LR(0) Item

An **LR(0) item** is of the form:

- $N \rightarrow \alpha.\beta$

The **dot** gives us the current location (a local view).

LR(0) Item

An LR(0) item with the dot at the end is called **reduce** item:

- $N \to \alpha\beta.$

Otherwise, it's a **shift** item:

- $N \to .\alpha\beta$
- $N \to \alpha.\beta$

LR(0) Item Closure Set

The **LR(0) closure set** of an LR(0) item i is a set S such that:

- $i \in S$
- If $A \rightarrow \alpha . N\beta \in S$ then for each rule $N \rightarrow \gamma$:
  - $N \rightarrow . \gamma \in S$

LR(0) Item Closure Set

For example, given the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = X$
- $E \rightarrow \{ID\}$
- $X \rightarrow INT$

the closure set of the $S \rightarrow . E\$$ contains:

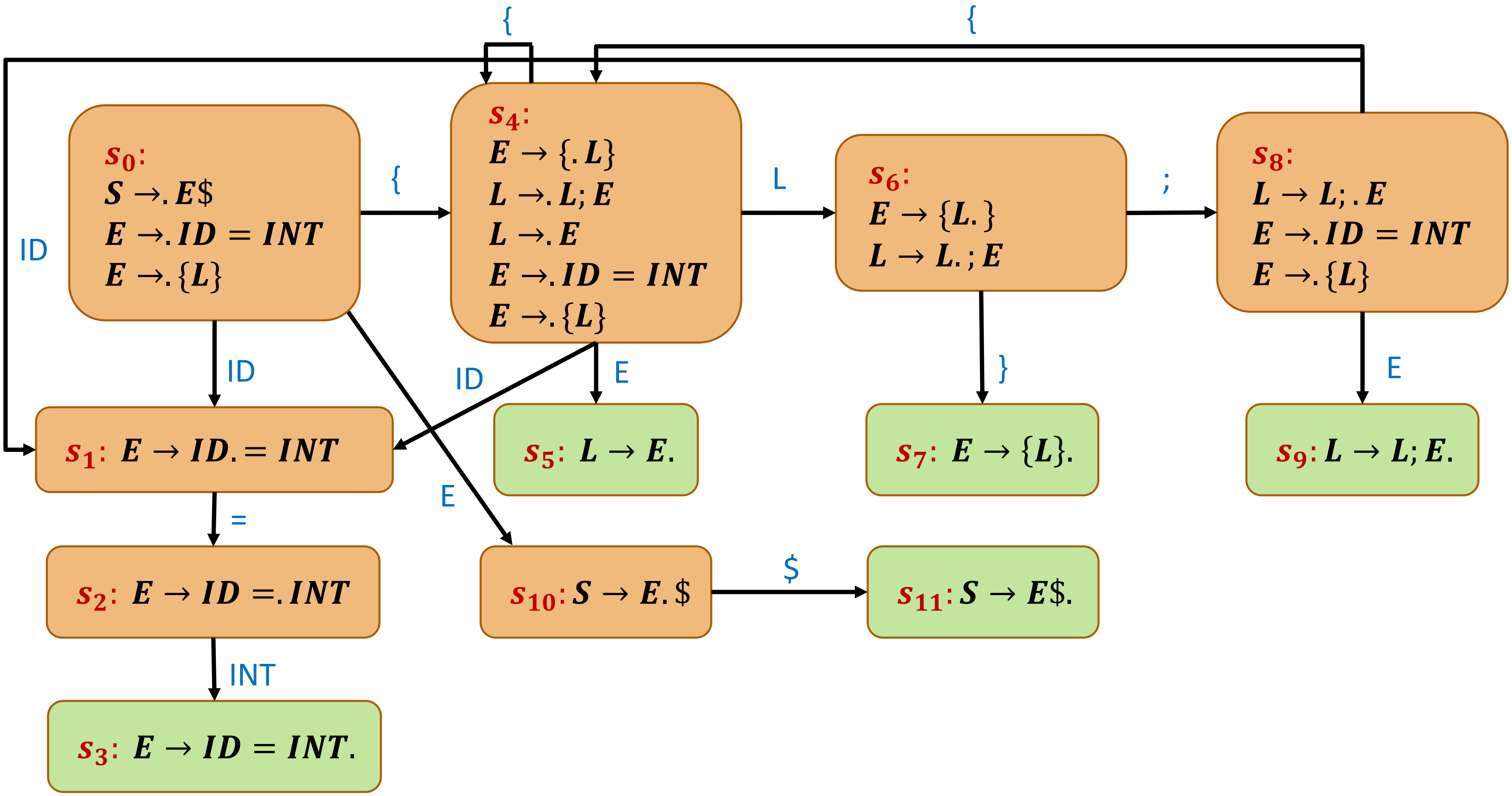
- $S \rightarrow . E\$$
- $E \rightarrow . ID = X$
- $E \rightarrow . \{ID\}$

LR(0) Parsing

Consider the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = INT$
- $E \rightarrow \{L\}$
- $L \rightarrow E$
- $L \rightarrow L; E$

What will be the **transition system** of the LR(0) parser for this CFG?



s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID =. INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L;E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.;E$
s_7: $E \to \{L\}.$
s_8: $L \to L;.E$ $E \to .ID = INT$ $E \to .\{L\}$
s_9: $L \to L;E.$
s_{10}: $S \to E.\$$
s_{11}: $S \to E\$.$
{
{
{
L
;
ID
ID
ID
E
E
E
}
=
INT
$

LR(0) Parser

We start with the initial LR(0) item (that comes from the initial rule):

- $S \rightarrow .E\$$

The initial state is the ϵ-**closure** of that item, which contains:

$$S \rightarrow E\$$$
$$E \rightarrow ID = INT$$
$$E \rightarrow \{L\}$$
$$L \rightarrow E$$
$$L \rightarrow L; E$$

LR(0) Parser

We start with the initial LR(0) item (that comes from the initial rule):

- $S \rightarrow . E\$$

The initial state is the ϵ-closure of that item, which contains:

- $S \rightarrow . E\$$
- $E \rightarrow . ID = INT$
- $E \rightarrow . \{L\}$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

s_0:

$S \rightarrow .E\$$

$E \rightarrow .ID = INT$

$E \rightarrow .\{L\}$

LR(0) Parser

From s_0, if we recognized ID, then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

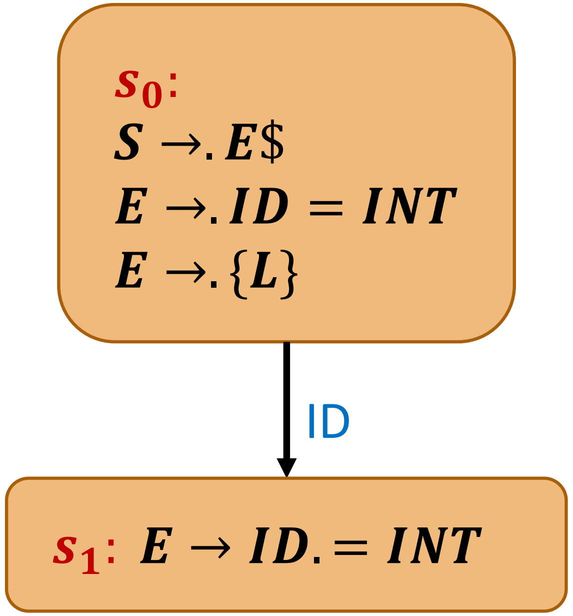
From s_0, if we recognized ID, then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID. = INT$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$

LR(0) Parser

From s_1, if we recognized $=$, then the next state will contain:

- $E \rightarrow ID = .INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

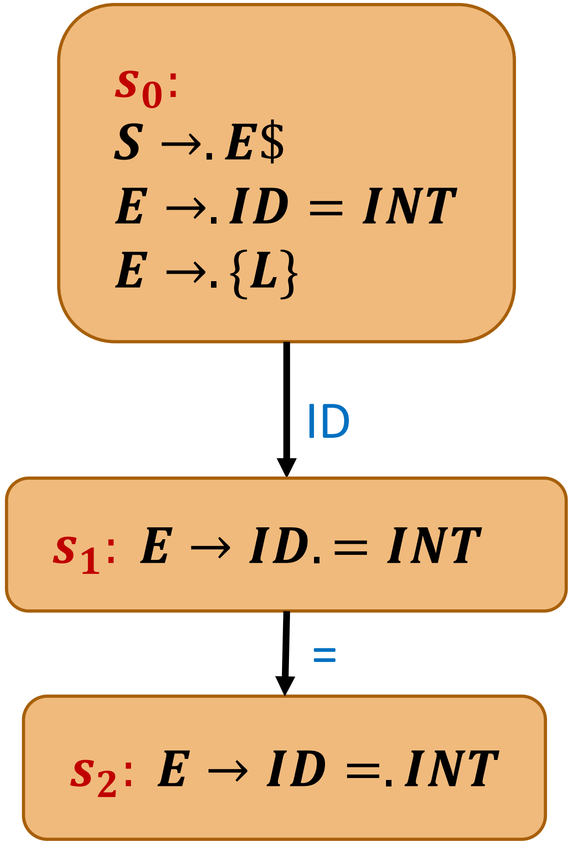
From s_1, if we recognized $=$, then the next state will contain:

- $E \rightarrow ID =. INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID =. INT$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID = .INT$

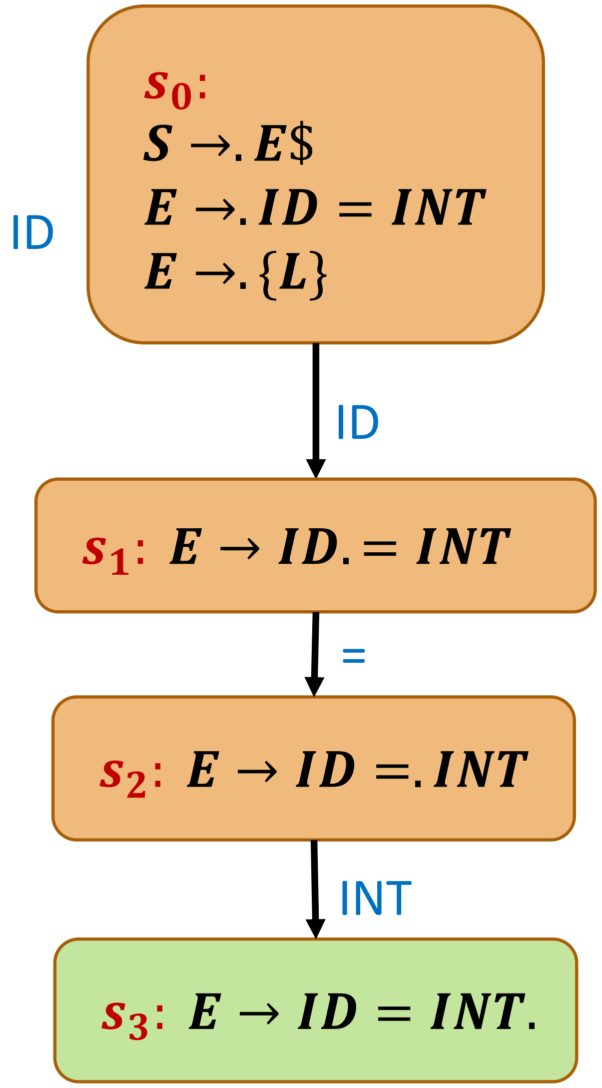
LR(0) Parser

From s_2, if we recognized INT, then the next state will contain:

- $E \to ID = INT.$

Which is a reduce state.

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_0, if we recognized $\{$, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$

$E \rightarrow ID = INT$

$E \rightarrow \{L\}$

$L \rightarrow E$

$L \rightarrow L; E$

LR(0) Parser

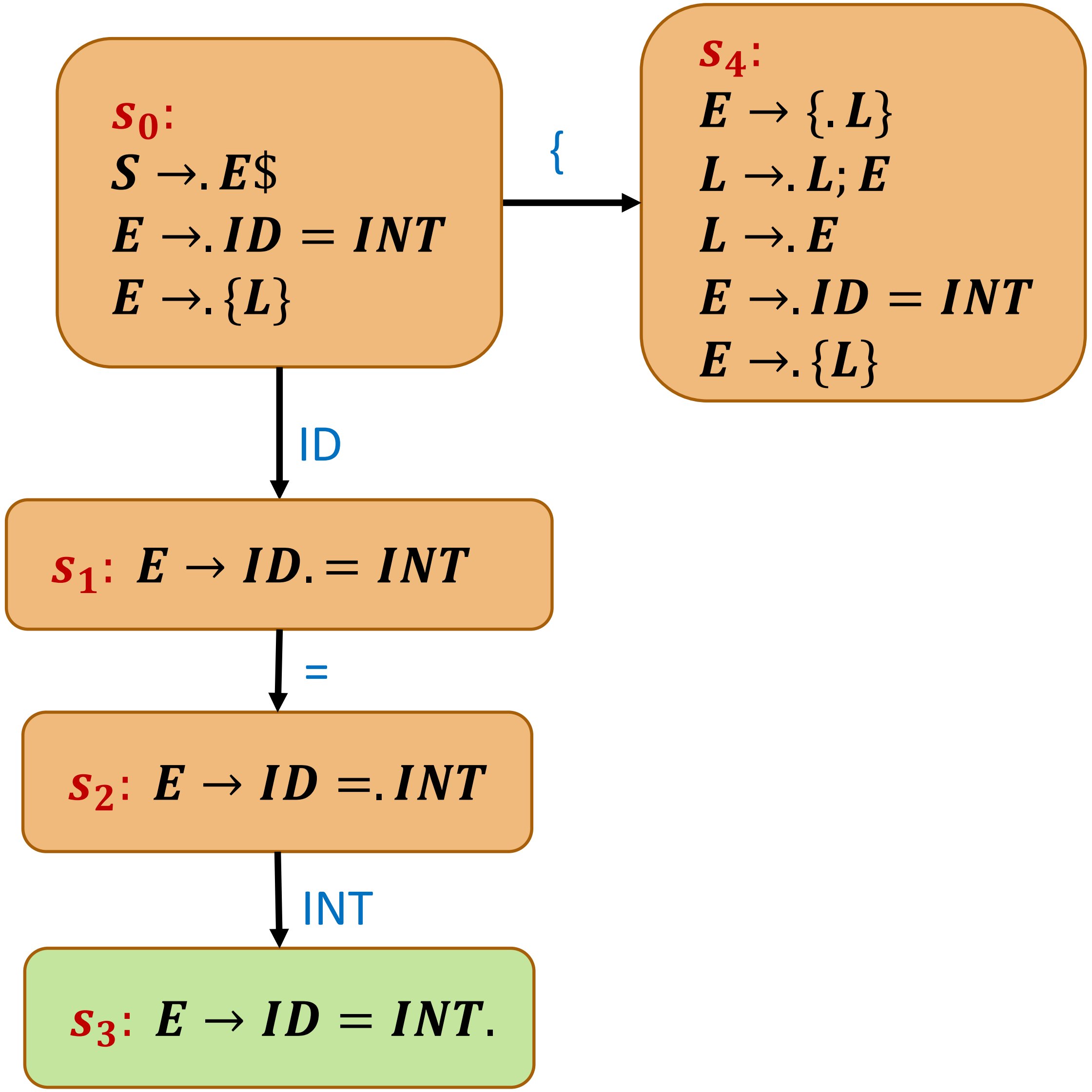
From s_0, if we recognized {, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L; E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s_0:
$S \to .E\$$
$E \to .ID = INT$
$E \to .\{L\}$
{
s_4:
$E \to \{.L\}$
$L \to .L; E$
$L \to .E$
$E \to .ID = INT$
$E \to .\{L\}$
ID
s_1: $E \to ID. = INT$
=
s_2: $E \to ID = .INT$
INT
s_3: $E \to ID = INT.$

LR(0) Parser

From s_4, if we recognized $\{$, then the next state will contain:

- $E \to \{.L\}$

So the next state (the ϵ-closure) contains:

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$

LR(0) Parser

From s_4, if we recognized {, then the next state will contain:

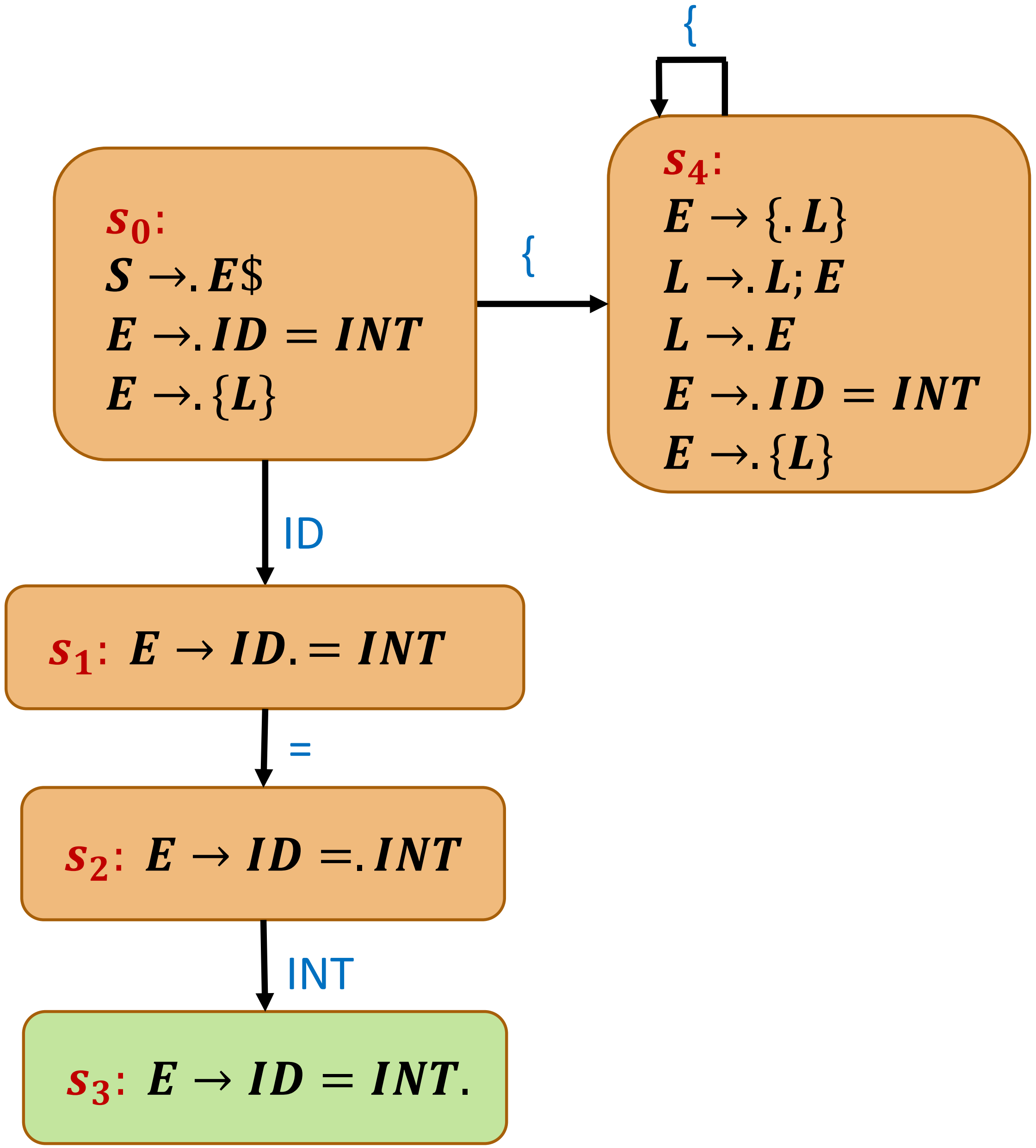
- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L;E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$



s_0:
$S \to .E\$$
$E \to .ID = INT$
$E \to .\{L\}$
{
s_4:
$E \to \{.L\}$
$L \to .L; E$
$L \to .E$
$E \to .ID = INT$
$E \to .\{L\}$
{
ID
s_1: $E \to ID. = INT$
=
s_2: $E \to ID =.INT$
INT
s_3: $E \to ID = INT.$

LR(0) Parser

From s_4, if we recognized ID, then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_4, if we recognized ID, then the next state will contain:

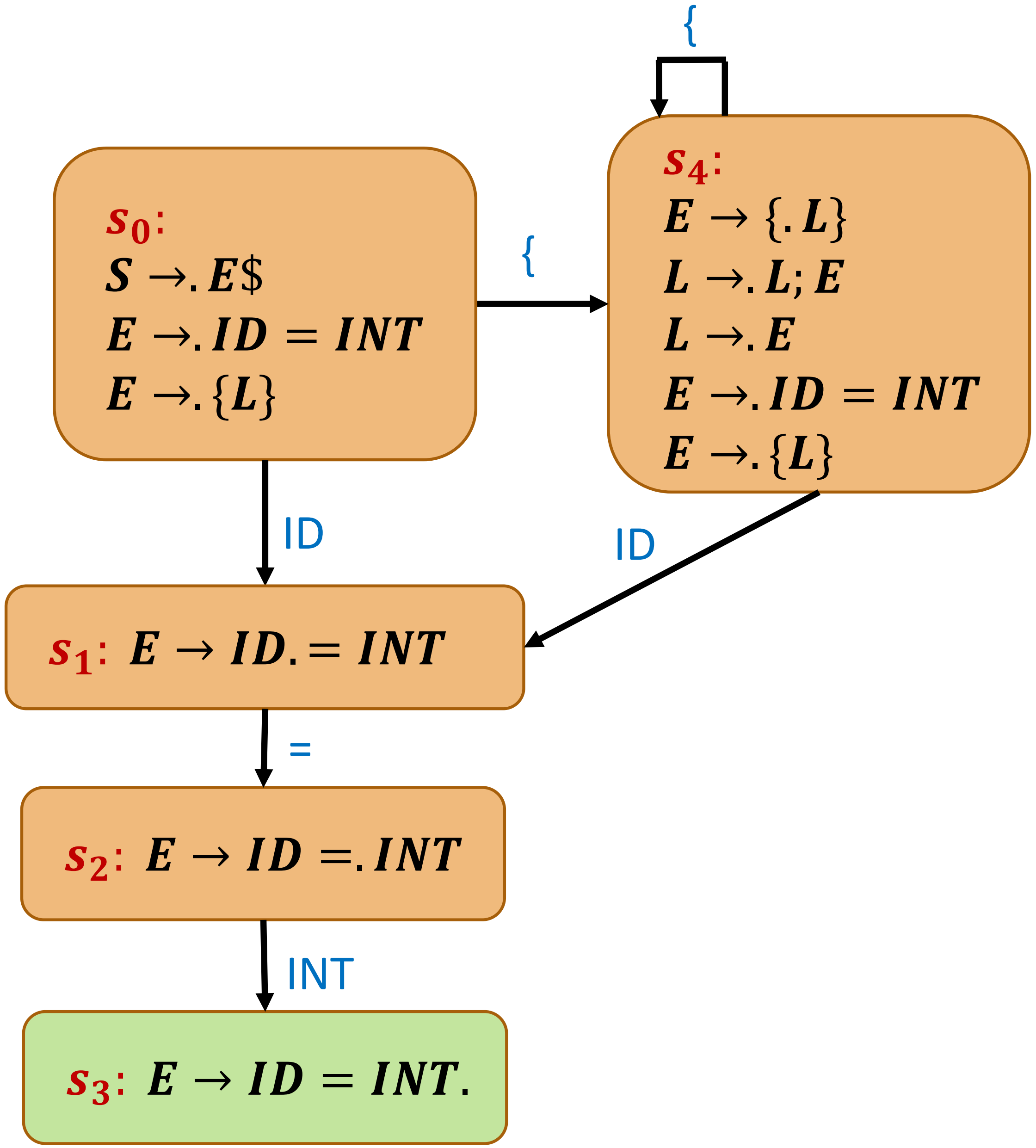
- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID. = INT$

which was already computed: s_1

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_4:
$E \to \{.L\}$
$L \to .L;E$
$L \to .E$
$E \to .ID = INT$
$E \to .\{L\}$
{
s_0:
$S \to .E\$$
$E \to .ID = INT$
$E \to .\{L\}$
{
ID
ID
s_1: $E \to ID. = INT$
=
s_2: $E \to ID =. INT$
INT
s_3: $E \to ID = INT.$

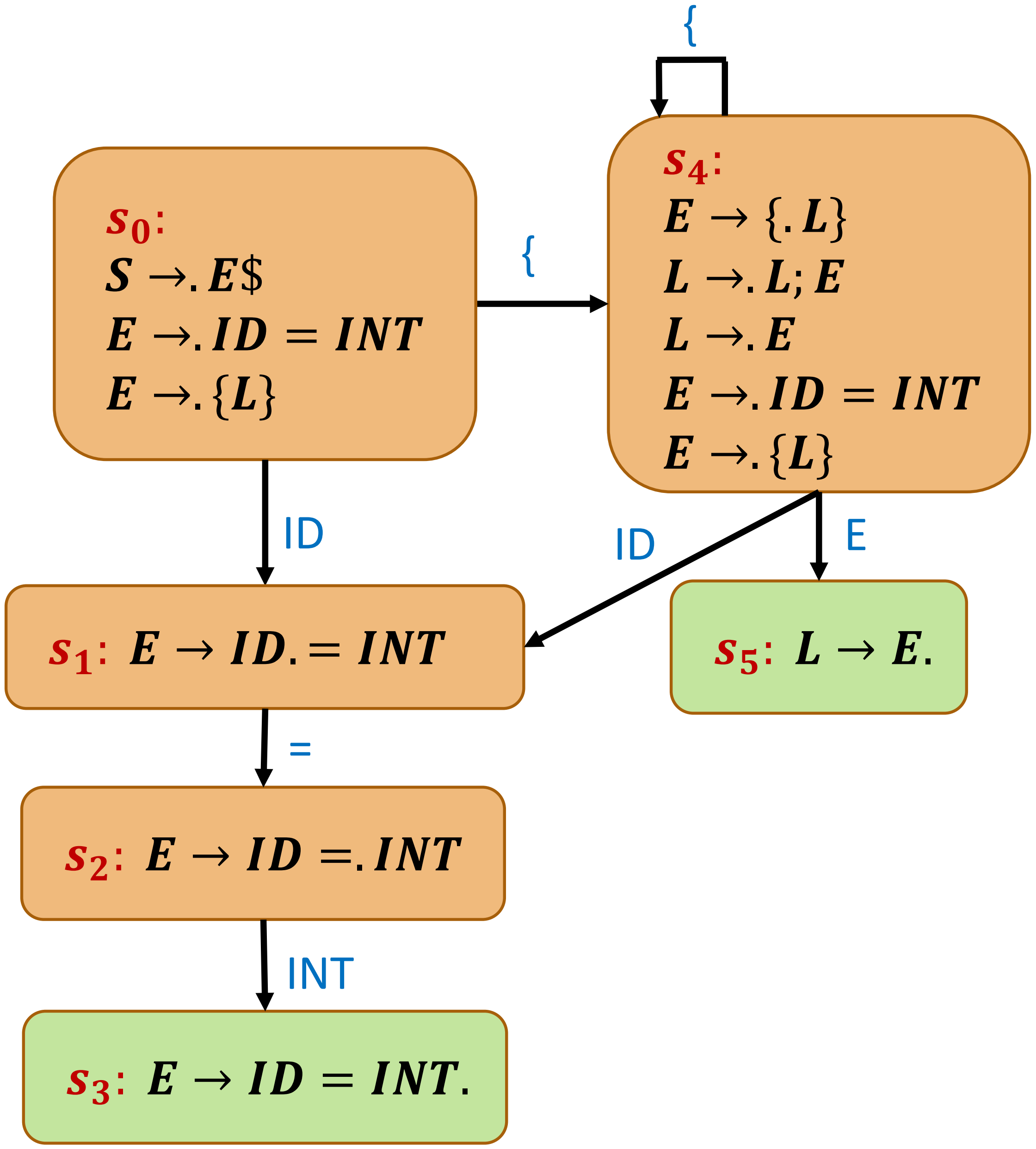
LR(0) Parser

From s_4, if we recognized E, then the next state will contain:

- $L \rightarrow E.$

which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s0:
S →. E$
E →. ID = INT
E →. {L}
{
s4:
E → {. L}
L →. L; E
L →. E
E →. ID = INT
E →. {L}
{
ID
ID
E
s1: E → ID. = INT
s5: L → E.
=
s2: E → ID =. INT
INT
s3: E → ID = INT.

LR(0) Parser

From s_4, if we recognized L, then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

So the next state (the ϵ-closure) contains:

$$S \rightarrow E\$$$
$$E \rightarrow ID = INT$$
$$E \rightarrow \{L\}$$
$$L \rightarrow E$$
$$L \rightarrow L;E$$

LR(0) Parser

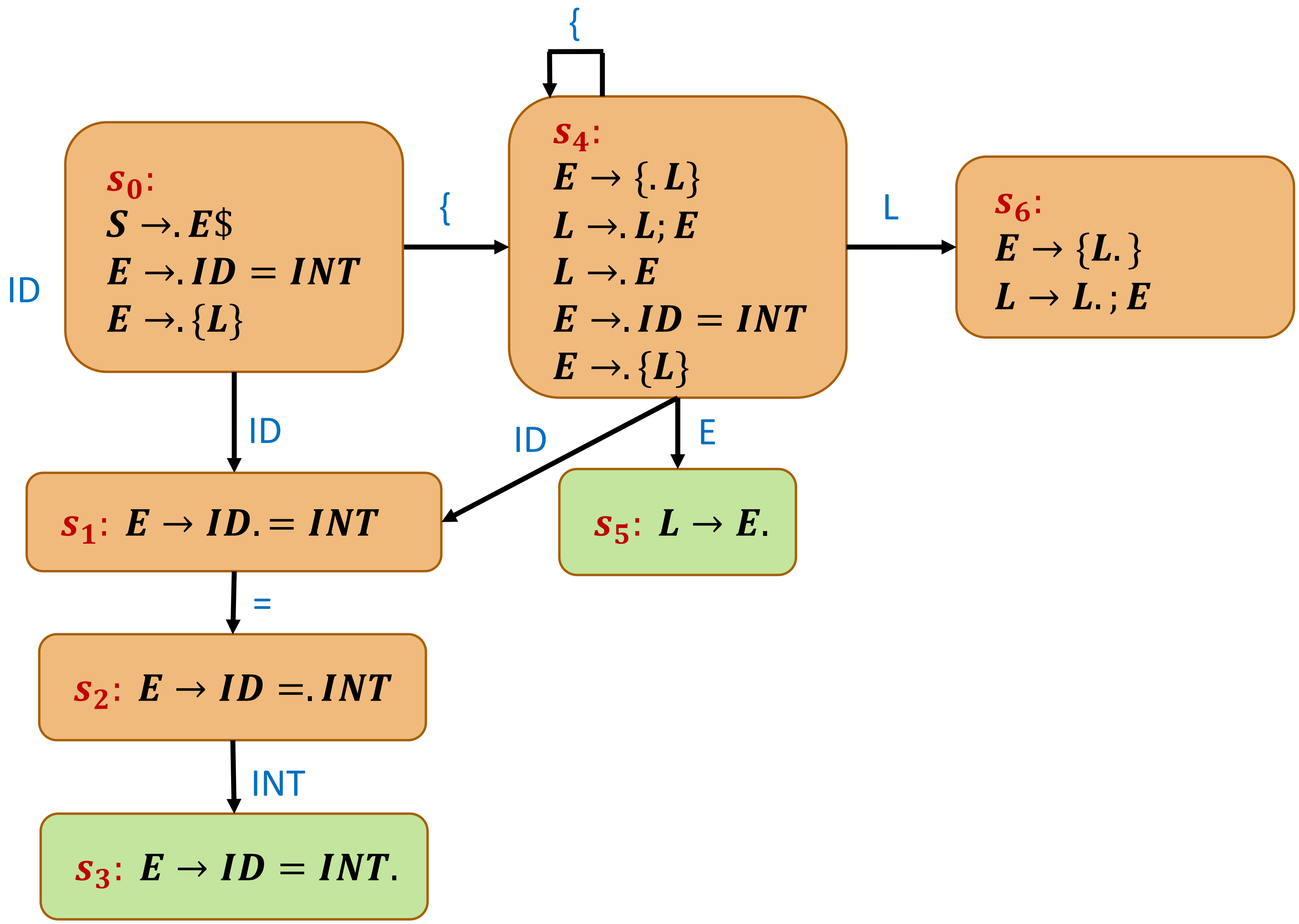
From s_4, if we recognized L, then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$



s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
ID
{
s_4: $E \to \{.L\}$ $L \to .L;E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
{
L
s_6: $E \to \{L.\}$ $L \to L.;E$
ID
ID
E
s_1: $E \to ID. = INT$
s_5: $L \to E.$
=
s_2: $E \to ID = .INT$
INT
s_3: $E \to ID = INT.$

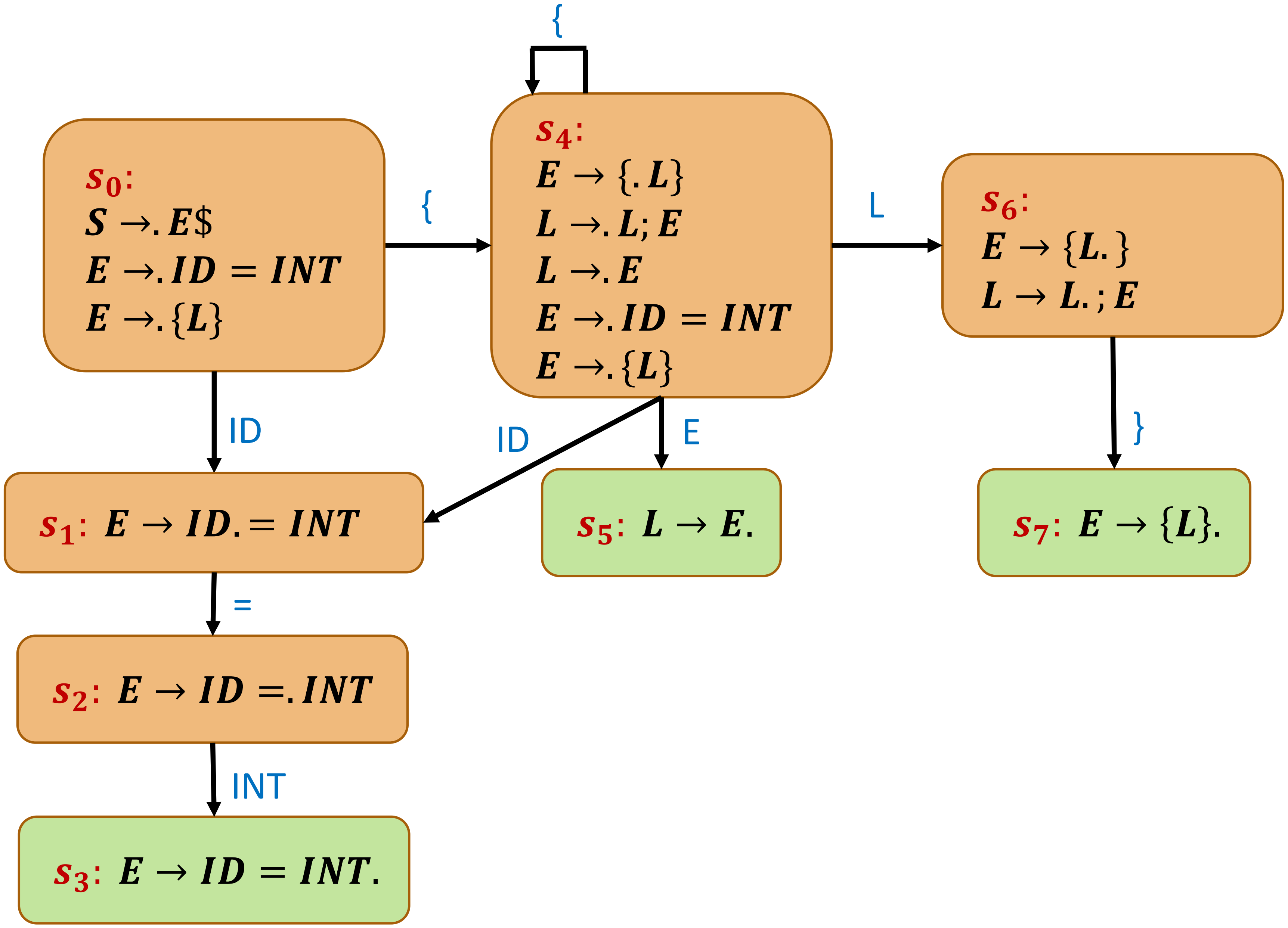
LR(0) Parser

From s_6, if we recognized $\}$, then the next state will contain:

- $E \rightarrow \{L\}.$

Which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_0: $S \rightarrow .E\$$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_4: $E \rightarrow \{.L\}$ $L \rightarrow .L;E$ $L \rightarrow .E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_6: $E \rightarrow \{L.\}$ $L \rightarrow L.;E$
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_2: $E \rightarrow ID =.INT$
s_3: $E \rightarrow ID = INT.$
{
{
L
ID
ID
E
}
=
INT

LR(0) Parser

From s_6, if we recognized ;, then the next state will contain:

- $L \rightarrow L; . E$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

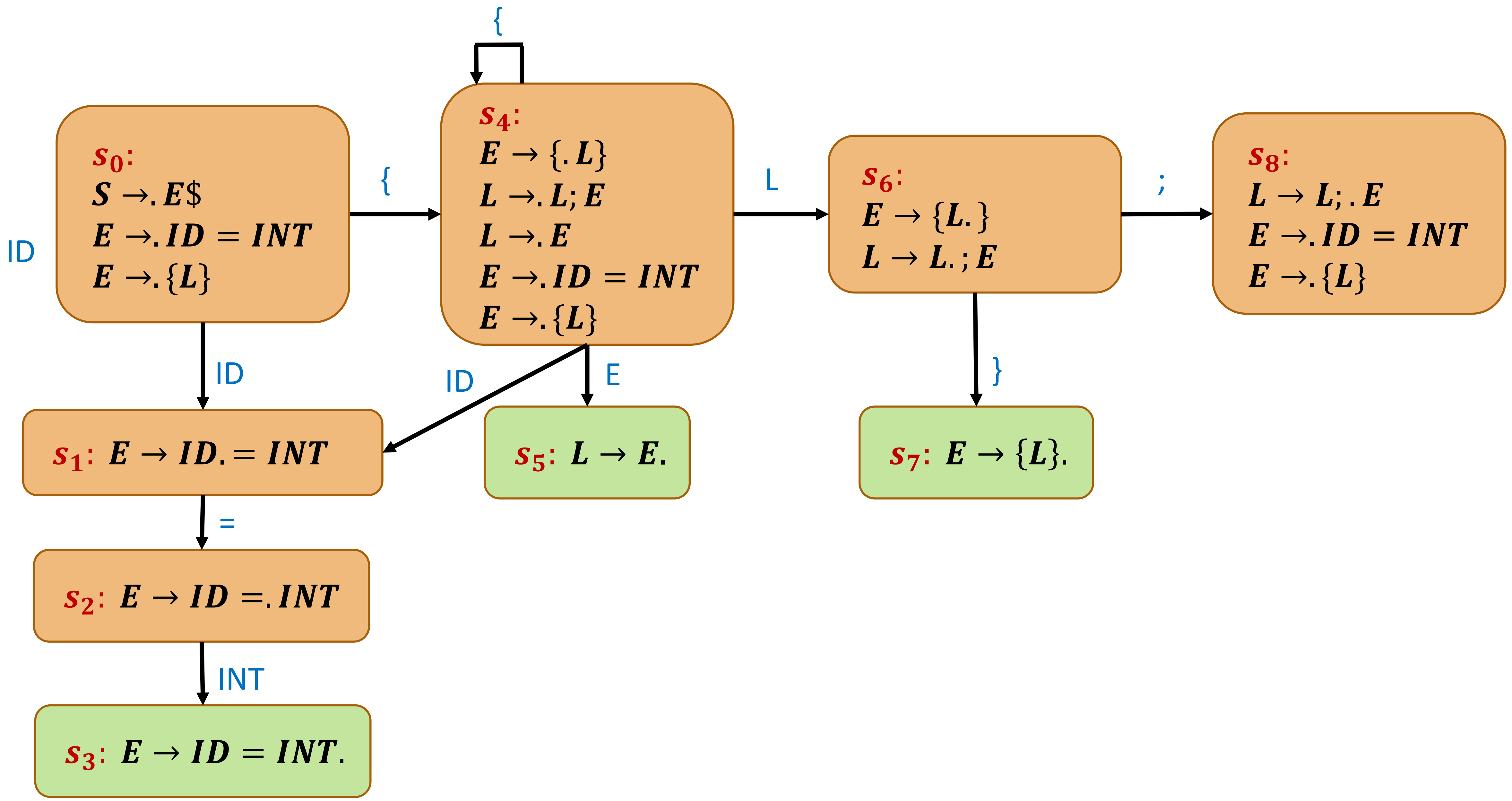
From s_6, if we recognized ;, then the next state will contain:

- $L \to L; . E$

So the next state (the ϵ-closure) contains:

- $L \to L; . E$
- $E \to . ID = INT$
- $E \to . \{L\}$

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$



s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID =. INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L; E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.; E$
s_7: $E \to \{L\}.$
s_8: $L \to L; .E$ $E \to .ID = INT$ $E \to .\{L\}$
ID
{
L
;
}
E
=
INT

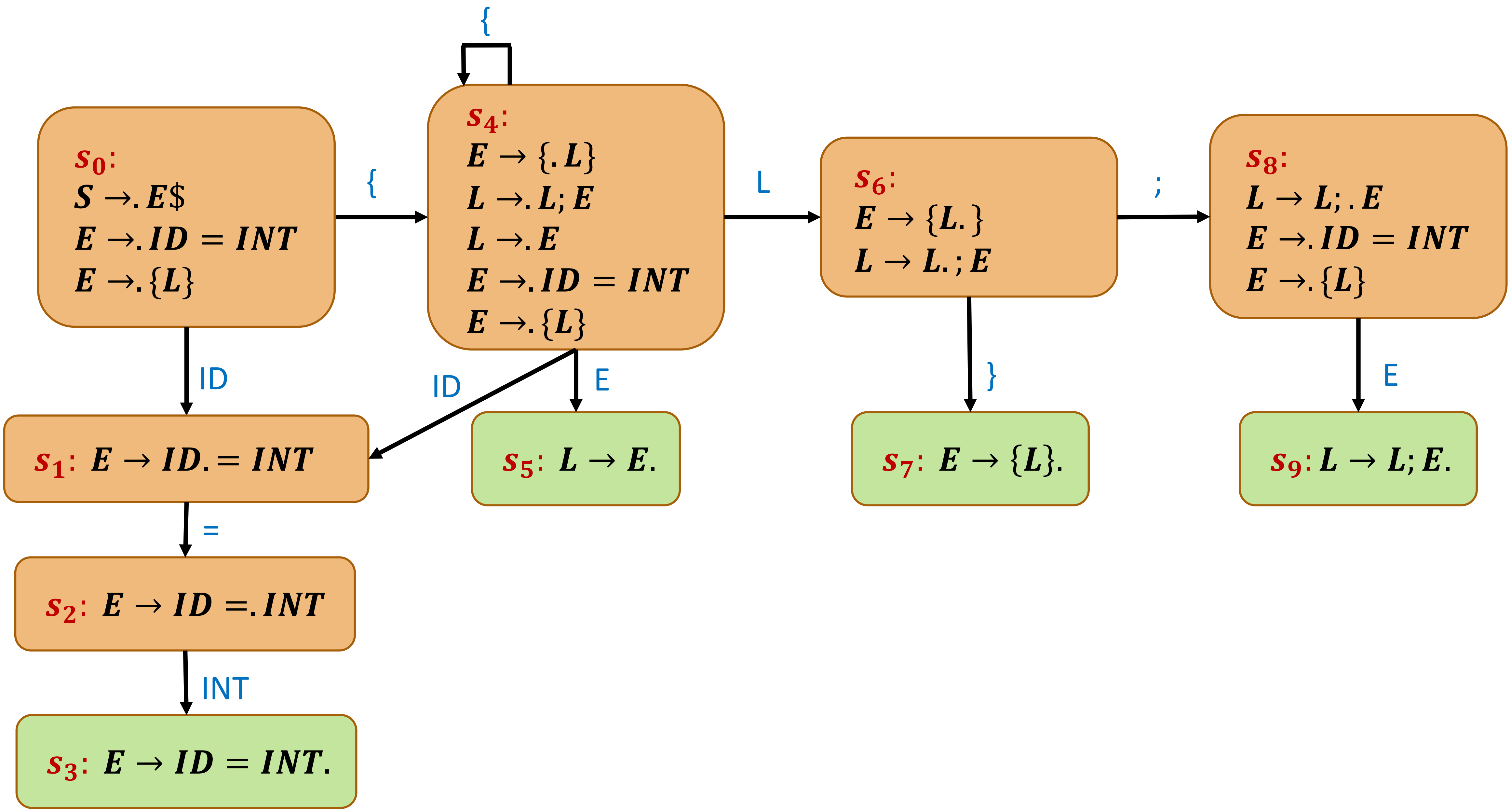
LR(0) Parser

From s_8, if we recognized E, then the next state will contain:

- $E \to L; E.$

which is a reduce state.

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$



s_0: $S \rightarrow .E\$$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
{
s_4: $E \rightarrow \{.L\}$ $L \rightarrow .L;E$ $L \rightarrow .E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
{
L
s_6: $E \rightarrow \{L.\}$ $L \rightarrow L.;E$
;
s_8: $L \rightarrow L;.E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L;E.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_8, if we recognized $\{$, then the next state will contain:

- $E \to \{.L\}$

So the next state (the ϵ-closure) contains:

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$

LR(0) Parser

From s_8, if we recognized $\{$, then the next state will contain:

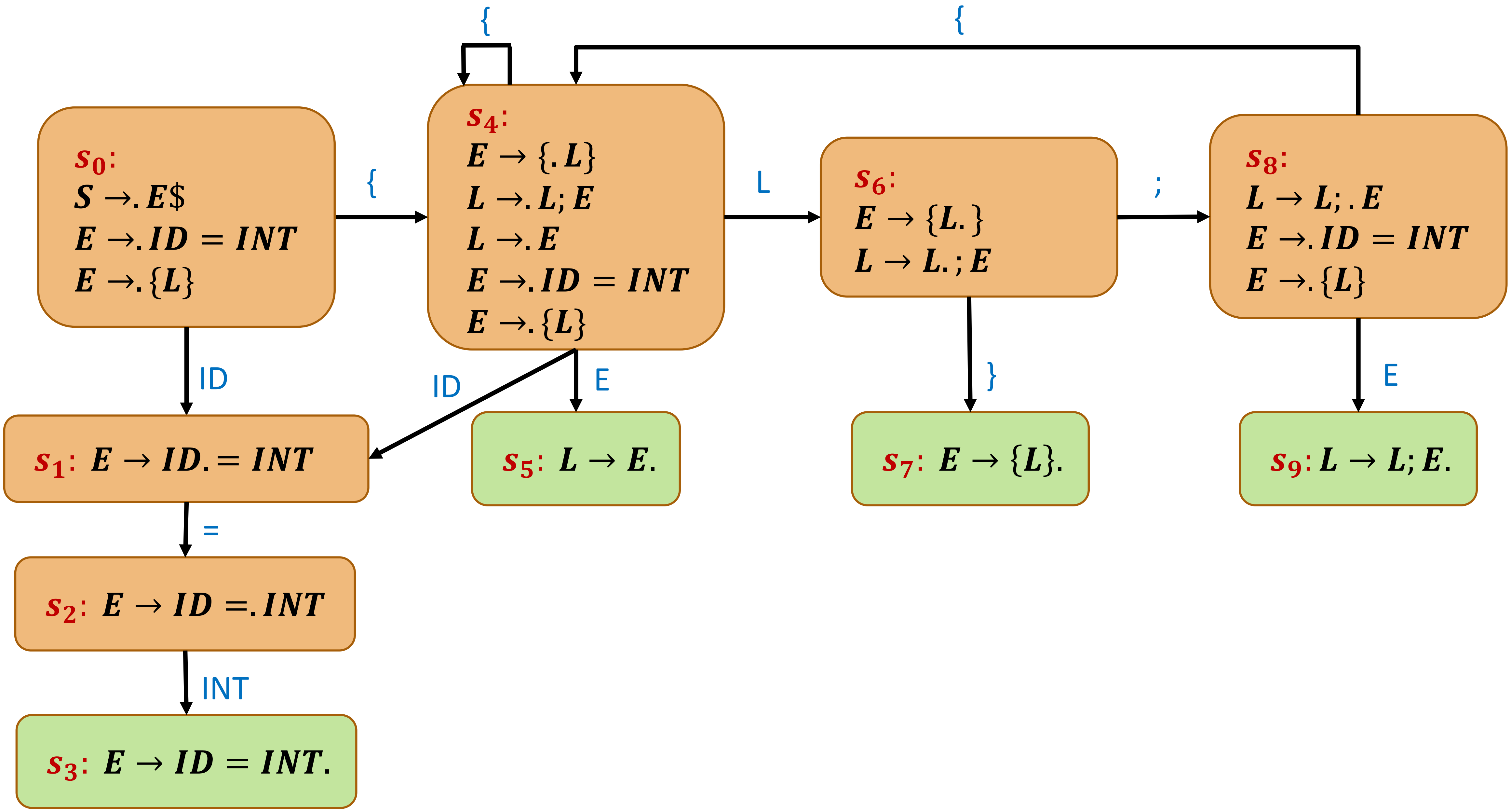
- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L; E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_1: $E \rightarrow ID. = INT$
s_2: $E \rightarrow ID =. INT$
s_3: $E \rightarrow ID = INT.$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
{
{
{
L
;
ID
ID
E
}
E
=
INT

LR(0) Parser

From s_8, if we recognized ID, then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_8, if we recognized ID, then the next state will contain:

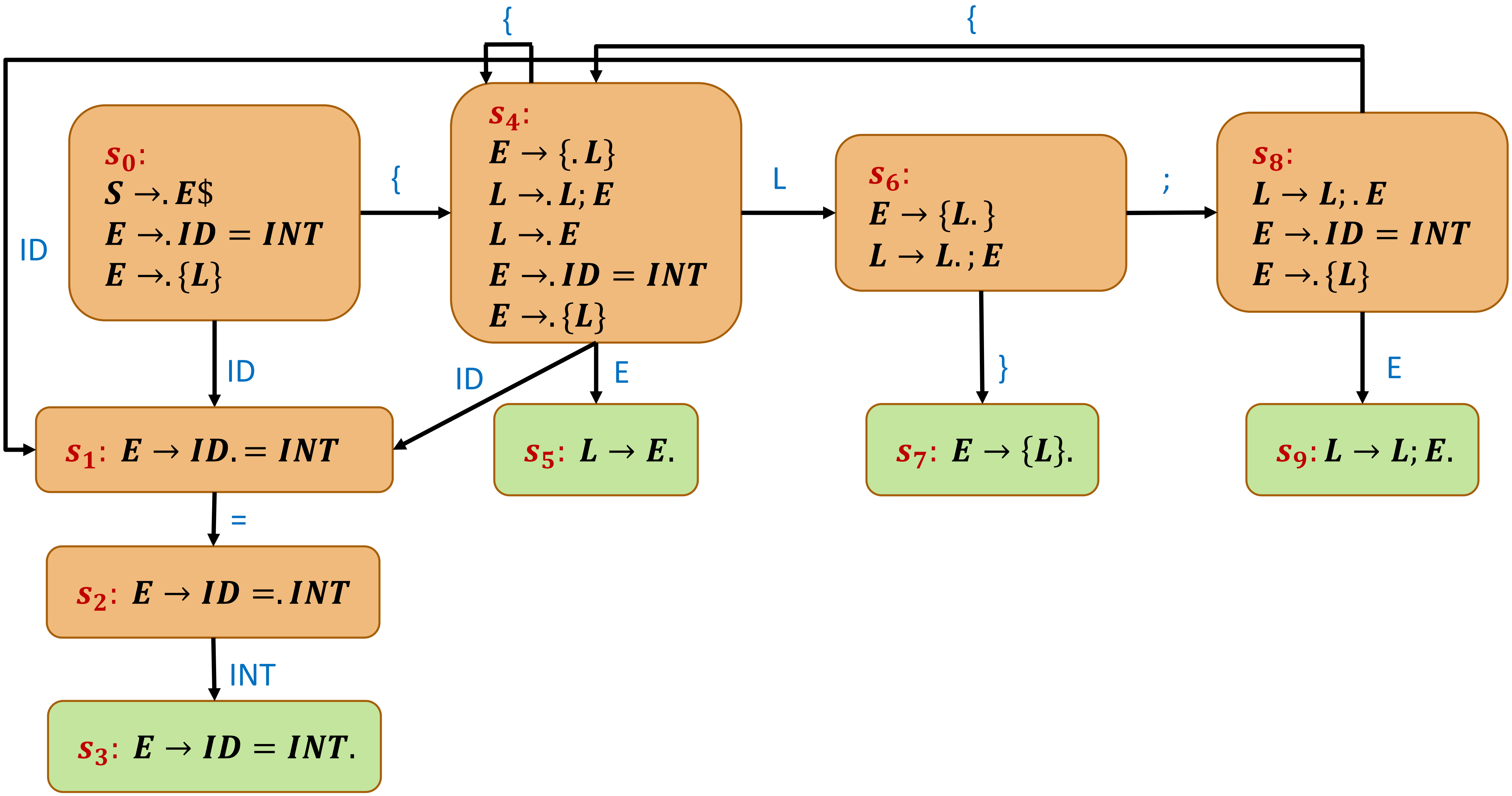
- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID. = INT$

which was already computed: s_1

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_0: $S \rightarrow .E\$$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_1: $E \rightarrow ID. = INT$
s_2: $E \rightarrow ID =. INT$
s_3: $E \rightarrow ID = INT.$
s_4: $E \rightarrow \{.L\}$ $L \rightarrow .L; E$ $L \rightarrow .E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_5: $L \rightarrow E.$
s_6: $E \rightarrow \{L.\}$ $L \rightarrow L.; E$
s_7: $E \rightarrow \{L\}.$
s_8: $L \rightarrow L;.E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_9: $L \rightarrow L; E.$
{
{
{
ID
ID
ID
E
L
}
;
E
=
INT

LR(0) Parser

From s_0, if we recognized E, then the next state will contain:

- $S \to E.\$$

So the next state (the ϵ-closure) contains:

$S \to E\$$
$E \to ID = INT$
$E \to \{L\}$
$L \to E$
$L \to L; E$

LR(0) Parser

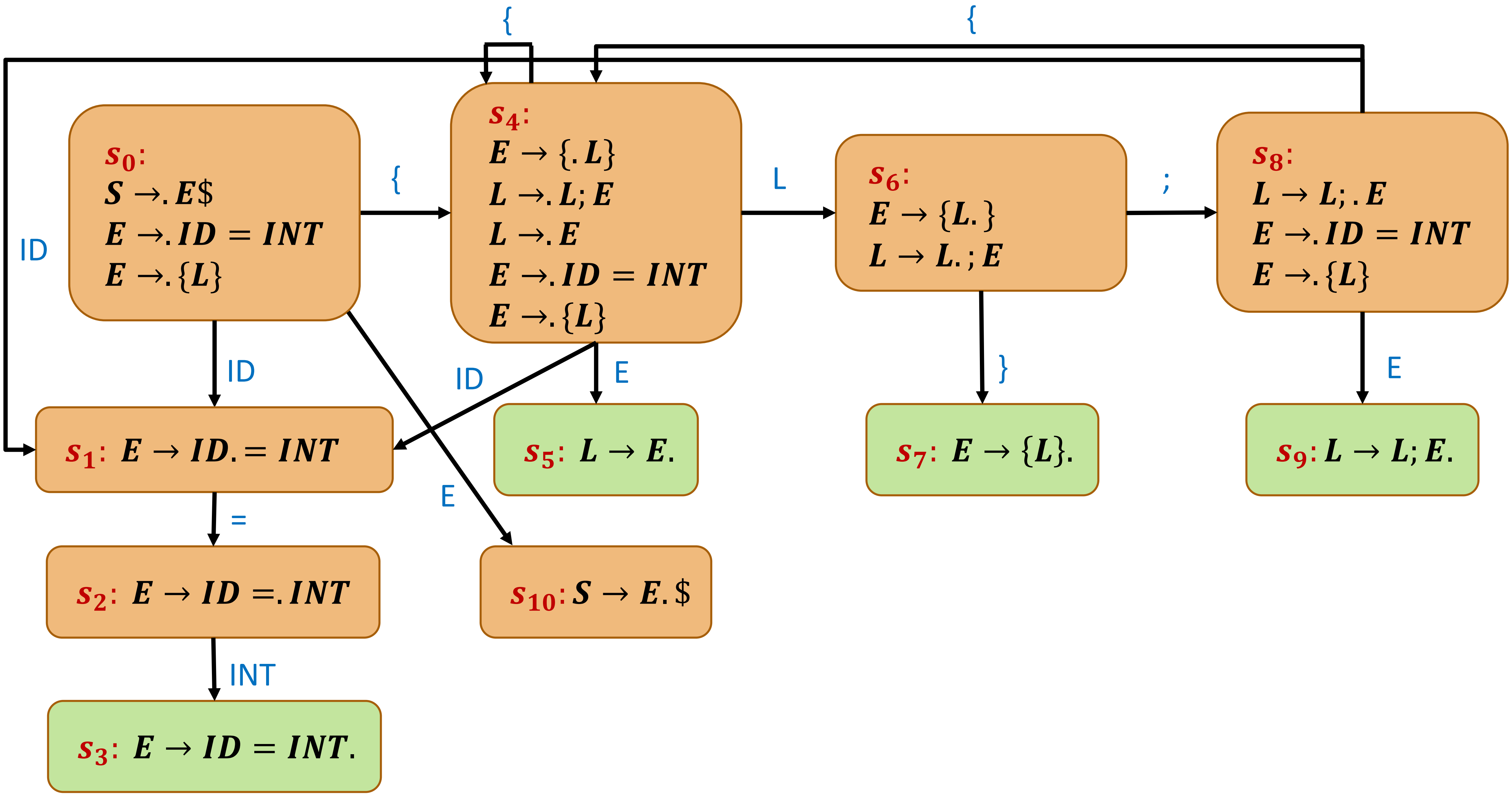
From s_0, if we recognized E, then the next state will contain:

- $S \rightarrow E.\$$

So the next state (the ϵ-closure) contains:

- $S \rightarrow E.\$$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$



s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID =. INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L;E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.;E$
s_7: $E \to \{L\}.$
s_8: $L \to L;.E$ $E \to .ID = INT$ $E \to .\{L\}$
s_9: $L \to L;E.$
s_{10}: $S \to E.\$$
{
{
{
L
;
ID
ID
ID
E
E
E
}
=
INT

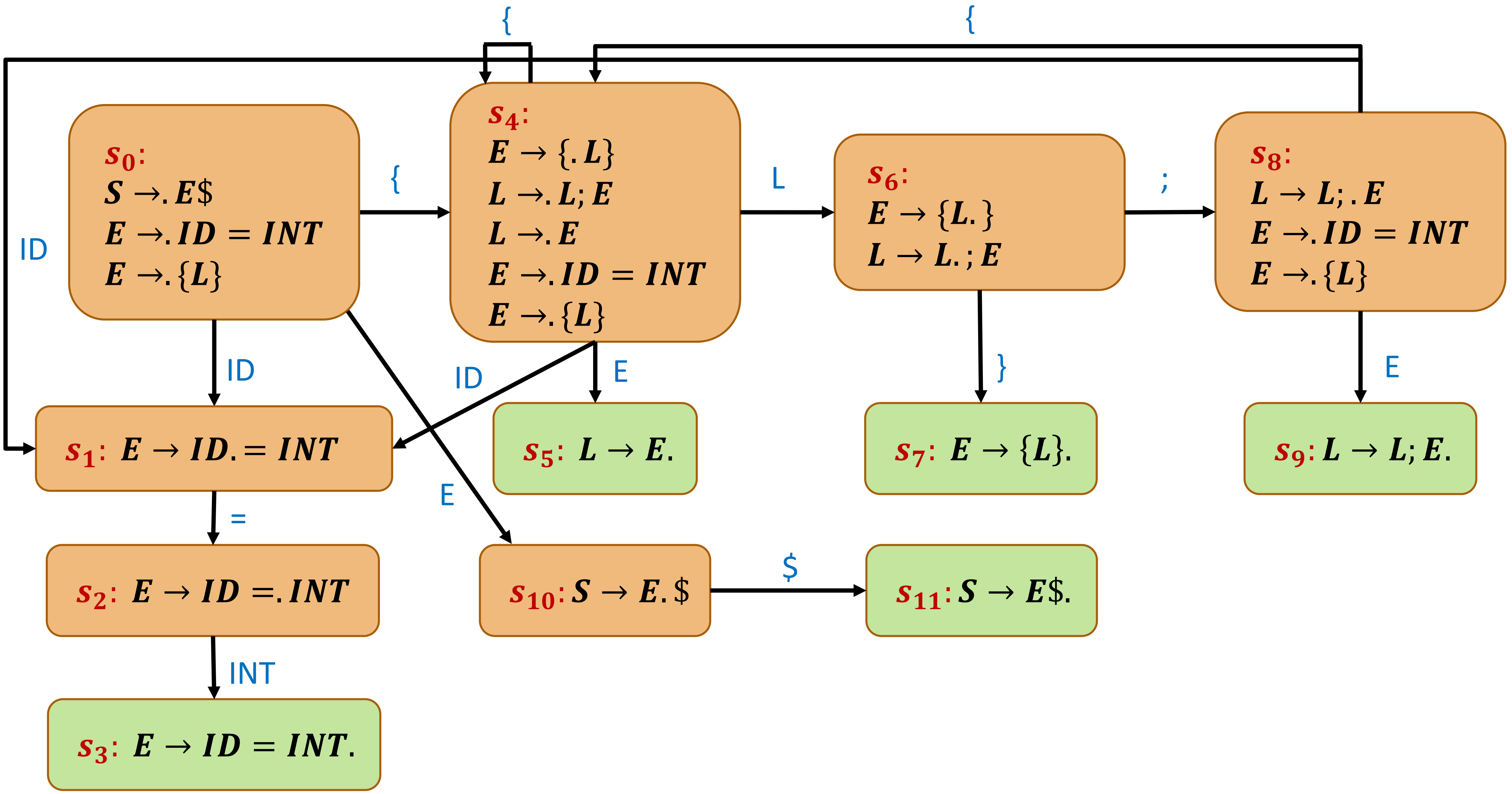
LR(0) Parser

From s_{10}, if we recognized $\$$, then the next state will contain:

- $S \rightarrow E\$.$

which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

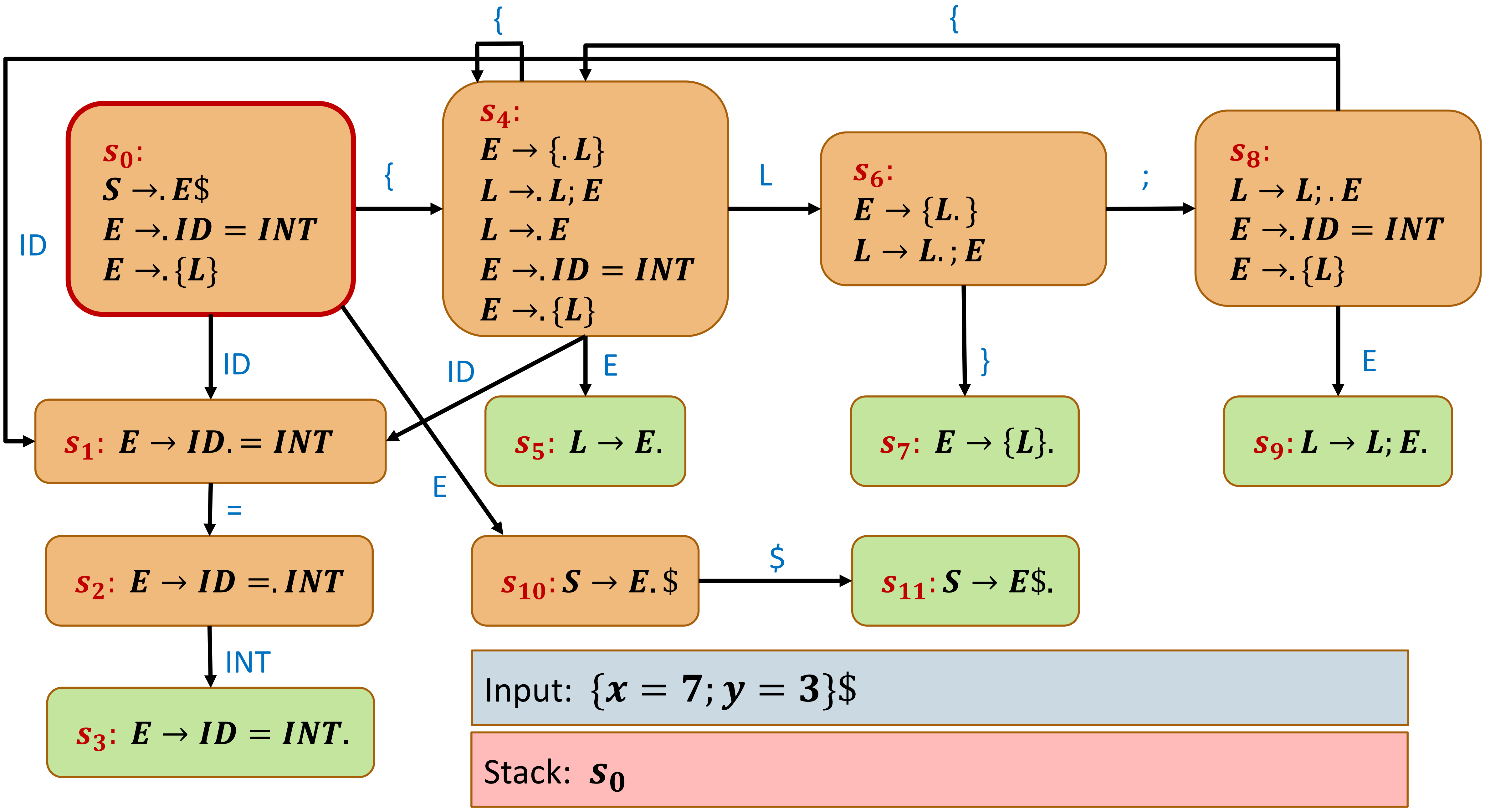


s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID = .INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L; E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.; E$
s_7: $E \to \{L\}.$
s_8: $L \to L; .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_9: $L \to L; E.$
s_{10}: $S \to E.\$$
s_{11}: $S \to E\$.$
{
ID
=
INT
E
L
;
}
$

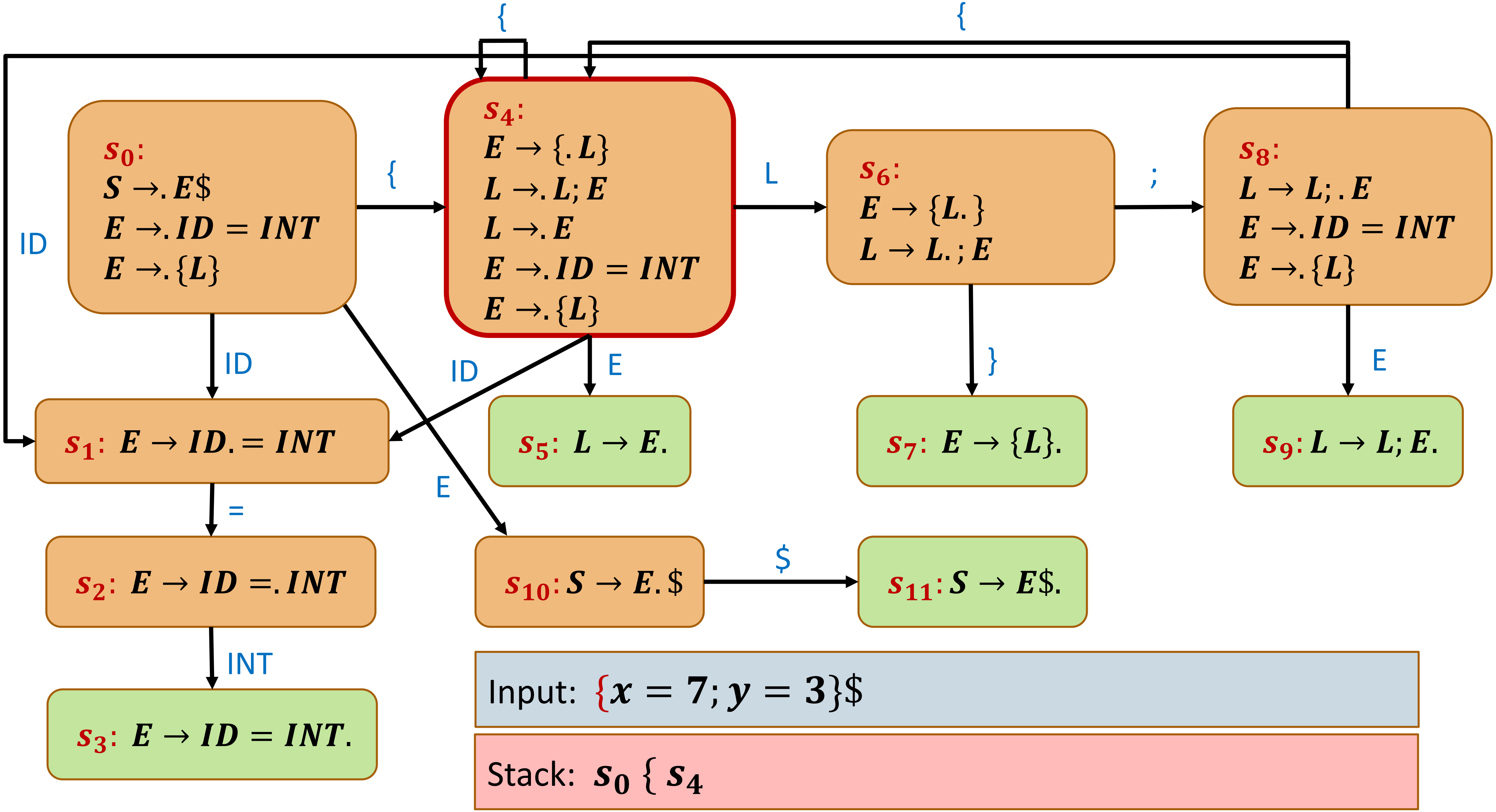
LR(0) Parser: Running Example

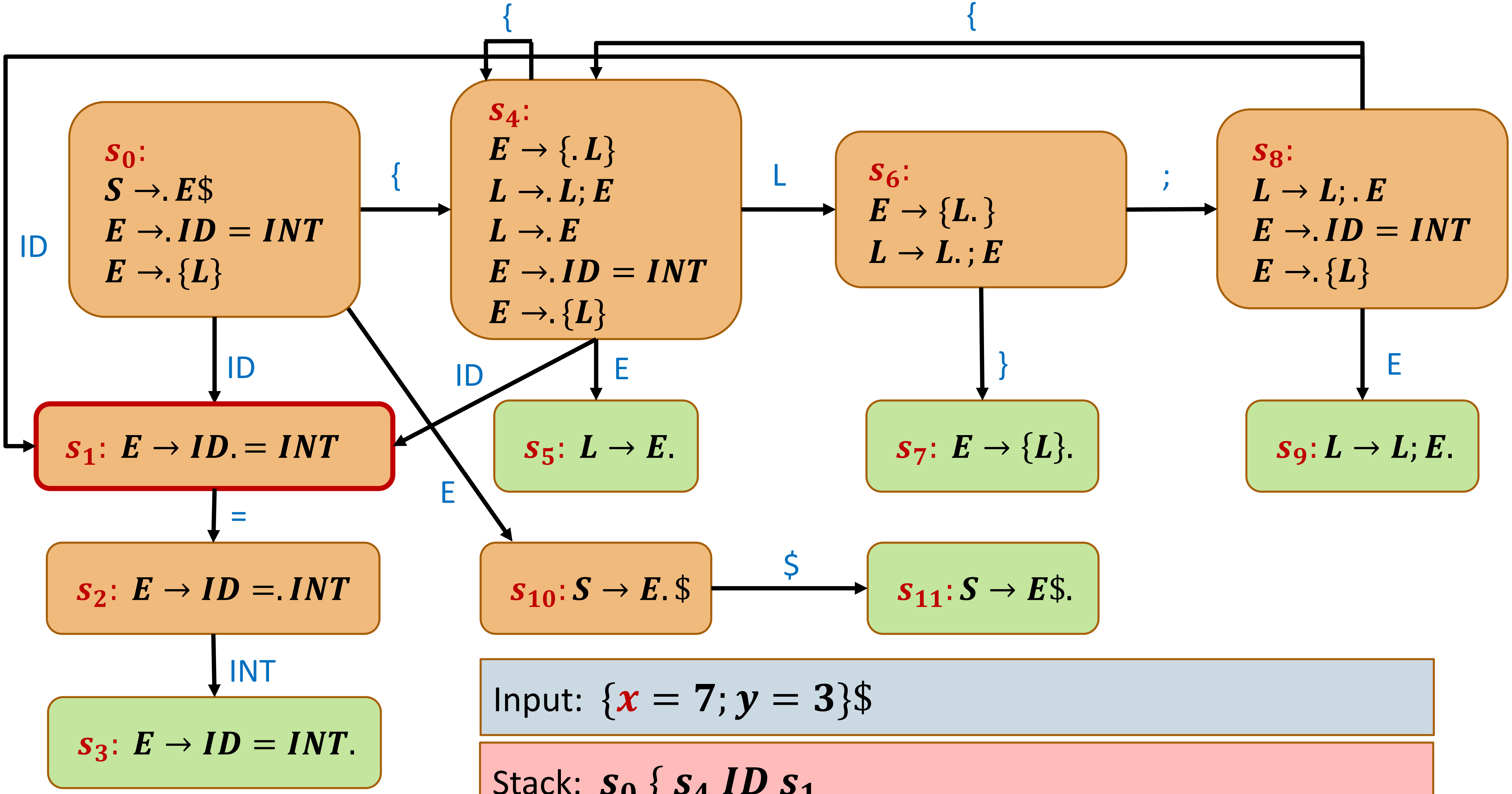
What will happen with the following input:

- $\{\mathrm{x} = 7; y = 3\}\$$



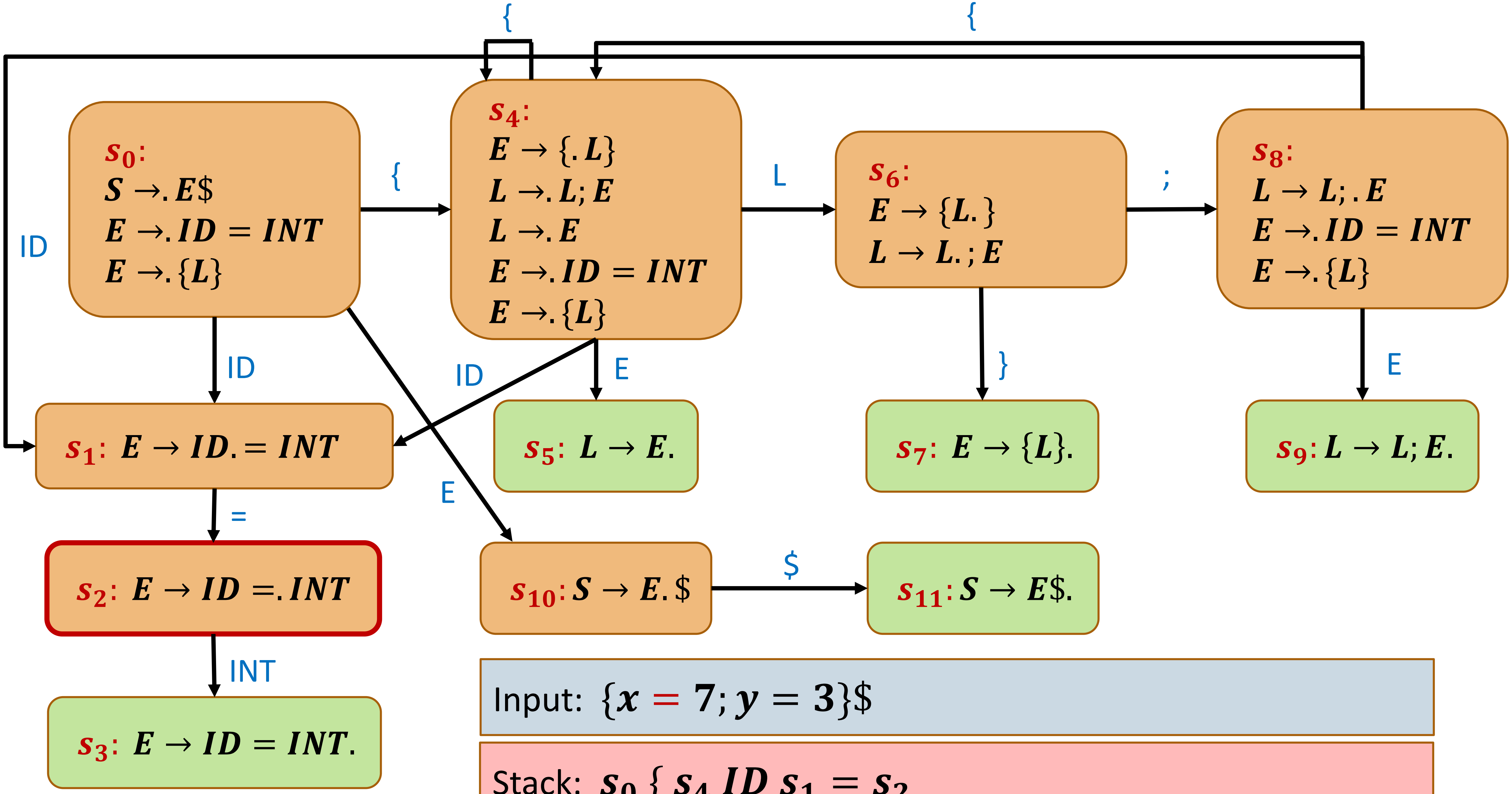
s_0: $S \rightarrow .E\$$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_1: $E \rightarrow ID. = INT$
s_2: $E \rightarrow ID =. INT$
s_3: $E \rightarrow ID = INT.$
s_4: $E \rightarrow \{.L\}$ $L \rightarrow .L;E$ $L \rightarrow .E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_5: $L \rightarrow E.$
s_6: $E \rightarrow \{L.\}$ $L \rightarrow L.;E$
s_7: $E \rightarrow \{L\}.$
s_8: $L \rightarrow L;.E$ $E \rightarrow .ID = INT$ $E \rightarrow .\{L\}$
s_9: $L \rightarrow L;E.$
s_{10}: $S \rightarrow E.\$$
s_{11}: $S \rightarrow E\$.$
{
{
{
ID
ID
ID
E
E
E
L
;
}
=
INT
$
Input: $\{x = 7; y = 3\}\$$
Stack: s_0


s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID =. INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L; E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.; E$
s_7: $E \to \{L\}.$
s_8: $L \to L; .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_9: $L \to L; E.$
s_{10}: $S \to E.\$$
s_{11}: $S \to E\$.$
{
{
{
L
;
ID
ID
ID
E
E
E
}
=
INT
$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4$



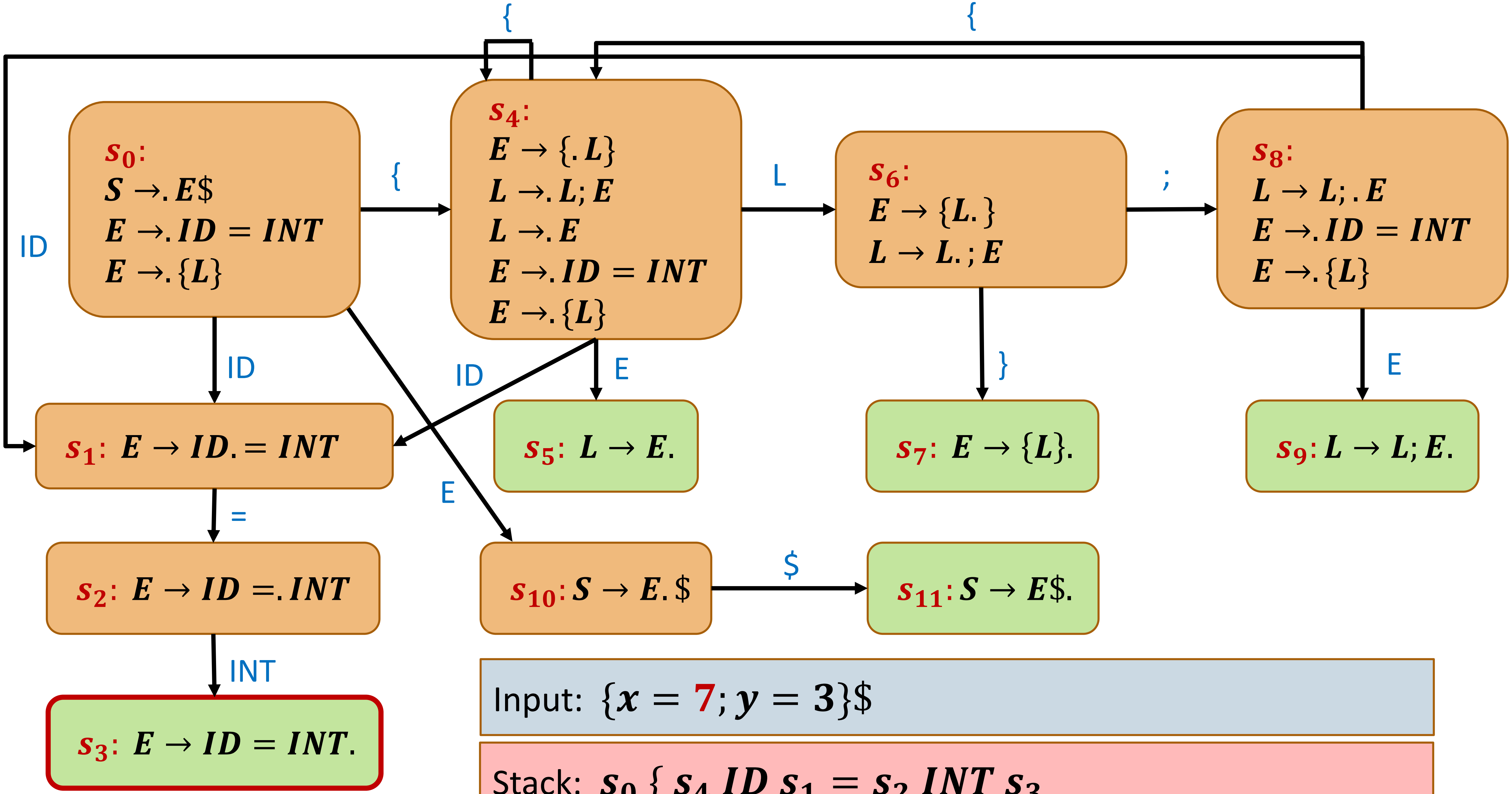
Input: $\{\boldsymbol{x} = 7; y = 3\}\$$

Stack: $s_0 \; \{ \; s_4 \; ID \; s_1$



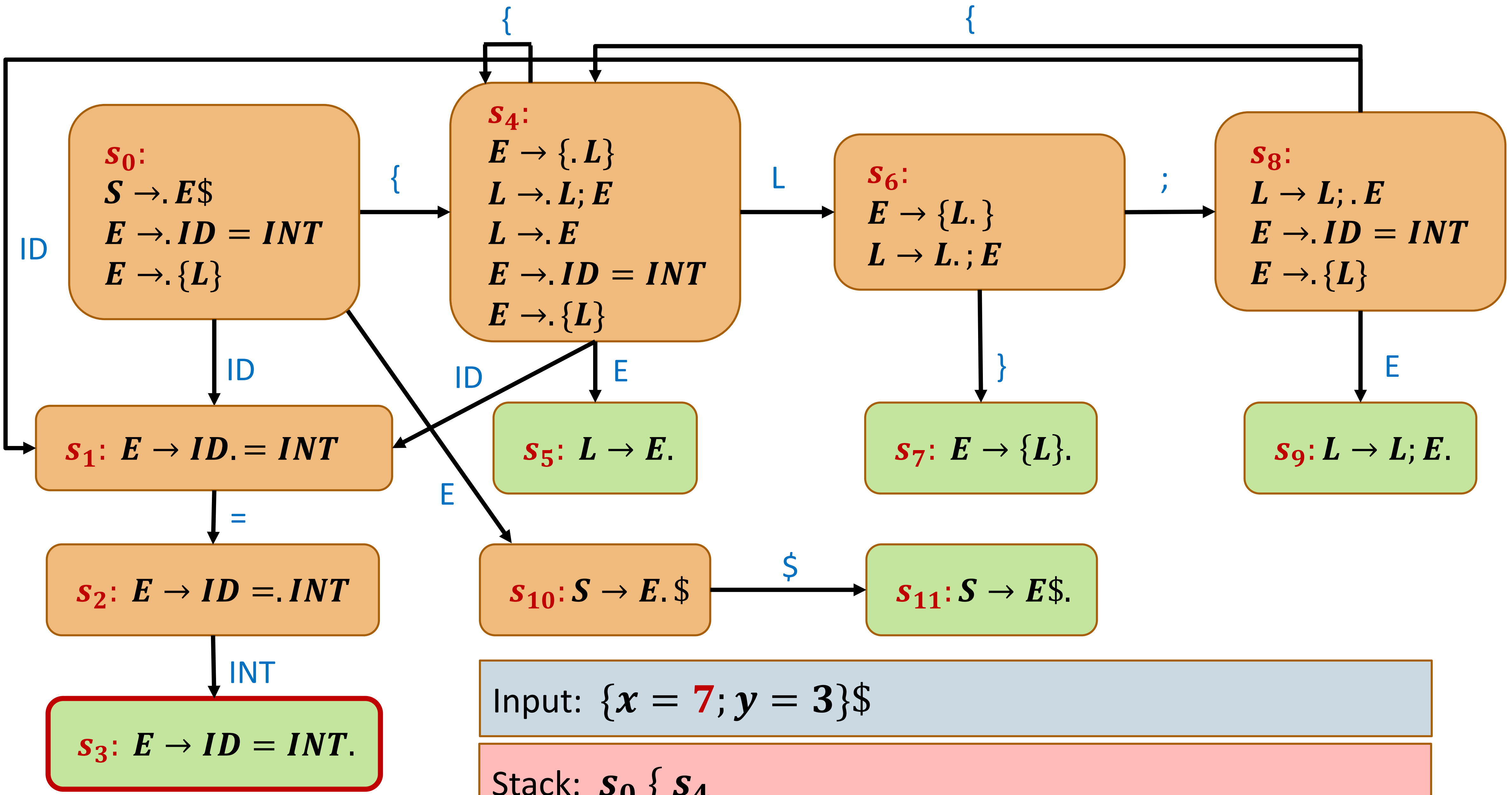
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \{ s_4\ ID\ s_1 = s_2$



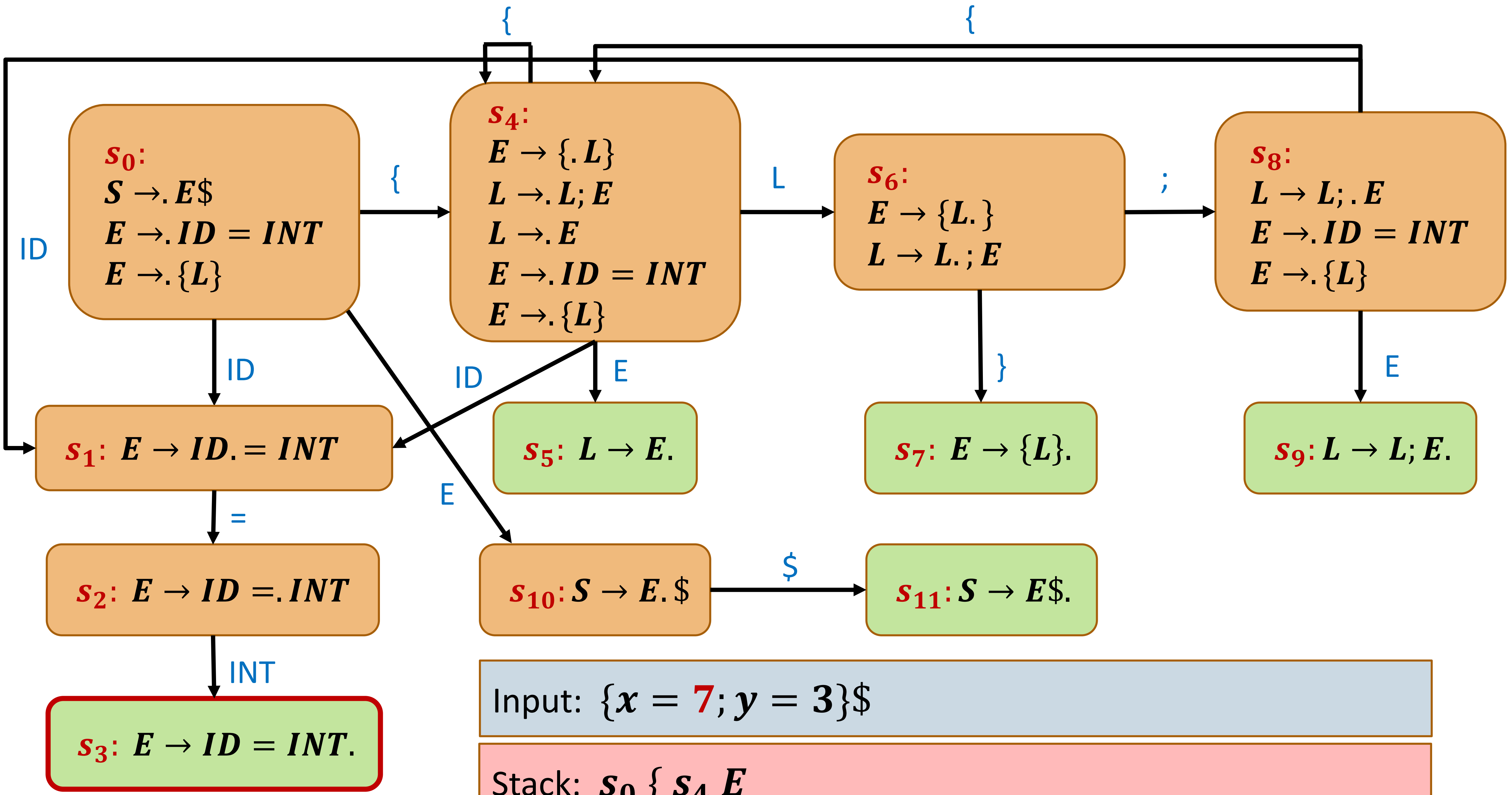
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \{ s_4\ ID\ s_1 = s_2\ INT\ s_3$



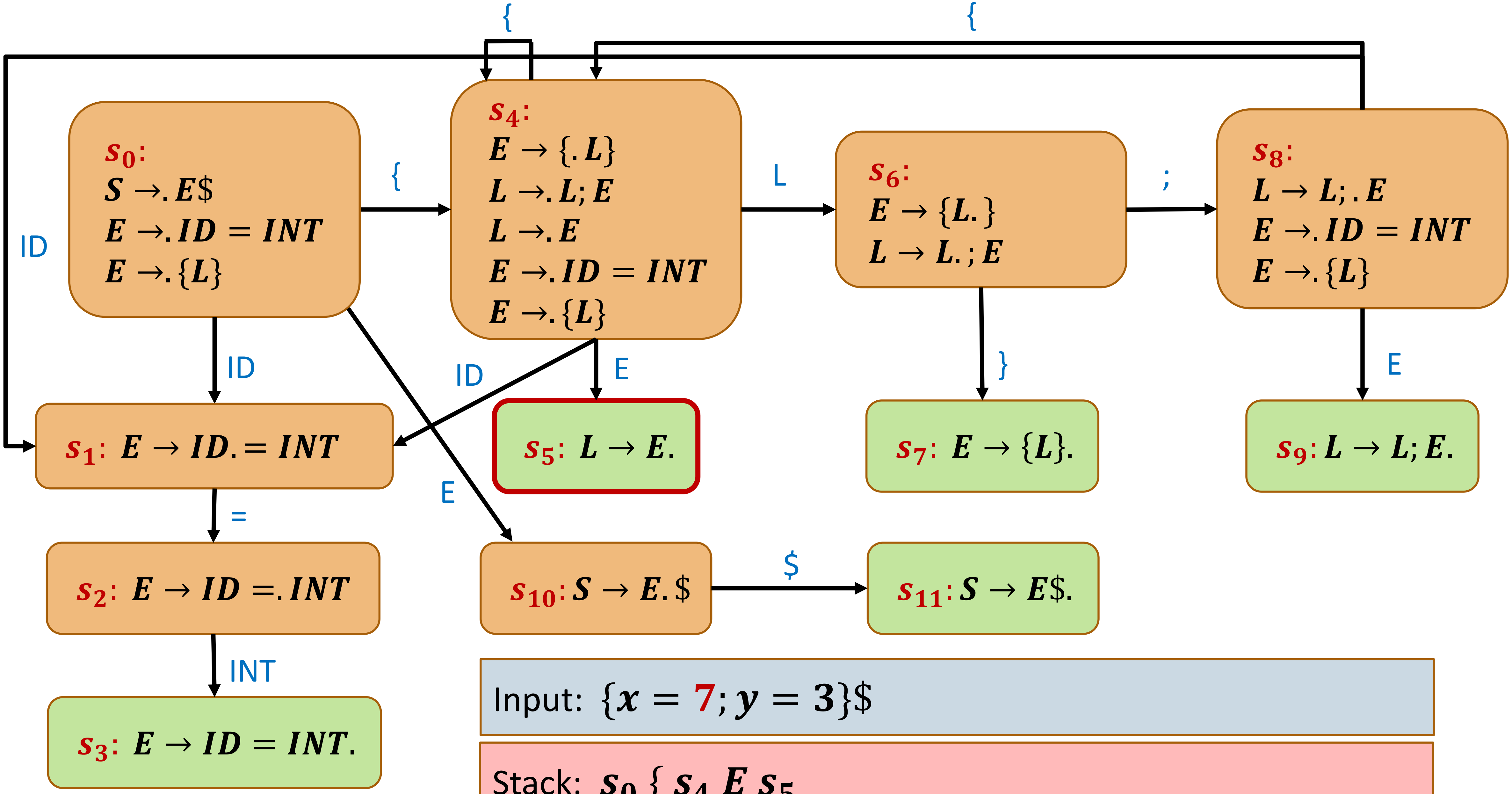
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \; \{ \; s_4$



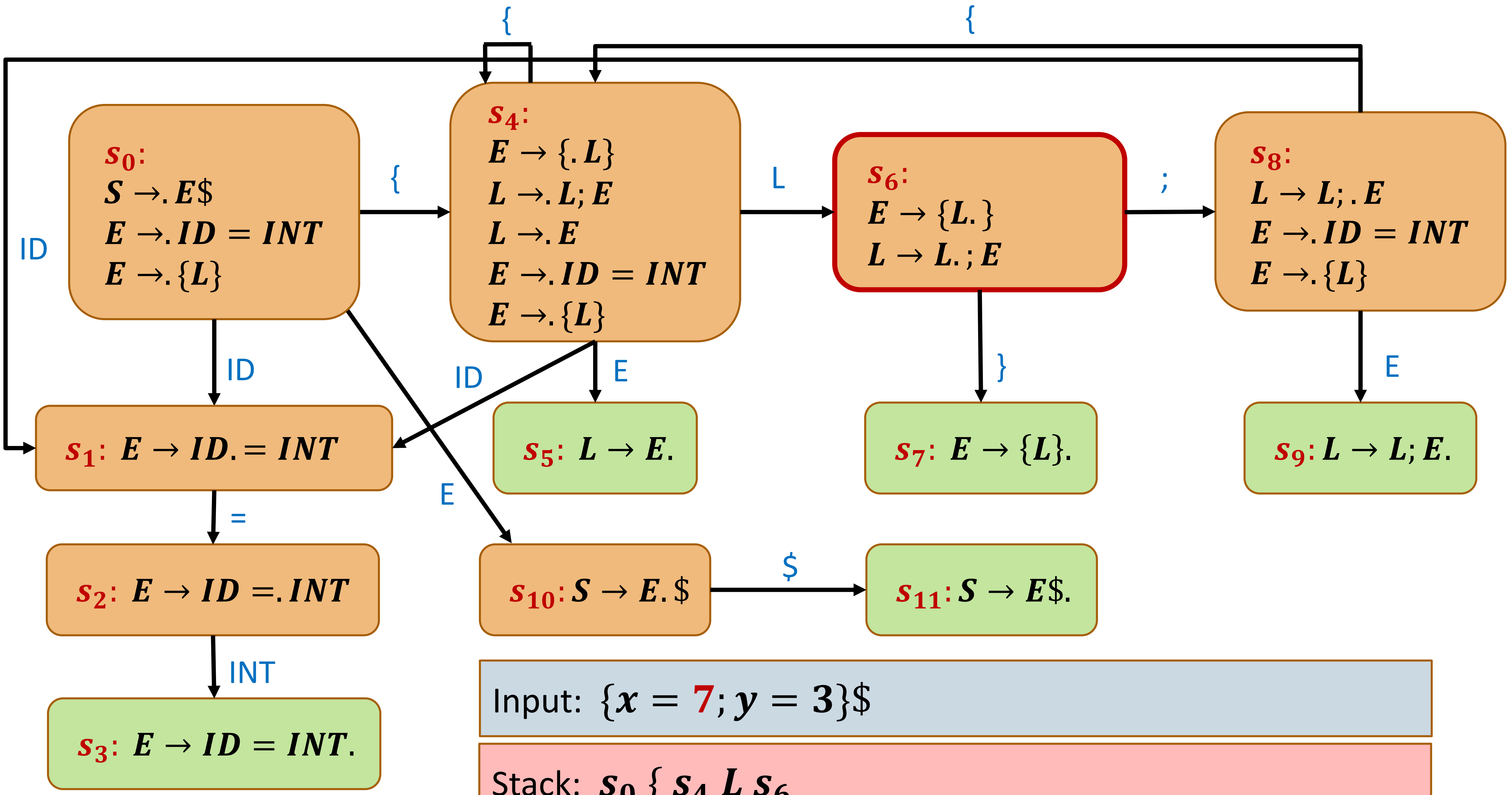
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \, \{ \, s_4 \, E$



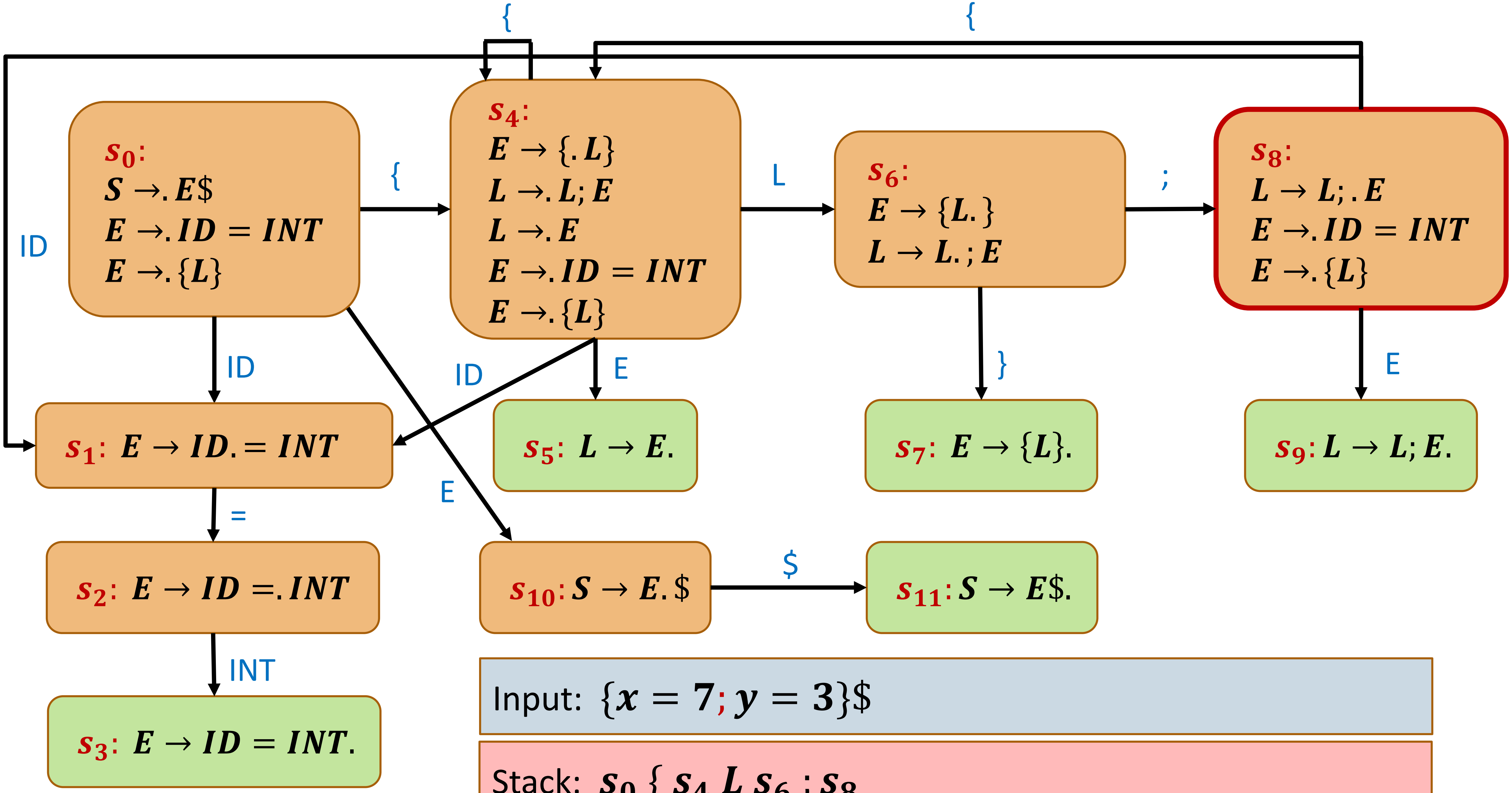
Input: $\{x = \color{red}{7}; y = 3\}\$$

Stack: $s_0 \ \{ \ s_4 \ E \ s_5$



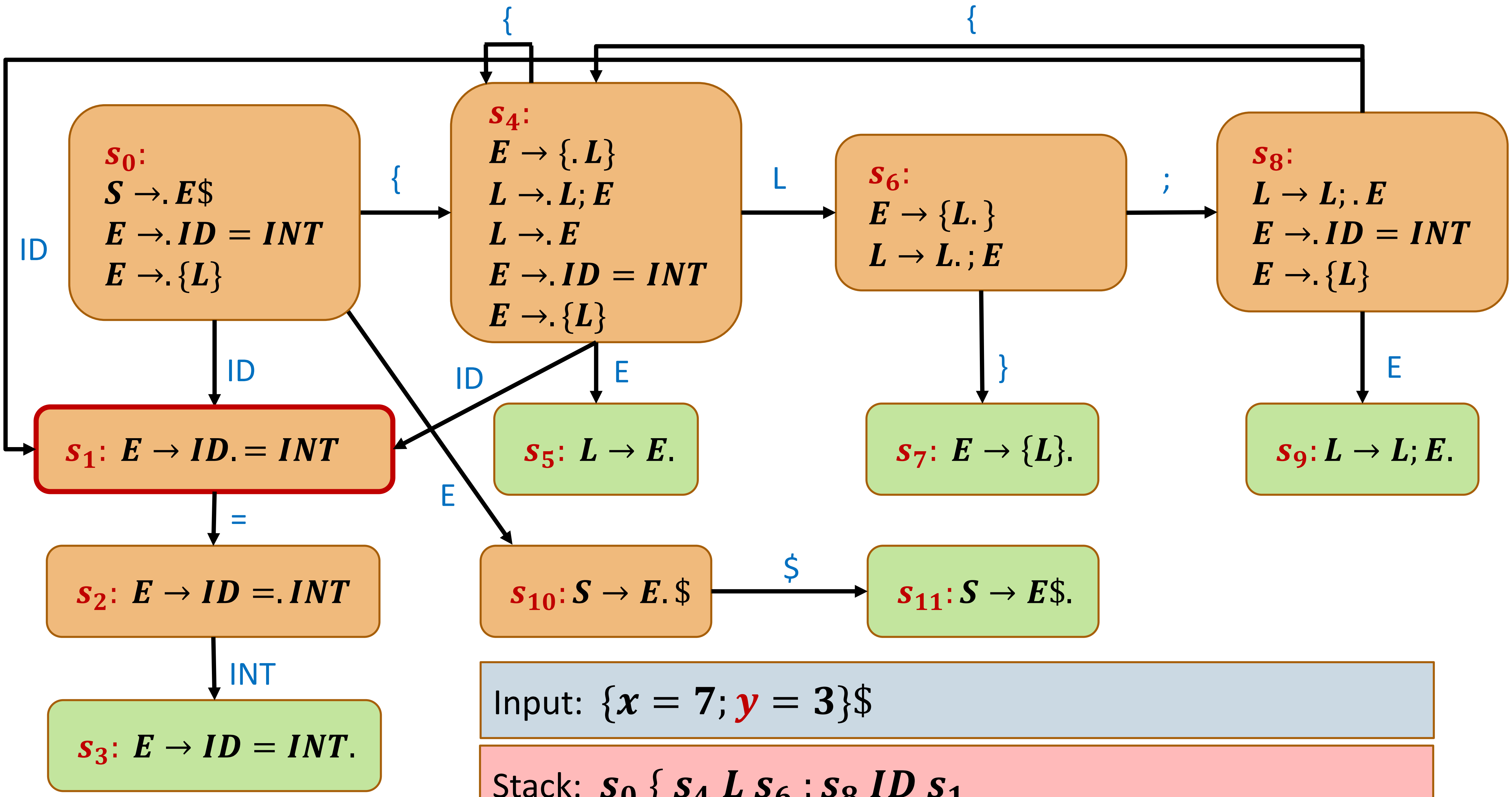
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \{ s_4 L s_6$



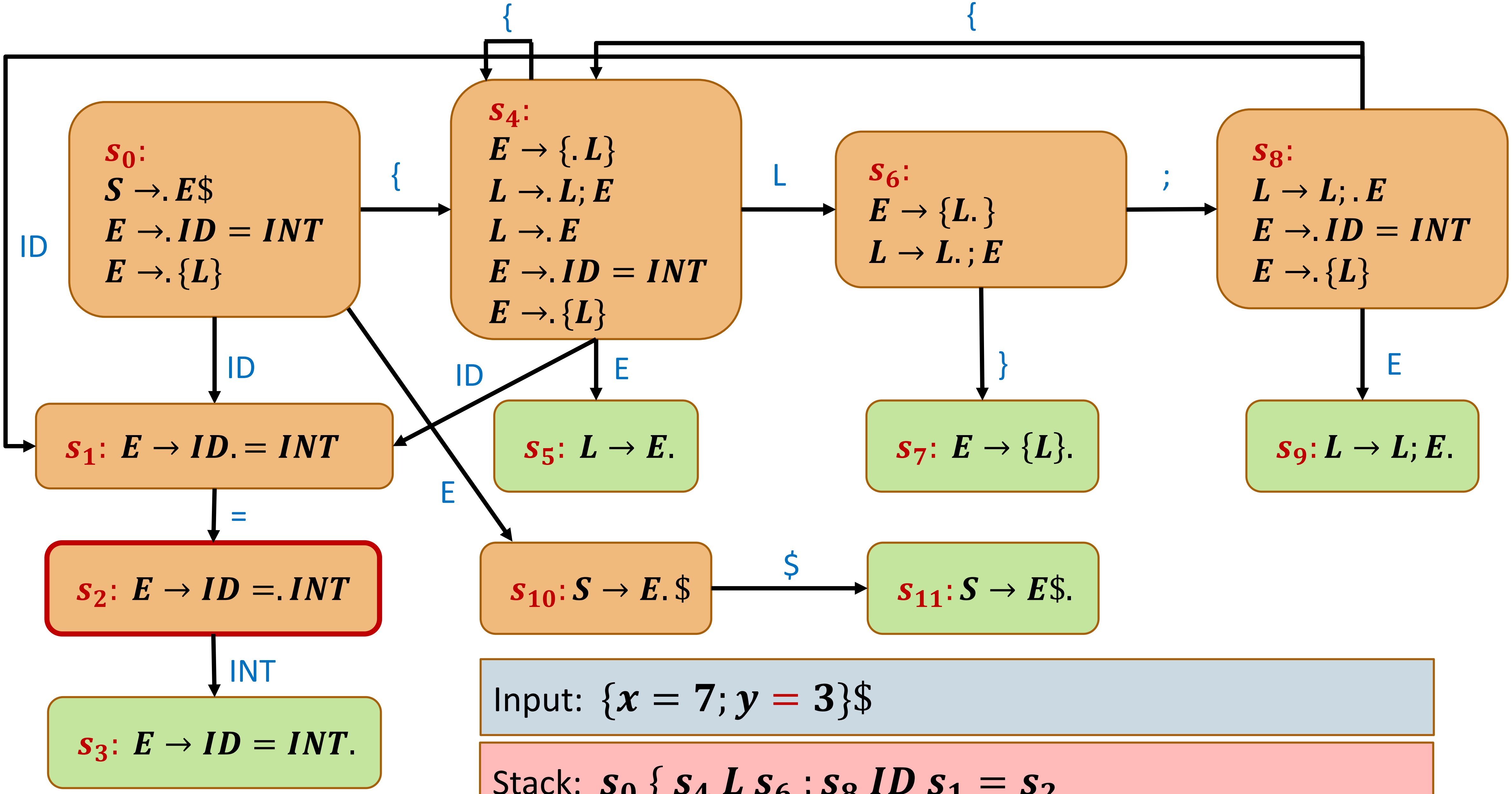
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \; \{ \; s_4 \; L \; s_6 \; ; \; s_8$



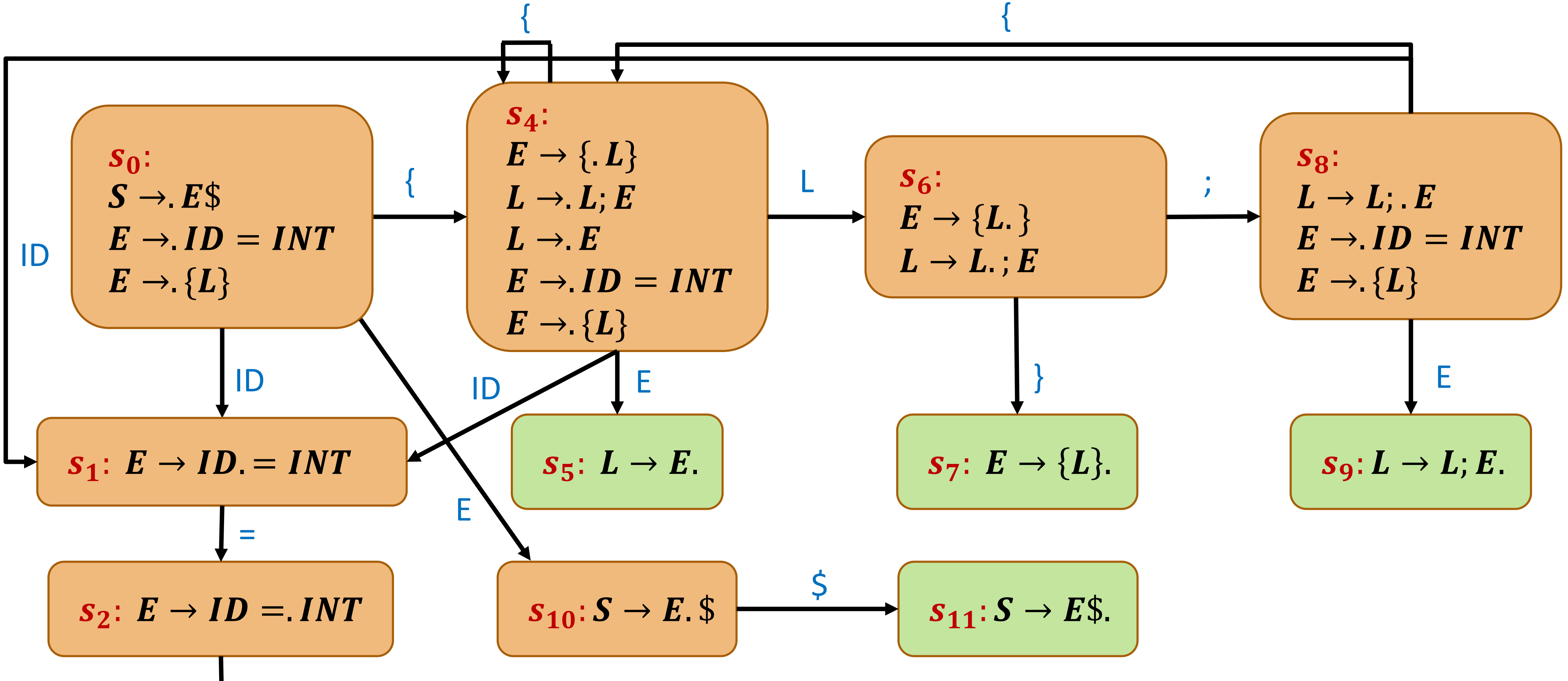
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \; \{ \; s_4 \; L \; s_6 \; ; \; s_8 \; ID \; s_1$



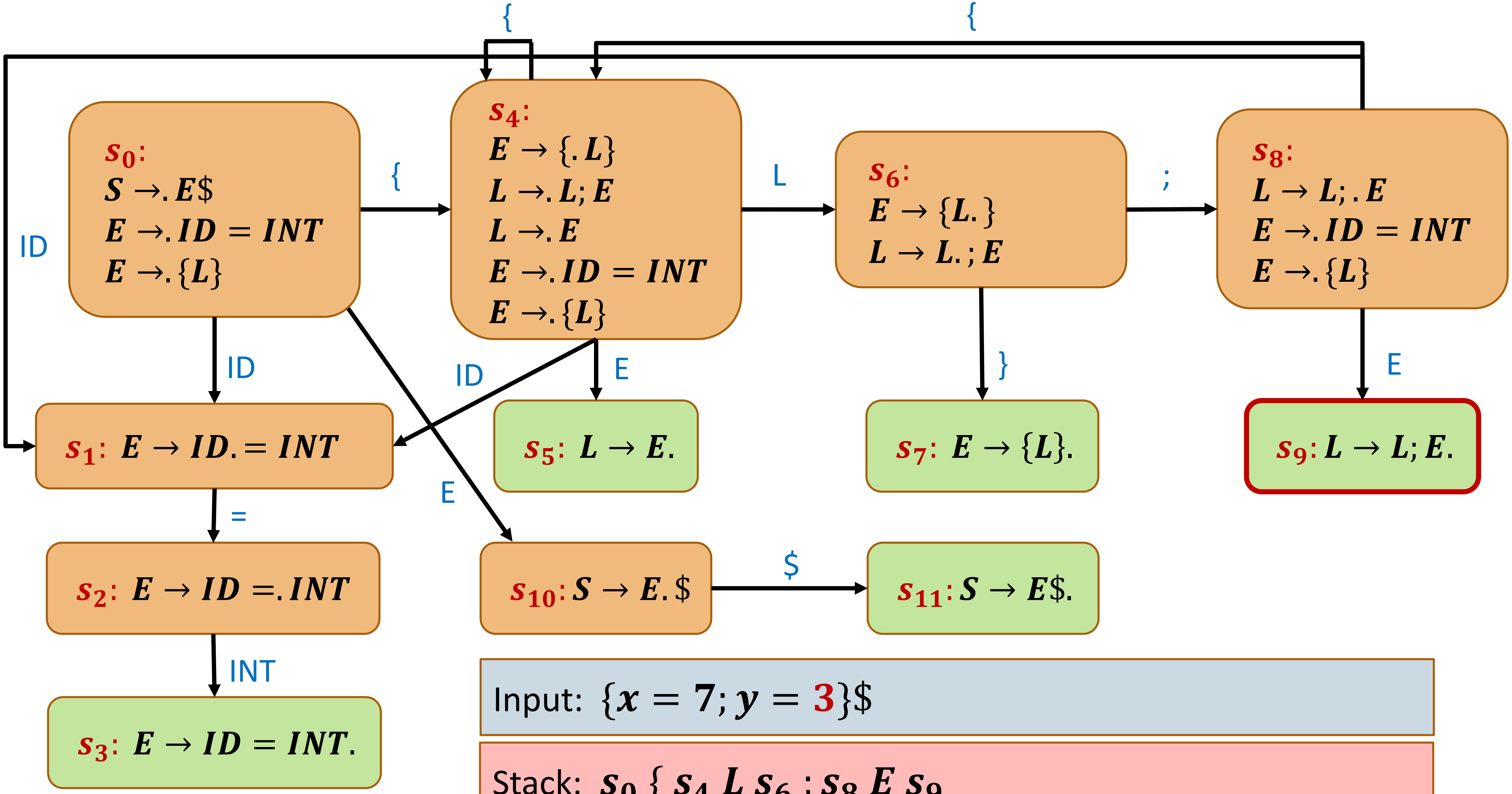
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \, \{ \, s_4 \, L \, s_6 \, ; s_8 \, ID \, s_1 = s_2$



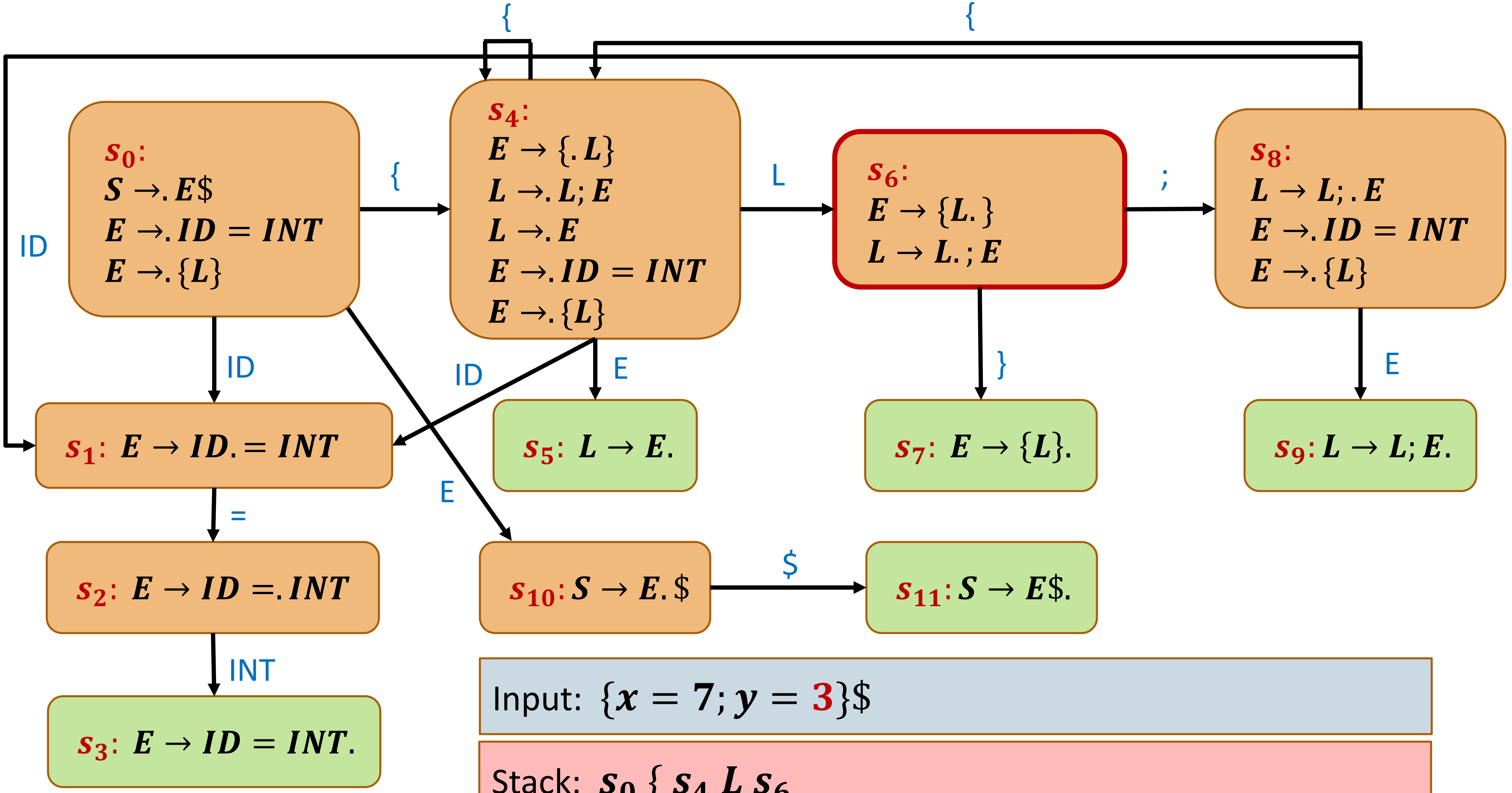
Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \{ s_4 L s_6 ; s_8 ID s_1 = s_2 INT s_3$



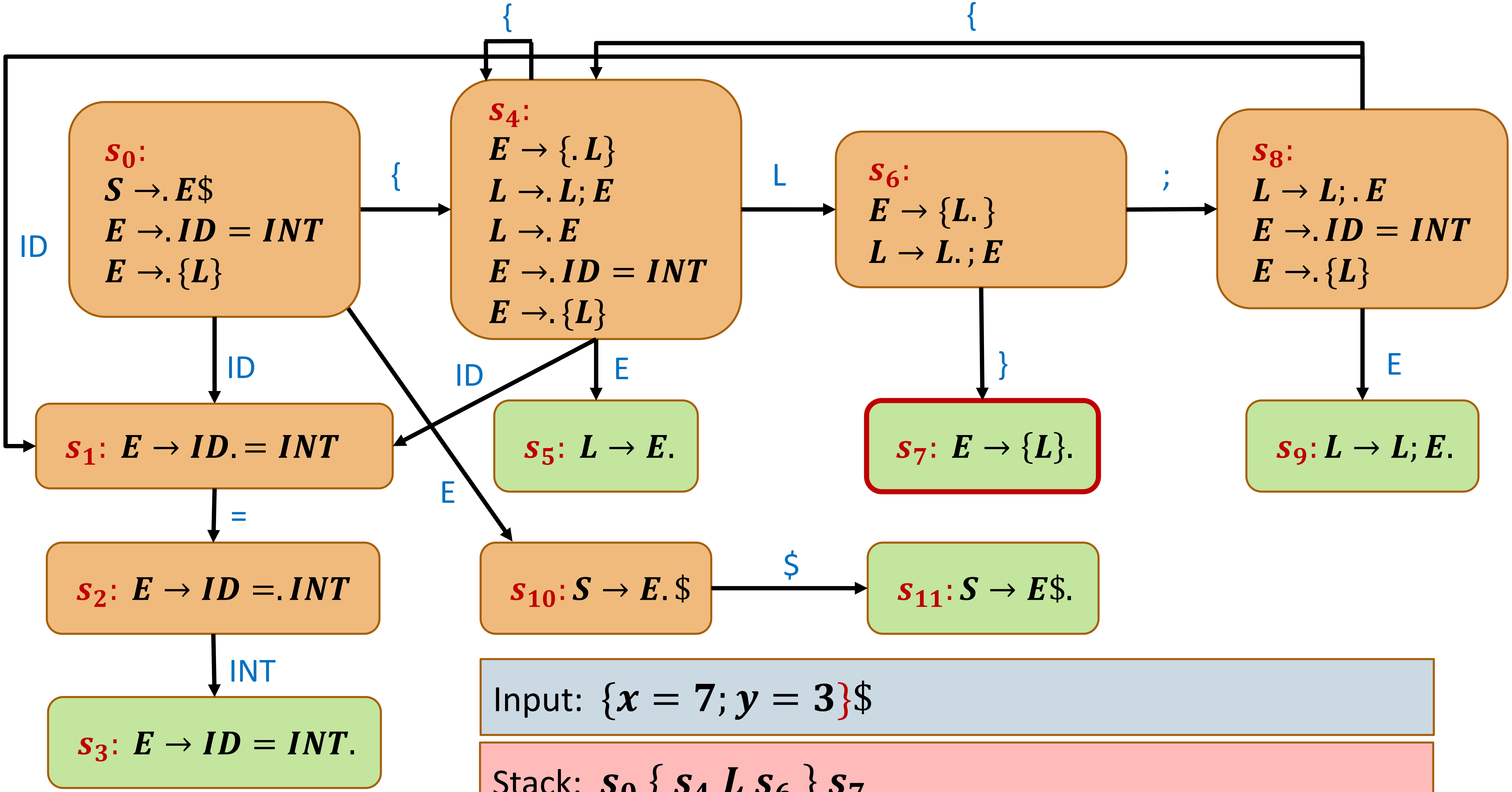
Input: $\{x = 7; y = \mathbf{3}\}\$$

Stack: $s_0 \; \{ \; s_4 \; L \; s_6 \; ; \; s_8 \; E \; s_9$



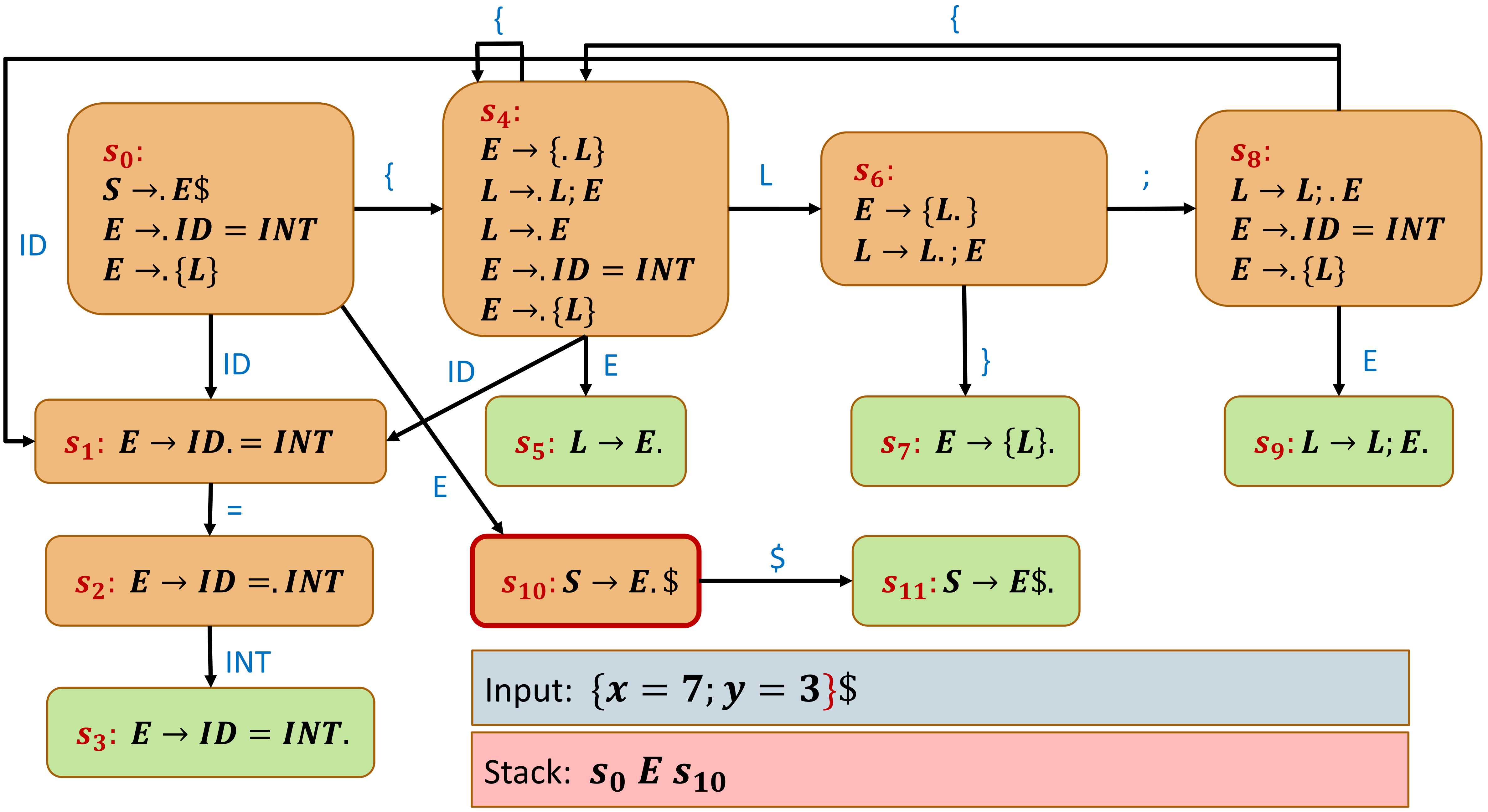
Input: $\{x = 7; y = \mathbf{3}\}\$$

Stack: $s_0 \{ s_4 \, L \, s_6$

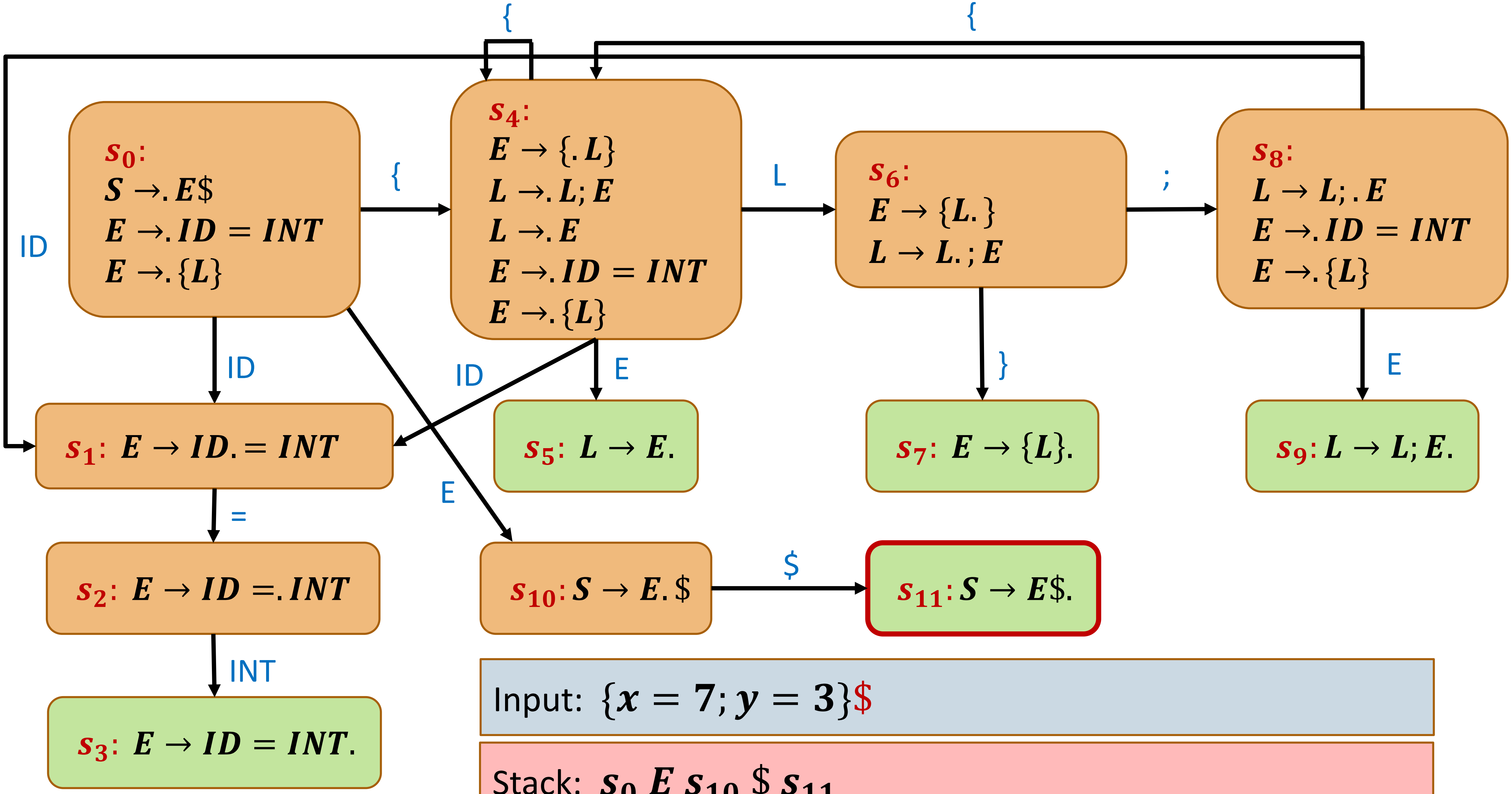


Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \, \{ \, s_4 \, L \, s_6 \, \} \, s_7$

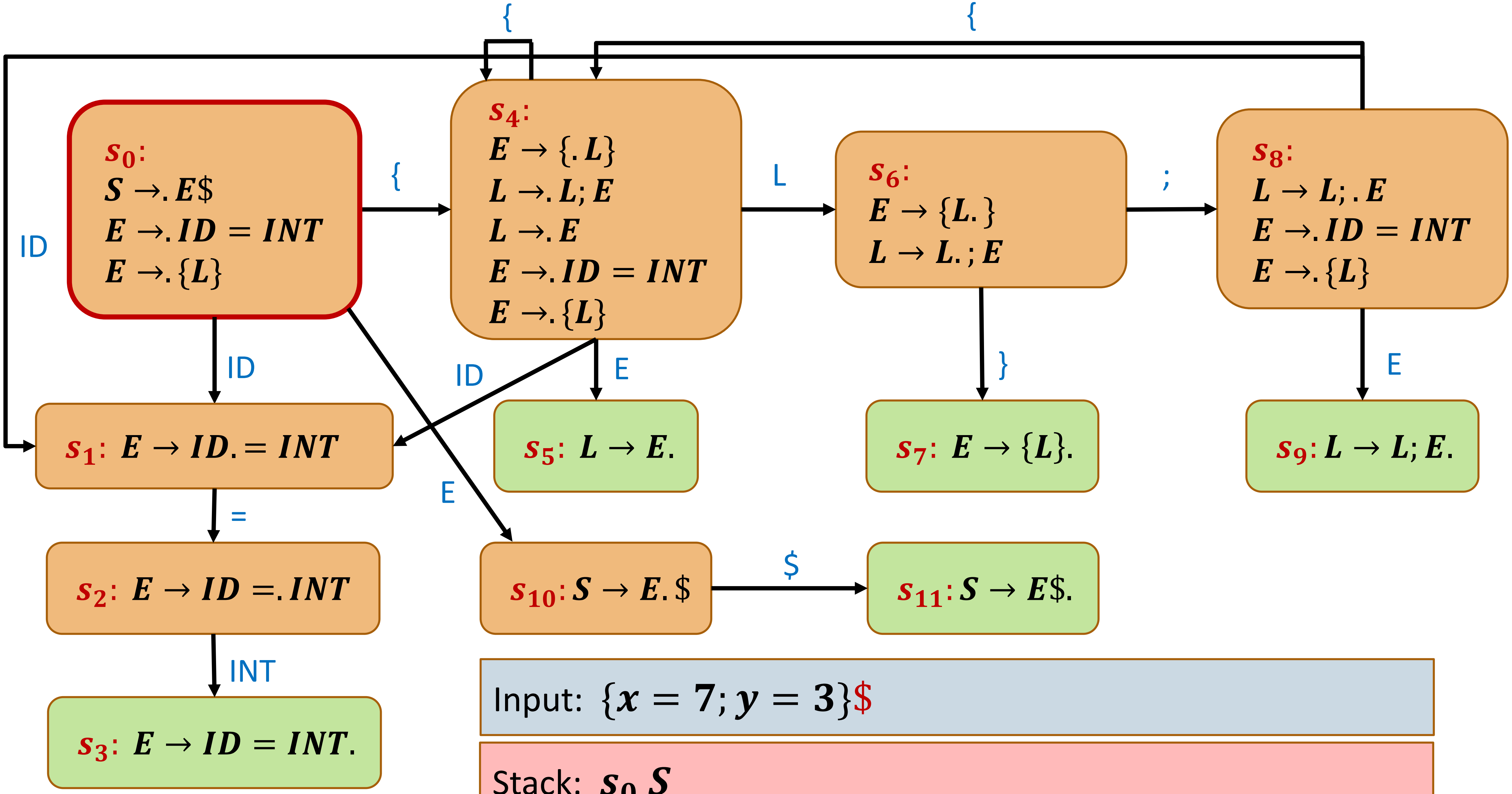


s_0: $S \to .E\$$ $E \to .ID = INT$ $E \to .\{L\}$
s_1: $E \to ID. = INT$
s_2: $E \to ID =. INT$
s_3: $E \to ID = INT.$
s_4: $E \to \{.L\}$ $L \to .L; E$ $L \to .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_5: $L \to E.$
s_6: $E \to \{L.\}$ $L \to L.; E$
s_7: $E \to \{L\}.$
s_8: $L \to L; .E$ $E \to .ID = INT$ $E \to .\{L\}$
s_9: $L \to L; E.$
s_{10}: $S \to E.\$$
s_{11}: $S \to E\$.$
{
ID
L
;
}
E
=
INT
$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ E\ s_{10}$



Input: $\{x = 7; y = 3\}\$$

Stack: $s_0 \; E \; s_{10} \; \$ \; s_{11}$



Input: $\{x = 7; y = 3\}\$$

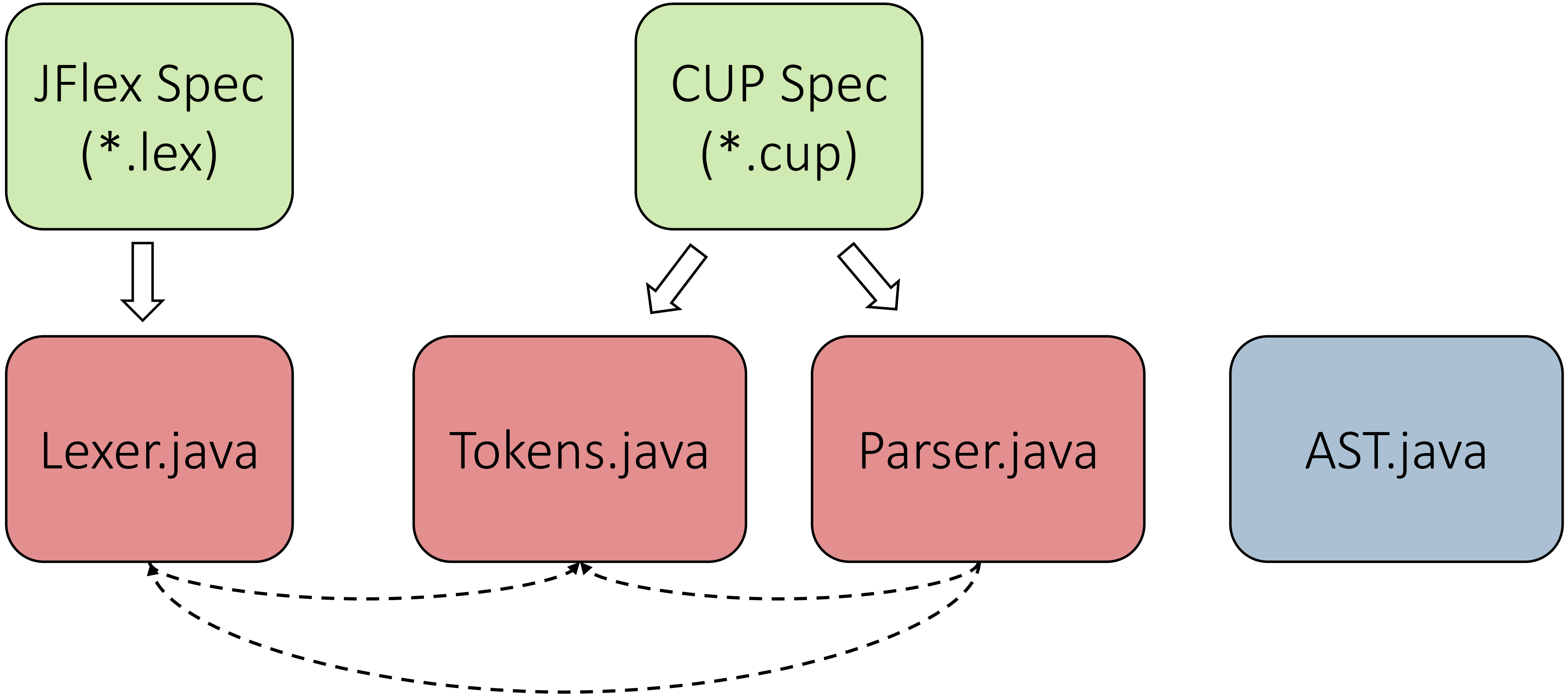
Stack: $s_0\ S$

Parsing with CUP

CUP

- Given a user-specified grammar, generates an LALR parser
- Works with JFlex, which provides the parsed tokens
- Other tools:
  - Bison (for C)

CUP/JFlex Workflow



CUP Format

parser setup

```
parser code {:
…
:}
```

lexer setup

```
scan with {:
…
:}
```

grammar

```
terminal …
non terminal …
start with …
<derivation rules…>
```

CUP Spec: Parser Setup

```
parser code {:
	public Lexer lexer;
	public Parser(Lexer lexer) {
		super(lexer);
		this.lexer = lexer;
	}
	public void report_error(String message, Object info) {
		System.exit(0);
	}
:}
```

CUP Spec: Lexer Setup

```
scan with {:
        Symbol s;
        s = lexer.next_token();
        // print token…
        return s;
:};
```

CUP Spec: Terminals

terminal T1;
terminal T2;
terminal T3;
terminal T4;
…

CUP Spec: Non-Terminals

non terminal AST_NODE_1 E1;
non terminal AST_NODE_2 E2;
non terminal AST_NODE_3 E3;
…

CUP Spec: Operator Precedence

precedence left OP1;
precedence left OP2;
precedence left OP3;
precedence left OP4;
…

These are token names…

CUP Spec: Grammar

```
start with S;
S ::=
  E1:v1           {: RESULT = new AST_NODE_CLASS_1(v1);  :} ;

S ::=
  E1:v1 E2:v2 {: RESULT = new AST_NODE_CLASS_2(v1, v2);  :}

…
```

CUP Spec: AST Nodes

- We need to **decide** which node types we have in our AST
- We need to **define** the classes for these AST nodes

CUP Example

Consider the following CFG:

- $E \rightarrow INT$
- $E \rightarrow V$
- $E \rightarrow E + E$
- $E \rightarrow E \ - E$
- $V \rightarrow ID$
- $V \rightarrow V \, .ID$

CUP Example: Terminals

```
terminal Integer INT;
terminal String ID;
terminal PLUS;
terminal MINUS;
terminal DOT;
```

CUP Example: Non-Terminals

non terminal AST_EXP EXP;
non terminal AST_VAR VAR;

CUP Example: Operator Precedence

precedence left PLUS;
precedence left MINUS;

CUP Example: Grammar

```
start with EXP;
EXP ::=
    INT:i {: RESULT = new AST_EXP_INT(i); :} |
    VAR:v {: RESULT = new AST_EXP_VAR(v); :} |
    EXP:e1 PLUS  EXP:e2 {: RESULT = new AST_EXP_BINOP(e1, e2, 0); :} |
    EXP:e1 MINUS EXP:e2 {: RESULT = new AST_EXP_BINOP(e1, e2, 1); :};
VAR ::=
    ID:name {: RESULT = new AST_VAR_SIMPLE(name); :} |
    VAR:v DOT ID:fieldName {: RESULT = new AST_VAR_FIELD(v, fieldName); :};
```

CUP Example: AST Nodes

For the non-terminal ***VAR***:

```
public abstract class AST_VAR extends AST_Node {

}
```

CUP Example: AST Nodes

For the rule ***VAR ::= ID:name***:

```
public class AST_VAR_SIMPLE extends AST_VAR {
	public String name;
	public AST_VAR_SIMPLE(String name) {
		this.name = name;
	}
}
```

CUP Example: AST Nodes

For the rule ***VAR ::= VAR:v DOT ID:fieldName*** :

```
public class AST_VAR_FIELD extends AST_VAR {
	public AST_VAR var;
	public String fieldName;
	public AST_VAR_FIELD(AST_VAR var, String fieldName) {
		this.var = var;
		this.fieldName = fieldName;
	}
}
```

CUP Example: AST Nodes

For the non-terminal ***EXP***:

```
public abstract class AST_EXP extends AST_Node {

}
```

CUP Example: AST Nodes

For the rule ***EXP ::= INT:i***:

```
public class AST_EXP_INT extends AST_EXP {
	public int value;
	public AST_EXP_INT(int value)    {
		this.value = value;
	}
}
```

CUP Example: AST Nodes

For the rule ***EXP ::= VAR:v***:

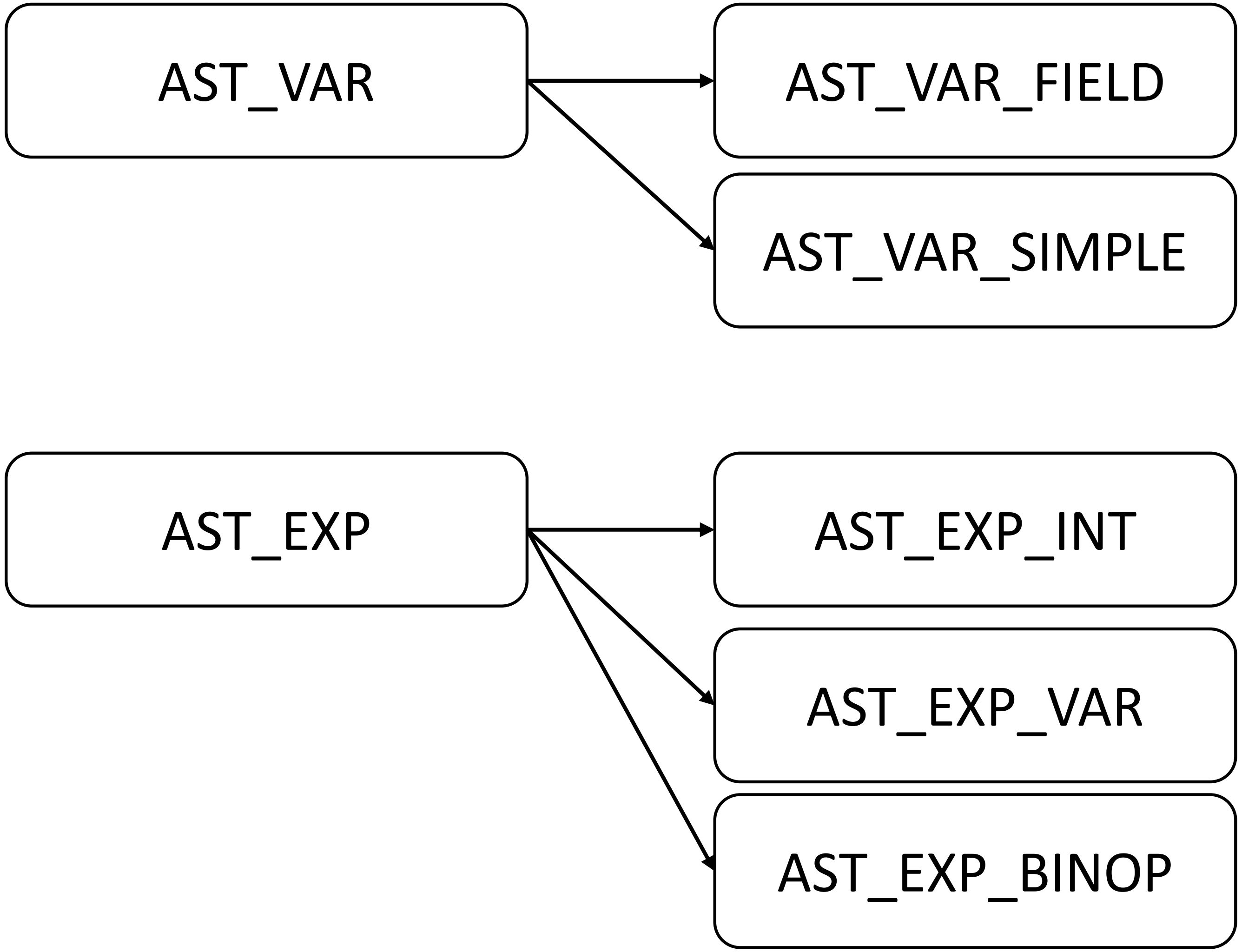
```
public class AST_EXP_VAR extends AST_EXP {
	public AST_VAR var;
	public AST_EXP_VAR(AST_VAR var) {
		this.var = var;
	}
}
```

CUP Example: AST Nodes

For the rule ***EXP ::= EXP:e1 <OP> EXP:e2*** :

```
public class AST_EXP_BINOP extends AST_EXP {
	int OP;
	public AST_EXP left;
	public AST_EXP right;
	public AST_EXP_BINOP(AST_EXP left, AST_EXP right, int OP) {
		this.left = left;
		this.right = right;
		this.OP = OP;
	}
}
```

Class Hierarchy (Inheritance)



CUP Example: Debugging

We can generate an image of the AST (using the exercise template)

For the input foo + 3 + obj.field we have: