

CS 61B: Lecture 22
Monday, October 18, 2010

Today's reading: Goodrich & Tamassia, Sections 9.1, 9.2, 9.5-9.5.1.

DICTIONARIES =====

Suppose you have a set of two-letter words and their definitions. You want to be able to look up the definition of any word, very quickly. The two-letter word is the `_key_` that addresses the definition.

Since there are 26 English letters, there are $26 * 26 = 676$ possible two-letter words. To implement a dictionary, we declare an array of 676 references, all initially set to null. To insert a Definition into the dictionary, we define a function `hashCode()` that maps each two-letter word (key) to a unique integer between 0 and 675. We use this integer as an index into the array, and make the corresponding bucket (array position) point to the Definition object.

```
public class Word {
    public static final int LETTERS = 26, WORDS = LETTERS * LETTERS;
    public String word;

    public int hashCode() {
        return LETTERS * (word.charAt(0) - 'a') + (word.charAt(1) - 'a');
    }
}

public class WordDictionary {
    private Definition[] defTable = new Definition[Word.WORDS];

    public void insert(Word w, Definition d) {
        defTable[w.hashCode()] = d;
    }

    Definition find(Word w) {
        return defTable[w.hashCode()];
    }
}
```

What if we want to store every English word, not just the two-letter words? The table "defTable" must be long enough to accommodate pneumonoultramicroscopicsilicovolcanoconiosis, 45 letters long. Unfortunately, declaring an array of length 26^{45} is out of the question. English has fewer than one million words, so we should be able to do better.

Hash Tables (the most common implementation of dictionaries) -----

Suppose n is the number of keys (words) whose definitions we want to store, and suppose we use a table of N buckets, where N is perhaps a bit larger than n , but much smaller than the number of `_possible_` keys. A hash table maps a huge set of possible keys into N buckets by applying a `_compression_function_` to each hash code. The obvious compression function is

$$h(\text{hashCode}) = \text{hashCode} \bmod N.$$

Hash codes are often negative, so remember that mod is not the same as Java's `"%"` operator. If you compute `hashCode % N`, check if the result is negative, and add N if it is.

With this compression function, no matter how long and variegated the keys are, we can map them into a table whose size is not much greater than the actual number of entries we want to store. However, we've created a new problem: several keys are hashed to the same bucket in the table if `h(hashCode1) = h(hashCode2)`. This circumstance is called a `_collision_`.

How do we handle collisions without losing entries? We use a simple idea called `_chaining_`. Instead of having each bucket in the table reference one entry, we have it reference a linked list of entries, called a `_chain_`. If several keys are mapped to the same bucket, their definitions all reside in that bucket's linked list.

Chaining creates a second problem: how do we know which definition corresponds to which word? The answer is that we must store each key in the table with its definition. The easiest way to do this is to have each listnode store an `_entry_` that has references to both a key (the word) and an associated value (its definition).

```
defTable |.--->| . | . | X | . | X | . | . | ...
          v   v       v       v       v
          |   |       |       |       |
          |.+>pus |.+>swirl |.+>tough |.+>cool|.+.>mud
          |.+>goo |.+>vortex |.+>too bad |.+>jrs |.+>wet dirt
          |.|    |X|       |X|       |.|    |X|
          +-+    ---      ---      +-+    ---
          |      |       |       |      |
          v      ^       ^       v      v
          |.+>sin      < chains > |.+>gigantic
          |.+>have fun          |.+>very big
          |X|                   |X|
          ---                   ---
```

Hash tables usually support at least three operations. An Entry object references a key and its associated value.

```
public Entry insert(key, value)
    Compute the key's hash code and compress it to determine the entry's bucket.
    Insert the entry (key and value together) into that bucket's list. I'm not
    sure why, but Goodrich & Tamassia's interface also returns the Entry object.
public Entry find(key)
    Hash the key to determine its bucket. Search the list for an entry with the
    given key. If found, return the entry; otherwise, return null.
public Entry remove(key)
    Hash the key to determine its bucket. Search the list for an entry with the
    given key. Remove it from the list if found. Return the entry or null.
```

What if two entries with the same key are inserted? There are two approaches. Following Goodrich and Tamassia, we can insert both, and have `find()` or `remove()` arbitrarily return/remove one. Goodrich and Tamassia also propose a method named `findAll` that returns all the entries with a given key. The other approach is to replace the old value with the new one, so only one entry with a given key exists in the table. Which approach is best? It depends on the application.

WARNING: When an object is stored in a hash table, an application should never change the object in a way that will change its hash code. If you do so, the object will thenceforth be in the wrong bucket.

The `_load_factor_` of a hash table is n/N , where n is the number of keys in the table and N is the number of buckets. If the load factor stays below one (or a small constant), and the hash code and compression function are "good," and there are no duplicate keys, then the linked lists are all short, and each operation takes $O(1)$ time. However, if the load factor grows too large ($n \gg N$), performance is dominated by linked list operations and degenerates to $O(n)$ time (albeit with a much smaller constant factor than if you replaced the hash table with one singly-linked list). A proper analysis requires a little probability theory, so we'll put it off until near the end of the semester.

Hash Codes and Compression Functions

Hash codes and compression functions are a bit of a black art. The ideal hash code and compression function would map each key to a uniformly distributed random bucket from zero to $N - 1$. (By "random", I don't mean that the function is different each time; a given key always hashes to the same bucket. I mean that two different keys, however similar, will hash to independently chosen integers, so the probability they'll collide is $1/N$.) This ideal is tricky to obtain.

In practice, it's easy to mess up and create far more collisions than necessary. Let's consider bad compression functions first. Suppose the keys are integers, and each integer's hash code is itself, so $\text{hashCode}(i) = i$.

Suppose we use the compression function $h(\text{hashCode}) = \text{hashCode} \bmod N$, and the number N of buckets is 10,000. Suppose for some reason that our application only ever generates keys that are divisible by 4. A number divisible by 4 mod 10,000 is still a number divisible by 4, so three quarters of the buckets are never used! We have at least four times as many collisions as necessary.

The same compression function is much better if N is prime. With N prime, even if the hash codes are always divisible by 4, numbers larger than N often hash to buckets not divisible by 4, so all the buckets can be used.

For reasons I won't explain (see Goodrich and Tamassia Section 9.2.4 if you're interested),

$$h(\text{hashCode}) = ((a * \text{hashCode} + b) \bmod p) \bmod N$$

is a yet better compression function. Here, a , b , and p are positive integers, p is a large prime, and $p \gg N$. Now, the number N of buckets doesn't need to be prime.

I recommend always using a known good compression function like the two above. Unfortunately, it's still possible to mess up by inventing a hash code that creates lots of conflicts even before the compression function is used. We'll discuss hash codes next lecture.