CS61B: Lecture 41
Friday, December 3, 2010

Generational Garbage Collection

Studies of memory allocation have shown that most objects allocated by most
programs have short lifetimes, while a few go on to survive through many
garbage collections. This observation has inspired generational garbage
collectors, which separate old from new objects.

A generational collector has two or more generations, which are like the
separate spaces used by copying collectors, except that the generations can be
of different sizes, and can change size during a program's lifetime.

Sun's 1.3 JVM divides objects into an old generation and a young generation.
Because old objects tend to last longer, the old generation doesn't need to be
garbage collected nearly as often. Hence, the old generation uses a compacting
mark-and-sweep collector, because speed is not critical, but memory efficiency
might be. Because old objects are long-lived, and because mark and sweep only
uses one memory space, the old generation tends to remain compact.

The young generation is itself divided into three areas. The largest area is
called "Eden", and it is the space where all objects are born, and most die.
Eden is large enough that most objects in it will become garbage long before it
gets full. When Eden fills up, it is garbage collected and the surviving
objects are copied into one of two _survivor_spaces_. The survivor spaces are
just the two spaces of a copying garbage collector.

If an unexpectedly large number of objects survive Eden, the survivor spaces
can expand if necessary to make room for additional objects.

Objects move back and forth between the two survivor spaces until they age
enough to be _tenured_ - moved to the old generation. Young objects benefit
from the speed of the copying collector while they're still wild and prone to
die young.

Thus, the Sun JVM takes advantage of the best features of both the
mark-and-sweep and copying garbage collection methods.

There are two types of garbage collection: minor collections, which happen
frequently but only affect the young generation - thereby saving lots of time -
and major collections, which happen much less often but cover all the objects
in memory.

This introduces a problem. Suppose a young object is live only because an old
object references it. How does the minor collection find this out, if it
doesn't search the old generation?

References from old objects to young objects tend to be rare, because old
objects are set in their ways and don't change much. Since references from old
objects to young are so rare, the JVM keeps a special table of them, which it
updates whenever such a reference is created. The table of references is added
to the roots of the young generation's copying collector.
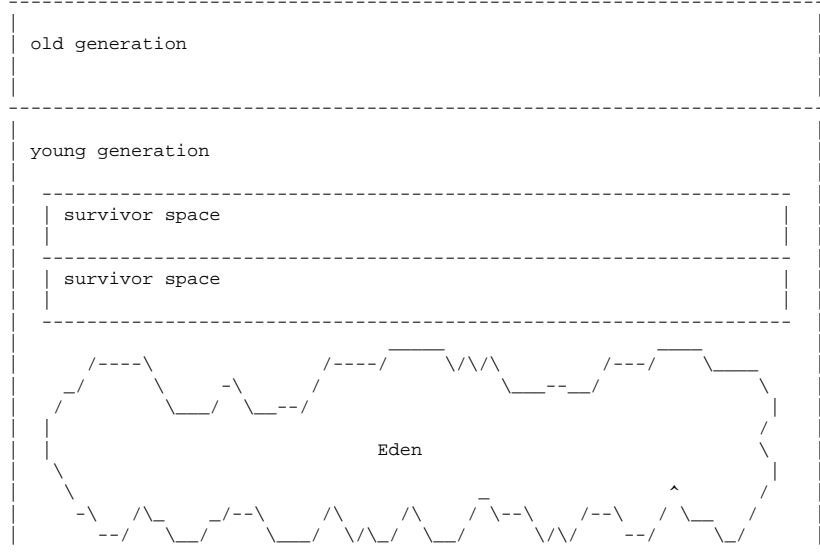
AUGMENTING DATA STRUCTURES
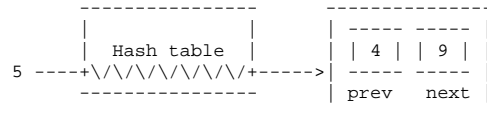==========================
Once you know how to design one of the data structures taught in this class,
it's sometimes easy to augment it to have "extra" abilities.

You've already augmented data structures in Project 3. For example, the set E
of edges is stored as both a hash table and an adjacency list. The hash table
allows you to test set membership in O(1) time, unlike the adjacency list. The
adjacency list tells you the edges adjoining a vertex in O(degree) time, unlike
the hash table.

2-3-4 Trees with Fast Neighbors

Suppose you have a 2-3-4 tree with no duplicate keys. Given a key k, you want
to be able to determine whether k is in the tree, and what the next smaller and
larger keys are, in O(1) time. The insert() and delete() operations must still
take O(log n) time. Can you do it?

It's easy if you combine the 2-3-4 tree with a hash table. The hash table maps
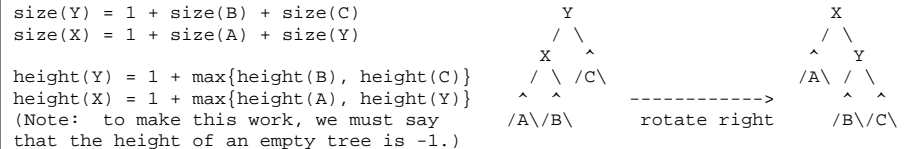each key to an record that stores the next smaller and next larger keys in the
tree.

```
        ----------------      ---------------
        |              |      | ----- ----- |
        |  Hash table  |      | | 4 | | 9 | |
   5 ----+\/\/\/\/\/\/\/+----->| ----- ----- |
        ----------------      | prev   next |
                              ---------------
```

The trick is that when you insert a key into the tree, you can determine by
tree search in O(log n) time what the next smaller and larger keys are. Then,
you update all three keys' records in the hash table in O(1) time.

Similarly, when you delete an key from the tree, you must delete it from the
hash table too, and update the records for the two neighboring keys. This too
takes O(1) time.

Splay Trees with Node Information

Sometimes it's useful for a binary search tree to record extra information in
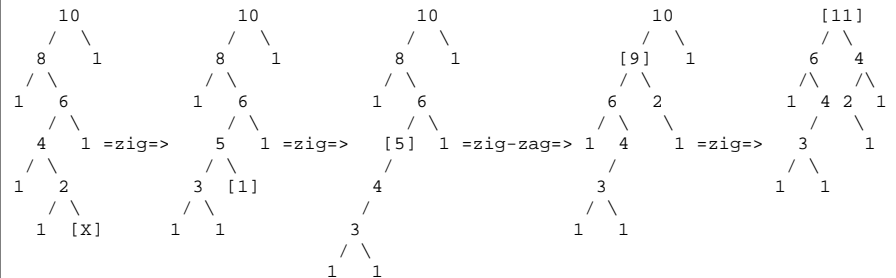each node, like the size and height of each subtree at each node.

In splay trees, this is easy to maintain. Splaying is just a sequence of tree
rotations. Each rotation changes the sizes of only two subtrees, and we can
easily compute their new sizes after the rotation. Let size(Y) be the number
of nodes in the subtree rooted at node Y. After a right rotation (for
instance) you can recompute the information as follows:

```
size(Y) = 1 + size(B) + size(C)                Y                    X
size(X) = 1 + size(A) + size(Y)               / \                  / \
                                             X   ^                ^   Y
height(Y) = 1 + max{height(B), height(C)}   / \ /C\              /A\ / \
height(X) = 1 + max{height(A), height(Y)}   ^ ^    ------------>     ^ ^
(Note:  to make this work, we must say     /A\/B\    rotate right   /B\/C\
that the height of an empty tree is -1.)
```

Be forwarned that a rotation does not just change the heights of X and Y--it
also can change the heights of all their ancestors. But X gets splayed all the
way to the root, so all the ancestors' heights are fixed on the way up.

Likewise, inserting or removing an item changes the subtree sizes of all the
ancestors of the affected item, and possibly their heights as well. But a
newly inserted item gets splayed to the top; and a deleted node's parent is
splayed to the top. So again, all the sizes and heights will get fixed during
the rotations. Let's watch the size fields as we insert a new node X into a
splay tree. (The following numbers are sizes, _not_ keys.)

Note that the very first rotation is at the grandparent of node X (zig-zig).

```
   10              10              10               10           [11]
  /  \            /  \            /  \             /  \          /  \
 8    1          8    1          8    1          [9]   1        6    4
/ \             / \             / \              / \           /\   /\
1   6           1   6           1   6            6   2         1 4 2  1
   / \             / \             / \          / \   \          / \
  4   1 =zig=>    5   1 =zig=>   [5]  1 =zig-zag=> 1 4   1 =zig=>  3    1
 / \             / \             / \             /             / \
1   2           3  [1]          4              3             1   1
   / \         / \             /              / \
  1  [X]      1   1           3              1   1
                             / \
                            1   1
```

How can we use this information? We can answer the query "How
many keys are there between x and y?" in O(log n) amortized
time if the splay tree has no duplicate keys and we label every
subtree with its size. Our strategy is to set c = n, then
deduct from c the number of keys outside the range [x, y].

```
                                                          3  find(4)
                                                         / \
                                                        2   5
                                                       /     \
                                                      1       8
                                                             / \
                                                            6   9
```

  find(x); // After the splaying, the keys in the root's left
            // subtree are all less than x, so subtract their number from c.
  c = c - size(root's left subtree);
  if (root key < x) // Only possible if x is not in the tree--
    c--;   // otherwise x was splayed to the root.

```
                                                          6  find(7)
                                                         / \
                                                        3   8
                                                       / \   \
                                                      2   5   9
                                                     /
                                                    1
```

  find(y); // After the splaying, the keys in the root's
            // right subtree all exceed y.
  c = c - size(root's right subtree);
  if (root key > y) c--;

Now, c is the number of keys in [x, y].