

# A Hasty Overview of pbdR, with an Application to Matrix Exponentiation

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<http://r-pbd.org/NIMBioS>



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## 1 The pbdR Project



## Recall: Parallel R Packages

### Shared Memory

- 1 **foreach**
- 2 **parallel**
- 3 **snow**
- 4 **multicore**

### Distributed

- 1 **Rmpi**
- 2 **RHIPE, RHadoop**
- 3 **pbdR**

(and others...)

## Programming with Big Data in R (pbdR)

Striving for *Productivity, Portability, Performance*

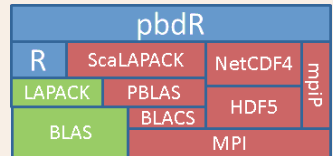
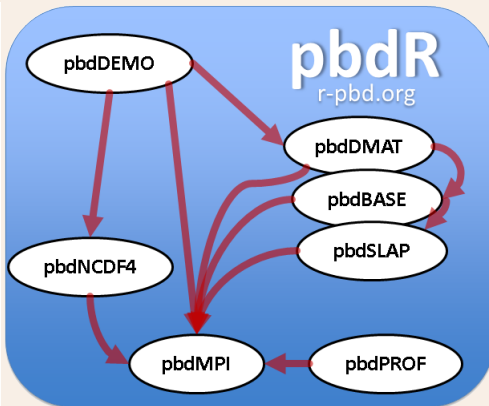


- *Free<sup>a</sup>* R packages.
- Bridging high-performance compiled code with high-productivity of R
- Scalable, big data analytics.
- Offers implicit and explicit parallelism.
- Methods have syntax *identical* to R.

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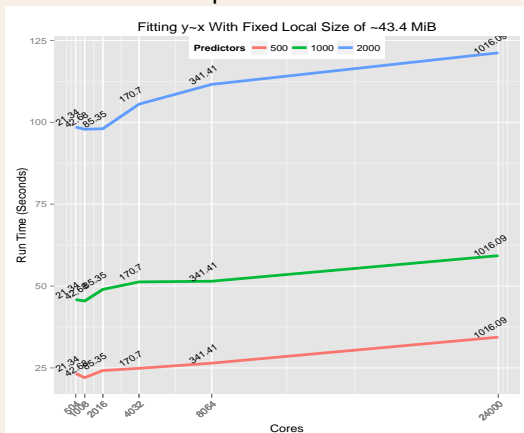
<sup>a</sup>MPL, BSD, and GPL licensed

## pbdR Packages



# Distributed Matrices and Statistics with pbdDMAT

## Least Squares Benchmark



```
x <- ddmatrix("rnorm", nrow=m, ncol=n)
y <- ddmatrix("rnorm", nrow=m, ncol=1)
mdl <- lm.fit(x=x, y=y)
```

## Profiling with pbdPROF

### 1. Rebuild pbdR packages

```
R CMD INSTALL
  pbdMPI_0.2-1.tar.gz \
  --configure-args= \
  "--enable-pbdPROF"
```

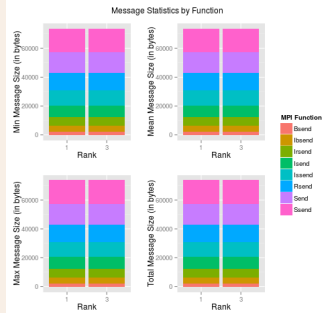
### 2. Run code

```
mpirun -np 64 Rscript
  my_script.R
```

### 3. Analyze results

```
1 library(pbdPROF)
2 prof <- read.prof(
  "profiler_output.mpiP")
3 plot(prof)
```

## Publication-quality graphs





## pbdR Scripts

- They're just R scripts.
- Can't run interactively (with more than 1 rank).
- We can use **pbdinline** to get “pretend interactivity”.

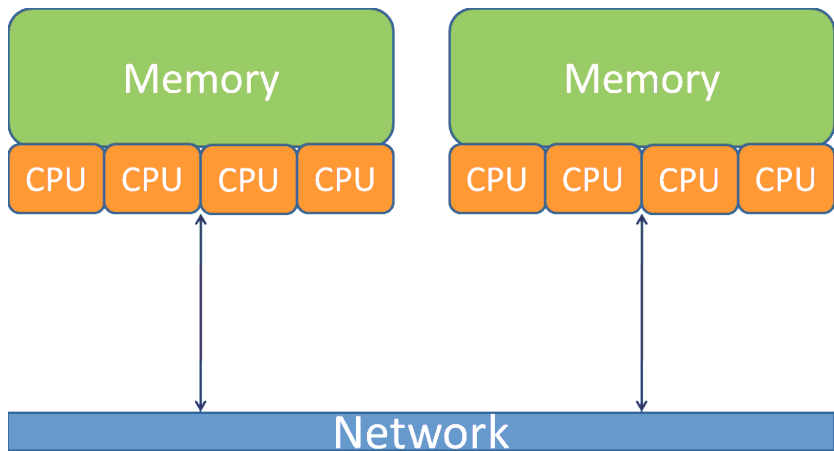
## 2 A Hasty Introduction to MPI



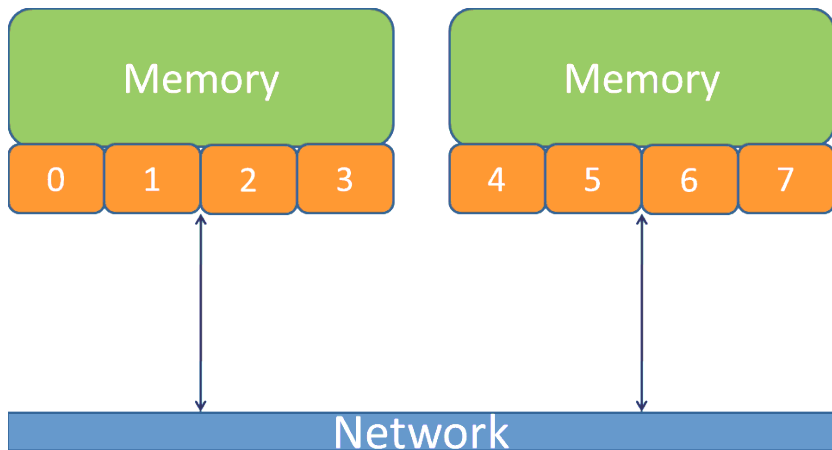
## Message Passing Interface (MPI)

- *MPI*: Standard for managing communications (data and instructions) between different nodes/computers.
- *Implementations*: OpenMPI, MPICH2, Cray MPT, ...
- Enables parallelism (via communication) on distributed machines.
- *Communicator*: manages communications between processors.

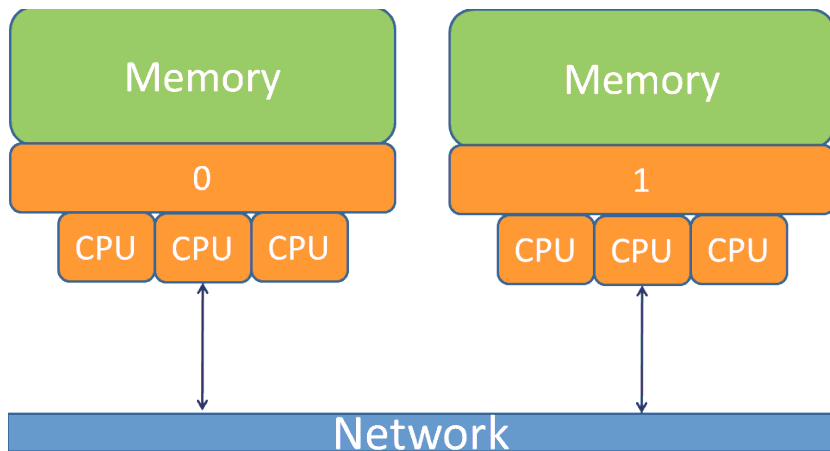
# MPI Communicators



# MPI Communicators



# MPI Communicators



## MPI Operations (1 of 2)

- **Managing a Communicator:** Create and destroy communicators.  
`init()` — initialize communicator  
`finalize()` — shut down communicator(s)
- **Rank query:** determine the processor's position in the communicator.  
`comm.rank()` — “who am I?”  
`comm.size()` — “how many of us are there?”
- **Printing:** Printing output from various ranks.  
`comm.print(x)`  
`comm.cat(x)`  
**WARNING:** only use these functions on *results*, never on yet-to-be-computed things.



## Quick Example 1

### Rank Query: 1\_rank.r

```
1 library(pbdMPI, quietly = TRUE)
2 init()
3
4 my.rank <- comm.rank()
5 comm.print(my.rank, all.rank=TRUE)
6
7 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 1_rank.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] 0
3 COMM.RANK = 1
4 [1] 1
```



## Quick Example 1: pbdinline

```
library(pbdinline)
body <- "
  my.rank <- comm.rank()
  comm.print(my.rank, all.rank=TRUE)
"

pbdRscript(body, cores=2)
```

## Quick Example 2

### Hello World: 2\_hello.r

```
1 library(pbdMPI, quietly=TRUE)
2 init()
3
4 comm.print("Hello, world")
5
6 comm.print("Hello again", all.rank=TRUE, quietly=TRUE)
7
8 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 2_hello.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] "Hello, world"
3 [1] "Hello again"
4 [1] "Hello again"
```

## Quick Example 2: pbdinline

```
library(pbdinline)
body <- "
  comm.print("Hello, world")

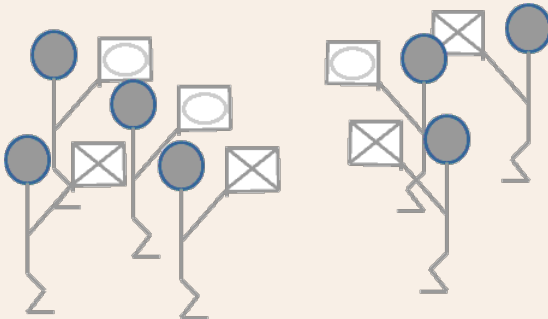
  comm.print("Hello again", all.rank=TRUE, quietly=TRUE)
"

pbdRscript(body, cores=2)
```

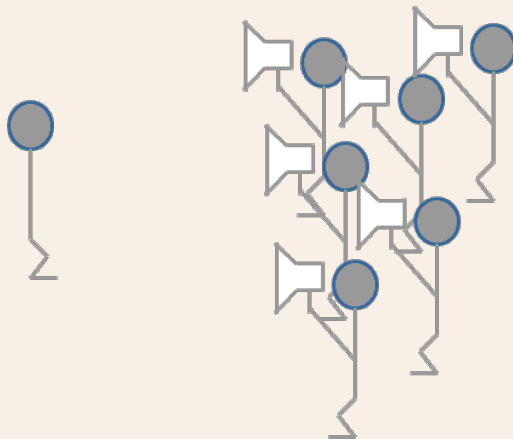
## MPI Operations

- 1 Reduce
- 2 Gather
- 3 Broadcast
- 4 Barrier

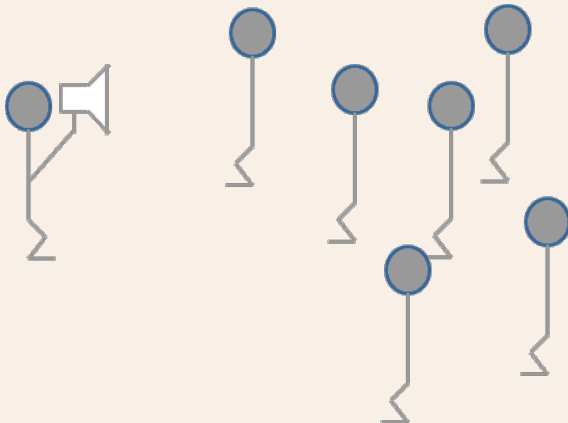
## Reductions — Combine results into single result



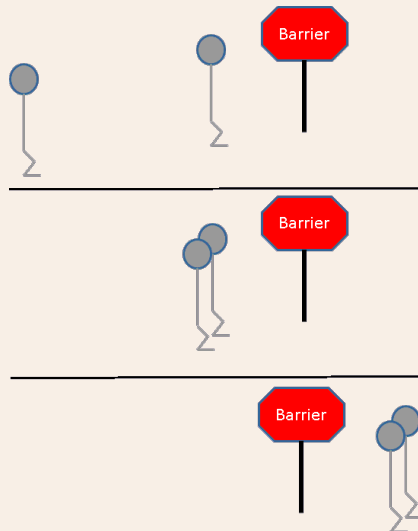
## Gather — Many-to-one



## Broadcast — One-to-many



## Barrier — Synchronization





## MPI Operations (2 of 2)

- **Reduction:** each processor has a number  $x$ ; add all of them up, find the largest/smallest, ....  
`reduce(x, op='sum')` — reduce to one  
`allreduce(x, op='sum')` — reduce to all
- **Gather:** each processor has a number; create a new object on some processor containing all of those numbers.  
`gather(x)` — gather to one  
`allgather(x)` — gather to all
- **Broadcast:** one processor has a number  $x$  that every other processor should also have.  
`bcast(x)`
- **Barrier:** “computation wall”; no processor can proceed until *all* processors can proceed.  
`barrier()`

## Quick Example 3

### Reduce and Gather: 3\_gt.r

```
1 library(pbdMPI, quietly=TRUE)
2 init()
3
4 comm.set.seed(1234, diff=TRUE)
5
6 n <- sample(1:10, size=1)
7
8 gt <- gather(n)
9 comm.print(unlist(gt))
10
11 sm <- allreduce(n, op='sum')
12 comm.print(sm, all.rank=T)
13
14 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 3_gt.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] 2 8
3 COMM.RANK = 0
4 [1] 10
5 COMM.RANK = 1
6 [1] 10
```

## Quick Example 3: pbdinline

```
library(pbdinline)
body <- "
  comm.set.seed(1234, diff=TRUE)

  n <- sample(1:10, size=1)

  gt <- gather(n)
  comm.print(unlist(gt))

  sm <- allreduce(n, op='sum')
  comm.print(sm, all.rank=T)
"

pbdRscript(body, cores=2)
```

## Quick Example 4

### Broadcast: 4\_bcast.r

```
1 library(pbdMPI, quietly=T)
2 init()
3
4 if (comm.rank()==0){
5   x <- matrix(1:4, nrow=2)
6 } else {
7   x <- NULL
8 }
9
10 y <- bcast(x, rank.source=0)
11
12 comm.print(y, rank=1)
13
14 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 4_bcast.r
```

Sample Output:

```
1 COMM.RANK = 1
2   [,1] [,2]
3 [1,]   1   3
4 [2,]   2   4
```

## Quick Example 4: pbdinline

```
library(pbdinline)
body <- "
  if (comm.rank()==0){
    x <- matrix(1:4, nrow=2)
  } else {
    x <- NULL
  }

  y <- bcast(x, rank.source=0)

  comm.print(y, rank=1)
"

pbdRscript(body, cores=2)
```

## Other Helper Tools

**pbdMPI** Also contains useful tools for Manager/Worker and task parallelism codes:

- **Task Subsetting:** Distributing a list of jobs/tasks  
`get.jid(n)`
- **\*ply:** Functions in the \*ply family.  
`pbdApply(X, MARGIN, FUN, ...)` — analogue of `apply()`  
`pbdLapply(X, FUN, ...)` — analogue of `lapply()`  
`pbdSapply(X, FUN, ...)` — analogue of `sapply()`

## 3 Distributed Matrices

## Distributed Matrices

Most problems in data science are matrix algebra problems, so:

Distributed matrices  $\implies$  Handle Bigger data



## ddmatrix: 2-dimensional Block-Cyclic with 6 Processors

$$X = \begin{bmatrix} \begin{array}{cc|cc|cc|cc|c} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ \hline x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ \hline x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ \hline x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ \hline x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{array} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \left| \begin{array}{ccc} 0 & 1 & 2 \\ 3 & 4 & 5 \end{array} \right| = \left| \begin{array}{cc} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{array} \right|$$

## Understanding ddmatrix: Local View

$$\begin{array}{ccc}
 \left[ \begin{array}{cc|cc} x_{11} & x_{12} & x_{17} & x_{18} \\ x_{21} & x_{22} & x_{27} & x_{28} \\ \hline x_{51} & x_{52} & x_{57} & x_{58} \\ x_{61} & x_{62} & x_{67} & x_{68} \\ \hline x_{91} & x_{92} & x_{97} & x_{98} \end{array} \right]_{5 \times 4} & 
 \left[ \begin{array}{cc|c} x_{13} & x_{14} & x_{19} \\ x_{23} & x_{24} & x_{29} \\ \hline x_{53} & x_{54} & x_{59} \\ x_{63} & x_{64} & x_{69} \\ \hline x_{93} & x_{94} & x_{99} \end{array} \right]_{5 \times 3} & 
 \left[ \begin{array}{cc} x_{15} & x_{16} \\ x_{25} & x_{26} \\ \hline x_{55} & x_{56} \\ x_{65} & x_{66} \\ \hline x_{95} & x_{96} \end{array} \right]_{5 \times 2} \\
 \\ 
 \left[ \begin{array}{cc|cc} x_{31} & x_{32} & x_{37} & x_{38} \\ x_{41} & x_{42} & x_{47} & x_{48} \\ \hline x_{71} & x_{72} & x_{77} & x_{78} \\ x_{81} & x_{82} & x_{87} & x_{88} \end{array} \right]_{4 \times 4} & 
 \left[ \begin{array}{cc|c} x_{33} & x_{34} & x_{39} \\ x_{43} & x_{44} & x_{49} \\ \hline x_{73} & x_{74} & x_{79} \\ x_{83} & x_{84} & x_{89} \end{array} \right]_{4 \times 3} & 
 \left[ \begin{array}{cc} x_{35} & x_{36} \\ x_{45} & x_{46} \\ \hline x_{75} & x_{76} \\ x_{85} & x_{86} \end{array} \right]_{4 \times 2}
 \end{array}$$

$$\text{Processor grid} = \left| \begin{array}{ccc} 0 & 1 & 2 \\ 3 & 4 & 5 \end{array} \right| = \left| \begin{array}{ccc} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{array} \right|$$

## Methods for class `ddmatrix`

**pbdDMAT** has over 100 methods with *identical* syntax to R:

- ``[, rbind(), cbind(), ...`
- `lm.fit(), prcomp(), cov(), ...`
- ``%*%`, solve(), svd(), norm(), ...`
- `median(), mean(), rowSums(), ...`

### Serial Code

```
1 cov(x)
```

### Parallel Code

```
1 cov(x)
```

## ddmatrix Syntax

```
1 cov.x <- cov(x)
2 pca <- prcomp(x)
3 x <- x[, -1]
4 col.sd <- apply(x, MARGIN=2, FUN=sd)
```

## 4 Matrix Exponentiation



## Exponential Function

Recall from calculus that if  $x \in \mathbb{R}$ :

$$\exp(x) = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

## Matrix Exponentiation

For a square matrix  $X_{n \times n}$ , we define the matrix exponential:

$$\text{expm}(X) = \frac{1}{1!}X + \frac{1}{2!}X^2 + \frac{1}{3!}X^3 + \dots$$

when  $X \neq \mathbf{0}_{n \times n}$ ; in this case, we take:

$$\text{expm}(\mathbf{0}_{n \times n}) = \mathbf{id}_{n \times n}$$

## Computing the Matrix Exponential

- The naive implementation leads to a loss of accuracy for many matrices.
- This problem has been vigorously argued for 30+ years.
- Moler and Van Loan, *Nineteen Dubious Ways to Compute the Exponential of a Matrix*.



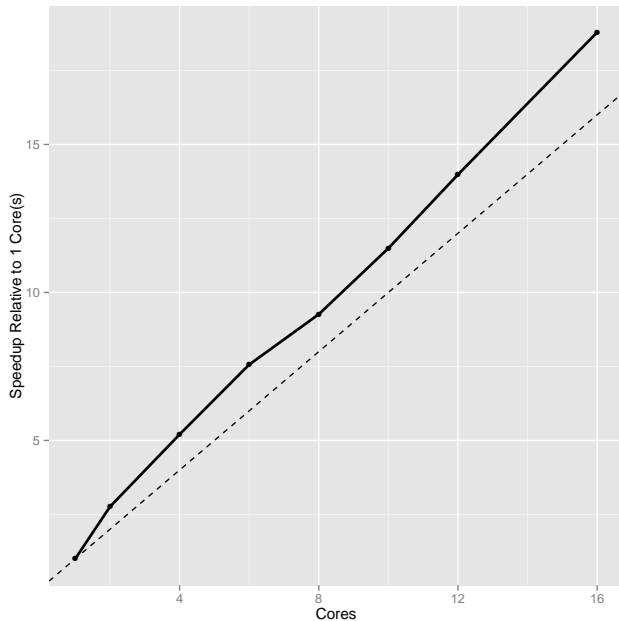
## Scaling and Squaring

We use an improvement from Al-Mohy and Higham, *A New Scaling and Squaring Algorithm for the Matrix Exponential*.

```
1 expm <- function(x)
2 {
3   n <- 2^j
4   x <- x/n
5
6   S <- matexp_pade(x)
7   S <- matpow_by_squaring(S, n)
8
9   return( S )
10 }
```

## expm()

```
1 library(pbdDMAT)
2
3 x <- matrix(rnorm(25), 5, 5)
4 expm(x)
5
6 dx <- as.ddmatrix(x)
7 expm(dx)
```



## 5 Wrapup



## Where to Learn More

- The **pbdDEMO** package: <http://cran.r-project.org/web/packages/pbdDEMO/index.html>
- The **pbdDEMO** vignette, *Speaking Serial R with a Parallel Accent*: <http://cran.r-project.org/web/packages/pbdDEMO/vignettes/pbdDEMO-guide.pdf>
- Full tutorial at UseR 2014

Thanks for coming!

Questions?