Lab Report - Free Fall

Objective:
An object in free fall is only affected by the force of gravity. If velocity is known, then the motion of the object can be predicted using kinematic equations. The goal of this lab is to use the Bullet physics simulation to predict the motion of an object in free fall.

Method:
There are two objects in the scene. One object acts as the ground. To account for volume, the ground has to be offset downwards. The restitution of both objects should be set to 0, so that the object does not bounce. The position and velocity of the falling object are reported every simulation update using the printf function, which records to six decimal places. Each simulation update takes about 1/60 seconds on average, but can vary. Time is calculated using the processor's time stamp counter, which is accurate to <1 us. To simplify the calculations, the falling object starts at rest. The simulation can also use different values for gravitational acceleration, so the experiment was repeated twice using the surface gravitational acceleration on Jupiter and Mars. The time of impact is recorded by observing when there is a sudden change in velocity.

The results are linearized using the kinematic equation:

$$\Delta d = v\Delta t + 0.5a\Delta t^2$$

$$\Delta d = 0.5a\Delta t^2$$

$$\Delta t^2 = \Delta d\frac{2}{a}$$

The experimental slope is used as the experimental value, and then compared to the theoretical value.

Raw Data:
Table 1: Height and Time for Gravitational Acceleration 9.8 m/s^2

Constants: **Mass:** 1.000 ± 0.001 kg, **Gravitational Acceleration:** 9.800 ± 0.001 m/s^2							
	Time (± 0.001 s)					Time (s)	
Height (± 0.001 m)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
10.000	1.450	1.450	1.450	1.449	1.450	1.4498	0.0005
20.000	2.033	2.033	2.034	2.033	2.100	2.05	0.03
30.000	2.483	2.484	2.483	2.483	2.484	2.4834	0.0005
40.000	2.867	2.867	2.867	2.867	2.868	2.8672	0.0005
50.000	3.216	3.217	3.219	3.217	3.267	3.23	0.03

Table 2: Height and Time for Gravitational Acceleration 24.8 m/s^2

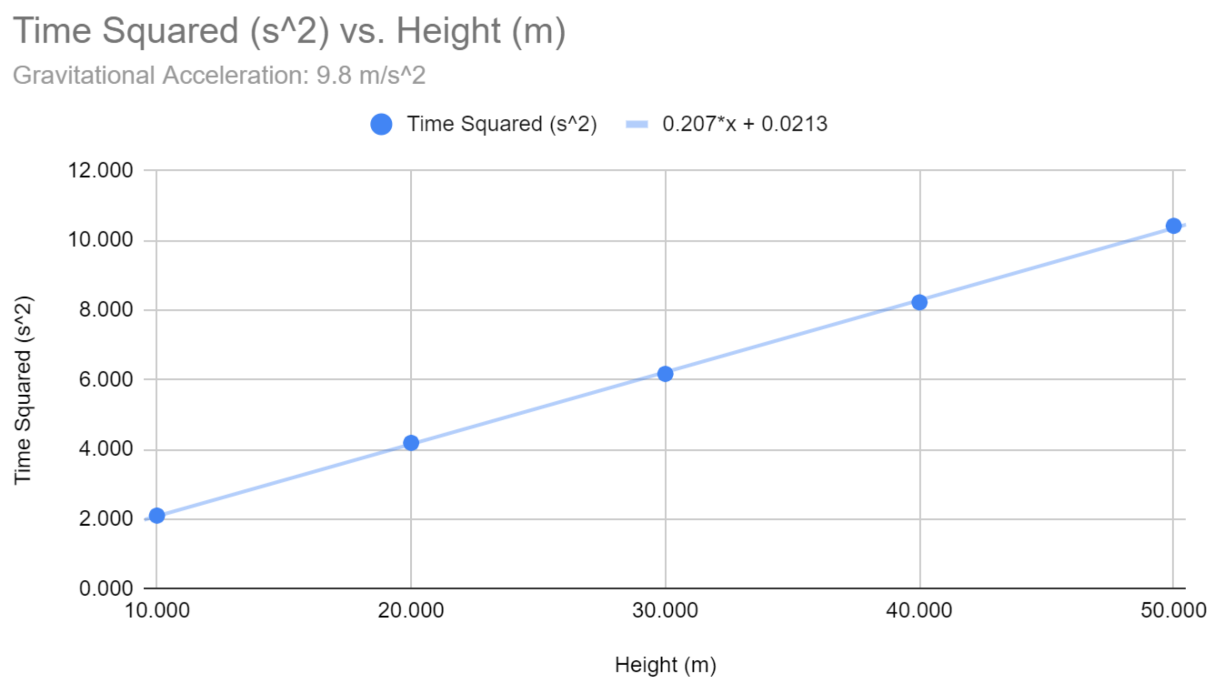
Constants: Mass: 1.000 ± 0.001 kg, Gravitational Acceleration: 24.800 ± 0.001 m/s^2							
	Time (± 0.001 s)					Time (s)	
Height (± 0.001 m)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
10.000	0.916	0.917	0.917	0.917	0.917	0.9168	0.0005
20.000	1.283	1.283	1.285	1.283	1.286	1.284	0.002
30.000	1.567	1.567	1.567	1.567	1.567	1.567	0
40.000	1.817	1.817	1.817	1.817	1.817	1.817	0
50.000	2.017	2.017	2.017	2.017	2.017	2.017	0

Table 3: Height and Time for Gravitational Acceleration 3.7 m/s^2

Constants: Mass: 1.000 ± 0.001 kg, Gravitational Acceleration: 3.700 ± 0.001 m/s^2							
	Time (± 0.001 s)					Time (s)	
Height (± 0.001 m)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
2.000	1.067	1.050	1.050	1.051	1.050	1.054	0.008
4.000	1.484	1.500	1.484	1.517	1.551	1.51	0.03
6.000	1.817	1.817	1.817	1.833	1.818	1.820	0.008
8.000	2.117	2.101	2.101	2.100	2.101	2.104	0.008
10.000	2.333	2.334	2.334	2.334	2.334	2.334	0.0005

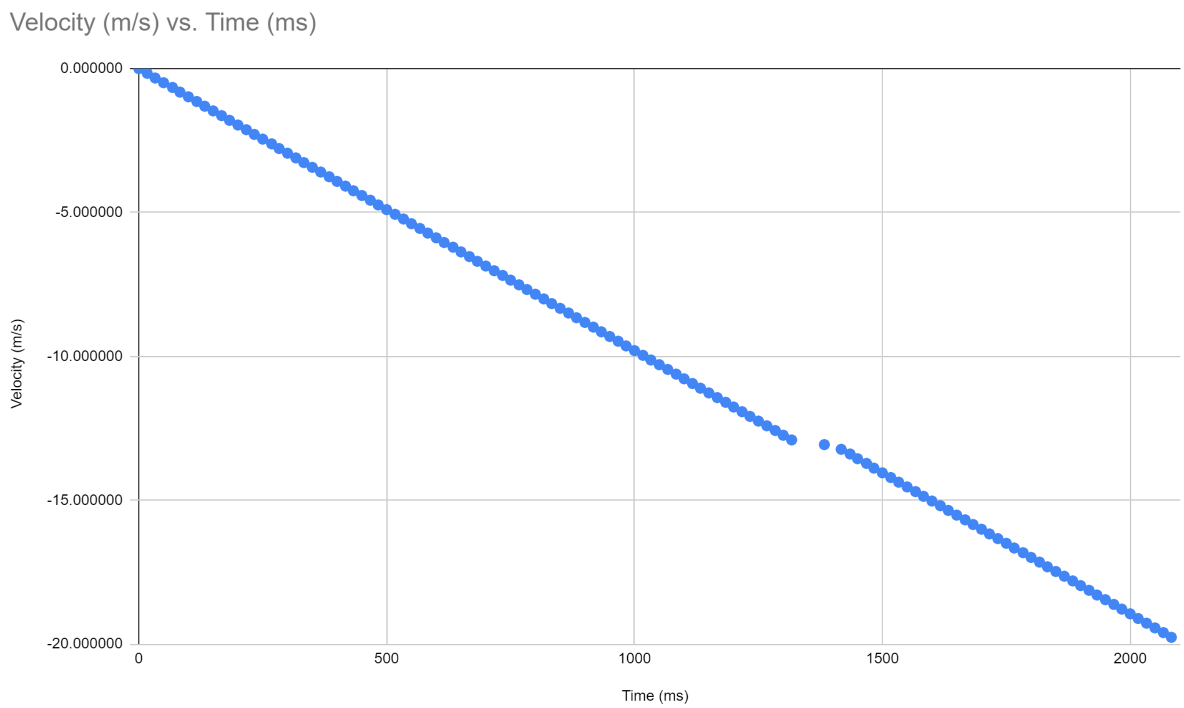
Data Analysis:

Figure 1: Time Squared vs Height for Gravitational Acceleration 9.8 m/s^2



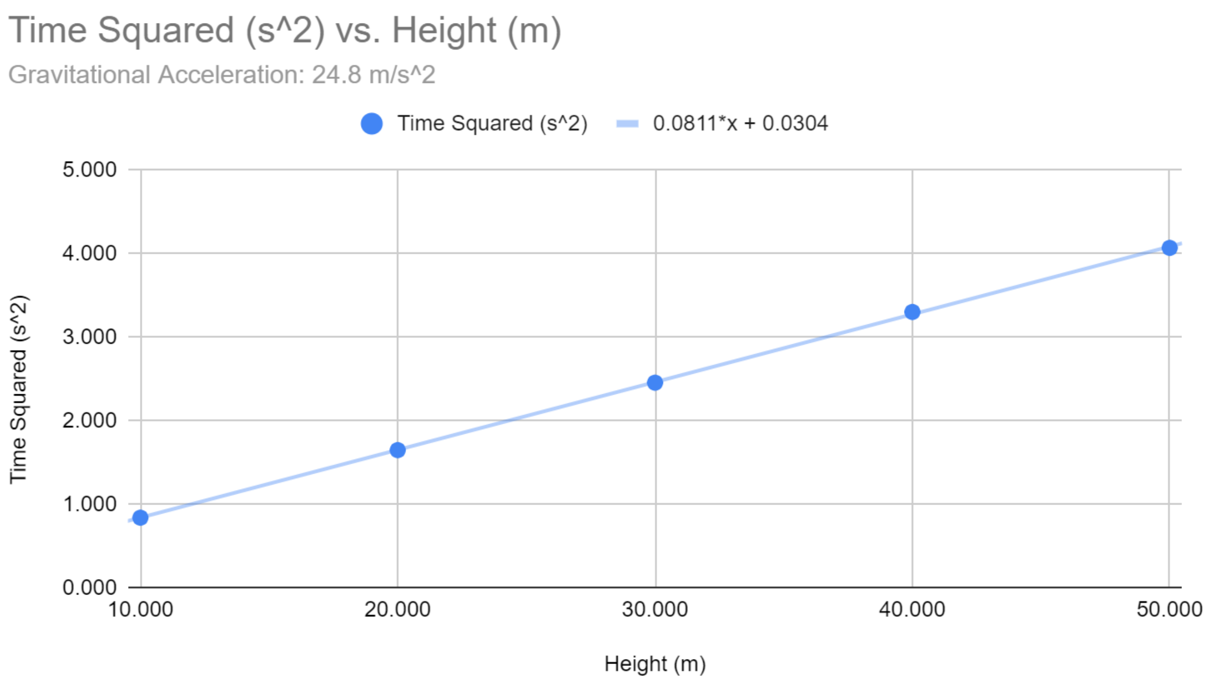
The slope of the graph represents inverse gravitational acceleration multiplied by 2 in s^2/m. The experimental value of 0.207 s^2/m agrees closely with the theoretical value of 0.204 s^2/m, with a percent error of 1.47%. The simulation is only able to update in discrete intervals, resulting in the experimental values being greater than the theoretical values. Most values were within 17 ms, which is the average update frequency. However, some values were significantly greater than the theoretical values. The fifth trial of the simulation using height = 20 m results in a predicted time 66 ms greater than the median.

Figure 2: Velocity vs. Time for Height 20 m and Gravitational Acceleration 9.8 m/s^2



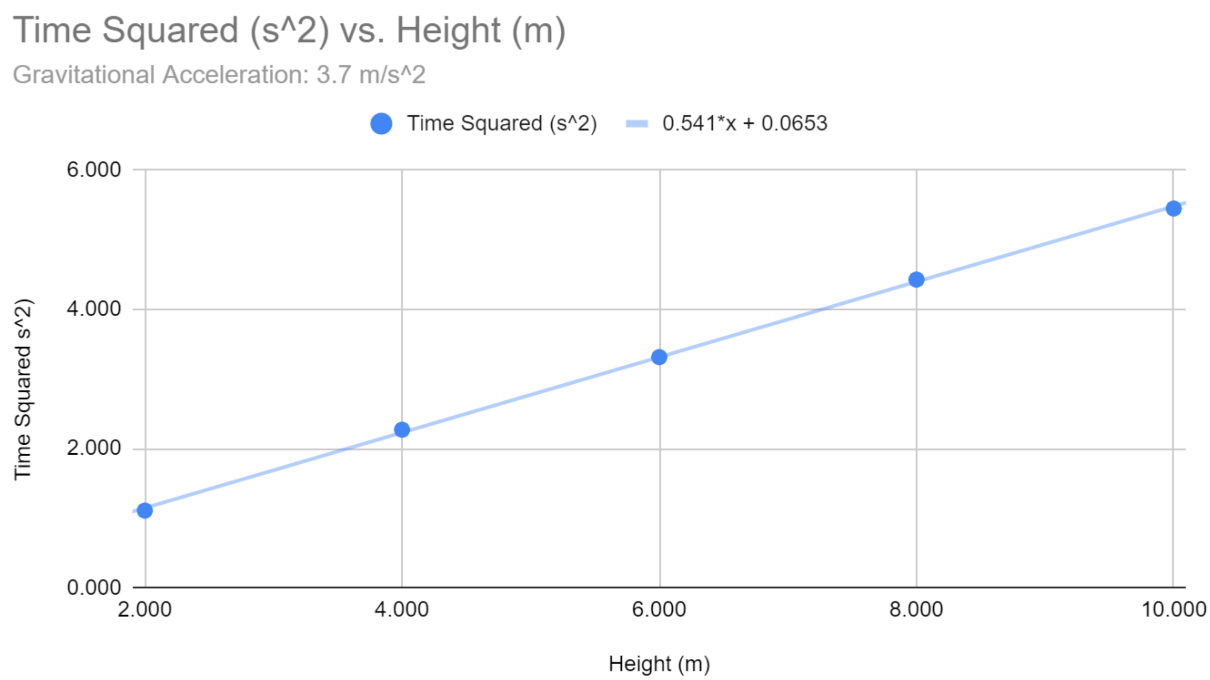
The graph appears to have a displacement around t = 1300. The raw data shows that around this time the update frequency was abnormally slow. This indicates that the simulation updates using 1/60 seconds for Δt whenever it is updated, rather than using the provided time elapsed between updates. The raw data indicates that the abnormal time period resulted in a time 66 ms greater than the expected value, which accounts for the discrepancy.

Figure 3: Time Squared vs. Height for Gravitational Acceleration 24.8 m/s^2



The experimental value using Jupiter's surface gravitational acceleration agrees closely with the theoretical value of 0.0806, with a percent error of 0.56%. The experimental values were greater than the theoretical values as expected, but within 17 ms.

Figure 4: Time Squared vs. Height for Gravitational Acceleration 3.7 m/s^2



The experimental slope using Mars' surface gravitational acceleration matches closely with the theoretical value of 0.54, with a percent error of 0.09%. As expected, the experimental values were greater than the theoretical values. Most of the values were within 17 ms, however some values were slightly greater due to slower update frequencies.

Because the simulation updates discretely, with an average frequency of 60 hz, the experimental values are greater than the theoretical values. The impact on the experimental values are somewhat random, due to random variations in the update frequency, however tends to be within one simulation step of the theoretical value. The results are linearized by squaring the time. With the error due to the discrete simulation steps, the time can be modeled as $(a + b)^2$, where a is the theoretical value and b is the error. Expanding gives $a^2 + 2ab + b^2$; b tends to be small compared to a, so will have little effect. Since b also tends to be constant, it affects all values equally. The term 2ab has a greater effect on the value with greater values of a. Therefore, it is expected that the difference between linearized values of the experimental values for time and the theoretical values will be greater for greater values of time. This would result in a greater slope, and an experimental value greater than the theoretical value. This correlates with the observations, however the difference between the experimental and theoretical slopes are so insignificant that it could easily be due to randomness.

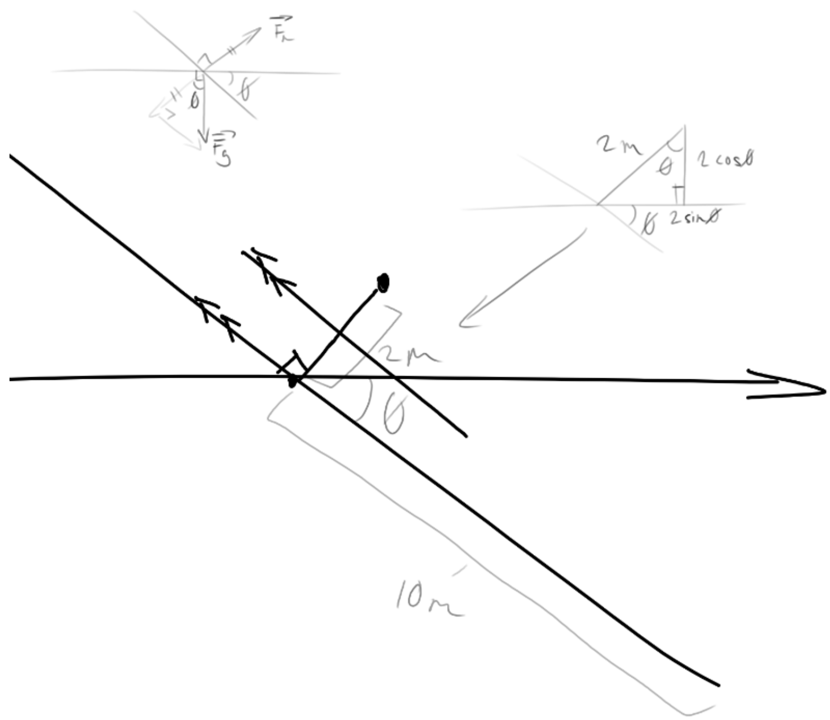
Conclusion:

The Bullet physics simulation was able to accurately simulate the motion of an object in free fall using different values of gravity. The error in the experimental values were a result of an inherent limitation in the simulation: that it must be updated discretely, rather than continuously. The experiment also revealed that the physics simulation does not properly take into account sudden increases in time between updates. This can result in accumulative error if the update frequencies are unstable, which can occur frequently.

Lab Report - Incline Plane

Objective:
The normal force on an object is perpendicular to the surface that the object is in contact with. When an object is on a ramp, the normal force does not fully cancel out the force of gravity; this results in downward and forward acceleration. The angle of the ramp changes the direction of the normal force, which impacts the direction of acceleration. The goal of this lab is to use the bullet physics simulation to predict the motion of an object on a frictionless ramp.

Method:
An object is placed on a frictionless ramp with a known distance. The time it takes for the object to travel down the ramp due to gravity is then timed. The ground is offset downwards so that the moment when the object has traveled down the ramp can be determined by looking at the instant when there is no forward acceleration. The object needs to be offset slightly because it has a volume, so the position must be calculated.



The results are linearized using the relationship:

$$\Delta d = v\Delta t + 0.5a\Delta t^2$$

$$\Delta d = 0.5a\Delta t^2$$

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{mg\sin\theta}{m} \\ &= g\sin\theta \end{aligned}$$

$$\Delta d = 0.5 \times g\sin\theta \times \Delta t^2$$

$$\begin{aligned} \Delta t^2 &= \frac{2\Delta d}{g\sin\theta} \\ &= \csc\theta \times \frac{2\Delta d}{g} \\ &= \csc\theta \times \frac{20\ \mathrm{m}}{9.8\ \mathrm{m/s}^2} \end{aligned}$$

The accuracy of the physics simulation can then be determined by comparing the experimental value with the theoretical value.

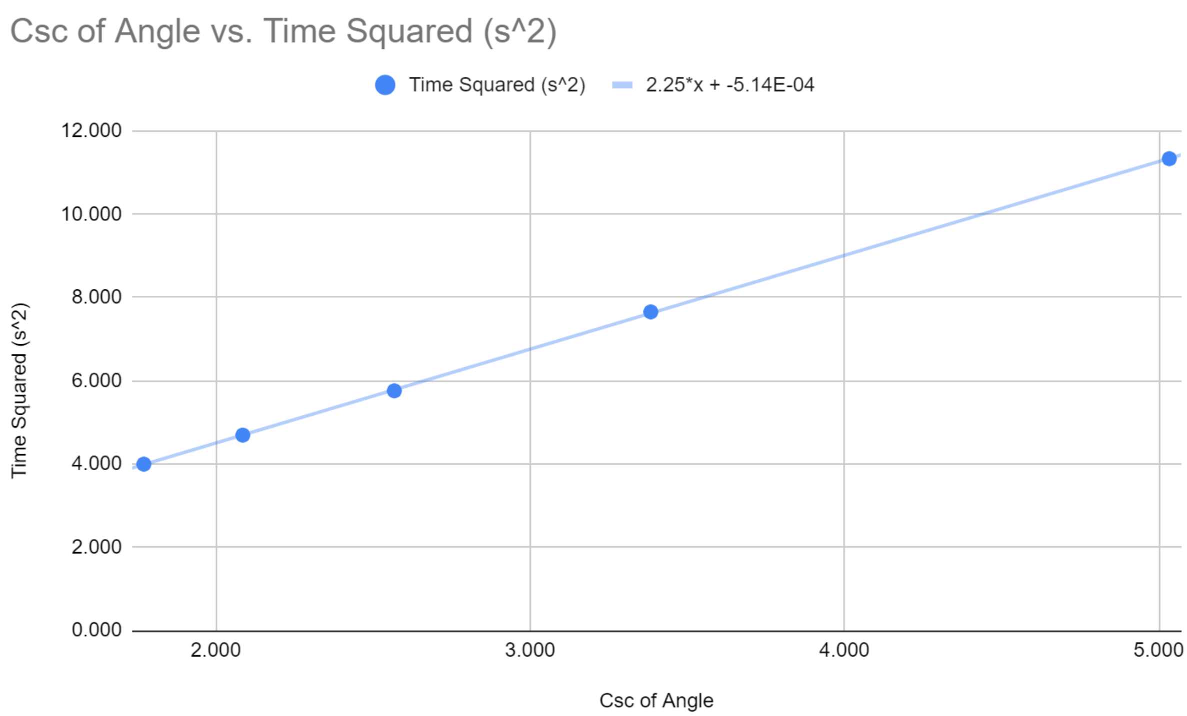
Raw Data:

Table 1: Angle and Time

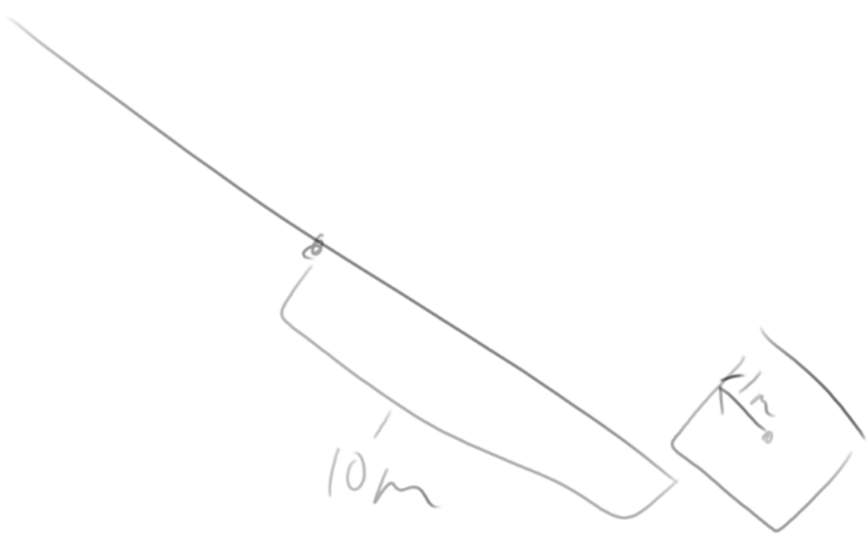
Constants: **Mass:** 1.000 ± 0.001 kg, **Gravitational Acceleration:** 9.800 ± 0.001 m/s^2, **Distance:** 10.000 ± 0.001 m							
	Time (± 0.001 s)					Time (s)	
Angle (± 0.001)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
0.200	3.366	3.367	3.367	3.367	3.367	3.3668	0.0005
0.300	2.767	2.767	2.767	2.767	2.767	2.767	0
0.400	2.401	2.400	2.400	2.400	2.400	2.4002	0.0005
0.500	2.167	2.168	2.166	2.167	2.167	2.167	0.001
0.600	2.000	2.000	2.000	2.000	1.999	1.9998	0.0005

Data Analysis:

Figure 1: Time vs. Angle



The theoretical value is roughly 2.04 s^2. The percent error of the experimental value is significant, at 10.2%. This error can be attributed to an oversight in the design of the scene. Because the cube has volume, the cube travels a longer distance than expected.



Adjusting for the additional one meter traveled, the theoretical value is roughly 2.25 s^2, with a percent error of 0.2%. All of the experimental values were within 17 ms of the theoretical value, consistent with the error expected due to simulation update intervals. Time is linearized by taking the square. With the error, time can be modeled as $(a + b)^2$, where a is the theoretical value and b is the error. Expanding the term gives $a^2 + 2ab + b^2$. The term 2ab has a greater effect on the value with greater values of a. Therefore, greater values of time are affected more than smaller values. This would result in a greater slope, and an experimental value greater than the theoretical value. This correlates with the observations, however the difference between the experimental and theoretical values are insignificant and could have other causes, such as randomness due to the discrete updates.

Conclusion:

The Bullet physics simulation was able to accurately predict the motion of an object on a frictionless ramp. Some error due to the discrete simulation intervals is expected in the experimental results, and were observed. Originally, there was some discrepancy between the experimental and theoretical results, but that was due to an oversight in the design of the lab. Adjusting for the discrepancy, the theoretical and experimental results matched closely, with a percent error of 0.2%.

Lab Report - Mechanical Energy

Objective:
The law of conservation of mechanical energy states that mechanical energy is conserved. The goal of this lab is to determine whether the bullet physics simulation properly conserves energy.

Method:
A cube is placed at some height on a ramp, giving it gravitational potential energy. Since energy is conserved, the velocity of the cube can be predicted using the law of conservation of mechanical energy.
Using the law of conservation of mechanical energy, height and velocity are related by:

$$E_g = E_k$$

$$mgh = 0.5mv^2$$

$$v^2 = (2g)h$$

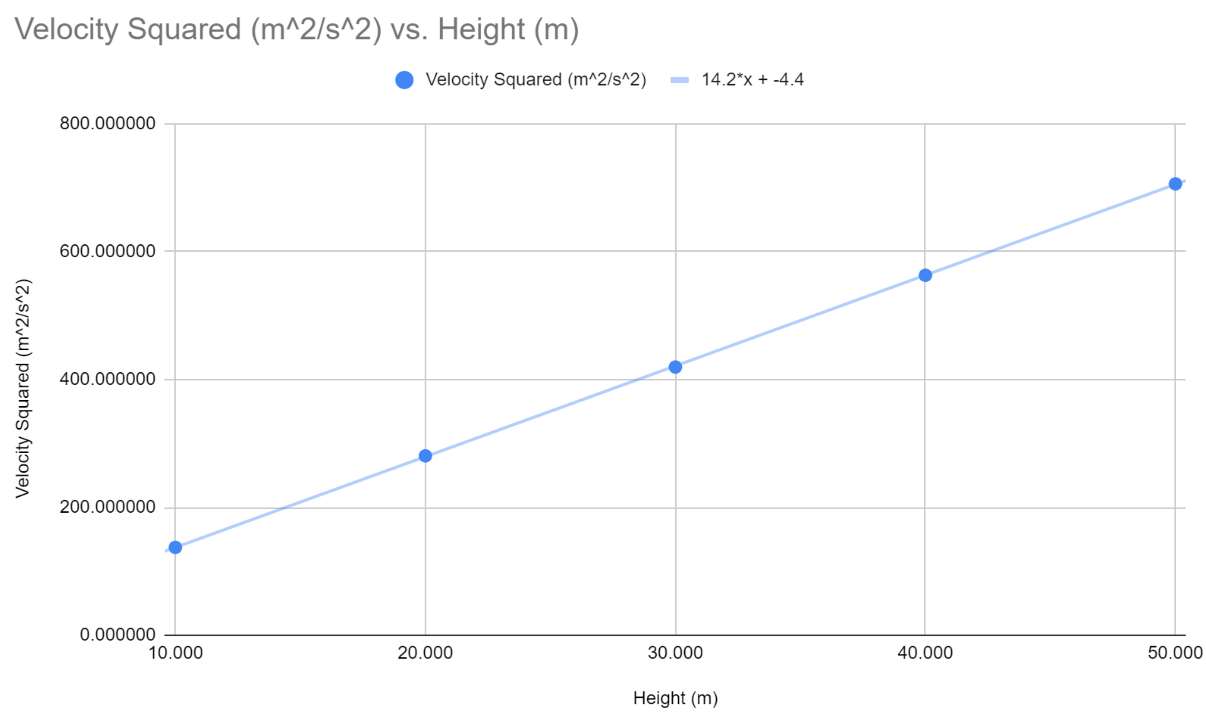
Therefore it is expected that the experimental slope be close to the value of 2g.

Raw Data:
Table 1: Height and Velocity Squared

Constants: **Mass:** 1.000 ± 0.001 kg, **Gravitational Acceleration:** 9.800 ± 0.001 m/s^2, **Angle:** 0.550 ± 0.001							
	Velocity (± 0.000001 m/s)					Velocity (m/s)	
Height (± 0.001)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
10.000	11.729712	11.729712	11.729712	11.729712	11.729712	11.729712	0
20.000	16.753786	16.753786	16.753786	16.753786	16.753786	16.753786	0
30.000	20.485489	20.485489	20.485489	20.485489	20.485489	20.485489	0
40.000	23.730259	23.730259	23.730259	23.730259	23.730259	23.730259	0
50.000	26.571943	26.571943	26.571943	26.571943	26.571943	26.571943	0

Data Analysis:

Figure 1: Velocity Squared vs.Height



The experimental value is significantly lower than the theoretical value of 19.6, with a percent error of 27.6%. This discrepancy suggests that energy is lost in the simulation. The velocity of the object is measured after the cube collides with the ground, leaving it with only a forward velocity component. Since the collision is inelastic, energy is not conserved, and the velocity in the downward direction is lost.

The vertical component of velocity is equal to $(2gh)^{0.5}\cos(\theta)$. Therefore as velocity increases, more energy is lost. This affects larger values of velocity more than smaller values of velocity, therefore the slope will be shallower, resulting in a lower experimental value.

Taking into account the component vectors:

$$v^2 = (2g)h$$

$$v = \sqrt{2gh}$$

$$\cos\theta \times v = \cos\theta \times \sqrt{2gh}$$

$$\cos^2\theta \times v^2 = \cos^2\theta \times 2gh$$

The theoretical value is therefore roughly 14.25. The experimental value therefore has a percent error of 0.3%. This suggests that energy is at least conserved in the forward direction.

Conclusion:
Due to an error in the experimental design, no conclusion can be made on whether or not the physics simulation properly conserved energy. However, it is known that energy was properly conserved in the forwards direction, as the simulation accurately predicted the forwards component of velocity of the cube.

Lab Report - Elastic Collision

Objective:
The law of conservation of momentum states that momentum is conserved in a system. In addition, perfectly elastic collisions have the property that kinetic energy is conserved. The goal of this lab is to observe whether the bullet physics simulation properly conserves momentum and energy.

Method:
A variety of masses and velocities should be chosen in order to gauge whether the physics simulation properly conserves momentum and energy. After the collision occurs, the final velocities are recorded in order to calculate energy and momentum. The initial energy and momentum should match the final energy and momentum.

Raw Data:
Table 1: Cube 1 Initial and Final Velocity

Cube 1								
		Final Velocity (± 0.000001 m/s)					Final Velocity (m/s)	
Mass (± 0.001 kg)	Initial Velocity (± 0.001 m/s)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
1.000	-5.000	4.999314	4.999314	4.999314	4.999314	4.999314	4.999314	0
1.000	-10.000	9.998953	9.999372	9.999372	9.999372	9.999372	9.9993	0.0002
1.000	-10.000	6.292553	6.292553	10.000683	6.292553	6.292553	7	2
1.000	-10.000	1.193489	1.193489	1.193489	1.193489	1.193489	1.193489	0
1.000	-20.000	6.667564	6.667564	6.667564	6.667564	6.667564	6.667564	0

Table 2: Cube 2 Initial and Final Velocity

Cube 2								
		Final Velocity (± 0.000001 m/s)					Final Velocity (m/s)	
Mass (± 0.001 kg)	Initial Velocity (± 0.001 m/s)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
1.000	5.000	-4.999314	-4.999314	-4.999314	-4.999314	-4.999314	-4.999314	0
1.000	10.000	-9.998953	-9.999372	-9.999372	-9.999372	-9.999372	-9.9993	0.0002
2.000	5.000	-3.146276	-3.146276	-5.000341	-3.146276	-3.146276	-3.5	0.9
2.000	0.000	-5.596745	-5.596745	-5.596745	-5.596745	-5.596745	-5.596745	0
2.000	0.000	-13.333782	-13.333782	-13.333782	-13.333782	-13.333782	-13.333782	0

Data Analysis:

Table 3: Initial and Final Momentum and Initial and Final Energy

Initial Momentum (Ns)	Final Momentum (Ns)	Initial Energy (J)	Final Energy (J)
0	0	25	24.99314047
0	0	100	99.98576451
0	0.000001	75	37.10975214
-10	-10.000001	50	32.03576259
-20	-20	200	200.0179473

The physics simulation was able to conserve momentum in all test cases, however, in two cases kinetic energy was not conserved. The third trial in the third test case had an interesting result. While the other trials did not conserve energy, the third trial did conserve energy. In addition the trial was significantly different from the others. While not recorded in the raw data, after the collision in the third test case, the cubes both had an upward impulse applied on them, in addition to some angular impulse.

Conclusion:

The bullet physics simulation failed to conserve kinetic energy in all test cases, breaking the law of conservation of energy. Some of the velocities given were erroneous, due to strange errors in the simulation such as applying impulses improperly.

Lab Report - Inelastic Collision

Objective:
The law of conservation of momentum states that momentum is conserved in a system. The goal of this lab is to evaluate whether the bullet physics simulation properly conserves momentum.

Method:
A variety of mass and velocities for the cubes should be chosen in order to gauge whether the physics simulation properly conserves momentum across a range of mass and velocities. Velocity is recorded before and after the collision in order to determine whether momentum is conserved.

Raw Data:
Table 1: Cube 1 Initial and Final Velocity

Cube 1								
		Final Velocity (± 0.000001 m/s)					Final Velocity (m/s)	
Mass (± 0.001 kg)	Initial Velocity (± 0.001 m/s)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
1.000	-10.000	0.033888	0.033888	0.000000	0.000000	0.000000	0.01	0.02
1.000	-20.000	0.000000	0.000000	0.526559	0.526559	0.526559	0.3	0.3
1.000	-20.000	-6.535725	-6.535725	-6.535725	-6.535725	-6.535725	-6.535725	0.000
1.000	-20.000	-4.700407	-4.700407	-4.700407	-4.700407	-4.700407	-4.700407	0.000
1.000	-5.000	5.218104	5.218104	5.218104	5.218104	5.218104	5.218104	0.000

Table 2: Cube 2 Initial and Final Velocity

Cube 2								
		Final Velocity (± 0.000001 m/s)					Final Velocity (m/s)	
Mass (± 0.001 kg)	Initial Velocity (± 0.001 m/s)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Uncertainty
1.000	10.000	-0.033888	-0.033888	0.000000	0.000000	0.000000	-0.01	0.02
2.000	10.000	0.000000	0.000000	-0.263279	-0.263279	0.000000	-0.1	0.1
2.000	0.000	-6.732137	-6.732137	-6.732137	-6.732137	-6.732137	-6.732137	0
2.000	2.500	-5.149797	-5.149797	-5.149797	-5.149797	-5.149797	-5.149797	0
2.000	10.000	4.890948	4.890948	4.890948	4.890948	4.890948	4.890948	0

Data Analysis:

Table 3: Initial and Final Momentum and Initial and Final Energy

Initial Momentum (Ns)	Final Momentum (Ns)	Initial Energy (J)	Final Energy (J)
0	0	100	0.000183743447
0	0.1053122	300	0.06099812158
-20	-19.999999	200	66.67951922
-15	-15.000001	206.25	37.56732212
15	15	112.5	37.53567702

In most test cases the physics simulation was able to conserve momentum. However, in the second test case, the physics simulation gave some inconsistent results. The first and second trial conserved momentum, however, the rest of the trials did not. In addition, they gave inconsistent final velocities for the cubes. Energy was not conserved, however that was an expected result. Initial energy was greater than final energy in all test cases, so energy was not added into the system. The results for the velocities were also different than expected. In an elastic collision, the two objects are expected to stick together, however this was not observed in the simulation.

Conclusion:

The physics simulation was able to conserve momentum most of the time, however the velocities given were different from expectations. After an inelastic collision, the two objects are expected to stick together, however this was not observed.