The Hong Kong University of Science and Technology
Department of Electronic and Computer Engineering
ELEC2600 Spring 2019 Homework-3
Please submit the soft copy of your homework solutions to Canvas
Due at 17:00 on April 16, 2019

1. (22 pts) X and Y are discrete random variables with joint pmf shown below. Assume $p_{X,Y}(j,k) = 0$ for all values not shown.

$p_{X,Y}(j,k)$		j			
		1	2	3	4
k	0	c	0.1	0	0.1
	1	0.2	0.1	0.2	0
	2	0	0.1	0	0.1

(a) Find the value of c.
(b) Find the value of the joint cdf for X and Y at the point (2, 1), $F_{X,Y}(2,1)$.
(c) Find the marginal pmfs of X and Y, respectively.
(d) Are X and Y independent? Justify your answer.
(e) Are X and Y correlated? Justify your answer.
(f) Find the correlation coefficient between X and Y.
(g) Find the conditional pmf of X given $Y < 1$.
(h) Find the conditional expected value of X given $Y < 1$.

2. (Lec 13 & 15 & 16 pairs of discrete R.V., conditional pmf and conditional moments, 17 pts) We are studying the flow of packets at a switch, which receives packets from two transmission paths, during a given period of time. Let X and Y be the numbers of packets arriving at the switch from Transmission Path 1 and Transmission Path 2, respectively. The total number of packets that arrives at the switch is thus $Z = X + Y$. Assume the joint pmf of X and Z is given by:

$$p_{X,Z}(x,z) = \begin{cases} \dfrac{3^z}{x!\,(z-x)!}(0.4)^x(0.6)^{z-x}e^{-3}, & 0 \le x \le z \\ 0, & otherwise \end{cases}$$

(a) Find the marginal pmfs of X. Identify this distribution as a distribution known in class, and give the explicit parameters for the known distribution. (Hint: $\sum_{k=0}^{\infty}\frac{\alpha^k}{k!} = e^{\alpha}$.)
(b) Find the marginal pmfs of Z. Identify this distribution as a distribution known in class, and give the explicit parameters for the known distribution.
(c) Find the marginal pmf of Y. Identify this distribution as a distribution known in class, and give the explicit parameters for the known distribution.
(d) Are X and Z independent? Justify your answer.
(e) Are X and Y independent? Justify your answer.

(f) Find the conditional pmf of X given Z. Identify this conditional distribution as a distribution known in class, and give the explicit parameters for the known distribution.
(g) Find the conditional expectation of X given Z.

3. (Lec 14 & 15 & 16 pairs of continuous R.V., conditional pdf and conditional expectation, 15 pts)
The joint pdf of X and Y is given by:

$$f_{X,Y}(x,y) = \begin{cases} c, & 0 \le x^2 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find and plot the region in the X, Y plane where the pdf is non-zero.
(b) Find the value of the constant c.
(c) Find the marginal pdfs of X and Y, respectively.
(d) Find the conditional pdf of Y given X, $f_{Y|X}(y|x)$. Plot the pdf for $x = -0.5$.
(e) Find the conditional expectation of Y given X, $E[Y|X]$. Plot $E[Y|X]$ for X from -1 to 1.

4. (Lec 14 pairs of continuous R.V., 8 pts) Alice and Bob play a game together. Assume that the scores of Alice and Bob, X and Y, are continuous and uniformly distributed in [5, 15] and [5, 20], respectively. Further assume that the scores of Alice and Bob are independent.
(a) Find the joint pdf of X and Y. Plot the region where the PDF is non-zero on the X, Y plane.
(b) Find the probability that Bob's score is higher than Alice's score.
(c) Find the probability that the sum of their scores is less than 20.

5. (Lec 17 function of pairs of R.V., 8 pts) Let X be the lifetime of a critical and expensive component in a system, which is exponentially distributed with mean 2 years. The system also has a cheaper backup component that can take over when the expensive component fails so that the system can provide continuous service while the more expensive system is being repaired. Let Y be the lifetime of the backup system, which is also exponentially distributed, but with a shorter mean 0.5 years. Assume that the lifetimes of the expensive and backup components are independent.
(a) Suppose that it turns out that the expensive system cannot be repaired. Find the pdf of the total lifetime of the system $Z = X + Y$.
(b) Let $U = X - Y$ be the difference in the lifetimes of the two systems. Note that U can be both positive and negative, since it is possible (although unlikely) that the more expensive component may last for less time than the backup. Find the pdf of U.

6. (10 pts) Download the Jupyter notebook associated with his homework and complete the exercises listed there.