ELEC 2600 HW4 solution

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1.

a.

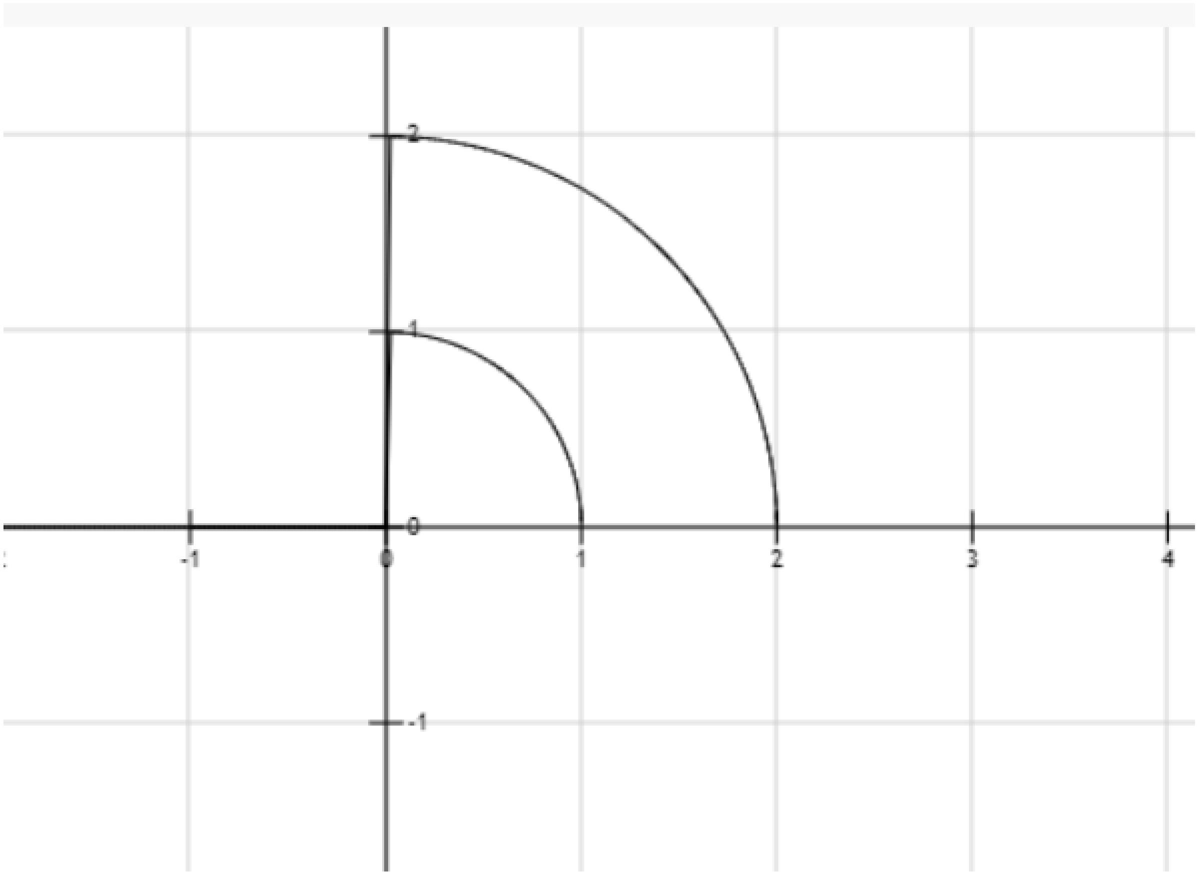
- From Chebyshev Inequality, we have $P(|X-m|>0.05) \leq \frac{p(1-p)}{n0.05^2} \leq 0.02$, so $n \geq p(1-p)*50*400$, for all possible p
- From $p(1-p) \leq 0.25$, we have $n \geq 0.25 * 50 * 400 = 5,000$

  b.
- From Central Limit Theorem, we have $P(|M_n - p| < 0.05) = 1 - 2Q(\frac{0.05}{(p(1-p)/n)^{1/2}}) \geq 0.98$
- Refer to Q function table, we have $\frac{0.05}{(p(1-p)/n)^{1/2}} \geq 2.30, n \geq 2116 * p(1-p)$
- From $p(1-p) \leq 0.25$, we have $n \geq 0.25 * 2116 = 529$

  c.
- The Chebyshev Inequality gives an absulote number that makes sure the estimated value falls in in its neighbourhood, while the Central Limit Theorem does not guarantee this.
- From this definition(and intuition), Chebyshev Inequality should result in larger number to satisfy a stronger condition.

2.

a. The plot below shows the possible values of $(X(1), Y(1))$ and $(X(2), Y(2))$, where the inner circle is the former and the outer circle the latter.



b. From the question, we have $(X(t), Y(t)) = (t\cos\theta, t\sin\theta)$, so $P[Y(2) > 1] = P[2\sin\theta > 1] = (\pi/3)/(\pi/2) = 2/3$

c. Similar to b., we have $P[X(2) > \sqrt{2}] = (\pi/4)(\pi/2) = 1/2$

d. $P[\{Y(2) > 1\} \cap \{X(2) > \sqrt{2}\}] = P[\theta \in [\pi/6, \pi/4]] = (\pi/12)/(\pi/2) = 1/6$

e. We have $P[\{Y(2) > 1\} | \{X(2) > \sqrt{2}\}] = (\pi/12) / (\pi/4) = 1/3$

3.

a. Let X_i be a Bernoulli random varible with $p = 0.15$, $E[S_{10}] = E[\sum_1^{10} X_i] = \sum_1^{10} E[X_i] = 0.15 * 10 = 1.5$. $Var[S_{10}] = Var[\sum_1^{10} X_i] = \sum_1^{10} Var[X_i] = 10 * 0.15 * (1 - 0.15) = 1.275$

$Cov(S_5, S_{10}) = 5 * 0.15 * 0.85 = 0.6375$

b. $P[S_5 \leq 1] = P[S_5 = 0] + P[S_5 = 1] = 0.85^5 + 5 * 0.85^4 * 0.15 = 0.83521$

c. From the definition of binomial counting process, we have $P[S_{10} = 3 | S_5 = 1] = P[S_5 = 2] = 0.85^3 * 0.15^2 * 10 = 0.138178125$

d. From the porperty of binomial counting process, we have $P[S_{10} = 3 \,\cap\, S_5 = 1] = P[S_5 = 1] * P[S_5 = 2] = 0.5296828125$

4.

a. the number of requests is given by $P[N(t)=k]=\frac{(\lambda t)^k}{k!}e^{-\lambda t}=\frac{(10t)^k}{k!}e^{-10t}$, where t is measured in minutes, so $P[N(0.5)=8]=\frac{(10*0.5)^8}{8!}e^{-10*0.5}=0.0652780393481587$

b. from the porperty of Poission random process, we have
$P[N(0.5)=8\ \cap\ N(1)=16]=P[N(0.5)=8]^2=0.0652780393481587^2$
$=0.00426122242113975551479308038569$

c. From the fact that $P[N(3/60)=0]=\frac{(10/20)^0}{0!}e^{-10/20}=e^{-1/2}$, we have $P[N(3/60)>0]=1-e^{-1/2}$

d. $P[N(1/60)=0\ \cap\ N(3/60)\geq 1]$
$=P[N(1/60)=0]*(1-P[N(2/60)=0])=e^{-1/6}(1-e^{-1/3})$

e. $P[N(6/60)\geq 2]=1-P[N(1/10)=0]-P[N(1/10)=1]=1-e^{-1}-e^{-1}=1-2e^{-1}$

5.

a. The mean of Y_n is $E[Y_n]=E[0.6X_n+0.4X_{n-1}]=0.6*0+0.4+0=0$

The autocorrelation $R_Y(n_1,n_2)=E[Y_{n_1}Y_{n_2}]=E[(0.6X_{n_1}+0.4X_{n_1-1})(0.6X_{n_2}+0.4X_{n_2-1})]$

- when $n_1=n_2$, it is $E[Y_{n_1}^2]=Var[Y_{n_1}]+E[Y_{n_1}]^2=2.08$
- when $|n_1-n_2|=1$, it is \$0.24E[X_{n}^2] = 0.96\$
- otherwise, 0

  The auto covariance function is the same as autocorrelation function, since $E[Y_n]=0$

b. The mean of Y_n is also 0.

The varience of Y_n is $Var[Y_n]=E[Y_n^2]=E[(0.6X_n+0.4X_{n-1})^2]$

- From the autocorrelation function of X,we have $E[X_n^2]=R_X(n,n)=4$, and $E[X_nX_{n-1}]=4e^{-1/2}$

  thus $Var[Y_n]=(0.36+0.16)*4+0.24*2*4e^{-1/2}=2.08+1.92e^{-1/2}$

  $E[Y_nY_{n-1}]$
  $=E[(0.6X_n+0.4X_{n-1})(0.6X_{n-1}+0.4X_{n-2})]$
  $=(0.36+0.16)R_X(n,n-1)+0.24R_X(n,n)+0.24R_X(n,n-2)$
  $=0.24*4+0.52*4e^{-1/2}+0.24*4e^{-1}$
  $=0.96+2.08e^{-1/2}+0.96e^{-1}$