The Hong Kong University of Science and Technology
Department of Electronic and Computer Engineering
ELEC2600 Spring 2019 Homework-1
Please submit the soft copy of your homework solutions to Canvas
Due at 17:00 on Feb 26, 2019

1. (Lec 2 event and its probability, 4 pts) A box contains 100 transistors, 5% of which are defective. Suppose you pick up 2 transistors at random from the box without replacement and with ordering. You record the condition, defective or non-defective, of the transistors picked up.
   (a) Write down the sample space of this experiment.
   (b) Define one event for this experiment other than the elementary events, null event and certain event.
   (c) Find the probability of the event you defined in (b).

2. (Lec 3 counting method, 13 pts) Consider an experiment where you pick 5 cards at random out of a deck of cards one at a time and without replacement. Note that the deck contains 52 cards comprising 13 ranks (the numbers 2, …, 10, as well as Jack, Queen, King, Ace), and for each rank there are 4 suits (hearts, diamonds, spades, clubs). Compute the sizes (number of outcomes in) and probabilities of the following events:
   (a) A: {exactly 1 of the 5 cards picked is a heart}.
   (b) B: {at least one of the 5 cards picked is a heart}.
   (c) C: {the first card picked is a heart}.
   (d) D: {the second card picked is a heart}.
   (e) E: {the first two cards picked are hearts}.
   (f) F: {at least one of the first two cards picked is a heart}.

3. (Lec 4, Bayes' rule & total probability theorem, 6 pts) A type I die is a fair six-sided die with the numbers of dots on its sides as 1, 2, 3, 4, 5 and 6, respectively. A type II die is a fair six-sided die with the numbers of dots on its sides as 1, 3 and 5, respectively. In addition, the number of dots on one side of type II is the same as the opposite side. The number of dots on each side of type III die is 1.

   A box contains 5 type I dice, 3 type II dice and 4 type III dice. We pick a die from the box at random, roll the die once and record down the number of dots that faces up.
   (a) Find the probability that the number of dots faces up is one.
   (b) Given that the number of dots faces up is one, find the probability that the die we picked and rolled is a type I die.

4. (Lec 4 & 5 Bayes' rule & independent sequential experiment, 7 pts)
Consider an experiment in which Machine A sends a sequence of binary digits to Machine B through a communication channel. A "0" digit is sent with probability 0.4; otherwise, a "1" digit is sent.

The communication channel introduces errors due to noise.
The probability that a "1" is received given that a "0" is transmitted is 0.15
The probability that a "0" is received given that a "1" is transmitted is also 0.15.

Answer the following questions:
(a) Find the probability that the first binary digit received by Machine B is "1".
(b) If a "1" is received by Machine B, what is the probability that a "1" was actually transmitted by Machine A?
(c) Assuming all transmissions are independent, find the probability that 2 errors occur if 4 digits are transmitted.
(d) Find the probability that the first time a "1" is transmitted and received correctly by Machine B is on the third transmission.

5. (Lec 5 dependent sequential experiment, 4 pts) Alice and Bob are playing chess. From Alice's point of view, a game can result in a victory (denoted by *V*) or a loss (denoted by *L*). Assume that the outcome of one game only depends on the color Alice played that game with. When Alice plays with white (denoted by *W*), she wins with probability 0.8, loses with probability 0.2. When she plays with black (denoted by *B*), she wins with probability 0.4, loses with probability 0.6. We also assume that the color Alice plays with in one game only depends on the outcome of the previous game.
Alice and Bob decide to play a three-game match with the following rules:
- For the first game, Alice plays white.
- If Alice wins a game, she plays black in the following game;
- If Alice loses a games, she plays white in the following game;

Find the probability that Alice wins the first game and loses the second game and the third game.

6. (Python & conditional probability, 10 pts) Please submit a pdf file of your completed notebook on Canvas. You can do this in one of two ways:
(1) Go to File, click Download as, click PDF via LaTeX (.pdf).
(2) Go to File, click Download as, click HTML (.html), then convert the html file to pdf file.

Use Python and data about rainfall in Hong Kong in 2018 to design a weather estimator. Based on the rainfall distribution in Hong Kong in 2018, the estimator can estimate the probability that it will rain on a day in 2019.

The file “rainfall2018.xlsx”, which is extracted from The Hong Kong Observatory https://www.hko.gov.hk/wxinfo/pastwx/mws/mws.htm, stores the rainfall in Hong Kong in 2018. Each element in the column “rainfall” in the file “rainfall2018.xlsx” indicates the rainfall on a day. A positive value indicates it rained on that day, while a 0 indicates it did not rain on that day. The column “seasons” stores the season labels. Assume that the lengths of spring in 2019 and 2018 are the same.

Your Python code should be able to output the following values:

(a) The number of rainy days in 2018.
(b) The number of spring days in 2018.
(c) The number of rainy days in the spring season in 2018.
(d) The probability that a day is rainy.
(e) The probability that a day is a spring day.
(f) The probability that a day is a spring rainy day.
(g) Given it rained on a day, the probability that this day is a spring day.
(h) The conditional probability of rain in spring.

Hint: the estimated probability of an event can be computed by the relative frequency of the event.