The Hong Kong University of Science and Technology
Department of Electronic and Computer Engineering
ELEC2600 Spring 2019 Homework-4
Please submit the soft copy of your homework solutions to Canvas
Due at 17:00 on May 10, 2019

1. (Lec 20 & 21 Law of large numbers & CLT, 10 pts) Suppose a factory produces electronic devices which have a probability p of being defective. A technician wants to estimate the probability p, by testing n devices and estimating the probability p as the fraction of devices found to be defective, i.e. if k devices are found to be defective, then the estimate of p is k/n. Assume that the defects occur independently from device to device.
   (a) Use the Chebyshev Inequality to find the number of electronic devices that should be inspected, n, such that the estimated value of p is within 0.05 of its actual value with probability greater than or equal to 98%.
   (b) Use the Central Limit Theorem to approximate the number of electronic devices that should be inspected, n, such that the estimated probability is within 0.05 of its actual value with probability greater than or equal to 98%.
   (c) Explain the difference in the estimated sample size.

2. (Lec 22 R.P., 10 pts) Suppose that the robot starts at its base (0,0) at time $t = 0$ and moves away from its base at the speed of 1cm/s in a random direction θ, which is an angle measured counterclockwise from the X axis. Suppose θ is uniformly distributed in $\left[0, \frac{\pi}{2}\right]$.
   Let $\big(X(t), Y(t)\big)$ be the position of a robot at time t.

   (a) Sketch the possible values of $(X(1), Y(1))$ and $(X(2), Y(2))$ on the (x, y) plane.
   (b) Find the probability $P[Y(2) > 1]$.
   (c) Find the probability $P[X(2) > \sqrt{2}]$.
   (d) Find the probability $P[\{Y(2) > 1\} \cap \{X(2) > \sqrt{2}\}]$
   (e) Find the probability $P[\{Y(2) > 1\} \mid \{X(2) > \sqrt{2}\}]$

   Hint. For this question, find the equivalent events in terms of θ.

3. (Lec. 24, Discrete Time Random Process, 9 pts) Suppose that S_n is a binomial counting process with parameter $p = 0.15$.

   (a) Find $E[S_{10}]$, $Var[S_{10}]$and $C_s(5,10) = Cov(S_5, S_{10})$.
   (b) Find $P[S_5 \leq 1]$.
   (c) Find $P[S_{10} = 3 | S_5 = 1]$.
   (d) Find $P[S_{10} = 3 \cap S_5 = 1]$.

4. (Lecture 25, Continuous Time Random Process, 9 pts) Suppose that the total number of requests to a web server received between time 0 and time t, $N(t)$, is given by a Poisson random process with rate $\lambda = 10$ requests per minute.

   (a) Find the probability that exactly 8 requests are received in the first 30 seconds.
   (b) Find the probability that exactly 8 requests are received in the first 30 seconds and that exactly 8 requests are received in the next 30 seconds.
   (c) Find the probability that the first request occurs before $t = 3$ seconds. Note that more than one request can occur before 3 seconds.
   (d) Find the probability that the first request occurs between $t = 1$ and $t = 3$ seconds.
   (e) Find the probability that the second request occurs before $t = 6$ seconds.

5. (Lec 22-26 mean and autocorrelation of R.P. and WSS R.P., 12 pts) In digital communication systems, the moving average (MA) filter is commonly used to reduce the effects of additive noise. Assume we designed a moving average filter whose output Y_n is given by

$$Y_n = 0.6X_n + 0.4X_{n-1},$$

   where X_n represents the input samples to the filter. Assume that processes are defined for all integer n from $-\infty$ to ∞.

   (a) Assume that the input samples, X_n, are i.i.d random variables with zero mean and variance 4. Find the mean, autocorrelation and autocovariance functions of Y_n.
   (b) Assume that the input samples , X_n, are taken from a random process with zero mean and autocorrelation function given by $R_X(n_1, n_2) = 4e^{-\frac{1}{2}|n_1 - n_2|}$. Find the mean and variance of Y_n, and $E[Y_n Y_{n-1}]$.

      Hint: Use linearity of the expectation.

6. (10 pts) Download the Jupyter notebook associated with this homework and complete the exercises listed there.