Huang Daoji
20623420

Homework 1

Problem 1

a: Notations: D stands for *defective*, and N for *non-defective*. Then the sample space

$$S = \{< D, D >, < D, N >, < N, D >, < N, N >\}, \tag{1}$$

where $\{< D, N >\}$ means the first transistor picked up is defective while the second non-defective.

b: Define event B to be *the first transistor picked up is defective*, s.t.

$$B = \{< D, N >, < D, D >\}, \tag{2}$$

which is not a null, certain event, and is not an elementary event.

c: The probability of event B is the number of outcomes in B devided by the total number of outcomes, which is

$$P(B) = \frac{5 \times 99}{100 \times 99} = 0.05, \tag{3}$$

where 5×99 means the first one is defective(5 defective transistors in the box), and the second transistor could be a random one; while 100×99 is the number of outcomes we pick up two transistors from 100 ones.

□

Problem 2

From the description in the problem, we know the five cards are ordered. The total number of outcomes S is

$$\|S\| = 52 \times 51 \times 50 \times 49 \times 48 = 311875200 \tag{4}$$

a: Define events $A_i, i \in \{1, 2, 3, 4, 5\}$, which means the i-th card is a heart, while all of the other four are non-heart. Then we have

$$A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5, \tag{5}$$

and for each A_i,

$$\|A_i\| = 13 \times 39 \times 38 \times 37 \times 36 = 25662312, \tag{6}$$

that is for that we can view each A_i as an event where we pick up four non-heart cards(ordered!) and one heart card, given the fact that the "position" of the heart card is fixed by subscript i. So we have

$$\|A\| = 5 \times (13 \times 39 \times 38 \times 37 \times 36) = 128311560, \tag{7}$$

and the probability of event A is

$$P(A) = \frac{\|A\|}{\|S\|} = 0.4114 \tag{8}$$

b: Instead of computing the sizes and probability of event B directly, consider the complement of event B, denoted by B^c, which is none of the 5 cards picked is a heart. Its size is the same with the size of outcomes where we pick 5 cards from $39(= 3 \times 13)$ cards, which is

$$\|B^c\| = 39 \times 38 \times 37 \times 36 \times 35 = 69090840. \tag{9}$$

We then have

$$\|B\| = \|S\| - \|B^c\| = 242784360 \tag{10}$$

and the probability of event B is

$$P(B) = \frac{\|B\|}{\|S\|} = 0.7785 \tag{11}$$

c: We first pick up one heart card, then pick 4 cards from the remaining 51 cards, so the size of event C is

$$\|C\| = 13 \times 51 \times 50 \times 49 \times 48 = 77968800 \tag{12}$$

and the probability of event B is

$$P(C) = \frac{\|C\|}{\|S\|} = 0.25, \tag{13}$$

which is consistent to the intuition that we have $\frac{1}{4}$ of all the cards being heart at first.

d: From the equivalence between the first and the second card, the size and probability of event D are the same with event C.
Or we can simulate the experiment step by step, and we should pay special attention to whether the first card is a heart. So the size of event D is

$$\|D\| = 13 \times 12 \times 50 \times 49 \times 48 + 39 \times 13 \times 50 \times 49 \times 48 = 77968800, \tag{14}$$

where the first term calculates cases where the first card picked is a heart, while the second calculates other cases. And the probability of event D is

$$P(D) = \frac{\|D\|}{\|S\|} = 0.25, \tag{15}$$

e Event E specifies the first two cards while the last three can be random cards, so by definition, the size of event E is

$$\|E\| = 13 \times 12 \times 50 \times 49 \times 48 = 18345600, \tag{16}$$

and the probability of event E is

$$P(E) = \frac{\|E\|}{\|S\|} = 0.0588, \tag{17}$$

f: Event F can be considered to be the union of event C and D, and we observe the fact that event E is the intersection of event C and D, so we have:

$$\|F\| = \|C \cup D\| = \|C\| + \|D\| - \|C \cap D\| = \|C\| + \|D\| - \|E\| = 137592000 \tag{18}$$

and the probability of event F is

$$P(F) = \frac{\|F\|}{\|S\|} = 0.4412, \tag{19}$$

□

Problem 3

a: Notations: we use event I, II, III to represent the event the dice picked up is of the respective type. So we have

$$\begin{aligned} P(one) &= P(one|I) \times P(I) + P(one|II) \times P(II) + P(one|III) \times P(III) \\ &= \frac{1}{6} \times \frac{5}{12} + \frac{1}{3} \times \frac{3}{12} + 1 \times \frac{4}{12} \\ &= \frac{5 + 6 + 24}{72} \\ &= \frac{35}{72} \end{aligned} \tag{20}$$

b: From Bayesian's law, we derive

$$\begin{aligned} P(I|one) &= \frac{P(one|I) \times P(I)}{P(one)} \\ &= \frac{\frac{1}{6} \times \frac{5}{12}}{\frac{35}{72}} \\ &= \frac{5}{35} = \frac{1}{7} \end{aligned} \tag{21}$$

□

Problem 4

Notations: we use event s_i, r_j to represent Machine A sends a i and Machine B receive a j respectively.

a: From the law of total probability, we have

$$\begin{aligned} P(r_1) &= P(r_1|s_1) \times P(s_1) + P(r_1|s_0) \times P(s_0) \\ &= 0.85 \times 0.6 + 0.15 \times 0.4 \\ &= 0.57 \end{aligned} \tag{22}$$

b: From Bayesian's law, we derive

$$\begin{aligned} P(s_1|r_1) &= \frac{P(r_1|s_1) \times P(s_1)}{P(r_1)} \\ &= \frac{0.85 \times 0.6}{0.57} \\ &= 0.8947 \end{aligned} \tag{23}$$

c: No matter what digit is transmitted, the error probability is 0.15. So we only need to select two out of the four transmissions to be received correctly, and the other two received wrong answers. Thus we have

$$\begin{aligned} P(two\ errors) &= (P(correct))^2 \times (P(wrong1))^2 \times C_2^4 \\ &= 0.85^2 \times 0.15^2 \times 6 \\ &= 0.0975 \end{aligned} \tag{24}$$

d: *Based on my understanding of this question, it means in the first two transmissions, Machine A sends an "1" but Machine B received a "0", or Machine A sends a "0"*
The probability of event A: a "1" transmitted and received correctly is

$$\begin{aligned} P(A) &= P(s_1) \times P(r_1|s_1) \\ &= 0.6 \times 0.85 \\ &= 0.51 \end{aligned} \tag{25}$$

So the probability of event A's first occurance is the third time is:

$$\begin{aligned} P(third\ time) &= (1 - P(A))^2 \times P(A) \\ &= 0.1225 \end{aligned} \tag{26}$$

Problem 5

We use subscript 1, 2 and 3 to denote the first, second and third game. So the probability is

$$\begin{aligned} P(V_1 \cap L_2 \cap L_3) &= P(V_1) \times P(L_2|V_1) \times P(L_3|V_1, L_2) \\ &= P(V_1) \times P(L_2|V_1) \times P(L_3|L_2), \end{aligned} \tag{27}$$

for the color only depends on the previous game. Take the three rules into account, we have

$$\begin{aligned} P(V_1 \cap L_2 \cap L_3) &= P(V_1) \times P(L_2|V_1) \times P(L_3|L_2) \\ &= P(V|W) \times P(L|B) \times P(L|W) \\ &= 0.8 \times 0.6 \times 0.2 \\ &= 0.096 \end{aligned} \tag{28}$$