

## 1 definition

k-nearest neighbors algorithm is a non-parametric method for classification and regression. In k-NN classification, the output is a class membership classified by a majority vote of its neighbors, especially, if  $k = 1$  then the object is simply assigned to the class of its nearest neighbor. In k-NN regression, the output value is the average of the values of its  $k$  nearest neighbors.

## 2 algorithm

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### Algorithm 1 k-NN algorithm

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**Input:**  $\vec{x}$ , training set  $T = (x_1, y_1), \dots, (x_n, y_n)$ , where  $\vec{x}_i \in X$  being eigenvectors and  $y_i \in Y = \{c_1, c_2, \dots, c_k\}$  being classes

**Output:** class  $y$  that  $\vec{x}$  belongs to.

- 1: find  $k$  nearest neighbors using metric given, using  $N_k(x)$  to denote these dots
  - 2: use decision rule, e.g. majority rule, to decide the class of  $x$
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## 3 metric

usually we use Euclidean distance,  $L_2(x_i, x_j) = (\sum_{l=1}^n (x_i^{(l)} - x_j^{(l)})^2)^{1/2}$ , more generally,  $L_p$  distance is  $L_p(x_i, x_j) = (\sum_{l=1}^n (x_i^{(l)} - x_j^{(l)})^p)^{1/p}$ . If  $k = 1$ , it is called Manhattan distance, and when  $k = \infty$ ,  $L_{\infty}(x_i, x_j) = \max_l \|x_i^{(l)} - x_j^{(l)}\|$

## 4 weighted nearest neighbor classifier

the k-nearest neighbor classifier could be viewed as assigning the  $k$  nearest neighbors a weight of  $1/k$  and all others 0 weight, we can generalize it to weighted nearest neighbor classifiers with the  $i$ th nearest neighbor a weight  $w_{ni}$ , and  $\sum_{i=1}^n w_{ni} = 1$

## 5 CNN for data reduction

condensed nearest neighbor is an algorithm designed to reduce the data set for k-NN classification.

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### Algorithm 2 CNN for data reduction

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**Input:** training set  $X$

**Output:** the set of prototypes  $U$

- 1: scan all elements of  $X$ , looking for an element whose nearest element in  $U$  has a different label than  $x$ .
  - 2: remove  $x$  from  $X$ , and add it to  $U$ .
  - 3: repeat till no more prototypes are added to  $U$ .
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and use  $U$  instead of  $X$  for classification.