1 QRAM

You are given a random function $f : \{0,1\}^n \to \{0,1\}^n$. The goal is to find a pair of colliding inputs (x_1, x_2), i.e., such that $f(x_1) = f(x_2) = y$.

1.1 Briefly describe the Brassard-Høyer-Tapp (BHT) quantum algorithm for finding collisions in two-to-one functions. What is the quantum query complexity of this algorithm? How does it compare to the classical complexity? (6p)

1.2 Identify the specific step in the BHT algorithm that requires the use of quantum random access memory (QRAM) to achieve a time complexity comparable to the query complexity of $O(2^{n/3})$. Explain why QRAM is essential in this step. (4p)

2 Quantum Fourier transform and phase estimation

Consider a quantum system of n qubits, each in the computational basis state $|x\rangle$, where x is an n-bit binary number. The quantum Fourier transform transforms the basis state $|x\rangle$ into a superposition of all basis states with specific phase factors.

2.1 Write the general formula for $\mathrm{QFT}_n|x\rangle$. (2p)

2.2 Apply the QFT to the 2-qubit state $|01\rangle$ and show the resulting state. (3p)

2.3 Construct the quantum circuit for QFT_2, describing the sequence of gates required. Explain the role of each gate. (4p)

2.4 Provide the general formula for the inverse Quantum Fourier Transform QFT_n^{-1}. Explain how the quantum circuit for the inverse QFT relates to the circuit for the QFT. (3p)

2.5 Explain the phase estimation algorithm and how it utilizes the quantum Fourier transform. Describe the high-level steps of the algorithm. (4p)

2.6 Consider an eigenstate $|\psi\rangle$ of a unitary operator U with eigenvalue $e^{2\pi i\theta}$, where θ is an unknown phase. Illustrate how the phase estimation algorithm can be used to estimate θ to a precision of $1/2^n$, using n qubits for the phase register. (2p)

3 Quantum states

Which of the following are valid quantum states?

3.1 $\begin{pmatrix}1\\0\end{pmatrix}$ (1p)

3.2 $\frac{1}{3}|0\rangle + \frac{2}{3}|1\rangle$ (1p)

3.3 $\frac{1}{2}|++\rangle - \frac{i}{2}|--\rangle + \frac{1}{\sqrt{2}}|+-\rangle$ (1p)

3.4 $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|+\rangle$ (1p)

4 Quantum Machine Learning

4.1 Quantum machine learning is a debated subject in the NISQ era. Discuss the viability of quantum machine learning in the NISQ era. (2p)