TECHNICAL UNIVERSITY OF DENMARK

Trial written examination, May 12th 2025

Course title: Quantum Algorithms and Machine Learning

Course number: 02195

Aids allowed: Written material only

Exam duration: 2 hours

Weighting: All problems contribute towards the final grade based on their respective number of points.

This is a 2-hour written exam, written material allowed.
All questions contribute toward the final grade and are not sorted in order of difficulty. The maximum number of points awarded by each correctly answered question is listed next to the question. NB: The exam later in May will be four hour and scaled accordingly.

Partial answers are considered and may results in points!

If you make additional assumptions, remember to briefly explain them. For all problems, when answering the different questions, remember to show/explain your calculations and motivate your answers. Failure to do so may cause substantial reduction of the number of points given.

1 Fourier transform

Recall that F_N denotes the N -dimensional Fourier transform. Its square F_N^2 turns out to map computational basis states to computational basis states.

1.1 Describe this map, i.e., determine to which basis state a basis state $|k\rangle$ gets mapped. (3p)

1.1 Show that $F_4^4 = I$. (2p)

1.1 Show that $F_N^{-1} = F_N^3$. (1p)

2 Parallel search

This question is about parallelizing search. Let $p \geq 1$ be a fixed integer. Suppose you have an input $x \in \{0,1\}^N$ and you have a special kind of oracle Q_x that answers p binary queries to x in parallel:

$$Q_x : |i_1, b_1, i_2, b_2, ..., i_p, b_p\rangle \rightarrow |i_1, b_1 \oplus x_{i_1}, i_2, b_2 \oplus x_{i_2}, ..., i_p, b_p \oplus x_{i_p}\rangle, \quad (1)$$

where the i_j's are in $\{0, ..., N-1\}$ and the b_j's are bits.

2.1 Show how you can find a solution to the search problem (i.e., an $i \in \{0, ..., N-1\}$ such that $x_i = 1$, if such an i exists) using $O(\sqrt{N/p})$ applications of Q_x. You may assume for simplicity that N/p is a power of 2. A precise higher-level description suffices, no need to draw a circuit. (6p)

3 Miscellaneous

3.1 Write down a quantum circuit that produces the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

on input $|00\rangle$ (2p).

3.2 What are the differences and similarities between supervised and unsupervised machine learning. (2p)

3.3 What are the three steps in the Quantum Approximate Optimization Algorithm? How does these contribute to solving combinatorial problems? (4p)