

1 Miscellaneous

1.1 Show how you can compose a CNOT gate using only Hadamard and controlled-Z gates. (3p)

1.2 Write down a quantum circuit that produces the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle)$$

on input $|000\rangle$. (3p)

1.3

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

is the Hadamard gate. What does H^2 (Please note the square) map $|1\rangle$ to? (2p)

1.4 Why is both gate depth and accuracy of gates important in the NISQ era? (2p)

1.5 Support vector machines also have a quantum counterpart. What is the purpose of support vector machines? (2p)

2 Hamiltonians

Suppose you have a classical description of an n -qubit Hamiltonian H that is the sum of $m = n^2$ 2-local terms – for instance enumerating Pauli strings. Assume the eigenvalues of the Hermitian matrix H lie in $[0, 1]$.

2.1 Assume that the eigenvalues of H can all be written exactly with $2 \log(n)$ bits of precision. You aim to determine the largest eigenvalue λ_{\max} of H , corresponding to the unknown n -qubit eigenstate $|\varphi_{\max}\rangle$. You are given (as a quantum state) an n -qubit state $|\varphi\rangle$ that has a significant overlap with $|\varphi_{\max}\rangle$: $|\langle\varphi|\varphi_{\max}\rangle|^2 \geq 0.7$. Give a polynomial-size quantum circuit that, with probability $\geq 2/3$, outputs λ_{\max} exactly. (8p)

2.2 Suppose now that the Hamiltonian is k -local, (for example involving the product of up to k Pauli matrices in any given Hamiltonian term). How would this affect the scaling of the algorithm? (3p)

2.3 Suppose now that the eigenvalues cannot be exactly written with $2 \log(n)$ bits of precision. Does there still exist a polynomial size quantum circuit that outputs an approximation of λ_{\min} to additive precision ϵ ? (3p)

Note: You don't need to write down the circuits in full detail, or prove the statements rigorously; clear descriptions of the different parts of the circuit, and solid argumentation is sufficient.

3 Quantum Search

Consider the following variant of the Grover search problem: we are given query access to a string $x \in \{0, 1\}^N$, $N = 2^n$, and we know a set $S \subset [N]$ of $k < N$ elements such that $x_i = 0$ for all $i \notin S$.

3.1 Give a quantum algorithm that finds an i such that $x_i = 1$, if there is one, with success probability $\geq 2/3$ using $O(\sqrt{k})$ queries to x . (6p)

3.2 How many queries are necessary to certify that there is no solution, if no solutions exist? (2p)

4 Variational Algorithms

- 4.1 Assume you have a Hamiltonian of a molecule. What is the relationship between the lowest energy state of the molecule and the Hamiltonian? (2p)
- 4.2 What is the variational principle? (3p)

End of problem set