TECHNICAL UNIVERSITY OF DENMARK

Trial written examination, May 7st 2025

Course title: Quantum Compilers

Course number: 02196

Aids allowed: Written material only

Exam duration: 2 hours

Weighting: All problems contribute towards the final grade based on their respective number of points.

This is a 2-hour written exam, written material allowed.
All questions contribute toward the final grade and are not sorted in order of difficulty. The maximum number of points awarded by each correctly answered question is listed next to the question. NB: The exam later in May will be four hour and scaled accordingly.

Partial answers are considered and may results in points!

If you make additional assumptions, remember to briefly explain them. For all problems, when answering the different questions, remember to show/explain your calculations and motivate your answers. Failure to do so may cause substantial reduction of the number of points given.

1 Miscellaneous

1.1 What are the three main parts of a quantum neural network? (3p)

1.2 What is the role of the classical part in varational algorithms? (2p)

1.3 Many quantum oriented programming models for HPC-QC integration is leveraging C. Why is that so? (2p)

2 Szegedy algorithm

Consider a reversible, ergodic Markov chain on the state space $\Omega = \{1, 2, 3\}$. It has transition matrix

$$P = \begin{pmatrix} 0.6 & 0.2 & 0 \\ 0.4 & 0.5 & 0.6 \\ 0 & 0.3 & 0.4 \end{pmatrix}.$$

and its stationary distribution is $\pi = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$. Throughout the problem let $\delta = 1 - \lambda_1$ denote the spectral gap, where λ_1 is the second-largest eigenvalue of P. For each $x \in \Omega$ define the quantum states

$$|\psi_x\rangle = \sum_{y\in\Omega} \underset{yx}{\sqrt{P_{[illegible]}}}\, |y\rangle, \qquad \Pi := \sum_{x\in\Omega} |x\rangle\langle x| \otimes |\psi_x\rangle\langle\psi_x|.$$

The (swap) operator S acts as $S|x, y\rangle = |y, x\rangle$ and $\Lambda := S\Pi S$. The *Szegedy walk* unitary is

$$W := (2\Pi - I)\,(2\Lambda - I).$$

Assume you can implement controlled-W exactly and that preparing $|\psi_x\rangle$ controlled on the first register costs $O(1)$ elementary gate.

2.1 Show that every non-stationary eigenvalue $e^{i\theta}$ of W satisfies $|\theta| \geq 2\sqrt{\delta}$. (4p) (Hint: use $\cos\theta = \lambda$ where λ is an eigenvalue of P)

2.2 Describe a phase-estimation circuit that, given input state $|\varphi\rangle \in \mathbb{C}^9$, appends an ancilla register and marks the eigenvalue [illegible] 1 (stationary state) component of W with probability $\geq 1 - \varepsilon$. Specify the required phase precision $\varepsilon_{\mathrm{PE}}$ in terms of δ. (4p)

2.3 Argue that the circuit uses $O(\frac{1}{\sqrt{\delta}} \log \frac{1}{\varepsilon})$ controlled uses of W. (3p)

2.4 Start from the uniform superposition $|u'\rangle = \frac{1}{\sqrt{3}} \sum_{x=1}^{3} |x\rangle \otimes |\psi_x\rangle$, and compute the probability $p = |\langle\Phi|u'\rangle|^2$ with the phase-0 (stationary) eigenstate $|\Phi\rangle = \sum_x \sqrt{\pi_x}\, |x\rangle \otimes |\psi_x\rangle$. (4p)

2.5 Using the phase estimation oracle from 2.2 and a reflection about $|u'\rangle$, determine how many Grover iterations are needed to obtain fidelity ≥ 0.99 with $|\Phi\rangle$. (6p)

2.6 What is the overall complexity of the procedure? (3p)

3 QRAM

The collision finding problem for a random function can be stated as follows. Given oracle access to a random function $f : \{0,1\}^n \to \{0,1\}^n$ find a pair of colliding inputs (x_1, x_2), i.e., a pair such that $f(x_1) = f(x_2)$.

One of the quantum algorithms that solves this problem when quantum query access is given is the BHT algorithm. It uses Grover's search as a subroutine, and can be described as follows:

- Using ℓ queries, create a list $\mathcal{L} = ((x_0, f(x_0)), \ldots, (x_{\ell-1}, f(x_{\ell-1})))$. If the list contains a collision, output it and stop.
- Construct the function $f_{\mathcal{L}} : \{0,1\}^n \to \{0,1\}$ such that $f_{\mathcal{L}}(x) = 1 \leftrightarrow \exists i \in \{1, \ldots, \ell\} : f(x) = f(x_i)$. In words: the membership function $f_{\mathcal{L}}(x) = 1$ if $f(x)$ appears as an output value in the list $\mathcal{L}$, and $f_{\mathcal{L}}(x) = 0$ else.
- Use Grover's search with $\sqrt{\frac{2^n}{\ell}}$ steps to find an input x with $f_{\mathcal{L}}(x) = 1$. If $x = x_i$ for some $i \in \{1, \ldots, q\} : f(x) = f(x_i)$, repeat this step, else, find a pair $(x_i, f(x_i))$ in the list such that $f(x_i) = f(x)$ and output (x, x_i).

3.1 Identify the specific step in the BHT algorithm that requires the use of quantum random access memory (QRAM) to achieve a time complexity comparable to the query complexity of $O(2^{n/3})$ for $\ell = 2^{n/3}$. Explain why QRAM is essential in this step. (4p)

3.2 Suppose you have a piece of hardware that provides an intermediate functionality between a cicuit model quantum memory and an actual QRAM, for which each memory access to one out of ℓ addresses costs time $\sqrt{\ell}$. Determine the optimal value for the parameter ℓ in the BHT algorithm in this scenario, and the resulting runtime of the BHT algorithm. You can use the rule of thumb that the optimal value for ℓ is the value such that the runtime of the first and third step in the algorithm description above are equal. (6p)

End of problem set