

1 Quantum gates

$$|A\rangle = \alpha|0\rangle + \beta|1\rangle \text{ with } \alpha, \beta \in \mathbb{C}$$

$$\langle A| = \bar{\alpha}\langle 0| + \bar{\beta}\langle 1| = |A\rangle^\dagger$$

$$\langle 0|A\rangle = \alpha \quad \langle 1|A\rangle = \beta$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Gates	$ 0\rangle$	$ 1\rangle$	$ -\rangle$	$ +\rangle$
σ_x	$ 1\rangle$	$ 0\rangle$	$- -\rangle$	$ +\rangle$
σ_y	$i 1\rangle$	$-i 0\rangle$	$i +\rangle$	$-i -\rangle$
σ_z	$ 0\rangle$	$- 1\rangle$	$ +\rangle$	$ -\rangle$
H	$ +\rangle$	$ -\rangle$	$ 1\rangle$	$ 0\rangle$

$$\sigma_x \sigma_y = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i \sigma_z \quad \sigma_y \sigma_z = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i \sigma_x \quad \sigma_z \sigma_x = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = i \sigma_y$$

$$\sigma_y \sigma_x = -\sigma_x \sigma_y \quad \sigma_z \sigma_y = -\sigma_y \sigma_z \quad \sigma_x \sigma_z = -\sigma_z \sigma_x$$

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = H^2 = \mathbf{I}$$

$$\sigma_x \sigma_y \sigma_z = i \mathbf{I}$$

$$\sigma_y \sigma_z \sigma_x = i \mathbf{I}$$

$$\sigma_z \sigma_x \sigma_y = i \mathbf{I}$$

$$\sigma_x \sigma_z \sigma_y = -i \mathbf{I}$$

$$\sigma_y \sigma_x \sigma_z = -i \mathbf{I}$$

$$\sigma_z \sigma_y \sigma_x = -i \mathbf{I}$$

