Rubik Group

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Basic Notation

A single letter by itself, refers to a *clockwise* face rotation in 90^o (quarter turn). If it's followed by an apostrophe means to turn that face *counterclockwise* 90^o. A number after it, marks repeated turn.

- Side turns
  - L Left
  - D Down
  - F Front
  - R Right
  - U Upper
  - B Back
- Layer turns
  - M Middle layer, direction as L
  - E Equatorial layer, direction as D
  - S Standing layer, direction as F
- Full turns
  - X All the cube, direction as R
  - Y All the cube, direction as U
  - Z All the cube, direction as F

Identities

$\Sigma = L, D, F, R, U, B, M, E, S, X, Y, Z$

A basic turn is an element of the alphabet, and it could or not be followed by an apostrophe. All the valid moves are either basic turns or a sequence of basic ones. All movements can be executed as a sum of one lateral turn and two full turns.

The identity is equal to a neutral move. So, $I = T + T' = 4nT$. Let $m, n \in \Sigma$ then $m + n \neq n + m$. The sum isn't commutative. The inverse of a valid move is defined as:

$$inv(m) = \begin{cases} m' & , m \text{ is basic} \\ (\text{map inv} * \text{reverse})\, m & , m \text{ is sequence} \end{cases}$$

`I = foldl (+) I [X, Z, Y, Z']` $I = [X, Z, Y, Z'] = [X, Y, Z, Y, 2Z]$

Define all basic moves with X and (Y, D)

- $2T = 2T', 3T = T', 4T = I$
- $I \to X' = [2Y, X, 2Y]$
- $I_2 \to L = [Y, X, D] + [X', Y]$
- $I_2 \to R = [Y', X, D] + [X', Y]$
- $L_2 \to F = [2Y, X, D] + [X']$
- $L_2 \to B = [X, D] + [X']$
- $L_2 \to U = [2X, D] + [2X] = [Y, 2X, Y']$