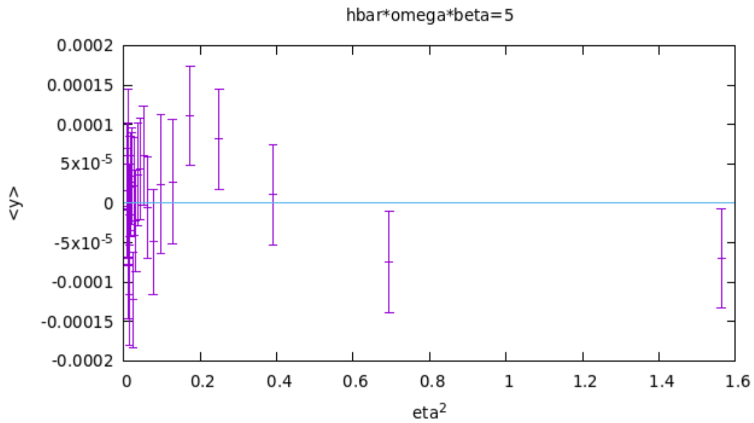
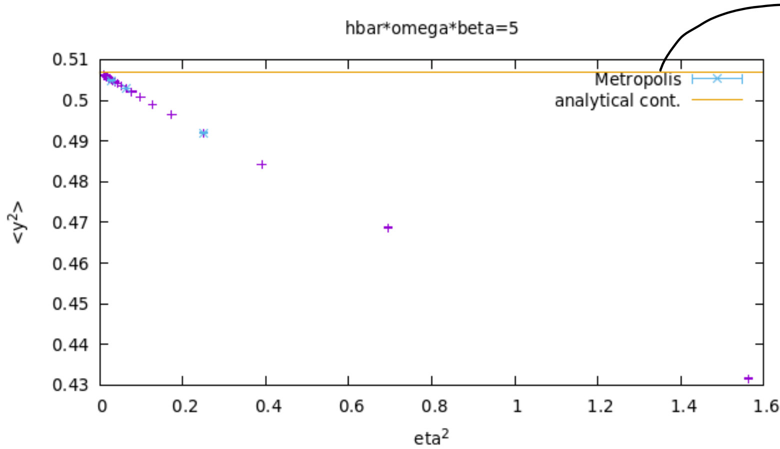
Thermodynamics of the 1-dim. harmonic oscillator

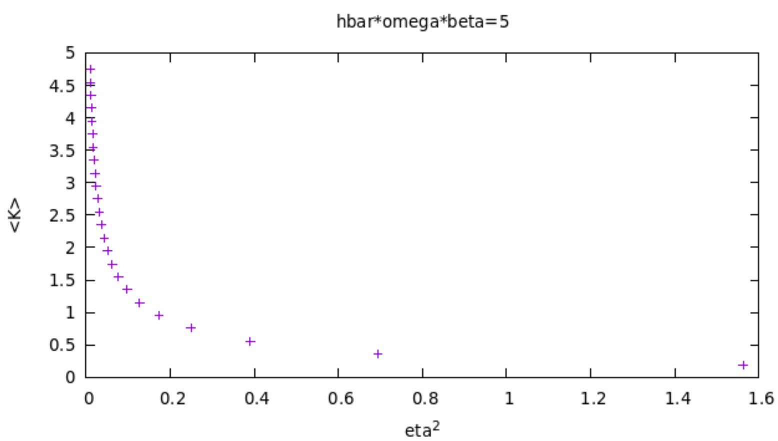


Check that the average position vanishes.



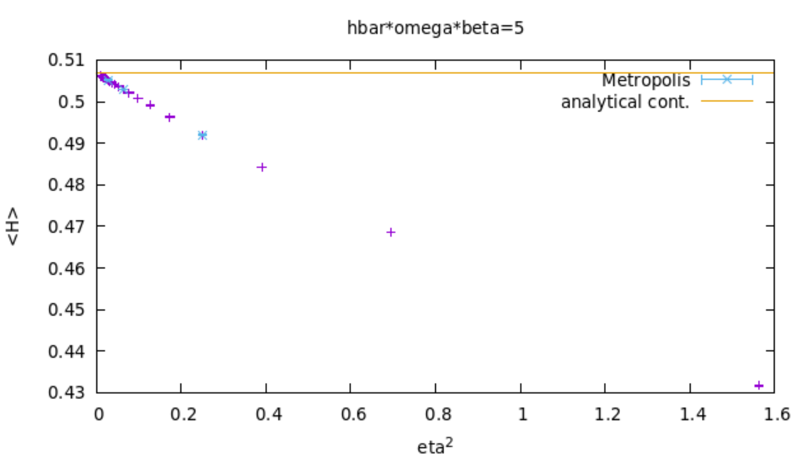
Heatbath was used and in some cases it was compared with Metropolis.

Average value of the position square as a function of η compared with the analytical continuum value.



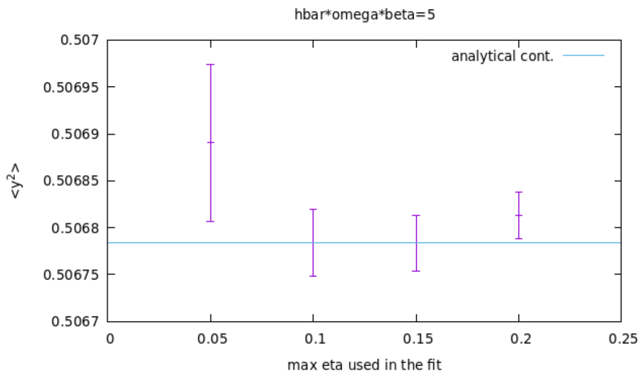
Wrong estimator for Kinetic energy

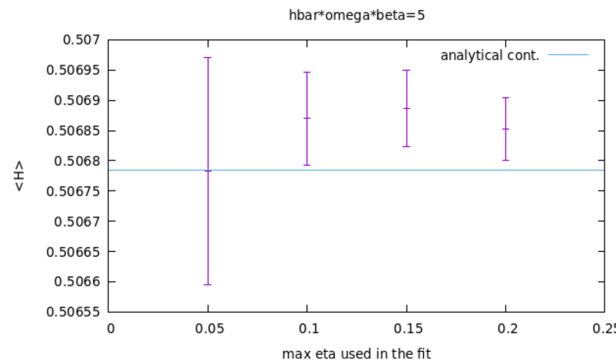
$$\langle K\rangle = \frac{1}{2\eta^2}\langle (y_1-y_0)^2\rangle$$



Average value of the energy

$$\langle H\rangle = \frac{1}{2}\langle y_0^2\rangle + \frac{1}{2\eta} - \frac{1}{2}\frac{\langle (y_1-y_0)^2\rangle}{\eta^2}$$





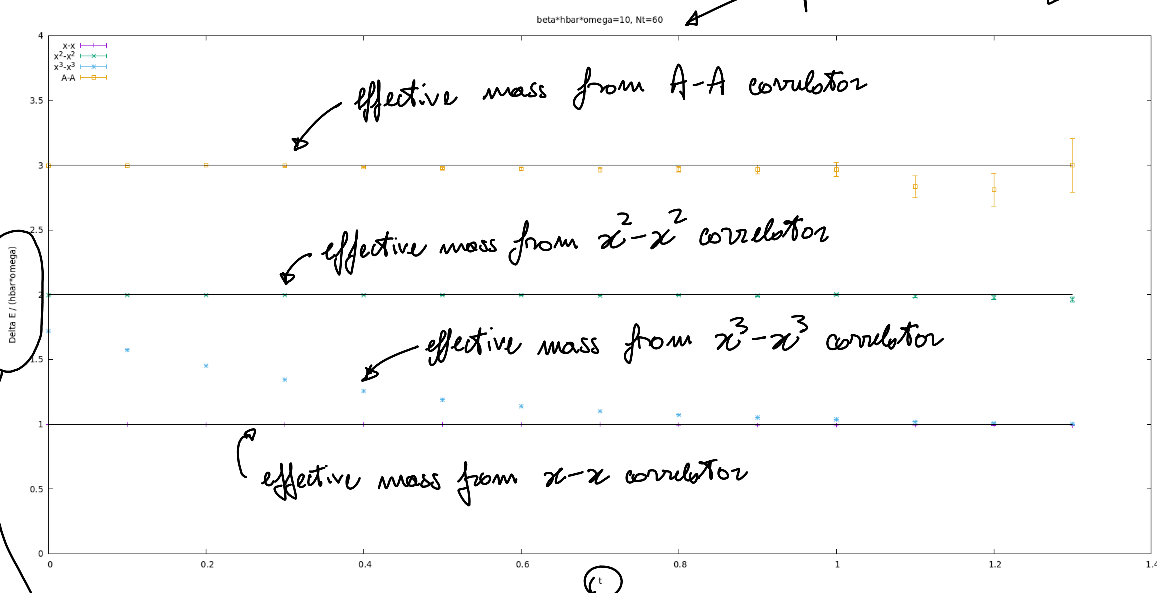
Values of $\langle y^2 \rangle$ and $\langle H \rangle$ extracted by using a fit $a + b\eta^2$ as a function of the maximum η used in the fit compared with the analytical continuum value.

Simulations have been performed usin lattices with N_t from 4 to 50, using $5 \cdot 10^8$ (heatbath + 5 microcanonical) updates of the whole lattice

Simulation times go from ≈ 3 minutes to ≈ 30 minutes for the different lattice sizes.

Spectrum of the 1-dim. harmonic oscillator

$\beta\hbar\omega = 10$ $N_t = 60$

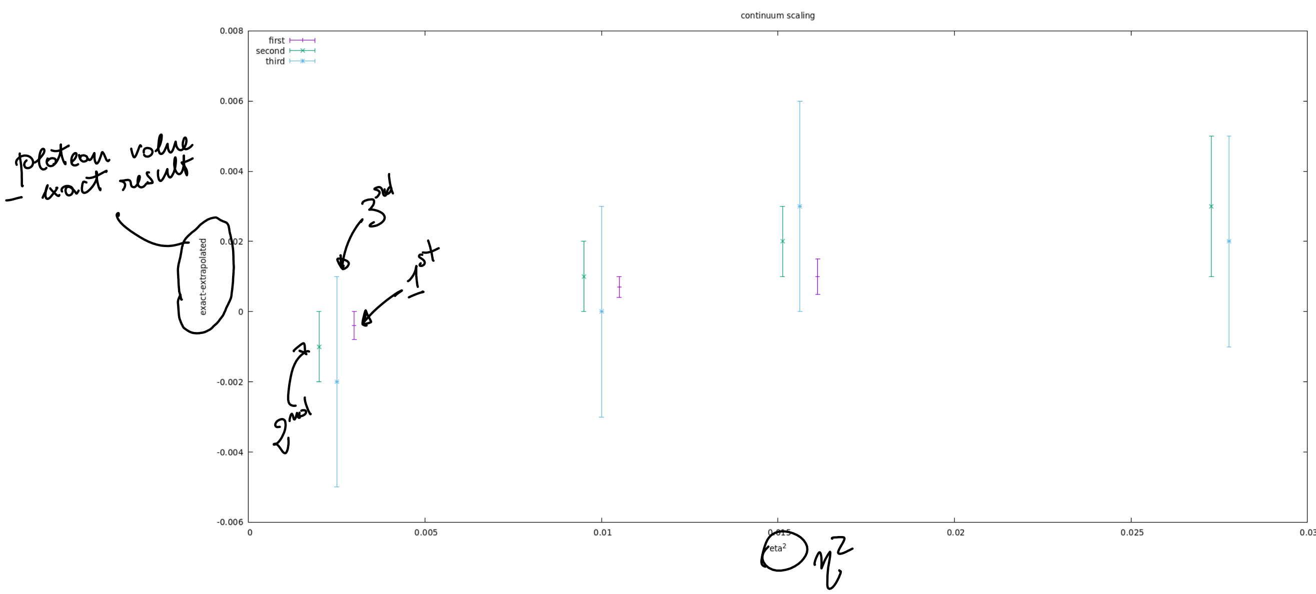


energy gap / $\hbar\omega$

temporal separation

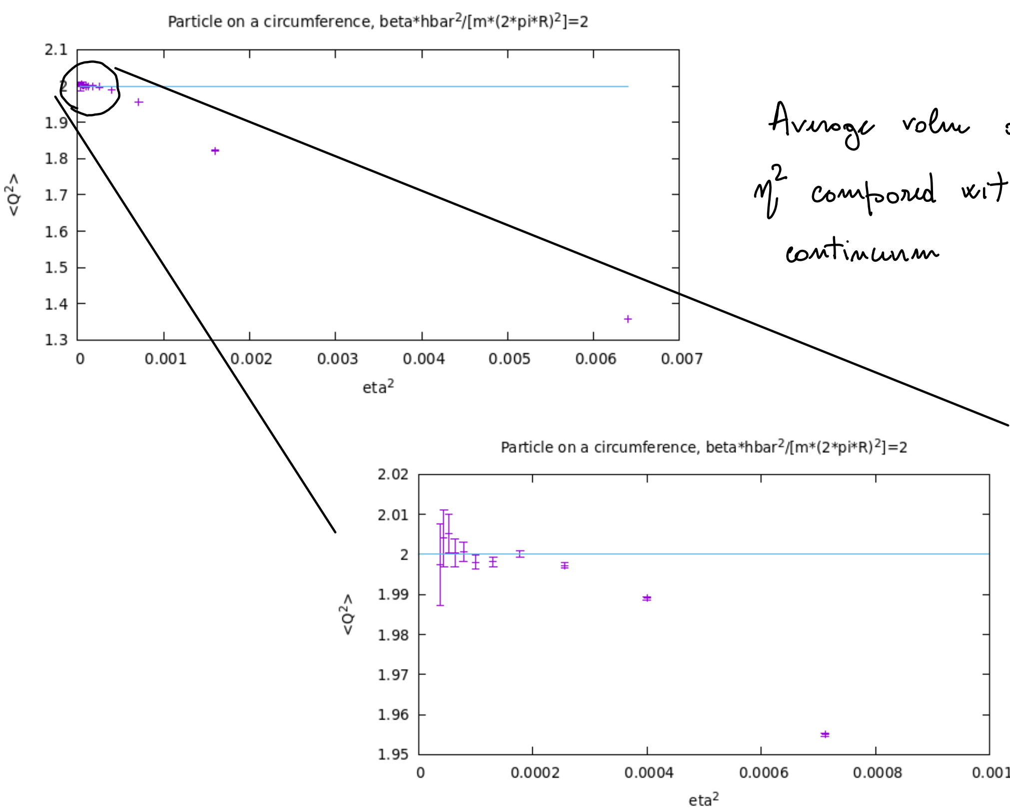
$A = y^3 - \frac{3}{2} y$ = "exact" interpolating operator for the state $|3\rangle$ in the continuum

10^8 updates (1 heatbath + 5 microcanonical) updates of the whole lattice where performed for $N_t = 60, 80, 100$ and 200, with execution times ranging from 13min to 60 min.



Scaling of the deviations from the exact continuum values as a function of η^2

Particle on a circumference: topology

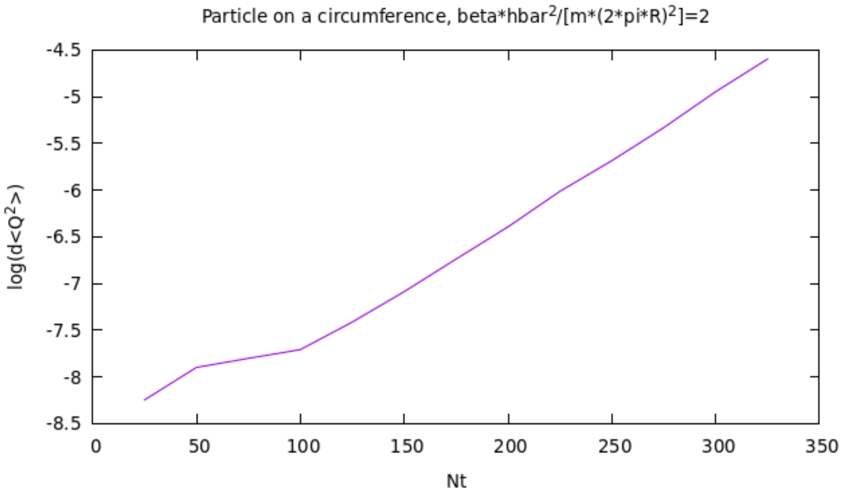


Average value of $\langle Q^2 \rangle$ as a function of η^2 compared with the exact value in the continuum

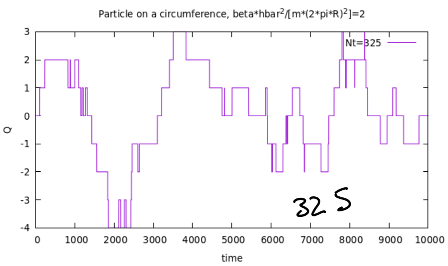
Zoom of the small η region

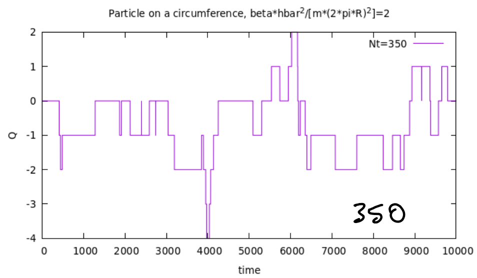
5×10^8 Metropolis updates ($\Delta=0.5$) of the whole lattice for N_t from 25 to 325

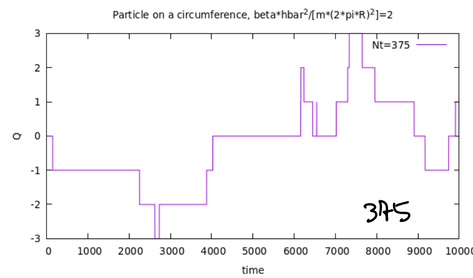
Simulation times go from ≈ 12 min to $\simeq 120$ min.

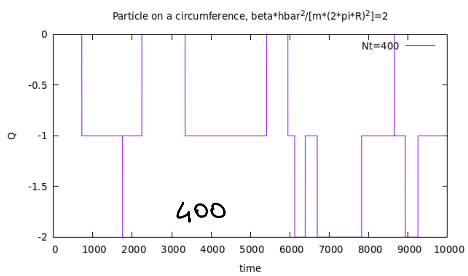


Scaling of the log of the error of $\langle Q^2 \rangle$ as a function of N_t



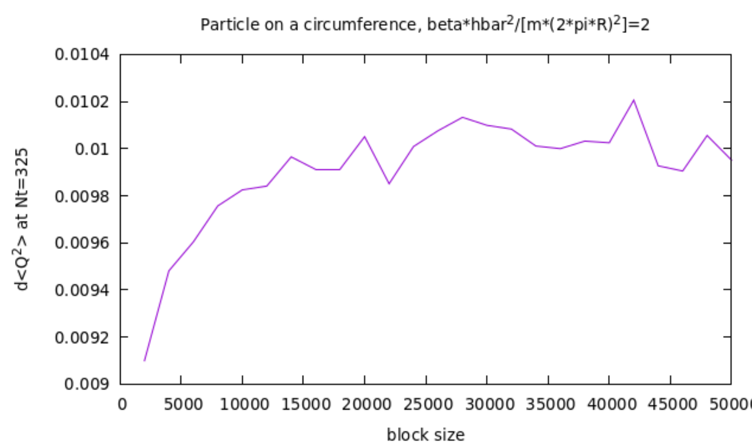


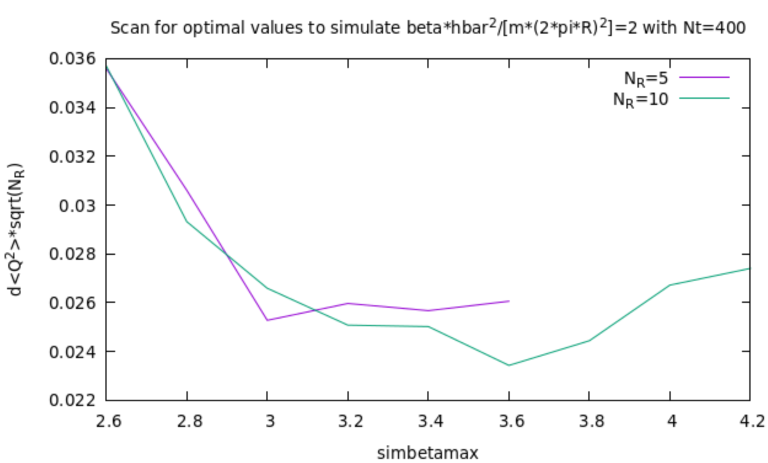




time histories of Q for $N_t = 325, 350, 375, 400$

Behavior of the error of $\langle Q^2 \rangle$ as a function of the block-size for $N_t = 325$



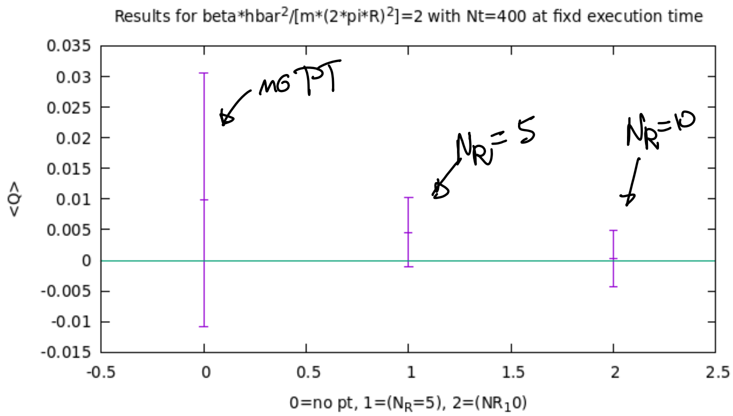


Scan of the quantity

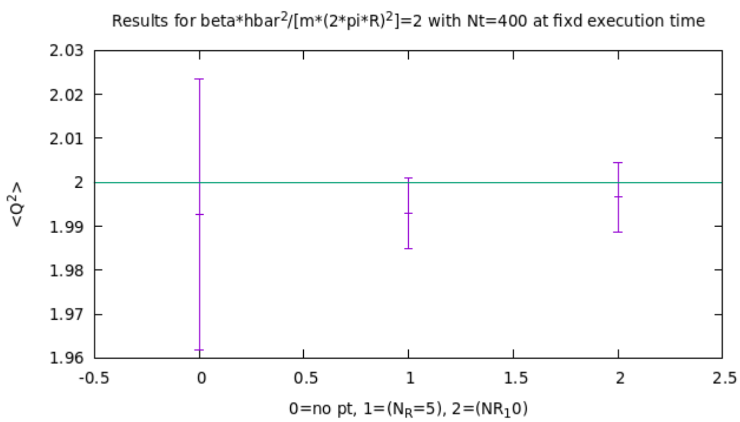
$(\text{error of } \langle Q^2 \rangle) \times \sqrt{N_R}$

for two values of N_R as a function of the maximum value of $\dfrac{\beta \hbar^2}{m(2\pi R)^2}$ to be used in parallel tempering

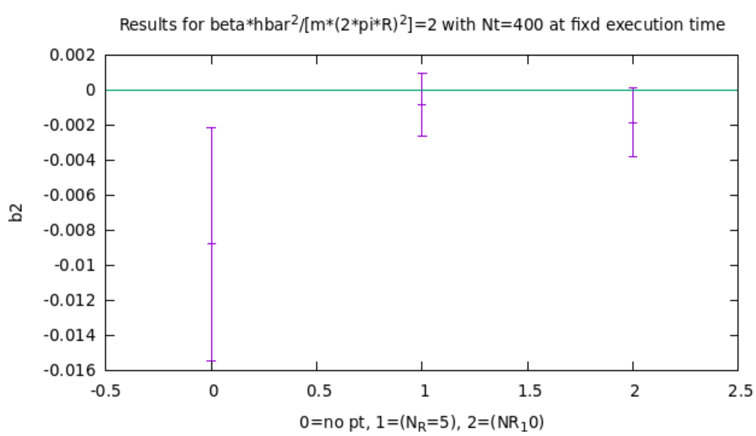
(5×10^6 updates, time ≈ 1 h for $N_R = 5$)



$\langle Q \rangle$ for $N_t = 400$ computed without and with parallel tempering



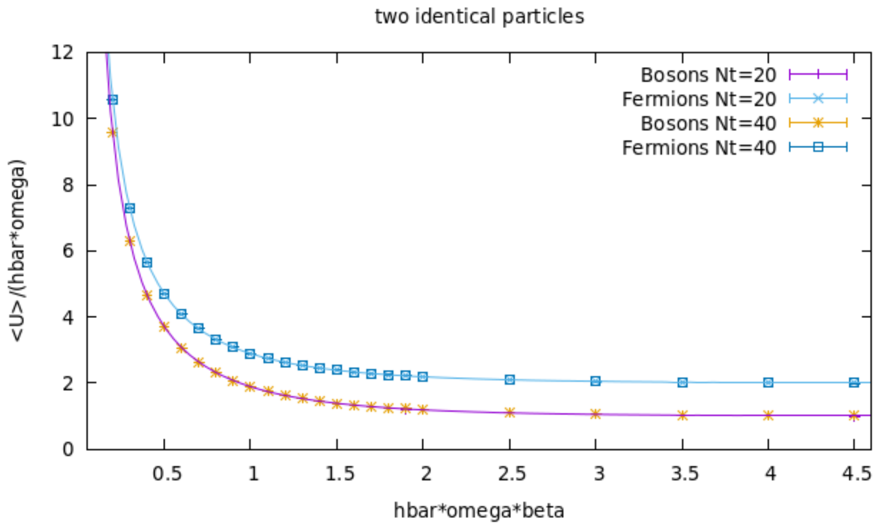
Values of $\langle Q^2 \rangle$



Values of $b_2 = -\dfrac{\langle Q^4 \rangle - 3\langle Q^2 \rangle^2}{12 \langle Q^2 \rangle}$

$$\left.\begin{array}{ll} \text{Without parallel tempering} & 5\times10^8 \text{ updates} \\ N_R = 5 & 10^8 \text{ updates} \\ N_R = 10 & 5\times10^7 \text{ updates} \end{array}\right\} \text{ in all cases simulation times } \approx 2h.$$

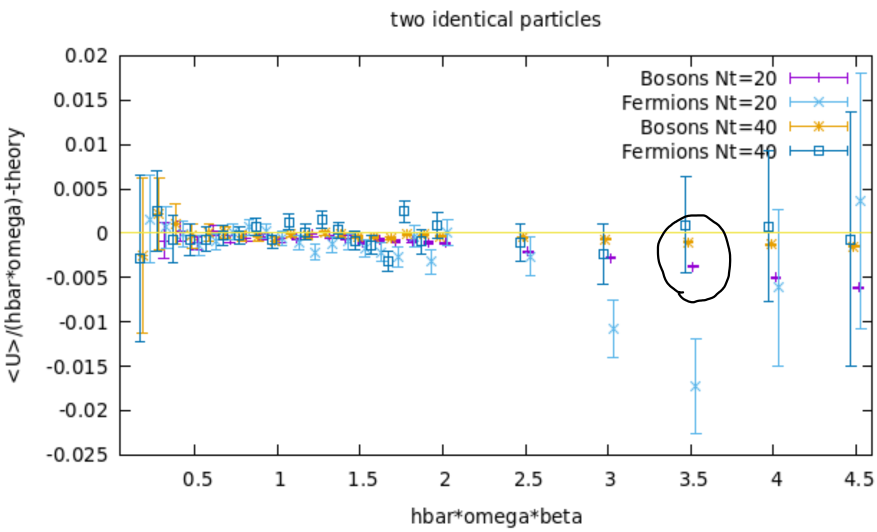
Two id. particles in harmonic trap



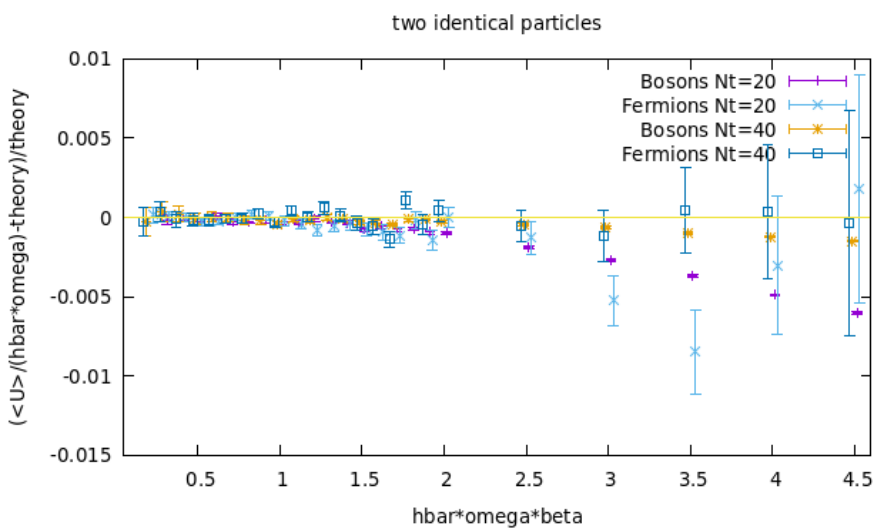
$$\langle U\rangle = \langle y_{(1)}^2 + y_{(2)}^2\rangle$$

where $y_{(i)}$ = position of the i-th particle

compared with continuum exact results (solid lines)

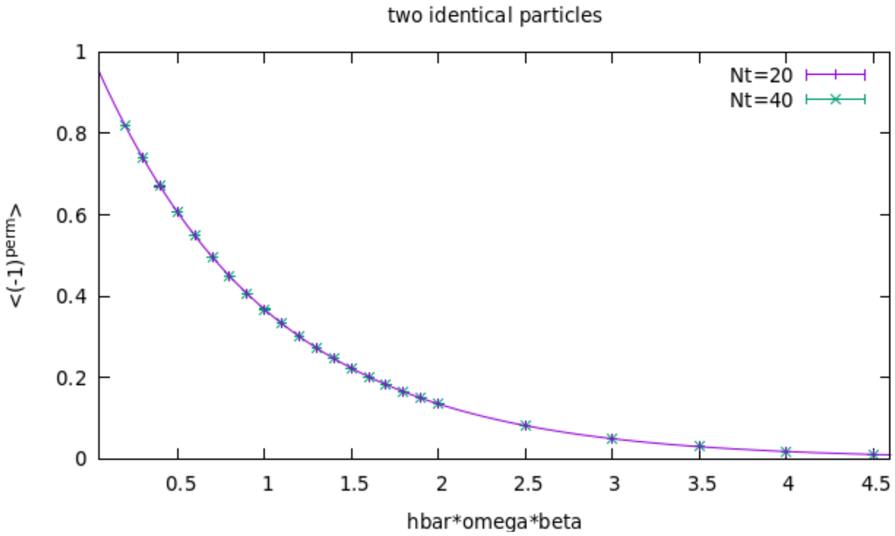


Same as before but $\langle U\rangle/\hbar\omega$ − theory is shown. Deviations from the continuum results are visible for bosons at low temperature.



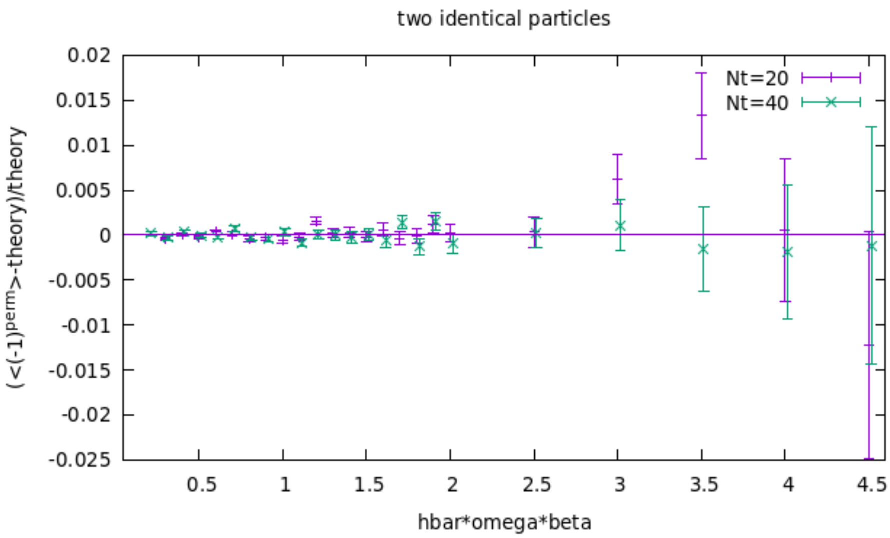
Same as before but normalized with the theoretical value:

$$\frac{\langle U\rangle/\hbar\omega - \text{theory}}{\text{theory}}$$



Average value of $(-1)^{\text{twist}}$ computed on configurations sampled with the bosonic weight

(solid line = continuum result)



$$\frac{\langle (-1)^{\text{perm}} \rangle - \text{theory}}{\text{theory}}$$

the relative error explodes when theory $\to 0$.

≈ 20 values of $\hbar\beta\omega$ where simulated. In each case $5 \cdot 10^8$ updates (1 heatbath + 5 microcanonical on the whole lattice + 1 exchange) where performed.

Simulation times: ≈ 20 min and ≈ 40 min for $N_t = 20$ and 40, respectively.