II. Exact diagonalization methods for quantum many-body systems on a lattice

Check the accuracy of the Krylov-space procedure to implement Lanczos diagonalization.

Ground-state properties of the quantum Ising chain:
[the analytic solution can be found in standard textbooks as the Sachdev's book on Quantum Phase Transitions. Some interesting details are contained, e.g., in G. M. Mbeng, A. Russomanno, G. E. Santoro, SciPost Phys. Lect. Notes **82** (2024).
A rough numerical analysis has been done in: J. Um, S.-I. Lee, B. J. Kim, J. Korean Phys. Soc. 50, 285 (2007).
More robust finite-size scaling analyses are contained in: M. Campostrini, A. Pelissetto, E. Vicari, Phys. Rev. B **89**, 094516 (2014); M. Campostrini, J. Nespolo, A. Pelissetto, E. Vicari, Phys. Rev. Lett. **113**, 070402 (2014)]

- Calculation of the gaps for the low-lying energy levels (at least first two gaps E_1 - E_0 and E_2 - E_0) at a fixed system size, for various strengths of the transverse field. Scaling with the system size, both at the critical point and away from it.
- Calculation of the longitudinal and the transverse local magnetization as a function of the magnetic fields, for different system sizes. The longitudinal magnetization (order parameter) can be computed either by explicitly breaking the Z_2 Ising symmetry or by evaluating it with the tricks discussed during the lectures.
- Finite-size scaling analysis of the order parameter across the critical point and/or across the first-order transition line.
- Calculation of the magnetic susceptibility, either along the coupling direction or along the transverse direction (the latter one being the analogue of the heat capacity).
- Finite-size scaling analysis of the susceptibility across the critical point.
- Calculation of the Binder parameter, estimate of the location of the critical point by analyzing the crossing of the various finite-size curves.

Spectral properties of quantum many-body systems:

- Study of the level spacing statistics (LSS) for the Ising model, when switching off/on the longitudinal field. A randomly varying field may help in breaking residual symmetries and also to increase the statistics by disorder-averaging.
  [cf. J. Karthik, A. Sharma, A. Lakshminarayan, Phys. Rev. A **75**, 022304 (2007)]
- Analysis of the many-body localization (MBL) transition for a XXZ Heisenberg quantum spin chain in the presence of a random magnetic field, when varying the field and/or the Z-anisotropy, through the LSS (Poisson means a localized regime; Wigner-Dyson means an ergodic regime).
  [cf. D. J. Luitz, N. Laflorencie, F. Alet, Phys. Rev. B **91**, 081103(R) (2015)]

Note that, in general, numerical results for the statistics of the ratio between two consecutive energy gaps may give cleaner results than those for the LSS.
[cf. V. Oganesyan, D. A. Huse, Phys. Rev. B **75**, 155111 (2007)]

Real-time dynamics of many-body quantum systems:

- Dynamic finite-size scaling analysis for soft quenches, close to the critical point and/or the first-order transition line in the quantum Ising chain.
  [cf. A. Pelissetto, D. Rossini, E. Vicari, Phys. Rev. E **97**, 052148 (2018)]
- Study of the magnetization dynamics for a qubit coupled to a many-body disordered and interacting spin chain: emergence of revivals in the MBL phase.
  [cf. R. Vasseur, S. A. Parameswaran, J. E. Moore, Phys. Rev. B **91**, 140202(R) (2015)].