№1.1

$$\frac{x^{n+2}}{x^{n-2}} = x^{n+2-\cancel{n}+2} = x^4$$

№1.2

$$x^{-1} \cdot 8 = 2$$

$$\frac{8}{x} = 2$$

$$2x = 8$$

$$x = 4$$

№1.3

If $a=5$
$b=10$ $\quad (a^b)^0 = ?$

$$(5^{10})^0 = 1$$

№1.4

$$\frac{\sqrt{4x}}{\sqrt{x}} = \sqrt{\frac{4\cancel{x}}{\cancel{x}}} = \sqrt{4} = 2$$

№1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

$$\cancel{x^2} + x^2 + 2x + 1 = \cancel{x^2} + 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$D = 4 + 12 = 16$$

$$x_1 = \frac{2+4}{2} = 3 \qquad x_2 = \frac{2-4}{2} = -1$$

№1.6

$$2^x > 1024$$

$$x > \log_2 1024 \Rightarrow x > 10$$

№2.1

$$\begin{array}{cc} C & F \\ 0 & 32 \\ 100 & 212 \end{array} \qquad +100 \qquad +180 \qquad F=C\ ?$$

$$32+1{,}8C=C \Rightarrow 1{,}8C-C=-32$$
$$0{,}8C=-32$$
$$C=-40 \Rightarrow C=F=-40$$

№2.2

$$f(x)=5x+4$$
$$f(3)=y$$
$$\Rightarrow y=5\cdot 3+4=19$$

№2.3

$$x^2-4x+3=0$$
$$D=16-12=4$$
$$x_1=\frac{4+2}{2}=3 \quad x_2=\frac{4-2}{2}=1$$

№2.4

$$10\cdot 1.02^{90}=59{,}43$$

№2.5

$$e^{\ln 5}=5$$

№ 3.1

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \underline{\underline{3}}$$

$$r = \frac{12}{36} = \frac{1}{3} \quad |r| < 1$$

$$\sum_{i=1}^{\infty} \frac{a}{1-r} \Rightarrow a = \frac{12}{6} = 2$$

$$= \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = \frac{2 \cdot 3}{2} = \underline{\underline{3}}$$

№ 3.2

$$\lim_{x \to 1} \frac{6^{1-x}}{x} = \cancel{\lim} \ \frac{1}{1} = 1$$

№ 3.3

$$f(x) = x^5 - 8 \quad at \ x = -3$$

$$f'(x) = 5x^4$$

$$f'(-3) = 5 \cdot (-3)^4 = 5 \cdot 81 = 405$$

№ 3.4

$$\frac{d}{dx} \frac{x^3+2x-1}{x-2} = \frac{(x^3+2x-1)'(x-2) - (x-2)'(x^3+2x-1)}{(x-2)^2} =$$

$$= \frac{(3x^2+2)(x-2) - (x^3+2x-1) \cdot 1}{(x-2)^2} = \frac{3x^3 - 6x^2 + 2x - 4 - x^3 - 2x + 1}{(x-2)^2} =$$

$$= \frac{2x^3 - 6x^2 - 3}{(x-2)^2}$$

№3.5

$$\frac{d^2}{dx^2}\, 4x^4+4x^2 = \frac{d}{dx}\, 16x^3+8x = \cancel{\frac{d}{d}}\, 48x^2+8$$

№3.6

$$\frac{d}{dx}\,\frac{\ln x}{e^x} = \frac{(\ln x)' \cdot e^x - (e^x)' \cdot (\ln x)}{(e^x)^2} =$$

$$= \frac{\frac{e^x}{x} - e^x \ln x}{(e^x)^2} = \frac{\frac{e^x - x e^x \ln x}{x}}{e^{2x}} = \frac{e^{-2x}\left(e^x - x e^x \ln x\right)}{x} =$$

$$= \frac{e^{-x}(1 - x\ln x)}{x}$$

№3.7

$f(x) = 3x^2 - 5x + 2 \Rightarrow f(x) = 0$

① $D = 25 - 24 = 1$

$x_1 = \frac{5+1}{6} = 1 \quad x_2 = \frac{5-1}{6} = \frac{2}{3}$

x	$-\infty$	$x < \frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6} < x < 1$	1	∞
$f(x)$	+	+	0	−	−	−	0	+
$f'(x)$	−	−	−	−	0	+	+	+
Shape	↘	↘	↘	↘	min	↗	↗	↗
$f''(x)$	+	+	+	+	+	+	+	+
Shape	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$

②

$f'(x) = 6x - 5$

$f'(0) \Rightarrow 6x - 5 = 0$

$x = \frac{5}{6}$

max/min $\rightarrow f''(x) = 6 \Rightarrow$ convex, we have minimum point

№ 3.8

$f(x,y) = x^2 + y^3$ $\quad f(2,3) - ?$

$f(2,3) = 2^2 + 3^3 = 4 + 27 = 31$

№ 3.9

$f(x,y) = \ln(x-y)$

$(x,y) \in \mathbb{R}^2 : x > y$

$\ln(-R)$ - doesn't exist

№ 3.10

$\frac{d}{dx}(x^5 + xy^3)$ Fand partial derivative

① $\frac{d}{dx}(x^5 + xy^3) = 5x^4 + y^3$

② $\frac{d}{dy}(x^5 + xy^3) = 3xy^2$

№ 3.11

$f(x,y) = x^2y^2 + 10$

① $f'(x) = 2xy^2 \Rightarrow 2xy^2 = 0$, $x = 0$

② $f'(y) = 2x^2y \Rightarrow 2x^2y = 0$, $y = 0$

$f(x,0) = 10$ - local min

$f(0,y) = 10$ - local min

Point $(0,0)$ - global min

$\cancel{№3.8}$

$\cancel{f(x,y) = x^2 + y^3}$

$\cancel{f(2,3) - ?}$

$\cancel{f(2,3) = 2^2 + 3^3 = 4 + 27 = 31}$

№3.12

Langrange max x^2y^2

s.t. $x+y=10$

$x^2 - y^2 - \lambda(x+y+10)$

① $\frac{d}{dx} = 2xy^2 - \lambda = 0$

② $\frac{d}{dy} = 2x^2y - \lambda = 0$

③ $\frac{d}{d\lambda} = -x-y+10 = x+y-10 = 0$

$\Downarrow$

$x+y=10$

④ $\begin{cases} 2xy^2 - \lambda = 0 \\ 2x^2y - \lambda = 0 \\ x+y = 10. \end{cases} \Rightarrow \begin{matrix} \lambda = 2xy^2 \\ \lambda = 2x^2y \end{matrix}$

$\hookrightarrow x = 10 - y$

$\Downarrow$

$2xy^2 = 2x^2y \;/:2$

$xy^2 = x^2y \;/:x\cdot y$

$y = x$

⑤ $x = 10 - x$

$2x = 10$

$x = 5$

$x = y = 5$

$x=5$

$y=5$

№4.1

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

$$A \cdot B = \begin{array}{c|ccc} & 1 & 1 & 7 \\ & 2 & 8 & 2 \\ \hline 2\,6 & 14 & 50 & 26 \\ 5\,1 & 7 & 13 & 37 \\ 1\,9 & 19 & 73 & 25 \end{array}$$

$2 \cdot 1 + 2 \cdot 6 = 2 + 12 = 14$

$2 \cdot 1 + 6 \cdot 8 = 2 + 48 = 50$

$14 + 12 = 26$

$5 + 2 = 7$

$5 + 8 = 13$

$35 + 2 = 37$

$1 + 18 = 19$

$1 + 72 = 73$

$7 + 18 = 25$

$$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

№4.2

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$B \cdot A = \begin{array}{c|cc} & 2 & 2 \\ & 4 & 6 \\ & 1 & 3 \\ \hline 1\,9\,1 & 39 & 59 \\ 2\,1\,2 & 10 & 16 \end{array}$$

$2 + 36 + 1 = 39$

$2 + 54 + 3 = 59$

$4 + 4 + 2 = 10$

$4 + 6 + 6 = 16$

$$B \cdot A = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

№4.3

$$A = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

N4.4

$$A = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$$

Det - ? $\Rightarrow 1 \cdot 8 - 2 \cdot 9 = -10$

N5.1

$$\Omega = \{(1,1),(1,2),(1,3);(1,4),(1,5);(1,6),(2,1),(2,2);(2,3),(2,4);(2,5);(2,6),(3,1),(3,2),(3,3);(3,4),(3,5),(3,6),(4,1);(4,2);(4,3);(4,4);(4,5);(4,6),(5,1);(5,2);(5,3);(5,4),(5,5);(5,6);(6,1);(6,2),(6,3),(6,4);(6,5),(6,6)\}$$

$\#\Omega = 36$

N5.2

Use Drugs = 1% = 0.01
Drug Free = 99% = 0.99
True Positive = 99% = 0.99
False Negative = 1% = 0.01
True Negative = 99.5% = 0.995
False Positive = 0.5% = 0.005

Positive result (random) $= 0.99 \cdot 0.01 + 0.005 \cdot 0.99 = 0.01485$

1.485%

N5.3

Use Bayes rule $= \mathbb{P} = \dfrac{0.99 - 0.01}{0.01485} = 0{,}6666 = 66{,}67\%$