Turbulence and Transport in Fusion Plasmas
Part VI

DIFFER

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Demonstration

Let's have a look at all that in action!

With the GENE Diagnostics Tool, we can

1. extract eigenvalues
2. look at evolving and converged mode structures; IV vs. EV
3. evaluate phases
4. compare moments, compute quasilinear ratios (e.g., Q/n^2)
5. identify different instabilities

See `gene-diag.pdf` for a summary

Questions & Discussion

Who has any questions and is not afraid to ask?

Group Work: Finite Differencing

1 hour group work:

Either find your own sources or look at
Pueschel CPC 2010 to learn

- how finite differencing works
- how the stencil notation works and what the stencil order is

Be prepared to present your findings.

Bonus for those who are not dissuaded by the math: learn

- *how to derive your own stencils*

Student Lecture

¿Voluntarios?

Explain what **finite differencing** means!

What is a **stencil**?

What can be done at the **domain boundary**?

Are there **alternatives** to finite differencing?

Group Work: Hyperdiffusion

45 minutes group work

With the Fourier transform of $\Phi(x \pm \Delta x)$ being $\Phi(k)\exp(\pm ik\Delta x)$,

1. evaluate the Fourier transform of $\partial_x \Phi$ with stencil $[-1\ 0\ 1]$
2. show that it produces a modified $ik\Phi \to i\tilde{k}\Phi$
3. evaluate $\tilde{k}$ in the limits of $k \to 0$ and $k \to \pi/\Delta x$

4.

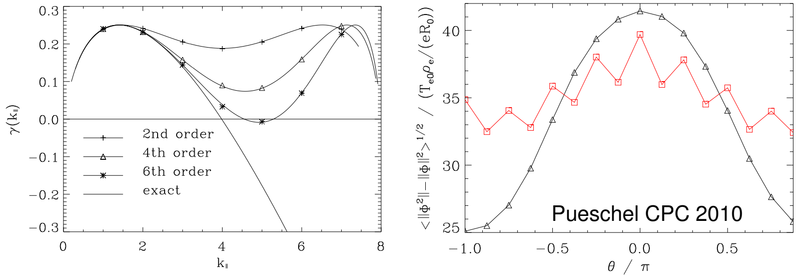
High-k Excitation

Solution:

$$k \to 0: \quad \tilde{k} = \frac{\sin k\Delta x}{\Delta x} \to k \quad \checkmark$$

$$k \to \frac{\pi}{\Delta x}: \quad \tilde{k} = \frac{\sin k\Delta x}{\Delta x} \to 0 \quad \times$$

As $k \to \pi/\Delta x$, this stencil **reproduces the physics of** $k \to 0$**!**
$\Rightarrow$ **high-k modes** (physically stable) **take over** simulations



What can we do about that?

Group Work: Hyperdiffusion

45 minutes group work

With the Fourier transform of $\Phi(x \pm \Delta x)$ being $\Phi(k)\exp(\pm ik\Delta x)$,

1. evaluate the Fourier transform of $\partial_x \Phi$ with stencil $[-1\ 0\ 1]$
2. show that it produces a modified $ik\Phi \rightarrow i\tilde{k}\Phi$
3. evaluate $\tilde{k}$ in the limits of $k \rightarrow 0$ and $k \rightarrow \pi/\Delta x$

4. show that adding a term $\propto k^n$ with even n to the Vlasov equation can solve this problem

Group Work: Time Stepping

45 minutes group work:

In our code, we will use an **explicit Runge-Kutta** method to evolve the distribution function in time.

Note: already in the code, but good to understand!

1. Figure out how that works (e.g., Wikipedia)
2. Find sources (lecture notes, online, ...) that show and explain the stability regions of explicit Runge-Kutta schemes in the eigenvalue plane
3. Rejoin the plenum and discuss your findings

What happens when we choose a too-small Δt?

Questions & Discussion

Any questions before we write our own simulation code?

Code Prep

Perhaps 30–60 minutes, but really as-long-as-it-takes effort:

Set up your system to be able compile and run code written in Fortran 90, e.g., by having the open-source compiler `gfortran` installed.

Familiarize yourself with basic Fortran 90 syntax, e.g. via `pages.mtu.edu/~shene/COURSES/cs201/NOTES/fortran.html` *(be sure to use the correct tilde character when pasting into browser)*.

If you are stuck, get help from one of the others in your group of from the lecturer!

Group Work: Vlasov Code

2 hours group work

1. take `dkVlasov.F90` and search for missing code marked "to be implemented"
2. implement our linear, μ-integrated drift-kinetic equations

$$\frac{\partial f}{\partial t} = -v_{\|}\frac{\partial f}{\partial z} - v_{\|}F_0\frac{\partial \Phi}{\partial z} - \left[\omega_n + \left(v_{\|}^2 - \frac{1}{2}\right)\omega_{T\mathrm{i}}\right] F_0 i k_y \Phi + \eta\frac{\partial^2 f}{\partial z^2}$$

$$\Phi = \int f \mathrm{d}v_{\|} \quad F_0 = \frac{1}{\pi^{1/2}}\mathrm{e}^{-v_{\|}^2} \quad \frac{\partial}{\partial z} \rightarrow [-1\ 0\ 1] \quad \frac{\partial^2}{\partial z^2} \rightarrow [1\ -2\ 1]$$

3. come up with an initial condition $F(t=0)$, either for a single or a spectrum of $k_{\|}$
4. run the code and perform **convergence** checks: **local independence of numerical parameters**
5. reproduce the $k_{\|}$ dependence in Pueschel CPC 2010 for different diffusion coefficients

Questions & Discussion

Anything that we covered in need of clarification?