Turbulence and Transport in Fusion Plasmas
Part VIII

DIFFER

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Tuesday Recap

Yesterday, we went over

- saturation of linear instabilities:
  - zonal flows & stable modes
  - ITG vs. ∇T-TEM vs. ∇n-TEM vs. KBM saturation
  - loss of saturation in the non-zonal transition
- quasilinear transport modeling using linear gyrokinetics
- improved modeling with the τ correction

Now, let's finish the quasilinear tasks with our Vlasov code . . .

Group Work Help: $T_{\rm i}$

How to evaluate the ion temperature fluctuation?

Definitions for density & pressure fluctuations (normalized):

$$
\begin{aligned}
n_{\rm i} &= n_{\rm i0}\, \pi B_0 \int f_{\rm i}\, {\rm d}v_{\|} {\rm d}\mu \\
p_{\rm i} &= n_{\rm i0} T_{\rm i0}\, \pi B_0 \int v^2 f_{\rm i}\, {\rm d}v_{\|} {\rm d}\mu
\end{aligned}
$$

Further noting

$$\delta p = \delta(nT) = n_0 \delta T + T_0 \delta n \text{ and } v^2 = v_{\|}^2 + v_{\perp}^2$$

we can write a definition for $T_{\rm i}$ based on $n_{\rm i}$ and $p_{\rm i}$

Group Work: Scans & Quasilinear

2 hours group work

Using your drift-kinetic Vlasov code,

1. conduct a 2D parameter scan, over either k_y-$\omega_{T\mathrm{i}}$ or k_y-ω_n at some fixed $k_{||}$ (suggested range $0.1 \leq k_y \leq 0.8$)
2. based thereupon, construct a simple quasilinear model

$$Q_{\mathrm{i}}^{\mathrm{QL}} = \omega_{T\mathrm{i}} \sum_{k_y} \frac{\gamma(k_y)}{\langle k_y^2 \rangle} \,, \qquad \langle k_y^2 \rangle = \frac{\int k_y^2 |\Phi(k_y)|^2 \mathrm{d}z}{\int |\Phi(k_y)|^2 \mathrm{d}z} = k_y^2$$

   to gauge heat flux scalings of slab ITG turbulence

3. extract the $\Phi \times T_{\mathrm{i}}$ phase and include it in the model, normalized so $\alpha_{\Phi\times T} = \pi/2$ yields 100% $Q_{\mathrm{i}}^{\mathrm{QL}}$

Group Work: Fluid Quasilinear

2 hours group work:

Returning to the (linear) Horton-Holland model solution,

$$\omega_{\mathrm{c}1,2} = \frac{k_y}{2+2k_\perp^2} \pm i k_y \left(\frac{(1+\eta)\epsilon}{1+k_\perp^2} \right)^{1/2} - \frac{i\nu k_\perp^2}{2+2k_\perp^2}$$

1. construct a quasilinear $Q_{\mathrm{QL}} = \eta\epsilon \sum_{k_y} C(k_y)\gamma/k_\perp^2$ with $k_x = 0$
2. evaluate this model for scans over η and ν, assuming the base case $Q_{\mathrm{NL}}(k_y) \leftrightarrow C(k_y)$ to be an inverse parabola peaked at $k_y = 0.5$, with no flux below/above $k_y = 0.1/0.9$
3. add in the τ correction, assuming $\omega_{\mathrm{ZF}} = 0$ and $k_x^{\mathrm{ZF}} = 0.1$
4. repeat the scans and compare the two models

For those who are interested: P.-Y. Li *et al.*, Phys. Plasmas **28**, 102507 (2021) has a Horton-Holland comparison NL-QL(τ)

Discussion: Benchmark Point

Let us have a short discussion:

What would go into an “ideal” benchmark parameter point?
Benchmark: comparison of different simulation codes

- Why would we want to do this?
- Where in our vast parameter space should we do this?

Questions & Discussion

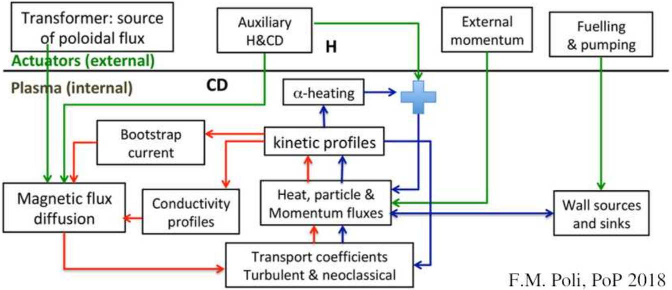
Any questions about anything nonlinear? Or linear?

Any feedback for the fella doin' so much talkin'?

Integrated Modeling

Now that we understand turbulence: **integrated modeling**

Fusion reactor: **multi-component**, **multi-physics** system
$\Rightarrow$ need to use many models, account for complex interactions



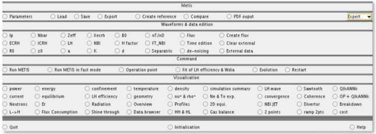
F.M. Poli, PoP 2018

Whole-Device Modeling: a few frameworks, e.g. Romanelli 2014

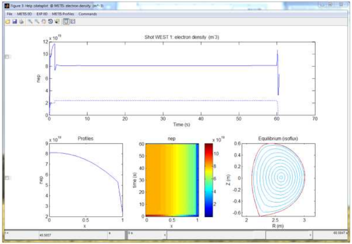
Shout-out DIFFER expert: Jonathan Citrin, head of IMT group

Flight Simulator

Aside from fundamental understanding, **goal: flight simulator**



Artaud NF 2018



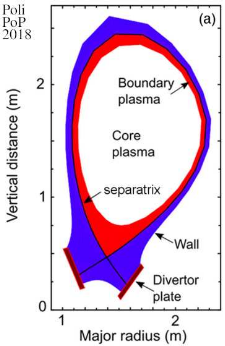
- simulate plasma evolution ahead of experiment
- ensure good performance, no disruptions
- determine good settings for heating, fueling, etc.
- *ideally modular*: many code coupling choices

See, e.g., Felici PPCF 2012, Artaud NF 2018, Janky FED 2021

Need for speed: **cannot use full-physics models**

Core vs. Edge

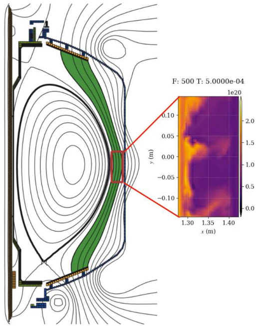
So far, this class has focused on **core plasmas**



Currently hot research topic: **plasma edge**

- pedestal
- separatrix
- scrape-off layer (SOL)

Edge gyrokinetics



(e.g., Shi JPP 2017)

For our purposes, will ignore scrape-off layer—2D transport!
(plus: open field lines, sheath boundary, full-f, neutrals, ...)

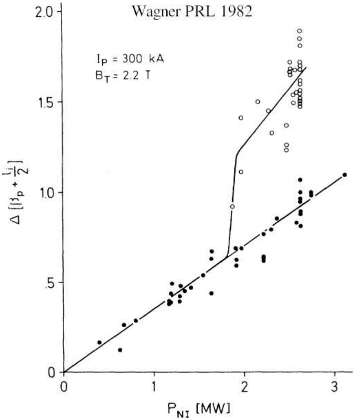
H-Mode & Pedestal

ASDEX: transition **L**ow- $\rightarrow$ **H**igh-confinement **mode**

Heating power boost

$\Rightarrow$ sudden increase in Φ_0

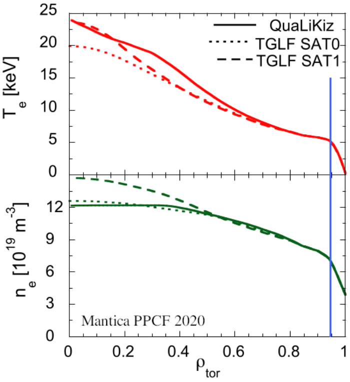
$\Rightarrow$ lower Q, higher τ_E



caused by ion orbit losses?

MHD pedestal models:
EPED (Snyder NF 2011)
IMEP (Luda NF 2020)

However, common to just put boundary inside pedestal:



Sources

Fusion reactors: need **heat, particle**, possibly momentum **sources** ⇒ compensate losses, refuel D-T

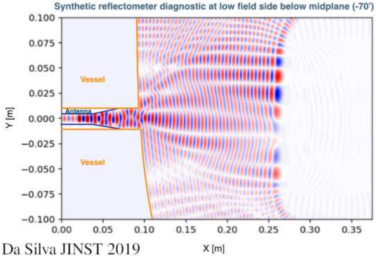
Heat sources:

- ICRH
- ECRH
- NBI

Particle sources:

- pellets
- gas puffing
- *wall erosion*

ECRH: rays; ICRH: need **full-wave modeling**



Da Silva JINST 2019

NBI: localized heat/particle/momentum source ⇒ **current drive**

Transport Equations: Particles

How to describe transport and profile evolution mathematically?

Vlasov: $$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f_j(\mathbf{x},\mathbf{v},t) = \mathcal{C}[f_j]$$

Term by term, take 0$^{\text{th}}$ moment:

$$\int \mathrm{d}\mathbf{v}\,\frac{\partial f_j}{\partial t} = \frac{\partial n_j}{\partial t} \qquad \int \mathrm{d}\mathbf{v}\,\mathcal{C}[f_j] = 0 \text{ (particle conservation)}$$

$$\int \mathrm{d}\mathbf{v}\,\mathbf{v}\cdot\frac{\partial f_j}{\partial \mathbf{x}} = \nabla\cdot(n_j\mathbf{u}_j) \equiv \nabla\cdot\mathbf{\Gamma}_j \quad (\mathbf{\Gamma}\text{: particle flux})$$

$$\int \mathrm{d}\mathbf{v}\left(E_a + \frac{v_b}{c}B_c\right)\frac{\partial f_j}{\partial v_a} = \underbrace{\left[\left(E_a + \frac{v_b}{c}B_c\right)f_j\right]_{-\infty}^{\infty}}_{=0\ [f(v\to\infty)=0]} - \int \mathrm{d}\mathbf{v}\,\underbrace{\frac{\partial}{\partial v_a}\left(E_a + \frac{v_b}{c}B_c\right)}_{=0\ (\partial_v E=0\,\&\,\partial v_a/\partial v_b=0)} f_j$$

$\Rightarrow$ particle balance $\dfrac{\partial n_j}{\partial t} + \nabla\cdot\mathbf{\Gamma}_j = S_j$ (S_j: external particle sources)

Transport Equations: Heat I

Similar to particle balance: **heat/energy balance**, $\frac{m}{2}\int v^2 \mathrm{d}\mathbf{v}$

Vlasov: $\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q_j}{m_j}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f_j(\mathbf{x},\mathbf{v},t) = \mathcal{C}[f_j]$

$$\frac{m_j}{2}\int \mathrm{d}\mathbf{v}\frac{\partial v^2 f_j}{\partial t} = \frac{3}{2}\frac{\partial p_j}{\partial t}$$

$$\frac{m_j}{2}\int \mathrm{d}\mathbf{v}\,\mathbf{v}^3\cdot\frac{\partial f_j}{\partial \mathbf{x}} = \nabla\cdot\mathbf{Q}_j \quad (\mathbf{Q}\text{: energy/heat flux})$$

$$\frac{q_j}{2}\int \mathrm{d}\mathbf{v}\,v^2\mathbf{E}\cdot\frac{\partial f_j}{\partial \mathbf{v}} = \underbrace{\frac{q_j}{2}\left[v^2 E f_j\right]_{-\infty}^{\infty}}_{=0\ [v^2 f(v\to\infty)=0]} - q_j\mathbf{E}\cdot\int \mathrm{d}\mathbf{v}\,\mathbf{v}f_j = -q_j n_j \mathbf{E}\cdot\mathbf{u}_j$$

$$\frac{m}{2c}\int \mathrm{d}\mathbf{v}\,v^2\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f_j}{\partial \mathbf{v}} = \ldots = 0 \text{ (the magnetic field does no work)}$$

However, the collision term does not vanish this time ...

Transport Equations: Heat II

$$\frac{m_j}{2}\int \mathrm{d}\mathbf{v}\, v^2 \mathcal{C}[f_j] = \frac{m_j}{2}\int \mathrm{d}\mathbf{v}\,(\mathbf{v}-\mathbf{u}_j)^2 \mathcal{C}[f_j] + m_j \mathbf{u}_j \cdot \int \mathrm{d}\mathbf{v}\,\mathbf{v}\mathcal{C}[f_j] \equiv Q_{\mathrm{ei}} + \mathbf{u}_j \cdot \mathbf{F}_j$$

with F: collisional friction force; Q_{ei}: collisional energy exchange

$\Rightarrow$ energy balance $\dfrac{3}{2}\dfrac{\partial p_j}{\partial t} + \nabla\cdot\mathbf{Q}_j = Q_{\mathrm{ei}} + Q_{j,\mathrm{fus}} + Q_{j,\mathrm{ext}} + \mathbf{u}_j \cdot (\mathbf{F} + q_j n_j \mathbf{E})$

(gray terms: negligible for purely **radial transport**)

$Q_{j,\mathrm{fus}}$:	fusion power source (j: fusion alphas)
$Q_{j,\mathrm{ext}}$:	external heating: ECRH, ICRH, NBI
$\mathbf{Q}, \boldsymbol{\Gamma}$:	**turbulent** (or neoclassical) **fluxes**

However, still in Cartesian coordinates
Need to **account for** $\mathrm{d}V = \mathrm{d}V(r) = A(r)\mathrm{d}r$ **dependence!**

Group Work: Transport Equations

30 minutes group work:

1. Perform volume integration combined with radial derivative on particle balance

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \mathbf{\Gamma}_j = S_j$$

Hint 1: use Gauss divergence theorem for flux term
Hint 2: can insert unity factors like $\frac{\mathrm{d}r}{\mathrm{d}r}$ *and* $\frac{\int \mathrm{d}A\mathrm{d}r}{\int \mathrm{d}A\mathrm{d}r}$

2. Express particle balance in terms of flux-surface averages

$$\langle \xi \rangle \equiv \frac{\int_A \int_0^r \xi \, \mathrm{d}A\mathrm{d}r}{\int_A \int_0^r \mathrm{d}A\mathrm{d}r} , \quad \text{noting that } V' \equiv \frac{\mathrm{d}V}{\mathrm{d}r}$$

(each resultant term should have a $\langle \cdot \rangle$ *and a* V'*)*

Group Work: Solution

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \mathbf{\Gamma}_j = S_j$$

$$\frac{\partial}{\partial r}\int_A \int_0^r \frac{\partial n_j}{\partial t}\mathrm{d}A\mathrm{d}r = \frac{\partial}{\partial t}\int_A n_j \mathrm{d}A = \frac{\partial}{\partial t}\int_A n_j\,\mathrm{d}A\,\frac{\mathrm{d}r}{\mathrm{d}r}\,\frac{\int_A\int_0^r \mathrm{d}A\mathrm{d}r}{\int_A\int_0^r \mathrm{d}A\mathrm{d}r} = \frac{\partial V'\langle n_j\rangle}{\partial t}$$

$$\frac{\partial}{\partial r}\int_A \int_0^r S_j\,\mathrm{d}A\mathrm{d}r = V'\langle S_j\rangle$$

$$\frac{\partial}{\partial r}\int_A \int_0^r \nabla\cdot\mathbf{\Gamma}_j\,\mathrm{d}A\mathrm{d}r = \frac{\partial}{\partial r}\int_A \mathbf{\Gamma}_j\cdot\hat{\mathbf{n}}\,\mathrm{d}A = \frac{\partial}{\partial r}\int_A \Gamma_{j,r}\,\mathrm{d}A\,\frac{\mathrm{d}r}{\mathrm{d}r}\,\frac{\int_A\int_0^r \mathrm{d}A\mathrm{d}r}{\int_A\int_0^r \mathrm{d}A\mathrm{d}r} = \frac{\partial V'\langle \Gamma_{j,r}\rangle}{\partial r}$$

Typical notation: brackets $\langle\cdot\rangle$ are dropped, $\Gamma_j \equiv \Gamma_{j,r}$

Overall : $\quad \dfrac{\partial V' n_j}{\partial t} + \dfrac{\partial V'\Gamma_j}{\partial r} = V' S_j$

Transport Equations: Summary

For two-component plasma, ignoring current diffusion:

$$\frac{\partial V' n_\mathrm{i}}{\partial t} + \frac{\partial V' \Gamma_\mathrm{i}}{\partial r} = V' S_\mathrm{i}$$

$$n_\mathrm{e} = n_\mathrm{i} \qquad \Gamma_\mathrm{e} = \Gamma_\mathrm{i} \qquad P_\mathrm{ei} = -P_\mathrm{ie}$$

$$\frac{3}{2}\frac{\partial V' n_\mathrm{i} T_\mathrm{i}}{\partial t} + \frac{\partial V' Q_\mathrm{i}}{\partial r} = V'(P_\mathrm{ie} + P_\mathrm{i,fus} + P_\mathrm{i,ext})$$

$$\frac{3}{2}\frac{\partial V' n_\mathrm{e} T_\mathrm{e}}{\partial t} + \frac{\partial V' Q_\mathrm{e}}{\partial r} = V'(-P_\mathrm{ie} + P_\mathrm{e,ext})$$

Circular flux surfaces: $V = 2\pi^2 r^2 R \Rightarrow V' = 4\pi^2 r R$,
thus, can replace $V' \to r$ in above expressions

Typically, need to evaluate $10^3 - 10^5$ times
$\Rightarrow$ require fast models for Q, Γ

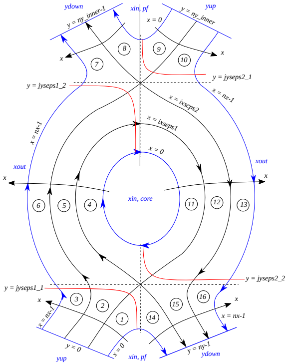
MHD & Neoclassical

Recall: Grad-Shafranov

$$R^2 \nabla \cdot \frac{\nabla \psi}{R^2} = -R^2 \frac{\partial p}{\partial \psi} - F \frac{\partial F}{\partial \psi}$$

used to get B_0 equilibrium

Transport modeling:
prescribe **shape of last closed flux surface** (LCFS) *or* (magnetic/inductive) **coil currents**



Complement Grad-Shafranov solver with **MHD stability code**

Also need code to **evaluate neoclassical fluxes**, especially when looking at stellarators (*any idea why?*)

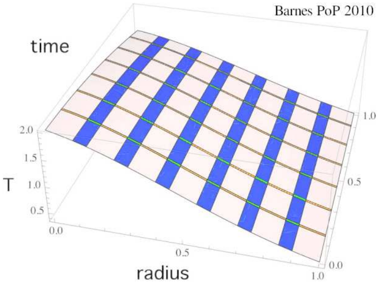
Turbulent Transport

Turbulent transport: **trade-off** between **speed** and **accuracy**

Can get turbulent Q, Γ from

- nonlinear gyrokinetics
- quasilinear gyrokinetics
- reduced QL models (QuaLiKiz, TGLF)
- simple transport coefficients and critical-gradient models

Coupling multiple nonlinear simulations at different r/a:

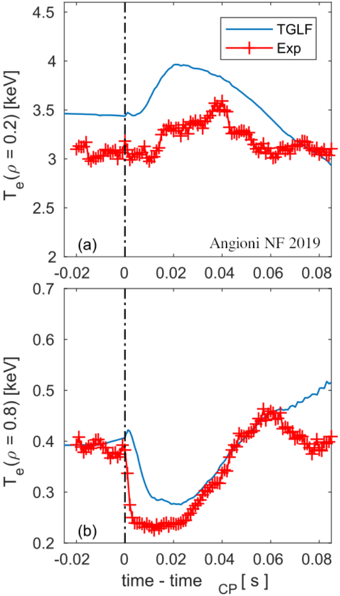


Trinity, Tango frameworks

For real-time control, flight simulators, need **machine learning/neural networks** (can be based off gyrokinetics and/or reduced QL models; e.g., van de Plassche PoP 2020)

Application I

Impurity injection: lowering edge T, expect inward propagation



Experiments (AUG/CMOD/DIII-D): edge "**cold pulse**" surprisingly **causes T increase in core**

Note: low n_e discharge, ∇n-TEM turbulence regime

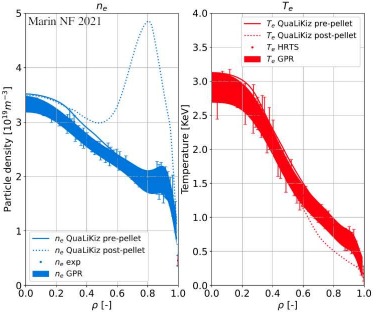
Poorly understood until recently (Angioni NF 2019):

1. impurities decrease T_{edge}, increase $n_{e,edge}$
2. impurities, electrons move inward
3. n_e boost flattens ∇n_e
4. TEM reduced $\Rightarrow$ lower Q_e $\Rightarrow$ core T_e increases

Application II

Key assumption for fusion reactors: **deuterium & tritium from pellets need to reach the plasma center**

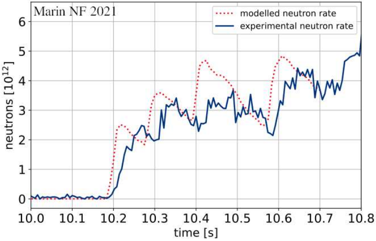
Marin NF 2021: use JINTRAC+QuaLiKiz to analyze D pellet in H plasma $\Rightarrow$ *does* D *move inward*?



Post-injection: massive $\pm\nabla n_{\mathrm{i}}$

$r/a > 0.8$: TEM; $r/a < 0.8$: ITG

ITG causes D **pinch**, boosts core n_{D} & D-D reactions $\Rightarrow$ **more fusion neutrons**



Caveat/future work: JINTRAC requires $T_{\mathrm{H}} = T_{\mathrm{D}}$

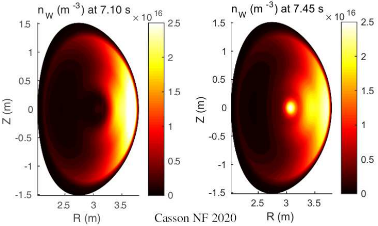
Application III

Recall: tritium retention means no C but W wall needed,
but W in core leads to radiative collapse of T_e

Casson NF 2020: study tungsten accumulation in JET

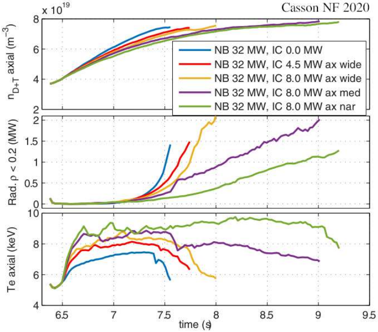
$\Gamma_\mathrm{W} < 0$ due to
neoclassical convection

$\Rightarrow$ fast core accumulation



ICRH known to help $\Rightarrow$ why?

By reducing main ion ∇n,
boost ITG $\Rightarrow$ turbulent $\Gamma_\mathrm{W} > 0$



ICRH against W not as important in ITER (AUG prefers ECRH)
Rather, the study shows we **can explain/predict the experiment**

Questions & Discussion

Any questions about integrated modeling? Other comments?