Turbulence and Transport in Fusion Plasmas
Part IV

M.J. Pueschel

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Wednesday Recap

Yesterday, we covered

- numerical treatment of the Horton-Holland dispersion relation
- different theory frameworks and their use
- Landau damping in kinetic theory
- what conditions have to be fulfilled so MHD and/or gyrokinetics can be used

Next: what coordinates and simulation domain should we use?

Group Work: Coordinates

45 minutes group work:

Find sources that explain

1. toroidal coordinates
2. the safety factor q as a measure of field-line pitch

and (roughly) familiarize yourself with those. Have a look at

3. all of
   `www-fusion.ciemat.es/wiki/Toroidal_coordinates`
4. as much as you feel like of
   `www-fusion.ciemat.es/wiki/Flux_coordinates`
5. bonus reading for those with a high pain threshold:
   pages 1–3 of P. Xanthopoulos *et al.*,
   Phys. Plasmas **13**, 092301 (2006)

Be prepared to present your findings.

Can you explain when/why/how field lines are (not) periodic?

Student Lecture

Who can tell the group . . .

How are **toroidal coordinates** defined?

When are field lines periodic? Why would we care?

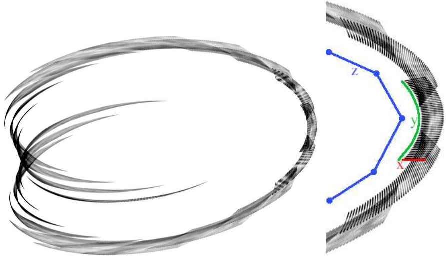
Is turbulence localized radially, toroidally, along the field?

What does all this mean for the stellarator?

Flux Tubes

Typical experiments/reactors: $k_\perp \rho_\mathrm{i} \sim 0.1 - 1 \leftrightarrow n \sim 30 - 300$
Strong radial/toroidal localization $\Rightarrow$ **flux tube** (Beer PoP 1995)

With radial domain $L_x \gtrsim \rho_{\mathrm{i,e}} \ll R, a$, can do Taylor expansion of n, T, q, etc. profiles, e.g., $T(r \approx r_0) \approx T_0 + (r - r_0)\mathrm{d}T/\mathrm{d}r$
A little confusing: in flux tube, both T and $\mathrm{d}T/\mathrm{d}r$ are constant!



Advantages

- cheaper (lower N_x)
- cleaner (Fourier)
- flexible (no fixed ρ^*)

Toroidal coordinates r, θ, ϕ transform to local x, y, z:

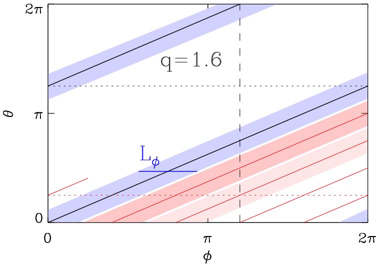
$$r = r_0 + x \qquad \theta = z - \pi \qquad \text{(circular flux surf.)}$$

$$\phi = -\frac{q_0}{r_0} y + q_0 \left(1 + \frac{\hat{s}}{r_0} x\right) \theta \qquad \hat{s} \equiv \frac{r_0}{q_0} \frac{\mathrm{d}q}{\mathrm{d}r}$$

Flux Tube vs. Flux Surface

Tokamak: a **single flux tube represents** entire **flux surface**

$\rho^* = \rho_\mathrm{i}/R_0 \ll 1$ free to pick,
$\mathcal{M} = 2\pi/L_\phi \in \mathbb{N}$ arbitrary
$\Rightarrow$ at $x = 0$, always periodic,
but not at $x \neq 0$ if $\hat{s} \neq 0$



Circular surface: parallel BC
$f(x, y, \pi) = f(x, y - 2\pi\hat{s}x, -\pi)$
(also called *twist-and-shift*)

Full system: entire flux surface (*but*: $n = 1$ means $k_y \sim k_\parallel$!)
$\Rightarrow$ need to test convergence only for $k_y^{\mathrm{min}} \propto L_y^{-1}$ in $-\pi \leq z < \pi$

Stellarator: **flux tubes** starting at different ϕ **differ**
$\Rightarrow$ for complete physics, need full-surface (or full-volume) code!

Ballooning Representation

Exercise: derive **parallel BC in $k_{x,y}$ Fourier space**

$$f(k_x, k_y, \pi) = (-1)^{\mathcal{N}} f(k_x + \mathcal{N} k_x^{\text{shift}}, k_y, -\pi)$$

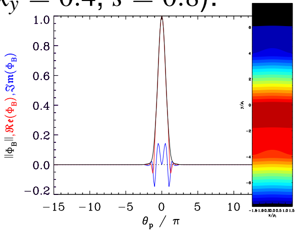
$$\mathcal{N} = 2\pi \hat{s} k_y / k_x^{\text{shift}} \overset{\text{commonly}}{=\!=\!=} \pm 1$$

- real space: y shift
- k-space: k_x shift
- can be used to stitch together k_x

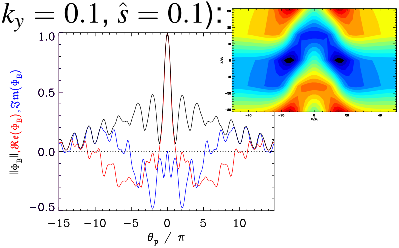
Ballooning space

Extended parallel coordinate: **ballooning angle** θ_p, usually $= 0$ at $k_x = 0$ (Candy PoP 2004)

"**ballooned**" mode ($k_y = 0.4$, $\hat{s} = 0.8$):



"**slab-like**" mode ($k_y = 0.1$, $\hat{s} = 0.1$):



Group Work: Locality

20 minutes group work

1. Determine whether the flux tube is likely to be valid for the same machines/radii as in the Orderings group work:
   Is the Taylor expansion a good approximation throughout $L_x \sim 100\rho_\mathrm{i}$?

Note: this is a simple estimate! For real applications, more thorough studies (e.g., comparing local, global) may be needed.

Questions & Discussion

Anything unclear so far?

Group Work: Equilibria

MHD: **magnetic equilibria** — no MHD instability,
fluxes from neoclassical collisions or microturbulence

1.5 hours group work:

1. Download J.W. Haverkort's write-up on equilibria:
   `http://homepage.tudelft.nl/20x40/documents/Equilibria.pdf`
2. Work through
   a. Sec. 1.1
   b. Sec. 1.2
   c. Appendix A

   for good understanding of the **Grad-Shafranov** equation
3. get R.L. Miller *et al.*, Phys. Plasmas **5**, 973 (1998),
   read Secs. 1–3, make notes about what is unclear,
   distill key findings

Then reconvene in the plenum to discuss everyone's findings

The Zoo of Instabilities

Microinstabilities: drift waves driven by pressure gradients
Note: all of them have **critical gradients**

	ITG	ETG	TEM	KBM	MT
drive	∇T_{i}	∇T_{e}	$\nabla T_{\mathrm{e}}, \nabla n$	$\nabla T_{\mathrm{i,e}} + \nabla n$	∇T_{e}
ρ_j scale	i	e	i	i	i
ω sign[1]	$+$	$-$	$(-)$	$+$	$-$
$\beta \nearrow$	$\gamma \searrow$	$\gamma \rightarrow$	$\gamma \rightarrow$	$\gamma \nearrow$	$\gamma \nearrow$
Φ vs. $A_\parallel$	$\gg$	$\gg$	$\gg$	$\gg$	$\lesssim$
parity[2]	$+(-)$	$+(-)$	$+(-)$	$+(-)$	$-$
slab branch[3]	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$
zonal flows[4]	$\checkmark$	$(\checkmark)$	$\times, \checkmark$	$\times$	$(\checkmark)$

Cause of turbulence & transport in fusion experiments

Each of the above is relevant to all of tokamak, stellarator, RFP

[1] $+(-)$ drifts in ion(electron)-direction; some use opposite nomenclature!
[2] $+(-)$ means even (odd) $\Phi(z)$ and odd (even) $A_\parallel$
[3] slab mode: parallel motion important, $|\theta_p| \gg \pi$
[4] nonlinear saturation mechanism, discussed later

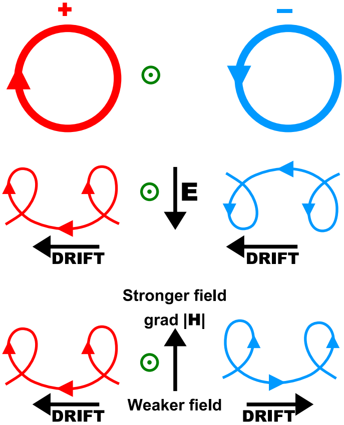
Plasma Drifts

To understand **drift-wave instabilities**, recall drifts: gradients cause perpendicular **drifts at constant** v

- electric field: "E cross B", $\mathbf{v}_E = c(\mathbf{E} \times \mathbf{B})/B^2$
- inhomogeneous guide field: "grad B" & curvature, $\mathbf{v}_{\nabla B} = v_\perp^2 (\mathbf{b} \times \nabla B)/(2B\Omega_j)$ $\mathbf{v}_\mathrm{c} = v_\parallel^2 (\nabla \times \mathbf{b})_\perp / \Omega_j$

Key properties

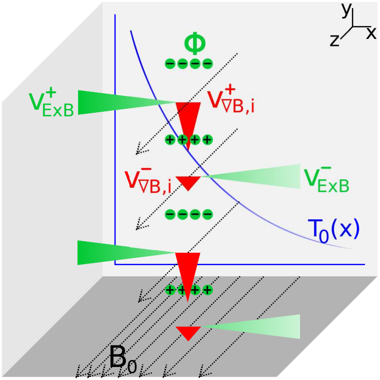
$\mathbf{v}_E$ in same direction for i, e
$\mathbf{v}_{\nabla B,\mathrm{c}}$ opposite for i, e



(adapted from: Wikipedia)

ITG & ETG Modes

ITG: Coppi PoF 1967 (linear), Dimits PoP 2000 (nonlinear)
ETG: Liu PRL 1971 (linear), Jenko PoP 2000 (nonlinear)



Toroidal ITG mode:

$\delta\Phi \to v_E \to \delta T_\mathrm{i} \to v_{\nabla B} \to \delta\Phi$

- slab: $k_\|$ instead of ∇B_0
- ETG linearly isomorphic
  $\Rightarrow \gamma_\mathrm{ITG}/v_\mathrm{ti} = \gamma_\mathrm{ETG}/v_\mathrm{te}$
- also called η_i (η_e) mode: ∇n can stabilize
- nonlinear toroidal ITG: zonal flows (ETG, slab ITG: somewhat less)

Characteristic scales

$k_y\rho_\mathrm{s,e} \sim 0.1 - 1$, $k_x \sim 0 - k_y$, $\gamma \sim 0.1 - 1 v_\mathrm{ti,te}/L_{T\mathrm{i,e}}$, $\omega \sim \pm\gamma$

Trapped-Electron Modes

Linear: Coppi PRL 1974, nonlinear: Ernst PoP 2004
∇T vs. ∇n drive: Ernst PoP 2009

∇n-driven TEM:
$\delta\Phi \rightarrow v_E \rightarrow \delta n \rightarrow v_{\nabla B}^{\mathrm{e,i}} \rightarrow \delta\Phi$

- no slab equivalent
- $\nu_{\mathrm{ei}} \gtrsim \gamma$: “dissipative” DTEM
- $\nu_{\mathrm{ei}} < \gamma$: “collisionless” CTEM
- “ion” iTEM: Plunk JPP 2017
- “ubiquitous” UTEM ($\omega > 0$): Coppi PoFB 1990

Electrons trapped on outboard ($\nabla B_0 \parallel \nabla n, T$)

∇T_{e}-driven TEM works correspondingly

- ∇T TEM and ETG driven by ∇T_{e}, can be indistinguishable
- trapped-ion TIM of limited relevance

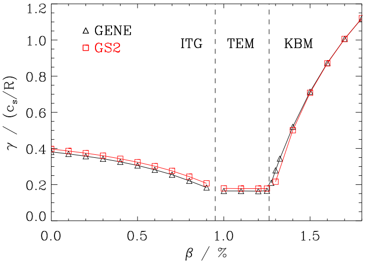
Characteristic scales

$k_y \rho_{\mathrm{s}} \sim 0.2 - 2$, $k_x \sim 0 - k_y$, $\gamma \sim 0.1 - 1 c_{\mathrm{s}}/L_{n,T\mathrm{e}}$, $\omega \sim -\gamma$

Kinetic Ballooning Modes

Linear: Tsai PoFB 1993, Hirose PRL 1994
Nonlinear: Pueschel PoP 2008 & 2010, Ishizawa NF 2013
KBM (also: "Alfvénic" AITG):
kinetic version of MHD ideal ballooning at high $n > 10$

Destabilized at high β



Driven by total gradient
$\nabla p \sim \nabla n + \nabla T_\mathrm{i} + \nabla T_\mathrm{e}$

$\beta_\mathrm{crit}^\mathrm{KBM}(k_y \to 0) \to \beta_\mathrm{crit}^\mathrm{MHD}$ at
$\alpha_\mathrm{MHD} = \beta q_0^2 R_0 (\mathrm{d}p/\mathrm{d}r)/p \approx 0.6\hat{s}$

McKinney JPP 2021: saturation requires $\beta < \beta_\mathrm{crit}^\mathrm{KBM}(k_y^\mathrm{min})$

Note: $\beta/\beta_\mathrm{crit}$ common figure of merit for electromagnetic effects

Characteristic scales

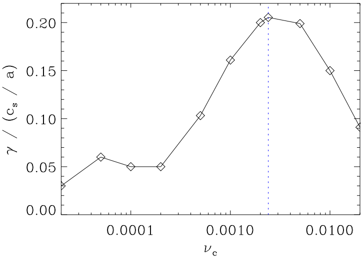
$k_y \rho_\mathrm{s} \sim 0 - 0.5$, $k_x \sim 0$, $\gamma \sim 0.1 - 1 c_\mathrm{s}/L_p$, $\omega \gg \gamma$ near β_crit

Microtearing Modes

Linear: Hazeltine PoF 1975, Drake PoF 1977
Nonlinear: Doerk PRL 2011, Guttenfelder PRL 2011

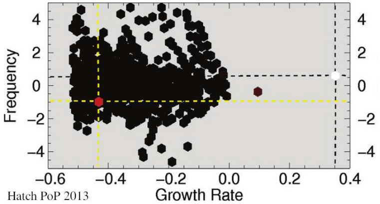
Global tearing: driven by ∇j, while **MT**: driven by β, ∇T_{e}

Energy access via ν_{c}
or curvature (collisionless)



Nonlinearly, pure $Q_{\mathrm{e}}^{\mathrm{em}}$
(no $Q_{\mathrm{e,i}}^{\mathrm{es}}$ or particle flux)

Hatch PRL 2012 & PoP 2013: Subdominant MT responsible for $Q_{\mathrm{e}}^{\mathrm{em}}$ in ITG turbulence



$\Rightarrow$ first-ever example of important stable mode

Characteristic scales

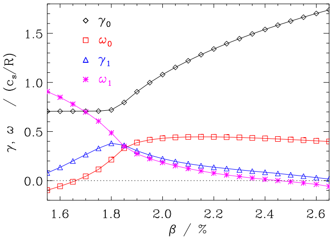
$k_y\rho_{\mathrm{s}} \sim 0.01 - 0.5$ (slab), $0.1 - 1$ (tor'l), $\gamma \sim 0.1 - 1 c_{\mathrm{s}}/L_{T\mathrm{e}}$, $\omega < 0$

Hybrid Modes

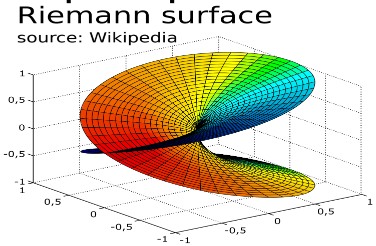
Already mentioned: ∇T TEM & ETG can join forces

Kammerer PoP 2008, Pueschel PoP 2008:
hybrid modes combining properties of **two instabilities**

E.g., can continuously **transform KBM into TEM**



Mathematically, related to **exceptional points**



Walking circle in parameter space can give different mode

Consequences:

- turbulence regime boundaries can have odd behavior
- **subcritical** linear excitation (e.g., KBM below β_{crit})

Group Work: Characteristic Scales

1 hour group work:

1. for same machines as earlier, calculate characteristic
   a. diamagnetic frequency
   b. parallel transit frequency
   c. wavelengths corresponding to $k_y \rho_{\mathrm{s}} = 0.3$, $k_y \rho_{\mathrm{e}} = 0.3$
   d. β and ballooning threshold $\beta_{\mathrm{crit}}^{\mathrm{MHD}}$

   in SI or cgs units (can look up diamagnetic frequency in A.J. Brizard, Rev. Mod. Phys. **79**, 421 (2007))

   Who can say what their physical relevance is?

2. where feasible, estimate importance of instabilities for the above machines/radii: ITG, ETG, TEM, KBM

Questions & Discussion

Anything unclear that we talked about?

Any feedback for the instructor?

Anyone still awake?