Turbulence and Transport in Fusion Plasmas
Part I

M.J. Pueschel

Ruhr-Universität Bochum, February 27 – March 10, 2023

Topics

In *Turbulence and Transport in Fusion Plasmas*,
you will receive an introduction on how

- fusion energy can help us reduce carbon emissions
- instabilities cause plasma microturbulence and anomalous heat/particle losses
- to use fluid and kinetic equations to describe plasmas and evaluate instabilities
- to derive and use the gyrokinetic framework
- to write and deploy plasma simulation code
- instabilities saturate, setting transport levels
- to use reduced quasilinear models for fast flux prediction
- transport depends on plasma parameters
- to solve transport equations to predict plasma profiles

Slides, Projects, Grades

Quizzes

Occasionally:
short quiz about recent subjects, self-evaluated

Not part of final grade

Research Projects

Pick topic by March 10th

Third week:
research in groups of 2–3

Present results on Friday, *March 17th*

Unless requested otherwise, **grades will be based on the research project** (67% joint) **and presentation** (33% individual)

Lecturer

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Dutch Institute for Fundamental Energy Research
leader of the *Plasma Microturbulence* research group

Priv. Doz. at the Ruhr-Universität Bochum
local collaborations with Tjus & Grauer groups

working on plasmas since 2004 and on fusion since 2005
formerly at University of Texas, University of Wisconsin,
& Max Planck Institute for Plasma Physics

Literature

- Individual papers covering specific topics will be cited throughout the class
- Textbook about plasma physics:
  Francis F. Chen, *Introduction to Plasma Physics*, Springer Science & Business Media
- Review paper on gyrokinetic turbulence simulations:
  X. Garbet, Y. Idomura, L. Villard, and T.H. Watanabe, Nucl. Fusion **50**, 043002 (2010)
- Background on quasilinear transport modeling:
  C. Bourdelle *et al.*,
  Plasma Phys. Control. Fusion **58**, 014036 (2016)
- Review paper on integrated modeling:
  F.M. Poli, Phys. Plasmas **25**, 055602 (2018)

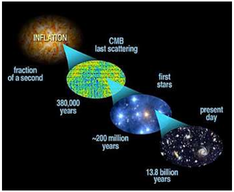
Questions

Any questions about the setup?

Cosmic Energy Sources

Given that energy in the Universe is conserved,
where does it come from and where can we find it?

- Big Bang



Unknown injection mechanism at $t = 0$

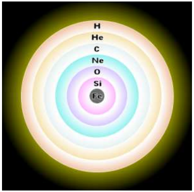
primordial nucleosynthesis

- Gravitation

Large scales:

Potential energy in planets, stars, galaxies . . .

- Fusion



Stars (+ Big Bang):

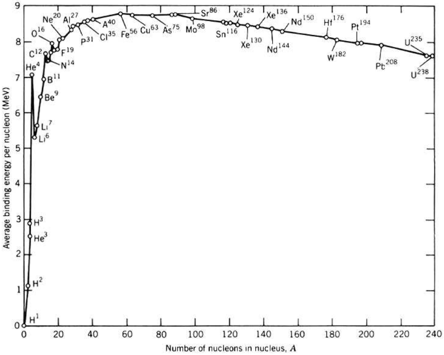
fusion of lighter into heavier elements

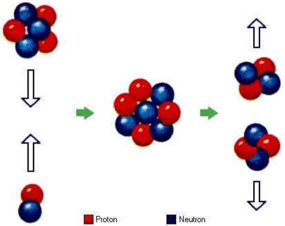
By comparison: the Sun radiates as much energy as 10^{17} large power plants would produce

Nuclear Fusion

Atomic nuclei: differences between elements in binding energy (due to quantum-mechanical energy states between nucleons)

⇒ *can convert less stable into more stable nuclei*



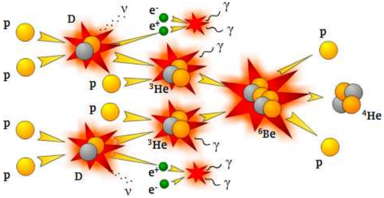


However, need to overcome Coulomb repulsion between positively charged nuclei via large collision speeds/temperature

Stellar Fusion Cycles

$\leftarrow$ M K G F A B O $\rightarrow$

lighter stars, Sun:



- **p-p cycle**:
  protons to helium (primary process in the sun)

heavier, hotter stars:

- **triple-alpha cycle**:
  key to the creation of heavier elements
- **CNO cycle**:
  carbon-catalyzed fusion of protons, highly efficient at high temperatures

Exotic Fusion Events

Big Bang

- initial state: only protons and neutrons exist
- fuse to D, T, He, Li
- heavier elements: only after the first generation of stars

Supernovae

- collapse due to missing core fuel
- fusion of heavy elements
- high energies: creation of radioactive elements

Fusion is the only process in the cosmos that can create life and sustain it in the long term

Terrestrial Considerations

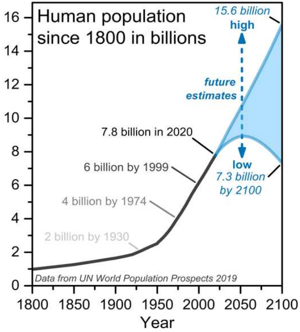
On Earth: most available energy stems (directly or indirectly) from the Sun (other sources: radioactivity, Earth's core), **exceeded by civilization's needs**

Historically: improved energy efficiency

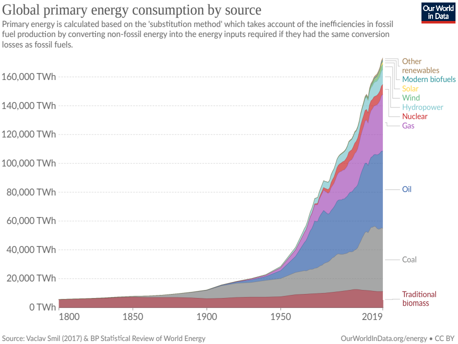
$\leftrightarrow$

accelerated growth of demand

Population growth:

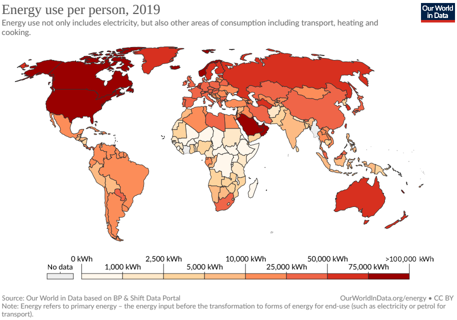


Energy use:



Distributive Justice

Per-capita energy use:



- enormous consumption in North America, Europe
- industrialization China and India
- future growth in Africa

$\leftrightarrow$ limited supply

$\leftrightarrow$ limited ability of nature to compensate the consequences

Moral and practical question:

Who should be allowed to use how much of what type of energy?

Climate Change

Causes:

- emission of greenhouse gases
- deforestation (esp. primordial)
- impact on marine life

$\Rightarrow$ global warming

Consequences:

- Reduction in biodiversity
- massive changes in precipitation
- food and water insecurity

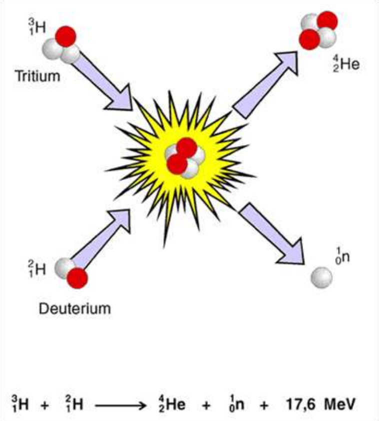
$\Rightarrow$ climate wars, refugee flows

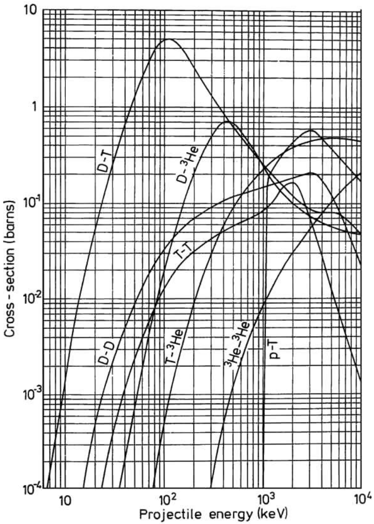
Without decisive and emcompassing action, hundreds of millions of people could die!

Presuming humans accept only minimal reductions in per-capita energy,
how do we sustain society without ruining the planet?

D-T Fusion

As we will see: **fusion** provides **safe long-term solution**
How about the most efficient fusion reaction?





- high reaction rate
- large released E
- safe ash: He

Magnetic Confinement

Sun's core ($T \approx 1.5 \times 10^7$ K): particles **confined gravitationally**

How to confine particles in a fusion reactor ($T \approx 10^8$ K)?

Similar to solar corona:

- at these temperatures, matter in **plasma state**
- Lorentz force: particles spiral about magnetic field lines

$\Rightarrow$ **magnetic confinement**

$\Rightarrow$ fusion reactors: vacuum chambers with strong magnetic fields containing hydrogen (isotope) plasmas

(*but*: density not too high, otherwise eruption/solar flares)

Lawson Criterion

If sufficiently many fusion reactions: energy released in alpha particles can compensate heat losses $\Rightarrow$ **“ignition”** (*note*: heat/particle losses = what this class is about!)

Requires high temperatures/densities, good confinement, or:

fusion triple product ($\sim$ Lawson criterion): $\quad nT\tau_{\mathrm{E}} \gtrsim 3{\times}10^{28}\,\frac{\mathrm{K\,s}}{\mathrm{m}^3}$

Stars: sufficient density to get to ignition (p-p cycle)
Planets: did not get there; note brown dwarfs (D-D)

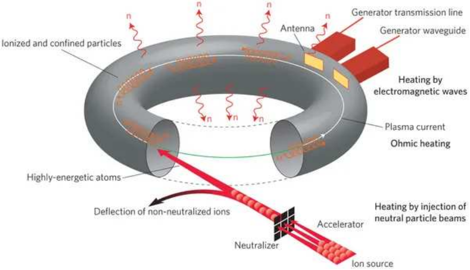
Different approaches to achieving fusion on Earth

- magnetic confinement (our focus): small-ish n, high T, τ_E
- inertial confinement: high n, high T, tiny τ_E

Note: ignition not necessary for commercial fusion!
$Q = P_{\mathrm{fusion}}/P_{\mathrm{heating}} \gtrsim 5$ suffices (ignition is $Q \to \infty$)

Plasma Heating

How can we heat a plasma to 10^8 degrees?



- **Ohmic heating**:
  induce current
  $\Rightarrow$ resistive
  heating
- **wave heating**:
  accelerate
  particles
  on Larmor orbits
  via radio waves
- **neutral-particle heating**:
  accelerate negative ions, strip excess electrons,
  inject into plasma $\Rightarrow$ collisional heating

Fueling: freeze hydrogen into pellets, accelerate in centrifuge, shoot into plasma

Timeline

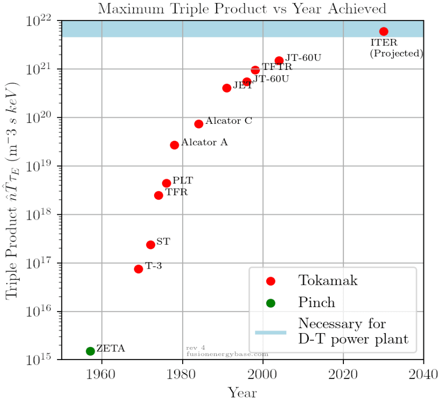
1950: Proyecto Huemul, first fusion scam

1950/60s: classified research in Western/Eastern Block

1970/80s: East-West collaboration

1990s: JET (Landshut LH181)

2000s: theory catches up



2010s:
development of breakthrough concepts: ITER, (SP)ARC, . . .

Also see:

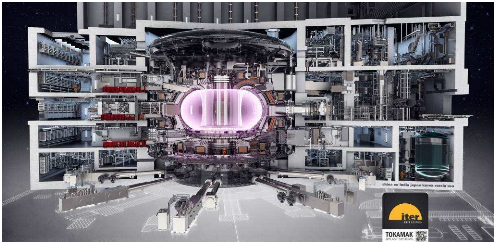
R. Herman, *Fusion – The Search for Endless Energy* (Cambridge University Press, 1990)

`https://www.fusionenergybase.com/article/measuring-progress-in-fusion-energy-the-triple-products/`

ITER – the Next Step

Key difficulty in fusion: insulation of hot plasma core

One approach: **build bigger reactors** (*but cost* $\propto$ *volume!*)



ITER:
International
Thermonuclear
Experimental
Reactor

International collaboration, biggest science experiment after ISS

Targets: $Q = 10$ for 30 minutes, $500\,\mathrm{MW}$ fusion power

Participants:

- China
- European Union
- India
- Japan
- Russia
- South Korea
- USA

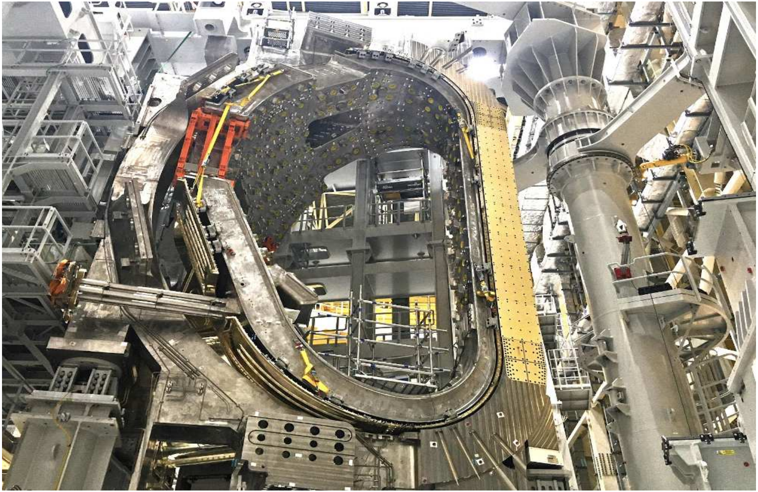
The ITER Site I

Cadarache, Southern France: ITER reactor hall completed

First experiments planned for December 2025

The ITER Site II

2021: First magnetic field coils and vacuum vessel segment installed



First experiments planned for December 2025

Power-Plant Operation

Fuel

Deuterium:
extract from water

Tritium: decays with half-life of 12 years
$\Rightarrow$ no natural sources
Create via Lithium breeding:
$\mathrm{Li} + \mathrm{n} \rightarrow \mathrm{T} + \mathrm{He}$

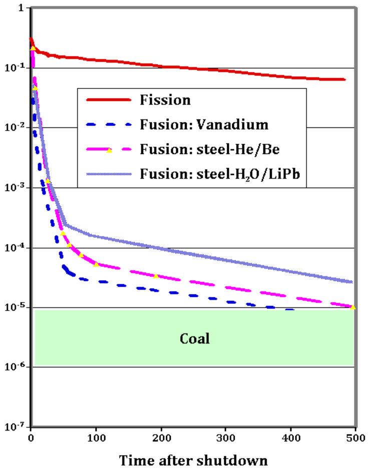
Blanket: Lithium layer absorbs fusion neutrons

- fuel available for millions of years:
  D: oceans,
  Li: Earth's crust, oceans
- operation: continuous or pulsed (ca. 12 hours, then ca. 15 minutes shut-down)
- operations inside reactor chamber: robots (e.g., repairing wall tiles)

Waste and Radioactivity

- **Fusion ash**: Helium (no waste)
- **Neutron irradiation**: transmutation of wall materials $\Rightarrow$ low-level radioactivity (compare radiotoxicity on right)
- **Radioactive fuel**: tritium burnt up in fusion reactions

Radiotoxicity of reactor components:



Group Work: Waste

45 minutes group work:

1. each group download one of
   a. K. Brodén *et al.*, Fusion Eng. Des. **42**, 1 (1998)
   b. M. Zucchetti *et al.*, Fusion Eng. Des. **136**, 1529 (2018)
2. digest the content & prepare a short presentations

After 45 minutes, each group will present one of the papers (10 minutes plus 5 minutes discussion)

Rule for all group work sessions: if you're stuck, just ask!

Risk

H. Bartels *et al.*,
Fusion Eng. Des. (1998):
accident scenarios

worst-case scenario:
airplane crash, pulverizing tritium in blanket,
airborne HTO or $\mathrm{T_2O}$

$\Rightarrow$ need to evacuate ca. $3\,\mathrm{km}$ radius for multiple days

F. Najmabadi *et al.*,
Fusion Eng. Des. (2006):
smart design can avoid even small-radius evacuation

Possible **cost problem**:

Solar-like eruptions:
"edge-localized modes", ELMs can damage wall tiles
$\Rightarrow$ *replacing very expensive*

Newer research:

ELMs can be split into many mini-ELMs (no wall damage) via

- controlled perturbation of magnetic field
- timed pellet injection

Solving the Energy Crisis?

Fusion holds enormous promise:

- hardly any CO_2
- little problematic waste
- globally available
- continually available
- compatible with current power grids
- near-inexhaustible fuel reserves

However: **no silver bullet against climate change**
requires all-of-the-above approach, with fusion playing key role

Research ongoing on

High-temperature and **high-field superconductors**
$\Rightarrow$ substantially smaller and more efficient reactors

Optimization using high-performance computing
$\Rightarrow$ reduce heat losses, improve efficiency via magnet shaping

Questions & Discussion

Who has unanswered questions about fusion energy?

Who has seen interesting fusion stuff on the news recently and wants to discuss?

For those of you who are not yet spoken for M.Sc.-wise:

The Eindhoven University of Technology (TU/e) offers a **Science and Technology of Nuclear Fusion** M.Sc. program

`www.tue.nl/en/education/graduate-school/master-science-and-technology-of-nuclear-fusion`

TU/e EINDHOVEN UNIVERSITY OF TECHNOLOGY

Ionized Gas

Want to confine & heat plasmas . . . **so what is a plasma?**

Common (*too simple*) definition: **plasma = ionized gas**

Saha equation: ion density in weakly ionized gases
Hydrogen gas:

$$\frac{n_{\text{ionized}}^2}{n_{\text{neutral}}} = \lambda_{\text{th}}^{-3}\, \mathrm{e}^{-E_{\text{ionization}}/T} \approx \frac{6\times 10^{21}}{\text{cm}^3} T^{3/2}\, \mathrm{e}^{-E_{\text{ionization}}/T}$$

(λ_{th}: electron de Broglie wavelength; $T[\text{eV}]$)

Quick exercise: calculate the ionization level $n_{\text{ionized}}/n_{\text{neutral}}$ for

- room temperature
- the Sun's core
- a fusion reactor plasma

What T to expect for the *Cosmic Microwave Background*?
Use $n \sim 300\,\text{cm}^{-3}$; how does T compare to the actual $\sim 1\,\text{eV}$?

Plasma Definition

Plasma = **fourth state of matter**, what is the **phase transition**?

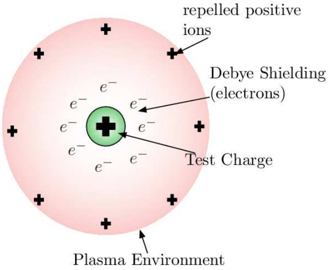
- **ionization**: very continuous, behavior of ionized gas does not change suddenly at very low/high ionization
- **weakly collisional**: when collisions completely dominate, dynamics follow physics of gases
- **collective effects**: long-range interactions of charged particles due to electromagnetic forces
  $\Rightarrow$ fundamentally new behavior, true phase transition

When and how do collective effects arise?

Debye Shielding

Consider single charge q:
repulsion of like q,
attraction of opposite q

$\Rightarrow$ "Debye shielding"
reduces effective charge



More quantitatively (1D):

$$f_{\mathrm{i,e}} = \mathrm{e}^{-m_{\mathrm{i,e}} v_{\mathrm{i,e}}^2/(2T_{\mathrm{i,e}}) - q_{\mathrm{i,e}}\Phi/T_{\mathrm{i,e}}} \quad \rightarrow \quad n_{\mathrm{i,e}} = n_0 \mathrm{e}^{-q_{\mathrm{i,e}}\Phi/T_{\mathrm{i,e}}}$$

Into Poisson equation, expand for small Φ, use $m_\mathrm{i} \gg m_\mathrm{e}$:

$$\frac{1}{4\pi}\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} = -q_\mathrm{i} n_\mathrm{i} - q_\mathrm{e} n_\mathrm{e} \approx e n_0 \left(\frac{e\Phi}{T_\mathrm{e}} - \frac{Z_\mathrm{i} e\Phi}{T_\mathrm{i}}\right) \approx n_0 \frac{e^2\Phi}{T_\mathrm{e}}$$

Solved by $\Phi \propto \exp(-|x|/\lambda_\mathrm{D})$ with Debye length $\lambda_\mathrm{D}^2 = T_\mathrm{e}/(4\pi e^2 n_\mathrm{e})$

Debye shielding reduces Φ_{eff}, inhibits collective effects!

Plasma Waves I

So what exactly are collective effects?
Plasmas can produce any number of waves and instabilities, we will discuss many of them later

Simplest plasma wave:

- move all electrons (density n_0) a little to one side ($m_\mathrm{i} \gg m_\mathrm{e} \Rightarrow$ ions static; $T_\mathrm{e} = T_\mathrm{i} = 0$)
- perturbed electron density δn creates electric field $E = E_x$
- perturbation travels at speed $\delta v = \delta v_x$

Continuity equation:	$\partial_t \delta n + n_0 \partial_x \delta v = 0$
electrostatic force:	$\partial_t m_\mathrm{e} \delta v = -eE$
Poisson equation:	$\partial_x E = -4\pi e \delta n$

Three equations, three unknowns $\Rightarrow$ *straightforward to solve*

Group Work: Plasma Frequency

15 minutes group work:

1. solve this set of equations with a wave ansatz $\mathrm{e}^{ikx-i\omega t}$

2.

Group Work: Plasma Frequency

15 minutes group work:

1. solve this set of equations with a wave ansatz $\mathrm{e}^{ikx-i\omega t}$

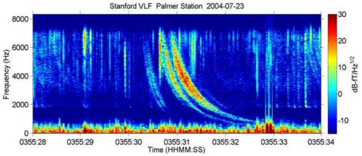
Solution: **plasma frequency** $\omega_\mathrm{p} = \sqrt{4\pi e^2 n_0/m_\mathrm{e}}$

2. evaluate ω_p for densities in a typical fusion plasma

Plasma Waves II

Before getting to instabilities, here is a sample of plasma waves (all in homogeneous magnetic fields, no pressure gradients)

Plasma oscillation	$\omega^2 = \omega_\mathrm{p}^2 + 3k^2 v_\mathrm{th}^2/2$	$\mathbf{k} \parallel \mathbf{B}$
Hybrid wave	$\omega^2 = \omega_\mathrm{p}^2 + \omega_\mathrm{c}^2$	$\mathbf{k} \perp \mathbf{B}$
Light wave	$\omega^2 = \omega_\mathrm{p}^2 + k^2 c^2$	$\mathbf{k} \perp \mathbf{B}$
Alfvén wave	$\omega^2 = k^2 v_\mathrm{A}^2 = k^2 \frac{B^2}{4\pi n_\mathrm{i} m_\mathrm{i}}$	$\mathbf{k} \parallel \mathbf{B}$
Whistler wave	$\omega^2 = k^2 c^2 + \frac{\omega_\mathrm{p}^2}{1-\omega_\mathrm{c}/\omega}$	$\mathbf{k} \parallel \mathbf{B}$



Whister wave: created by lightning, “whistling” detectable by radio

$\Rightarrow$ *interpreting frequency spectra in plasmas can be difficult!*

Quasineutrality

When dealing with ion-electron or pair plasmas, we often assume $n_\mathrm{i} = n_\mathrm{e}$ (**neutrality**), but still allow $\Phi \neq 0$...?!?

Underlying concept: only $\ll 1$ departures from $n_\mathrm{i} = n_\mathrm{e}$ allowed, otherwise truly extreme E fields

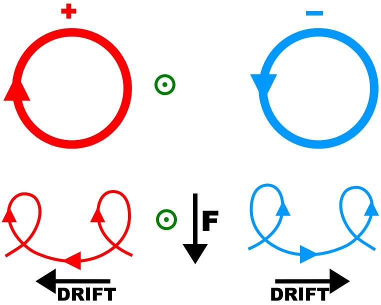
Correct vs. approximate derivation of ion sound waves (∇n):

$\delta n_\mathrm{i} = \delta n_\mathrm{e}$ **approximation**	**Correct treatment**
$m_\mathrm{i} n_0 \partial_t v_\mathrm{i} = -e n_0 \partial_x \Phi - T_\mathrm{i} \partial_x \delta n_\mathrm{i}$	$\longleftarrow$ same
adiabatic ("Boltzmann") electrons ($m_\mathrm{e} \to 0$):	Poisson equation $\partial_x E = 4\pi e(\delta n_\mathrm{i} - \delta n_\mathrm{e})$
$\delta n_\mathrm{e} = n_0 e\Phi/T = \delta n_\mathrm{i}$	$\delta n_\mathrm{e} = n_0 e\Phi/T \neq \delta n_\mathrm{i}$
continuity: $\partial_t \delta n_\mathrm{i} + n_0 \partial_x v_\mathrm{i} = 0$	$\longleftarrow$ same
$\Rightarrow \omega^2/k^2 = T_\mathrm{e}/m_\mathrm{i} + T_\mathrm{i}/m_\mathrm{i}$	$\omega^2/k^2 = \frac{1}{1+k^2\lambda_\mathrm{D}^2} T_\mathrm{e}/m_\mathrm{i} + T_\mathrm{i}/m_\mathrm{i}$

Valid for large scales — most fusion theory relies on approximate neutrality (**quasineutrality**) but uses Poisson

Particle Drifts

Fusion plasmas: **strong magnetic guide fields**
$\Rightarrow$ thermal motion along $\mathbf{B}$, slow **perpendicular drifting**



Some force $\mathbf{F} = F\hat{\mathbf{e}}_y$
(assuming $\partial_t F = 0$):

$$
\begin{aligned}
v_x &= v_\perp \exp(i\omega_c t) \\
v_y &= \pm i v_\perp \exp(i\omega_c t) + F/(qB) \\
\mathbf{v}_{\mathrm{drift}} &= \mathbf{F}\times\mathbf{B}/(qB^2)
\end{aligned}
$$

Possible F:

- gravitation
- polarization ($\partial_t E$)
  (*but*: ∂_t modifies v_x)
- inhomogeneous E field
- inhomogeneous B field

We will look at inhomogeneous fields in more detail later

Group Work: Drifts

45 minutes group work:

1. search online (e.g., in papers) for typical quantities in fusion plasmas: B, E (or Φ), turbulent frequencies ω
2. evaluate the gravitation drift for those parameters
3. derive the polarization drift for $E = E_x \propto \exp(i\omega t)$ (*need to add* E *to* $\partial_t v_x$ *force, get* $\partial_t^2 v_x = -\omega_c^2 v_x + i\omega\omega_c E_x/B$; use this for E-modified v_x ansatz and assume $\omega \ll \omega_c$)
4. evaluate the polarization drift for those parameters

After 45 minutes, report your findings to the class.
Which of these drifts, if any, is likely to be important?

Later, we will study **turbulence**, which is produced by **drift-wave instabilities**

Solution: Polarization Drift

To obtain **polarization drift**, consider $E = E_x$ acting on ions (electrons will have opposite drift sign)

No E: have $v_x = v_\perp \exp(i\omega_c t)$ and $\partial_t v_x = i\omega_c v_\perp \exp(i\omega_c t) = \omega_c v_y$
E enters via force equation $\partial_t v_x = \omega_c v_y + \omega_c E_x/B \leftarrow$ no drift in x if $\partial_t E = 0$

Thus, $$\partial_t^2 v_x = -\omega_c^2 \left(v_x - \frac{i\omega}{\omega_c} \frac{E_x}{B} \right)$$

Finite E: **ansatz** $v_x = v_\perp \exp(i\omega_c t) + v_{\text{test}}$ with $v_{\text{test}} = (i\omega/\omega_c)(E_x/B)$

$$\Rightarrow \quad \partial_t^2 v_x = -\omega_c^2 v_x + (\omega_c^2 - \omega^2) v_{\text{test}} \approx -\omega_c^2 v_x + \omega_c^2 v_{\text{test}} \qquad (1)$$

when $\omega \ll \omega_c \Rightarrow$ **solves equation**, can therefore define

$$v_{\text{test}} \equiv v_{\text{pol}} = \frac{1}{\omega_c} \frac{\partial_t E}{B} \qquad (2)$$

Note: E_x also causes drift in v_y; but E oscillates, so no net motion

Questions & Discussion

What is unclear/wrong/poorly explained,
and what else would you like to know?