Turbulence and Transport in Fusion Plasmas
Part VII

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Monday Recap

Yesterday, we covered

- the ins and outs of numerical schemes:
  - finite differencing
  - numerical (hyper-)diffusion
  - explicit time stepping
- how to implement equations in and then deploy a simulation code

Any questions, especially about the code implementation?

Quiz

10-minute self-evaluated quiz:

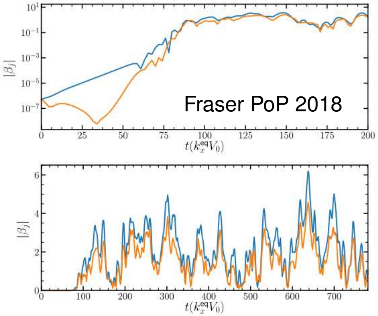
1. In the gyrokinetics derivation, what is $J_0(k_\perp \rho)$, and what does it do?
2. What is the excitation state ℓ of a mode?
3. When does a centered finite-differencing stencil fail?
4. What is the difference between diffusion and hyperdiffusion, which one do you prefer, and why?

Quiz – Answers

1. J_0 is the Bessel function, capturing how a particle with Larmor radius ρ "feels" a field (e.g., Φ) with correlation length $\sim k_\perp^{-1}$.
2. The mode's quantum state ℓ is related to how wiggly the mode is (*Hermite number*); even ℓ are ballooning parity, odd are tearing parity.
3. Centered stencils fail on scales not properly resolved by the grid spacing, $k \sim \Delta x^{-1}$.
4. Diffusion $\propto k^2$, hyperdiffusion $\propto k^{4,6,\cdots}$. Larger exponents can damp high k better but require larger stencils and more boundary points. k^4 is a common compromise.

Instability Saturation

Saturation: amplitudes stop growing



Two mechanisms:

- energy source depleted ("*quasilinear flattening*")
- nonlinearity balances linear terms ("*turbulent saturation*") $\Rightarrow$ heating, fueling maintains $\nabla T, n$

quasi-stationary: $\dfrac{\partial f}{\partial t} \approx 0 \;\Rightarrow\; \mathcal{L} = -\sum_{k'} \left(k'_x k_y - k'_y k_x\right) \Phi(k') f(k-k')$

$\Rightarrow$ **triplet interactions** shuffle energy from unstable to stable (*recall: Kolmogorov cascade vs. large-scale stable modes*)

Secondary Instability

fixed $\Phi(0, k'_y)$: $f(k_x, 0)$ and $f(k-k')$ grow exponentially $\Rightarrow \gamma_{\mathrm{sec}}$

Group Work: Zonal Flows

45 minutes group work:

Find sources that explain

- what zonal flows are and in what physical systems they matter
- whether – and if yes, when – zonal flows are important in fusion plasmas

and familiarize yourself with those.
Be prepared to present your findings.

Student Lecture: Zonal Flows

Please tell us:

What are zonal flows?

How do they affect turbulence?

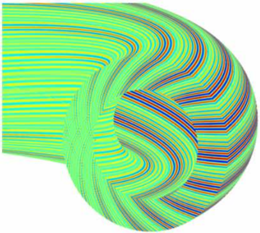
Aside from fusion plasmas, where do they exist?

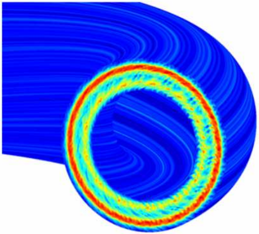
When do they and when do they not have an impact?

Perhaps some of you had run into Secondary Instability when researching zonal flows . . . who wants to tell us about that?

Higher-Order Instability

1.	background gradients	$\rightarrow$	linear instability
	zonal structure	$\rightarrow$	radial structure
2.	linear mode	$\rightarrow$	secondary instability
	radial structure	$\rightarrow$	zonal structure
3.	secondary mode	$\rightarrow$	tertiary instability
	zonal structure	$\rightarrow$	radial structure
	$\vdots$		$\vdots$



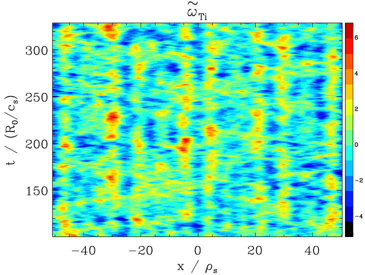


Tertiary Instability

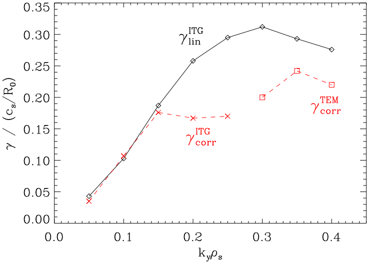
Rogers PRL 2000, Waltz PoP 2010, Pueschel PoP 2013:
Tertiary can be **new mode** or **modified primary**

Here: mixed ITG-TEM scenario (Pueschel PoP 2013)

Profile corrugations:



Tertiaries (full g corrugations):



⇒ locally/temporally enhanced gradients, may **influence linear modes**

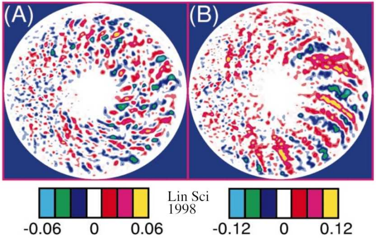
⇒ **little impact** near NL transport peak, zonal flows stabilize at higher k_y

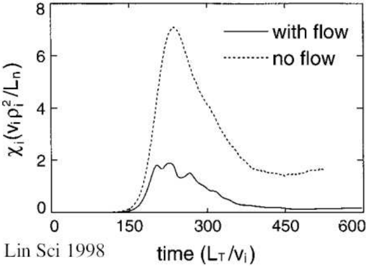
Note: zonal flows are effectively Φ corrugations

ITG Saturation

Toroidal ITG: strongly relies on **zonal flows** for saturation

Lin Sci 1998: first global nonlinear ITG simulations





- for two decades, *shearing paradigm*: ZFs shearing eddies, providing cascade to small scales
- now: **large-scale stable modes** responsible for saturation
- *note*: kinetic (vs. adiabatic) electrons affect zonal-flow dynamics $\Rightarrow \gamma_{\rm adi} \sim \gamma_{\rm kin}$ but $Q_{\rm adi} \ll Q_{\rm kin}$

Slab ITG: zonal flows less prominent but still matter

Questions & Discussion

Let us take a break and ponder all the things we learned so far:

- take 30 minutes to go over the lecture notes
- note down things that are conceptually opaque
- together, we will try to clear up anything that requires it

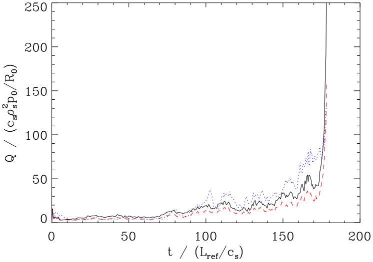
Hereafter, we will go into some current research into nonlinear aspects of fusion plasma turbulence …

The High-β Runaway

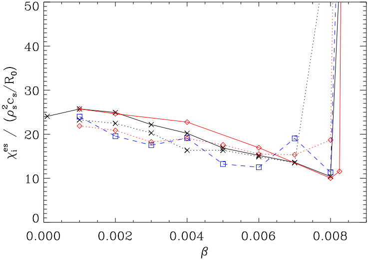
Need more proof that zonal flows matter?

For 10 years, strange, unexplained simulation behavior:

no saturation of turbulence above pressure $\beta_{\mathrm{crit}} < \beta_{\mathrm{KBM}}$



Seen in many codes: GENE, GYRO, GKW

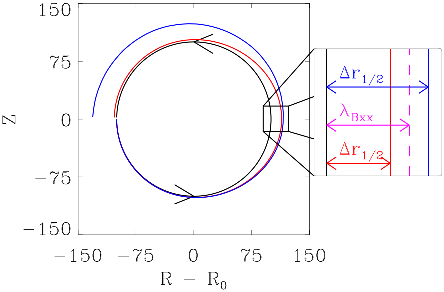


Key observation: **zonal flows absent** as fluxes take off

Field Line Decorrelation

Recall: ITG is ballooning-parity – finite β causes odd-parity B_x

Non-tearing B_x: field line leaves magnetic surface from inboard, maximum Δr at outboard, returns (almost) to B_0 surface



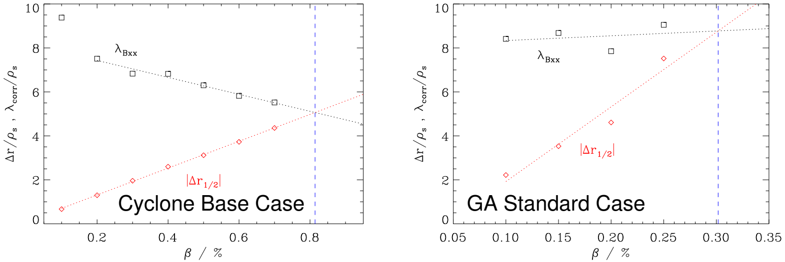
- $\Delta r_{1/2}$ is the radial displacement at $\theta = 0$
- λ_{Bxx} is the correlation length of B_x along x

However: if field line decorrelates, **second half turn** becomes **independent of first**, no return to original position

Zonal flows shorted out $\Rightarrow$ **Non-Zonal Transition** (NZT)

The Non-Zonal Transition

Can field line decorrelation really explain the runaway?



$\Rightarrow$ **excellent prediction** of runaway β by decorrelation!

Consequences for realistic applications:

heat transport time scale in NZT-marginal state can be $\sim \gamma^{-1}$
$\Rightarrow$ **stiff profiles**, cannot increase plasma pressure anymore

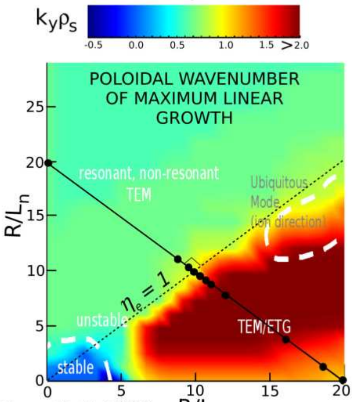
Note: indications that NZT may limit profiles in TCV, JT60-SA

TEM Saturation

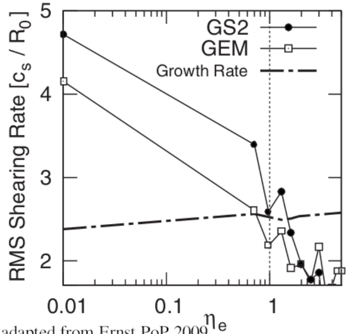
Merz PRL 2008: remove Φ_{ZF} in ∇T-TEM $\Rightarrow$ no change in Q
Ernst PoP 2009: scan $\eta_{\mathrm{e}} = \omega_{T\mathrm{e}}/\omega_n$, see ∇n-TEM $\rightarrow$ ∇T-TEM

Shearing Rate

Rule-of-thumb: $\omega_{\mathrm{s}} = k_x^2\Phi > 10\gamma \leftrightarrow$ ZFs matter; but: *ill-conditioned*



Ernst PoP 2009



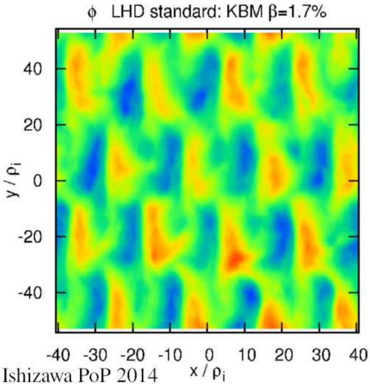
adapted from Ernst PoP 2009

- ∇T-TEM: ZFs irrelevant
- ∇n-TEM: ZFs important

Saturation without ZFs

*How do ∇T-**TEMs** saturate without zonal flows?*
Lang PoP 2008: **zonal density** = density corrugation

Ishizawa PoP 2013:
KBMs saturate via **diagonal band structures** in real and k-space



Ishizawa PoP 2014

But: **saturation without zonal** Φ mostly **poorly understood**, often through many different inefficient $(k, k', k - k')$ triplets

Why care? Nonlinear saturation very difficult to describe, being able to reduce it makes analytical approaches much easier!

Discussion: Mixing Length

Let us discuss how linear physics can describe turbulence:

1. What are the key observables in fusion plasma turbulence?
2.
3.

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   γ, ω, $k_\perp$, eigenmode Φ (or even f)
3. Dimensionally, how can we combine linear into nonlinear quantities?

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3. Dimensionally, how can we combine linear into nonlinear quantities?
   $[\chi] = [D] = \mathrm{m}^2/\mathrm{s} \Rightarrow \gamma/k_\perp^2$

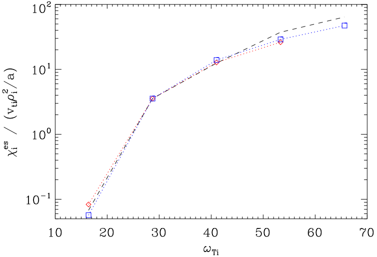
$\Rightarrow$ so-called **quasilinear**, or **mixing-length**, model:
much **faster** than nonlinear but less reliable,
whether QL is valid reveals key physics!

Quasilinear Flux Estimating

QL in Gyrokinetics

- perform scan over some parameter (e.g., ω_T or $\hat{s}$)
- use linear simulations to construct Q_{QL}
- fix Q_{QL} to nonlinear Q_{NL} at single point in scan

Example: high-∇T turbulence (Pueschel PPCF 2019)



More on QL in next course part

Base model used in the remainder of the turbulence part:

$$-(\nabla T)/T = R/L_T$$

growth rate of mode j

$$Q_{\mathrm{QL}} = \omega_T \sum_{j,k_y} C(k_y) \frac{\gamma(j,k_y) w(j,k_y)}{\langle k_\perp(k_y,j)^2 \rangle} \begin{array}{l} \leftarrow \text{QL weight } (\Phi \times T \text{ phase}) \\ \leftarrow \int_{-3\pi}^{3\pi} k_\perp^2 \Phi^2 \mathrm{d}\theta \,/\, \int_{-3\pi}^{3\pi} \Phi^2 \mathrm{d}\theta \end{array}$$

model constant

No stable mode modes in here, how to account for them?

Saturation Math

Terry PoP 2018: using Monday's Φ, p model for toroidal ITG, **statistical** "EDQNM" **closure** to get nonlinear properties

Conceptual derivation (playing fast and loose with the math):

- assume every k has one unstable, one stable eigenmode
- unstable (stable) mode amplitude $\beta_1 \sim |\Phi_1|$ ($\beta_2 \sim |\Phi_2|$)
- assume nonlinear interactions $\sim \beta'\beta'' = \beta(k')\beta(k-k')$ (sum over k' implied) dominated by zonal flow $\beta_\mathrm{z} \sim |\Phi_\mathrm{ZF}|$

energy $|\partial_t \beta_{1,2}|^2 \mp 2\gamma_{1,2}|\beta_{1,2}|^2 = 2\langle \beta'_\mathrm{z}\beta''\beta_{1,2}\rangle \quad |\partial_t\beta_\mathrm{z}|^2 + 2\nu|\beta_\mathrm{z}|^2 = 2\langle\beta'\beta''\beta_\mathrm{z}\rangle$

Evaluating $\langle\beta\beta'\beta''\rangle$ gives $\langle\beta\beta'\beta''\beta'''\rangle$, where one can assume only in-phase modes contribute ($\leftrightarrow$ *many metronomes*)

$$\frac{1}{2}\partial_t|\beta_1|^2 - \gamma_1|\beta_1|^2 = \frac{|\beta'_\mathrm{z}|^2(|\beta_1|^2 + \langle\beta_1\beta_2\rangle + \langle\beta''_1\beta''_2\rangle + |\beta''_2|^2)}{\underbrace{i\omega''_2 + i\omega'_\mathrm{z} - i\omega^*_1}_{\text{complex frequencies}}}$$

What does that mean and how can we use it?

Triplet Correlation Time

Triplet correlation time $\tau = \dfrac{1}{i\omega_{\text{stable}} + i\omega_{\text{ZF}} - i\omega_{\text{ITG}}^{*}}$

measures how resonantly ($\tau \to \infty$) modes interact nonlinearly;
large $\tau \Rightarrow$ strong transfer to stable modes $\Rightarrow$ **low flux**

Also from Terry '18:

$$|\beta_{1,2}|^2 \approx \frac{\nu}{\mathrm{Re}(\tau)}$$

and

$$Q = -\sum_k k_y \left(|\beta_1|^2 + |\beta_2|^2\right)$$

$$= \sum_k k_y \frac{\gamma_1}{k_\perp^2} \frac{\nu}{\mathrm{Re}(\tau)} \frac{1}{k_y^2 \epsilon^{1/2}}$$

Linear vs. Nonlinear τ

Essential: **linear** τ_{lin} **good proxy** for **nonlinear** τ_{NL}

$$\tau_{\text{lin}} = \frac{-i}{\omega_{\text{MITG}}(-k_x^{\text{ZF}}, k_y) - \omega_{\text{ITG}}^{*}(0, k_y)}$$

couples *streamer instability*, *sideband stable mode,* $\gamma_{\text{MITG}} = -\gamma_{\text{ITG}}$, and $\omega_{\text{ZF}} \approx 0$

Key modification $\mathrm{Re}(\tau)^{-1}$ can be incorporated in gyrokinetic QL

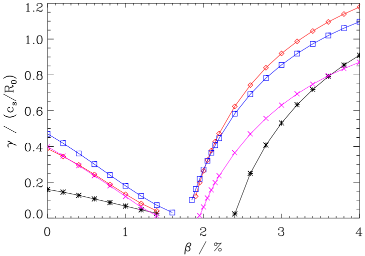
Improved quasilinear model $Q_{\text{i}}^{\text{es}} = \omega_{Ti} \sum_{k,j} \mathcal{C}_k \dfrac{\gamma_{kj} w_{kj}}{\langle k_{\perp kj}^2 \rangle \mathrm{Re}(\tau_{kj})}$

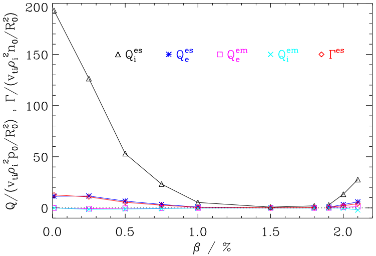
Electromagnetic Stabilization I

$\beta \equiv \frac{\text{kinetic pressure}}{\text{magnetic pressure}} > 0$ causes magnetic fluctuations

Hirose PoP 2000: ITG undergoes **linear electromagnetic stabilization** – field-line bending changes curvature

Pueschel PoP '08, '10: **nonlinearly**, even **more stabilization**





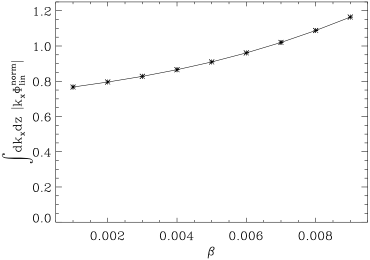
- stabilization up to $\beta_{\text{crit}}^{\text{KBM}}$; *note*: $\beta_{\text{crit}}^{\text{nonlinear}} = \beta_{\text{crit}}^{\text{linear}}$
- *here*, not considering β-induced Q_e^{em}

What causes **nonlinear electromagnetic stabilization** (NES)?

Electromagnetic Stabilization II

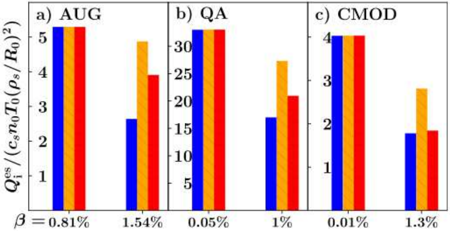
Application of τ-corrected QL model to this situation:

As β is increased, streamer and sideband ITG closer to resonance



Mode width increases
(Pueschel PoP 2013)

$\Rightarrow$ lower fluxes,
captured by τ correction
(Whelan PRL 2018, PoP 2019)



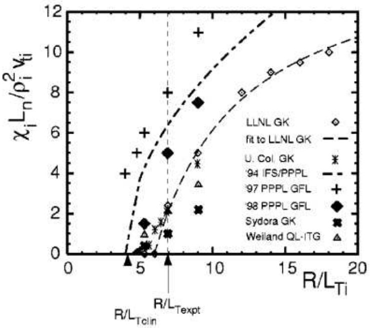
nonlinear, standard QL, corrected QL

$\Rightarrow$ can now **predict and explain** finite-β behavior

Note: further stabilization possible by fast ions
(Citrin PRL 2013, Di Siena NF 2018)

The Dimits Shift I

Dimits PoP 2000: **nonlinear upshift of critical gradient**



Dimits Regime

For $\omega^{\text{lin}}_{Ti,\text{crit}} < \omega_{Ti} < \omega^{\text{NL}}_{Ti,\text{crit}}$:

- strong zonal flows
- highly intermittent turbulence, transport
- small but finite amplitudes at $k_y > 0$

Many attempts to explain physics (e.g., secondary instability), but none predictive . . .

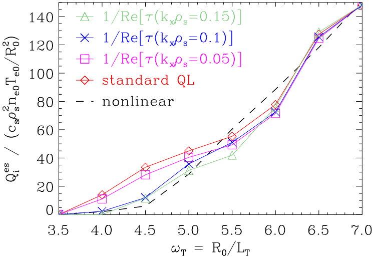
Up until now, **no QL model** that quantitatively **predicts** $\Delta\omega_{T,\text{crit}}$

How about our new model with stable modes and τ?

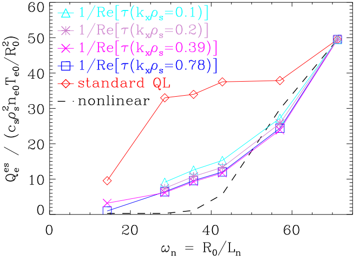
The Dimits Shift I

Pueschel NF 2021: test of τ correction for two cases,

pure ITG scenario:



reversed-field pinch case:



- new model performs much better at low gradients
- **upshift of critical gradient** is **captured**
- ITG case: also need to take **sideband stabilization** into account: perfect saturation efficiency when $\gamma_{\max}(k_x \neq 0) < 0$

Additional theory background: Terry PRL 2021, Li PoP 2021

Questions

Who has need for answers and further understanding?

Group Work: Scans & Quasilinear

2 hours group work

Using your drift-kinetic Vlasov code,

1. conduct a 2D parameter scan, over either k_y-$\omega_{T\mathrm{i}}$ or k_y-ω_n at some fixed $k_{||}$ (suggested range $0.1 \leq k_y \leq 0.8$)
2. based thereupon, construct a simple quasilinear model

$$Q_{\mathrm{i}}^{\mathrm{QL}} = \omega_{T\mathrm{i}} \sum_{k_y} \frac{\gamma(k_y)}{\langle k_y^2 \rangle}\,, \qquad \langle k_y^2 \rangle = \frac{\int k_y^2 |\Phi(k_y)|^2 \mathrm{d}z}{\int |\Phi(k_y)|^2 \mathrm{d}z} = k_y^2$$

   to gauge heat flux scalings of slab ITG turbulence
3. extract the $\Phi \times T_{\mathrm{i}}$ phase and include it in the model, normalized so $\alpha_{\Phi\times T} = \pi/2$ yields 100% $Q_{\mathrm{i}}^{\mathrm{QL}}$