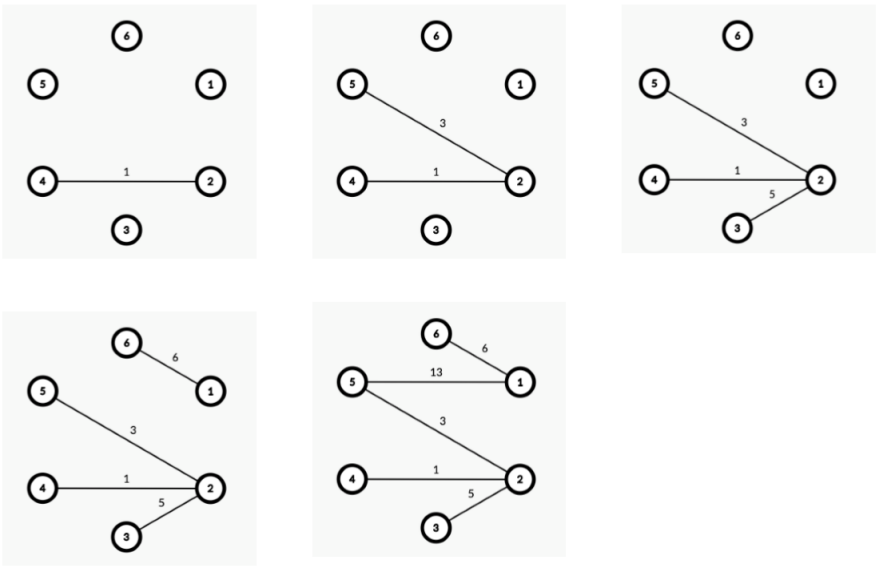
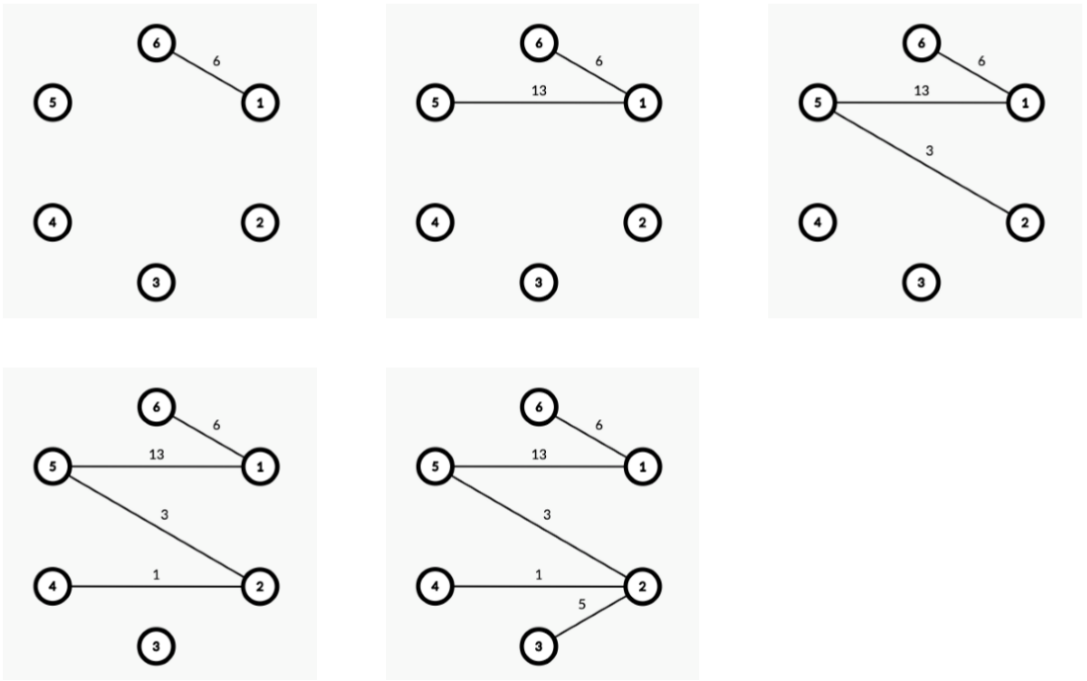
Chapter 23 Homework solution

1.
(a)



Above is the graphs after adding each edge into the spanning tree. Edge (3,4) with weight 8 is not included, since adding it into the spanning tree creates a cycle. Edge (4,5) with weight 10 is not included for the same reason. After adding edge (1,5) with weight 13, the graph becomes connected, so the process finishes. The total weight of the minimum spanning tree is 1+3+5+6+13=28.

(b)



Above is the graphs after adding each edge into the spanning tree. Edge (1,6) with weight 6 is added since it has the minimum weight among all edges adjacent to vertex 1. Edge (1,5) with weight 13 is added since it has the minimum weight among all

edges adjacent to vertex 1 or vertex 6, while not included in the minimum spanning tree yet. At the end, it generates the same minimum spanning tree as the Kruskal's algorithm.

2.

Let $T0$ and $T1$ be the two trees that are obtained by removing edge (u,v) from a MST. Suppose that $V0$ and $V1$ are the vertices of $T0$ and $T1$ respectively.

Consider the cut which separates $V0$ from $V1$. Suppose to a contradiction that there is some edge that has weight less than that of (u,v) in this cut. Then, we could construct a minimum spanning tree of the whole graph by adding that edge to $T1 \cup T0$. This would result in a minimum spanning tree that has weight less than the original minimum spanning tree that contained (u,v).