COM-303 - Signal Processing for Communications

Solutions for Homework #1

Solution 1. Digital signals

(a) Music recorded on a CD : **digital**

(b) Music listened by the audience at a live concert : **analog**

(c) Music recorded on a LP record (vinyl) : **analog**

(d) Photo recorded using a photographic film : **analog**

(e) Photo recorded using a CCD sensor : **digital**

(f) A page on a book : **analog**

(g) The image of a book page on a Kindle : **digital**

Solution 2. Sampling music

To compute the number of samples N, we need to multiply the length in seconds of the signal by the sampling frequency, i.e., the number of samples per second:

$$N = 44100 \times 2 \times 60 = 5,292,000$$

This assumes that the audio is mono; for stereo data, there are two independent channels so the number of samples is double.

Solution 3. Elementary signals and operators

$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Solution 4. Moving average

We compute $y[n]$ by substituting the expression for $x[n]$ into $y[n]=(x[n]+x[n-1])/2$:

$$y[n]=\frac{\delta[n]+2\delta[n-1]+3\delta[n-2]+\delta[n-1]+2\delta[n-2]+3\delta[n-3]}{2}$$

By collecting the delta signals that have the same delay we finally obtain:

$$y[n]=0.5\delta[n]+1.5\delta[n-1]+2.5\delta[n-2]+1.5\delta[n-3].$$

Solution 5. Operators and linearity

(a) $D\{\alpha x[n]\}=\alpha x[n-1]=\alpha D\{x[n]\}$
$D\{x[n]+y[n]\}=x[n-1]+y[n-1]=D\{x[n]\}+D\{y[n]\}.$

(b) $S\{\alpha x[n]\}=\alpha^2 x^2[n]=\alpha^2 S\{x[n]\}\neq\alpha S\{x[n]\}.$

Solution 6. Operators with matrix notation

$$\mathbf{\Delta}=\mathbf{I}-\mathbf{D}=\begin{bmatrix}1&0&0&-1\\-1&1&0&0\\0&-1&1&0\\0&0&-1&1\end{bmatrix}.$$

Solution 7. SP with Lego

To determine the input-output relationship of a complex circuit, it is a good strategy to first look at the intermediate signals that appear at key nodes in the graph. Once the expressions for these signals are computed, they can be combined to produce the global output of the circuit.

In the graph below, call $p[n]$, $q[n]$, $r[n]$ the internal signals in the circuit at the shown locations:

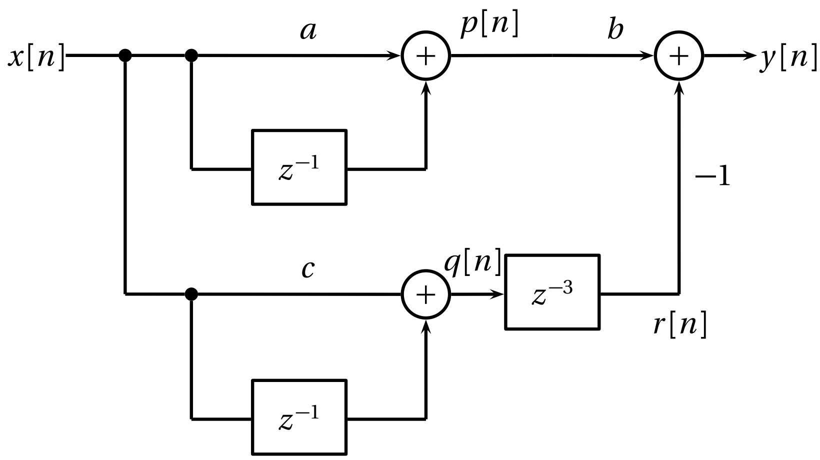
Simply by inspection, we can remark the following facts:

(a) $p[n]=a\,x[n]+x[n-1]$

(b) $q[n]=c\,x[n]+x[n-1]$

(c) $r[n]=q[n-3]$

(d) $y[n]=b\,p[n]-r[n]$



so that in the end

$$y[n] = a\,b\,x[n] + b\,x[n-1] - c\,x[n-3] - x[n-4]$$