Exam:

- written exam Tuesday July 3 from 8:15-11:15
- sample exams of previous years online
- miniproject counts 33 percent towards final grade

For written exam:

-bring 1 sheet A5 (double-sided) of own notes/summary

-HANDWRITTEN!

-no calculator, no textbook

LEARNING OUTCOMES

- Solve linear one-dimensional differential equations
- Analyze two-dimensional models in the phase plane
- Develop a simplified model by separation of time scales
- Analyze connected networks in the mean-field limit
- Formulate stochastic models of biological phenomena
- Formalize biological facts into mathematical models
- Prove stability and convergence
- Predict outcome of dynamics
- Describe neuronal phenomena
- Apply model concepts in simulations

Look at samples of past exams

Transversal skills

- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Collect data.
- Write a scientific or technical report.

Use a textbook, or video lectures. Don't use slides as only resources

miniproject

Your Questions for Exam?

LEARNING OUTCOMES (in red: repeated today)

- Solve linear one-dimensional differential equations
- Analyze two-dimensional models in the phase plane
- Develop a simplified model by separation of time scales
- Analyze connected networks in the mean-field limit
- Formulate stochastic models of biological phenomena
- Formalize biological facts into mathematical models
- Prove stability and convergence
- Predict outcome of dynamics
- Describe neuronal phenomena
- Apply model concepts in simulations

Look at samples of past exams

Transversal skills

- Plan and carry out activities in a way which makes optimal use of available time and other resources.
- Collect data.
- Write a scientific or technical report.

Use a textbook, (Use video lectures) don't use slides (only)

miniproject

Biological Modeling of Neural Networks

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Week XX

Optimizing Neuron Models

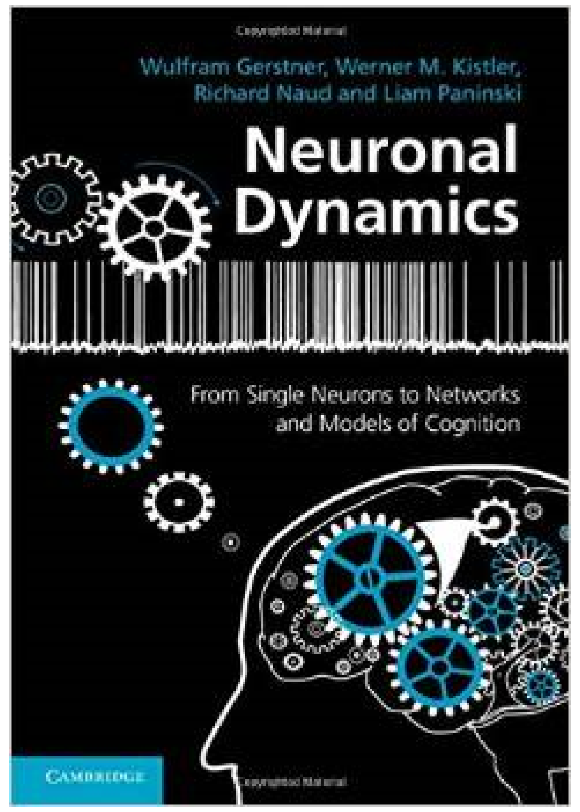
For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 4.6, 6.1,6.2,6.4, 9.2
- Ch. 10.2.3, 11.1. 11.3.3

Cambridge Univ. Press



9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

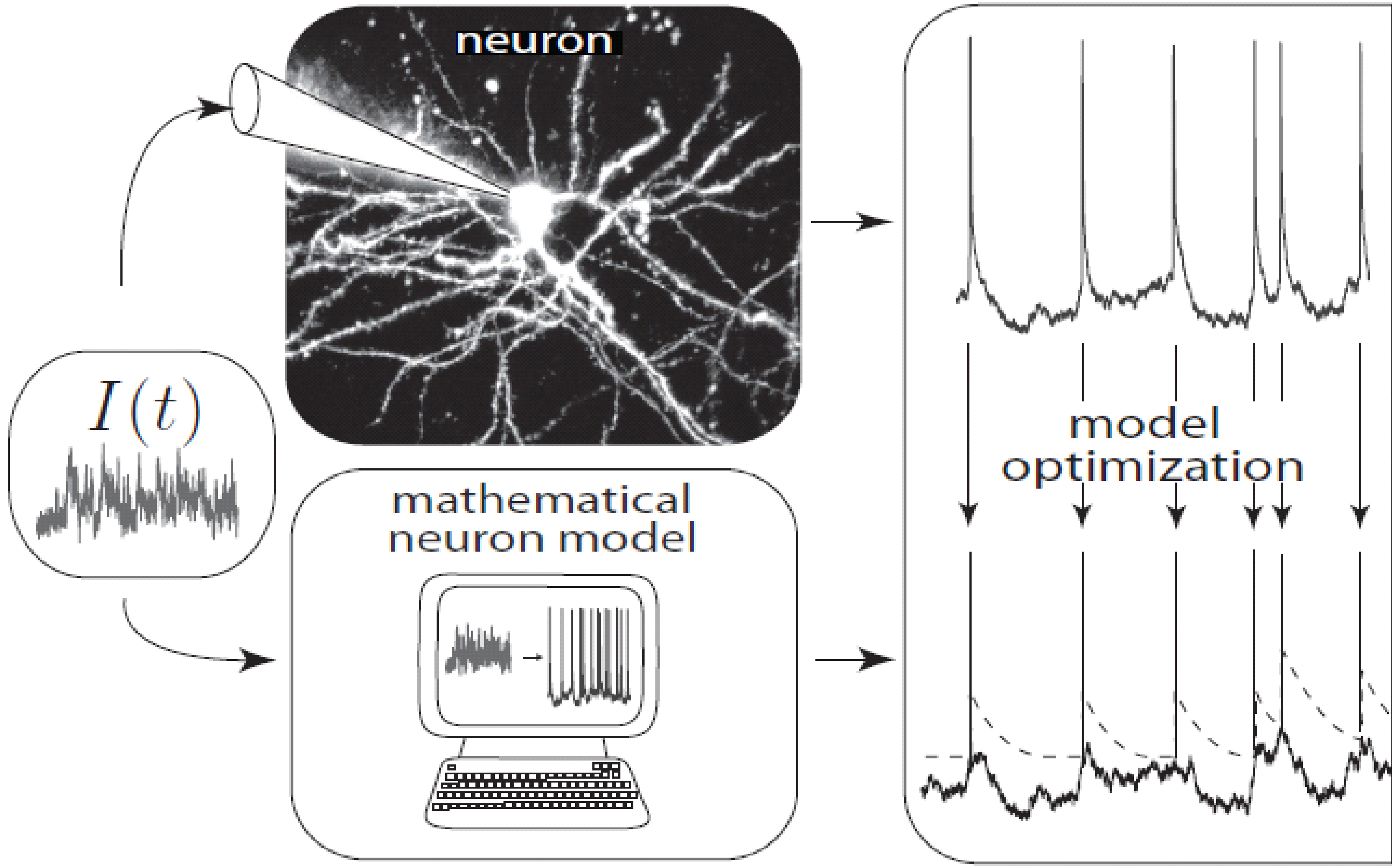
9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM
- Likelihood of a spike train

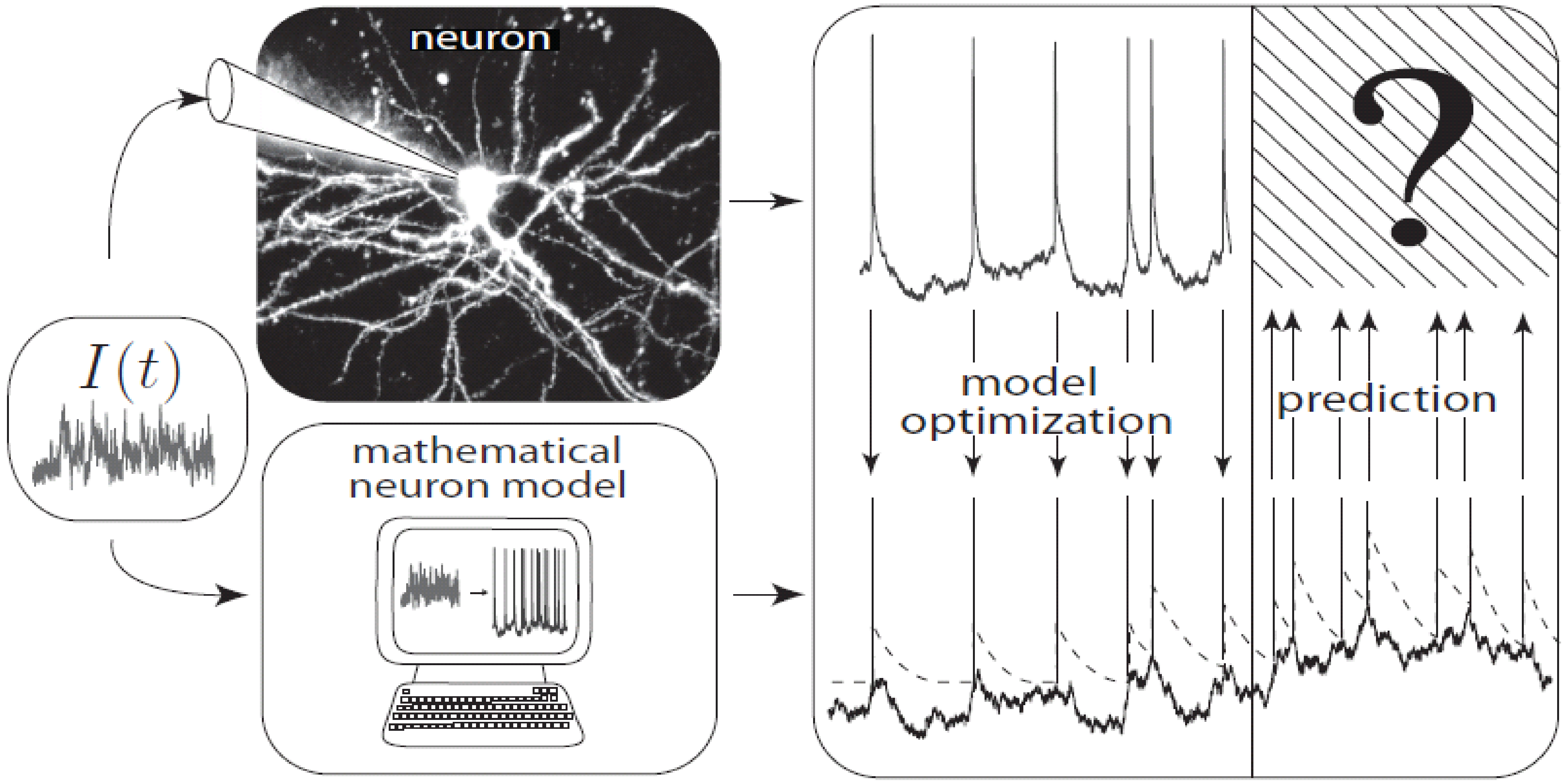
(9.5 Parameter Estimation)
(- Quadratic and convex optimization)

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

9.7. Helping humans – in vivo data



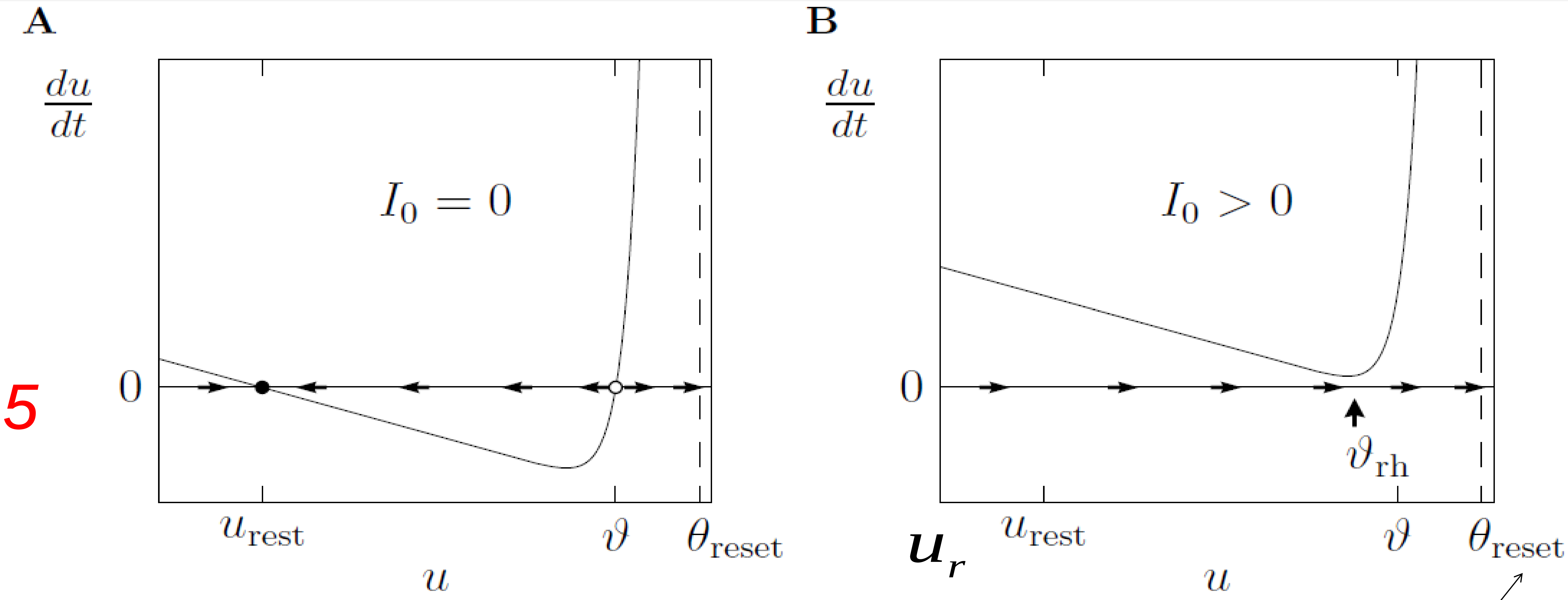
-What is a good neuron model?
-Estimate parameters of models?



A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible enough to account for a variety of phenomena
E) Systematic procedure to ‘optimize’ parameters

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

See: week 1, lecture 1.5



$$\tau \frac{du}{dt} = f(u) + R\, I(t)$$

What is a good choice of **f** ?

If $u = \theta_{reset}$

then reset to

$$u = u_r$$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $If \;\; u = \theta_{reset} \;\; then \;\; reset\,to\; u = u_r$

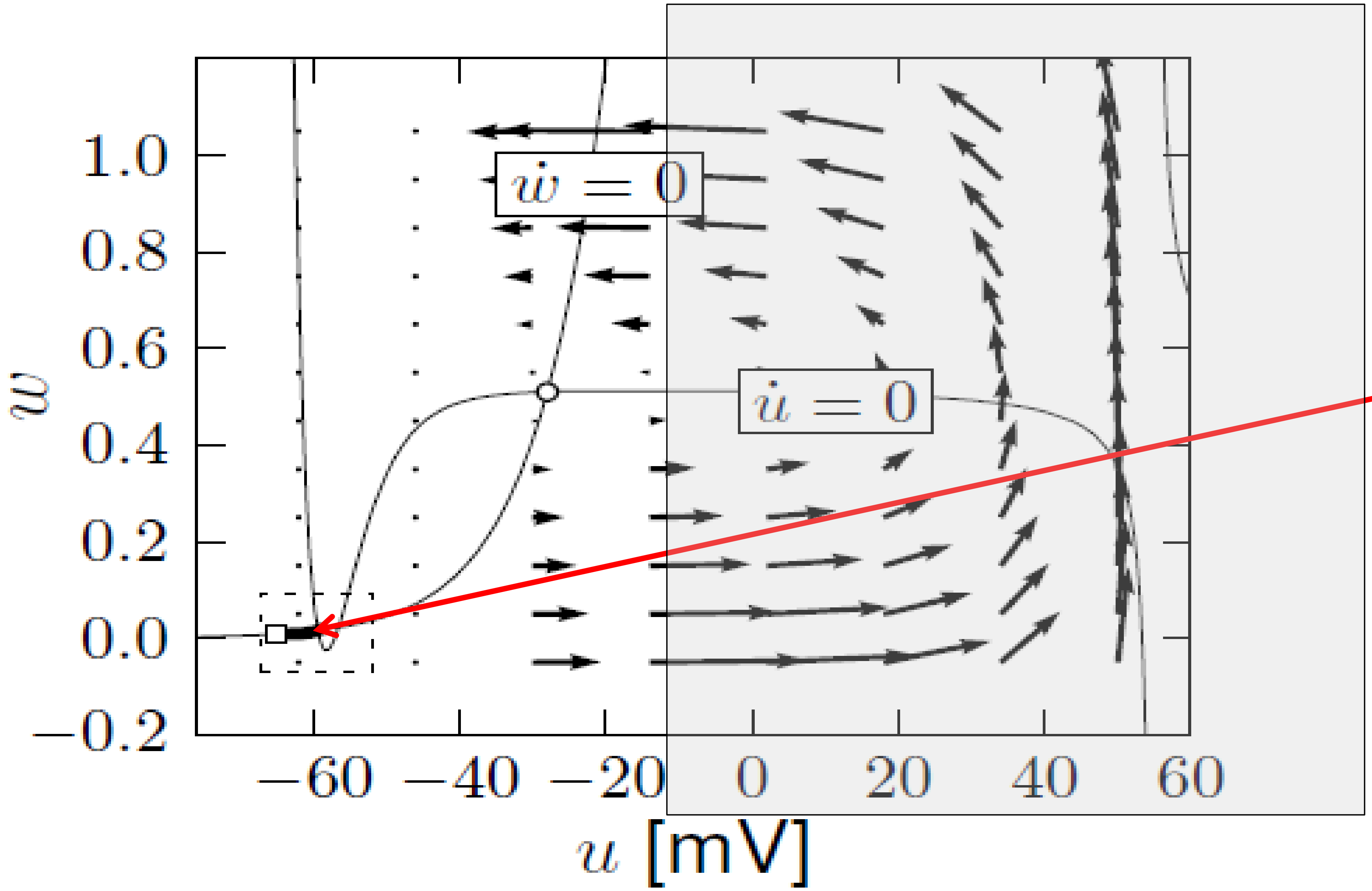
What is a good choice of *f* ?

(i) Extract *f* from more complex models

(ii) Extract *f* from data

(i) Extract *f* from more complex models

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$



A. detect spike and reset

resting state

Separation of time scales:
Arrows are nearly horizontal

Spike initiation, from rest

$$w \approx w_{rest}$$

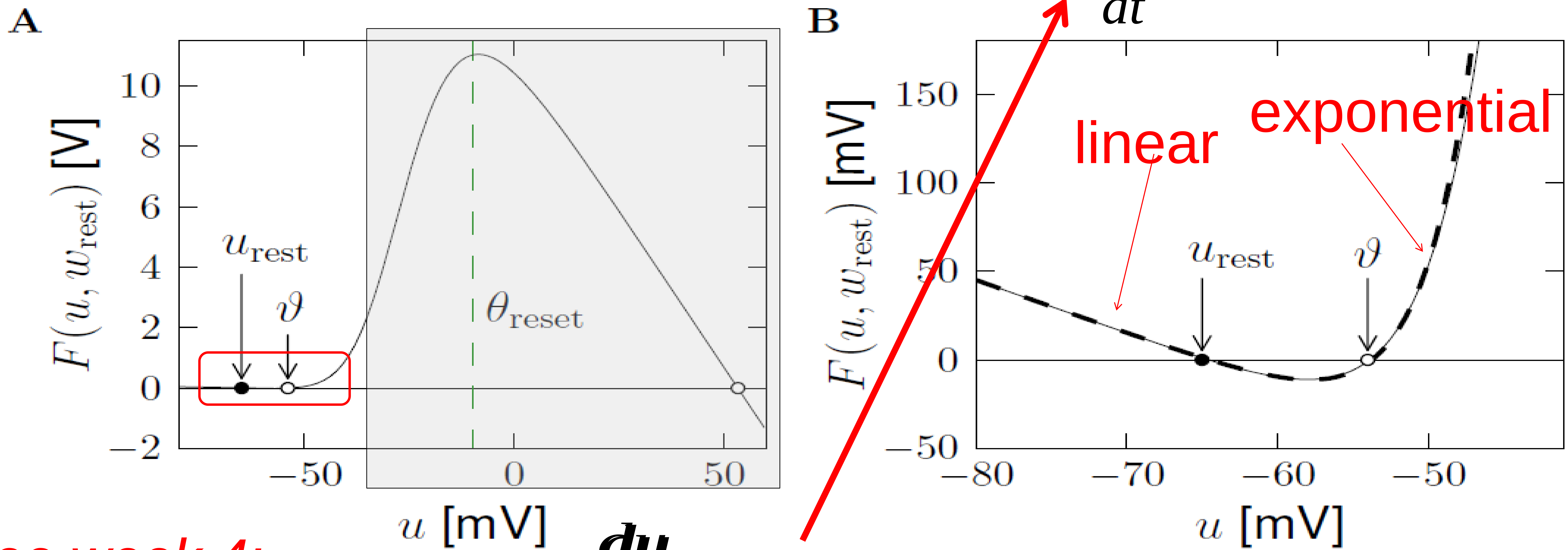
$$\tau \frac{du}{dt} = F(u,w) + R\,I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

B. Assume $w = w_{rest}$

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(i) Extract f from more complex models $\tau \frac{du}{dt} = f(u) + R\,I(t)$



See week 4:
2dim version of
Hodgkin-Huxley

$$\boldsymbol{\tau \frac{du}{dt} = F(u, w_{rest}) + R\,I(t)}$$

$$\tau_w \frac{dw}{dt} = G(u, w) \longrightarrow w \approx w_{rest}$$

Separation of time scales

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

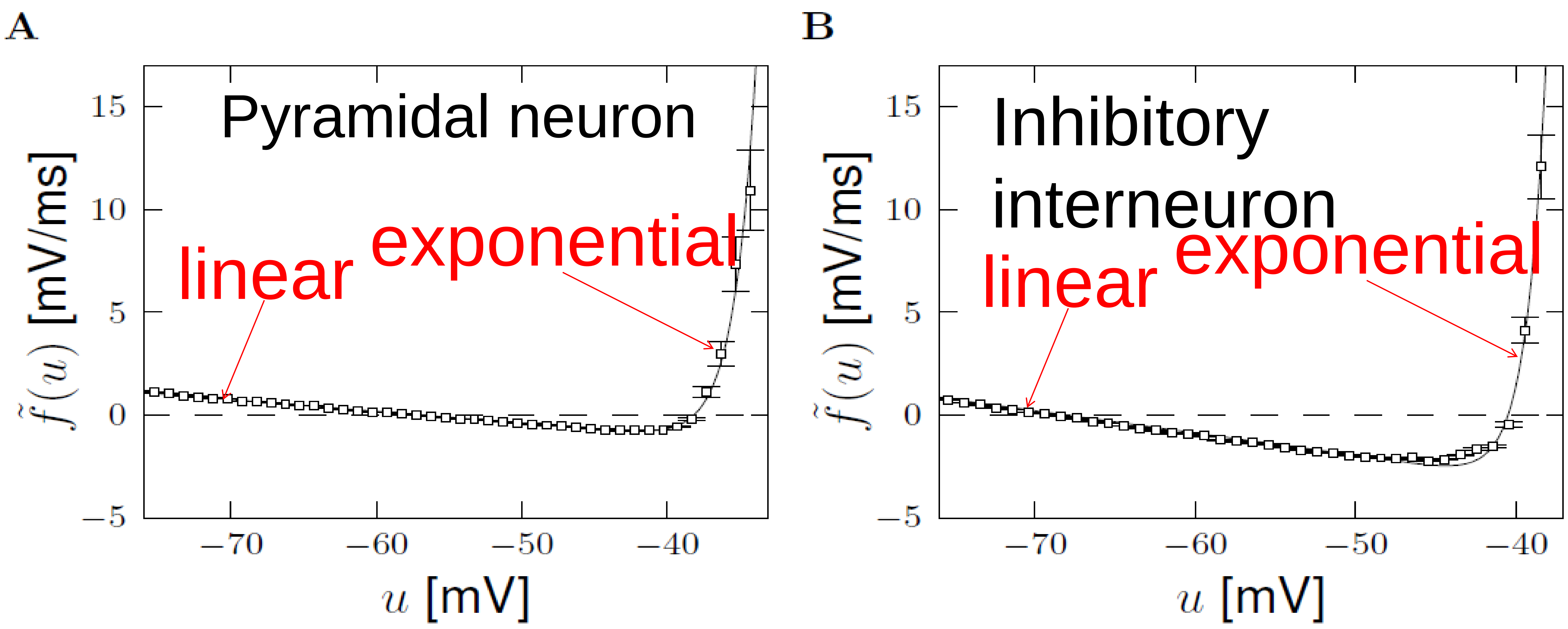
(ii) Extract *f* from data *Badel et al. (2008)*

$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$\tilde{f}(u) = \frac{f(u)}{\tau}$$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right)$$

Exp. Integrate-and-Fire, *Fourcaud et al. 2003*



Badel et al. (2008)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $\tau \frac{du}{dt} = f(u) + R\,I(t)$

(2) $If \;\; u = \theta_{reset} \;\; then \;\; reset\, to\; u = u_r$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta})$$

BUT: **Limitations – need to add**

-Adaptation on slower time scales

-Possibility for a diversity of firing patterns

-Increased threshold ϑ after each spike

-Noise

Week 9 – part 2 : Adaptive Expontential Integrate-and-Fire Model

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

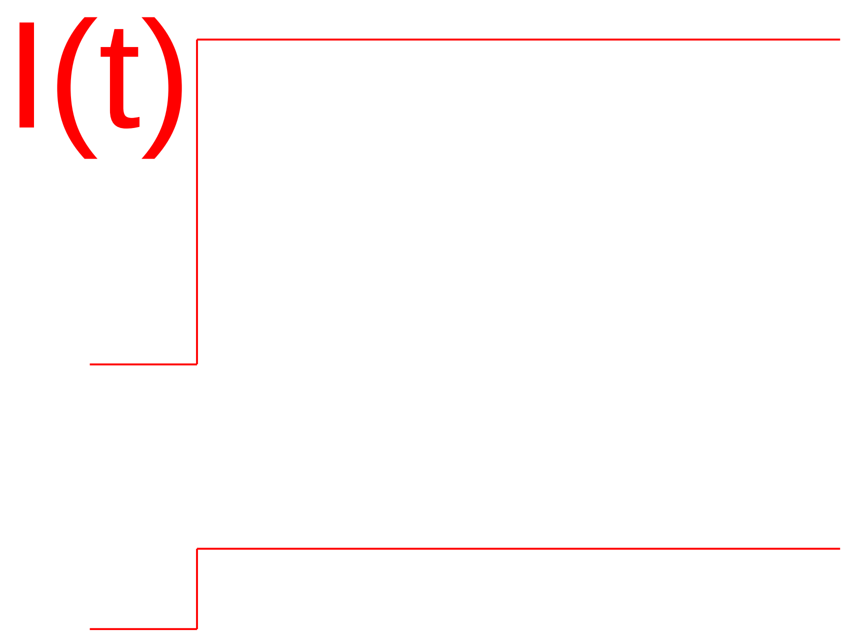
9.3 Spike Response Model (SRM)
- Integral formulation

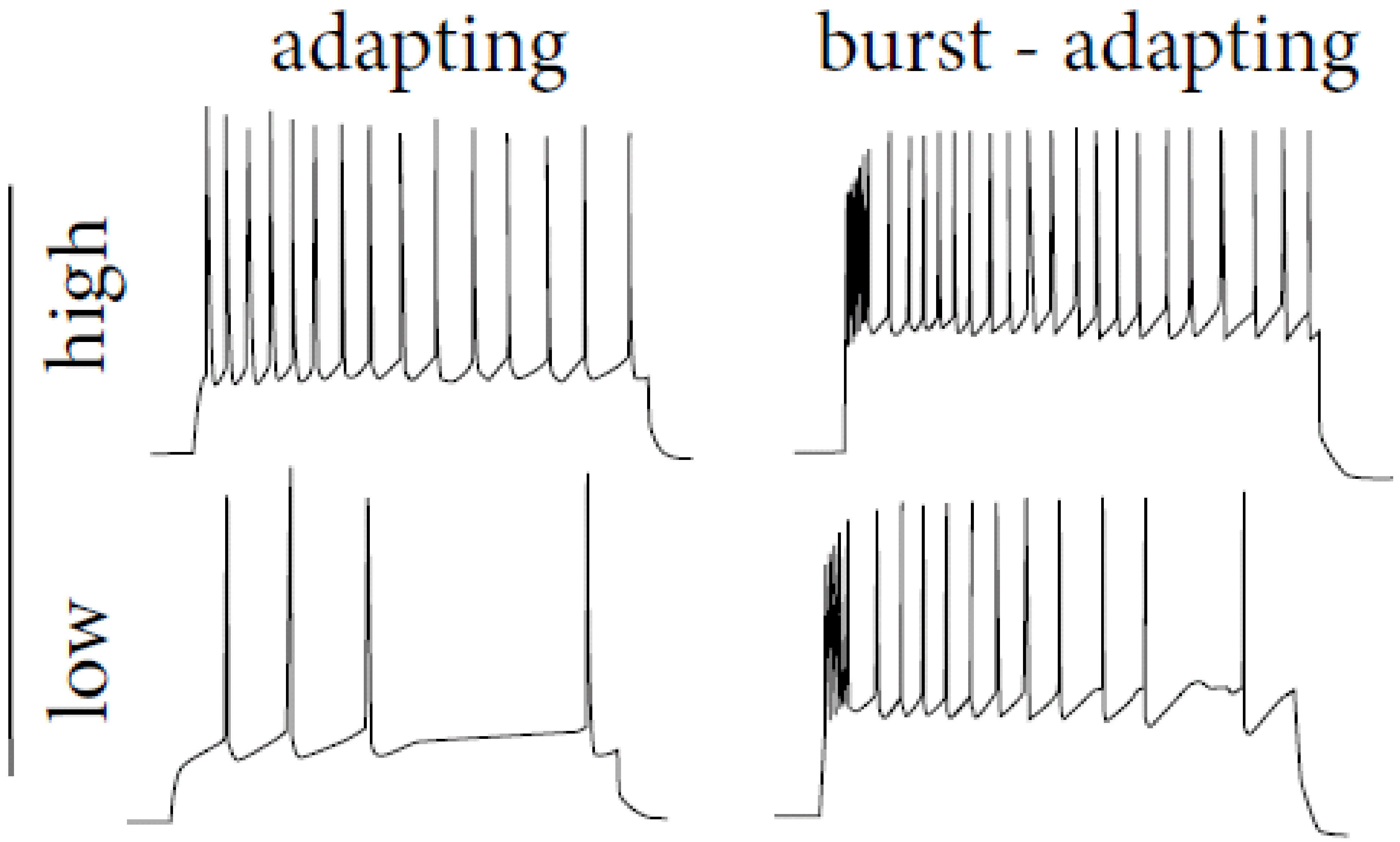
9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Step current input – neurons show adaptation





Data:
Markram et al.
(2004)

1-dimensional (nonlinear) integrate-and-fire model cannot do this!

Add adaptation variables:

Blackboard !

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) - R\sum_k w_k$$

Exponential I&F
+ **1 adaptation** var.
= AdEx

$$\tau_k \frac{dw_k}{dt} = a_k(u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

SPIKE AND RESET

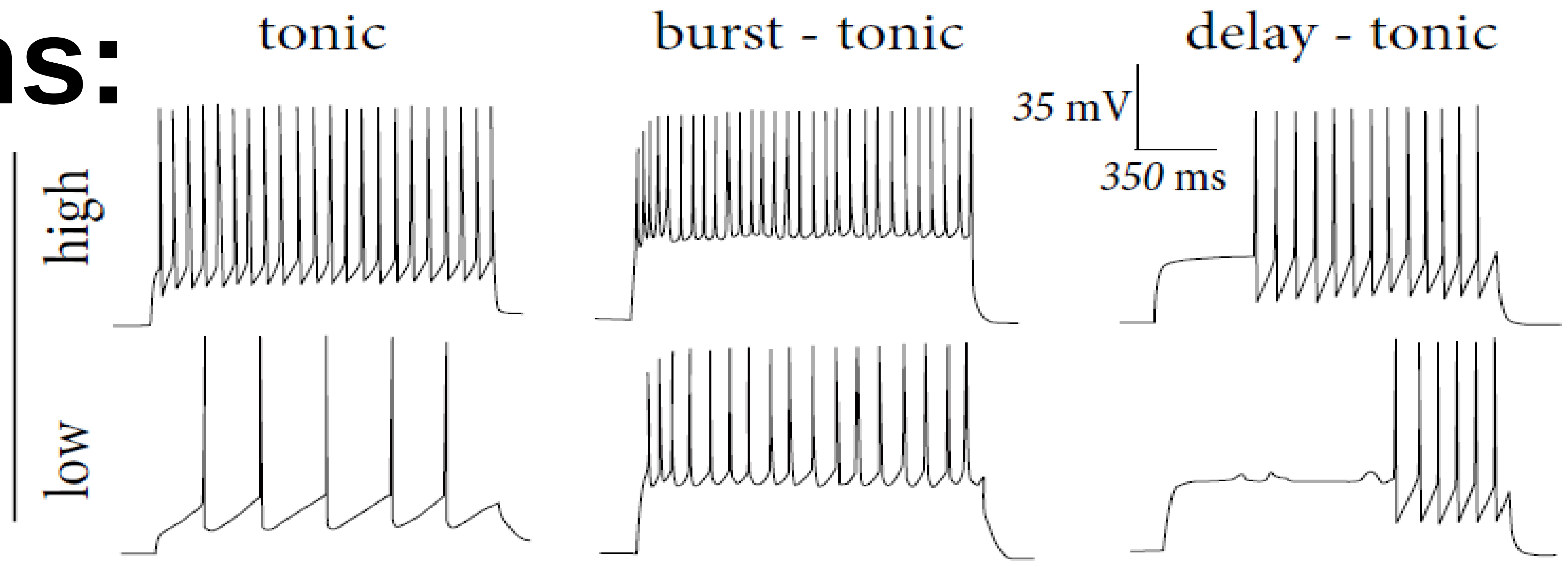
after each spike w_k jumps by an amount b_k

$$\text{If } \; u = \theta_{reset} \; \text{ then } \; \text{reset to } u = u_r$$

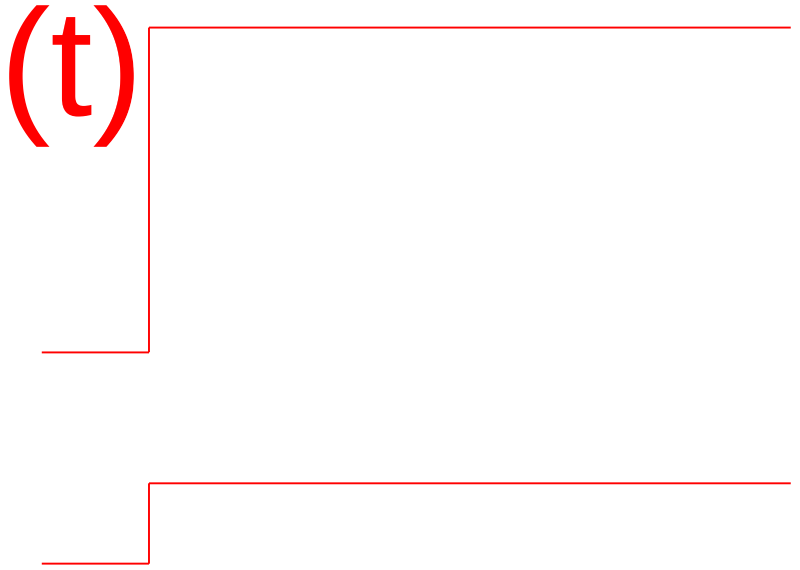
AdEx model, Brette&Gerstner (2005):

Firing patterns:

Response to
Step currents,
Exper. Data,
Markram et al.
(2004)

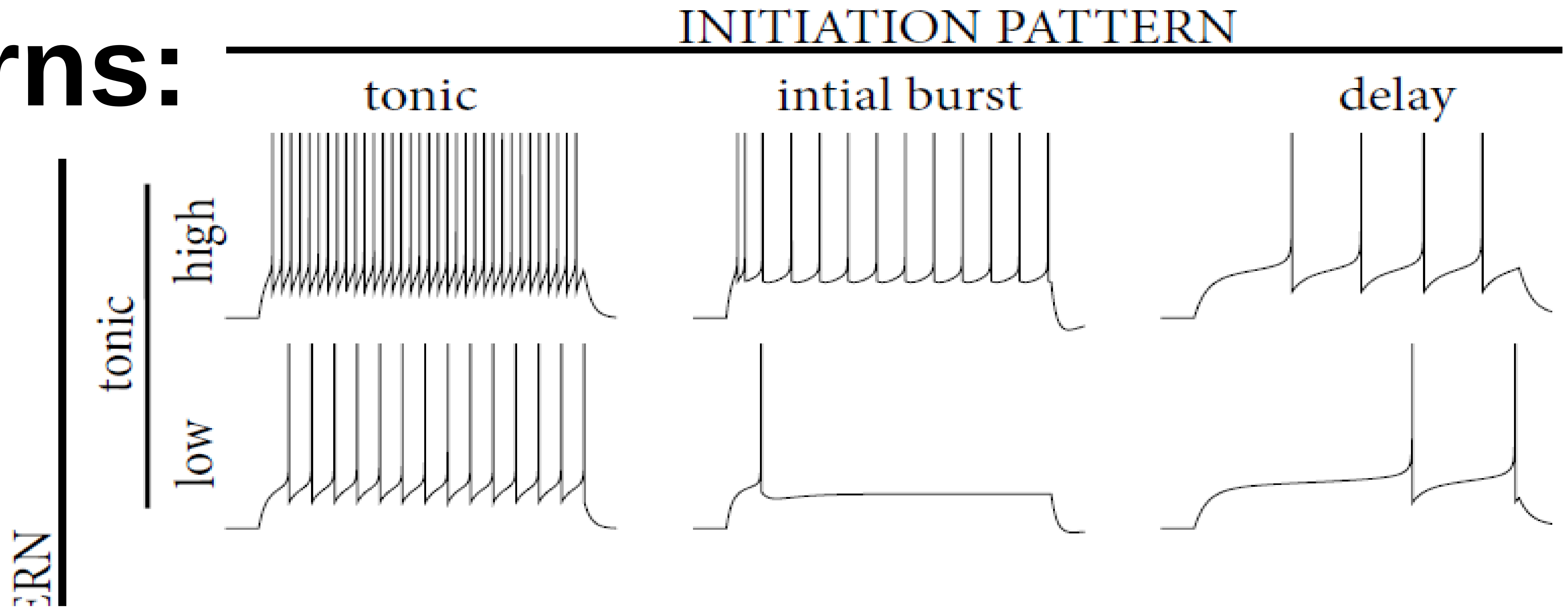


I(t)



Firing patterns:

Response to
Step currents,
AdEx Model,
Naud&Gerstner



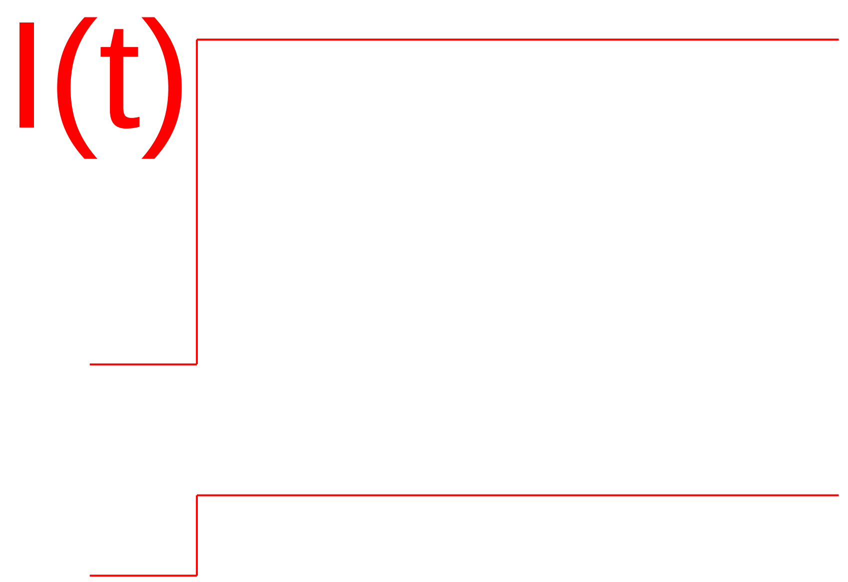


Image:
Neuronal Dynamics,
Gerstner et al.
Cambridge (2002)

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

AdEx model

Phase plane analysis!

Can we understand the different firing patterns?

Neuronal Dynamics – Quiz 9.1. Nullclines of AdEx

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w$$

A - What is the qualitative shape of the w-nullcline?

[ ] constant
[ ] linear, slope a
[ ] linear, slope 1
[ ] linear + quadratic
[ ] linear + exponential

B - What is the qualitative shape of the u-nullcline?

[ ] linear, slope 1
[ ] linear, slope 1/R
[ ] linear + quadratic
[ ] linear w. slope 1/R+ exponential

3 minutes
Restart at 9:40

Week 9 – part 2b : Firing Patterns

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

AdEx model

after each spike
u is reset to **u_r**

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\,(u - u_{rest}) - w + b\,\tau_w \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount ***b***

parameter a – **slope of w-nullcline**

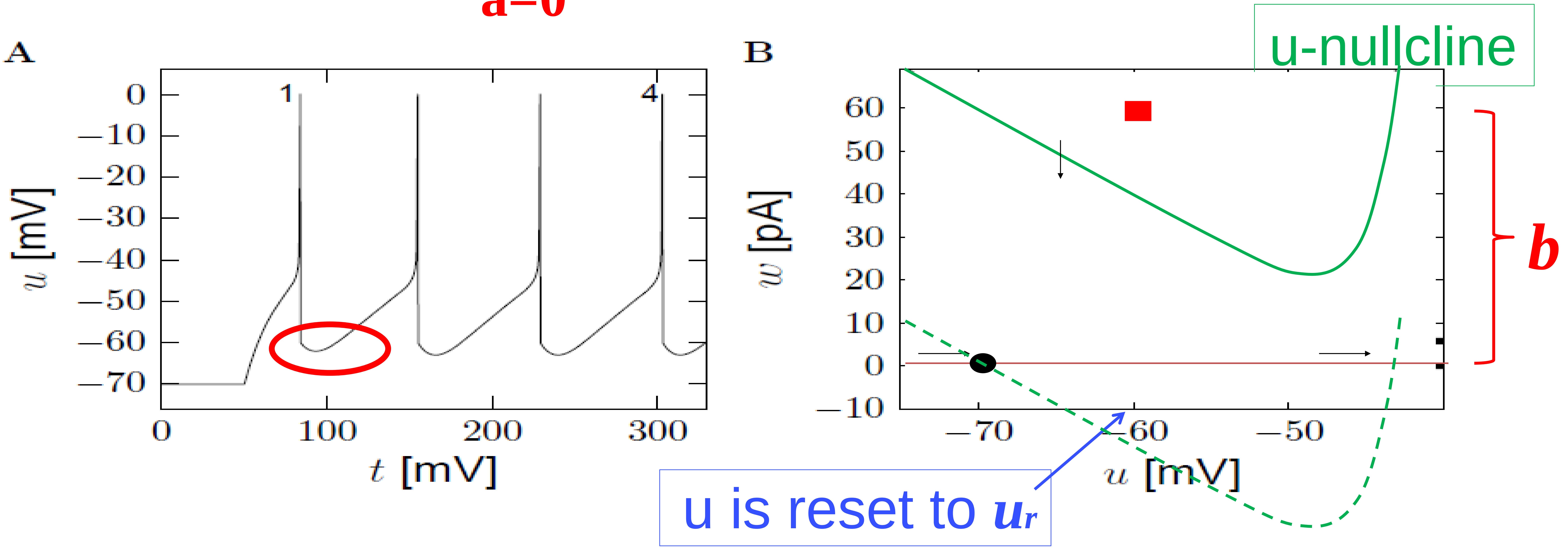
Can we understand the different firing patterns?

AdEx model – phase plane analysis: large *b*

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = \cancel{a\ (u - u_{rest})} - w + b\ \tau_w \sum_f \delta(t - t^f)$$

a=0

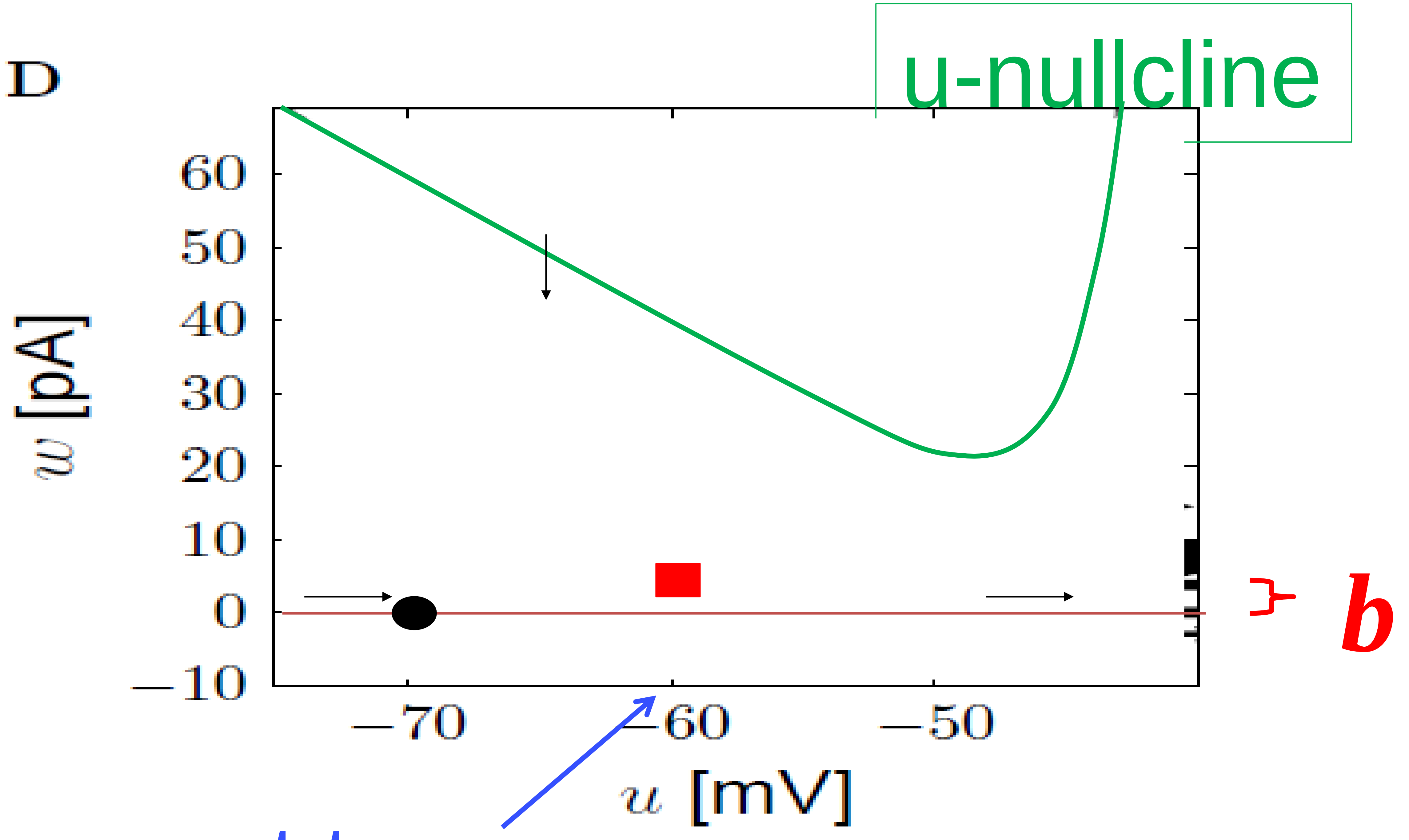


AdEx model – phase plane analysis: small b

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = \cancel{a\,(u - u_{rest})} - w + b\,\tau_w \sum_f \delta(t - t^f)$$

adaptation



u is reset to u_r

Quiz 9.2: AdEx model – phase plane analysis

$\tau_w >> \tau$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) + b\ \tau_w \sum_f \delta(t - t^f)$$

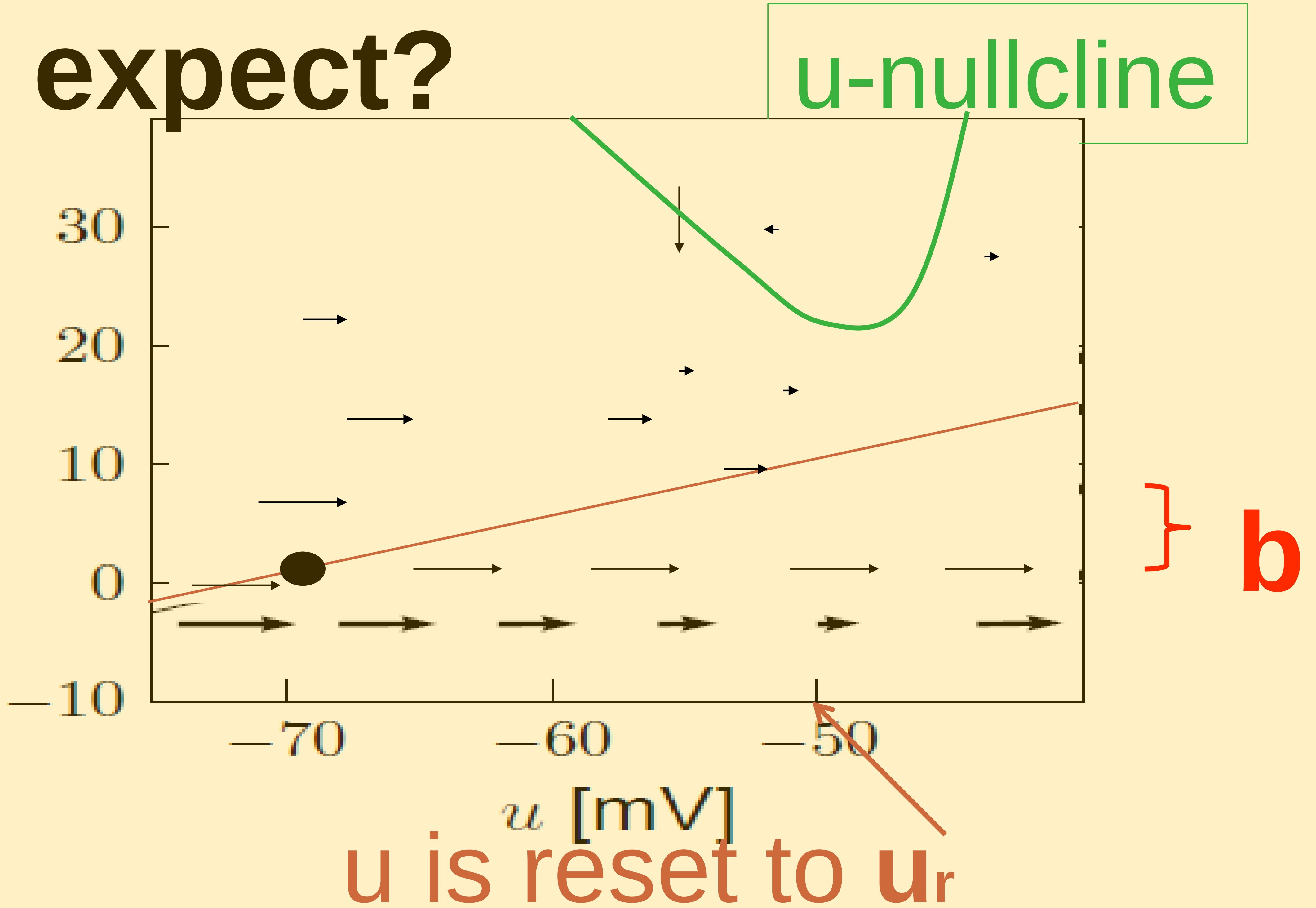
What firing pattern do you expect?

(i) Adapting

(ii) Bursting

(iii) Initial burst

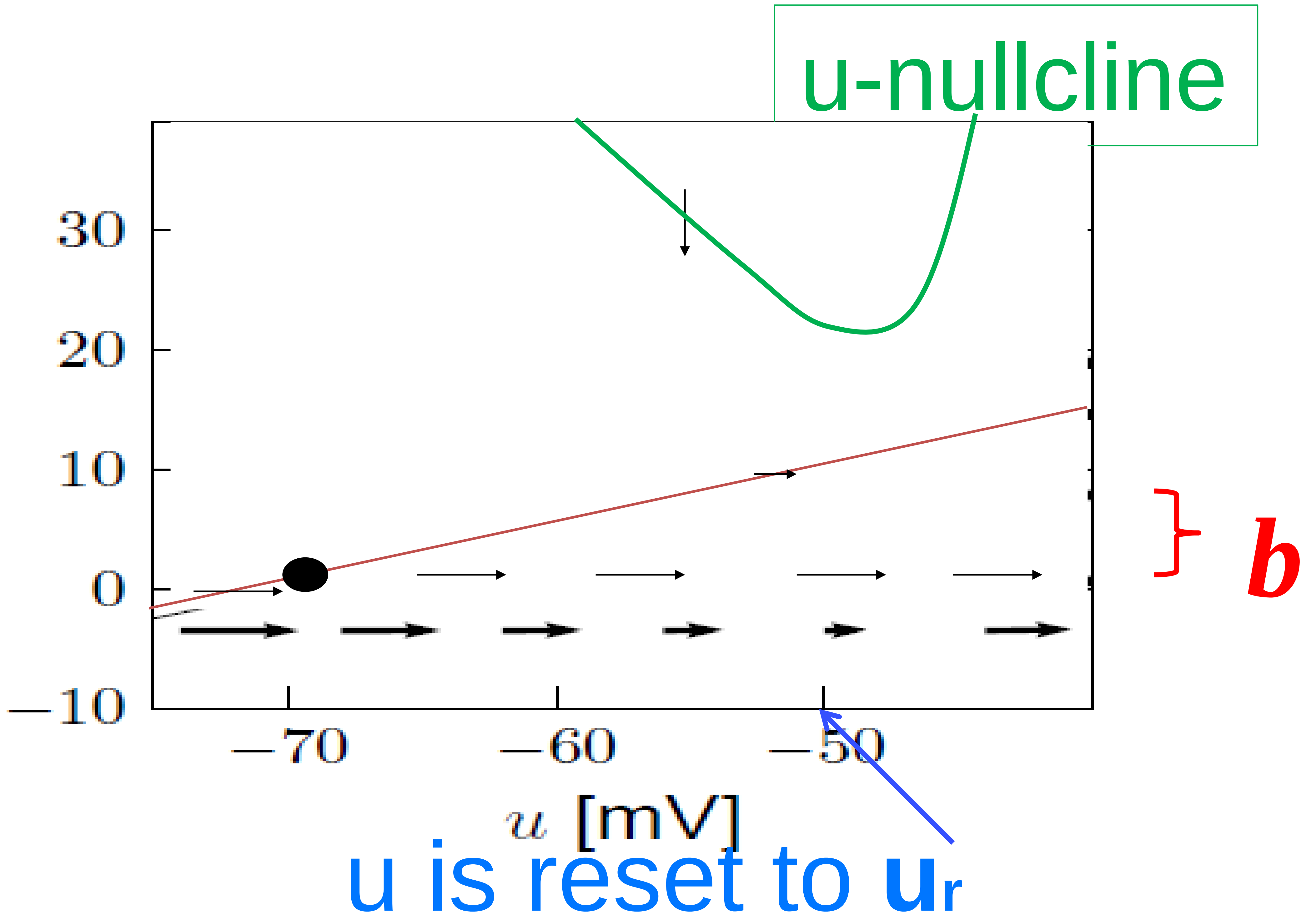
(iv)Non-adapting



AdEx model – phase plane analysis: a>0

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u-\vartheta}{\Delta}) + w + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$



after each spike u is reset to **u**$_r$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) - Rw + RI(t)$$

$$\tau_w \frac{dw}{dt} = a\ (u - u_{rest}) - w + b\ \tau_w \sum_f \delta(t - t^f)$$

after each spike
w jumps by an amount **b**

Blackboard:
Copy equations

parameter **a – slope of** w **nullcline**

Firing patterns arise from different parameters!

See Naud et al. (2008), see also Izikhevich (2003)

Neuronal Dynamics – Review: Nonlinear Integrate-and-fire

(1) $$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

(2) $$If \quad u = \theta_{reset} \quad then \quad reset\, to\; u = u_r$$

Best choice of ***f*** : linear + exponential

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp(\frac{u - \vartheta}{\Delta})$$

BUT: **Limitations – need to add**

✓ -Adaptation on slower time scales

✓ -Possibility for a diversity of firing patterns

-Increased threshold ϑ after each spike

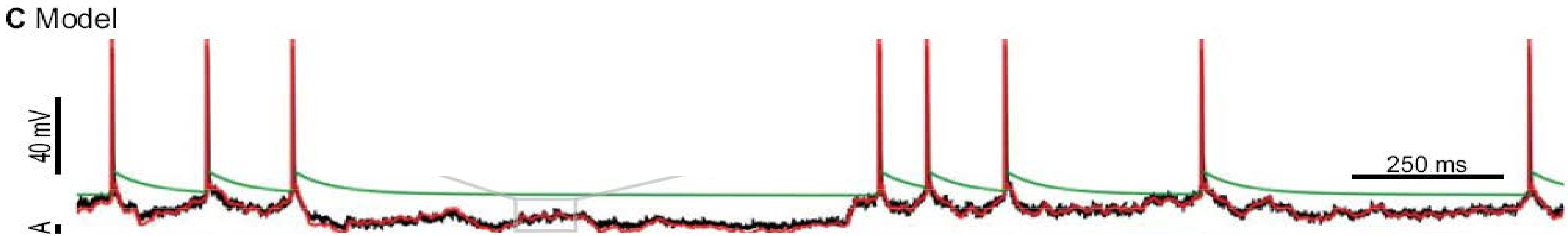
-Noise

Add dynamic threshold:

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) - R\sum_k w_k + RI(t)$$

Threshold increases after each spike

$$\boxed{\vartheta = \theta_0 + \sum_f \theta_1(t - t^f)}$$



$$\tau \frac{du}{dt} = f(u) + R\,I(t)$$

$$If \quad u = \theta_{reset} \quad then \quad reset\, to\ u = u_r$$

add

- ✓ -Adaptation variables
- ✓ -Possibility for firing patterns
- ✓ -Dynamic threshold ϑ
- -Noise

Use ‘escape noise’
(see earlier lecture)

Week 9 – part 3: Spike Response Model (SRM)

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

9.1 What is a good neuron model?
- Models and data

9.2 AdEx model
- Firing patterns and adaptation

9.3 Spike Response Model (SRM)
- Integral formulation

9.4 Generalized Linear Model
- Adding noise to the SRM

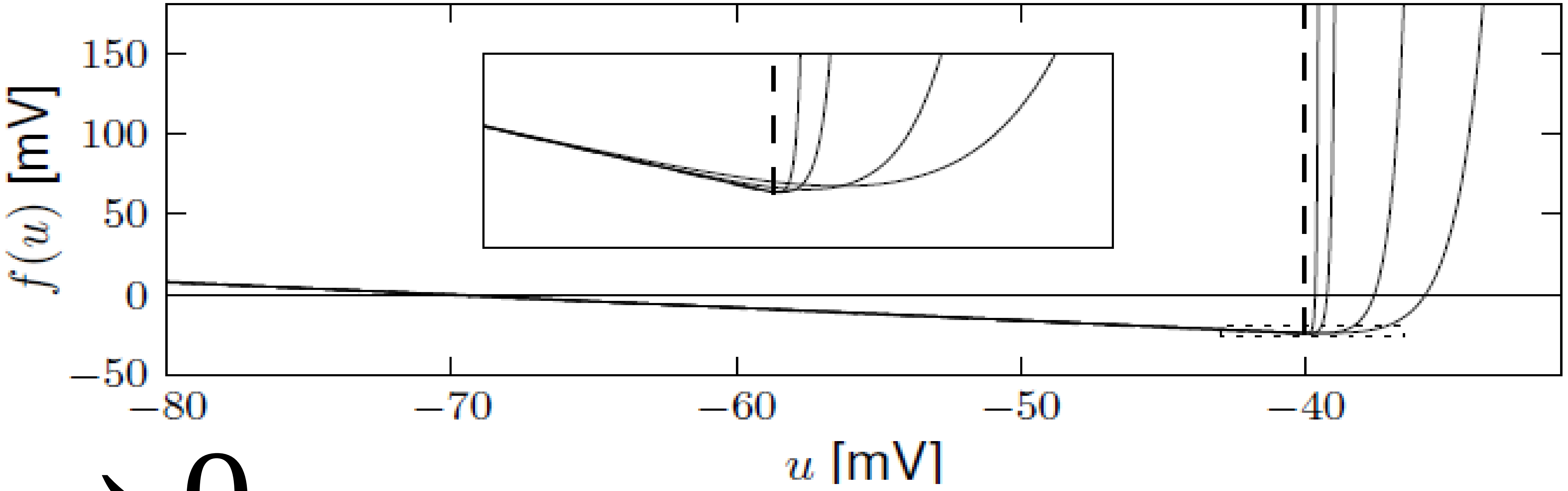
9.5 Parameter Estimation
- Quadratic and convex optimization

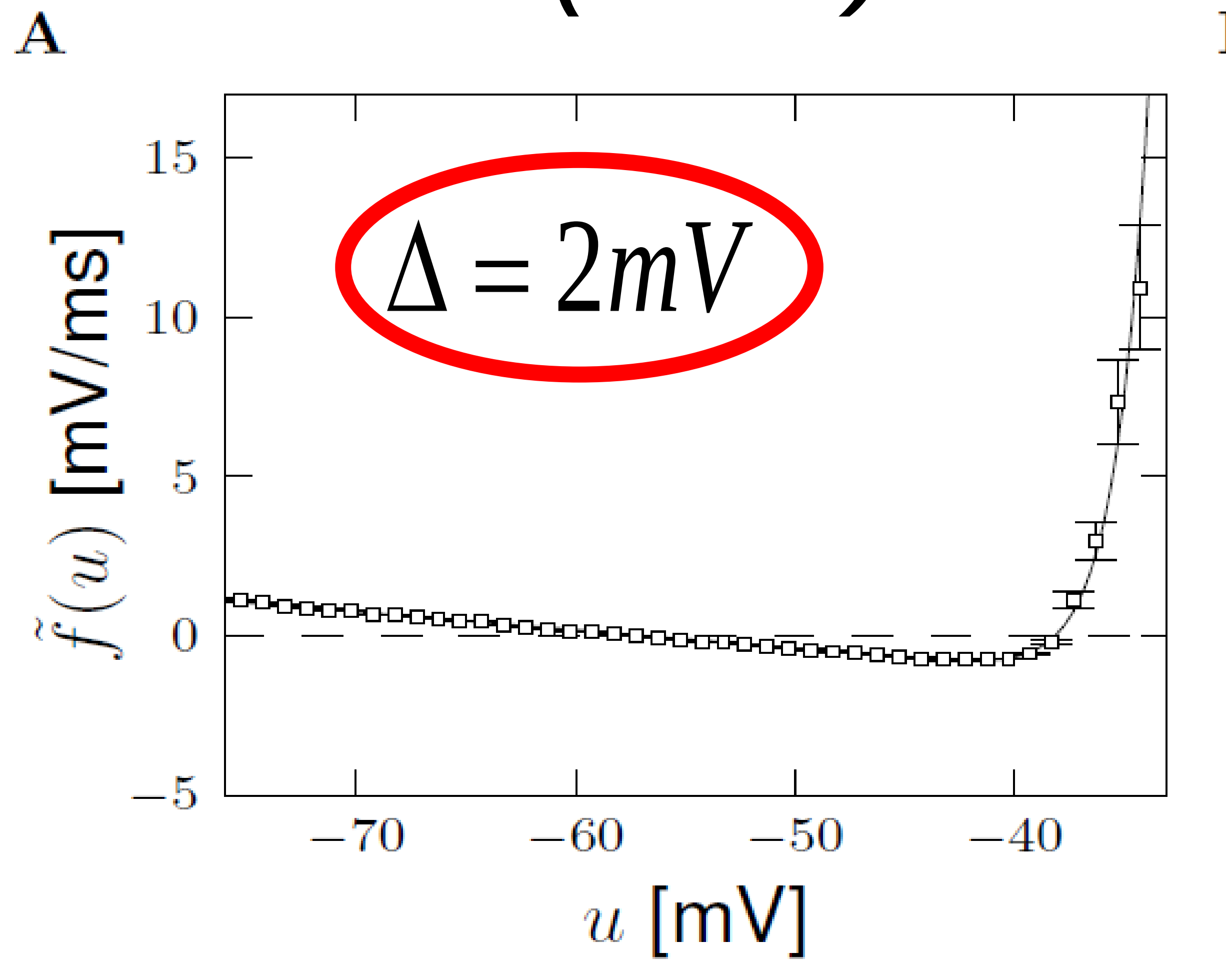
9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Exponential versus Leaky Integrate-and-Fire

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u - \vartheta}{\Delta}\right) + RI(t)$$

Badel et al (2008)





$$\Delta \to 0$$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + RI(t)$$

Leaky Integrate-and-Fire

Reset if u= ϑ

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k(u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

SPIKE AND RESET

after each spike

w_k jumps by an amount b_k

$If \quad u = \vartheta(t) \quad then \quad reset\, to\; u = u_r$

Dynamic threshold

Exercise 1: from adaptive IF to SRM

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\alpha w + RI(t), \qquad \alpha = \{0,1\}$$

$$If \quad u = \vartheta \ then \quad reset\, to \ u = u_r$$

$$\tau_w \frac{dw}{dt} = -w + b\,\tau_w \sum_f \delta(t - t^f)$$

Start before break

Next lecture at 10:20

Integrate the above system of two differential equations so as to rewrite the equations as

potential $$u(t) = \int_0^\infty \eta(s) S(t-s)\,ds + \int_0^\infty \varepsilon(s) I(t-s)\,ds + u_{rest}$$

Hint: voltage reset equivalent to short current pulse

A – what is $\eta(s)$? **(i)** $x(s) = \frac{R}{\tau}\exp(-\frac{s}{\tau})$ **(ii)** $x(s) = \frac{R}{\tau_w}\exp(-\frac{s}{\tau_w})$

B – what is $\varepsilon(s)$?

(iii) $x(s) = C[\exp(-\frac{s}{\tau}) - \exp(-\frac{s}{\tau_w})]$ **(iv) Combi of (i) + (iii)**

$$\tau \frac{du}{dt} = -(u - u_{rest}) - R\sum_k w_k + RI(t)$$

$$\tau_k \frac{dw_k}{dt} = a_k (u - u_{rest}) - w_k + b_k \tau_k \sum_f \delta(t - t^f)$$

Adaptive leaky I&F

Linear equation → can be integrated!

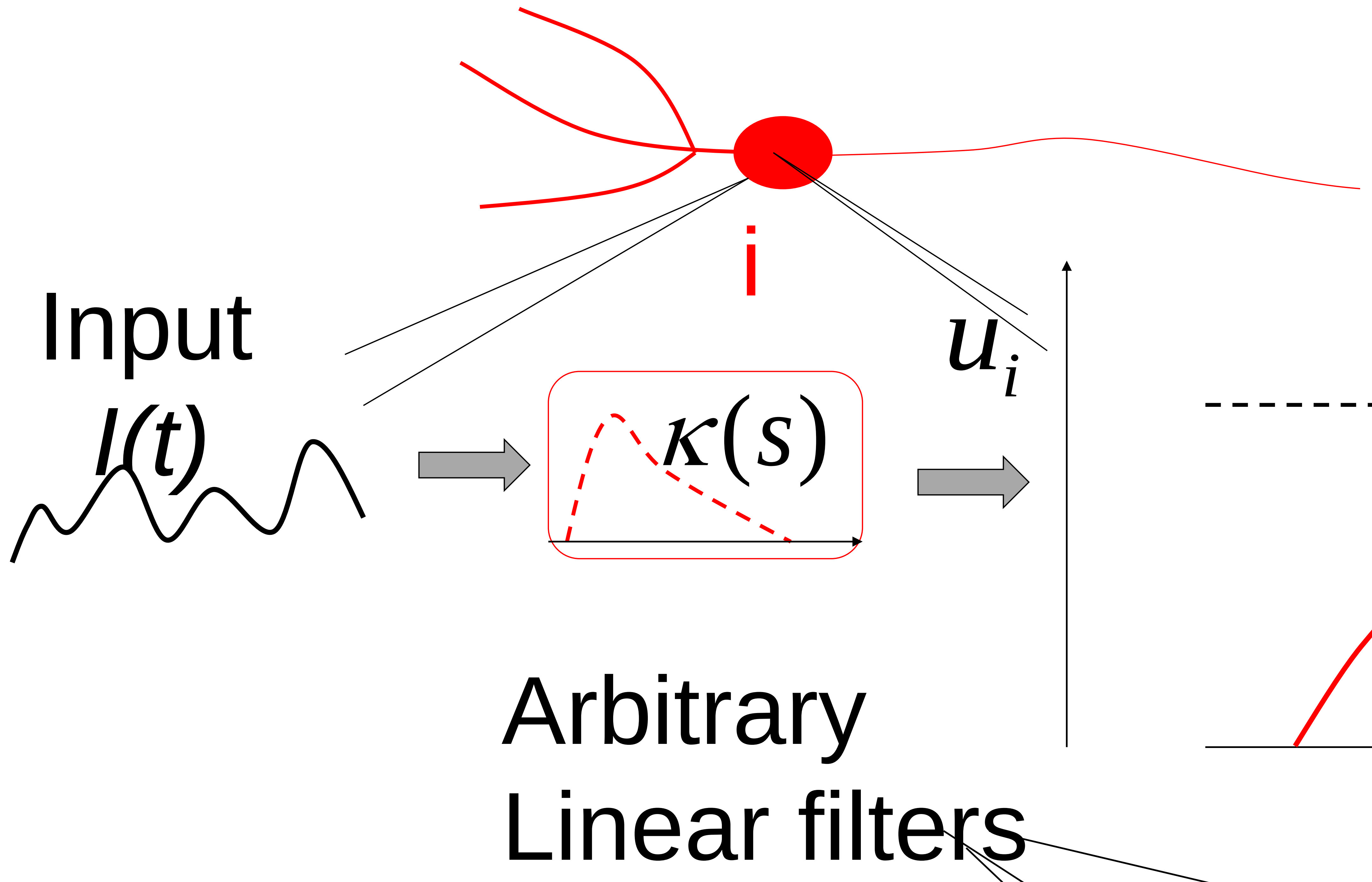
$$u(t) = \sum_f \eta\,(t - t^f) + \int_0^\infty ds\, \kappa(s) I(t - s)$$

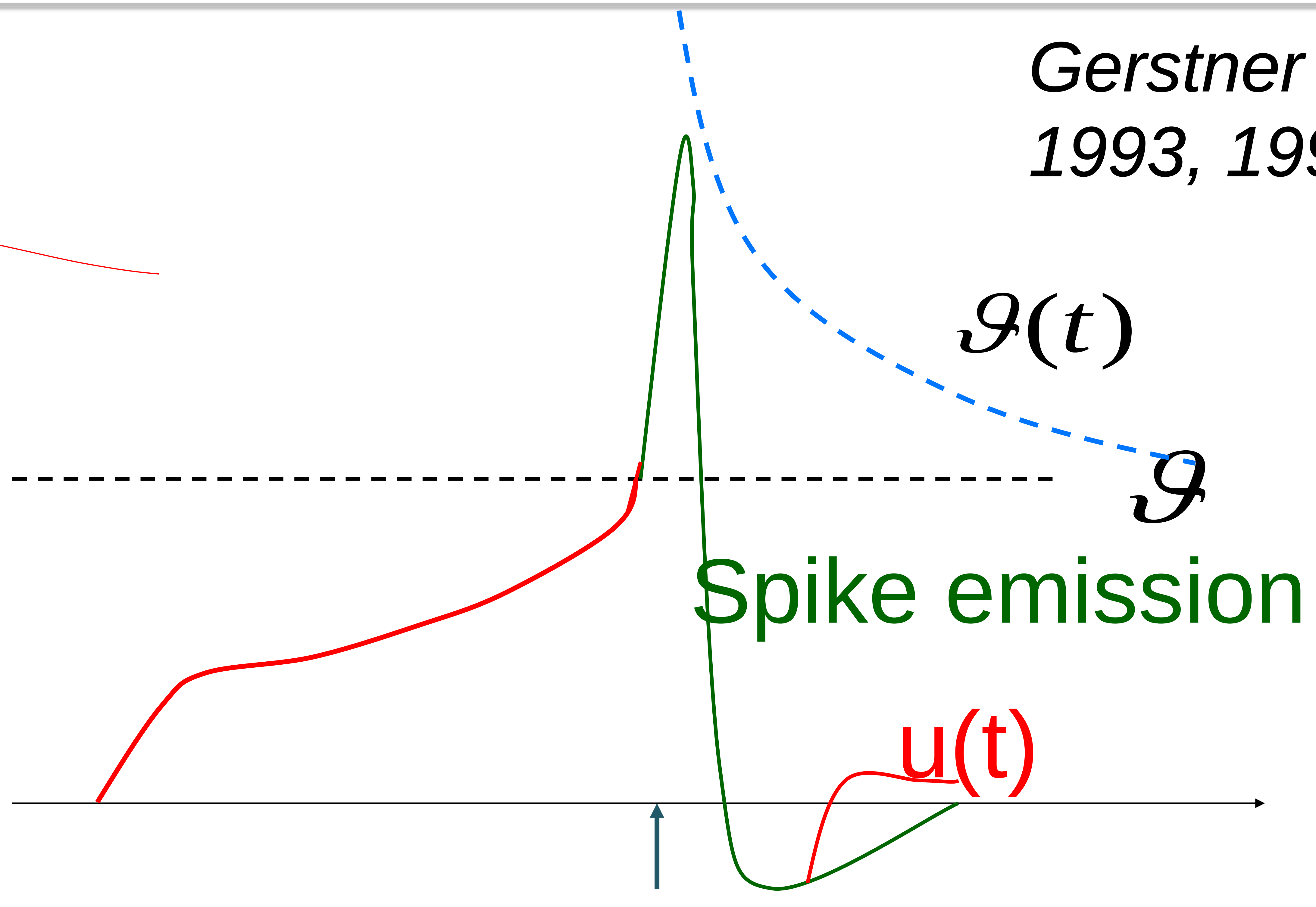
$$\vartheta(t) = \theta_0 + \sum_f \theta_1(t - t^f)$$

Spike Response Model (SRM)

Gerstner et al. (1996)

Gerstner et al., 1993, 1996





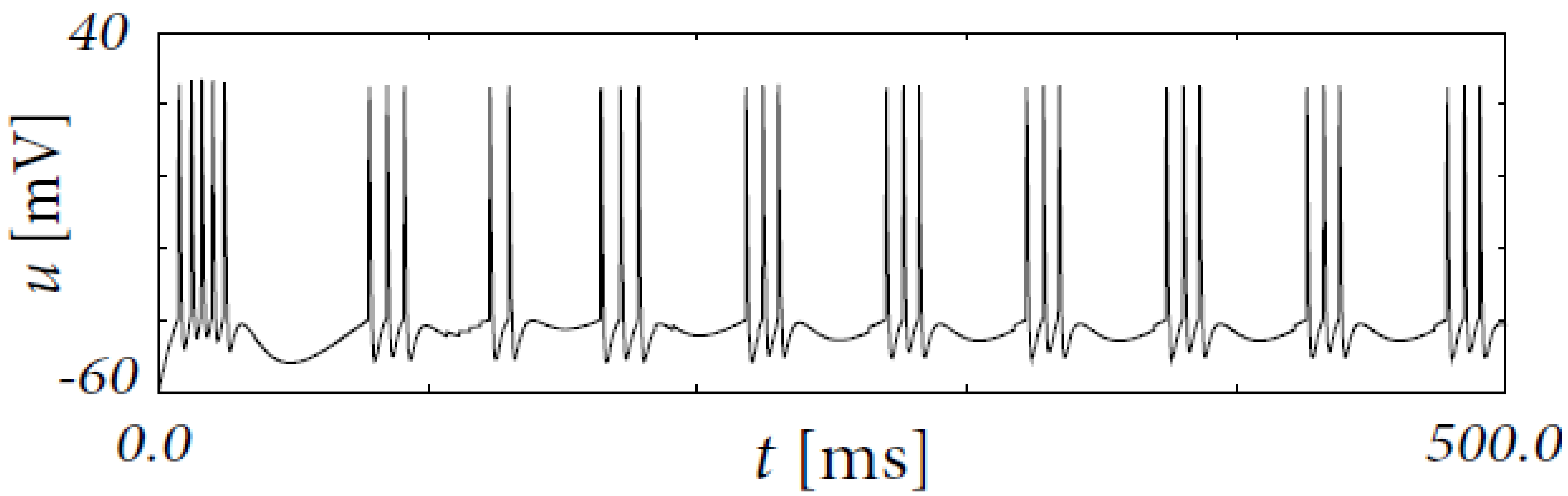
potential

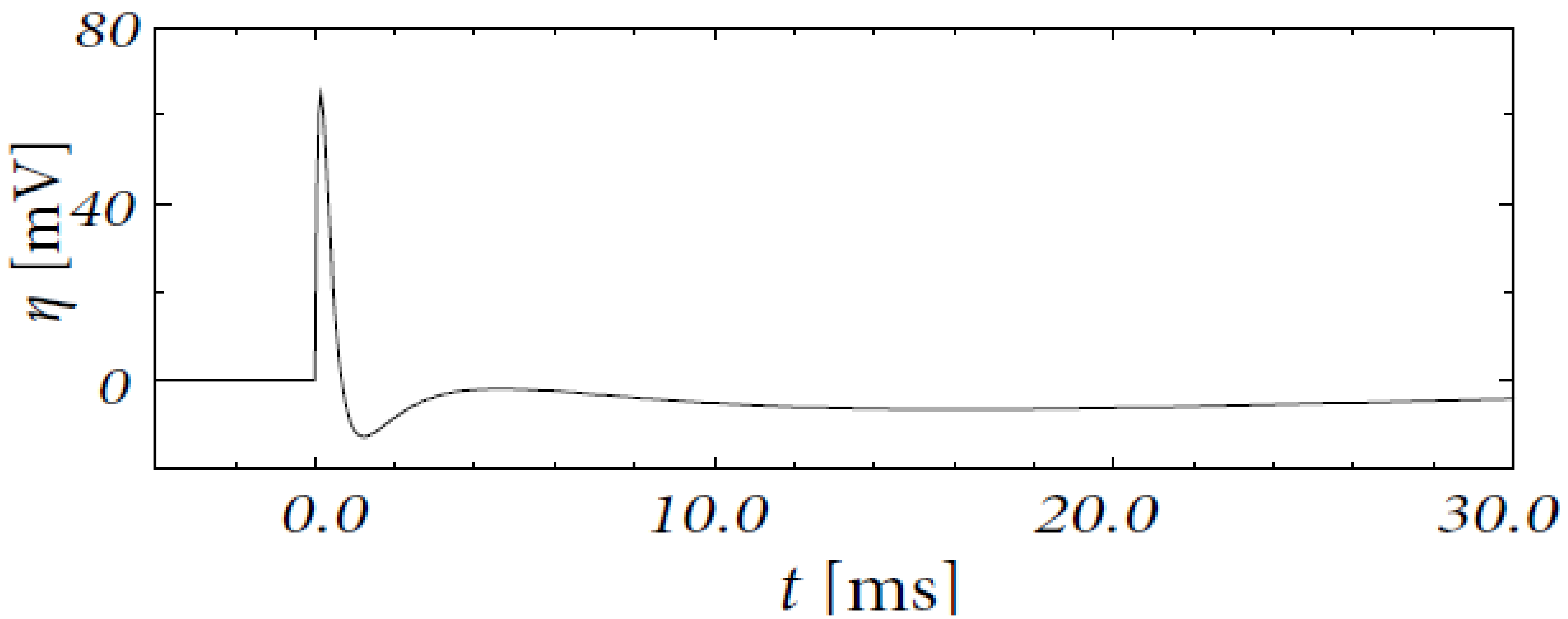
$$u(t)=\sum_{t'}\eta(t-t')+\int_0^\infty \kappa(s)I(t-s)ds+u_{rest}$$

threshold

$$\vartheta(t)=\theta_0+\sum_{t'}\theta_1(t-t')$$

SRM with appropriate η leads to bursting

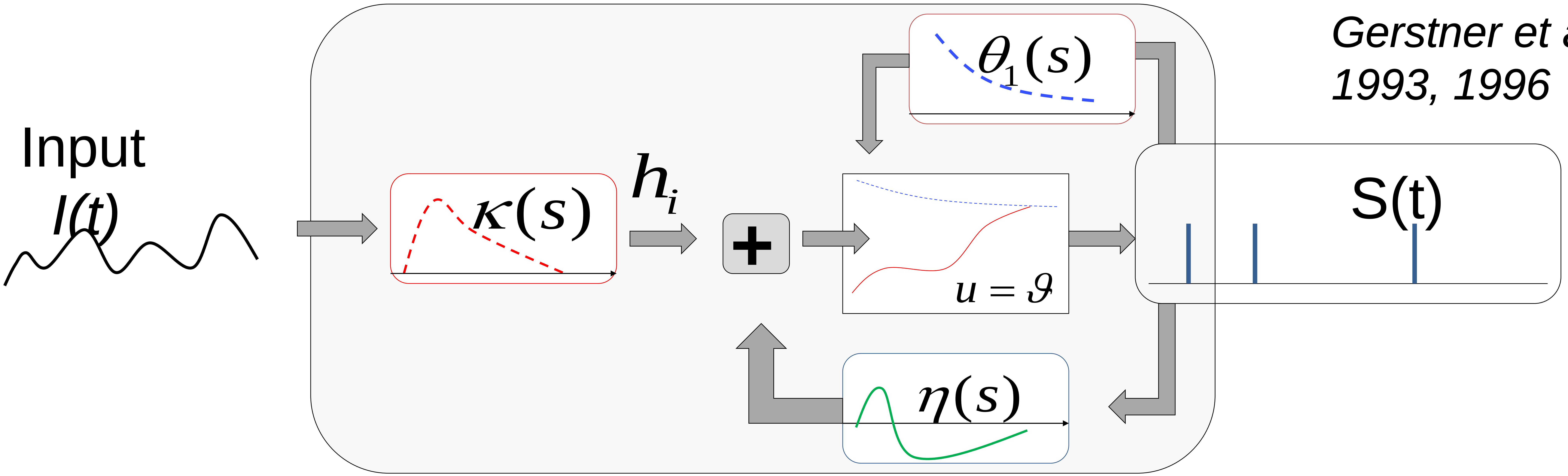




$$u(t) = \sum_f \eta\,(t - t^f) \; + \int_0^\infty ds\,\kappa(s) I(t-s) + u_{rest}$$

$$u(t) = \int_0^\infty ds\,\eta(s)\,S(t-s) + \int_0^\infty ds\,\kappa(s) I(t-s) + u_{rest}$$

Gerstner et al., 1993, 1996



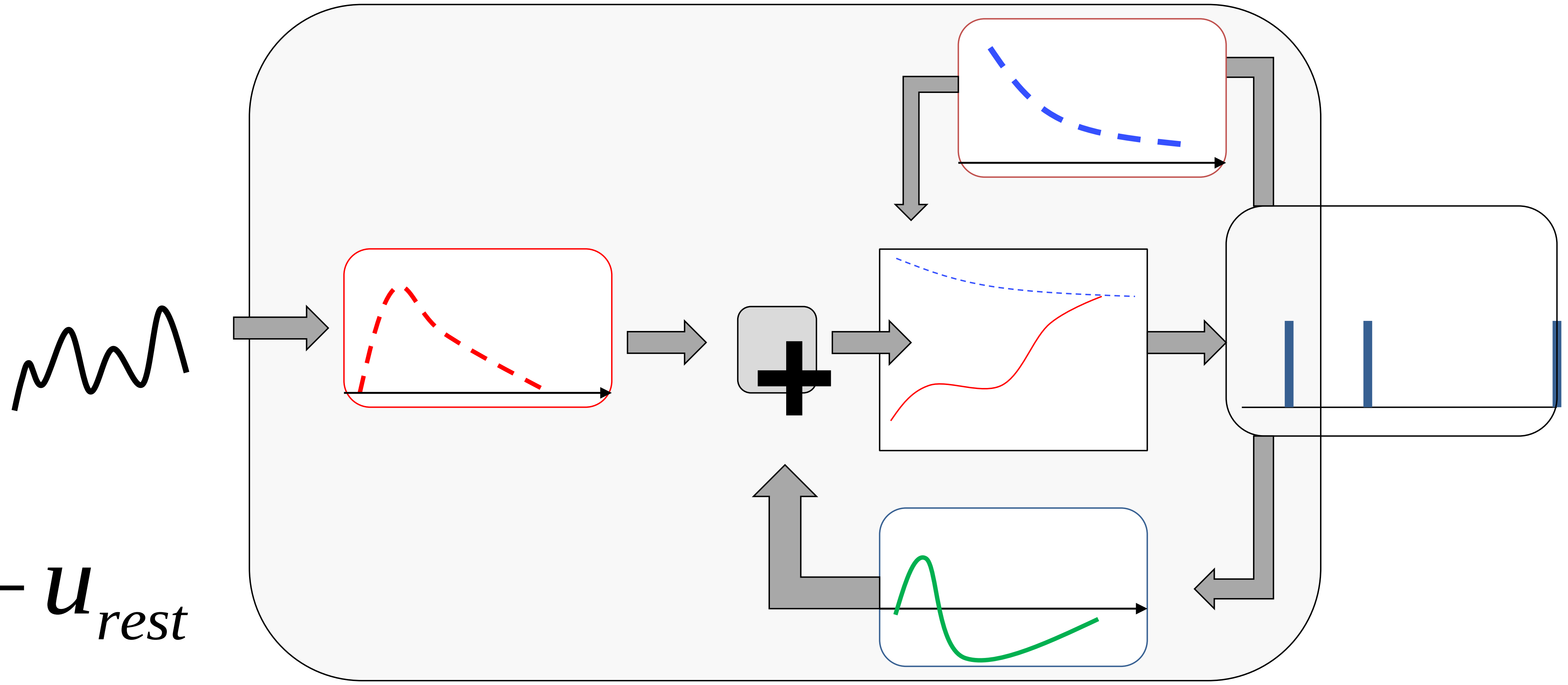
potential $$u(t) = \sum_{t'} \eta(t - t') + \int_0^{\infty} \kappa(s) I(t-s) ds + u_{rest}$$

threshold $$\vartheta(t) = \theta_0 + \sum_{t'} \theta_1(t - t')$$

firing if $$u(t) = \vartheta(t)$$

potential

$$u(t)=\sum_{t'} \underline{\eta(t-t')} + \int_0^\infty \underline{\kappa(s)} I(t-s)ds + u_{rest}$$



threshold

$$\vartheta(t)=\theta_0+\sum_{t'} \underline{\theta_1(t-t')}$$

Linear filters for
- input
- threshold
- refractoriness

Biological Modeling of Neural Networks:

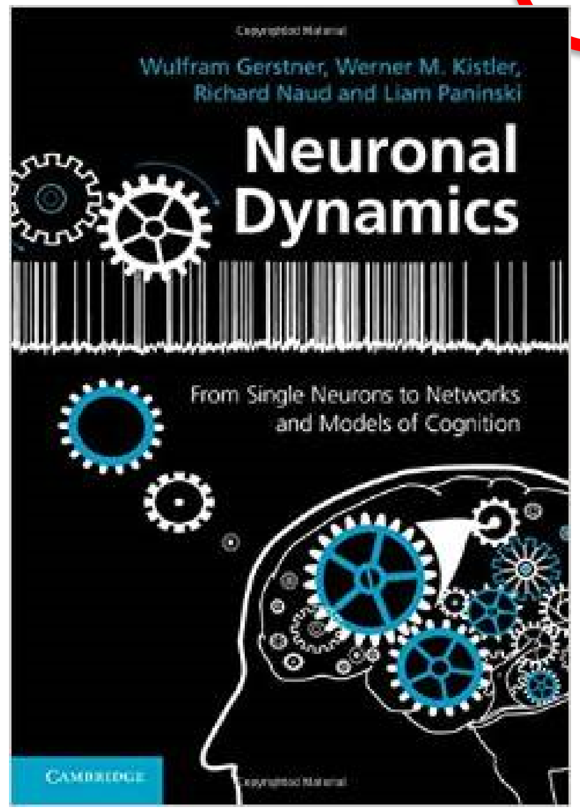
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for this week:
NEURONAL DYNAMICS
- Ch. 4.6, 6.1,6.2,6.4, 9.2
- Ch. 10.2.3, 11.1. 11.3.3



Cambridge Univ. Press

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

√ 9.3 Spike Response Model (SRM)
- Integral formulation

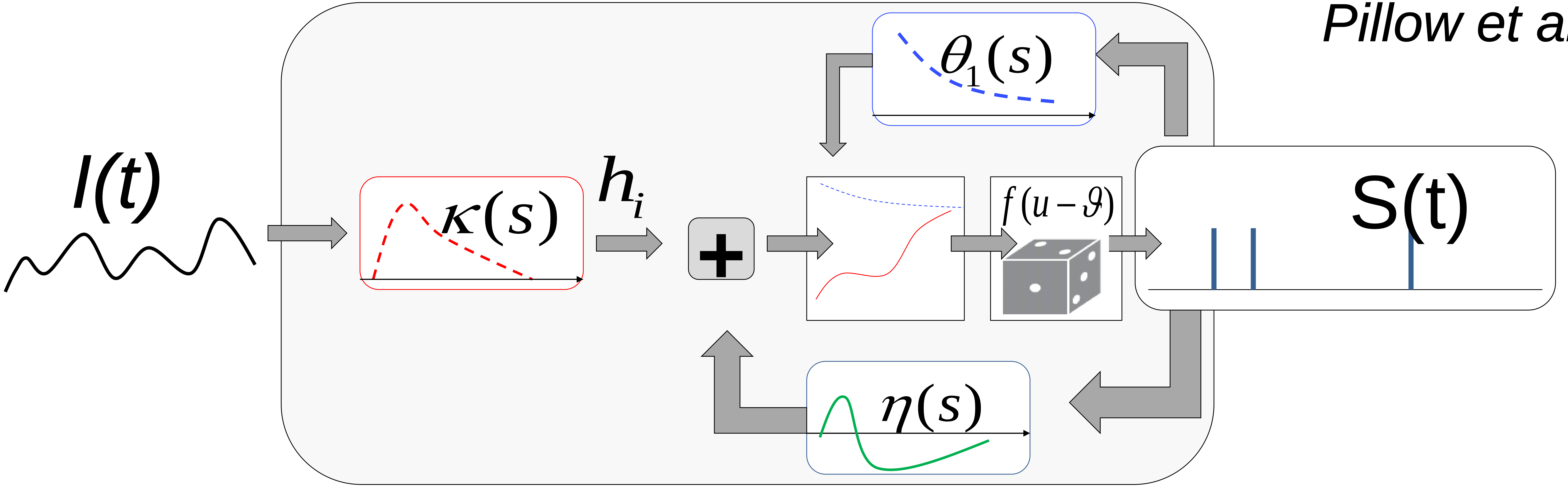
9.4 Generalized Linear Model
- Adding noise to the SRM
- Likelihood of a spike train

9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Spike Response Model (SRM)
Generalized Linear Model GLM

Gerstner et al., 1992,2000
Truccolo et al., 2005
Pillow et al. 2008



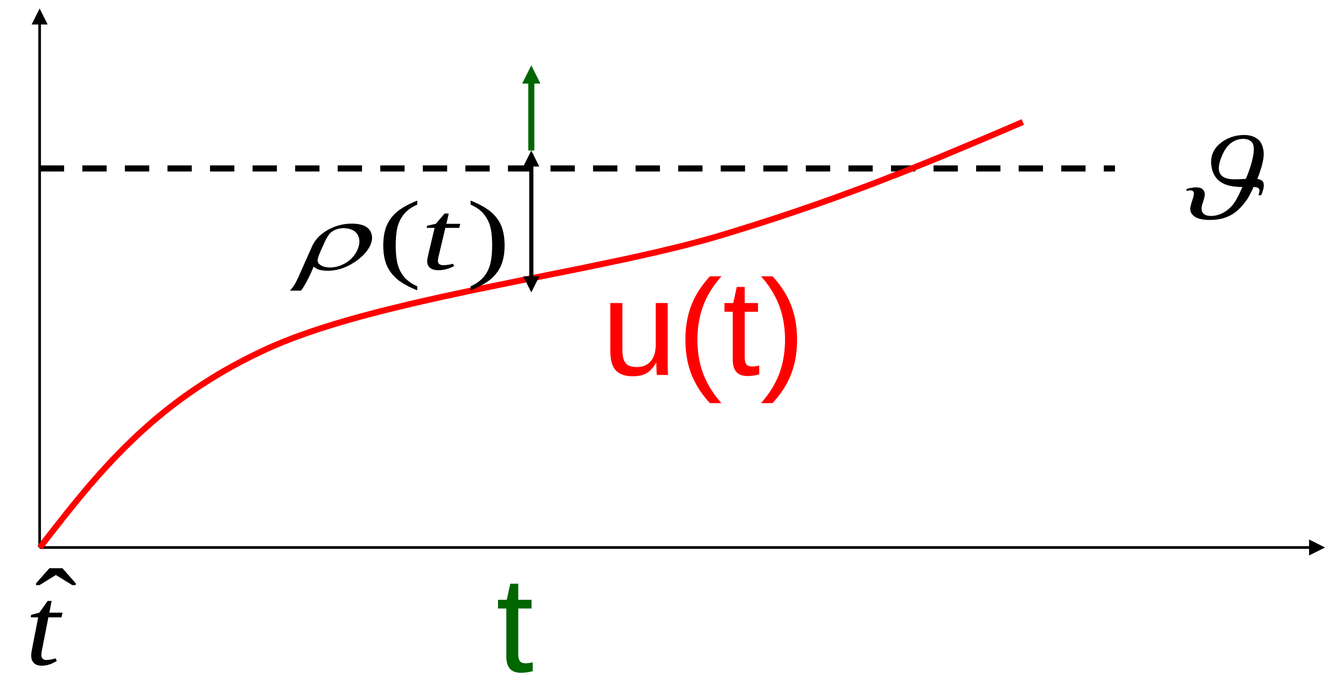
potential $u(t) = \int \eta(s) S(t-s)\,ds + \int_0^\infty \kappa(s) I(t-s)\,ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)\,ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Neuronal Dynamics – review: Escape noise

escape process

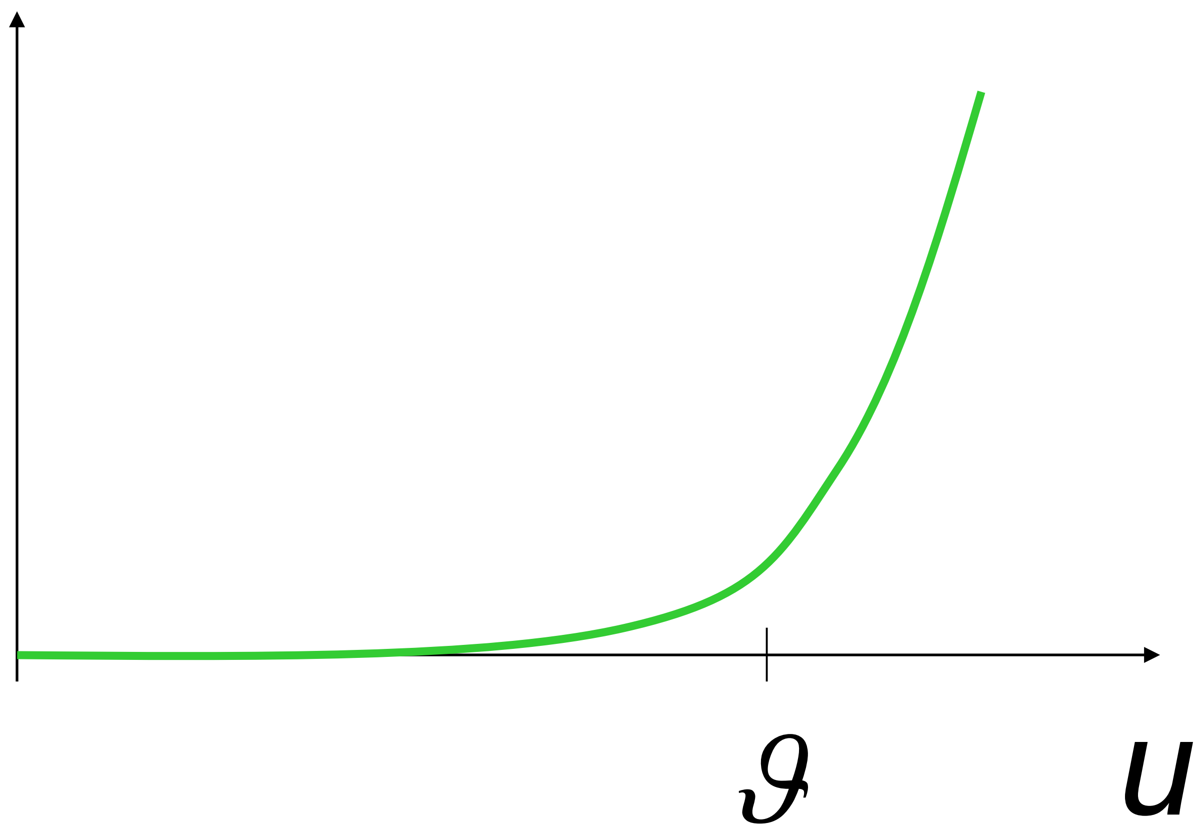


escape rate

$$\rho(t) = \rho_0 \exp\left(\frac{u(t)-\vartheta}{\Delta}\right)$$

escape rate

$$\rho(t) = f(u(t)-\vartheta)$$



Example: leaky integrate-and-fire model

$$\tau \cdot \frac{d}{dt} u = -(u - u_{rest}) + RI(t)$$

$$\text{if spike at } t^f \Rightarrow u\left(t^f + \delta\right) = u_r$$

Exerc. 2.1: Non-leaky IF with escape rates

$$\frac{du}{dt} = \frac{R}{\tau} I(t) = \frac{1}{C} I(t)$$ nonleaky

$$reset\ to\ u_r = 0$$

$$\tau \frac{du}{dt} = -(u - u_{rest}) + RI(t)$$

$$reset\ to\ u_{rest} = u_r = 0$$

Integrate for constant input (repetitive firing)

12 minutes,
Next lecture
at 10:55

Calculate

- potential $u(t-\hat{t})$

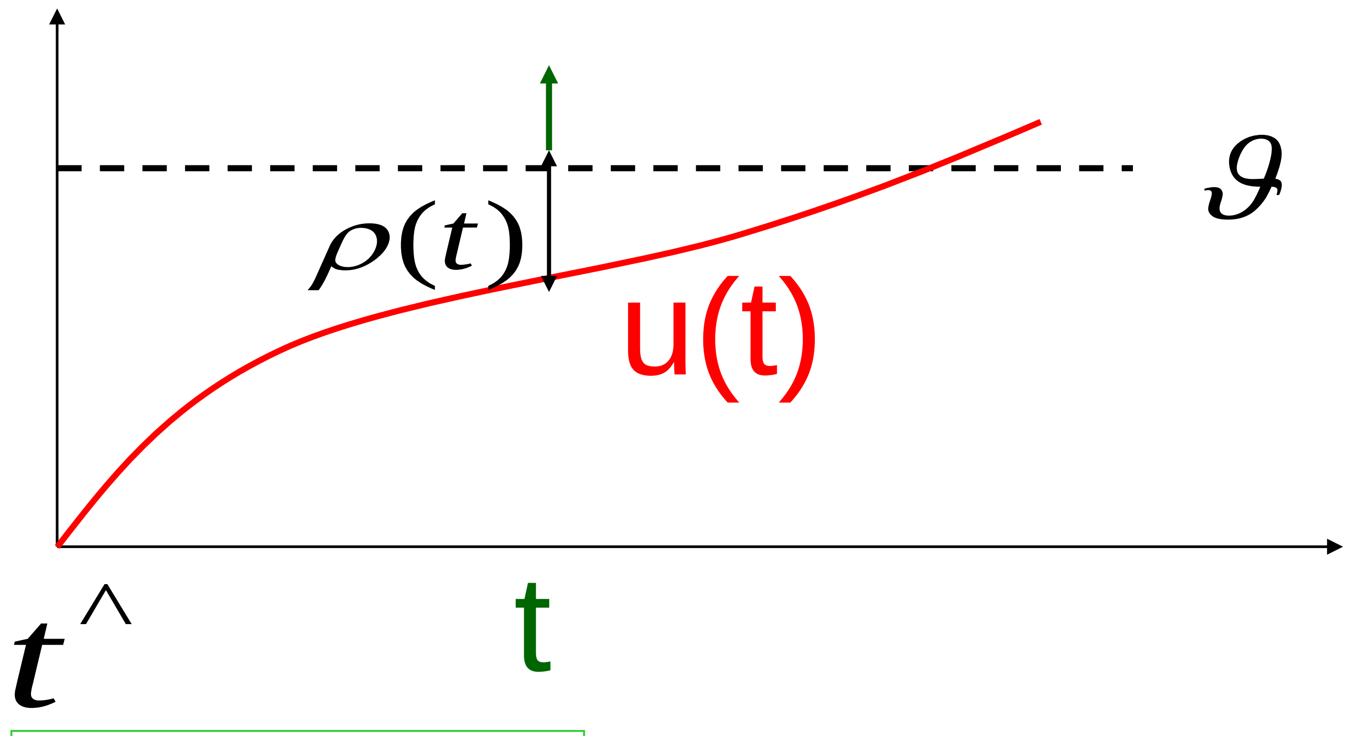
- hazard $\rho(t-\hat{t}) = \beta \cdot [u(t-\hat{t}) - \vartheta]_+$

- survivor function $S(t-\hat{t})$

- interval distrib. $P_0(t-\hat{t})$

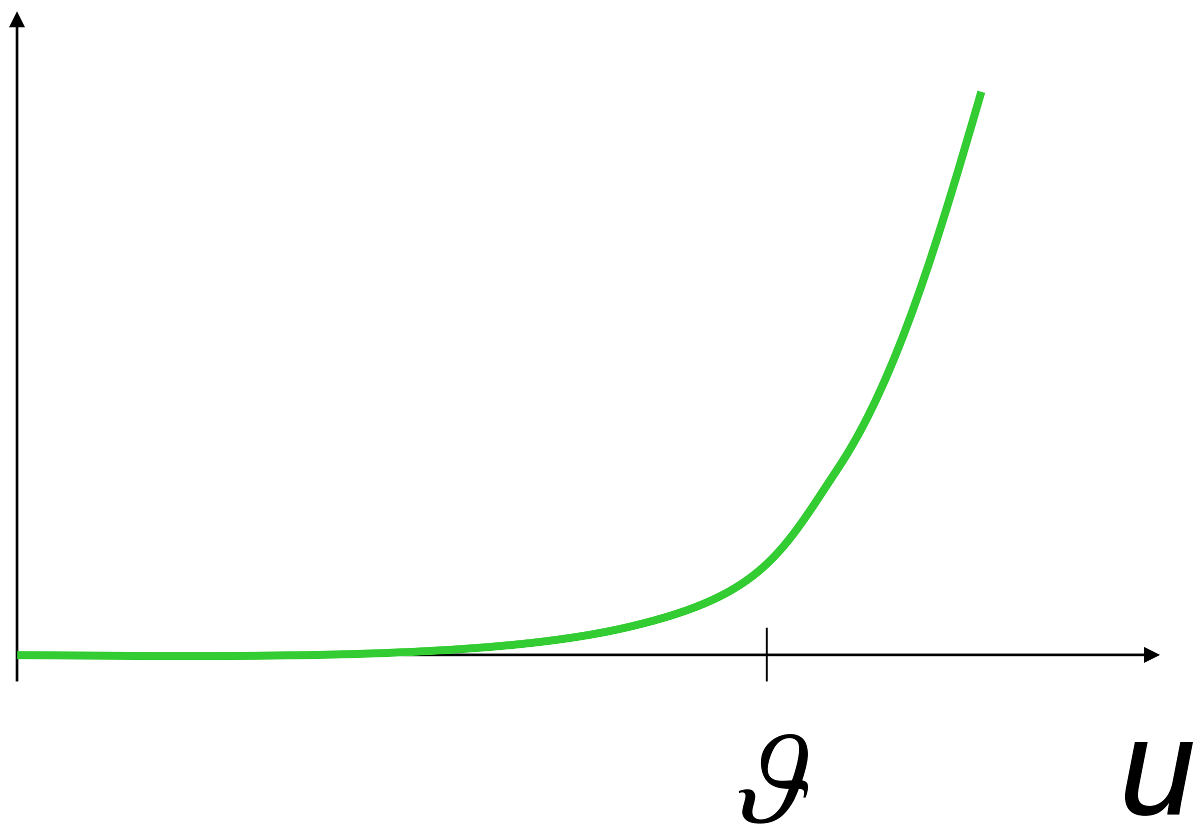
Neuronal Dynamics – review: Escape noise

escape process



escape rate

$$\rho(t) = f(u(t)-\vartheta(t))$$



Survivor function

$$\frac{d}{dt} S_I(t|\hat{t}) = -\rho(t)\, S_I(t|\hat{t})$$

$$S_I(t|\hat{t}) = \underbrace{\exp(-\int_{t^\wedge}^{t} \rho(t')dt')}$$

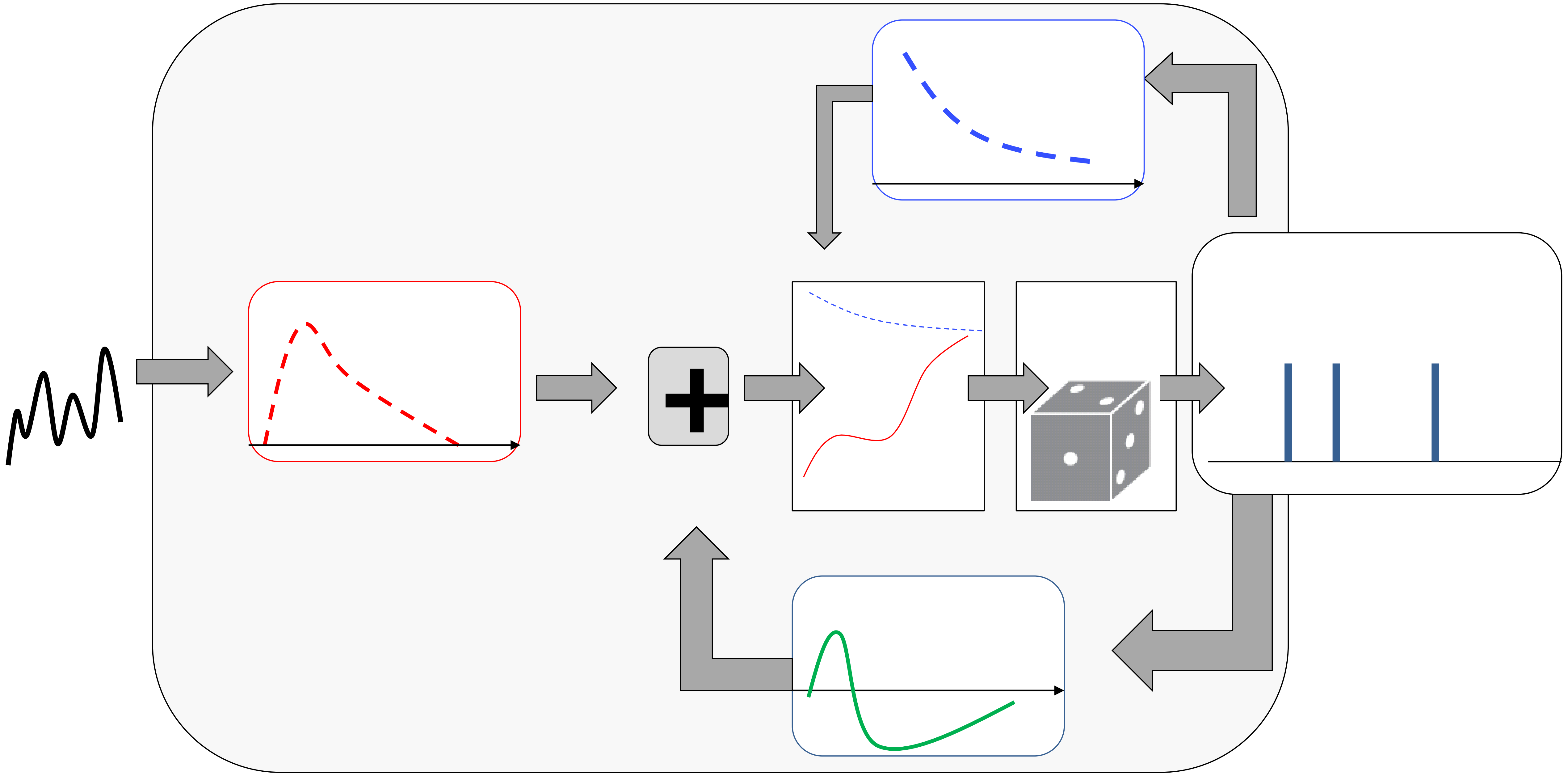
Interval distribution

$$P_I(t|\hat{t}) = \underbrace{\rho(t)}_{\text{escape rate}} \cdot \underbrace{\exp(-\int_{t^\wedge}^{t} \rho(t')dt')}_{\text{Survivor function}}$$

Good choice

$$\rho(t) = f(u(t)-\vartheta(t)) = \rho_0 \exp[\frac{u(t)-\vartheta(t)}{\Delta u}]$$

Neuronal Dynamics – Likelihood of spike train



-linear filters
-escape rate
→likelihood of observed spike train

Neuronal Dynamics – 9.4 Likelihood of a spike train in GLMs

$$S(t) = \sum_f \delta(t - t^f)$$

→Blackboard

$t^1, t^2, \ldots t^N$

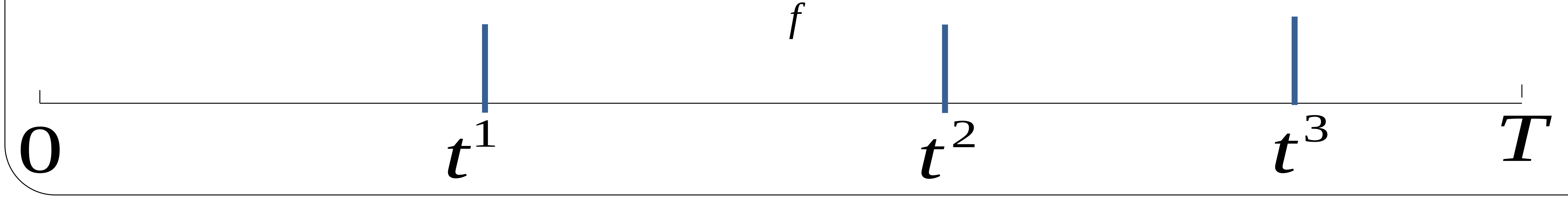
Measured spike train with spike times

Likelihood L that this spike train could have been generated by model?

$$L(t^1, \ldots, t^N) = \exp\left(-\int_0^{t^1} \rho(t')dt'\right)\rho(t^1) \cdot \exp\left(-\int_{t^1}^{t^2} \rho(t')dt'\right)\ldots$$

Neuronal Dynamics – 9.4 Likelihood of a spike train

$$S(t) = \sum_f \delta(t - t^f)$$

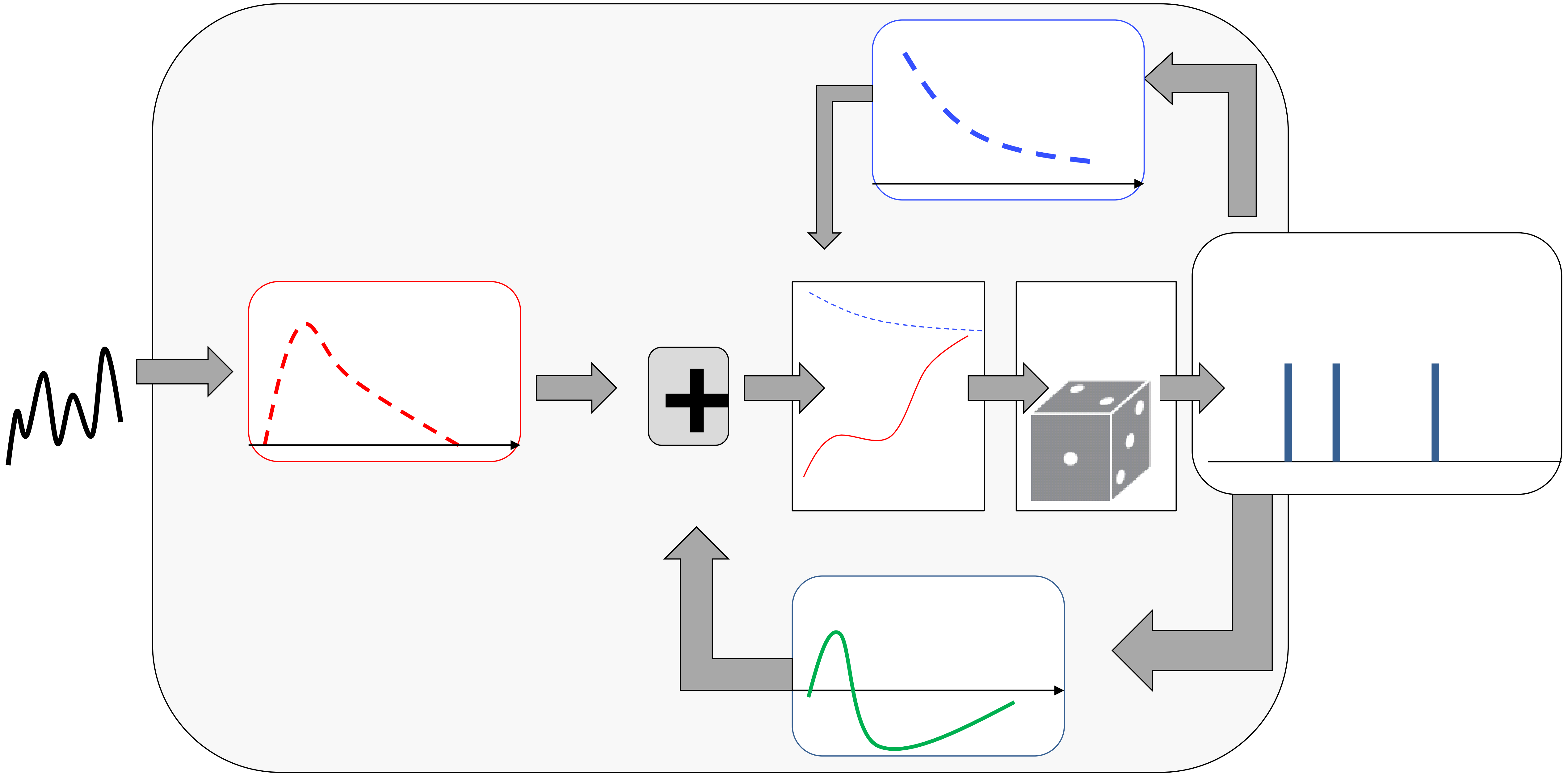


$$L(t^1,\ldots,t^N) = \exp\left(-\int_0^{t^1} \rho(t')dt'\right)\rho(t^1)\cdot\exp\left(-\int_{t^1}^{t^2} \rho(t')dt'\right)\rho(t^2)\ldots\cdot\exp\left(-\int_{t^N}^{T} \rho(t')dt'\right)$$

$$L(t^1,\ldots,t^N) = \exp\left(-\int_0^{T} \rho(t')dt'\right)\prod_f \rho(t^f)$$

$$\log L(t^1,\ldots,t^N) = -\int_0^{T} \rho(t')dt' + \sum_f \log\rho(t^f)$$

Neuronal Dynamics – 9.4 SRM with escape noise = GLM



-linear filters

-escape rate

→likelihood of observed spike train

→parameter optimization of neuron model

Week 9 – part 5: Parameter Estimation

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

√ 9.3 Spike Response Model (SRM)
- Integral formulation

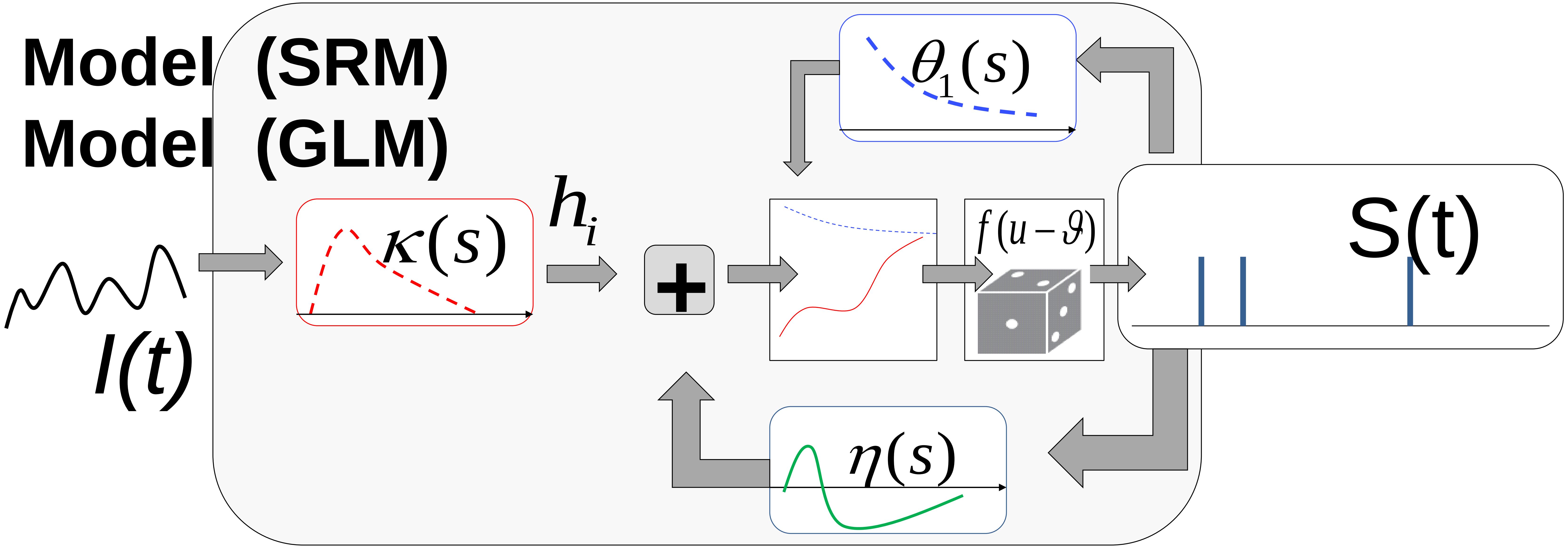
√ 9.4 Generalized Linear Model
- Adding noise to the SRM

$\boxed{\text{(9.5 Parameter Estimation) - Quadratic and convex optimization}}$

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Neuronal Dynamics – 9.5 Parameter estimation: voltage

Spike Response Model (SRM)
Generalized Lin. Model (GLM)



Subthreshold potential

$$u(t) = \int \underline{\eta(s)} S(t-s)\,ds + \int_0^\infty \underline{\kappa(s)} I(t-s)\,ds + u_{rest}$$

known spike train known input

Linear filters/linear in parameters

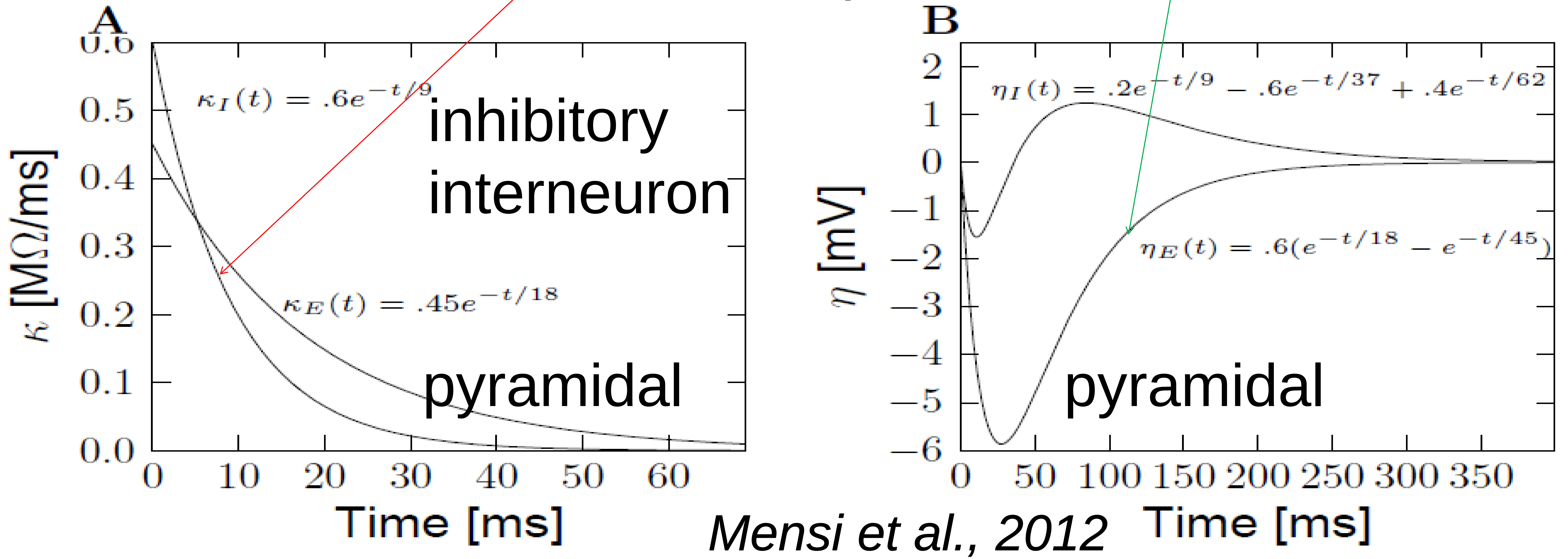
Neuronal Dynamics – 9.5 Extracted parameters: voltage

Subthreshold potential

$$u(t)=\int_0^\infty \kappa(s)\,I(t-s)\,ds+u_{rest}+\int \eta(s)\,S(t-s)\,ds$$

known input

known spike train



Mensi et al., 2012

Week 9 – part 5b: Quadratic and Convex Optimization

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

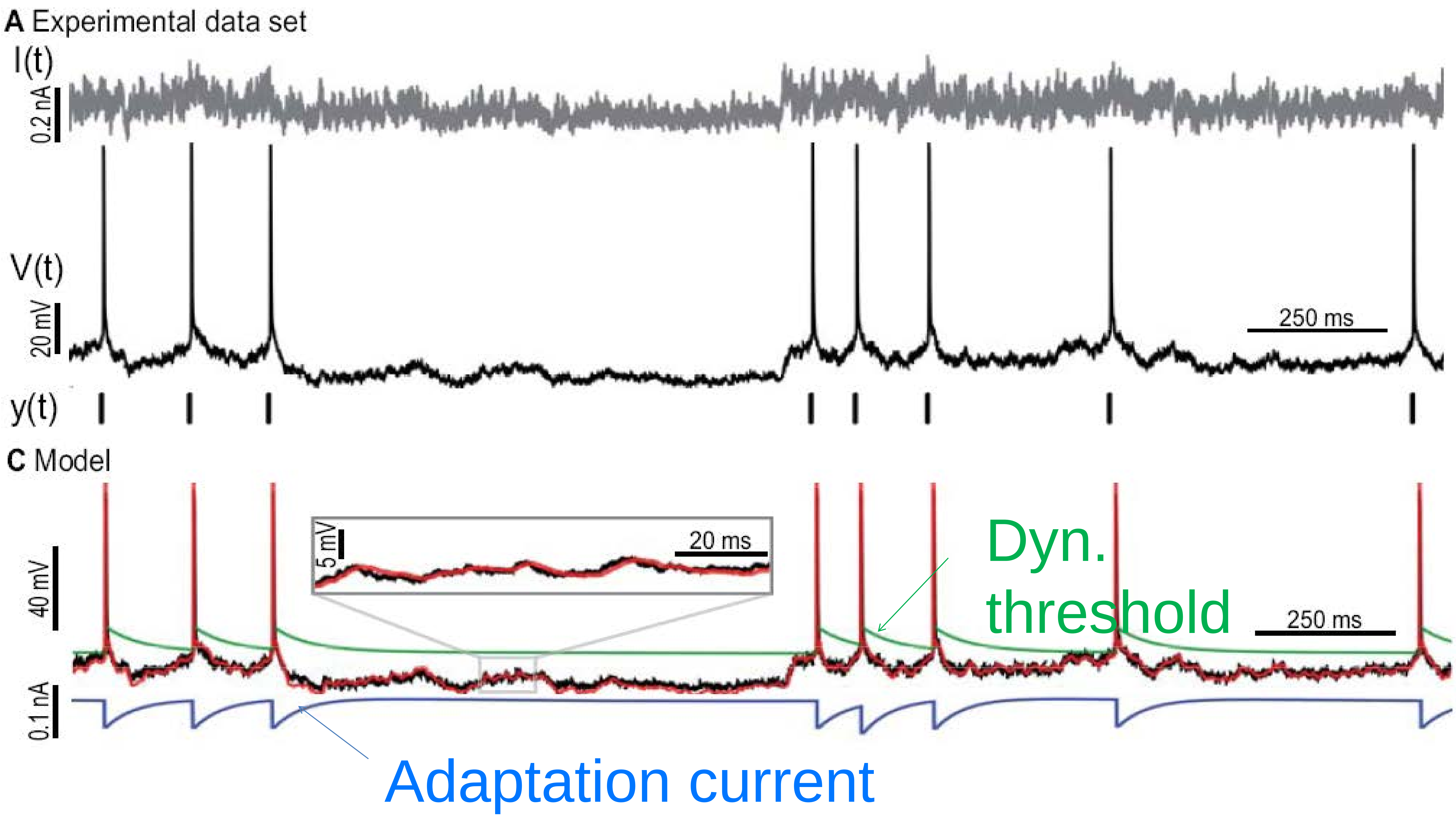
√ 9.3 Spike Response Model (SRM)
- Integral formulation

√ 9.4 Generalized Linear Model
- Adding noise to the SRM

9.5 Parameter Estimation
- Quadratic and convex optimization

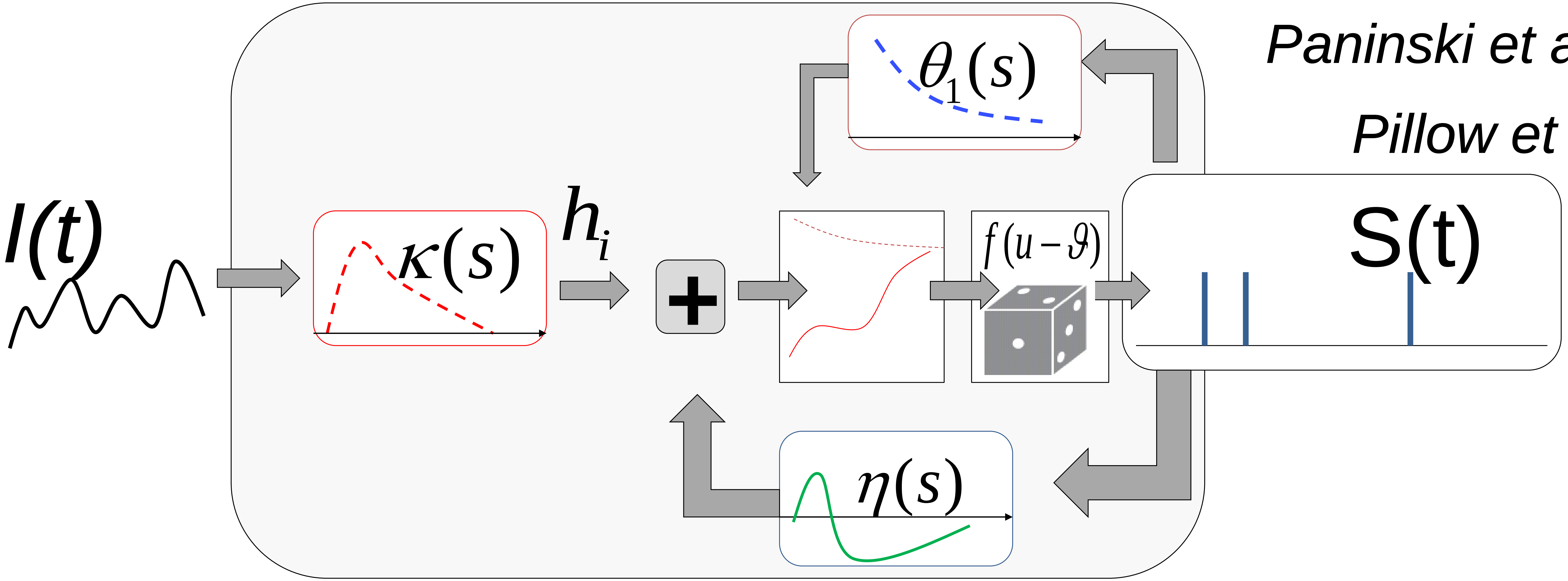
9.6. Modeling in vitro data
- how long lasts the effect of a spike?

Fitting models to data: so far ‘subthreshold’



Neuronal Dynamics – 9.5 Threshold: Predicting spike times

Jolivet&Gerstner, 2005
Paninski et al., 2004
Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

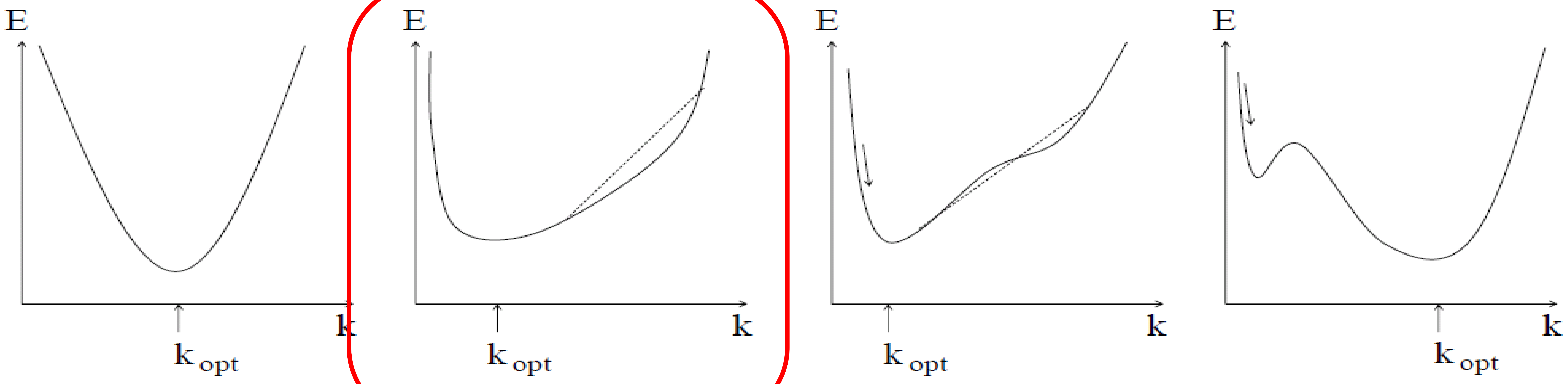
Neuronal Dynamics – 9.5 Generalized Linear Model (GLM)

$$\log L(t^1,\dots,t^N) = -\int_0^T \rho(t')dt' + \sum_f \log \rho(t^f) = -E$$

potential $$u(t) = \int \underline{\eta(s)} S(t-s)ds + \int_0^\infty \underline{\kappa(s)} I(t-s)ds + u_{rest}$$

threshold $$\vartheta(t) = \theta_0 + \int \underline{\theta_1(s)} S(t-s)ds$$

firing intensity $$\rho(t) = f(u(t) - \vartheta(t))$$

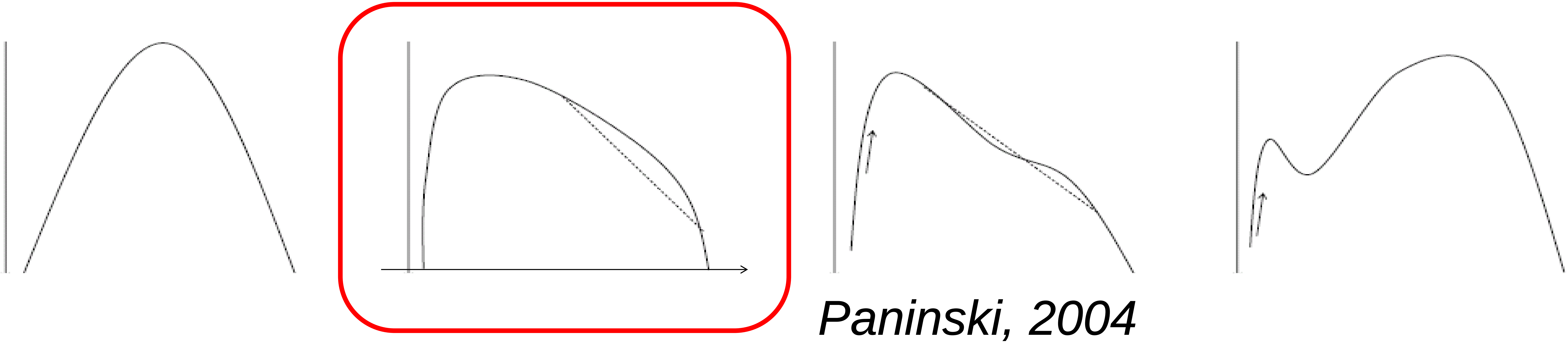


potential $$u(t) = \int \eta(s) S(t-s)\,ds + \int_0^\infty \kappa(s)\, I(t-s)\,ds + u_{rest}$$

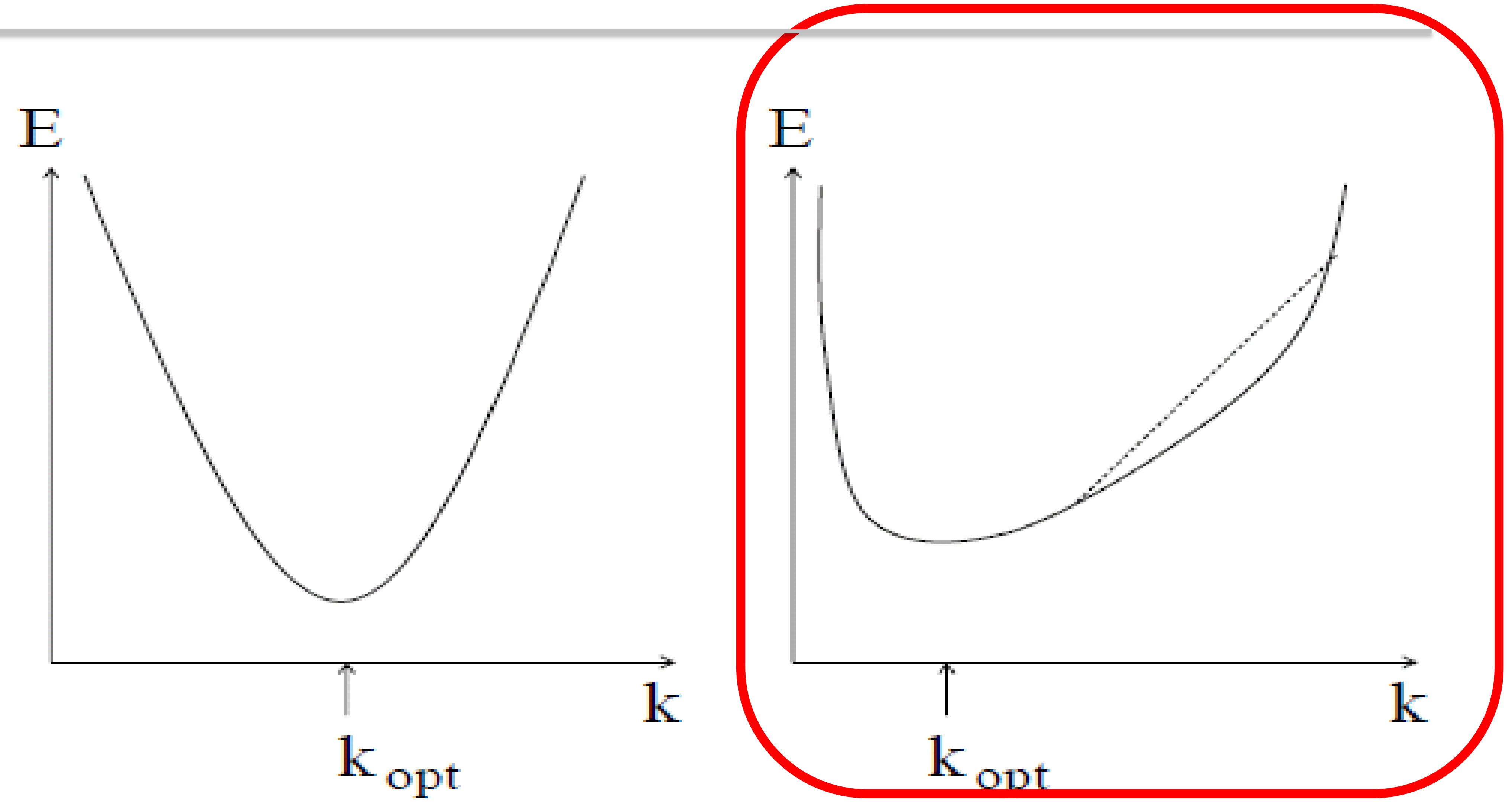
threshold $$\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s)\,ds$$

firing intensity $$\rho(t) = f(u(t) - \vartheta(t))$$

$$\log L(t^1, \ldots, t^N) = -\int_0^T \rho(t')\,dt' + \sum_f \log \rho(t^f)$$



Paninski, 2004



Voltage/subthreshold

- linear in parameters
  → quadratic error function

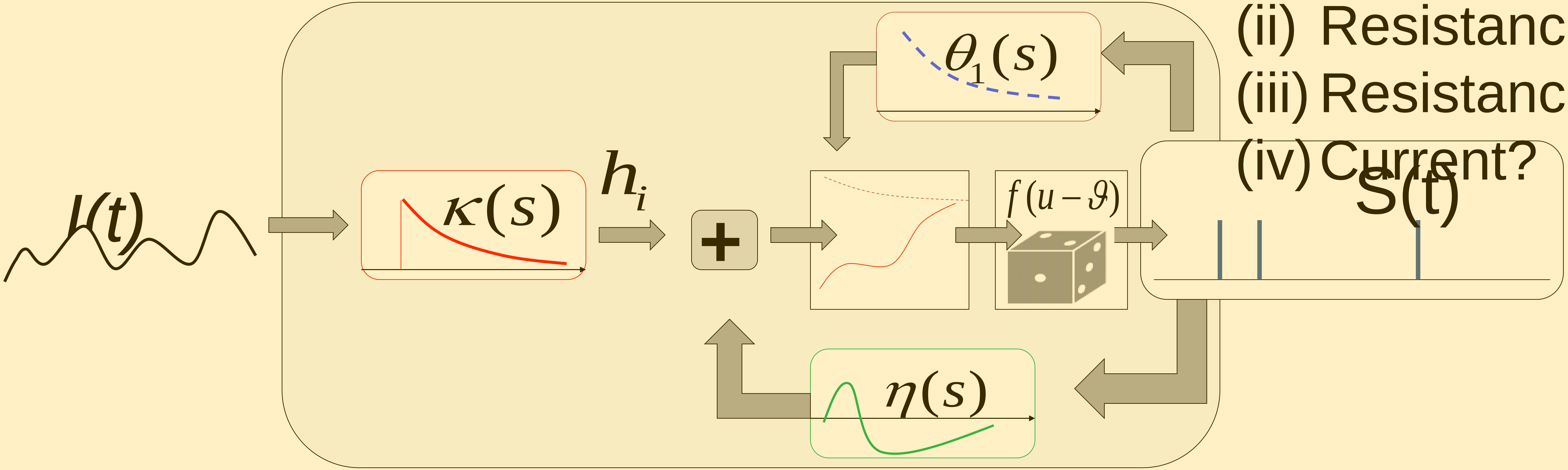
Spike times

- nonlinear, but GLM
  → convex error function

Quiz NOW :

What are the units of $\eta(s)$ **?**

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s?
(iv) Current?



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

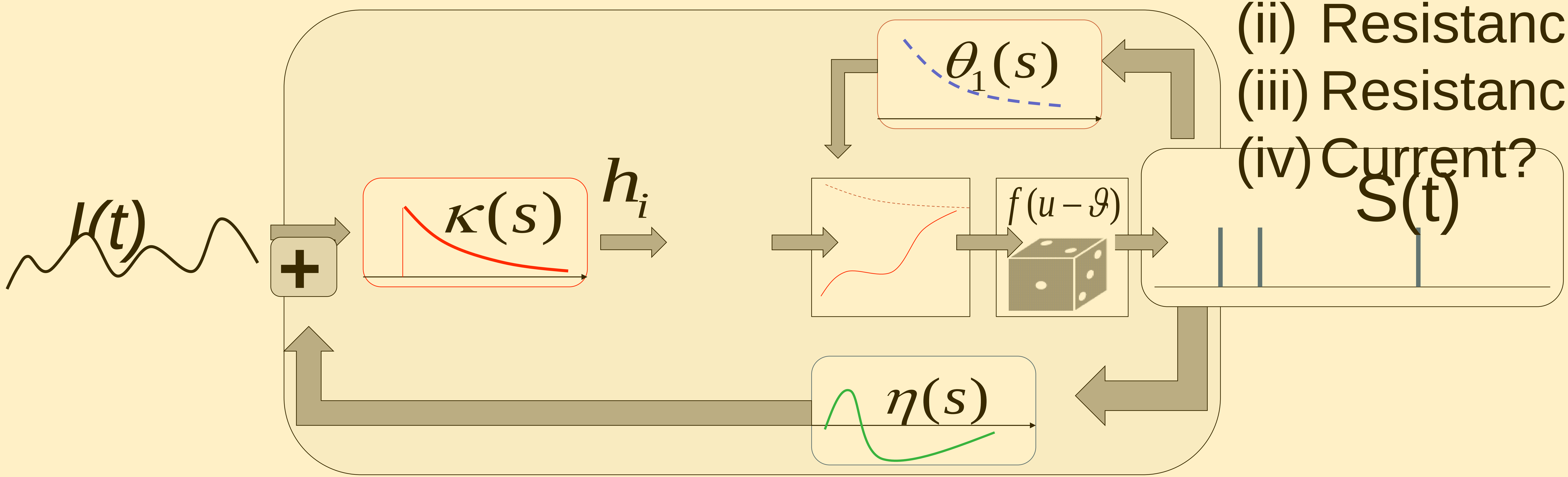
threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Quiz NOW:

What are the units of $\eta(s)$ **?**

(i) Voltage?
(ii) Resistance?
(iii) Resistance/s?
(iv) Current?



potential $C\frac{d}{dt}u(t) = -\frac{(u-u_{rest})}{R} + \int \eta(s)S(t-s)ds \quad + I(t-s)$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s)S(t-s)ds$

firing intensity $\rho(t) = f(u(t)-\vartheta(t))$

Week 9 – part 6: Modeling in vitro data

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

√ 9.3 Spike Response Model (SRM)
- Integral formulation

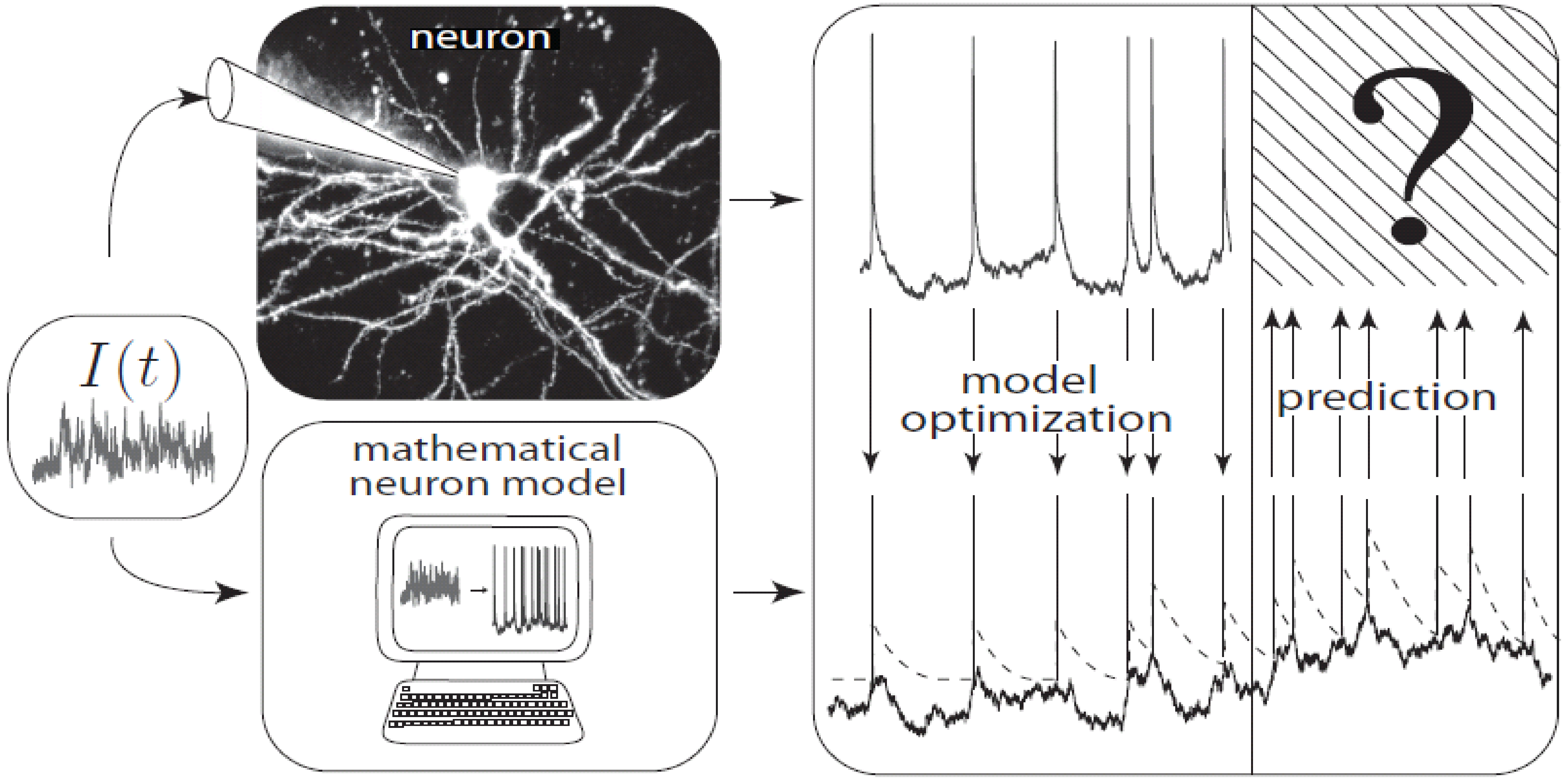
√ 9.4 Generalized Linear Model
- Adding noise to the SRM

√ 9.5 Parameter Estimation
- Quadratic and convex optimization

9.6. Modeling in vitro data
- how long lasts the effect of a spike?

9.7. Helping Humans

comparison model-data



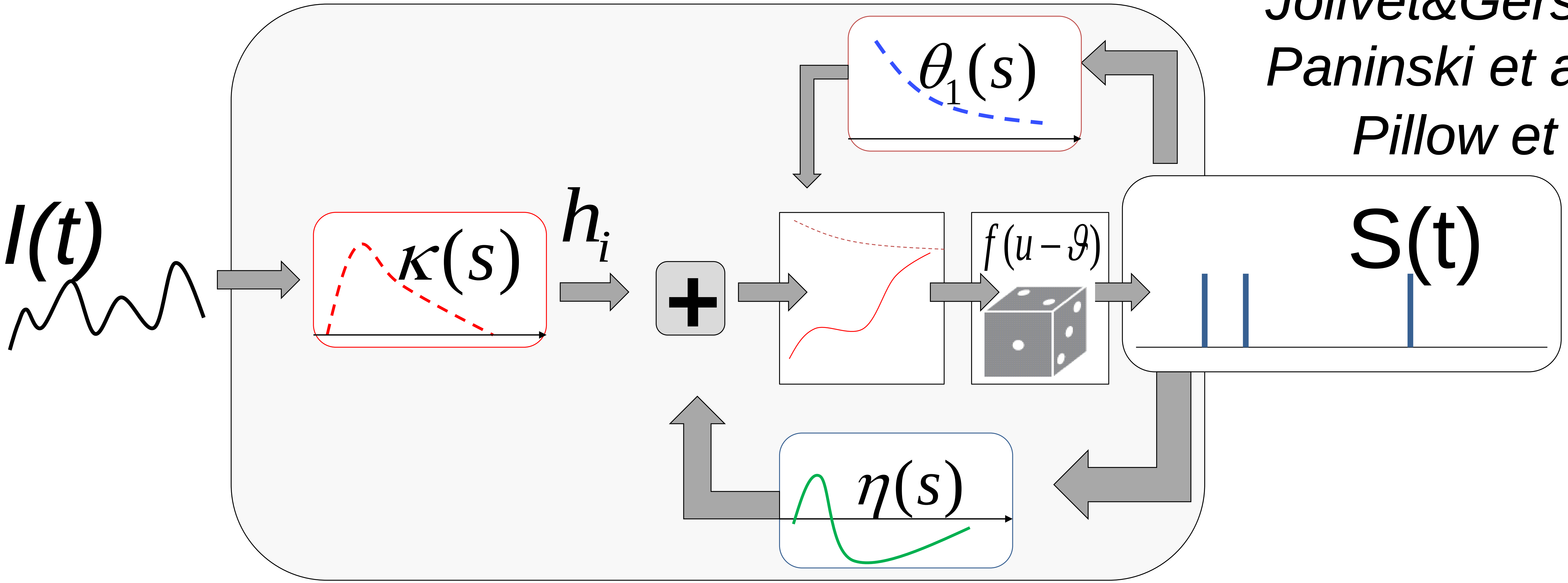
Predict

-Subthreshold voltage

-Spike times

Neuronal Dynamics – 9.6 GLM/SRM with escape noise

Jolivet&Gerstner, 2005
Paninski et al., 2004
Pillow et al. 2008



potential $u(t) = \int \eta(s) S(t-s) ds + \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$

threshold $\vartheta(t) = \theta_0 + \int \theta_1(s) S(t-s) ds$

firing intensity $\rho(t) = f(u(t) - \vartheta(t))$

Neuronal Dynamics – 9.6 GLM/SRM predict subthreshold voltage

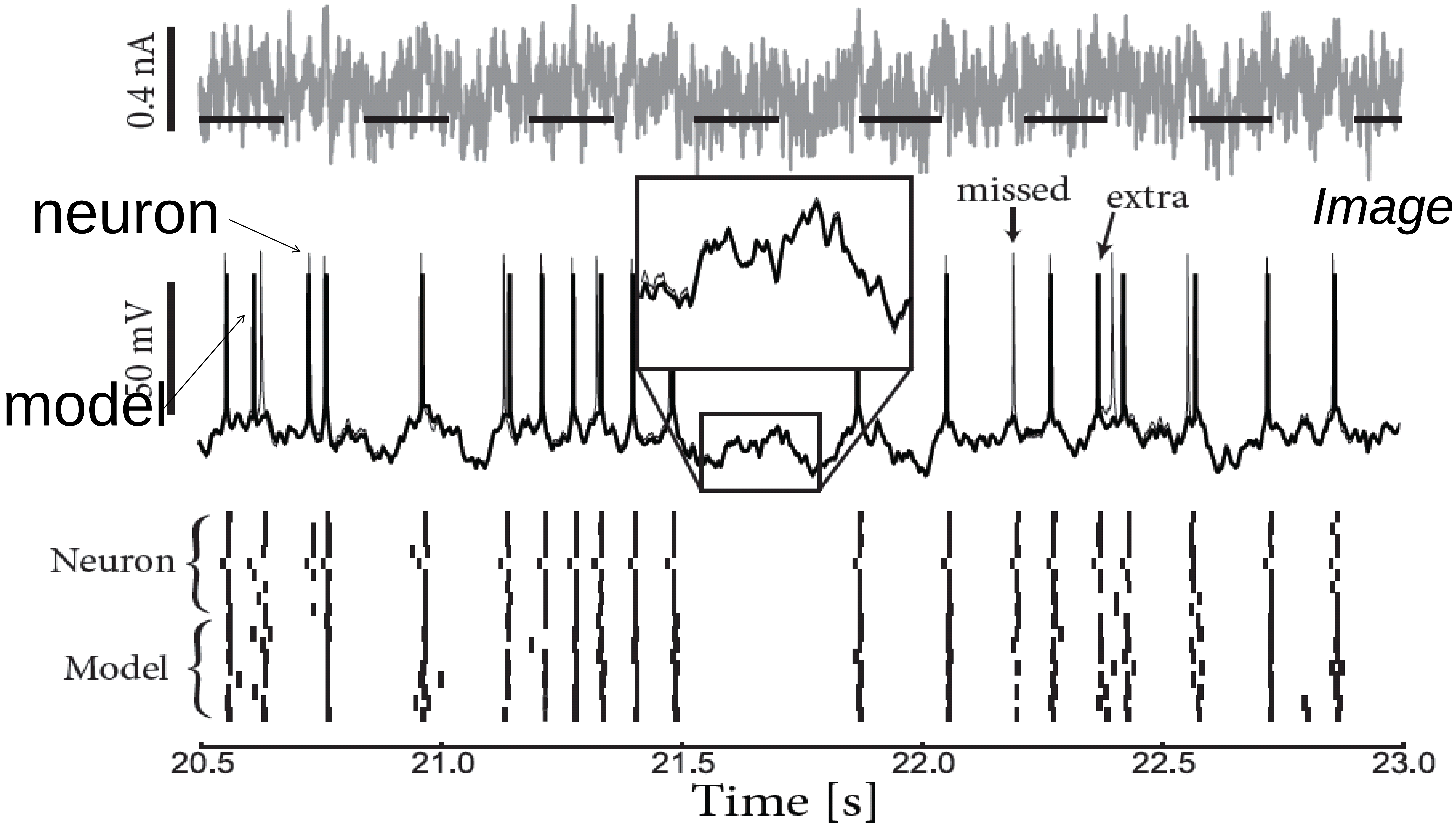
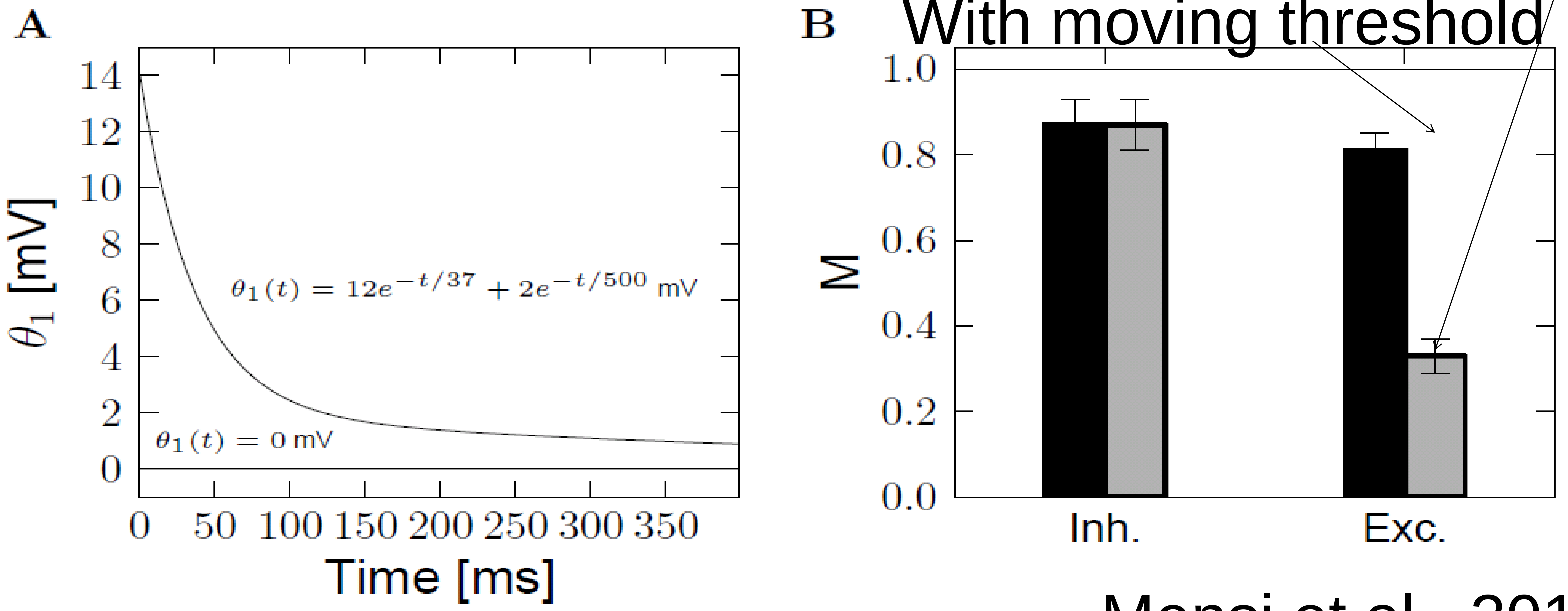


Image:Mensi et al.

Role of moving threshold

No moving threshold

With moving threshold



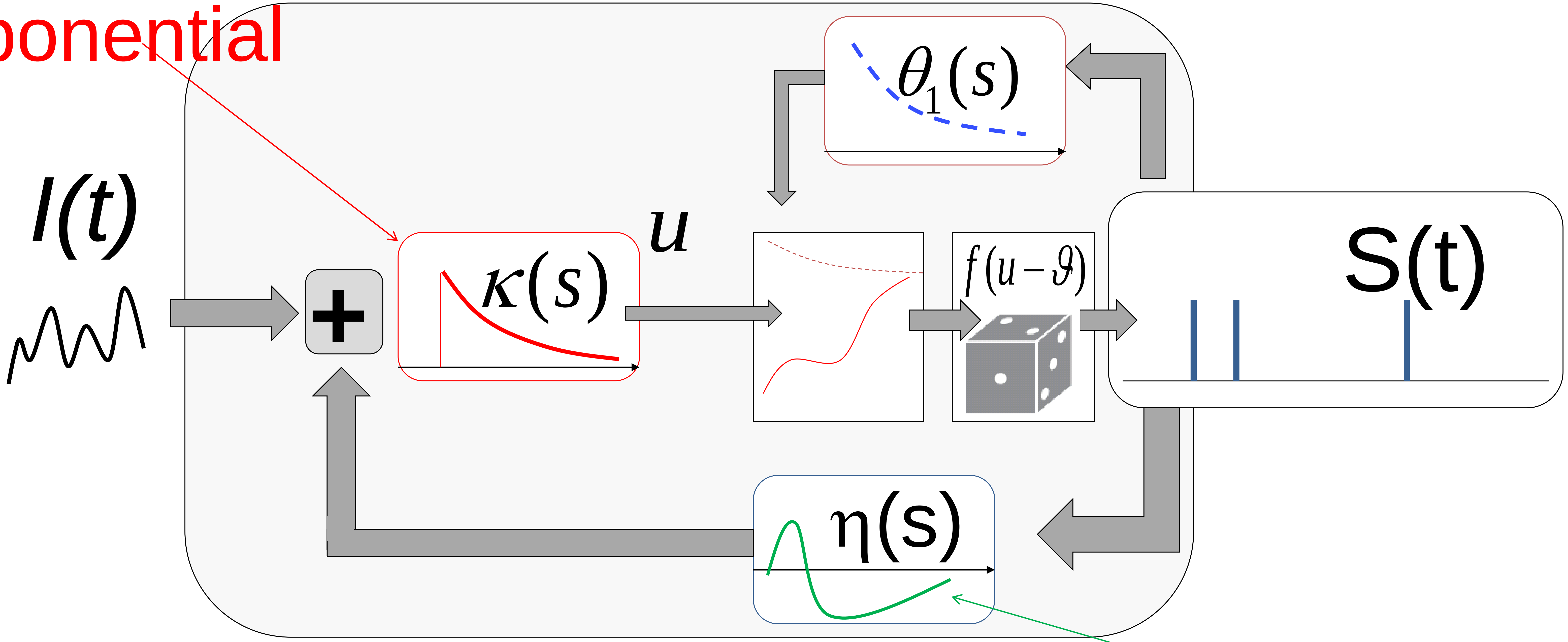
Mensi et al., 2012

Change in model formulation:

‘soft-threshold adaptive IF model’

What are the units of …. ?

exponential



adaptation current

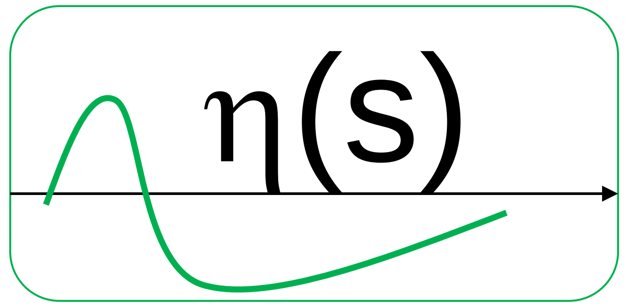
potential $$C\frac{d}{dt}u(t) = \int \underline{\eta(s)}S(t-s)ds \quad + I(t)$$

threshold $$\vartheta(t) = \theta_0 + \int \underline{\theta_1(s)}S(t-s)ds$$

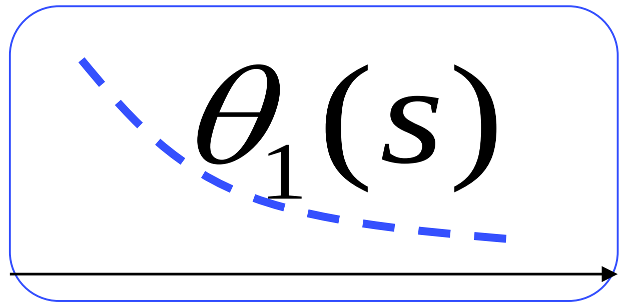
firing intensity $$\rho(t) = f(u(t) - \vartheta(t))$$

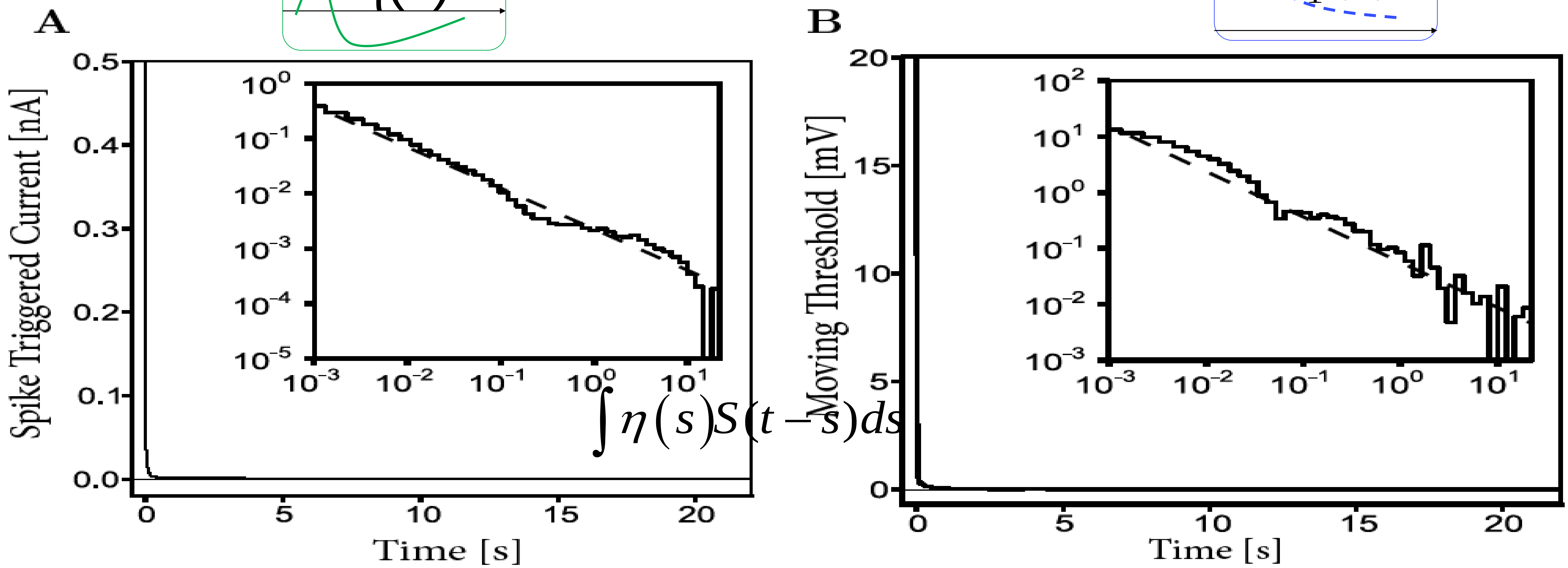
Neuronal Dynamics – 9.6 How long does the effect of a spike last?

Time scale of filters?



→ **Power law**

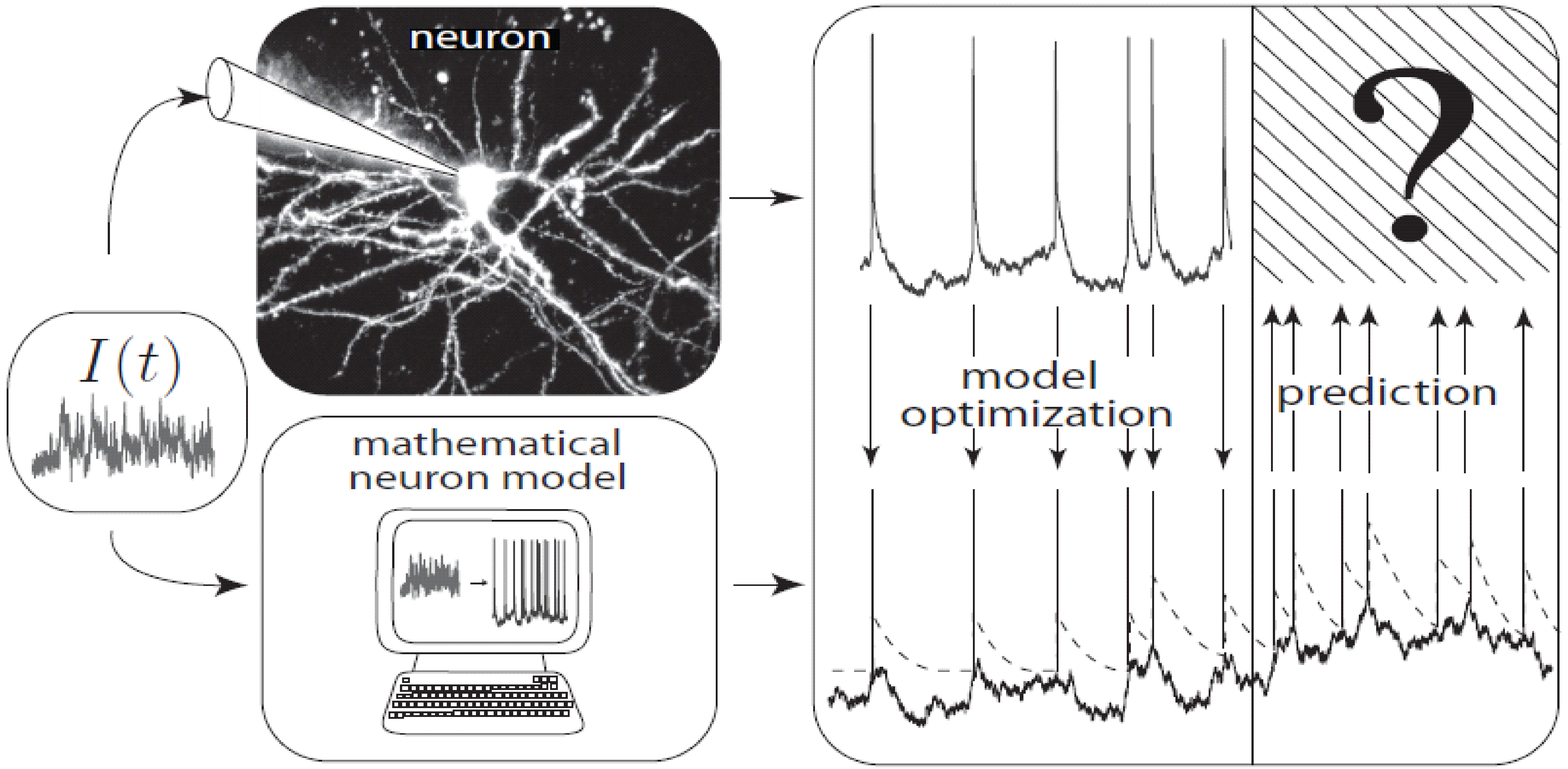




$$\int \eta(s) S(t-s) ds$$

A single spike has a measurable effect more than 10 seconds later!

Pozzorini et al. 2013



-Predict spike times
-Predict subthreshold voltage
-Easy to interpret (not a ‘black box’)
-Variety of phenomena
-Systematic: ‘optimize’ parameters

BUT so far limited to in vitro

Week 9 – part 6: Modeling in vitro data

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks:

Week 9 – Optimizing Neuron Models For Coding and Decoding

Wulfram Gerstner

EPFL, Lausanne, Switzerland

√ 9.1 What is a good neuron model?
- Models and data

√ 9.2 AdEx model
- Firing patterns and adaptation

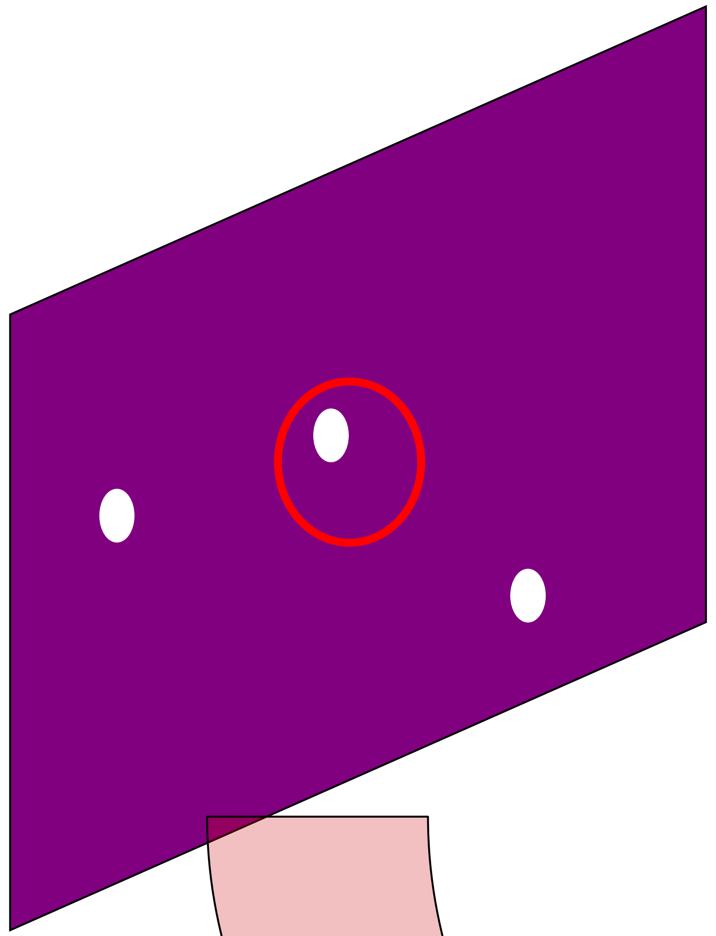
√ 9.3 Spike Response Model (SRM)
- Integral formulation

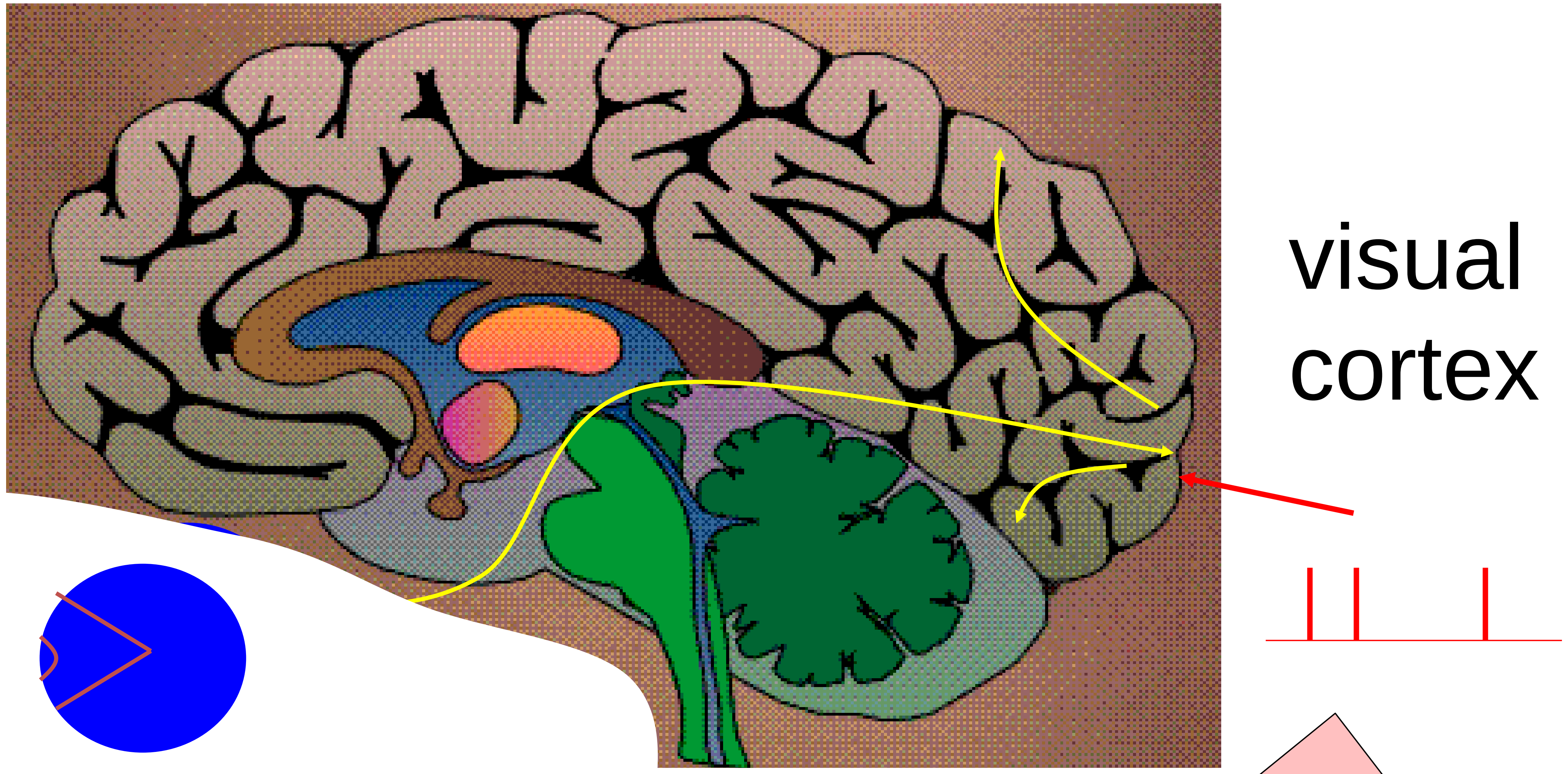
√ 9.4 Generalized Linear Model
- Adding noise to the SRM

√ 9.5 Parameter Estimation
- Quadratic and convex optimization

√ 9.6. Modeling in vitro data
- how long lasts the effect of a spike?

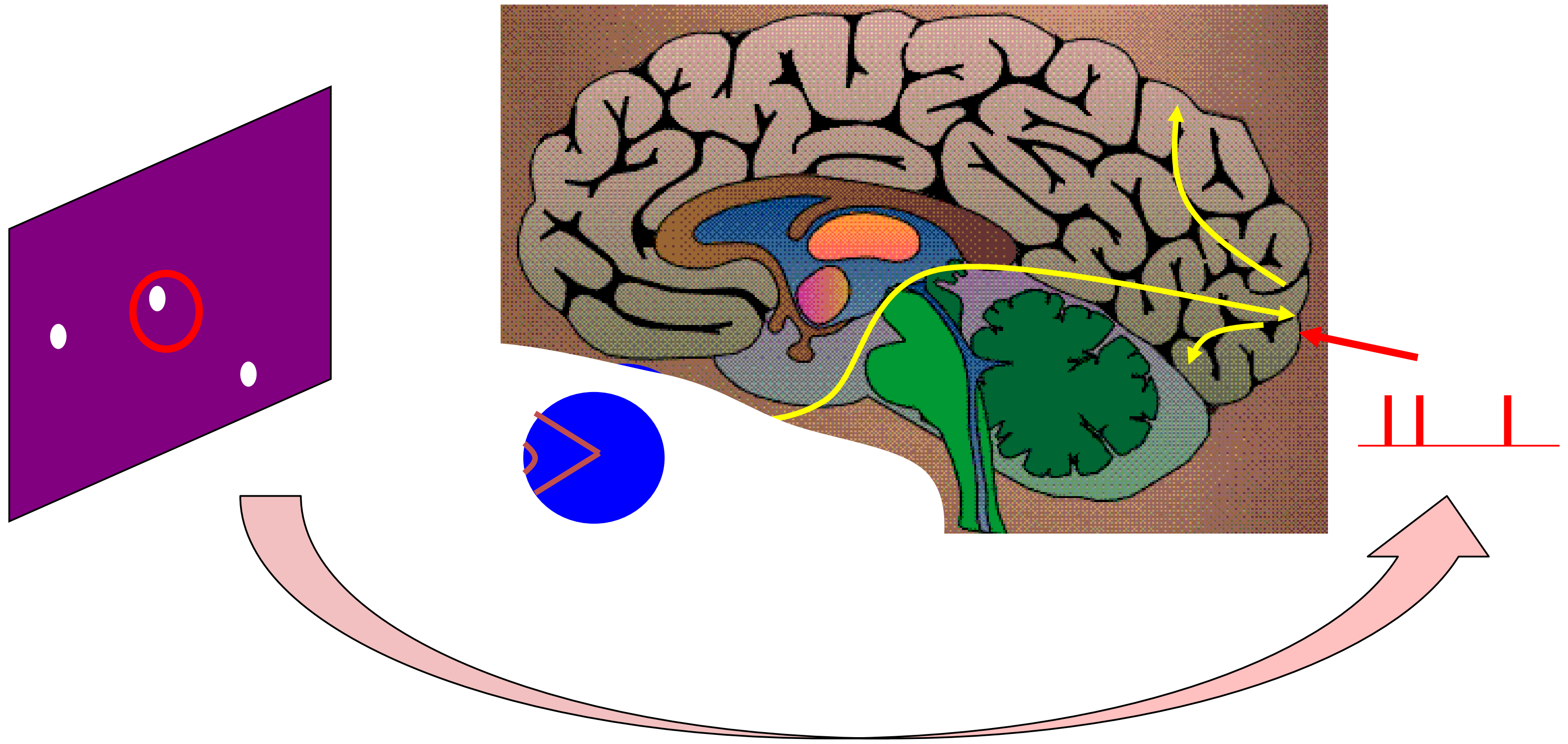
9.7. Helping Humans: in vivo data





A) Predict spike times, given stimulus
B) ~~Predict subthreshold voltage~~
C) Easy to interpret (not a ‘black box’)
D) Flexible enough to account for a variety of phenomena
E) Systematic procedure to ‘optimize’ parameters

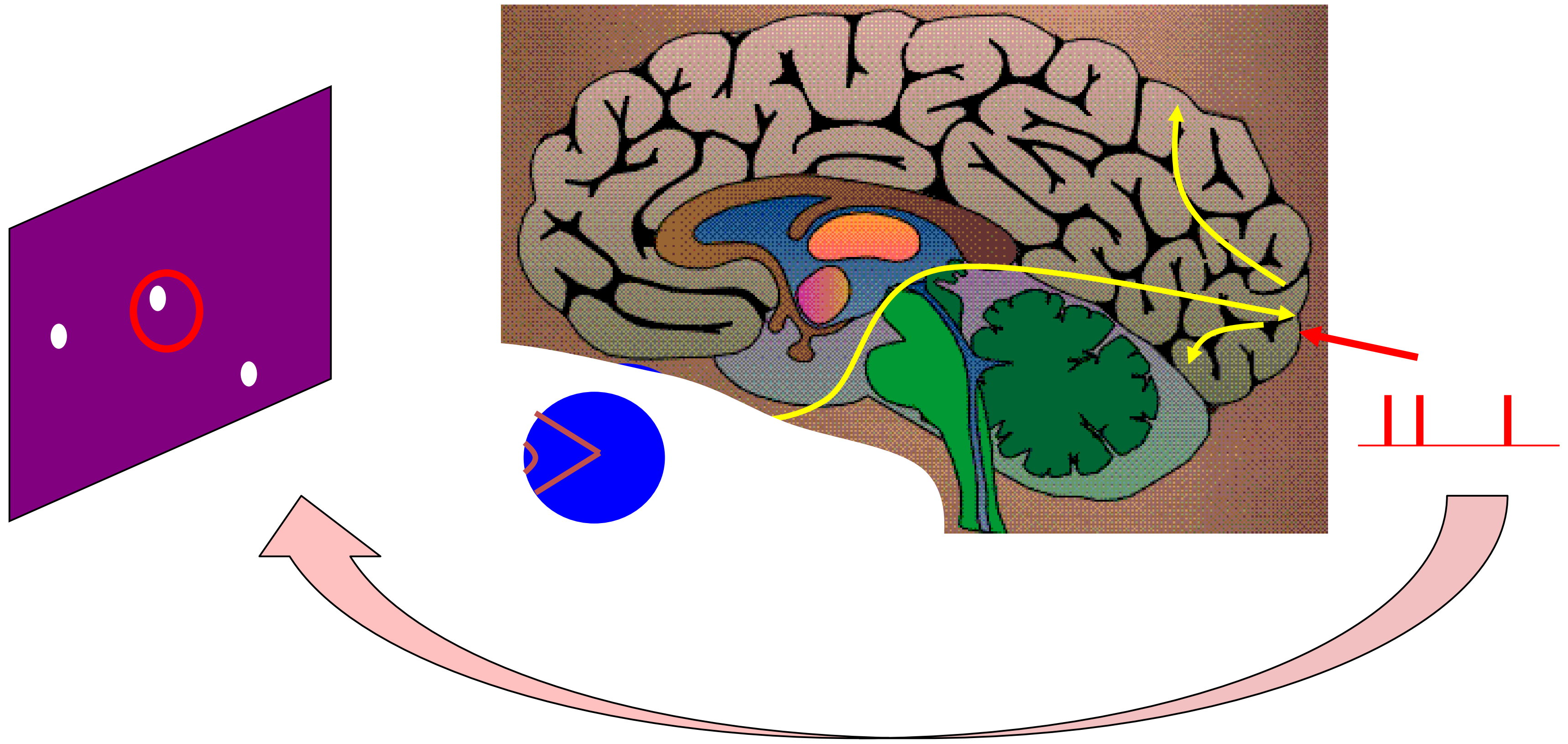
Model of ‘Encoding’



Model of 'Encoding'

Generalized Linear Model (GLM)

- flexible model
- systematic optimization of parameters



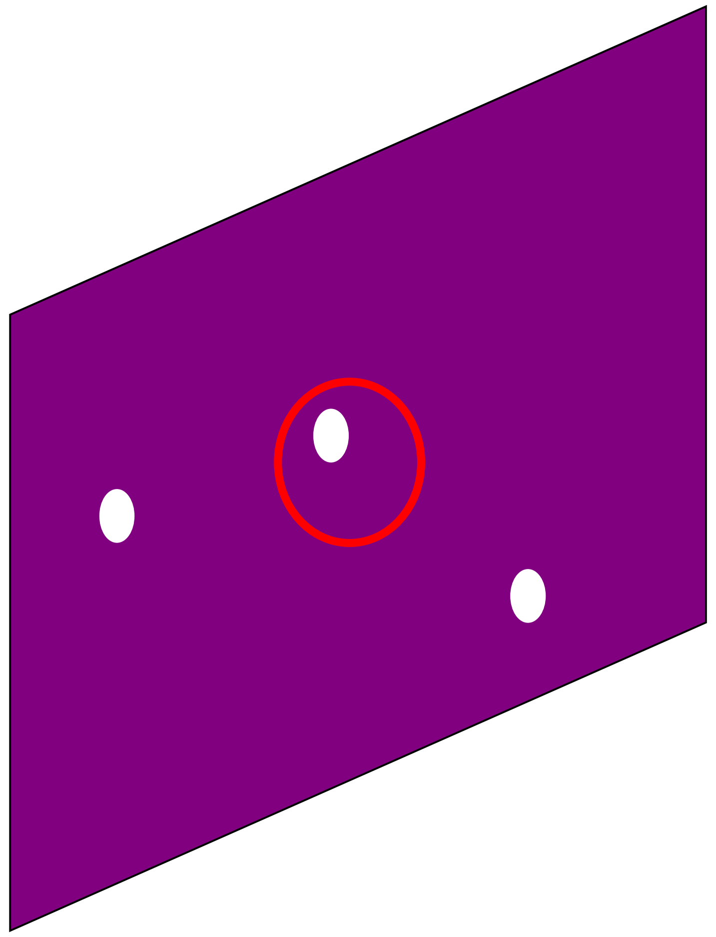
Model of 'Decoding'

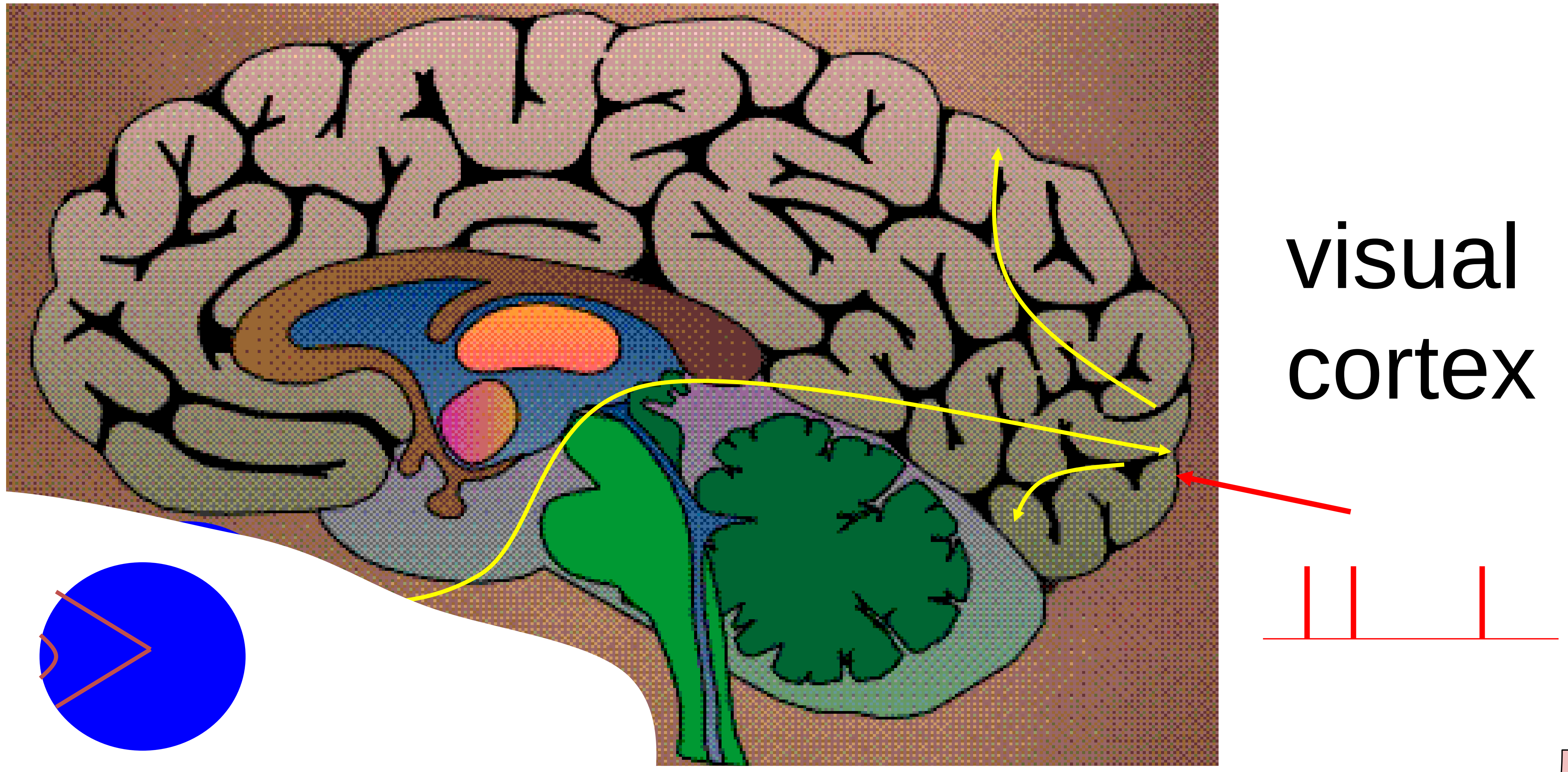
The same GLM works!

- flexible model
- systematic optimization of parameters

Neuronal Dynamics – 9.7 Model of DECODING

Predict stimulus!

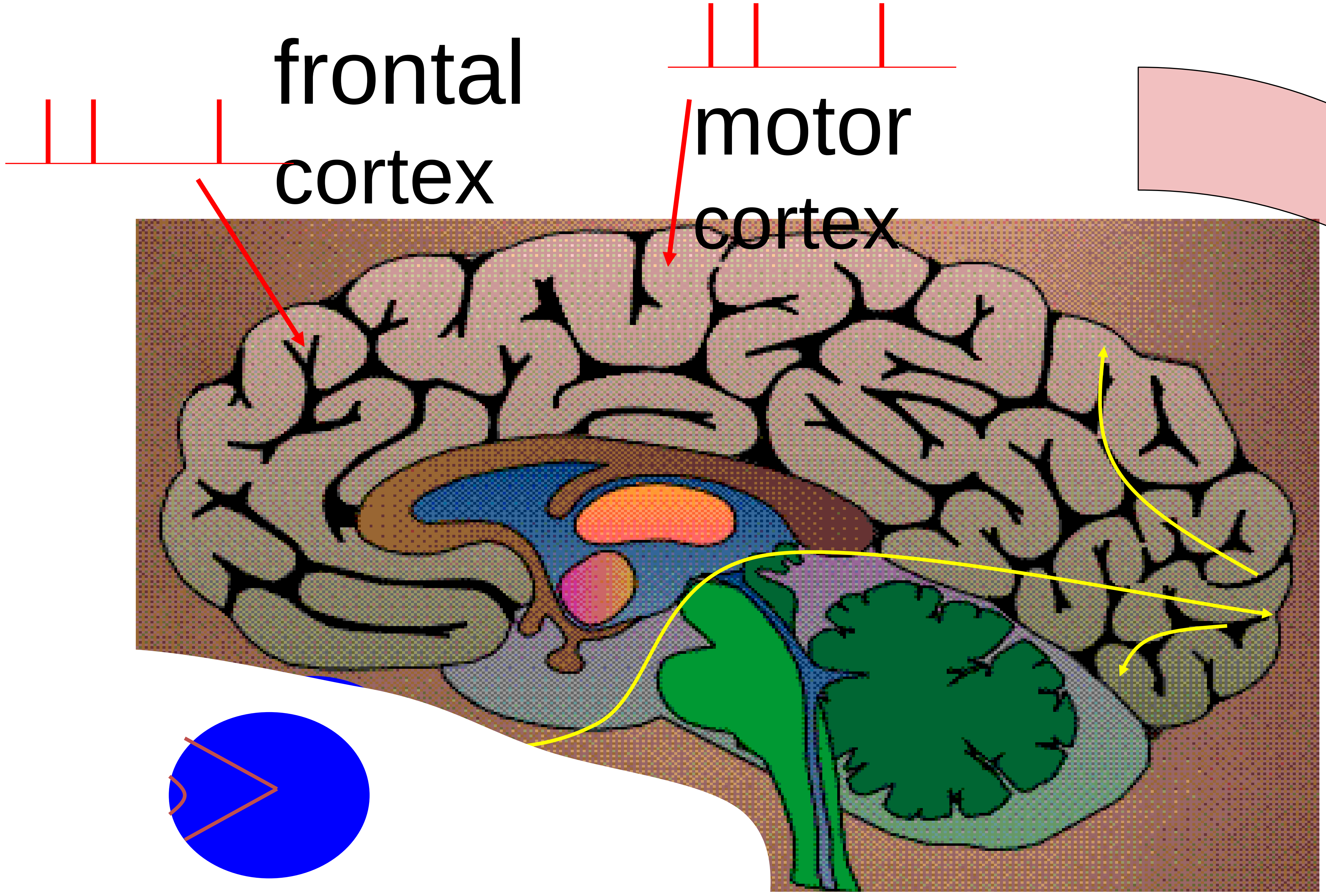




Model of ‘Decoding’:

predict stimulus, given spike times

Application: Neuroprosthetics



Many groups
world wide
work on this
problem!

Model of
'Decoding'

Predict intended arm movement,
given Spike Times

Application: Neuroprosthetics

Figure:
Neuronal Dynamics,
Cambridge Univ. Press;
See Truccolo et al. 2005

Decode the intended arm movement

Hand velocity

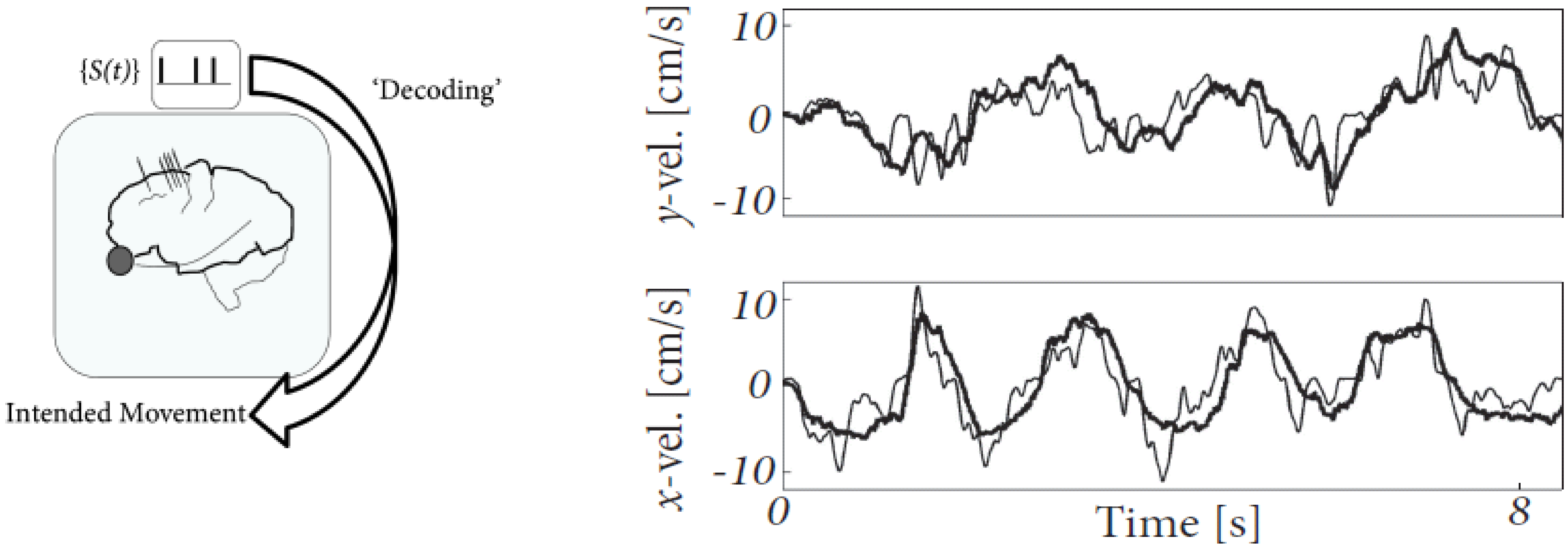


Fig. 11.12: Decoding had velocity from spiking activity in area MI of cortex. The real hand velocity (thin black line) is compared to the decoded velocity (thick black line) for the $x-$ (top) and the $y-$components (bottom). Modified from Truccolo et al. (2005).

Neuronal Dynamics week 7– Suggested Reading/selected references

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Ch. 6,10,11: Cambridge, 2014

Nonlinear and adaptive IF

Fourcaud-Trocme, N., Hansel, D., van Vreeswijk, C., and Brunel, N. (2003). How spike …. *J. Neuroscience*, 23:11628-11640.
Badel, L., et al. (2008a). Extracting nonlinear integrate-and-fire, *Biol. Cybernetics*, 99:361-370.
Brette, R. and Gerstner, W. (2005). Adaptive exponential integrate-and-fire *J. Neurophysiol.*, 94:3637- 3642.
Izhikevich, E. M. (2003). Simple model of spiking neurons. IEEE Trans Neural Netw, 14:1569-1572.
Gerstner, W. (2008). Spike-response model*. Scholarpedia*, 3(12):1343.

Optimization methods for neuron models, max likelihood, and GLM

-Brillinger, D. R. (1988). Maximum likelihood analysis of spike trains of interacting nerve cells. *Biol. Cybern*., 59:189-200.
-Truccolo, et al. (2005). A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects. Journal of Neurophysiology, 93:1074-1089.
- Paninski, L. (2004). Maximum likelihood estimation of … *Network: Computation in Neural Systems*, 15:243-262.
- Paninski, L., Pillow, J., and Lewi, J. (2007). Statistical models for neural encoding, decoding, and optimal stimulus design. In Cisek, P., et al. , *Comput. Neuroscience: Theoretical Insights into Brain Function*. Elsevier Science.
Pillow, J., ET AL.(2008). Spatio-temporal correlations and visual signalling… . Nature, 454:995-999.

Encoding and Decoding

Rieke, F., Warland, D., de Ruyter van Steveninck, R., and Bialek, W. (1997). Spikes - Exploring the neural code. MIT Press,
Keat, J., Reinagel, P., Reid, R., and Meister, M. (2001). Predicting every spike … *Neuron*, 30:803-817.
Mensi, S., et al. (2012). Parameter extraction and classication …. *J. Neurophys.,*107:1756-1775.
Pozzorini, C., Naud, R., Mensi, S., and Gerstner, W. (2013). Temporal whitening by . *Nat. Neuroscience,*
Georgopoulos, A. P., Schwartz, A.,Kettner, R. E. (1986). Neuronal population coding of movement direction. *Science*, 233:1416-1419.
Donoghue, J. (2002). Connecting cortex to machines: recent advances in brain interfaces. Nat. Neurosci., 5:1085-1088.

Next year a similar class will be taught.
What should be improved?

‘Exercises and Miniprojects take a lot of time,
more than other subjects at 4 ECTS.’

- workload for a 4 credit course (=6 h p. week, for 18 weeks)

In addition to class 9-12: 2h or less, 3h, 4h or more

[1 credit = 27 hours work total = 1.5 h p. week, for 18 weeks]

Projects: Next year a similar class will be taught. What should be improved?

‘We learn nothing while doing the projects’

‘We just connect the dots’

agree – not sure - disagree

‘The instructions of the graded exercises could be more precise’

agree – not sure - disagree

‘we spend more time coding than learning things which is not the goal of the course’

agree – not sure - disagree

Next year a similar class will be taught.
What should be improved?

Keep 3 projects?
Reduce to 1 project, but code ‘from scratch’?

More freedom in the projects?
Less freedom (clearer instructions) in the project?

Next year a similar class will be taught.
What should be improved?

‘I am more efficient in lectures than in MOOCs’

agree – not sure - disagree

‘2 hours of video needs 4 hours to watch and understand,

‘The inverted classroom is not efficient’

agree – not sure - disagree

- Keep video lectures as an available tool
- Offer at least 10 out of 13

Next year a similar class will be taught. What should be improved?

‘The slides should be redesigned’

‘The slides are too dull (almost without any text)’

agree – not sure - disagree

Quick feedback on course:

“*What can I do better and differently next year?*”

- support: link to book chapter, video, slides
  not sufficient, sufficient, good, excellent
- integrated exercises?
  repeat next year, do not repeat next year
- workload for a 4 credit course (=6 h p. week, for 18 weeks)
  In addition to class 9-12: 2h or less, 3h, 4h or more
- difficulty?
  easier than other theory classes,
  same, harder than other theory classes
- other points?

The END

Neuronal Dynamics – Quiz 9.2. Nullclines for constant input

$$\tau \frac{du}{dt} = -(u - u_{rest}) + \Delta \exp\left(\frac{u-\vartheta}{\Delta}\right) + w + RI(t)$$

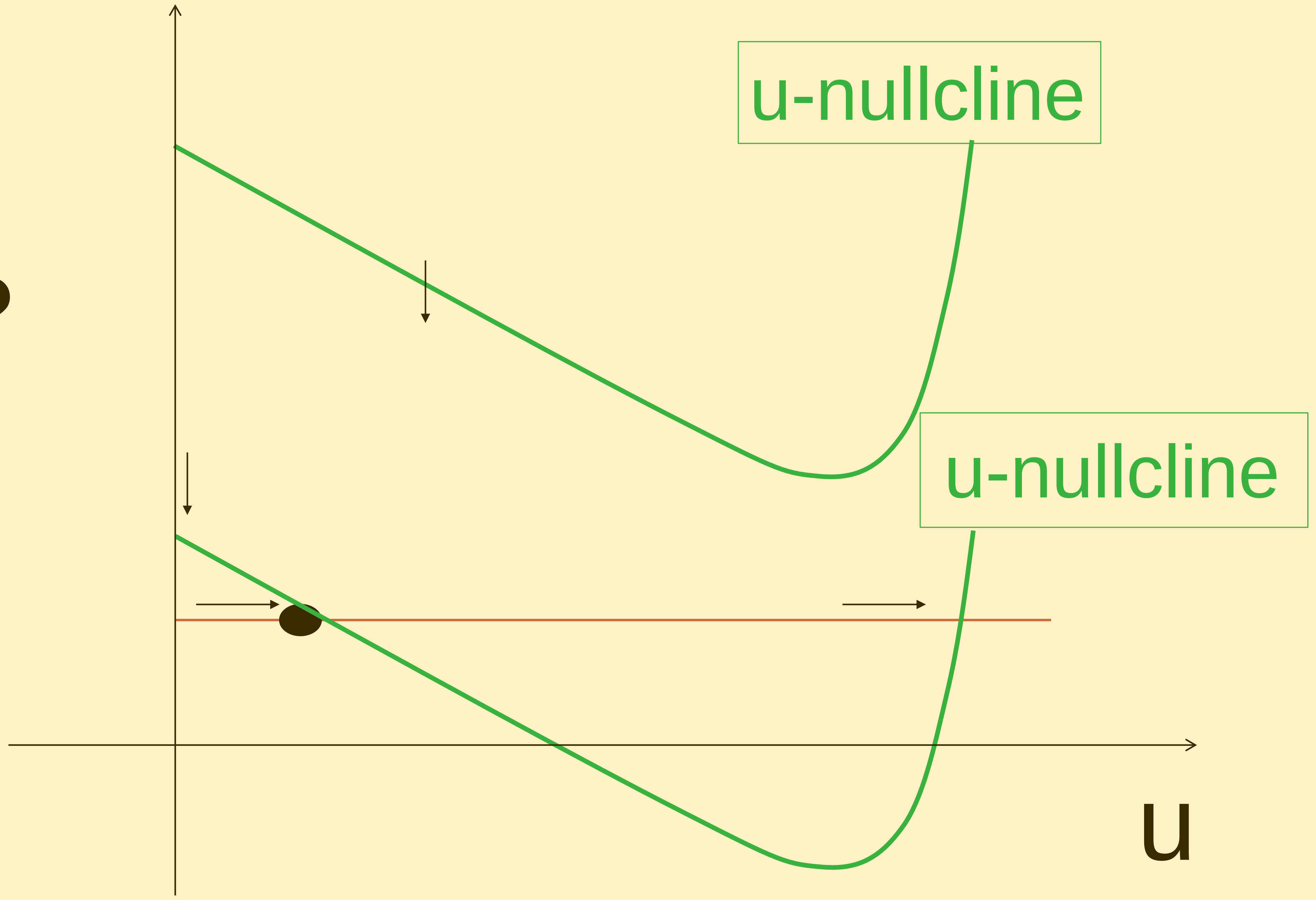
$$\tau_w \frac{dw}{dt} = \cancel{a\ (u - u_{rest})} - w + \cancel{b\ \tau_w \sum_f \delta(t-t^f)}$$

a=0

Only during reset

What happens if input switches from I=0 to I>0?

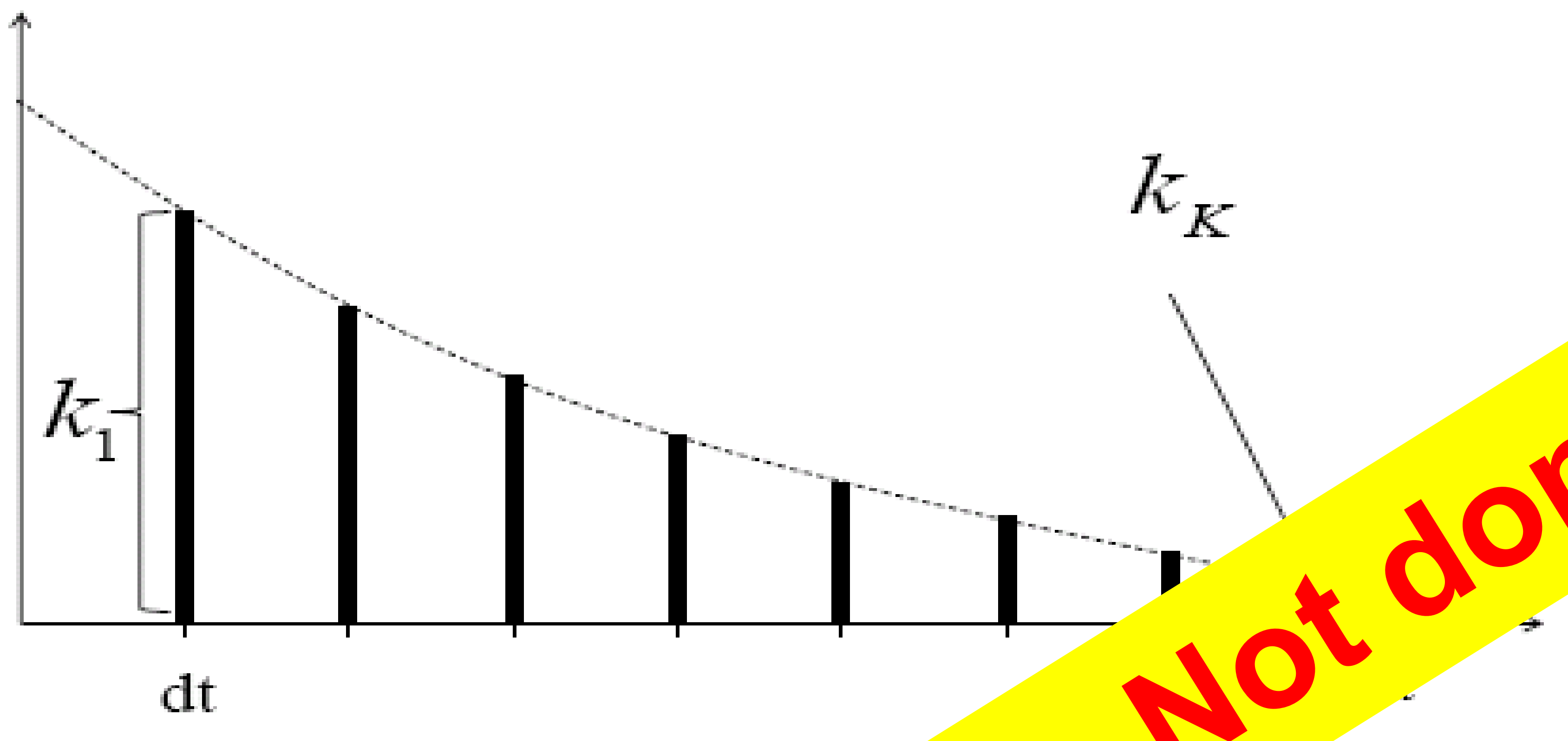
[ ] u-nullcline moves horizontally
[ ] u-nullcline moves vertically
[ ] w-nullcine moves horizontally
[ ] w-nullcline moves vertically



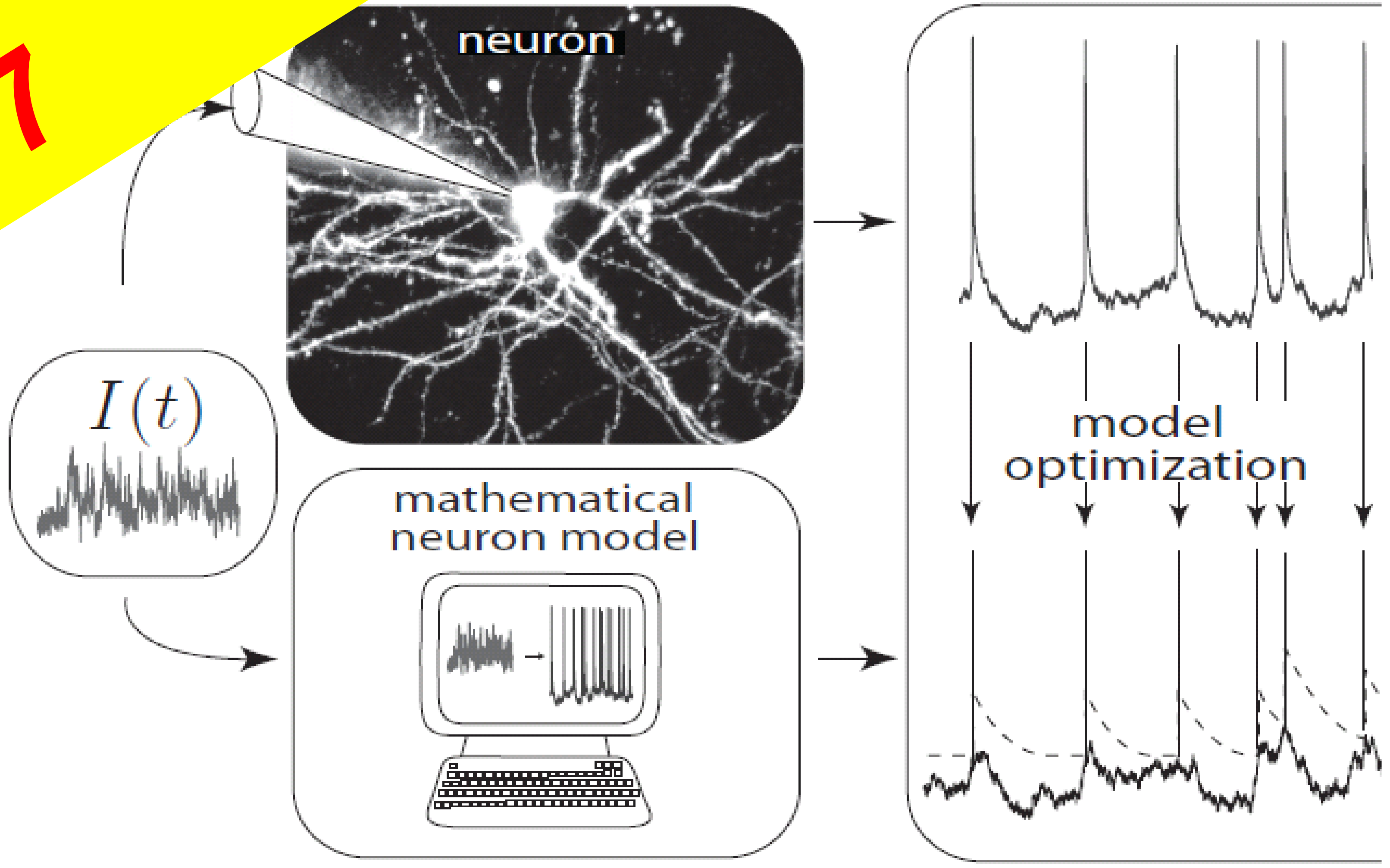
Linear in parameters = linear fit = quadratic problem

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$



comparison model-data



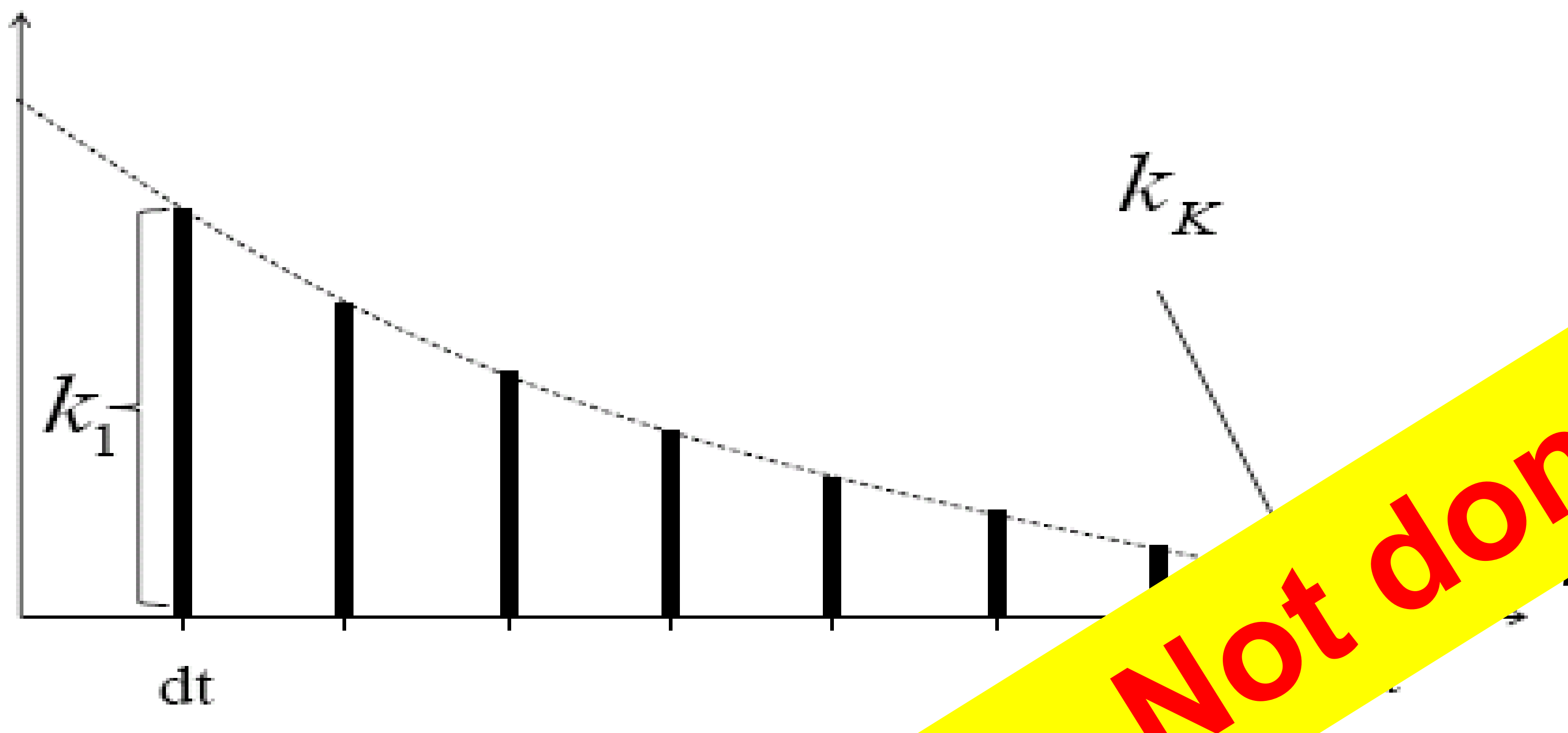
Not done in 2017

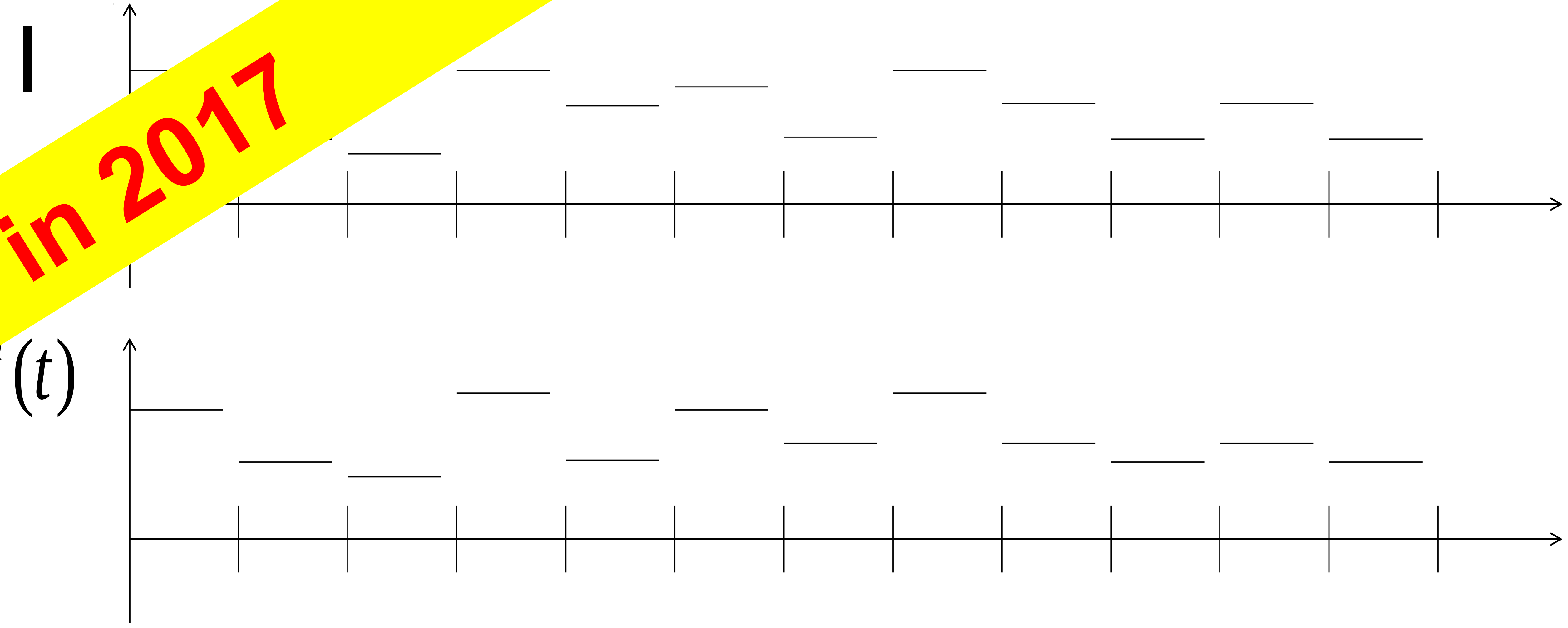
Blackboard: Riemann-sum

Linear in parameters = linear fit

$$u(t) = \int_0^\infty \kappa(s)\, I(t-s)\, ds + u_{rest}$$

$$u(t_n) = \sum k_k \, I_{n-k} + u_{rest}$$





Not done in 2017

Blackboard: Error function

$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k \, I_{n-k} - u_{rest}]^2$$

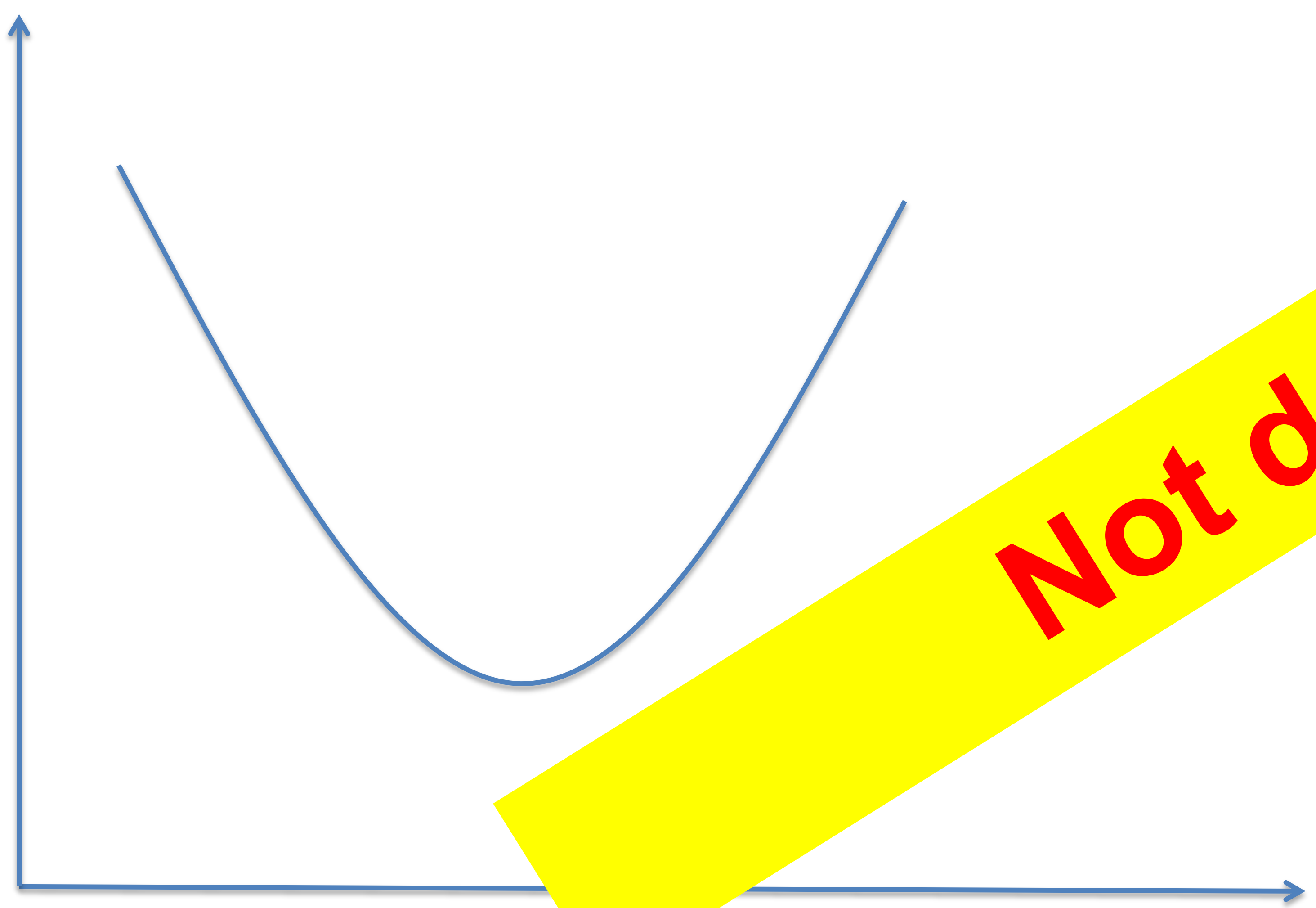
Linear in parameters = linear fit = quadratic optimization

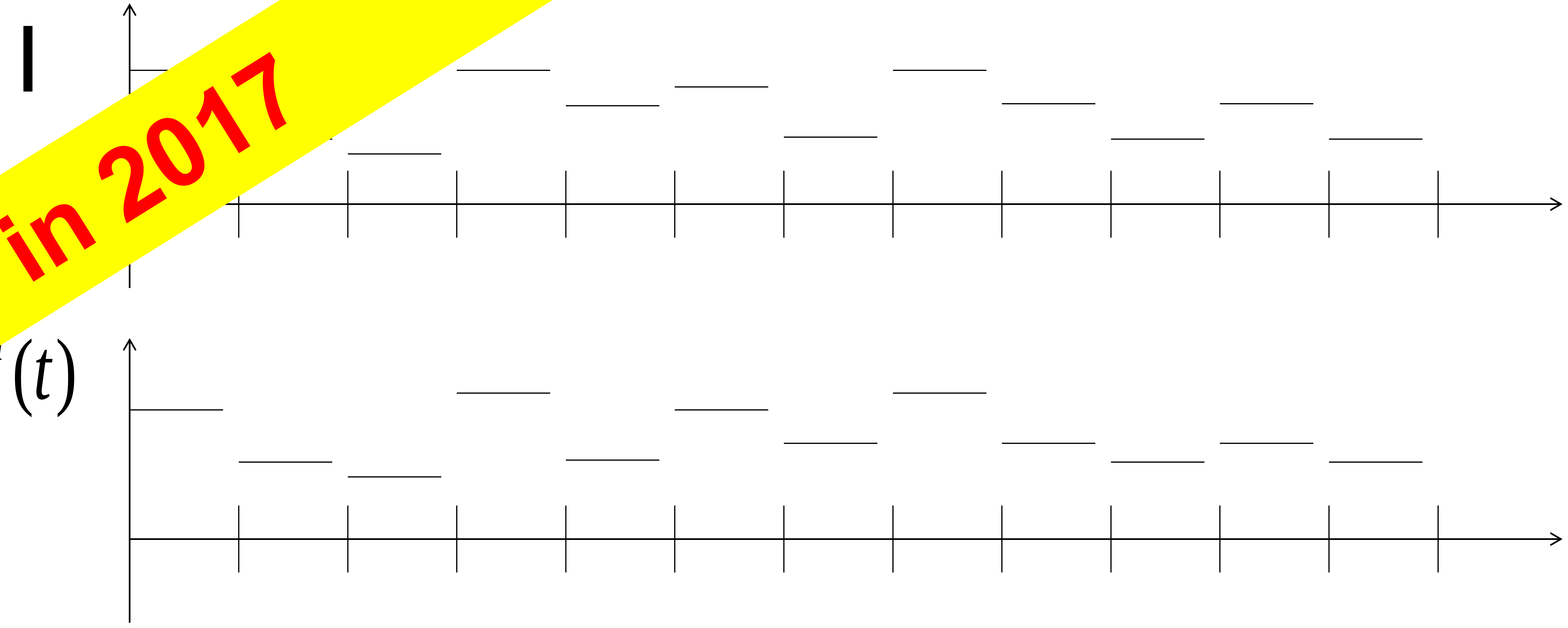
Model

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest}$$

$$u(t_n) = \sum_k k_k \, I_{n-k} + u_{rest}$$

$u^{data}(t)$





Not done in 2017

$$E = \sum_n \, [u^{data}(t_n) - \sum_{k=1}^{K} k_k \, I_{n-k} - u_{rest}]^2$$

Exercise 3 NOW: optimize 1 free parameter

Model

$$u_n = RI_n$$

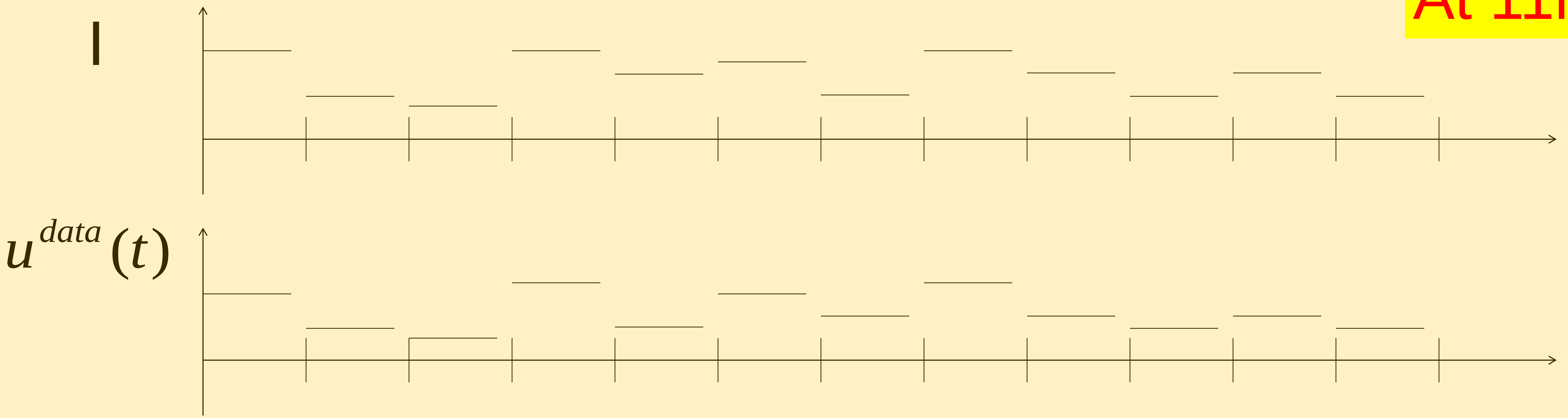
Data

$$u^{data}(t_n)$$

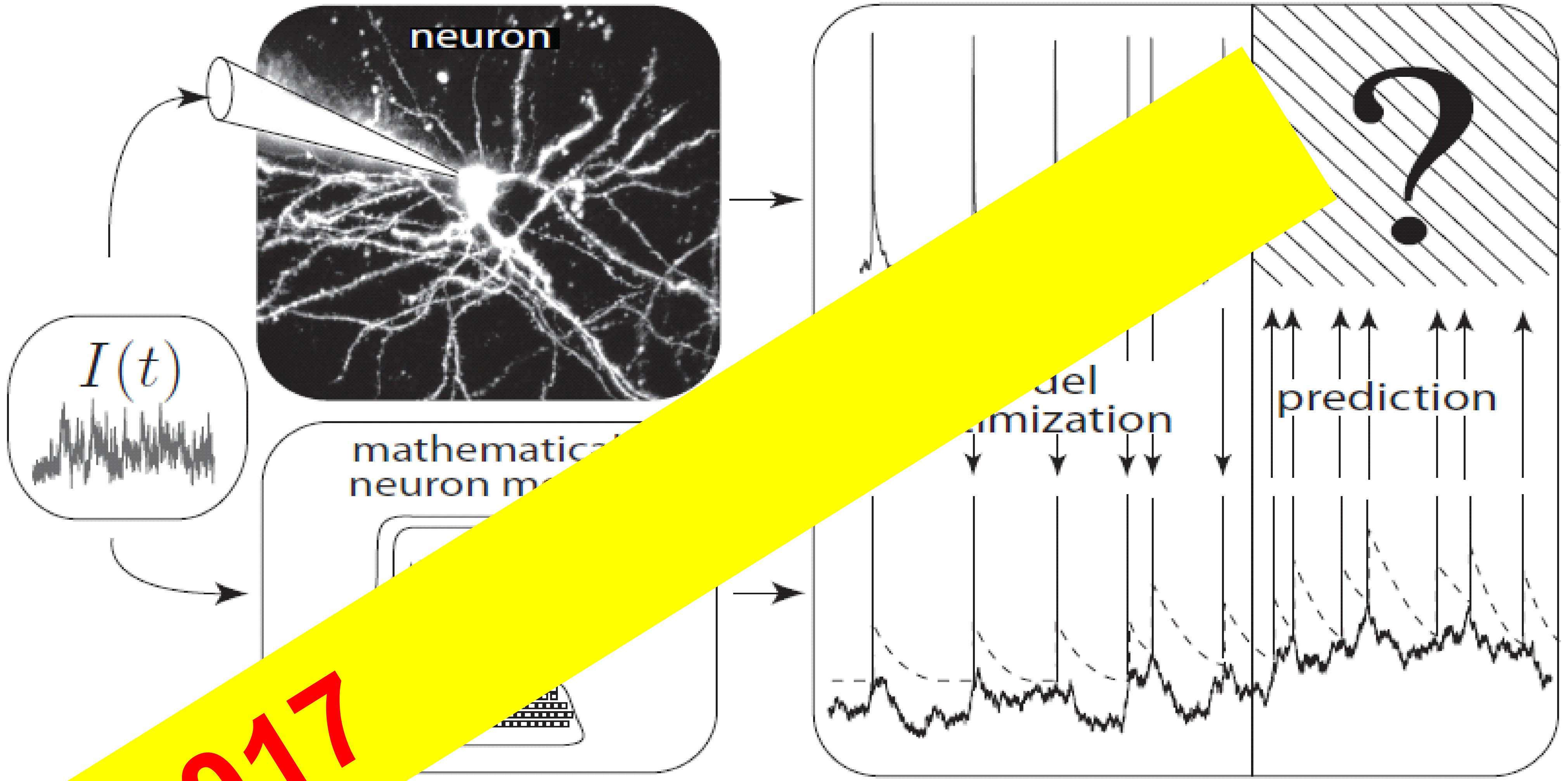
Optimize parameter R, so as to have a minimal error

$$E = \sum_n [u^{data}(t_n) - RI_n]^2$$

Next lecture
At 11h40



Neuronal Dynamics – What is a good neuron model?



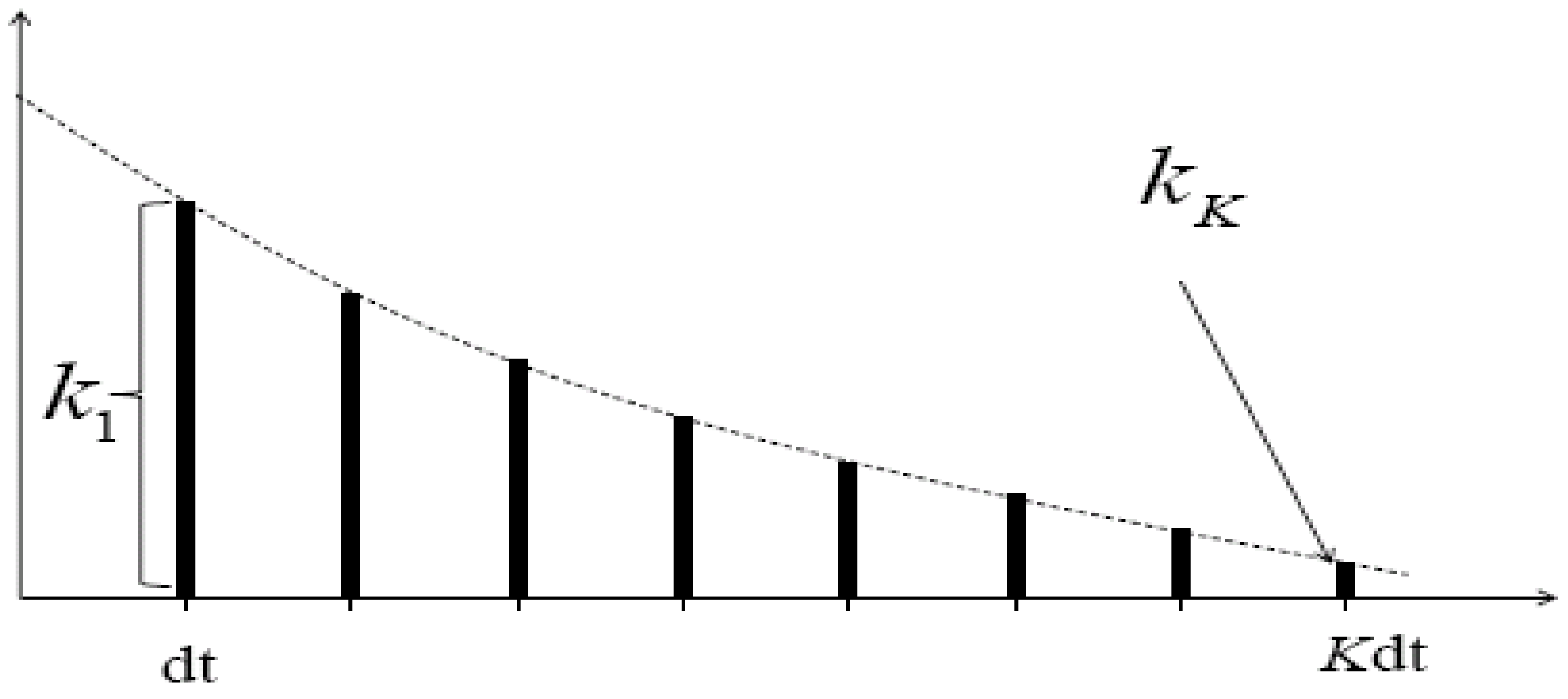
Not done in 2017

A) Predict spike times
B) Predict subthreshold voltage
C) Easy to interpret (not a ‘black box’)
D) Flexible
E) Systematic: ‘optimize’ parameters

Neuronal Dynamics – 9.5 Parameter estimation: voltage

Vector notation

$$u(t_n) = \sum_k k_k I_{n-k} + u_{rest}$$



$$u(t_n) = \vec{k} \cdot \vec{x}_n$$

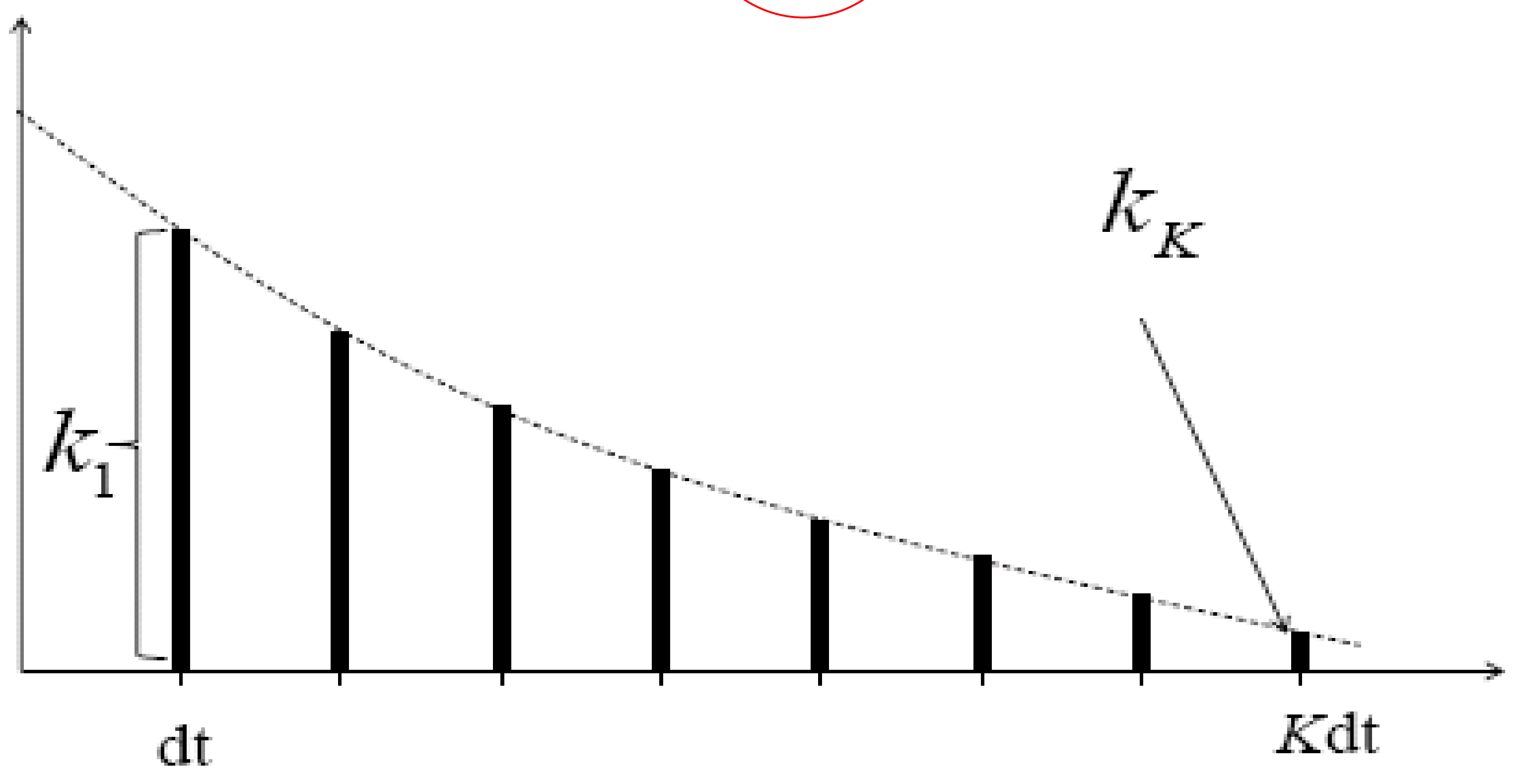
$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$

Linear in parameters = linear fit = quadratic problem

$$u(t) = \int_0^\infty \kappa(s) I(t-s) ds + u_{rest} \quad + \int_0^\infty \eta(s) S(t-s) ds$$

$$u(t_n) = \sum k_k I_{n-k} + u_{rest}$$

$$u(t_n) = \vec{k} \cdot \vec{x}_n$$



$$\begin{array}{c|c}
\text{time} \diagdown \text{input } \vec{\mathbf{x}} & \begin{array}{ccccc} x_1 & x_2 & x_3 & \dots & x_K \end{array} \\
\hline
\begin{array}{c} t{=}K{+}1 \\ t{=}K{+}2 \\ t{=}K{+}3 \\ . \\ . \\ . \\ t{=}T \end{array} & \left(\begin{array}{ccccc} I_K & I_{K-1} & I_{K-2} & \dots & I_1 \\ I_{K+1} & I_K & I_{K-1} & \dots & I_2 \\ I_{K+2} & I_{K+1} & I_K & \dots & I_3 \\ & & & & \\ & & & & \\ & & & & \\ I_T & I_{T-1} & I_{T-2} & \dots & I_{T-K+1} \end{array} \right) \mathrm{d}t
\end{array}$$

$$E = \sum_n [u^{data}(t_n) - \sum_{k=1}^{K} k_k I_{n-k} - u_{rest}]^2$$