COM-303 - Signal Processing for Communications

Homework #1

Exercise 1. Digital signals

Say which of the following signals are analog and which are digital:

(a) Music recorded on a CD.

(b) Music listened by the audience at a live concert.

(c) Music recorded on a LP record (vinyl).

(d) Photo recorded using a photographic film.

(e) Photo recorded using a CCD sensor.

(f) A page on a book.

(g) The image of a book page on a Kindle.

Exercise 2. Sampling music

A music song recorded in a studio is stored as a digital sequence on a CD. The analog signal representing the music is 2 minutes long and is sampled at a frequency $f_s = 44100\ s^{-1}$. How many samples should be stored on the CD?

Exercise 3. Elementary signals and operators

Using elementary operators, express the delta signal $\delta[n]$ in terms of the unit step $u[n]$ and conversely.

Exercise 4. Moving average

Consider the following signal,

$$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]. \tag{1}$$

Compute its moving average $y[n] = \frac{x[n]+x[n-1]}{2}$, where we call $x[n]$ the input and $y[n]$ the output.

Exercise 5. Operators and linearity

A *linear* operator is one for which the following holds:

$$\begin{cases} S\{\alpha x[n]\} = \alpha S\{x[n]\} \\ S\{x[n] + y[n]\} = S\{x[n]\} + S\{y[n]\} \end{cases}$$

(a) Show that the delay operator $D\{x[n]\} = x[n-1]$ is linear.

(b) Show that the squaring operator $S\{x[n]\} = x^2[n]$ is *not* linear.

Exercise 6. Operators with matrix notation

In $\mathbb{C}^N$, any linear operator on a finite-length signal $x[n]$ can be expressed as a matrix-vector multiplication. Let us see an example: in $\mathbb{C}^N$, define the delay operator as the left circular shift of a vector

$$D\{x[n]\} = [x[N-1]\ x[0]\ x[1]\ \dots\ x[N-2]]^T.$$

Assume $N = 4$ for convenience; it is easy to see that

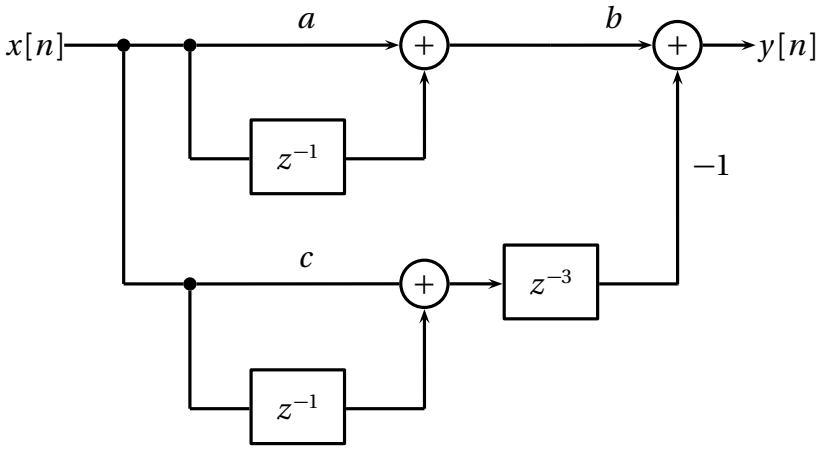
$$D\{x[n]\} = D\{\mathbf{x}\} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x} = \mathbf{D}\mathbf{x}$$

Write out the matrix form of the differentiation operator $\Delta\{x[n]\} = x[n] - D\{x[n]\} = x[n] - x[n-1]$ in $\mathbb{C}^4$.

Exercise 7. SP with Lego

Given the following filter:

What is the input-output relationship?


x[n]
a
b
y[n]
z^{-1}
−1
c
z^{-3}
z^{-1}