COM-303 - Signal Processing for Communications

Homework #4

Exercise 1. Implementing the DFT

You have been asked to implement a five-point DFT on a microprocessor that can only perform real-valued additions, subtractions and multiplications (in other words, there is no mathematical library with which to compute trigonometric functions, nor native support for complex numbers). In your code, you can store the values of just the two following numerical constants:

$$C = \cos(2\pi/5) \approx 0.309$$
$$S = \sin(2\pi/5) \approx 0.951$$

Write an algorithm that, for a real-valued input data vector $[x_0, x_1, x_2, x_3, x_4]$, computes the real and imaginary parts of its 5-point DFT using only additions, multiplications and the constants C, S. You can use any notation you prefer but try to be as clear as possible in your derivation.

Exercise 2. DTFTs

Compute the DTFTs of the following sequences:

(a) $x[n] = \frac{1}{2^n} u[n] - \frac{1}{4^n} u[n-1]$

(b) $x[n] = a^n \cos(\omega_0 n) u[n], \qquad |a| < 1$

Exercise 3. DTFT

Compute the DTFT of $x[n] = a^{|n|}$ for $n \in \mathbb{Z}$ and $|a| < 1$. Compute $X(e^{j\omega})$ and sketch its magnitude for a close to 1.

Exercise 4. DTFT

Consider a sequence $x[n] \in l_2(\mathbb{Z})$. Another sequence $y[n]$ is defined from $x[n]$ as

$$y[n] = \begin{cases} x[n] & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$$

Express $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$. [*Hint: the sequence* $(1+\cos(\pi n))/2$ *may prove useful.*]

Exercise 5. Modulation

Suppose $x[n]$ is a signal representing either voice or music. Since its energy is centered around $\omega = 0$ this is called a *baseband* signal. Baseband signals are good for our ears (which are baseband receivers) but they are not good for electromagnetic transmission (they don't travel far and they are all in the same region of the spectrum). Modulation is the process by which a baseband signal can be shifted up in frequency and transformed into a *passband* signal. Modulation is usually accomplished by multiplying the baseband signal by a cosine *carrier* at a given frequency.

(a) Consider now the following modulated signal and write its Fourier representation:

$$x[n] = \Re\{x[n]e^{j\omega_c n}\}$$

where ω_c is the carrier frequency. The signal $x[n]e^{j\omega_c n}$ is called the *complex passband signal* and, while not transmissible in practice, it is a very useful representation of the modulated signal in the derivations which will follow (especially in the case of Hilbert-filter demodulation as we have seen in class and as you will have to consider later). The actual passband signal is simply its real part.

(b) Assume that the baseband signal has spectral support $[-\omega_b/2,\ \omega_b/2]$ (i.e. its energy is zero for frequencies above $\omega_b/2$ – we also express this by saying that the *bandwidth* of the baseband is ω_b). What is the maximum carrier frequency we can use to create a passband signal?

(c) Consider the general AM system with $F_s = 48000$ and assume that the baseband signal has a bandwidth of 8 KHz. What is the maximum carrier frequency (in Hertz)?
