Auctions and Mechanism Design

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Social Choice

Many situations require a group of agents to agree on a *social choice*:

choose one of k outcomes $o_1, .., o_k$ *that affect all agents.*

- Who should be president?
- Which route to choose for a future highway?
- Who becomes owner of land parcel 325?
- What construction company gets the contract to build the highway?

Chosen outcome should reflect agent preferences.

Formal Model

- Each agent i has a preference order $\prec_i$ that orders the alternatives, for example for agent 5, it could be:

$$o_3 \prec_5 o_7 \prec_5 \ldots \prec_5 o_4$$

- The desired outcome is a *social choice function* $SCF(\prec_1, \ldots, \prec_k)$ that constructs a combined order $\prec^*$.
- However, agent preferences are private: known only to the agent.
- Agents do not have to tell them truthfully.
- Mechanism = game that can obtain $\prec^*$ with the *true* agent preference orders.

Expressing Preference Orders with Utilities

- Preferences can have different strengths.
- Reflected by *utilities* of each outcome $u(o_j)$.

$\Rightarrow$ allows to express risk attitude of agents.

- Assume preference orders expressed by each agent i declaring its utility $u_i(o_j)$ for each outcome j.

Auctions

Auctions = social choice:

- Decision = o_i = "agent i gets the resource"
- Preference orders specified by utilities for the resource.
- Agent actions = declare their utility for the resource (bid) $u_i(o_i)$, all other utilities $u_i(o_j) = 0, j \neq i$.
- Social choice function = resource should go to the agent that values it the most, i.e. agent i that has the strongest preference for o_i.

$\Rightarrow$ pick the agent that declared the highest value.

Misdeclarations

- Best strategies for agents: exaggerate their value!
- Picking the highest declaration does not implement social choice function.
- Solution: make agent *pay* the declared value.

$\Rightarrow$ exaggeration is costly.

Forward/reverse auctions

Forward auctions (e.g. art auctions):

- auctioneer = seller
- highest bid wins

Reverse auctions (e.g. procurement auctions):

- auctioneer = buyer
- lowest bid wins

Theory mostly identical: focus on forward auctions

Applications of e-auctions

- Stock and commodity markets
- Bandwidth and electricity allocation (band-x, Scandinavia)
- Unique consumer goods (e-bay)
- Internet Ad auctions
- Load balancing, scheduling

Success criteria for auctions

Optimal allocation (Pareto efficiency) =

resources end up with those who value them the most

Individual rationality =

every participant gains non-negative utility

Auction settings

Private value:
value of items depends only on agent's preferences, e.g. haircut
Common value:
value of items determined entirely by other's values, e.g. bank notes, gold bars
Correlated value:
value depends partly on own and other's values, e.g. a Picasso painting

Classifying auctions

- open-cry: bids are public vs.
  sealed-bid: bids are secret
- first-price: winner pays the highest bid vs.
  second-price: winner pays the second-highest bid

	open-cry	sealed-bid
first-price	Dutch	Discriminatory
second-price	(English)	Vickrey

Auction example

- A(lice), B(bob) and C(laude) are all interested in a painting.
- A values it a 3, B at 6, and C at 5.

$\Rightarrow$ B likes it the most: should win the auction.

Auction protocols: Dutch

- Auctioneer continuously lowers price until a bidder takes the item at the current price.
- Strategy: down-bias bid depending on other bidders. Since it is unlikely that another bidder will have a valuation just below ours, we can bid a bit less and still probably win the auction.
- Advantage: efficient, reveals only information about winner.
- Dutch flower market, fish markets.
- Example: start at 10, B should take the item when price dropped to 6 (but could speculate and wait until $5 + \epsilon$).

Auction protocols: English

- Bidders raise their bids until nobody is willing to go any higher.
  The item is then sold to the highest bid.
- Strategy: bid in small increments until own valuation is reached.
- Advantage: no speculation (but long process)
- Note: price paid = second highest + ϵ.
- Art auctions, etc.
- Example: starting at 1, all bid up the price; when it reaches 5 only B continues to bid and wins at 5 + ϵ.

Auction protocols: Discriminatory

- Each bidder submits one secret bid, without knowing the others' bids. The item is sold to the highest bidder.
- Strategy: downward bias depending on other bidders.
- Advantage: one round of bidding, no information revealed.
- Contracting in construction, etc.
- Example: all submit their value, B wins with $6 - \delta$.

Auction protocols: Vickrey

- Each bidder submits one secret bid, without knowing the others' bids. The item is sold to the highest bidder, but at the price of the *2nd-highest* bid.
- Strategy:
  - bidding less: lower probability of winning the bid, but winning amount unchanged
  - bidding more: same probability of profitable transaction, but possibility of unprofitable transaction

  $\Rightarrow$ bid true valuation.
- Advantage: no speculation, single round of bidding, no information revealed.
- Example: all submit their value, B wins with bid 6 but pays only 5.

Bidding strategies

- *Truthful* bidding = bidding one's true valuation.
- Optimal bidding: bid just enough to win the auction.

$\Rightarrow$ take into account other bidders:

  - valuations
  - budgets

Optimal bids (risk-neutral, private value)

	open-cry	sealed-bid
first-price	v(2nd)$+\epsilon$	v(2nd) $+\epsilon$
second-price	v	v

- v(2nd) $=$ estimate of next highest valuation.

$\Rightarrow$ equivalent revenue to auctioneer.
(revenue equivalence theorem)

- But bidders may make wrong estimate
$\Rightarrow$ suboptimal allocation.

Human bidders

- Risk-averse bidders:
  Dutch,Discriminatory $\geq$ English, Vickrey
- Because speculation involves risk.
- People sometimes have irrational behavior:
  Dutch auction: bid lower because of suspense.
  Vickrey auction: bid higher because you won't pay that price.

Non-private value settings

- Other bids influence own bid.
- Revenue non-equivalence:
  English $\geq$ Vickrey $\geq$ Dutch $=$ first-price sealed bid.

Vulnerability to collusion

- Collusion = group of buyers coordinate their bidding.
- Suppose $v_1 = 20$, $v_i = 18, i \geq 2$
- English auction: Agent 1 bids 6, all others bid 5; self-enforcing.
- Vickrey: Agent 1 bids 20, all others bid 5; self-enforcing.
- Dutch/Discriminatory: If Agent 1 bids below 18, others are motivated to break the agreement.

Bidding strategies for auctions

- For truthful protocols, straightforward: bidding true valuation is a *dominant* strategy.
- For non-truthful protocols, best bid depends on other bids, i.e. there is no dominant strategy.
- However, there are always Nash equilibra: no agent can do better given the others' strategy.
- Can be infinitely many: not clear how agents would synchronize...

Example: first price sealed bid auction

- n bidders have valuations drawn form the same distribution with density function F(x), with A the lowest valuation, then

$$b(t) = t - \frac{\int_A^t F(x)^{n-1} dx}{F(t)^{n-1}}$$

  is the bid for a bidder with valuation t. (McAfee and McMillan, 1987)
- For uniform distribution in [0..1]:

$$b(t) = \frac{n-1}{n} t$$

- Valid only if all bidders use this formula!

Example (first price)

valuations:

bidder	1	2	3	4	5
value	1	0.8	0.6	0.4	0.2
bid	0.8	0.64	0.48	0.32	0.16

Winning bidder pays 0.8
= second price (coincidence?)
Nobody can do better by unilaterally changing strategy.
(Nash equilibrum)

Multiple units

Example:

- 3 identical items
- 4 buyers willing to pay Fr. 400, 300, 200, 100

$\Rightarrow$ selling prices:

- first item sold for Fr. 300
- second item sold for Fr. 200
- third item for Fr. 100

Problems with multiple units

- Unfair! Everybody pays a different price.
- Every agent wants to pay the lowest possible price

$\Rightarrow$ non-truthful bidding behavior.

Generalizations

Uniform price auction:

n units for sale $\Rightarrow$ *each agent pays* $n+1$*st highest bid*

an instance of Multi-unit Vickrey auction:

Each agent pays price of the bid it displaced from the set of winning bids

Solves problem of strategic bidding

Example (multi-unit Vickrey)

- 3 identical items
- 4 buyers willing to pay Fr. 400, 300, 200, 100

$\Rightarrow$ Fr. 100 bid is displaced

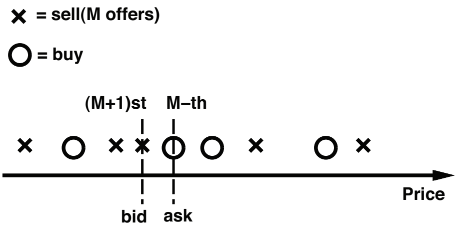
$\Rightarrow$ everyone pays Fr. 100

- Note: significantly lower revenue!
- But: without uniform price, some agents might not bid at all!
- When there are many buyers, loss with uniform price is small and outweighs loss because of fewer bidders.

Double auctions

- M identical items sold by M different sellers.
- Both N buyers *and* M sellers make bids:
  - Buy bid: maximum price to buy an item
  - Sell bid: minimum price to sell an item
- Every pair of sell bid and higher buy bid is a possible exchange.
- Multi-unit $\Rightarrow$ uniform price.
- What price allows a maximum number of transactions?

Clearing double auctions



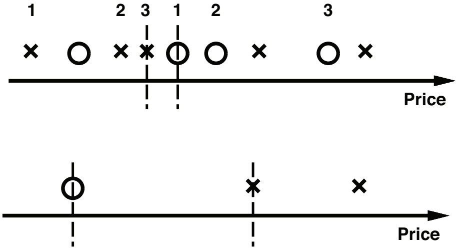
M sell and N buy bids $\Rightarrow$ price determination:

- M-th highest price (*ask* quote)
- (M+1)-st highest price (*bid* quote)
- fraction in between

Guarantees that the market will clear at this price.

Clearing the market

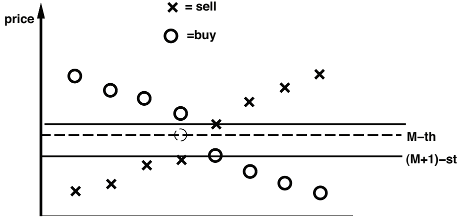
All buy bids $\geq$ M-th highest have a sell bid $\leq$ M-th highest
$\Rightarrow$ can always match buyers and sellers at price p,
(M+1)-st $\leq$ p $\leq$ M-th



$\Rightarrow$ new price quotes; no further transactions

Properties of the price

- Sort buy bids in decreasing, sell bids in increasing order



- Price is determined by first pair of non-trading bids.
- Sometimes one of the trading buy (M-th) or sell ((M+1)-st) bids is also involved.

Incentive-compatibility

M-th price is incentive-compatibile for sellers:

- if M-th price $=$ sell bid, the seller does not participate $\Rightarrow$ no gain from lying. However, could lead to unprofitable sale if underbid or loss of profitable sale if overbid.
- if buy bid, not influenced by any seller so no incentives to lie.

But not incentive-compatible for buyers if falls on buy bid!
Symmetric analysis for $(M+1)$-st price and buyers

Impossibility result

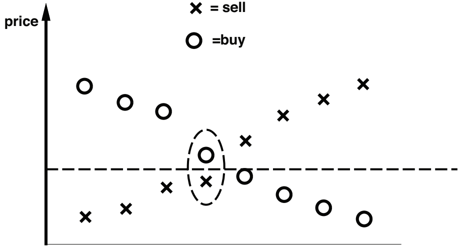
Incentive-compatible mechanism:

- sellers get M-th highest price (*ask* quote)
- buyers pay (M+1)-st highest price (*bid* quote)

requires subsidies: somebody has to pay the difference!
No mechanism satisfies all 4 properties:
IC, IR, efficiency, and budget-balance
(Myerson,Sattth‌erwaite)

McAffee auction

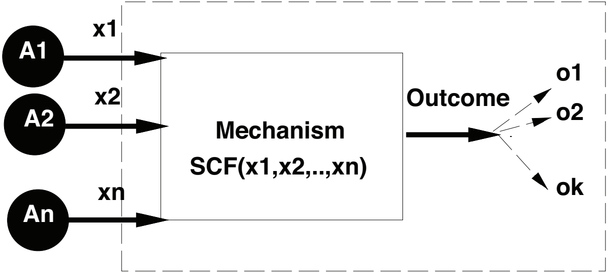
Price = average of last sell/buy combination that can still be matched up.



Exclude this combination from trading ⇒ incentive-compatible:

- do not trade ⇒ no gain from lying
- potential loss of trading opportunity or forced to accept unprofitable opportunity when lying

Mechanisms



- Mechanism = game between $A_1, .., A_n$ where agent agent actions determine the choice of outcome.
- Outcome determines utility to each agent $\Rightarrow$ actions chosen according to preference on outcome.
- Mapping from agent *actions* $x_1..x_n$ to *outcome* $\in \{d_1, .., d_k\}$ so that outcome implements the desired social choice function SCF.

Implementations of social choice functions

Dominant-strategy implementation of a social choice function = mechanism where

- for any utility function of the agents.
- the dominant strategy equilibrium will produce the outcome defined by the social choice function.

Bayes-Nash implementation of a social choice function = mechanism where

- for any utility function of the agents.
- there is a Bayes-Nash equilibrium where the agent actions produce the outcome defined by the social choice function.

Generalizing the Vickrey auction

- Vickrey auction: winning agent pays next highest bid.
- $=$ the lowest bid that would allow it to win the auction.
- The auction protocol does the speculation!

$\Rightarrow$ Truthful utility declaration is the best strategy.

- Can we generalize this?

Revelation principle

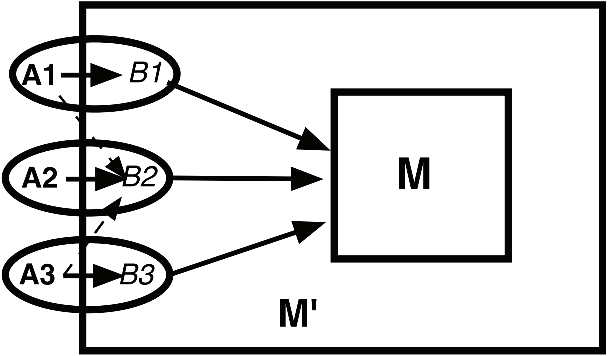
Revelation principle:

> *For any mechanism, there is a* truthful *mechanism with the same outcome and payments.*

Constructive proof:

- given a mechanism (possibly non-truthful) M and a set of participating agents A_i.
- for each agent A_i, construct a mechanism agent B_i. A_i truthfully reports its valuations to B_i and B_i will interact with M as A_i would.
- now construct a truthful mechanism M' that consists of M and the mechanism agents B_i for all agents.
- each A_i has the best interest to report its true valuations to B_i, so M' is truthful.

Revelation principle



$\Rightarrow$ can restrict attention to truthful mechanisms only.

Quasi-linear preferences

- Consider a set of k alternatives $o_1, o_2, .., o_k$.
- Each agent A_i has a numerical valuation $v_i(o_j)$ for each alternative.
- Furthermore, the mechanism makes each agent A_i pay a price p_i.
- The agent has quasilinear preferences if its utility in the mechanism making decision o^m is $u_i(m) = v_i(o^m) - f_i(p_i)$.
- Assume here that f_i is identity function, i.e. agent is risk-neutral and utility is transferable.

$\Rightarrow$ influence agent actions through payments.

Direct revelation mechanism design

Direct revelation mechanism:

1. agents declare their value for each outcome.
2. mechanism applies social choice function $\Rightarrow$ outcome.
3. agents make/receive payments.

Goal is incentive-compatibility:

- best strategy is to act so that outcome achieves the social choice function.
- achieved through payments that align incentives.
- here: incentive-compatibility $\simeq$ *truthful* declaration.

Constraints on payment scheme

- Agents are often free to not participate in the mechanism.

$\Rightarrow$ *individual rationality*: the agent should be better off participating in the mechanism than not.

- Budget-balance: mechanism should not require external subsidies.

$\Rightarrow$ not all social choice functions can be implemented!

Existence of truthful mechanisms

Some social choice functions can be implemented truthfully, e.g.:

- random dictator: pick an agent at random and let it make the decision.
- maximizing social welfare with quasilinear utilities.
- auctions: sell an item to the one who has the highest utility (special case).

but many can not:

- Nash bargaining solution or Shapley values.
- voting.
- ...

Consequence of revelation principle: there is no mechanism where rational agents with private utilities agree on the Nash bargaining solution!

The Clarke tax mechanism

- Social choice function: maximize the sum of declared valuations (affine maximizer).
- The only social choice function for quasi-linear preferences for which a general incentive-compatible scheme exists (Roberts 1979).
- Idea of the mechanism: make every agent pay a *tax* that punishes it for the damage it does to the other agents.
- o_{all} = choice that is optimal for all agents
  o_{-i} = choice that is optimal for all agents except agent A_i

$$tax(A_i) = \sum_{A_j \in A, A_j \neq A_i} (v_j(o_{-i}) - v_j(o_{all}))$$

- Also called *Vickrey-Clarke-Groves (VCG)* tax.

Auctions as VCG tax mechanisms

An auction is a VCG mechanism:

- outcomes = who gets the item.
- if winning agent a_i is not present, item should go to agent with next valuation u_j: social welfare of remaining agents is $= u_j$.
- if winning agent is present, social welfare of remaining agents is $= 0$, as none of them gets the item.

$\Rightarrow$ payment $= u_j - 0 =$ second price.

- VCG tax generalizes Vickrey auction.

Properties of the VCG tax

The VCG tax is:

- non-negative: since agents could use o_{all} in place of o_{-i}, so if o_{-i} is valued less it would not be optimal.
- individually rational: for each agent A_i,

$$\begin{aligned}
v_i(o_{all}) - tax(A_i) &= v_i(o_{all}) - \sum_{A_j \in A, A_j \neq A_i} (v_j(o_{-i}) - v_j(o_{all})) \\
&= (\sum_{A_j \in A} v_j(o_{all})) - (\sum_{A_j \in A, A_j \neq A_i} v_j(o_{-i})) \\
&\geq (\sum_{A_j \in A} v_j(o_{-i})) - (\sum_{A_j \in A, A_j \neq A_i} v_j(o_{-i})) \\
&= v_i(o_{-i})
\end{aligned}$$

The VCG tax aligns incentives

Total utility of agent i:

$$v_i(o_{all}) - tax(A_i) = (\sum_{A_j \in A} v_j(o_{all})) - (\sum_{A_j \in A, A_j \neq A_i} v_j(o_{-i}))$$

The first term:

$$(\sum_{A_j \in A} v_j(o_{all}))$$

is the sum of all agents' valuations, the goal to be optimized

The second term:

$$(\sum_{A_j \in A, A_j \neq A_i} v_j(o_{-i}))$$

is not at all influenced by A_i

$\Rightarrow A_i$'s interest: optimize sum of all valuations!

The VCG tax is truthful

- Let $v_i(o_j)$ be the true valuation and $v_i'(o_j)$ the declared valuation.
- Note that the tax paid by agent A_i depends only on the declarations of the other agents and the chosen solution.

$\Rightarrow$ if misdeclaration does not change the chosen solution, it has no effect on the agent's utility.

The VCG tax is truthful (2)

Case 1): $v_i'(o_j) < v_i(o_j)$ (underdeclaration): this will make a difference only if $v_i'(o_{all}) < v_i(o_{all})$, and there is another solution d' such that:

$$\begin{aligned}
v_i'(d') + \sum_{A_k, k \neq i} v_k(d') &> v_i'(o_{all}) + \sum_{A_k, k \neq i} v_k(o_{all}) \\
v_i(d') + \sum_{A_k, k \neq i} v_k(d') &< v_i(o_{all}) + \sum_{A_k, k \neq i} v_k(o_{all}) \\
v_i(d') + \sum_{A_k, k \neq i} v_k(d') - v_k(o_{all}) &< v_i(o_{all}) \\
v_i(d') + tax(o_{all}) - tax(d') &< v_i(o_{all}) \\
v_i(d') - tax(d') &< v_i(o_{all}) - tax(o_{all})
\end{aligned}$$

so that the agent looses utility from the speculation.

The VCG tax is truthful (3)

Case 2): $v_i'(o_j) > v_i(o_j)$ (overdeclaration): this will make a difference only $v_i'(d') > v_i(d')$ and d' thus becomes better than o_{all}:

$$\begin{aligned}
v_i'(d') + \sum_{A_k, k \neq i} v_k(d') &> v_i'(o_{all}) + \sum_{A_k, k \neq i} v_k(o_{all}) \\
v_i(d') + \sum_{A_k, k \neq i} v_k(d') &< v_i(o_{all}) + \sum_{A_k, k \neq i} v_k(o_{all}) \\
v_i(d') + \sum_{A_k, k \neq i} v_k(d') - v_k(o_{all}) &< v_i(o_{all}) \\
v_i(d') + tax(o_{all}) - tax(d') &< v_i(o_{all}) \\
v_i(d') - tax(d') &< v_i(o_{all}) - tax(o_{all})
\end{aligned}$$

so the agent again looses utility from the speculation.

Problems with the VCG tax for Social Choice

- Collusion: if I can convince another agent to claim exactly the same valuations as I do, we can both exaggerate our valuations. I get my solution and neither I nor the other agent pay anything.
- What to do with the tax: the tax cannot be returned to the agents, it must be wasted.
- Not pareto-efficient: the agents loose utility through paying the tax.
- Requires that solutions are truly optimal, not just approximations $\Rightarrow$ hard to apply to large problems.

Are there alternatives to VCG?

Theorem (Roberts): for at least 3 outcomes and unrestricted preference profiles, the only social choice functions that can be implemented in incentive-compatible mechanisms are affine maximizers:

$$f = argmax_{o \in O' \subset O}(c_o + \sum_i w_i v_i(o))$$

where:

- O is the set of alternatives.
- c_o is some constant for each alternative.
- i ranges over all agents.
- w_i is the weight of agent i.
- $v_i(o)$ is the utility agent i assigns to alternative o.

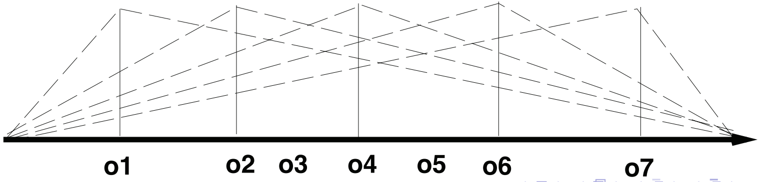
Monotonicity

- A social choice function F is *weakly monotone* if whenever it chooses o_j under agent preference profile $\preceq$ and o_l under agent preference profile $\preceq'$ that differ only in the preference of agent A_i, A_i prefers o_j more strongly over o_l in $\preceq$ than in $\preceq'$.
- Theorem: all incentive-compatible social choice functions F are weakly monotone, and any weakly montone function F with convex preferences is incentive-compatible.
- Theorem: for quasi-linear preferences, the only incentive-compatible social choice functions are affine maximizers.
- General characterization of incentive-compatible mechanisms with payments!

Alternative IC mechanisms

Restrict possible preference profiles:

- assume alternatives have a total order $o_1, ..., o_k$.
- Allow only *single-peaked* preferences, i.e. Agent A_i prefers o_j the most and $o_1 \prec ... \prec o_j$ and $o_j \succ ... \succ o_k$.
- Example: location of the fire station along a road.
- Agents declare their most preferred outcome.
- Choose as joint outcome the *median* $= o^*$ such that at most half the agents prefer an outcome below and at most half an outcome above.



Why the median rule is IC

Suppose A_i prefers o_j, and the median of the other agents' preferences is o_{-i}.
Suppose $o_j > o_{-i}$, for example $o_j = o_6$ and $o_{-i} = o_4$.

- a truthful report o_j moves the median to a value x such that $o_{-i} \leq x \leq o_j$, such as o_5.
- suppose A_i reports $o_j' > o_j$. Since o_j is already $\geq x$, this does not affect x.
- suppose A_i reports $o_j' < o_j$, and $o_j' \geq x$. Then again x is not affected.
- suppose A_i reports $o_j' < o_j$, and $o_j' < x$. Then the median may move to a different value $x' \leq o_j' < x$, for example o_3. Since already $x < o_j$, x' will be less preferable than x'.

$\Rightarrow$ never profitable for A_i to not be truthful!

Symmetric analysis for $o_j < o_{-i}$.

Truthful Information Extraction

- Problem solved by mechanisms: extract private information for use in decision mechanisms.
- Can also design incentives just for extracting information.
- Example:
  - ask people to label an image.
  - pay a reward if two people give the same label.
  - scale the reward according to difficulty.

$\Rightarrow$ incentive to report truthfully.

Output agreement

Multiple agents observe the same signal $x \in \{0, 1\}$ and report to center:

- quality of a product.
- correctness of an exam question.
- etc.

For report r, randomly choose *peer* report s:

- if $r = s$, pay reward, otherwise pay nothing.
- agent strategies: report truthfully, always report 0, etc.

$\Rightarrow$ truthful equilibrium: if s is truthful, then reward is highest if r is truthful as well.

Subjective Data

What if agents observe samples from a distribution?

- Reviewing a restaurant: everyone gets a different plate.
- Sample drawn by peer agent not necessarily of same quality.
- Assume agents have prior belief $p(i)$ for each possible value i.
- Bayesian updating: agent observes "0" $\Rightarrow$ posterior belief q that peer also observes "0" increases over prior:

$$
\begin{aligned}
q(0) &= p(0)(1-\epsilon)+\epsilon \\
q(1) &= p(1)(1-\epsilon)
\end{aligned}
$$

- But if $p(1) >> p(0)$, could still be that $q(1) > q(0)$

$\Rightarrow$ output agreement not truthful.

Peer Truth Serum

- Payment scheme:

$$pay(r, s) = \begin{cases} 1/p(r) - 1 & \text{if } r = s \\ -1 & \text{otherwise} \end{cases}$$

- Observed "0":
  reported "0": $E[pay] = q(0)/p(0) - 1 = \epsilon p(1)/p(0) > 0$;
  reported "1": $E[pay] = q(1)/p(1) - 1 = -\epsilon < 0$

$\Rightarrow$ Bayes-Nash (ex-ante) equilibrium: report truthfully.

- Example prior: 95% positive (1), 5% negative (0), $\epsilon = 0.1$
- Observed "0":
  expected payoff for "0": 0.1(20-1) = 1.9, for "1": - 0.1

Uninformative Equilibria

- Another equilibrium: everyone reports "0"!

$\Rightarrow$ Payment when reporting "0": $p(1)/p(0) = 19$; when reporting "1": -1.

- Higher payoff than truthful equilibrium!
- Avoid by taking prior = distribution of reports.

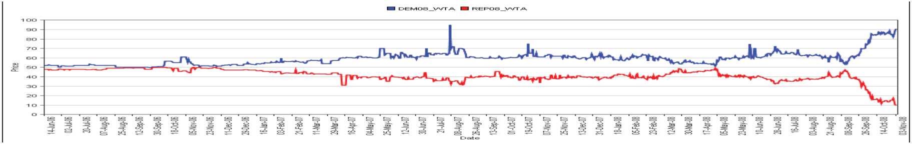
$\Rightarrow$ payoff of any heuristic equilibrium = 0, only truthful strategy pays a reward.

Prediction Markets

- Like a stock market: agents trade *securities* that pays 1 if event happens, 0 if it does not.

$\Rightarrow$ price of security converges to p(event).

- Example: US 2008 presidential election (Iowa Electronic Market):



Summary

- Auction protocols and their properties.
- Bidding strategies.
- Mechanisms for social choice.
- Truthful mechanisms.
- VCG mechanisms.
- Truthful information extraction.