COM-303 - Signal Processing for Communications

Homework #7

Exercise 1. Interleaving

Consider two two-sided sequences $h[n]$ and $g[n]$ and consider a third sequence $x[n]$ which is built by interleaving the values of $h[n]$ and $g[n]$:

$$x[n] = \dots, h[-3], g[-3], h[-2], g[-2], h[-1], g[-1], h[0], g[0], h[1], g[1], h[2], g[2], h[3], g[3], \dots$$

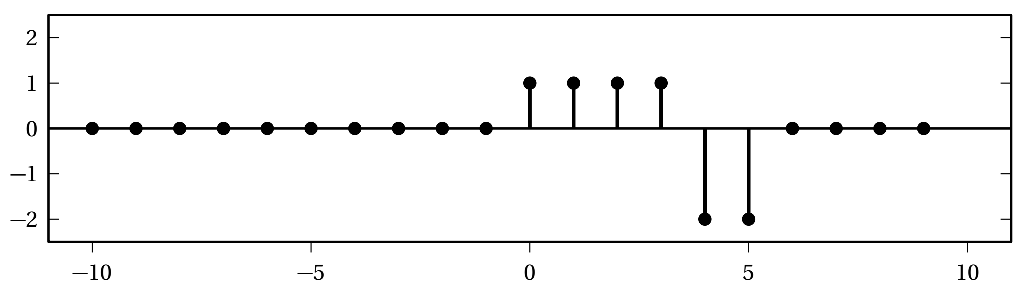
with $x[0] = h[0]$.

(a) Express the z-transform of $x[n]$ in terms of the z-transforms of $h[n]$ and $g[n]$.

(b) Assume that the ROC of $H(z)$ is $0.64 < |z| < 4$ and that the ROC of $G(z)$ is $0.25 < |z| < 9$. What is the ROC of $X(z)$?

Exercise 2. Impulse Response

Consider an LTI system with the following FIR impulse response:

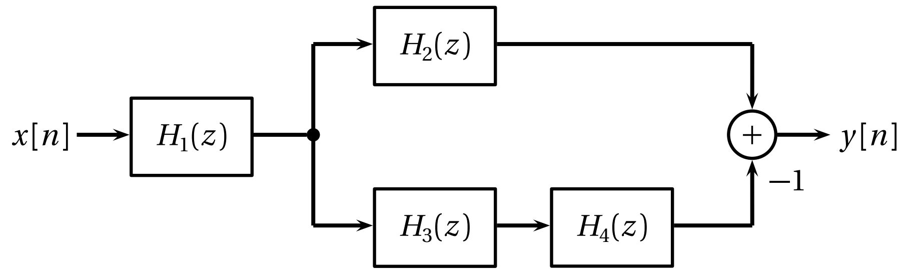


(a) Determine and carefully sketch the response of this system to the input $x[n] = u[n-4]$.

Now consider the causal system shown here where the impulse responses of the separate blocks are:

- $h_1[n] = 3(-1)^n(\frac{1}{4})^n u[n-2]$

- $h_2[n] = h_3[n] = u[n+2]$
- $h_4[n] = \delta[n-1]$



(b) Calculate the impulse response of the system

(c) Determine system's BIBO stability.

Exercise 3. Generalized Linear Phase Filters

Consider the filter given by $H(z) = 1 - z^{-1}$.

(a) Show that $H(z)$ is a generalized linear phase filter, i.e. that it can be written as

$$H(e^{j\omega}) = |H(e^{(j\omega)})|e^{-j(\omega d - \alpha)}.$$

Find the delay d and the phase factor α.

(b) What type of filter is it (I, II, III or IV)? Explain.

(c) Give the expression of $h[n]$ and show that it satisfies

$$\sum_n h[n]\sin(\omega(n-d)+\alpha) = 0$$

Exercise 4. Zero-Phase Filtering

Consider an operator $\mathscr{R}$ which turns a sequence into its time-reversed version:

$$\mathscr{R}\{x[n]\} = x[-n].$$

(a) The operator is clearly linear. Show that it is *not* time-invariant.

Suppose you have an LTI filter $\mathscr{H}$ with impulse response $h[n]$ and you perform the following sequence of operations in order:

1) $s[n] = \mathscr{H}\{x[n]\}$
2) $r[n] = \mathscr{R}\{s[n]\}$
3) $w[n] = \mathscr{H}\{r[n]\}$

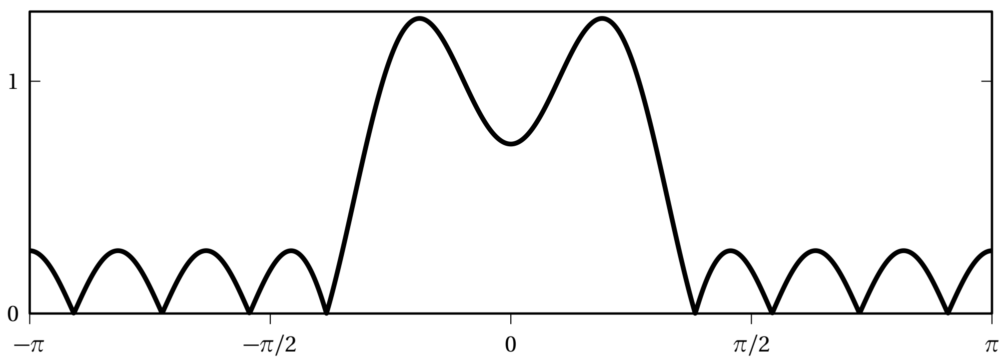
4) $y[n] = \mathscr{R}\{w[n]\}$

(b) Show that the input-output relation between $x[n]$ and $y[n]$ is an LTI transformation.

(c) Give the frequency response of the equivalent filter realized by the series of transformations and show that it has zero phase.

Exercise 5. Optimal FIR Filters

Consider the FIR filter $h[n]$ whose magnitude response $|H(e^{j\omega})|$ is plotted in the following figure:



The filter is optimal (in the sense of Parks-McClellan);

(a) What type is it (I, II, III, IV)?

(b) What is the length of the filter (the number of taps) ?

(c) Can you determine if the filter is causal?

(d) Sketch the magnitude response of a filter $h_1[n]$ whose impulse response is

$$h_1[n] = h[n]\cos(\pi n)$$

Exercise 6. Demodulation

Demodulation can be achieved in several ways:

A) Classic demodulation: Multiply the received signal by a carrier at the same frequency and filter with a lowpass filter with cutoff at least $\omega_b/2$.

B) Complex demodulation: Create a complex signal $c[n] = a[n] + jb[n]$ where $a[n]$ is the received signal and $b[n]$ is obtained by filtering the received signal with a zero-delay Hilbert filter; then multiply $c[n]$ by the complex exponential $e^{-j\omega_c n}$ and take the real part.

C) Galena demodulation Pass the received signal through a nonlinearity (e.g. compute $|y[n]|$) and filter the result with a 1-pole IIR lowpass filter (i.e. a leaky integrator). This method was actually used in early radio receivers.

With respect to the above demodulation schemes, answer to the following questions:

(a) For the demodulation schemes A and B, write a detailed derivation of the demodulation process (in the frequency domain, of course) and thus prove that it works. In particular, for scheme A, explain the constraints on the lowpass filter design. Along the line, don't forget to point out the various problems you might encounter in a practical realization of the system.

(b) For demodulation scheme C, try to explain why it works by sketching the various waveforms in the time domain.