COM303: Digital Signal Processing

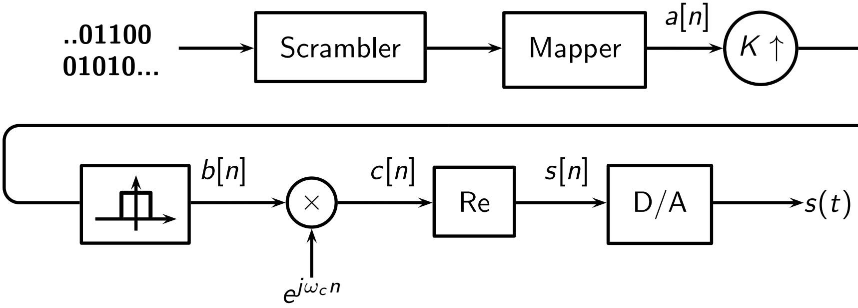
Lecture 23: Digital Communication Systems (II)

overview

- QAM receiver design
- ADSL

receiver design

QAM transmitter, final design



It's a dirty job...

but a receiver has to do it:

- propagation delay
- channel effects
- interference
- clock drift

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- propagation delay $\rightarrow$ handshake and delay estimation
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- propagation delay $\rightarrow$ handshake and delay estimation
- channel effects $\rightarrow$ adaptive equalization
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- propagation delay $\rightarrow$ handshake and delay estimation
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but a receiver has to do it:

- propagation delay $\rightarrow$ handshake and delay estimation
- channel effects $\rightarrow$ adaptive equalization
- interference $\rightarrow$ line probing
- clock drift $\rightarrow$ timing recovery

A blast from the past

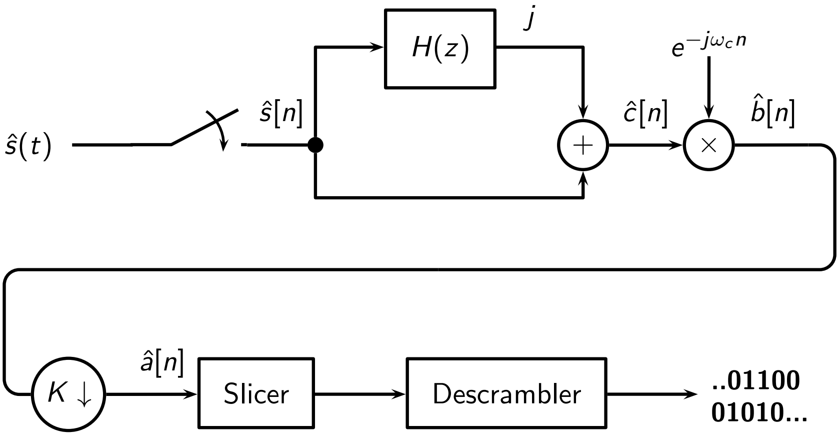
Play

A blast from the past

Play

- a sound familiar to anyone who's used a modem or a fax machine
- what's going on here?

Remember the (simplified) receiver



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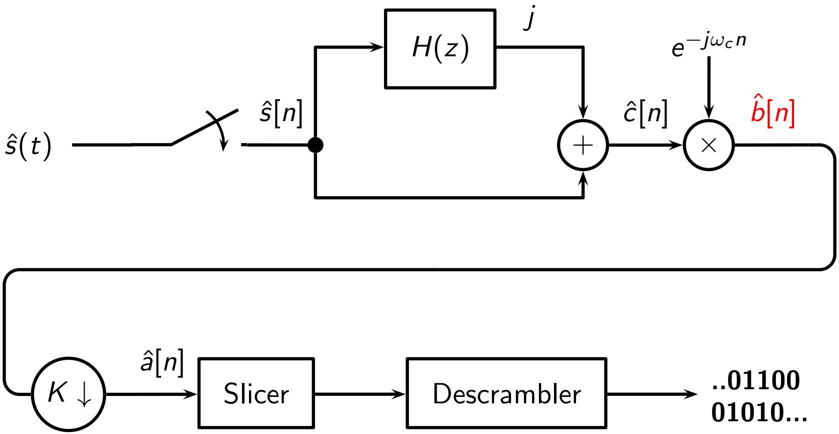
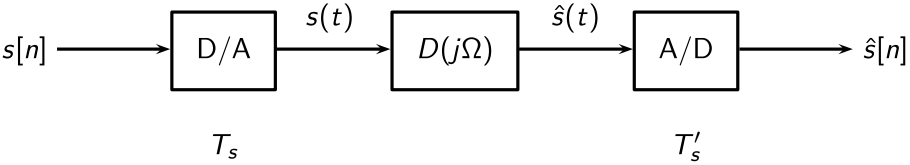


Visually, in slow motion, plotting $b[n]$

Pilot tones

$$\text{if } \hat{s}[n] = \cos((\omega_c + \omega_0)n) \text{ then } \hat{b}[n] = e^{j\omega_0 n}$$

The main problems



- noise
- propagation delay
- channel distortion $D(j\Omega)$
- different clocks ($T'_s \neq T_s$)

Delay compensation

Assume the channel is a simple delay: $\hat{s}(t) = s(t-d) \Rightarrow D(j\Omega) = e^{-j\Omega d}$

- channel introduces a delay of d seconds
- we can write $d = (L + \tau)T_s$ with $L \in \mathbb{N}$ and $|\tau| < 1/2$
- L is called the *bulk delay*
- τ is the fractional delay

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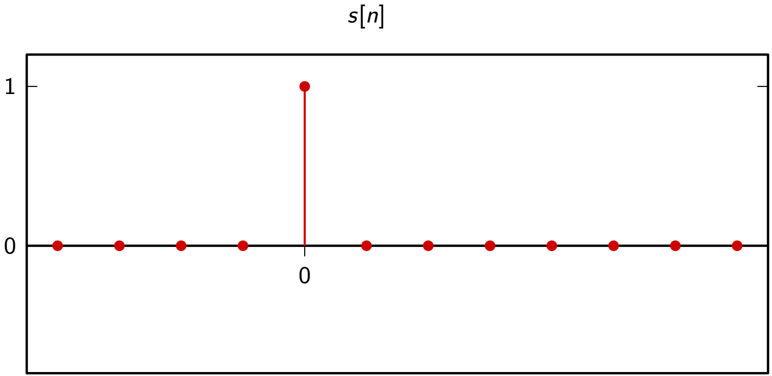
Estimating the bulk delay ($T_s = 1$)



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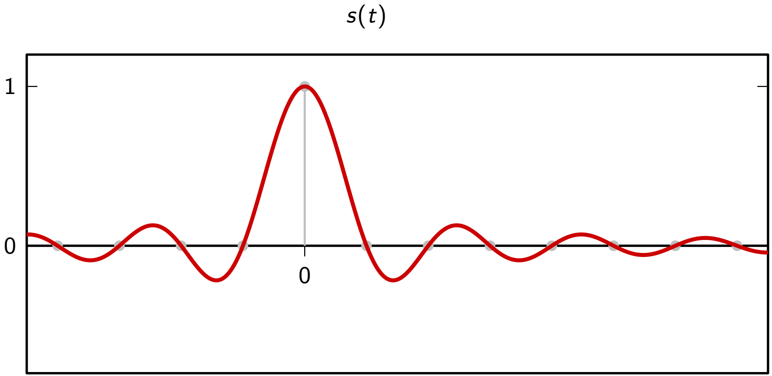

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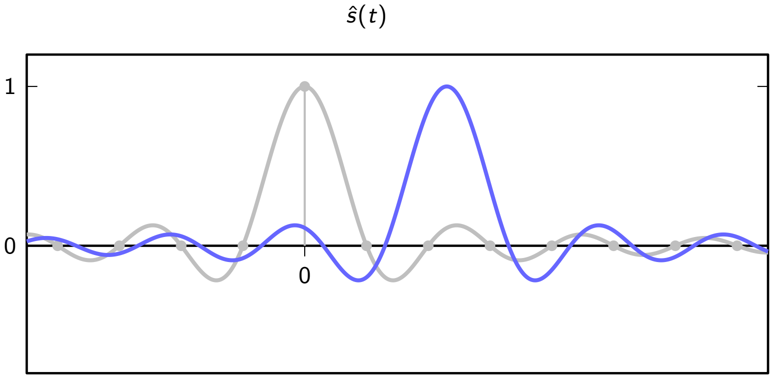

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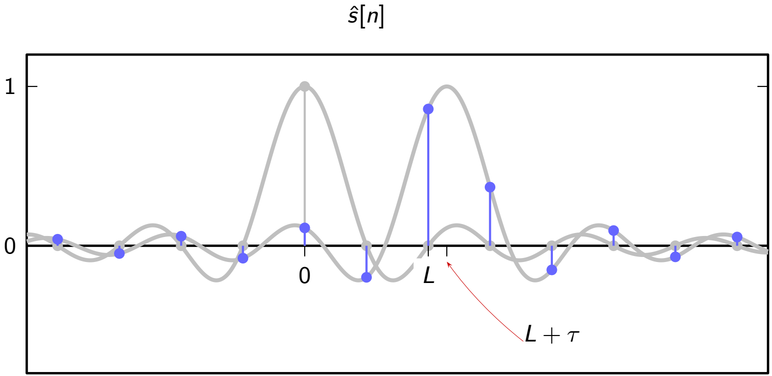

Estimating the fractional delay

- transmit $b[n] = e^{j\omega_0 n}$ (i.e. $s[n] = \cos((\omega_c + \omega_0)n)$)
- receive $\hat{s}[n] = \cos((\omega_c + \omega_0)(n - L - \tau))$
- after demodulation and bulk delay offset:

$$\hat{b}[n] = e^{j\omega_0(n-\tau)}$$

- multiply by known frequency

$$\hat{b}[n]\, e^{-j\omega_0 n} = e^{-j\omega_0 \tau}$$

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Compensating for the fractional delay

- $\hat{s}[n] = s((n - \tau)T_s)$ (after offsetting bulk delay)
- we need to compute subsample values
- in theory, compensate with a sinc fractional delay $h[n] = \mathrm{sinc}(n + \tau)$
- in practice, use local Lagrange approximation

$$x_L(n; t) = \sum_{k=-N}^{N} x[n-k] L_k^{(N)}(t)$$

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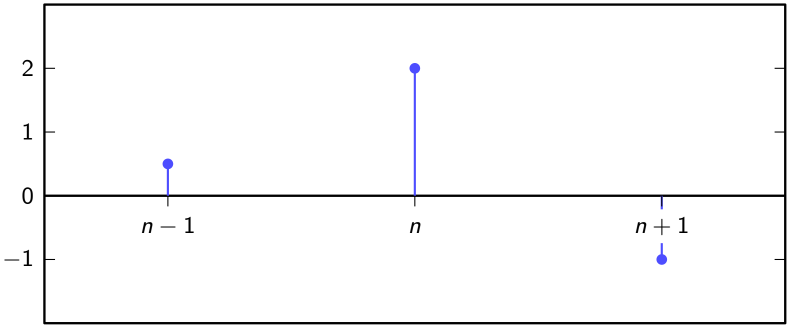

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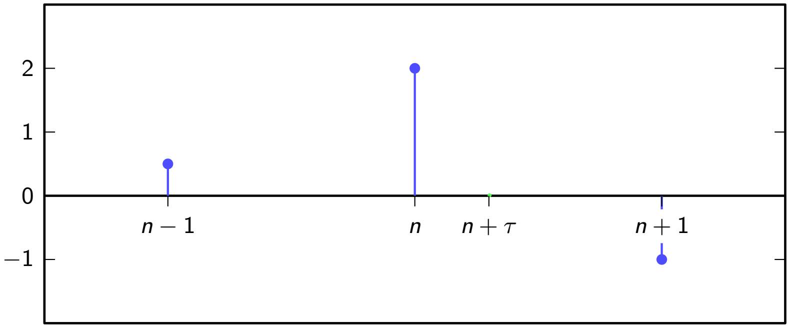

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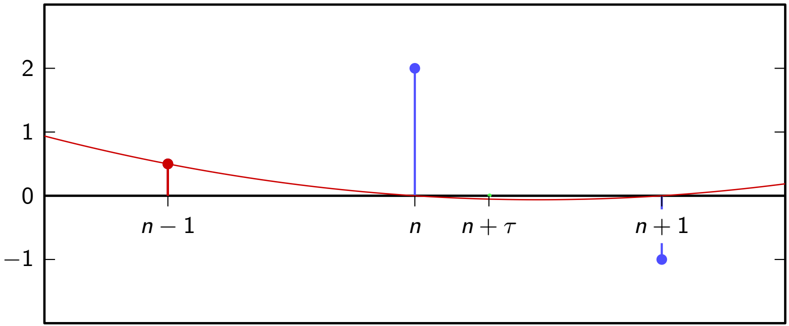

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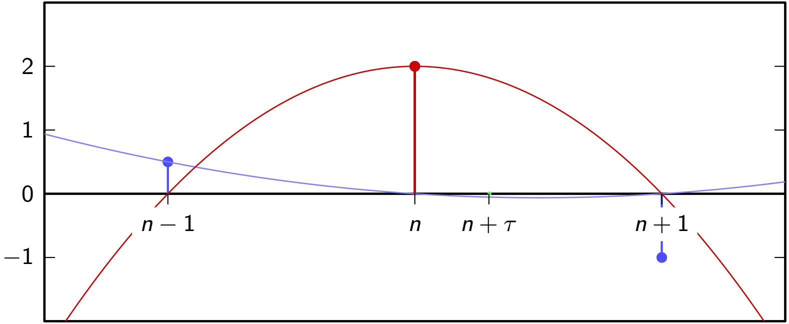

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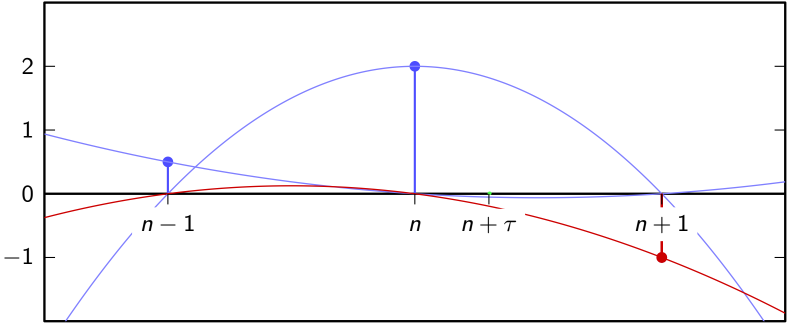

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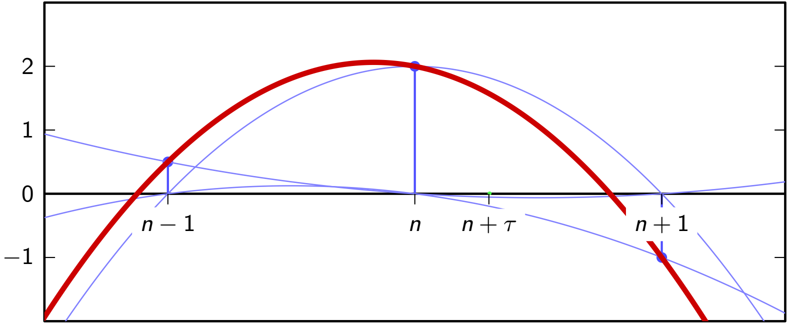

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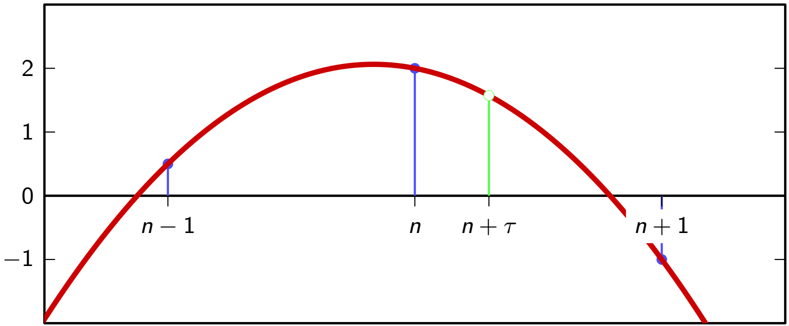

Lagrange interpolation as an FIR

- $x(n + \tau) \approx x_L(n; \tau)$
- $x_L(n; \tau) = \sum_{k=-N}^{N} x[n-k] L_k^{(N)}(\tau)$
- define $d_\tau[k] = L_k^{(N)}(\tau)$, $k = -N, \ldots, N$
- $x_L(n; \tau) = \sum_{k=-N}^{N} x[n-k] d_\tau[k]$
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Example ($N = 1$, second order approximation)

$$L_{-1}^{(1)}(t) = t\frac{t-1}{2}$$

$$L_{0}^{(1)}(t) = (1-t)(1+t)$$

$$L_{1}^{(1)}(t) = t\frac{t+1}{2}$$

Example ($N = 1$, second order approximation)

$$d_{0.2}[n] = \begin{cases} -0.08 & n = -1 \\ 0.96 & n = 0 \\ 0.12 & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

Delay compensation algorithm

- estimate the delay τ
- compute the $2N + 1$ Lagrangian coefficients
- filter with the resulting FIR

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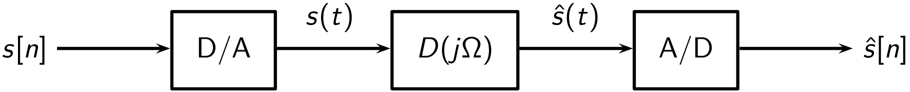
Compensating for the distortion



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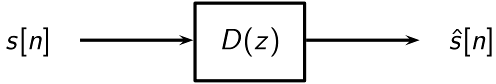
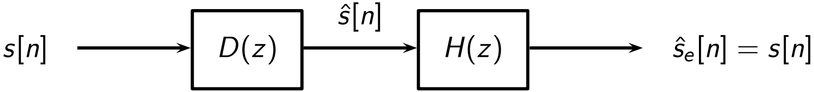


Example: adaptive equalization



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- but we don't know $D(z)$ in advance
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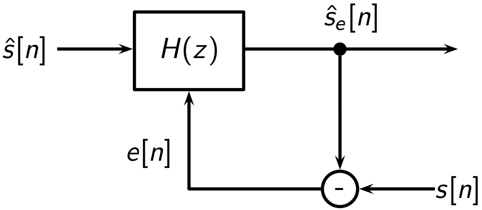
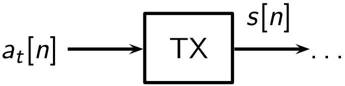


Adaptive equalization: bootstrapping via a training sequence



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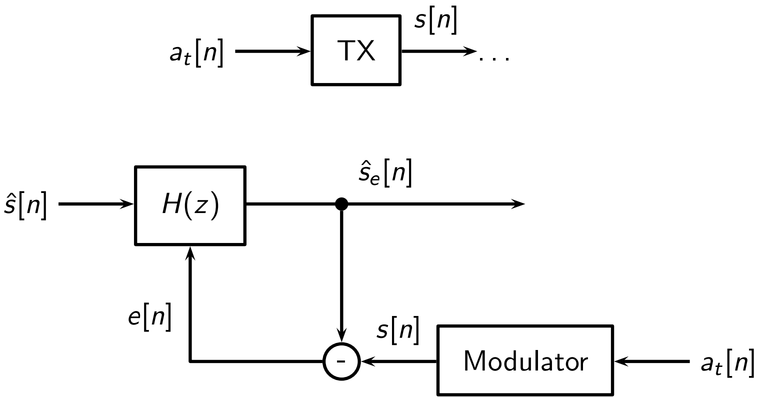
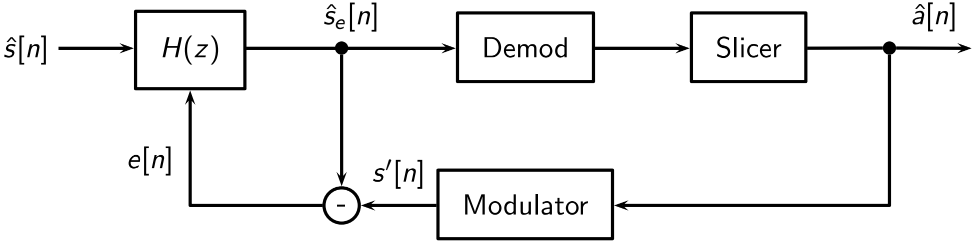


Adaptive equalization: online mode



Adaptive equalization: the LMS algorithm

FIR equalization:

$$\hat{s}_e[n] = \sum_{k=0}^{N-1} h_n[k]\hat{s}[n-k]$$

$$e[n] = \hat{s}_e[n] - s[n]$$

adapting the coefficients:

$$h_{n+1}[k] = h_n[k] + \alpha e[n] x[n-k], \qquad k = 0, 1, \ldots, N-1$$

So much more to do...

- how do we compensate for differences in clocks?
- how do we recover from interference?
- how do we improve resilience to noise?

advanced topics in communication system design

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ADSL

Overview:

- Channel
- Signaling strategy
- Discrete Multitone Modulation (DMT)

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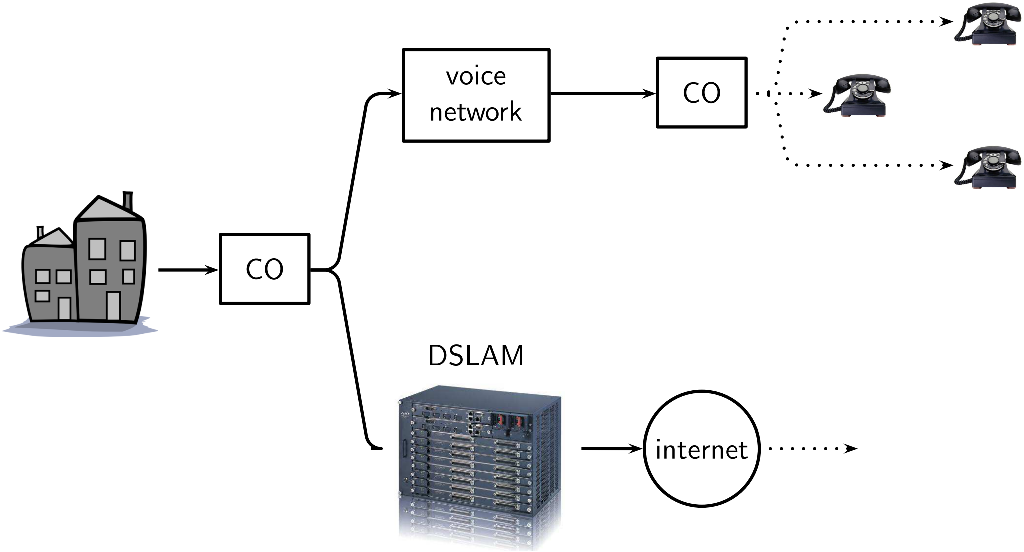
The telephone network today



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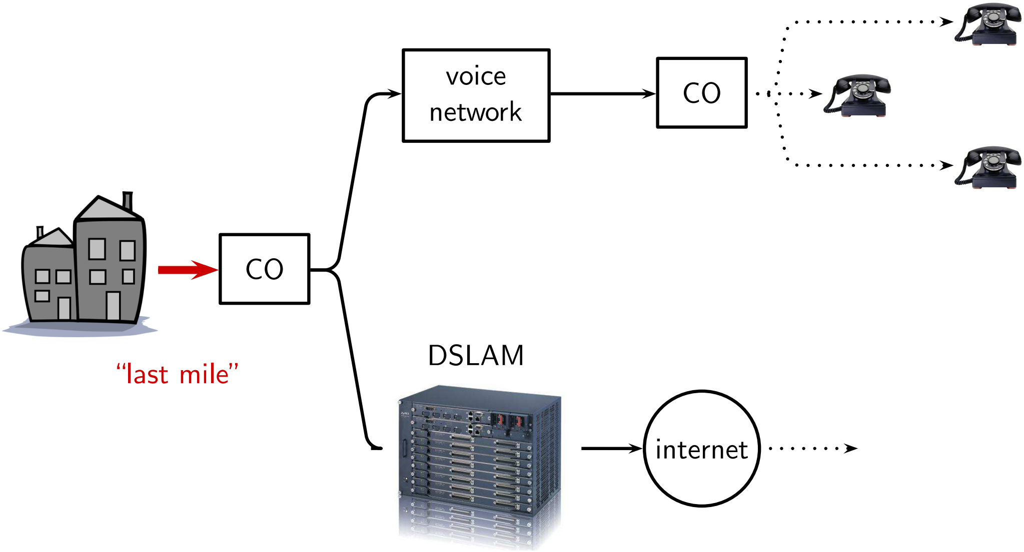

The last mile

- copper wire (twisted pair) between home and nearest CO
- very large bandwidth (well over 1MHz)
- very uneven spectrum: noise, attenuation, interference, etc.

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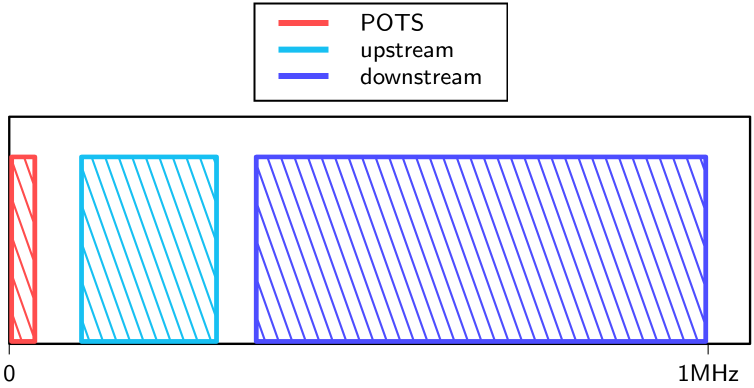
The ADSL channel



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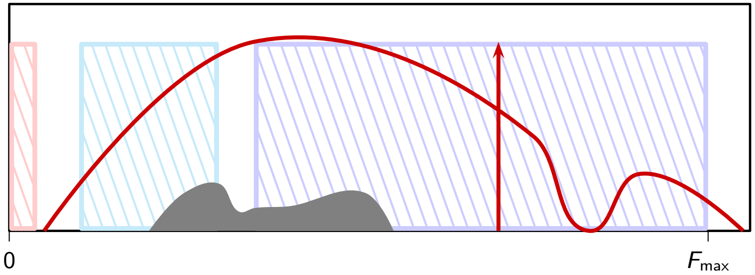
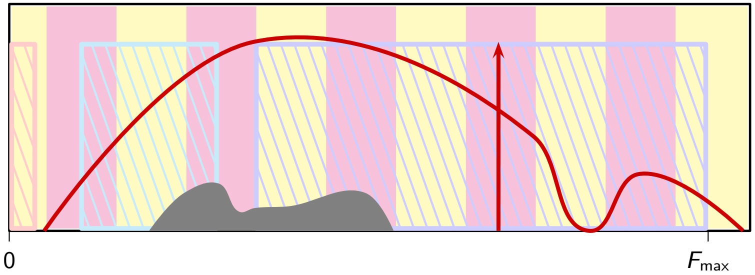


Idea: split the band into independent subchannels



Subchannel structure

- allocate N subchannels over the total positive bandwidth
- equal subchannel bandwidth $W = F_{\text{max}}/N$
- equally spaced subchannels with center frequency kF_{max}/N, $k = 0, \ldots, N-1$

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The digital design

- pick $F_s = 2F_{\max}$ ($F_s = 2NW = 2F_{\max}$. i.e. F_s multiple of W)
- center frequency for each subchannel $\omega_k = 2\pi \frac{kW}{F_s} = \frac{2\pi}{2N} k$
- bandwidth of each subchannel $\frac{2\pi}{2N}$
- to send symbols over a subchannel: upsampling factor $K = 2N$

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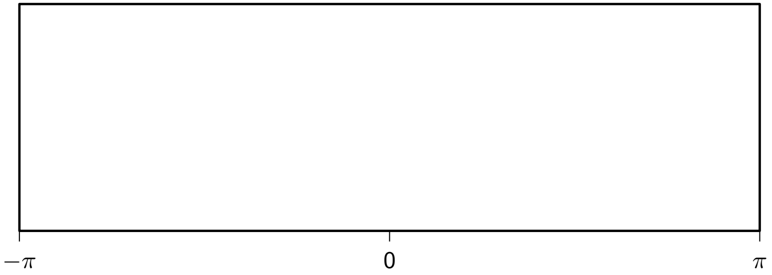
- to send symbols over a subchannel: upsampling factor $K = 2N$

The digital design ($N = 3$)



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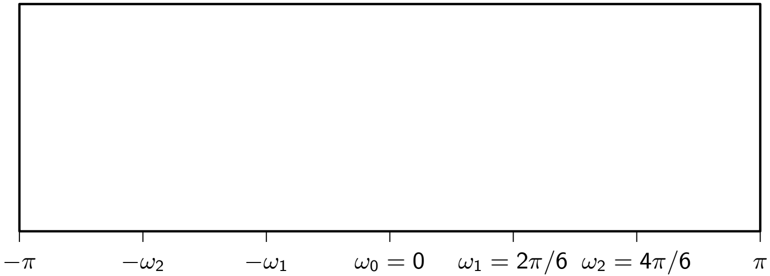

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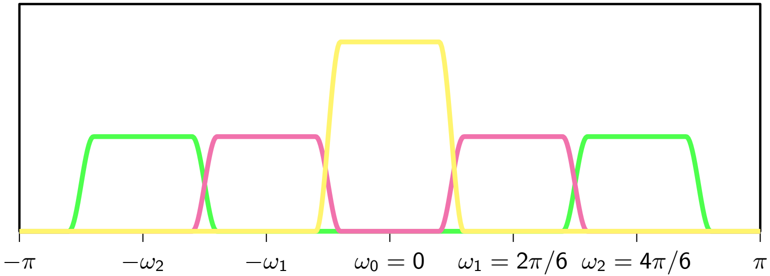

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- decide on constellation size independently
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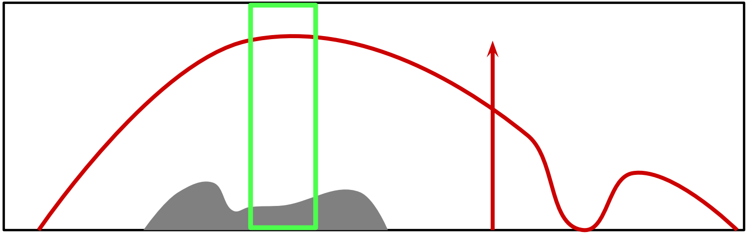
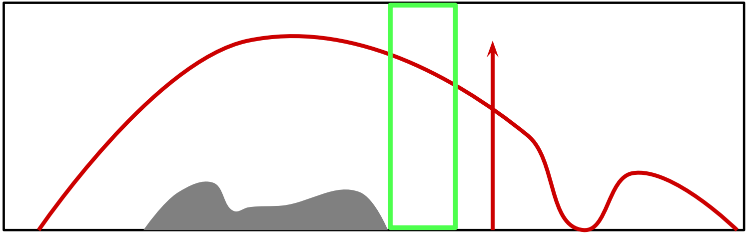
Idea: split the band into independent subchannels

channel #5, SNR 20dB



Idea: split the band into independent subchannels

channel #7, SNR 50dB



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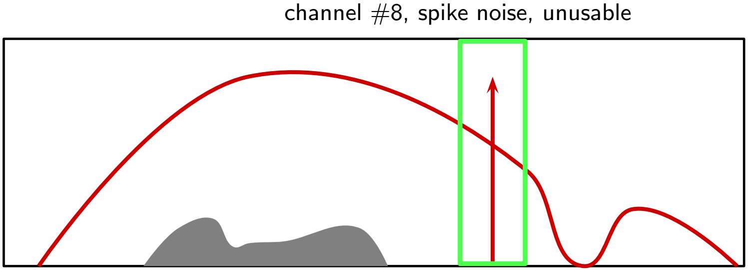
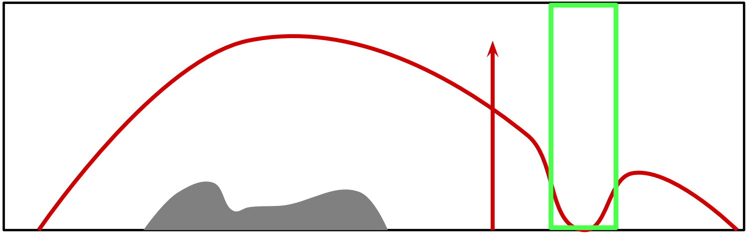


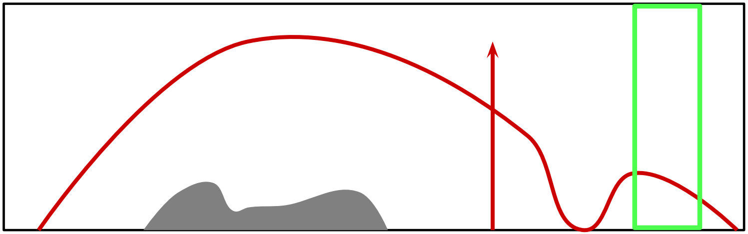
Idea: split the band into independent subchannels

channel #9, too much attenuation, unusable

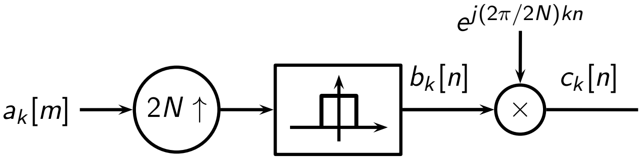


Idea: split the band into independent subchannels

channel #10, SNR 10dB



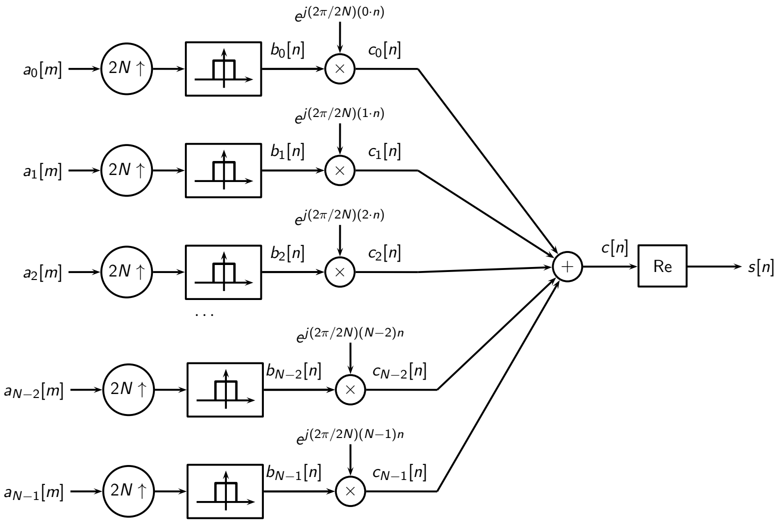
The subchannel modem



rate: W symbols/sec

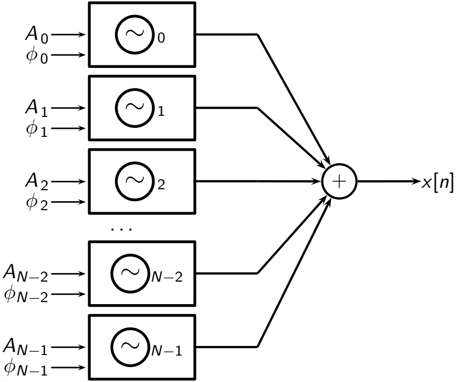
$2NW = F_s$ samples/sec

The bank of modems



If it looks familiar...

remember the DFT reconstruction formula?



DMT via IFFT

- we will show that transmission can be implemented efficiently via an IFFT
- Discrete Multitone Modulation

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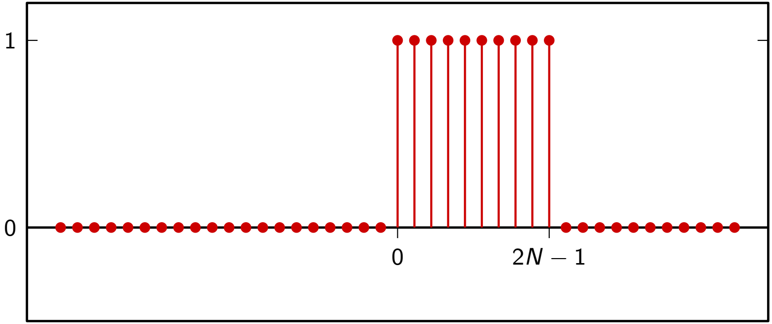
The great ADSL trick

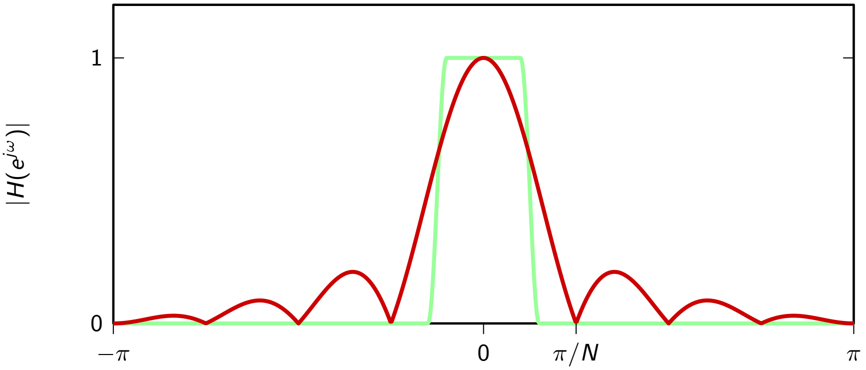
instead of using a “semi-ideal” digital interpolator (e.g. raised cosine), use a zero-order hold:

$$h[n] = \begin{cases} 1 & \text{for } 0 \leq n < 2N \\ 0 & \text{otherwise} \end{cases}$$

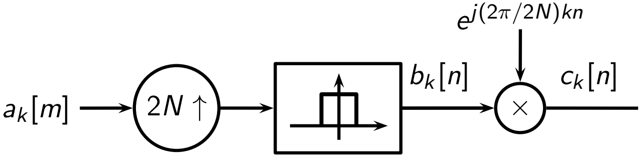
Digital ZOH: interval indicator signal



DTFT of interval signal



Back to the subchannel modem



rate: W symbols/sec

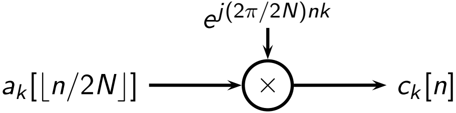
m changes every $1/W$ sec

$2NW = F_s$ samples/sec

n changes every $1/F_s = (1/W)/(2N)$

Back to the subchannel modem

by using the ZOH interpolator:

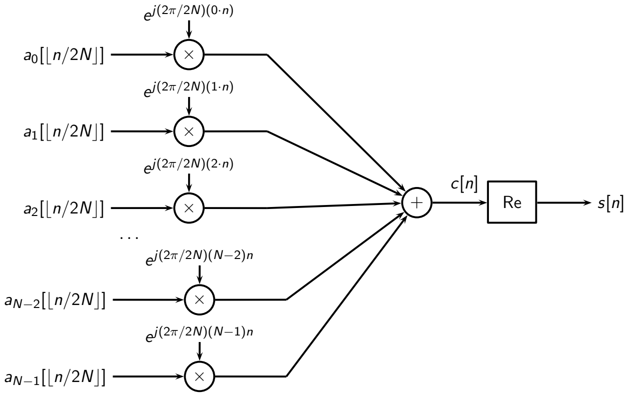


rate: W symbols/sec

$2NW = F_s$ samples/sec

n changes every $1/F_s$

The bank of modems, revisited



rate: $NW = F_{\max}$ symbols/sec

$2NW = F_s$ samples/sec

The complex output signal

$$c[n] = \sum_{k=0}^{N-1} a_k[\lfloor n/2N \rfloor] e^{j\frac{2\pi}{2N}nk}$$

for all $2N$-long output chunks, i.e. for $2pN \leq n < 2(p+1)N$:

- $a_k[\lfloor n/2N \rfloor] = a_k[p]$ don't change
- $c[n] = 2N \cdot \mathrm{IDFT}_{2N}\left\{\begin{bmatrix} a_0[p] & a_1[p] & \ldots & a_{N-1}[p] & 0 & 0 & \ldots & 0 \end{bmatrix}\right\}[n]$

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We can do even better!

- we are interested in $s[n] = \mathrm{Re}\{c[n]\} = (c[n] + c^*[n])/2$
- it is easy to prove (exercise) that:

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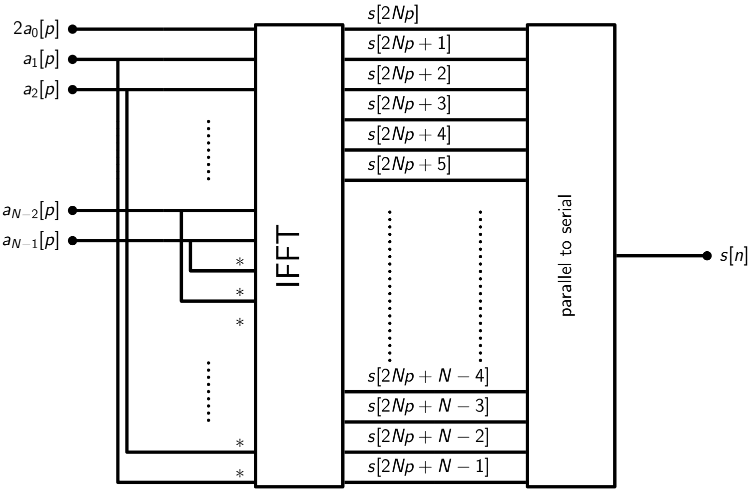
ADSL transmitter



rate: NW symbols/sec

$2NW = F_s$ samples/sec

ADSL specs

- $F_{max} = 1104$KHz
- $N = 256$
- each QAM can send from 0 to 15 bits per symbol
- forbidden channels: 0 to 7 (voice)
- channels 7 to 31: upstream data
- max theoretical throughput: 14.9Mbps (downstream)

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