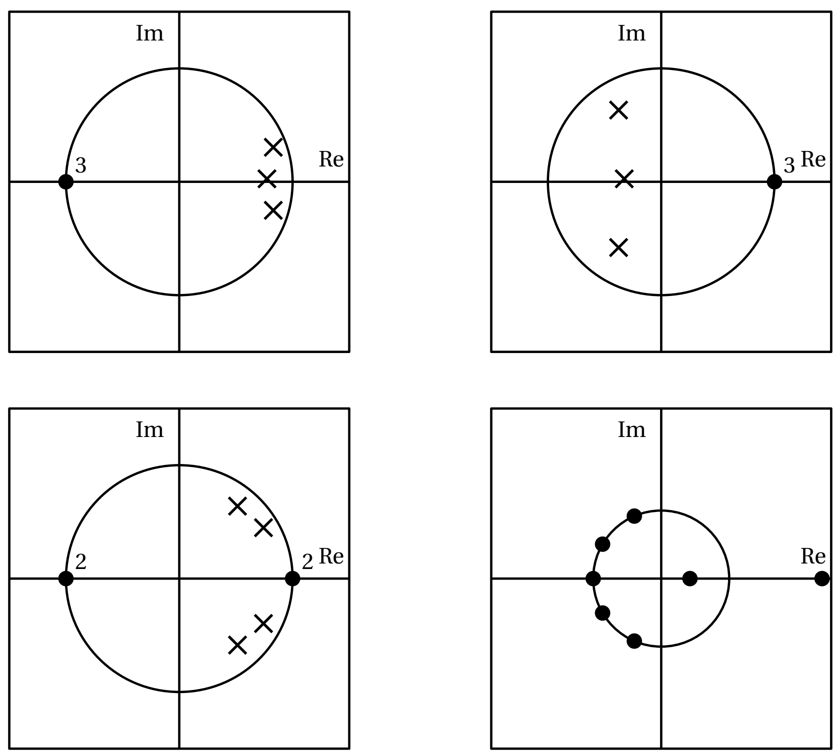
COM-303 - Signal Processing for Communications

Homework #6

Exercise 1. Transfer functions, zeros and poles

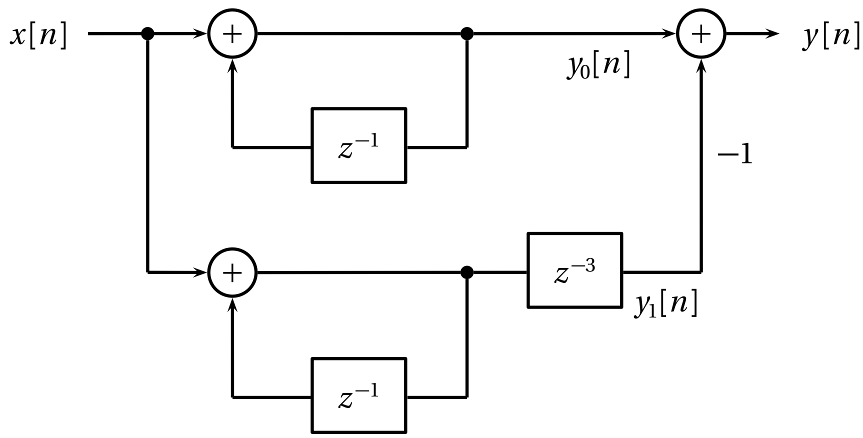
For each of the following pole-zero plots, sketch the magnitude frequency response of the corresponding filter.



(In the plots, poles are represented by crosses and zeros by circles; if applicable, the multiplicity of each pole and zero is indicated by a number. The circle indicates the unit circle on the complex plane).

Exercise 2. Discrete-time systems and stability

Consider the system in the picture below. Assume a causal input ($x[n] = 0$ for $n < 0$) and zero initial conditions.



(a) Find the constant-coefficients difference equations linking $y_0[n], y_1[n]$ and $y[n]$ to the input $x[n]$.

(b) Find $H_0(z)$, $H_1(z)$ and $H(z)$, the transfer functions relating the input $x[n]$ to the signals $y_0[n]$, $y_1[n]$ and $y[n]$, respectively.

(c) Consider the relationship between the input and the output; is the system BIBO stable?

(d) Is the system stable internally? (i.e. are the subsystems described by $H_0(z)$ and $H_1(z)$ stable?)

(e) Consider the input $x[n] = u[n]$, where, as usual, $u[n] = 1$ for $n \geq 0$ and $u[n] = 0$ for $n < 0$. How do $y_0[n], y_1[n]$ and $y[n]$ evolve over time? Sketch their values.

Exercise 3. Filter properties I

Assume G is a stable, causal IIR filter with impulse response $g[n]$ and transfer function $G(z)$. Which of the following statements is/are true for any choice of $G(z)$?

(a) The inverse filter, $1/G(z)$,is stable.

(b) The inverse filter is FIR.

(c) The DTFT of $g[n]$ exists.

(d) The cascade $G(z)G(z)$ is stable.

Exercise 4. Filter properties II

Consider $G(z)$, the transfer function of a causal stable LTI system. Which of the following statements is/are true for any such $G(z)$?

(a) The zeros of $G(z)$ are inside the unit circle.

(b) The ROC of $G(z)$ includes the curve $|z| = 0.5$.

(c) The system $H(z) = (1 - 3z^{-1})G(z)$ is stable.

(d) The system is an IIR filter.

Exercise 5. FIR Filters

Consider the following set of complex numbers

$$z_k = e^{j\pi(1-2^{-k})} \qquad k = 1, 2, \ldots, M$$

For $M = 4$,

(a) Plot z_k, $k = 1, 2, 3, 4$, on the complex plane.

(b) Consider an FIR whose transfer function $H(z)$ has the following zeros:

$$\{z_1, z_2, z_1^*, z_2^*, -1\}$$

and write out explicitly the expression for $H(z)$.

(c) How many nonzero taps will the impulse response $h[n]$ have at most?

(d) Sketch the magnitude of $H(e^{j\omega})$.

(e) What can you say about this filter:

   (a) What FIR type is it? (I, II, etc.)

   (b) Is it lowpass, bandpass, highpass?

   (c) Is it equiripple?

   (d) Is this a "good" filter? (By "good" we mean a filter which is close to 1 in the passband, close to zero in the stopband and which has a narrow transition band).
