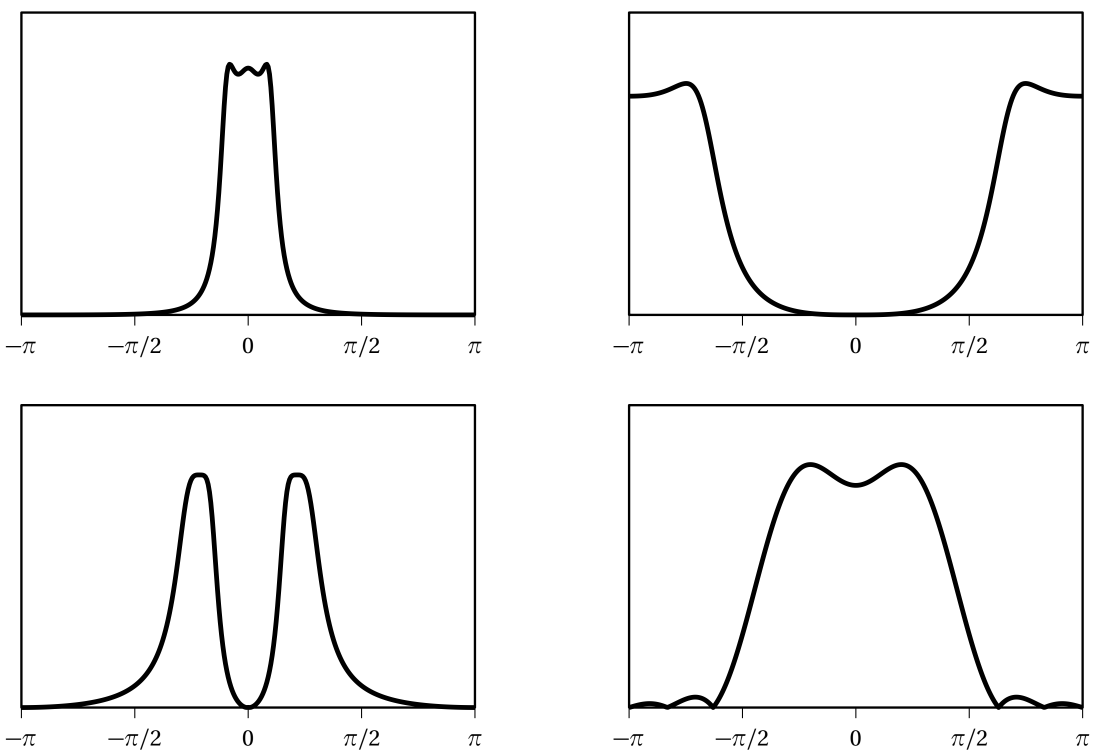
COM-303 - Signal Processing for Communications

Solutions for Homework #6

Solution 1. Transfer functions, Zeros and Poles



Solution 2. Discrete-time systems and stability.

(a)

$$
\begin{aligned}
y_0[n] &= x[n] + y_0[n-1] \\
y_1[n+1] &= y_1[n] + x[n] && \Rightarrow y_1[n] = y_1[n-1] + x[n-1] \\
y[n] &= y_0[n] - y_1[n] && \Rightarrow y[n] = (y_0[n-1] - y_1[n-1]) + x[n] - x[n-1] \\
& && \quad = y[n-1] + x[n] - x[n-1]
\end{aligned}
$$

(b)

$$\begin{aligned} H_0(z) &= \frac{1}{1-z^{-1}} \\ H_1(z) &= \frac{z^{-1}}{1-z^{-1}} \\ H(z) &= \frac{1-z^{-1}}{1-z^{-1}} = 1 \end{aligned}$$

(c) The system is BIBO stable since $y[n] = x[n]$.

(d) The system is not internally stable (the subsystems have poles on the unit circle.

(e) According to the CCDES,

$$\begin{aligned} y_0[n] &= (n+1)\,u[n] \to \infty \\ y_1[n] &= n\,u[n] \to \infty \\ y[n] &= u[n] \end{aligned}$$

Solution 3. Filter properties (I).

- False. The inverse filter is stable only if all the zeros of $G(z)$ are inside the unit circle; this is not true in general.
- False. The inverse filter is FIR only if $G(z)$ has no zeros; this is not true in general.
- True. If the filter is stable, the ROC of $G(z)$ includes the unit circle.
- True. The poles of the cascade double their multiplicity but remain inside the unit circle.

Solution 4. Filter properties (II).

- False. Zeros do not affect stability, therefore they can be anywhere for a stable system.
- False. The ROC includes the unit circle and extends outwards, therefore it includes all circles of radius greater than one, but not necessarily a circle of radius 0.5.
- True. Adding a zero does not affect stability.
- False. The system described does not necessarily have poles.

Solution 5. FIR filters

(a) First of all note that $1-2^{-k} = (2^k-1)/2^k$. With this we find that

$$\begin{aligned} z_1 &= e^{j\frac{1}{2}\pi} \\ z_2 &= e^{j\frac{3}{4}\pi} \\ z_3 &= e^{j\frac{7}{8}\pi} \\ z_4 &= e^{j\frac{15}{16}\pi} \end{aligned}$$

which are simply four points in the second quadrant on the unit circle.

(b) Each couple of complex-conjugate zeros contributes a factor of the form $(1-2z^{-1}\cos\theta + z^{-2})$ to the transfer function, where θ is the angle of the complex zero. We have in the end:

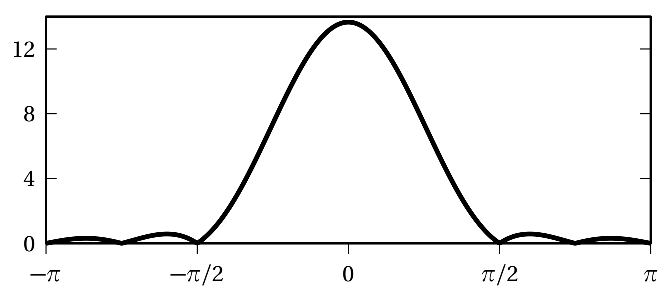
$$H(z) = (1+z^{-1})(1+z^{-2})(1-2\cos(\frac{3}{4}\pi)\, z^{-1} + z^{-2})$$

(c) $H(z)$ is a 5$^{\text{th}}$ degree polynomial in z^{-1} and therefore it has at most 6 nonzero coefficients. The impulse response will have 6 nonzero taps.

(d) You don't even need a numerical package to do this. First of all, the impulse response is real and therefore the magnitude of $H(e^{j\omega})$ is symmetric. Consider now the values of the frequency response at zero and π; these are computed from the z-transform for $z = 1$ and $z = -1$ respectively; we have:

$$\begin{aligned} H(e^{j0}) &= H(1) = 2\cdot 2\cdot 2(1-\cos(\frac{3}{4}\pi)) \approx 13.6 \\ H(e^{j\pi}) &= H(-1) = 0 \end{aligned}$$

Next, you need to consider that $H(z)$ is zero on the unit circle at z_1 and z_2, i.e. at $\omega = \pi/2$ and $\omega = 3\pi/4$. Now you can plot the magnitude:



(e) First of all, is the filter linear phase? You can compute the coefficient of the transfer function and verify that $h[n] = 1,\ 2.4142,\ 3.4142,\ 3.4142,\ 2.4142,\ 1$ for $n = 0, \ldots, 5$. In a simpler way, you can simply notice that $H(z^{-1}) = z^5 H(z)$ and therefore the filter is linear phase, symmetric. The filter has an even number of taps and therefore it is Type II.

Because of the zero in π and the large value in zero, the filter is lowpass. However, it is not equiripple since the magnitude at the peak of the first sidelobe in the stopband is higher than the peak of the second sidelobe. The filter is clearly not a good filter: the transition band is very large, it is not flat in the passband and the magnitude is rather large in the stopband.