COM-303 - Signal Processing for Communications

Homework #3

Exercise 1. DFT in matrix form

Consider the DFT expressed as a as a matrix/vector multiplication and call $\mathbf{W}$ the $N \times N$ DFT matrix. Is $\mathbf{W}$ Hermitian-symmetric for all values of N?

Exercise 2. DFS and DFT

Given a finite-length discrete signal $x[n]$ of length N $(n = 0, \cdots, N-1)$ and its DFT $X[k]$. Derive the DFS $\widetilde{X}[k]$ of $x[n]$'s periodization $\tilde{x}[n] = x[n \bmod N]$.

Exercise 3. Derivative in frequency

Let $x[n] \longleftrightarrow X(e^{j\omega})$ be a DTFT transform-pair.

(a) Assume $X(e^{j\omega})$ to be differentiable, compute the inverse DTFT of $j\frac{d}{d\omega}X(e^{j\omega})$.

(b) Compute the inverse DTFT of $\frac{d}{d\omega}\left(\frac{X(e^{j\omega})}{\pi}\right) - 2$. Which property of the DTFT allows you to simplify the calculation?

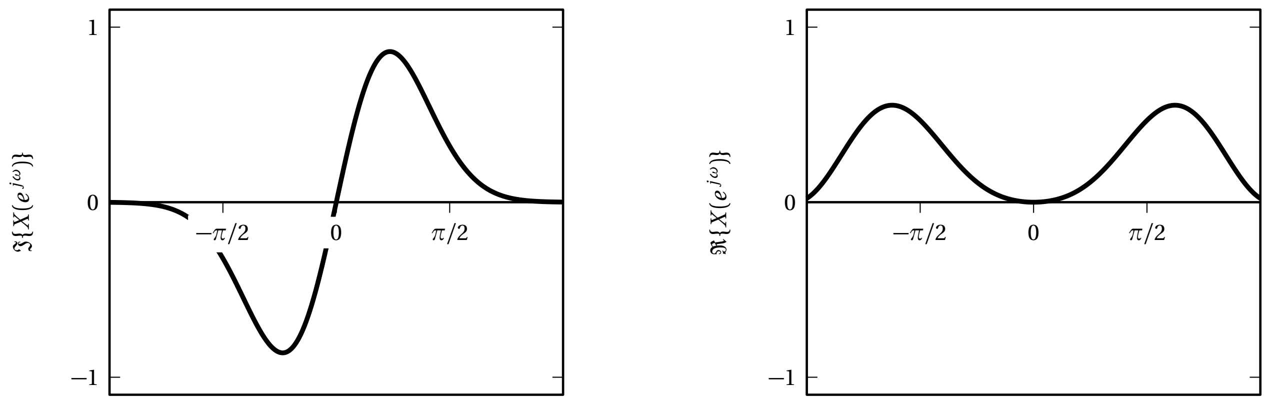
Hint: use integration by parts.

Exercise 4. DTFT visual inspection

The real and imaginary parts of $X(e^{j\omega})$ are shown in the figure below. By visual inspection of the plots, prove that

(a) $x[n]$ is 0-mean, i.e., $\sum_{n\in\mathbb{Z}} x[n] = 0$;

(b) $x[n]$ is real valued.



Exercise 5. DTFT properties

Derive the time-reversal and time-shift properties of the DTFT.

Exercise 6. Placherel-Parseval equaltiy

Let $x[n]$ and $y[n]$ be two complex valued sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their corresponding DTFTs.

(a) Show that

$$\left\langle x[n], y[n]\right\rangle = \frac{1}{2\pi}\left\langle X(e^{j\omega}), Y(e^{j\omega})\right\rangle$$

where we use the inner products for $\ell_2(\mathbb{Z})$ and $\mathscr{L}_2([-\pi,\pi])$ respectively.

(b) What is the physical meaning of the above formula when $x[n] = y[n]$?

Exercise 7. DTFT,DFT, and numerical computations

Consider the following infinite non-periodic discrete time signal, where $M \in \mathbb{N}$:

$$x[n] = \begin{cases} 0 & n < 0, \\ 1 & 0 \le n < M, \\ 0 & n \ge M. \end{cases}$$

The goal is to plot the magnitude of $X(e^{j\omega})$ using a numerical package. Using a numerical package implies that we will only obtain an approximation of the true value.

(a) Compute $X(e^{j\omega})$ analytically

(b) Using a numerical package plot $|X(e^{j\omega})|$ over $[-\pi\pi]$ for $M = 20$ using 10,000 points.

(c) Now generate a finite sequence $\hat{x}[n]$ of length $N = 30$ such that $\hat{x}[n] = x[n]$ for $n = 0, 1, \ldots, N-1$. Compute its DFT using the numerical package's FFT algorithm and plot its magnitude. Compare it with the plot obtained previously.

(d) Repeat now for different values of $N = 50, 100, 1000$. What can you conclude?