Biological Modeling of Neural Networks

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

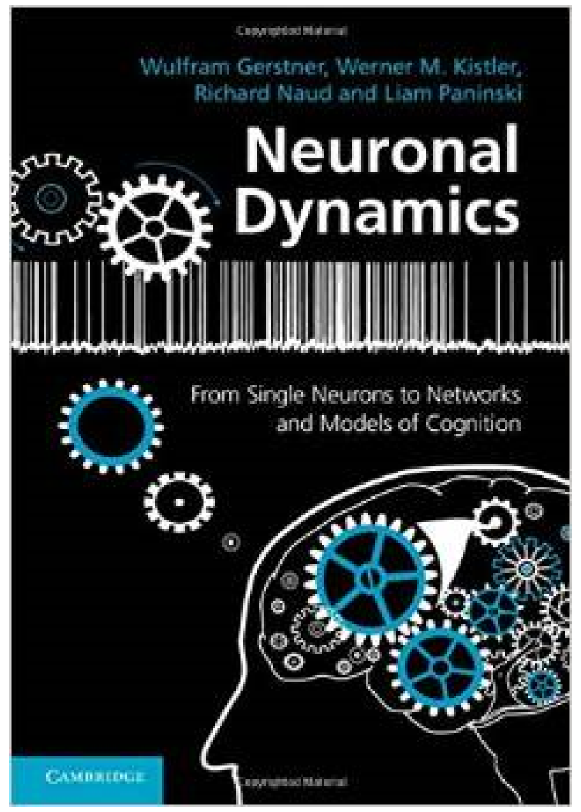
Week 2 – Biophysical modeling: The Hodgkin-Huxley model

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Reading for week 2:
NEURONAL DYNAMICS
- Ch. 2 (without 2.3.2 - 2.3.5)

Cambridge Univ. Press



2.1 Biophysics of neurons
- Overview

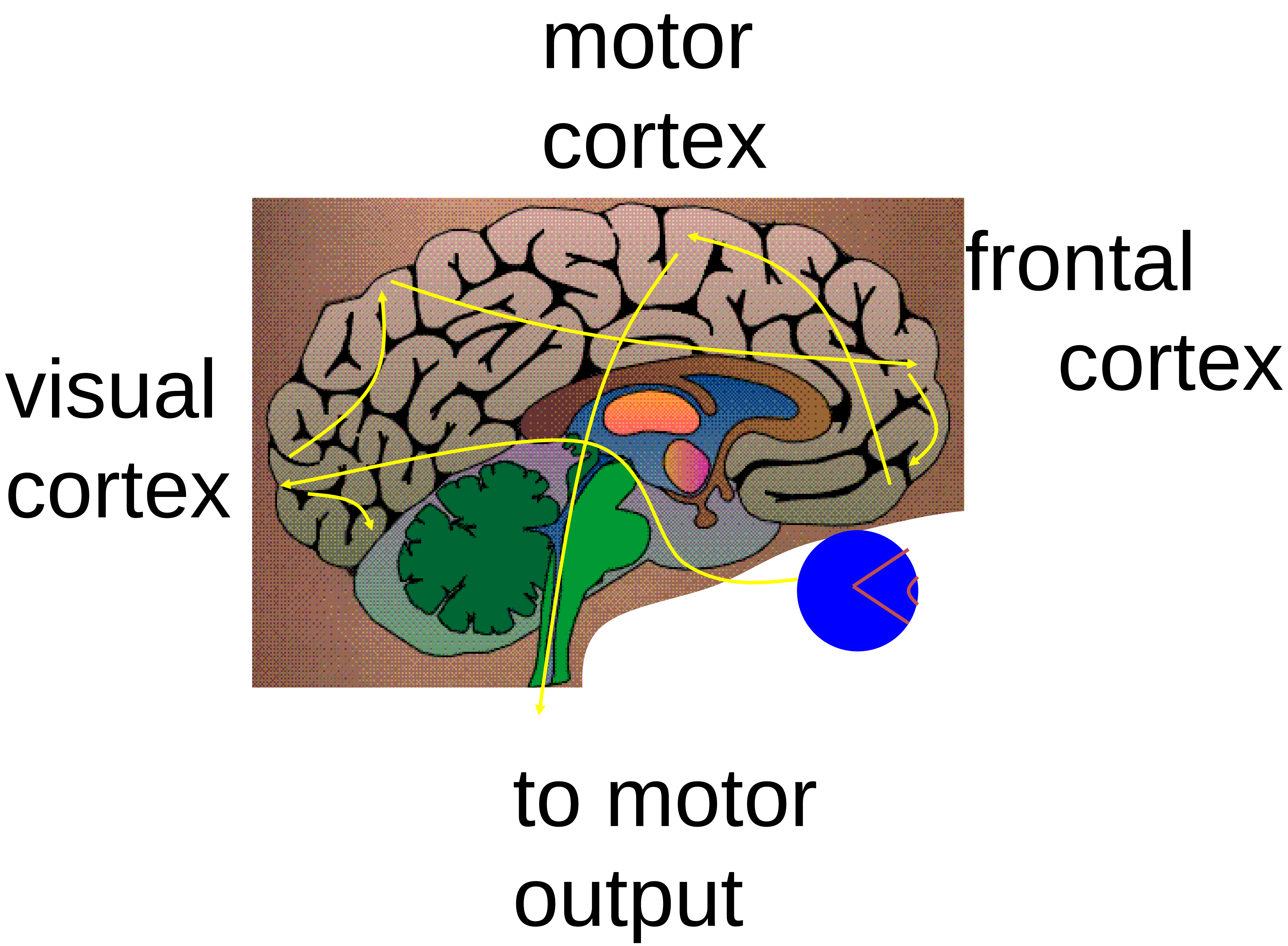
2.2 Reversal potential
- Nernst equation

2.3 Hodgin-Huxley Model

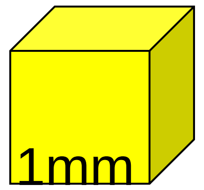
2.4 Threshold in the Hodgkin-Huxley Model
- where is the firing threshold?

2.5. Detailed biophysical models
- the zoo of ion channels

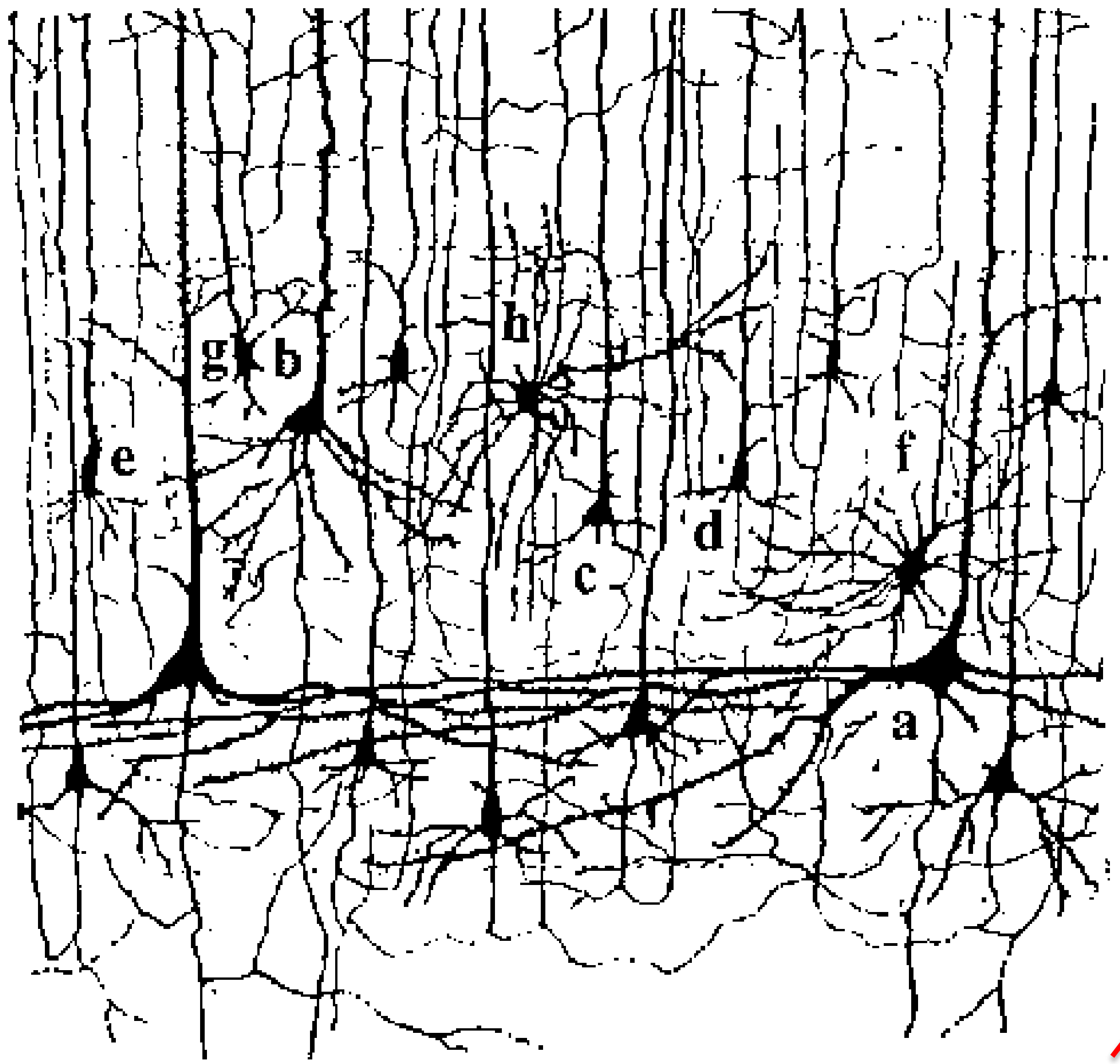
Review of week 1: Neurons and synapses

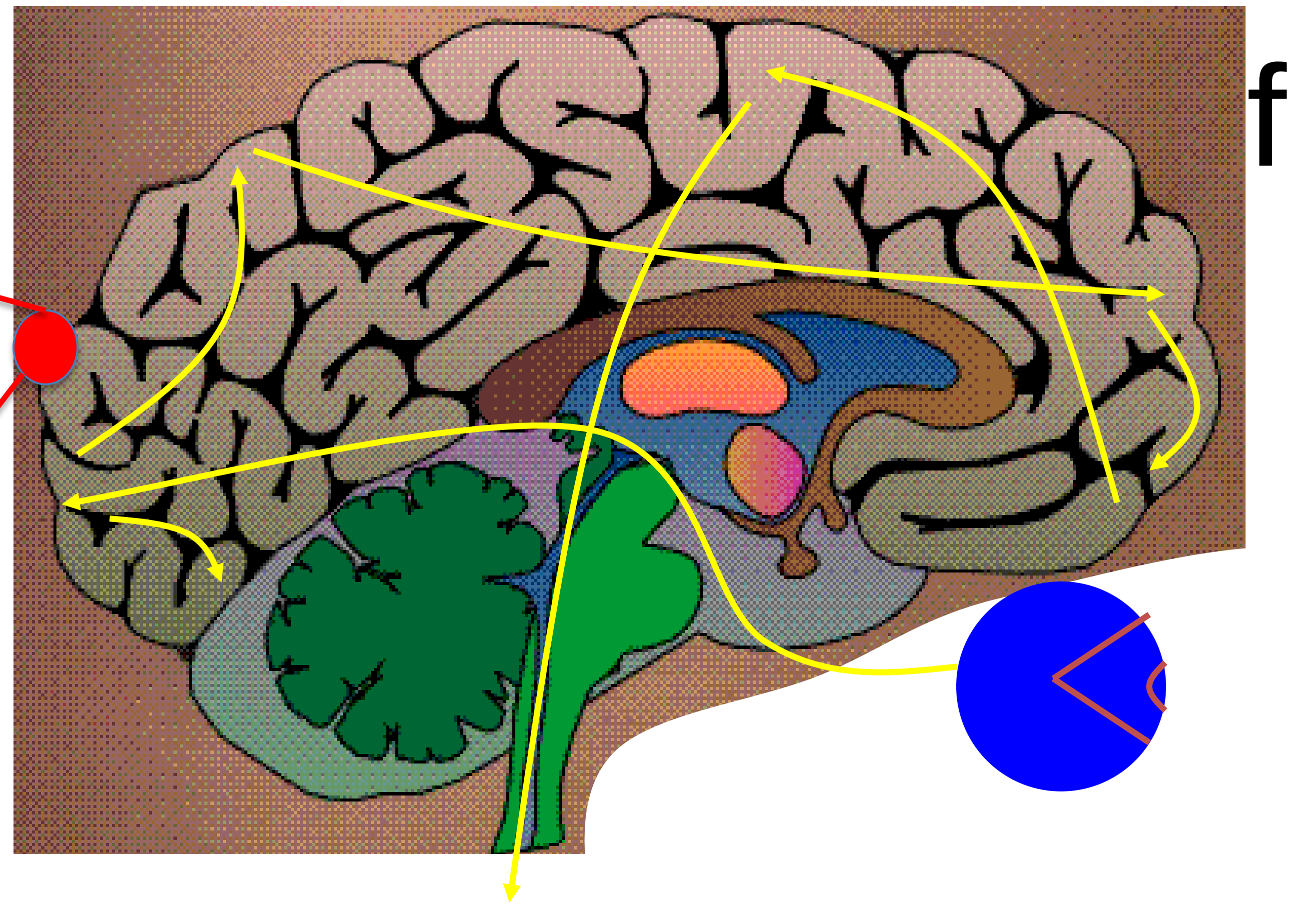


Review of week 1: Neurons and synapses



10 000 neurons
3 km of wire





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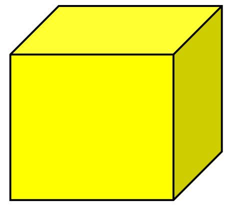

10 000 neurons
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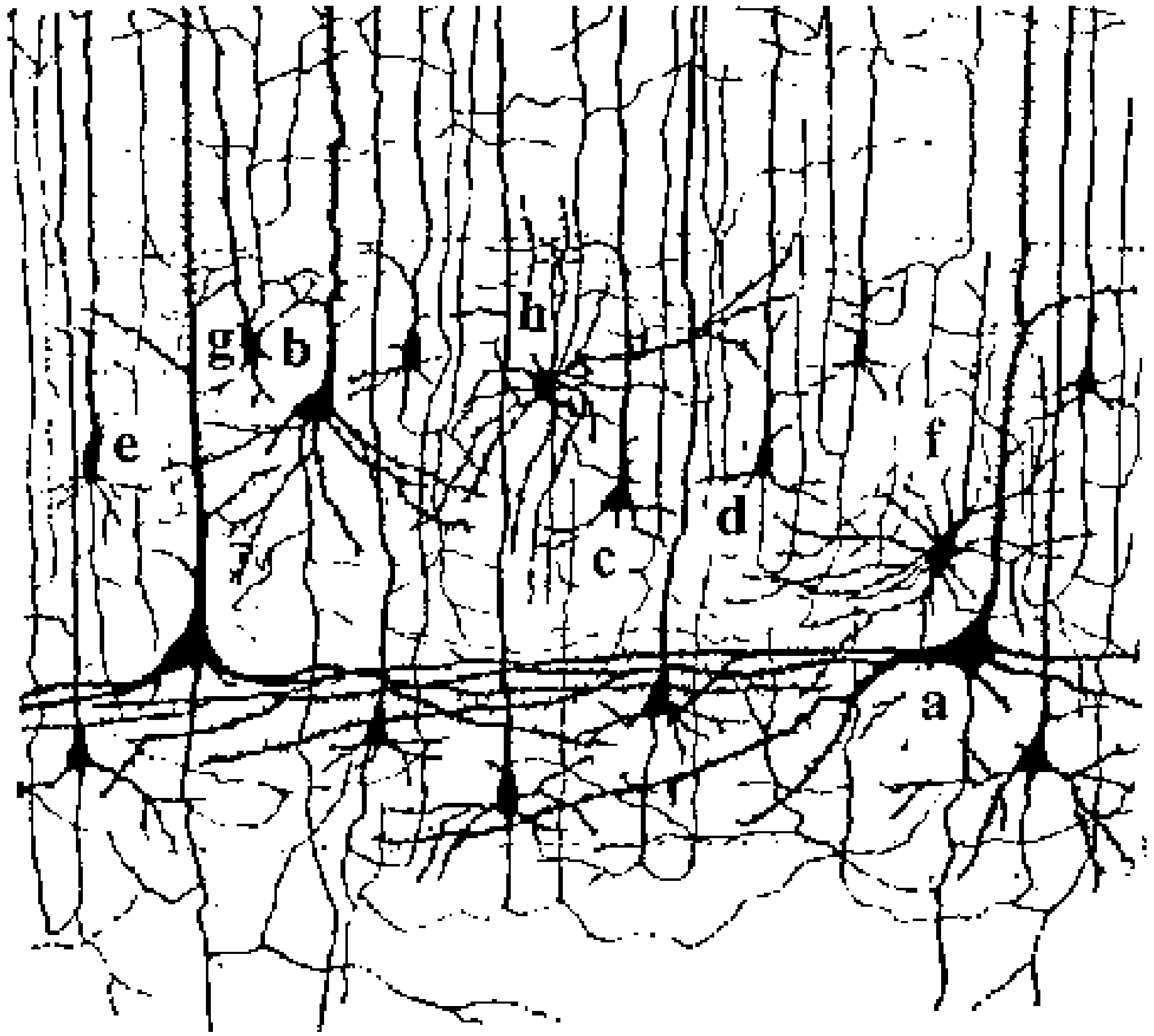
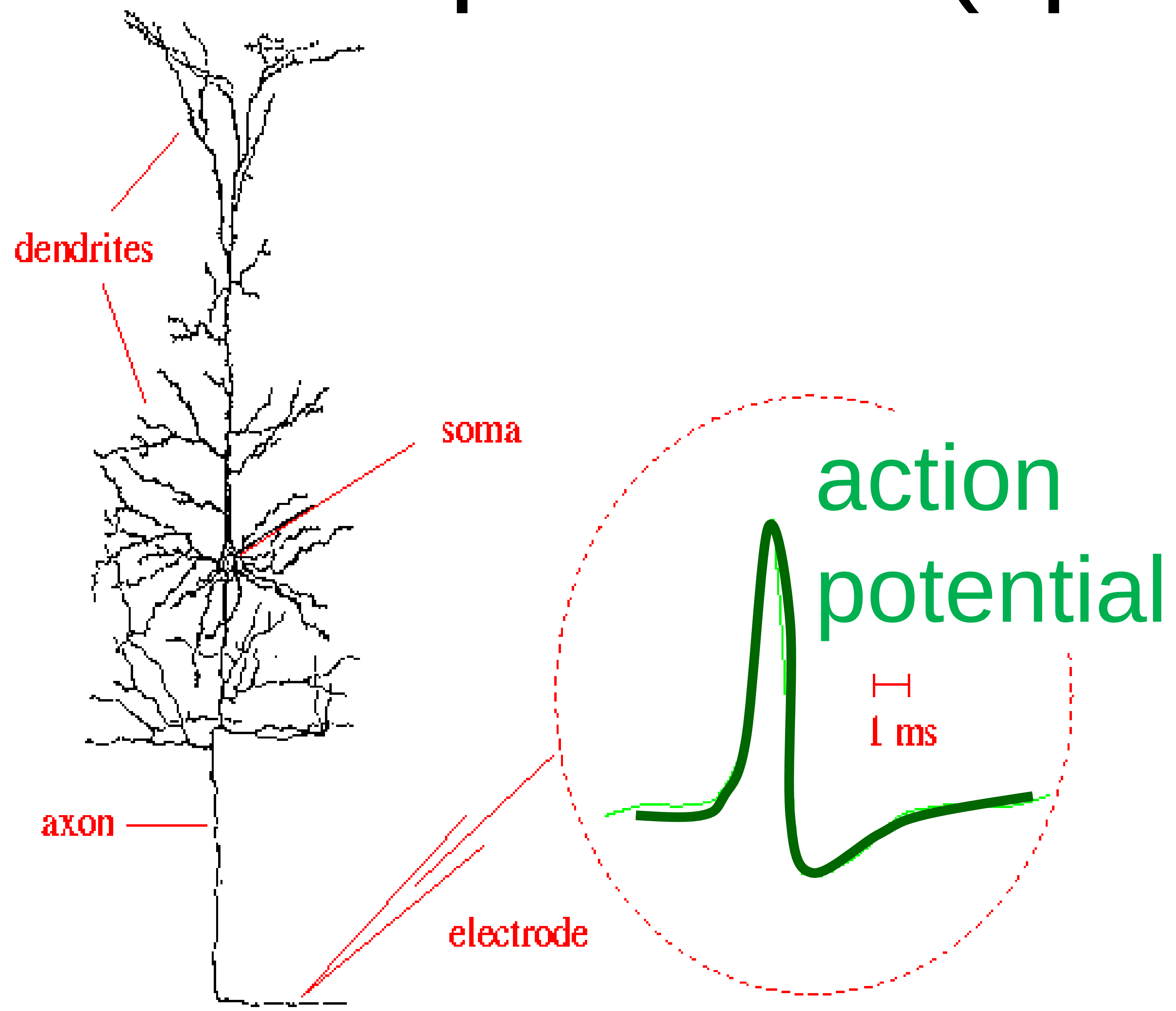


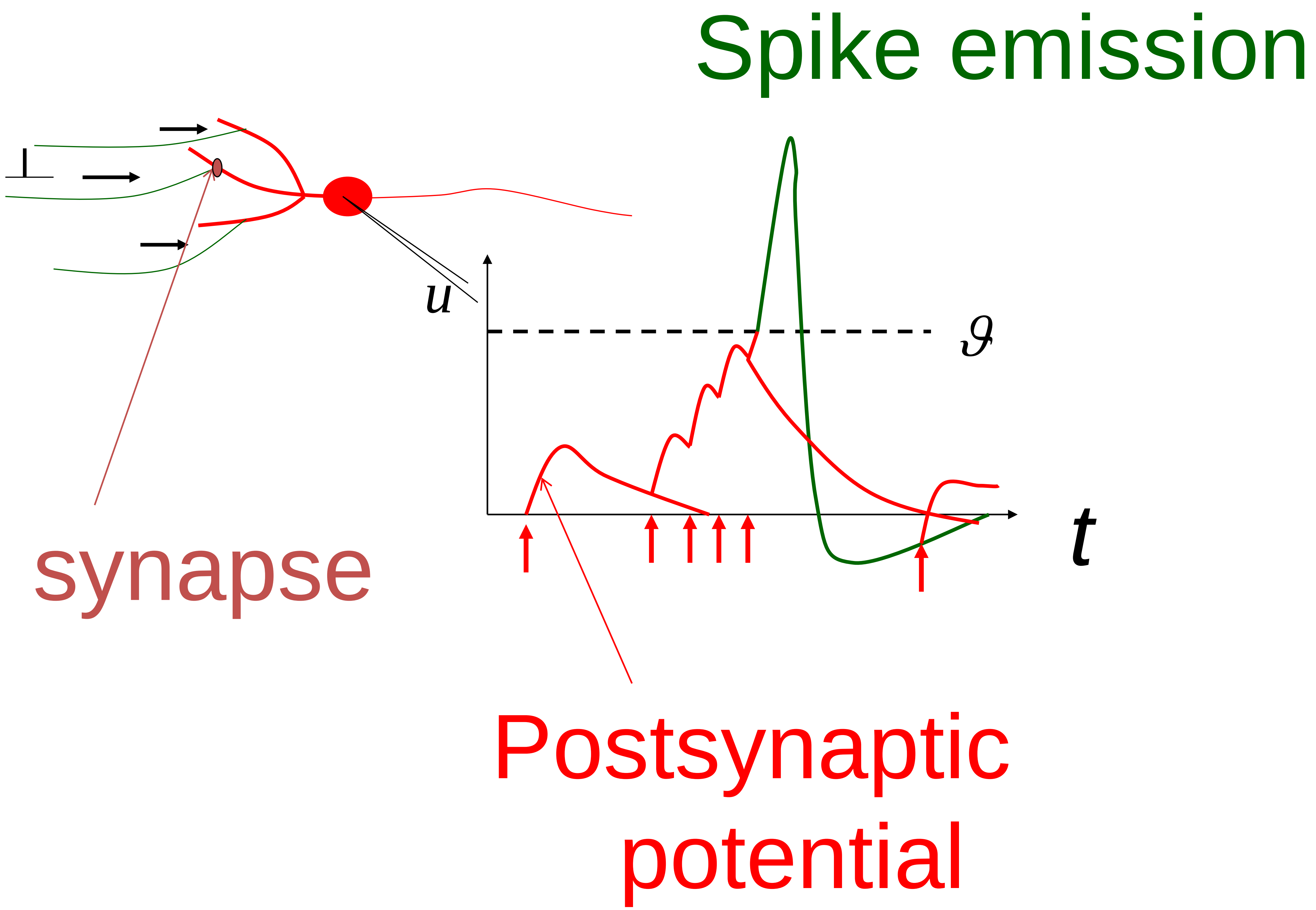
Ramon y Cajal

Signal:
action potential (spike)



How is a spike generated?

Review of week 1: Integrate-and-Fire models



-spikes are events
-triggered at threshold
-spike/reset/refractoriness

Neuronal Dynamics – week 2: Biophysics of neurons

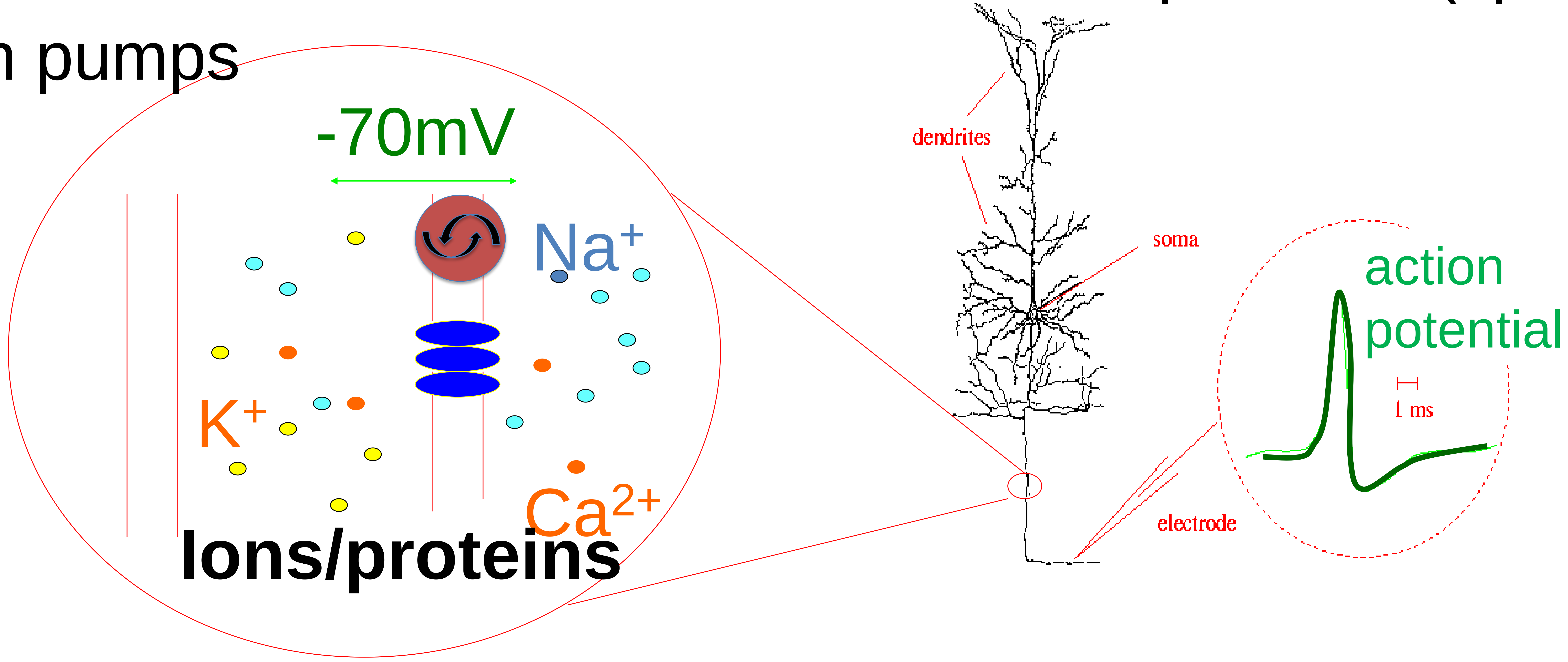
Cell surrounded by membrane

Membrane contains

- ion channels
- ion pumps

Signal:

action potential (spike)



Neuronal Dynamics – week 2: Biophysics of neurons

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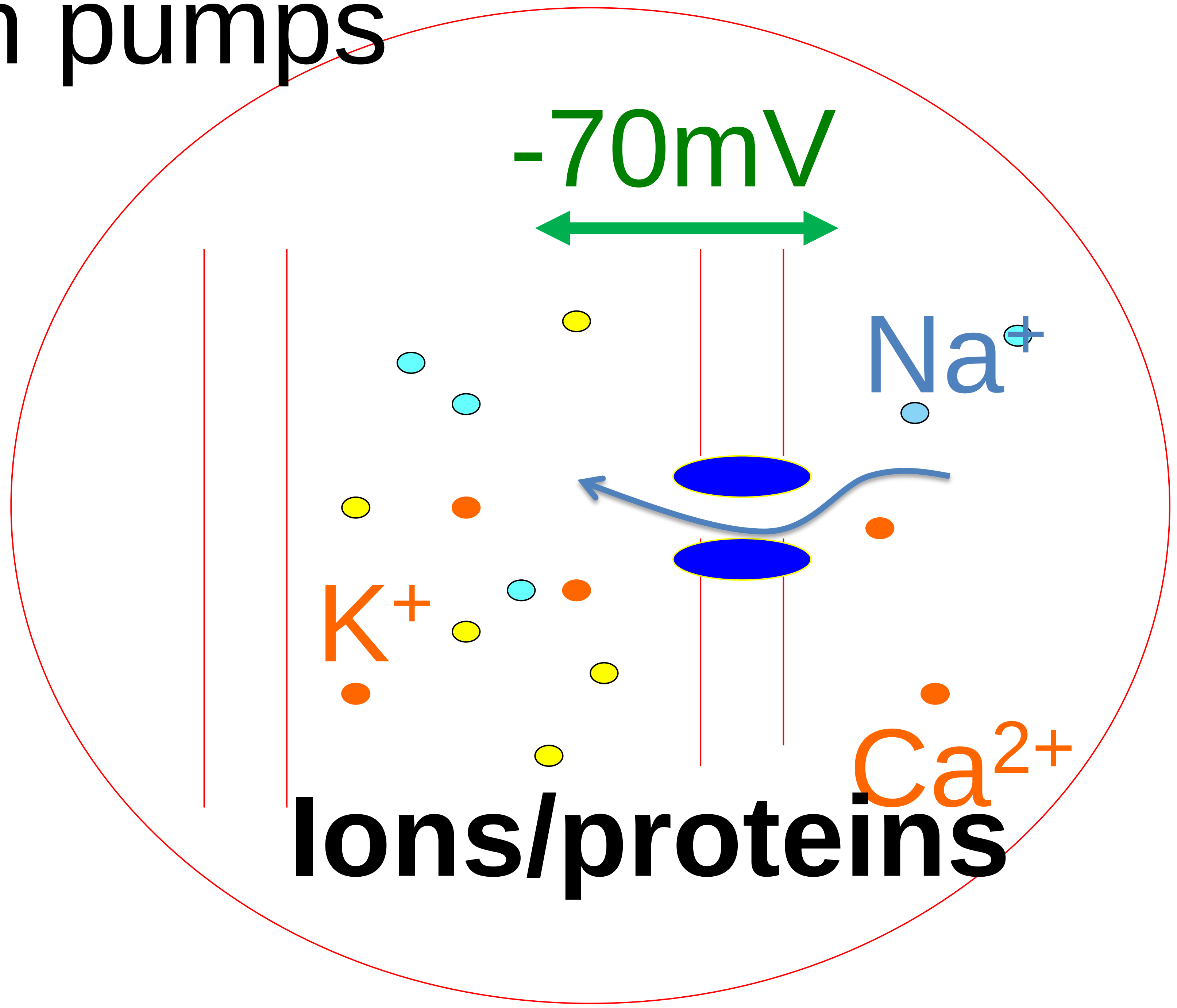


Resting potential -70mV

→ how does it arise?

Ions flow through channel

→ in which direction?

Neuron emits action potentials

→ why?

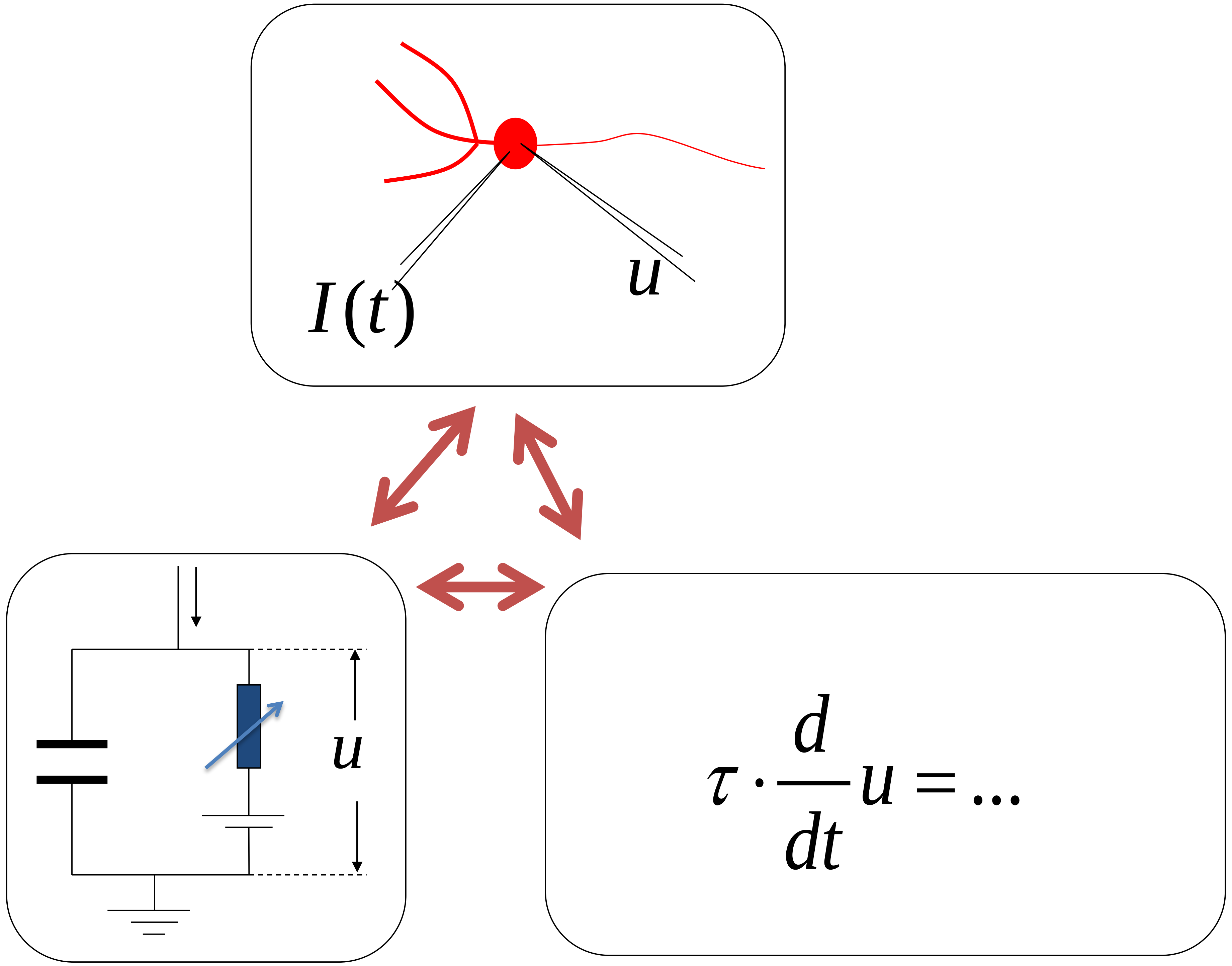
Resting potential -70mV
→ how does it arise?

Ions flow through channel
→ in which direction?

Neuron emits action potentials
→ why?
→Hodgkin-Huxley model

Hodgkin&Huxley (1952)
Nobel Prize 1963

Neuronal Dynamics – 2. 1. Biophysics of neurons



→Hodgkin-Huxley model

Hodgkin&Huxley (1952)

Nobel Prize 1963

Week 2 – Quiz

In a natural situation, the electrical potential inside a neuron is
- [ ] the same as outside
- [ ] is different by 50-100 microvolt
- [ ] is different by 50-100 millivolt

Neurons and cells
- [ ] Neurons are special cells because they are surrounded by a membrane
- [ ] Neurons are just like other cells surrounded by a membrane
- [ ] Neurons are not cells

Ion channels are
- [ ] located in the cell membrane
- [ ] special proteins
- [ ] can switch from open to closed

If a channel is open, ions can
- [ ] flow from the surround into the cell
- [ ] flow from inside the cell into the surrounding liquid

Multiple answers possible!

Week 2 – part 2: Reversal potential and Nernst equation

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Biological Modeling of Neural Networks

Week 2 – Biophysical modeling: The Hodgkin-Huxley model

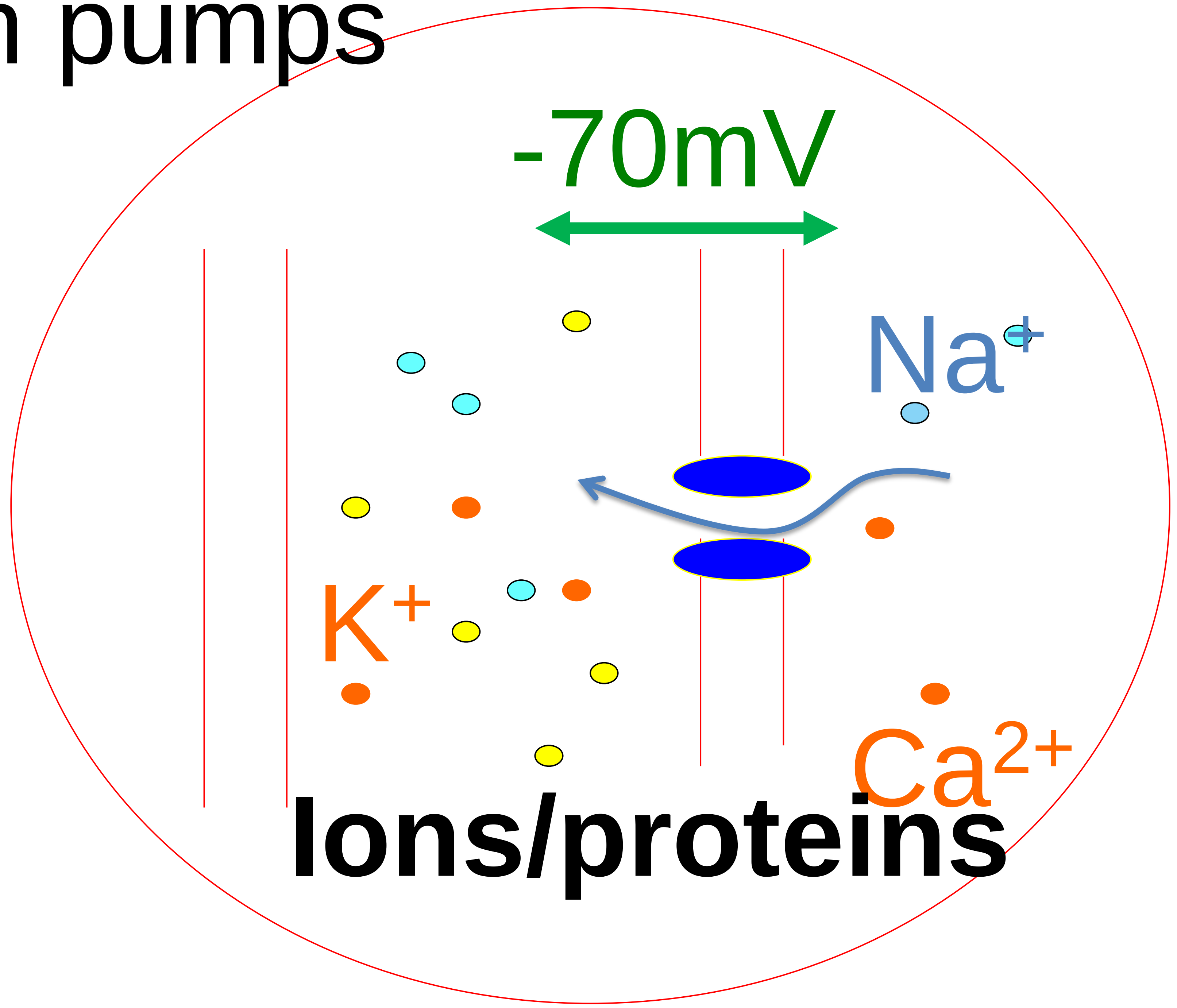
Wulfram Gerstner

EPFL, Lausanne, Switzerland

Cell surrounded by membrane

Membrane contains

- ion channels
- ion pumps



Resting potential -70mV
→ how does it arise?

Ions flow through channel
→ in which direction?

Neuron emits action potentials
→ why?

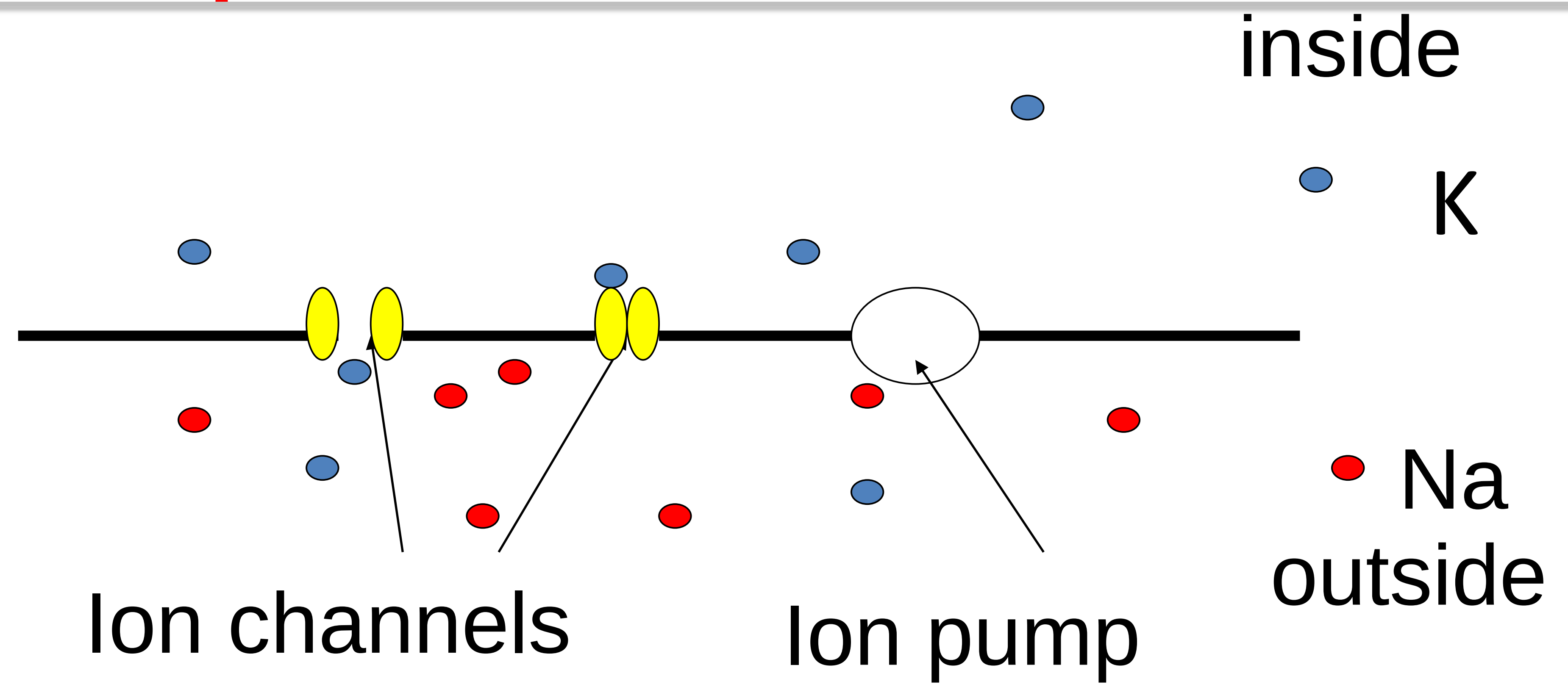
Neuronal Dynamics – 2. 2. Resting potential

Resting potential -70mV
→ how does it arise?

Ions flow through channel
→ in which direction?

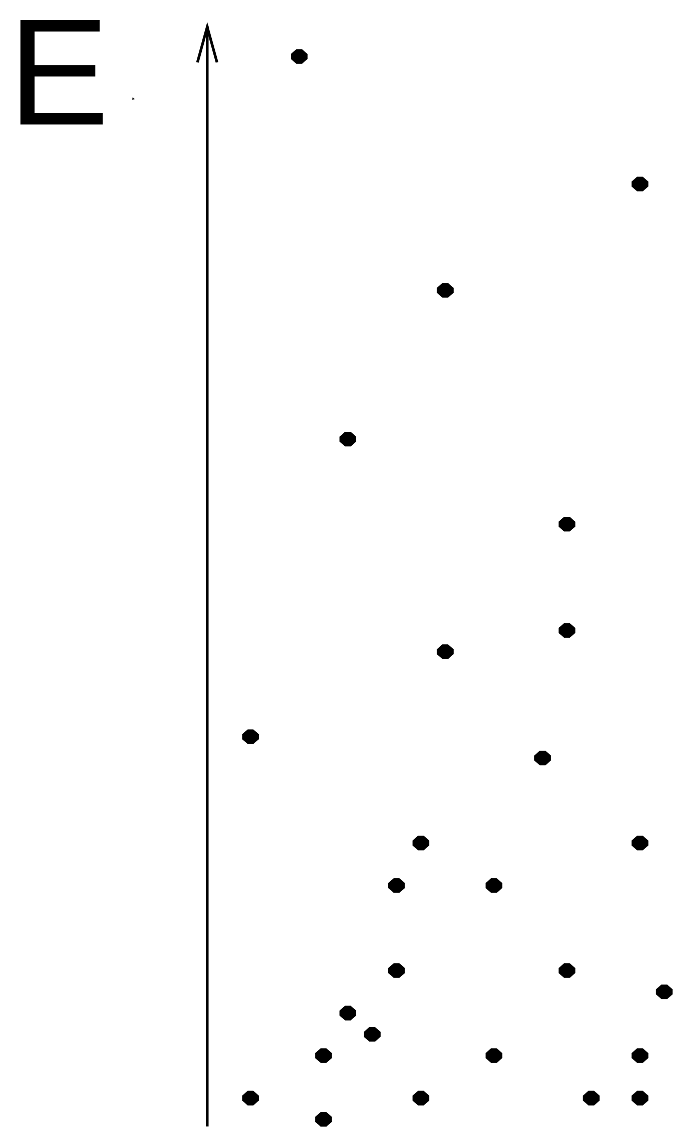
Neuron emits action potentials
→ why?
→Hodgkin-Huxley model

Hodgkin&Huxley (1952)
Nobel Prize 1963



density

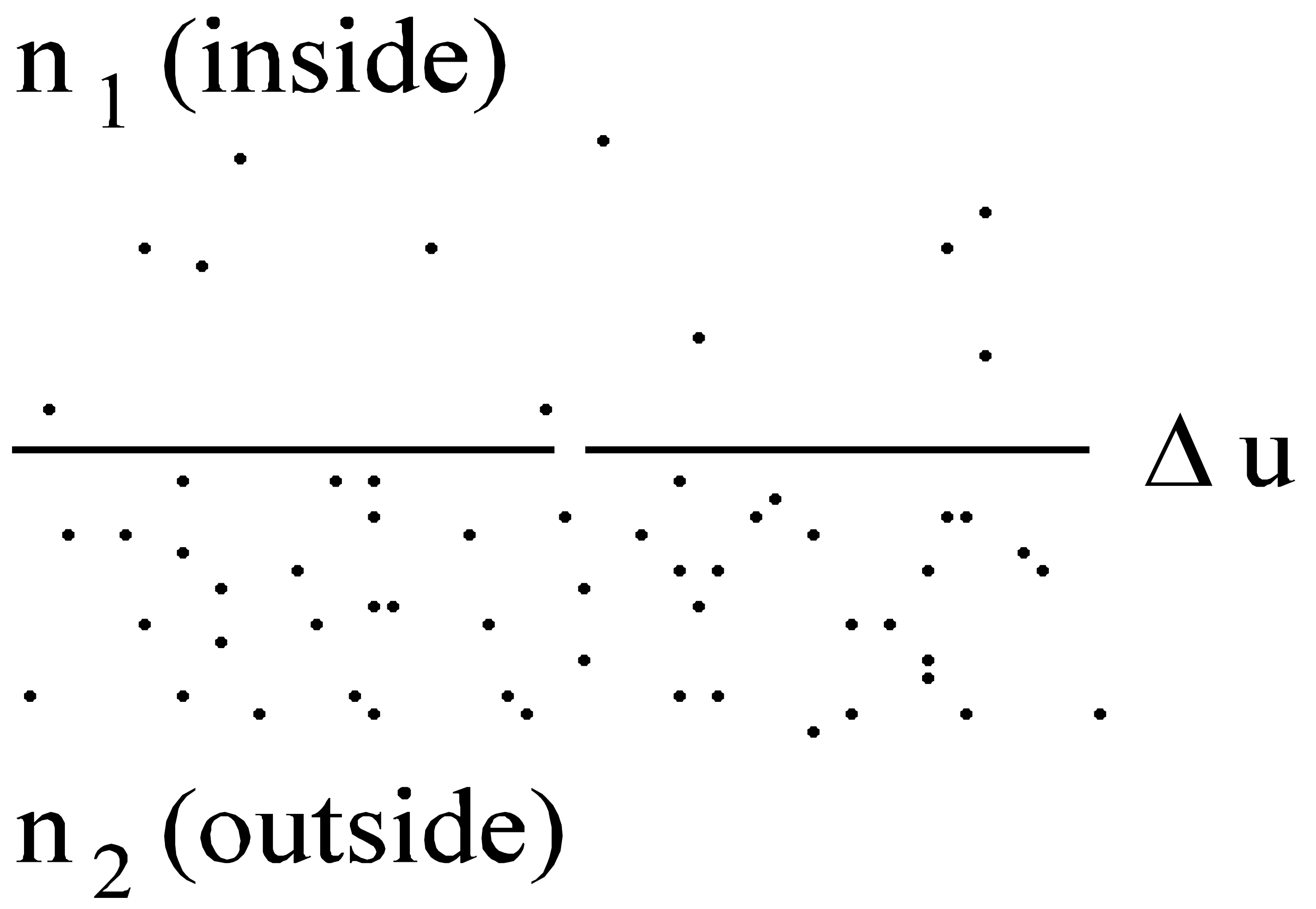
$$n \propto e^{-\frac{E}{kT}}$$



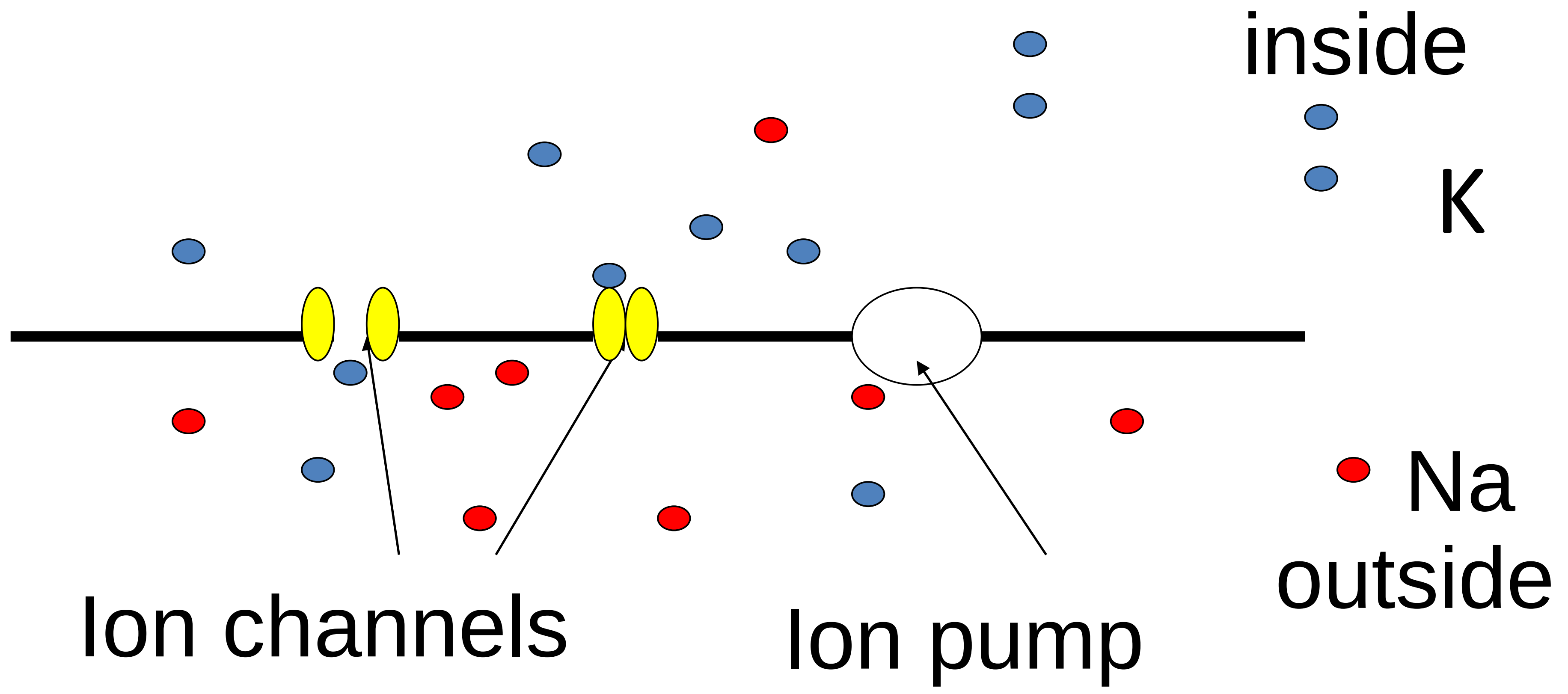
Ion pump ⇔ Concentration difference

Mathetical derivation

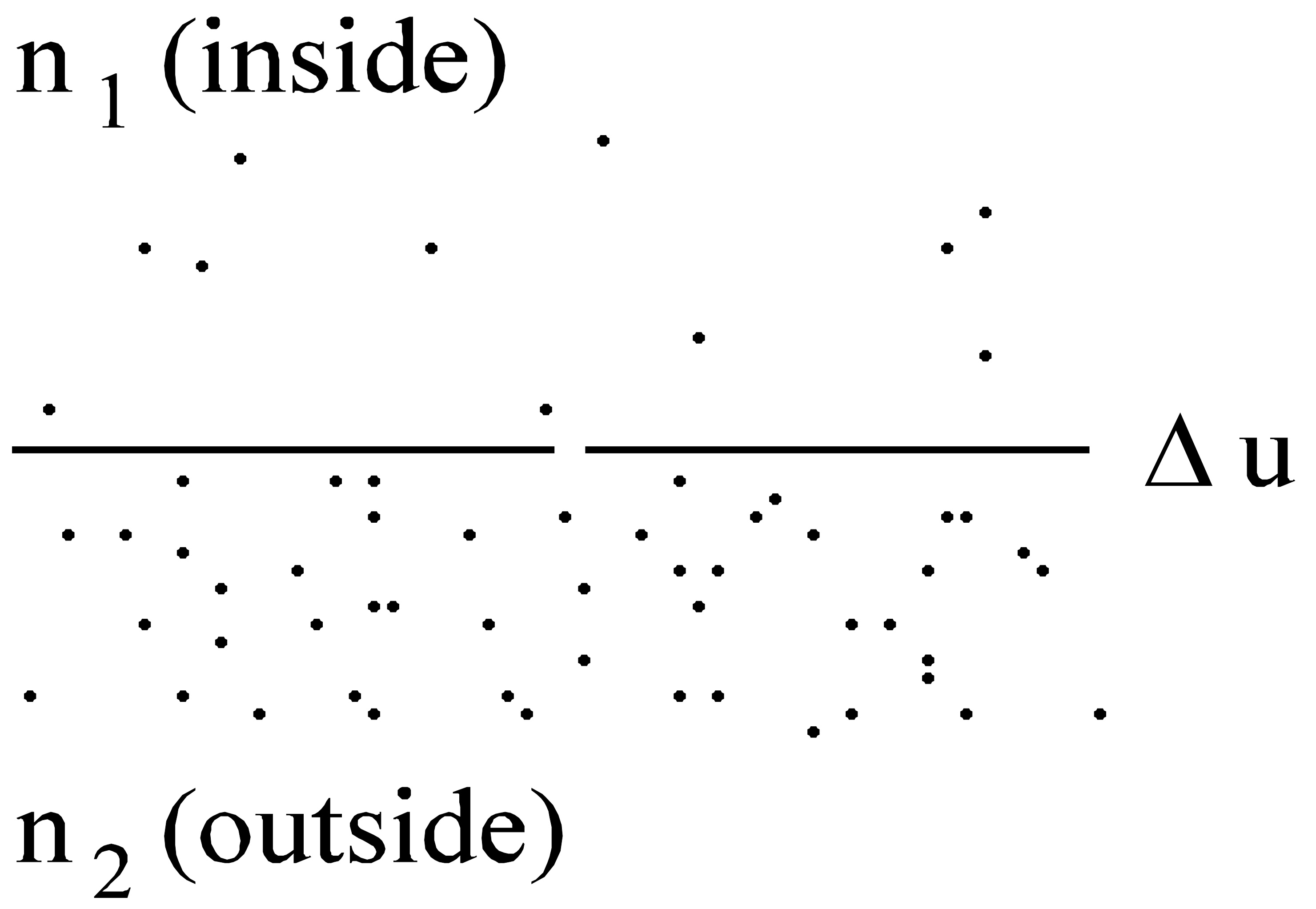
Neuronal Dynamics – 2. 2. Nernst equation

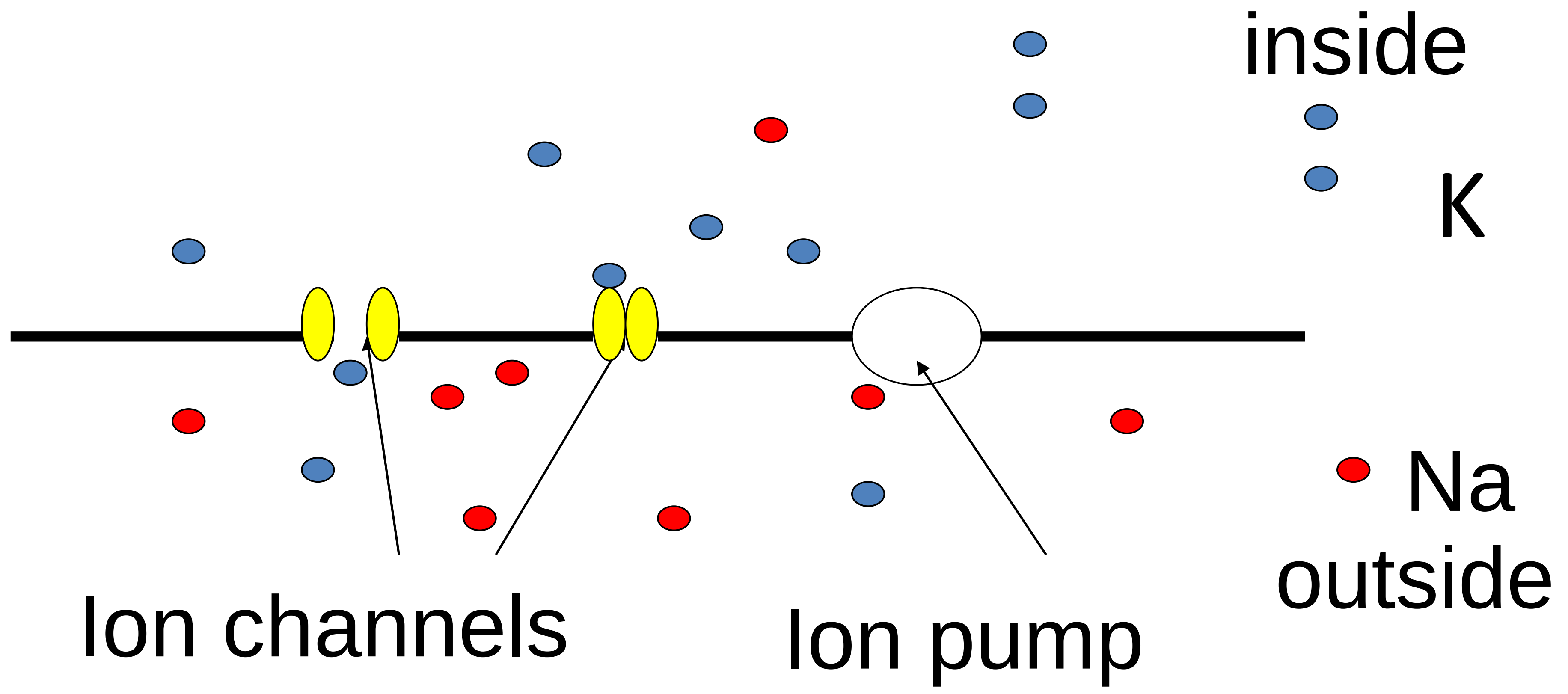


$$n \propto e^{-\frac{E}{kT}}$$



Neuronal Dynamics – 2. 2. Nernst equation





$$\Delta u = u_1 - u_2 = \frac{-kT}{q}\ln\frac{n(u_1)}{n(u_2)}$$

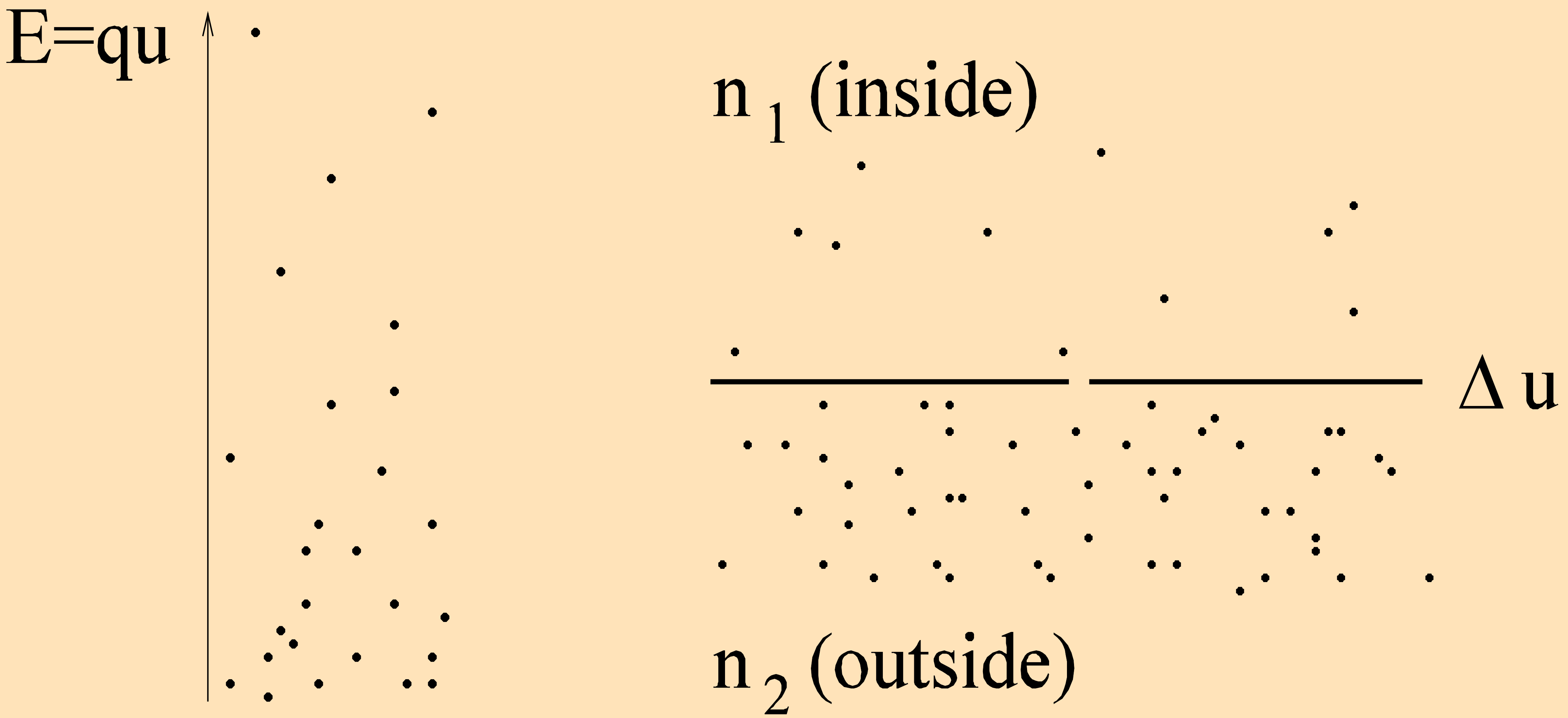
Reversal potential

Concentration difference ⇔ voltage difference

Exercise 1.1– Reversal potential of ion channels

Reversal potential

$$\Delta u = u_1 - u_2 = -\frac{kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

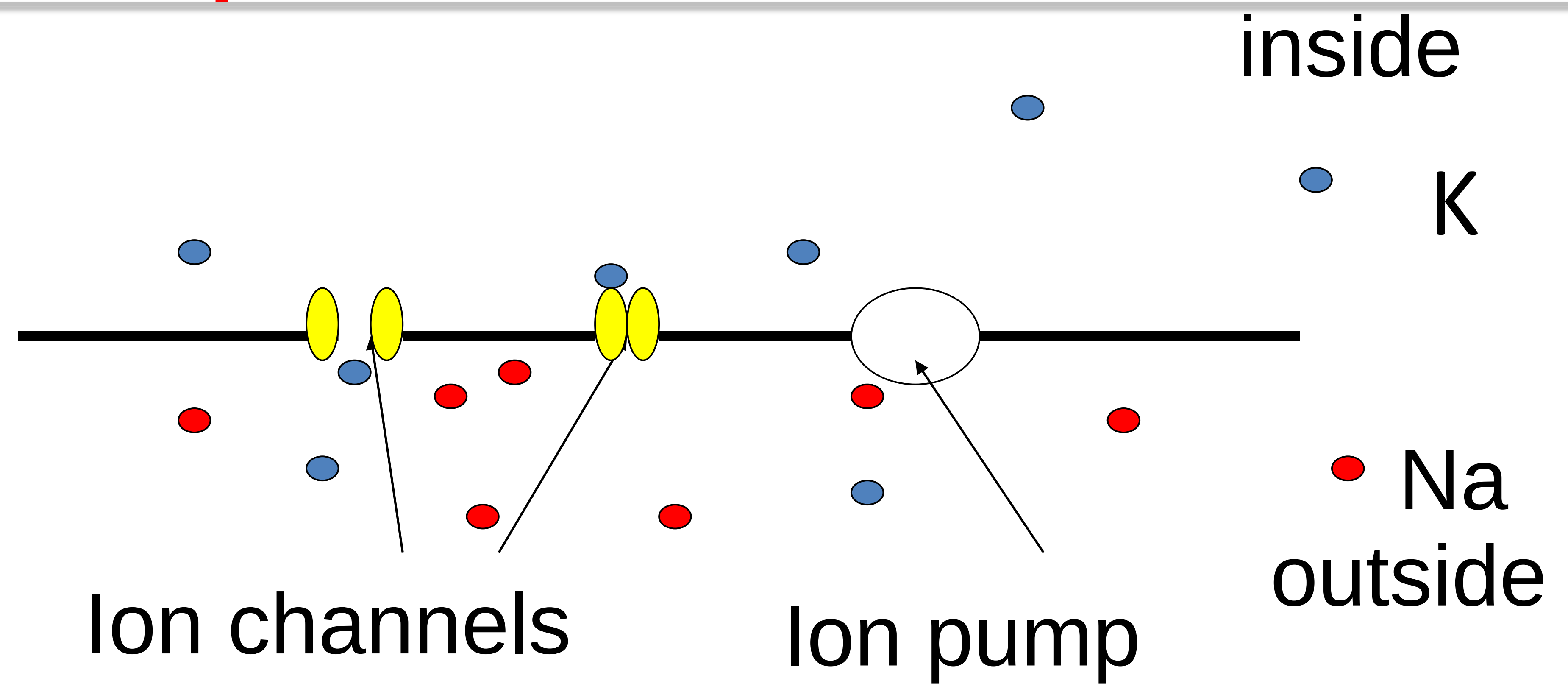


Calculate the reversal potential
for Sodium
Postassium
Calcium
given the concentrations

What happens if you change the temperature T from 37 to 18.5 degree?

Start exercise at 9:35
Next Lecture 9:48

Neuronal Dynamics – 2. 2. Reversal potential



Ion pump → Concentration difference

Concentration difference ⇔ voltage difference

Reversal potential

Nernst equation

Week 2 – part 3 : Hodgkin-Huxley Model

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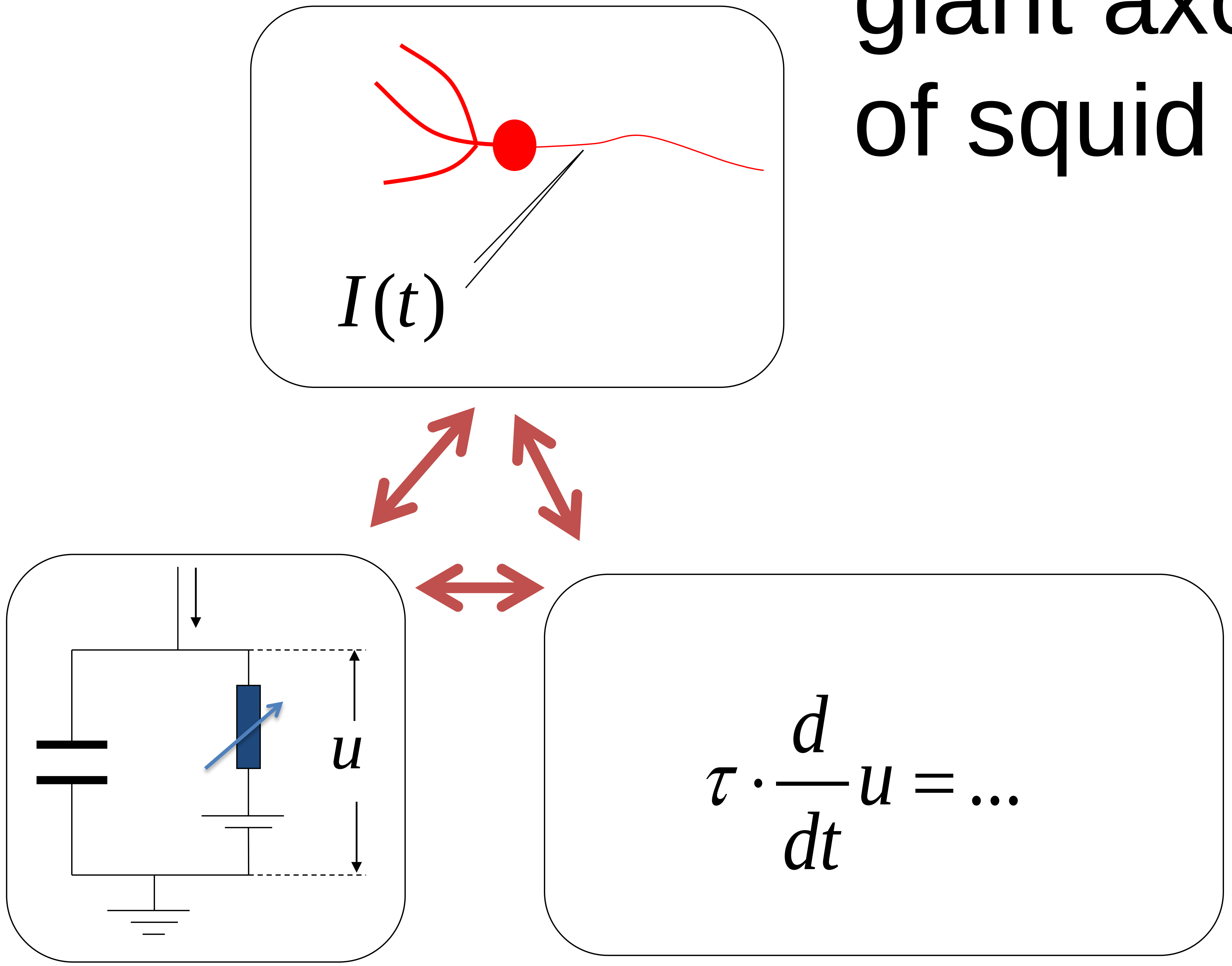
Neuronal Dynamics:
Computational Neuroscience
of Single Neurons

Week 2 – Biophysical modeling:
The Hodgkin-Huxley model

Wulfram Gerstner
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 2. 3. Hodgkin-Huxley Model

giant axon of squid



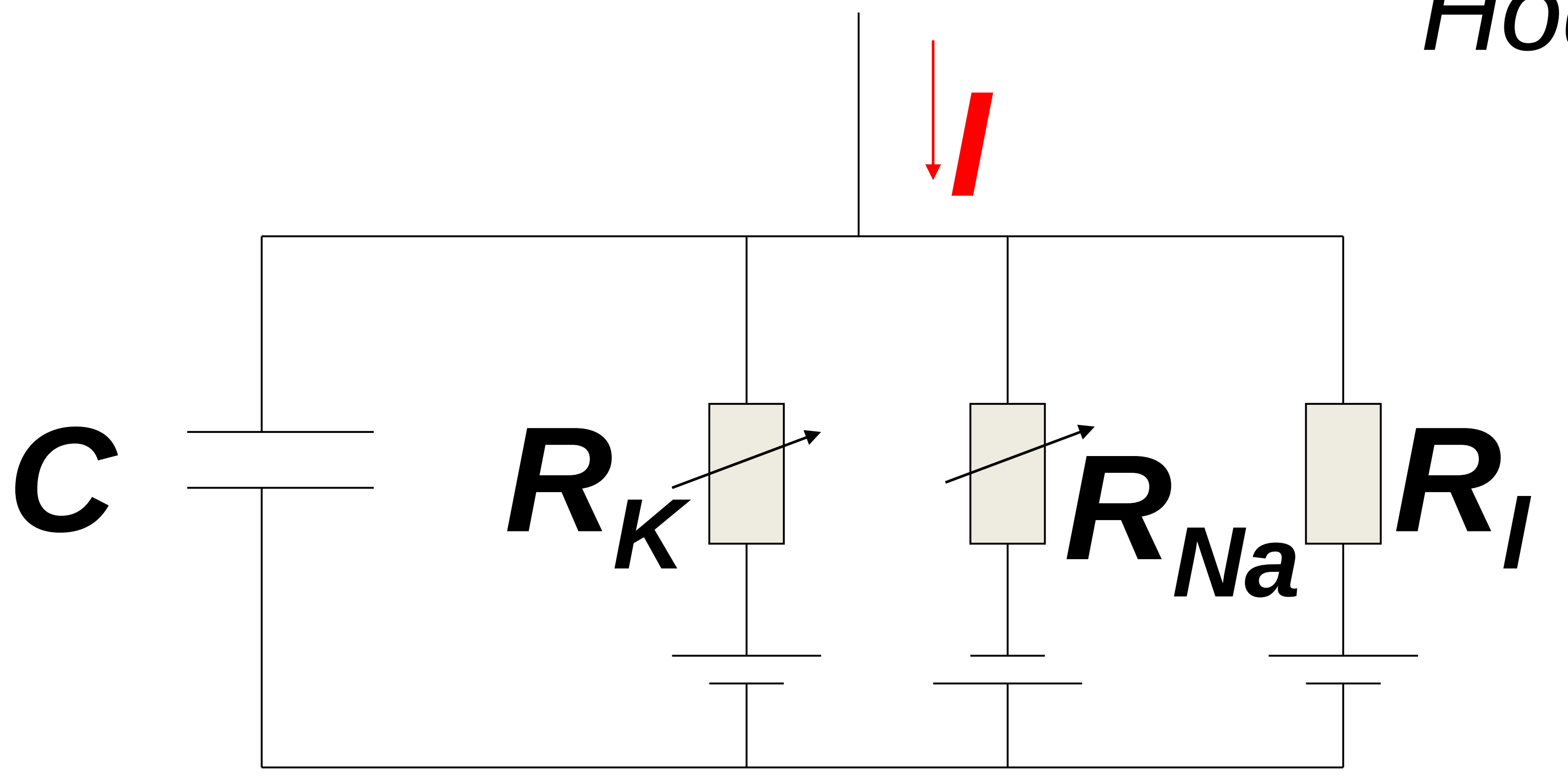
→Hodgkin-Huxley model

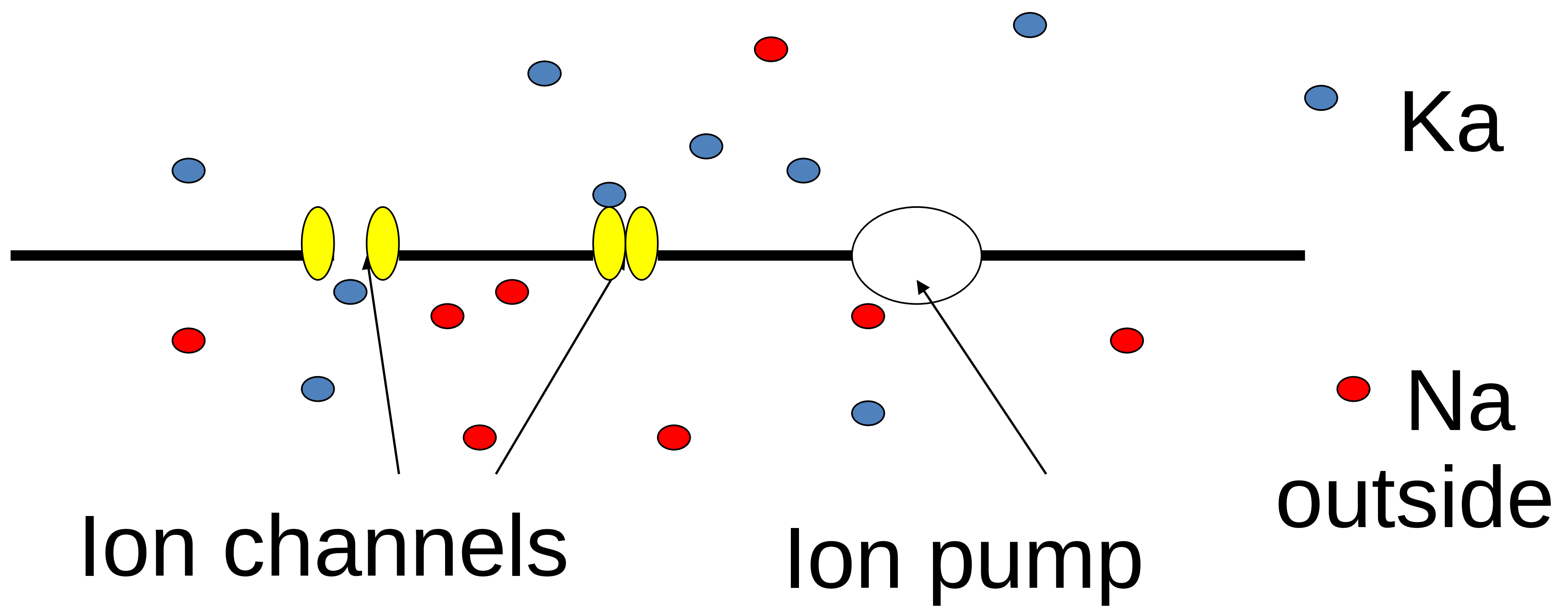
Hodgkin&Huxley (1952)

Nobel Prize 1963

Neuronal Dynamics – 2.3. Hodgkin-Huxley Model

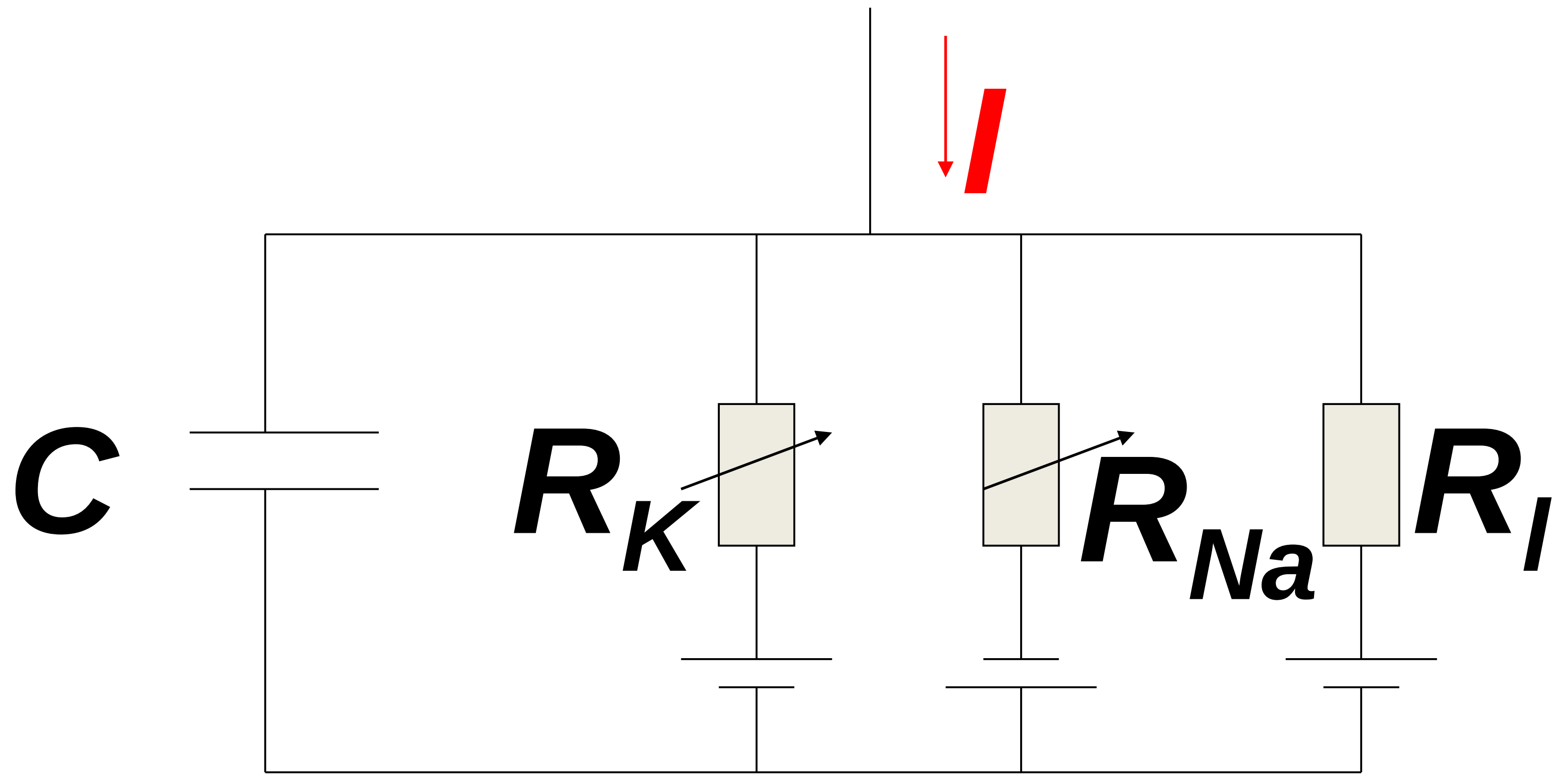
Hodgkin and Huxley, 1952





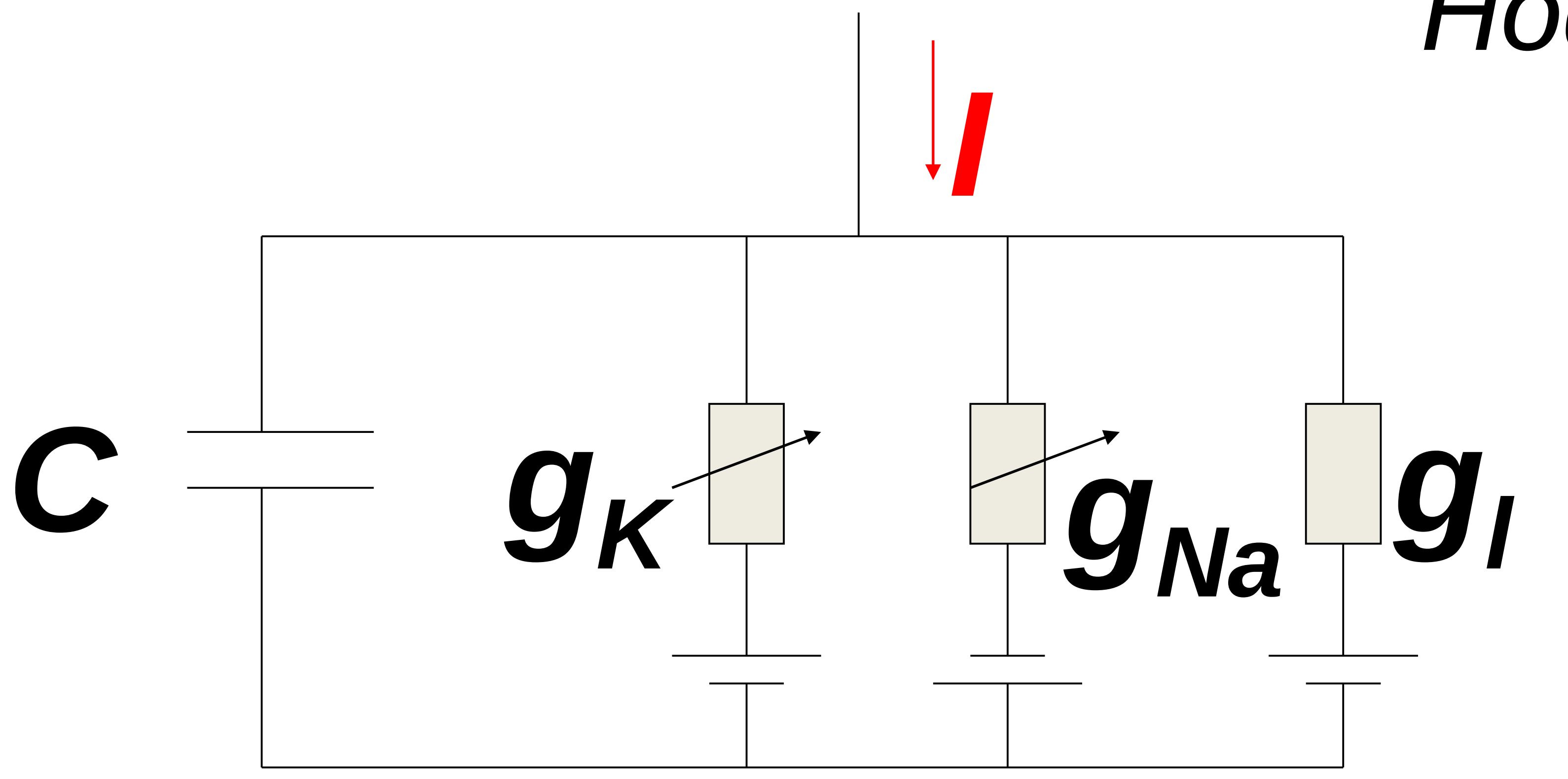
Mathematical derivation

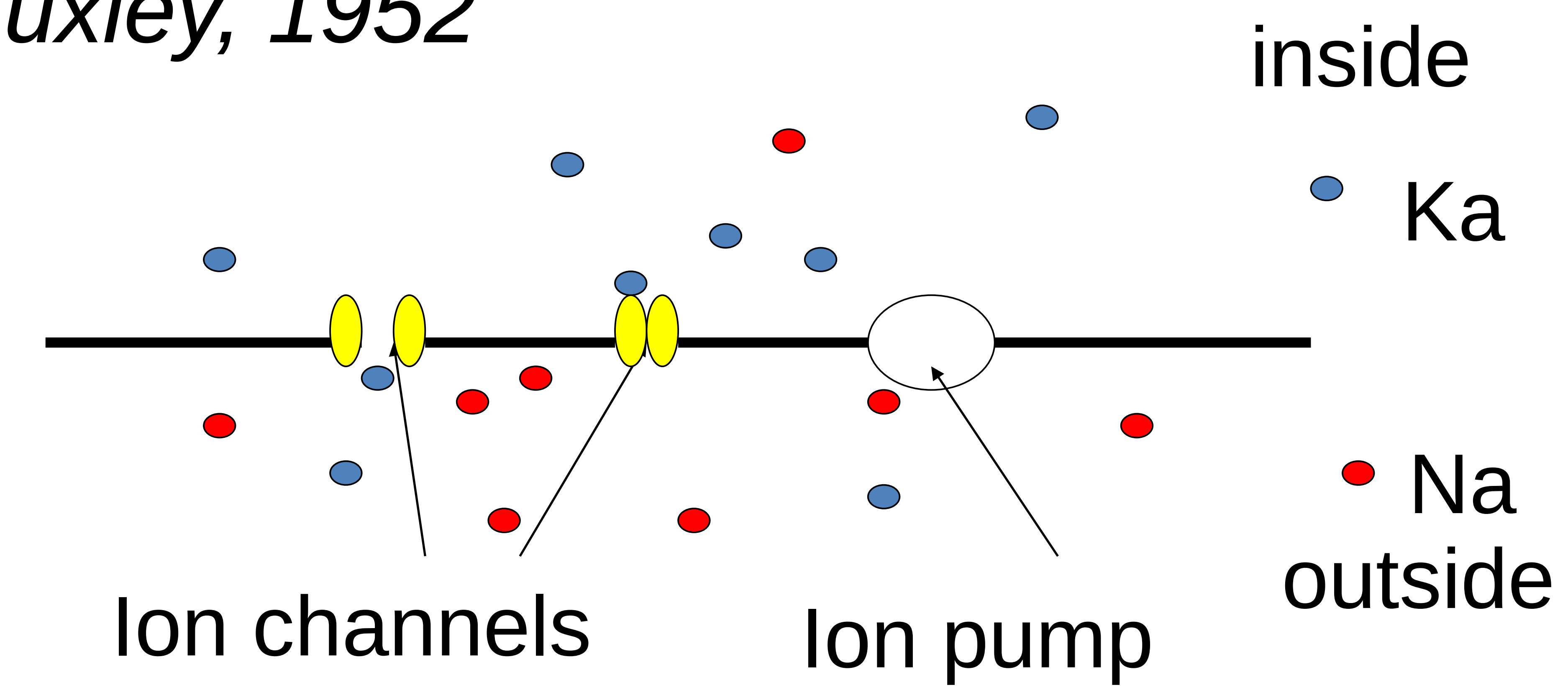
Neuronal Dynamics – 2.3. Hodgkin-Huxley Model



Neuronal Dynamics – 2.3. Hodgkin-Huxley Model

Hodgkin and Huxley, 1952

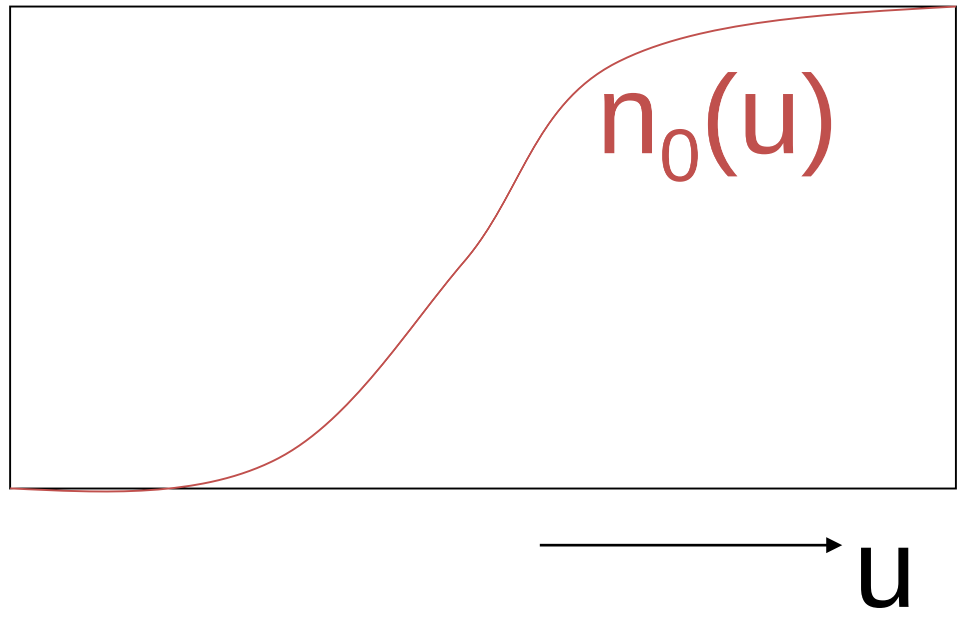


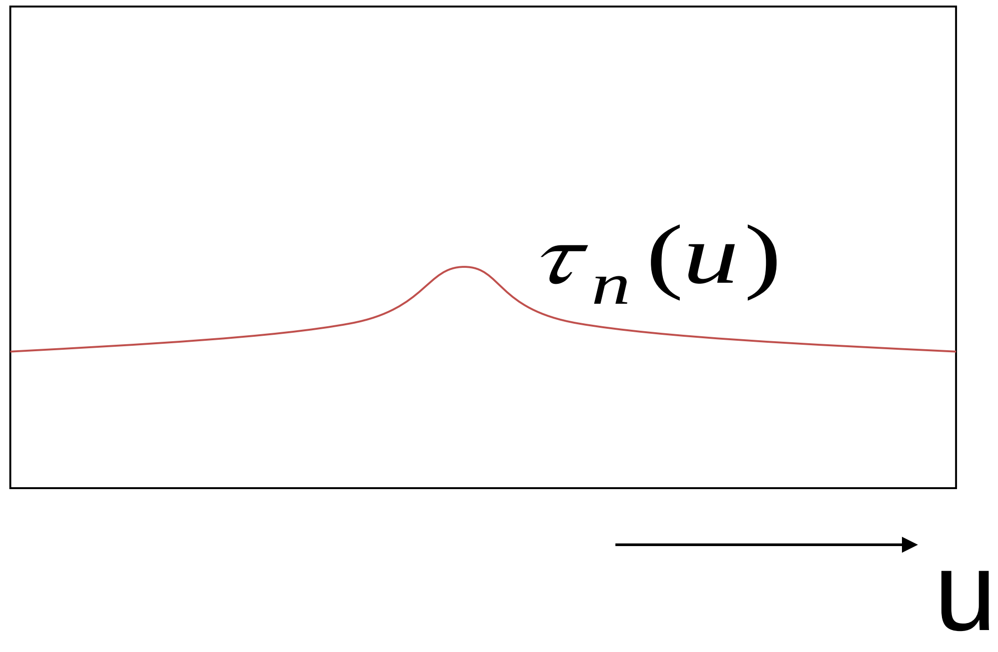


$$C\frac{du}{dt} = \overbrace{-g_{Na}m^3h(u-E_{Na})}^{I_{Na}} \overbrace{-g_K n^4(u-E_K)}^{I_K} \overbrace{-g_l(u-E_l)}^{I_{leak}} + \overset{\text{stimulus}}{\overset{\downarrow}{I(t)}}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

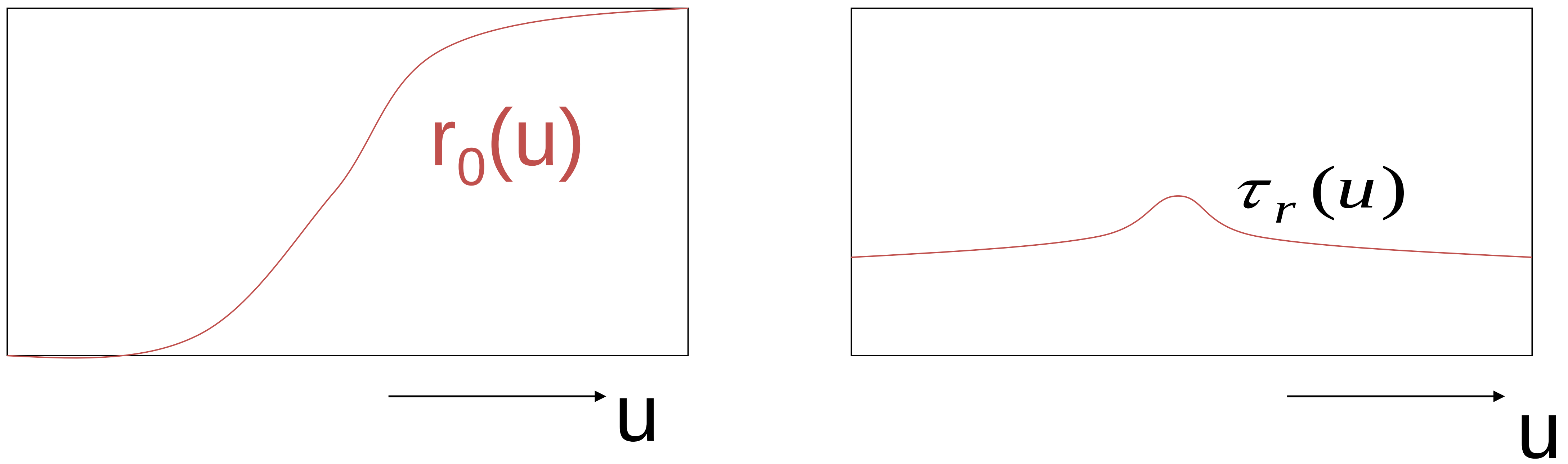
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$





Neuronal Dynamics – 2.3. Ion channel

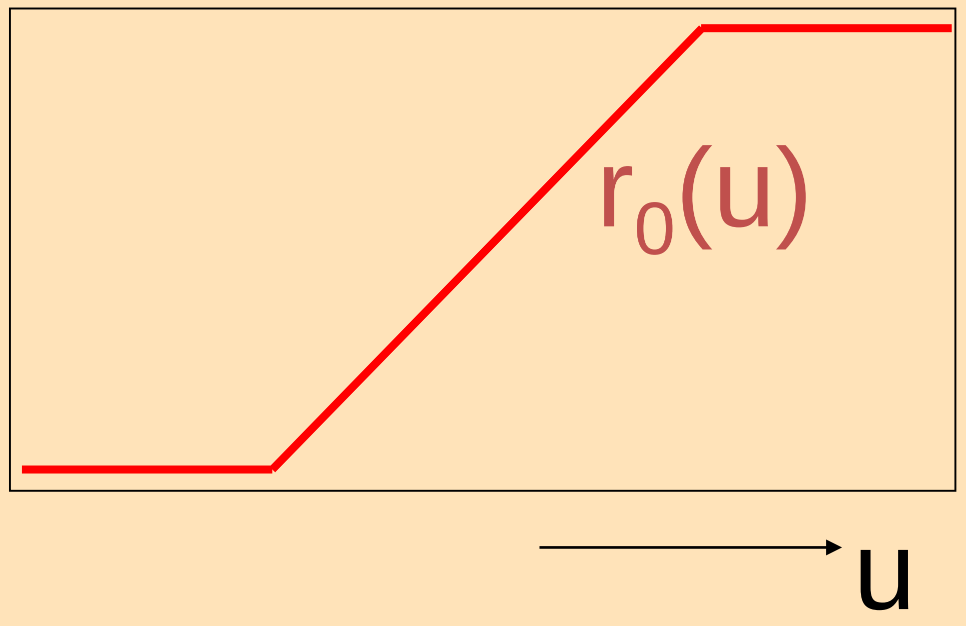
$$C\frac{du}{dt} = -\sum_k I_{ion,k} + I(t)$$

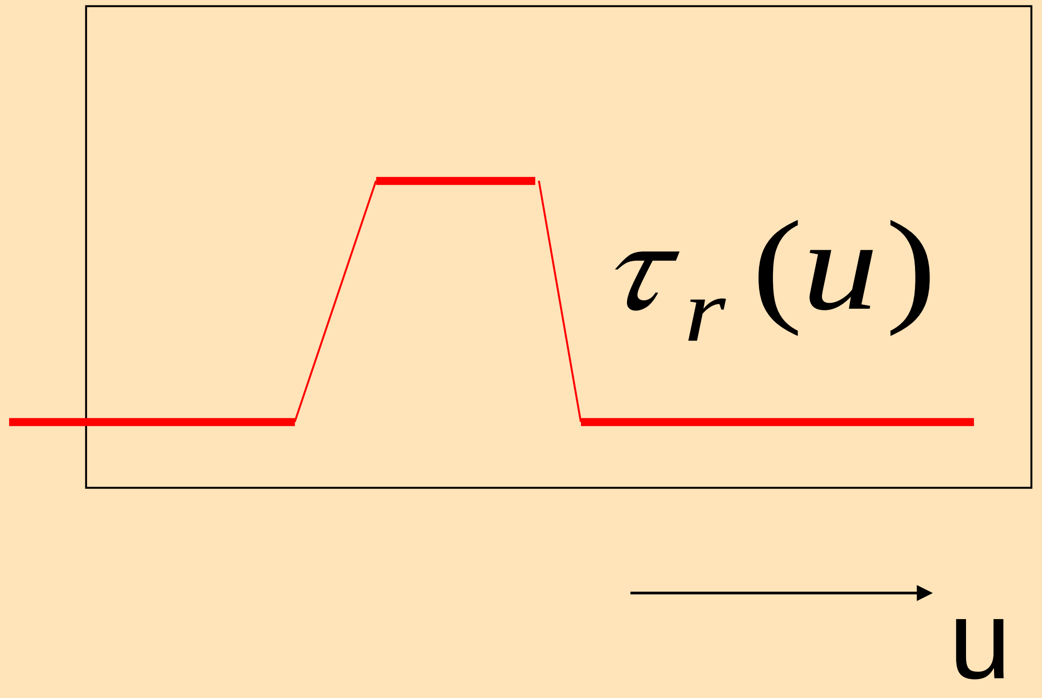


$$I_{ion} = -g_{ion} r^{n_1} s^{n_2} (u - E_{ion})$$

$$\frac{dr}{dt} = -\frac{r - r_0(u)}{\tau_r(u)} \qquad \frac{ds}{dt} = -\frac{s - s_0(u)}{\tau_r(u)}$$

Exercise 2 and 1.2 NOW!! - Ion channel





$$C\frac{du}{dt} = -g_{ion} r^{n_1} s^{n_2} (u - E_{ion}) + I(t)$$

$$\frac{dr}{dt} = -\frac{r - r_0(u)}{\tau_r(u)}$$

Exercises
1 and 2 NOW!
If finished, start
Exercise 3.
This will be a preparation
For Next lecture
At 11:15-12:05

Start Exercise 2 at 10:30.
Continue with Exercise 1.2

Next lecture at:
10H50

Week 2 – part 4: Threshold in the Hodgkin-Huxley Model

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Biological Modeling of Neural Networks

Week 2 – Biophysical modeling: The Hodgkin-Huxley model

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EPFL, Lausanne, Switzerland

√ 2.1 Biophysics of neurons
- Overview

√ 2.2 Reversal potential
- Nernst equation

√ 2.3 Hodgin-Huxley Model

2.4 Threshold in the Hodgkin-Huxley Model
- where is the firing threshold?

2.5. Detailed biophysical models
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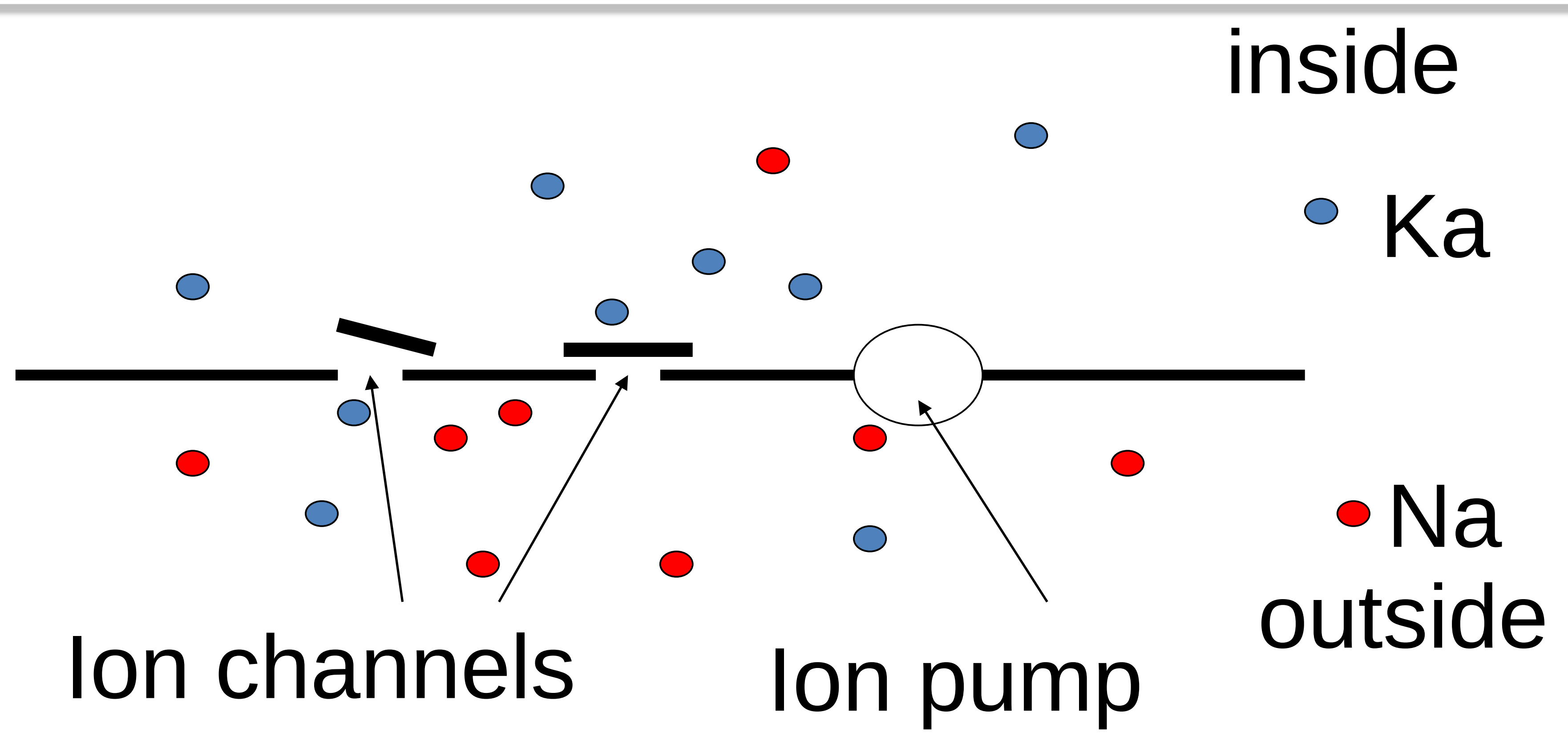
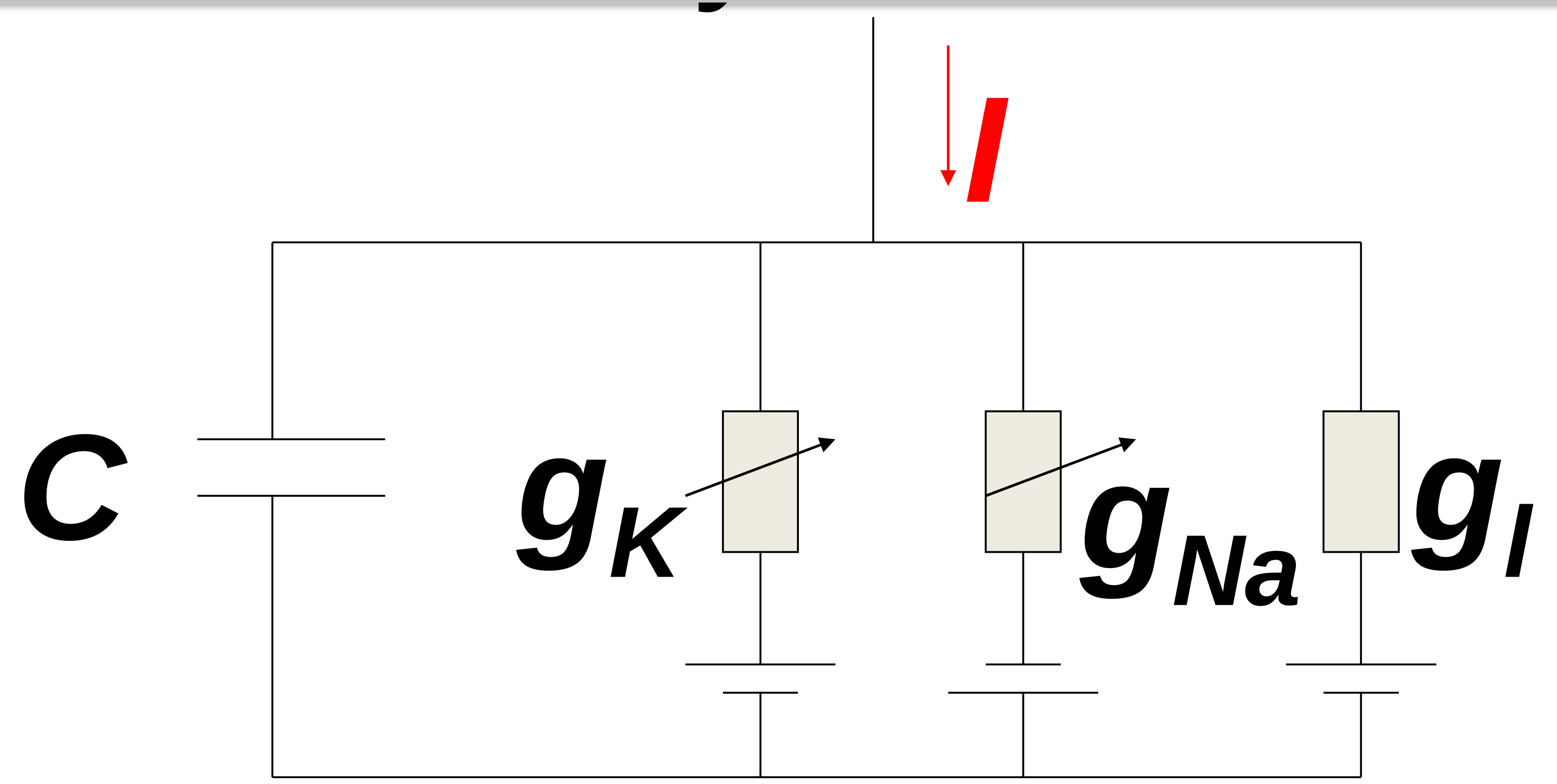
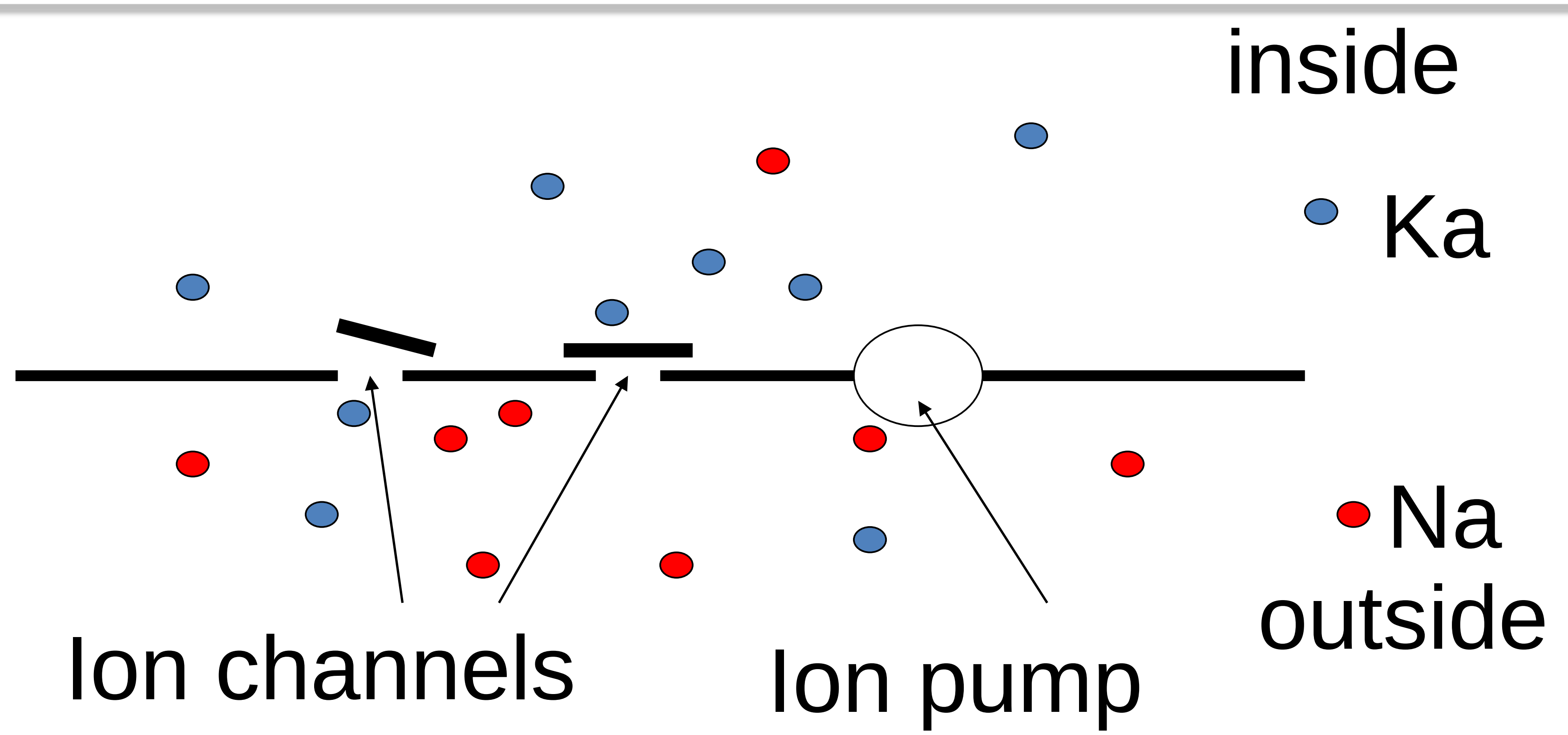
Neuronal Dynamics – 2.4. Threshold in HH model



Neuronal Dynamics – 2.4. Threshold in HH model



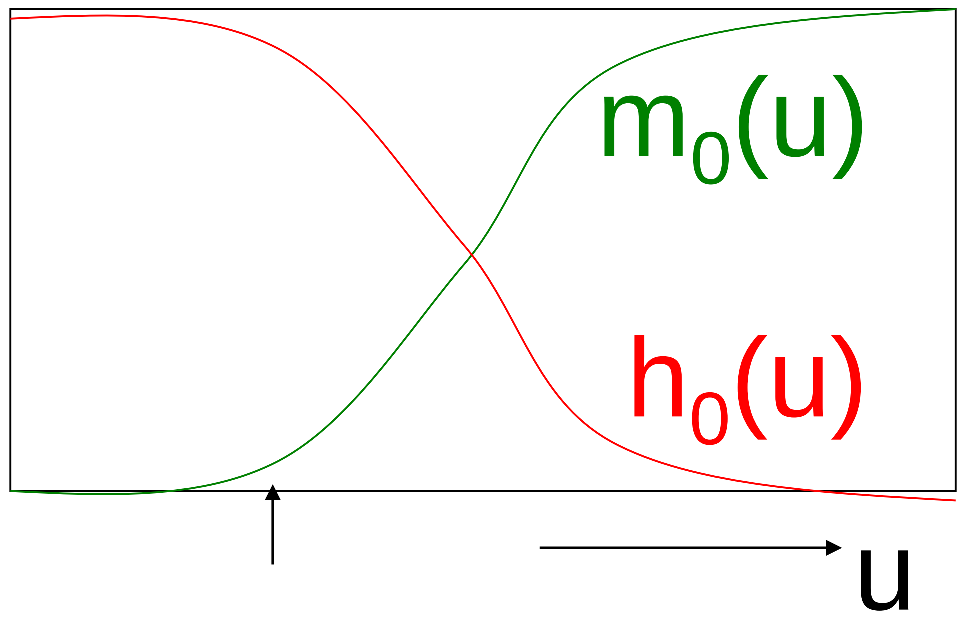


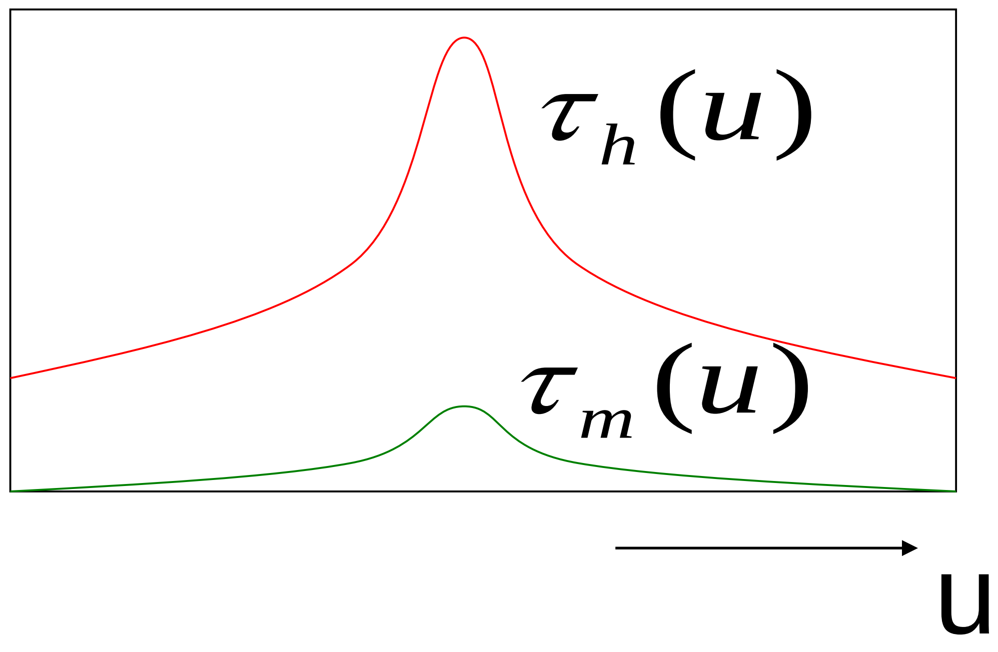
$$C\frac{du}{dt} = -\overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} - g \dots \overbrace{\dots}^{I_K} \dots \overbrace{\dots}^{I} g_l(u-E_l) + I(t)$$

$$\frac{dm}{dt} = -\frac{m \dots}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

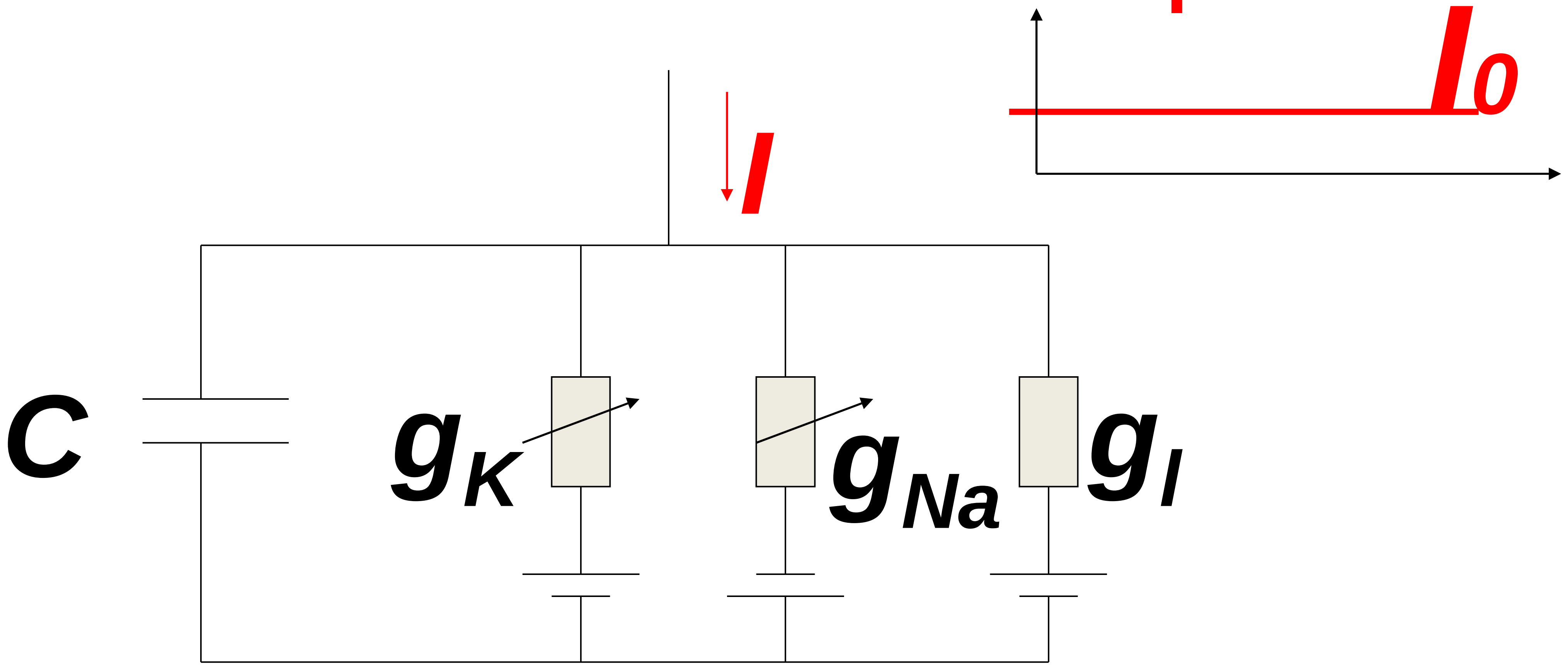
Where is the threshold for firing?

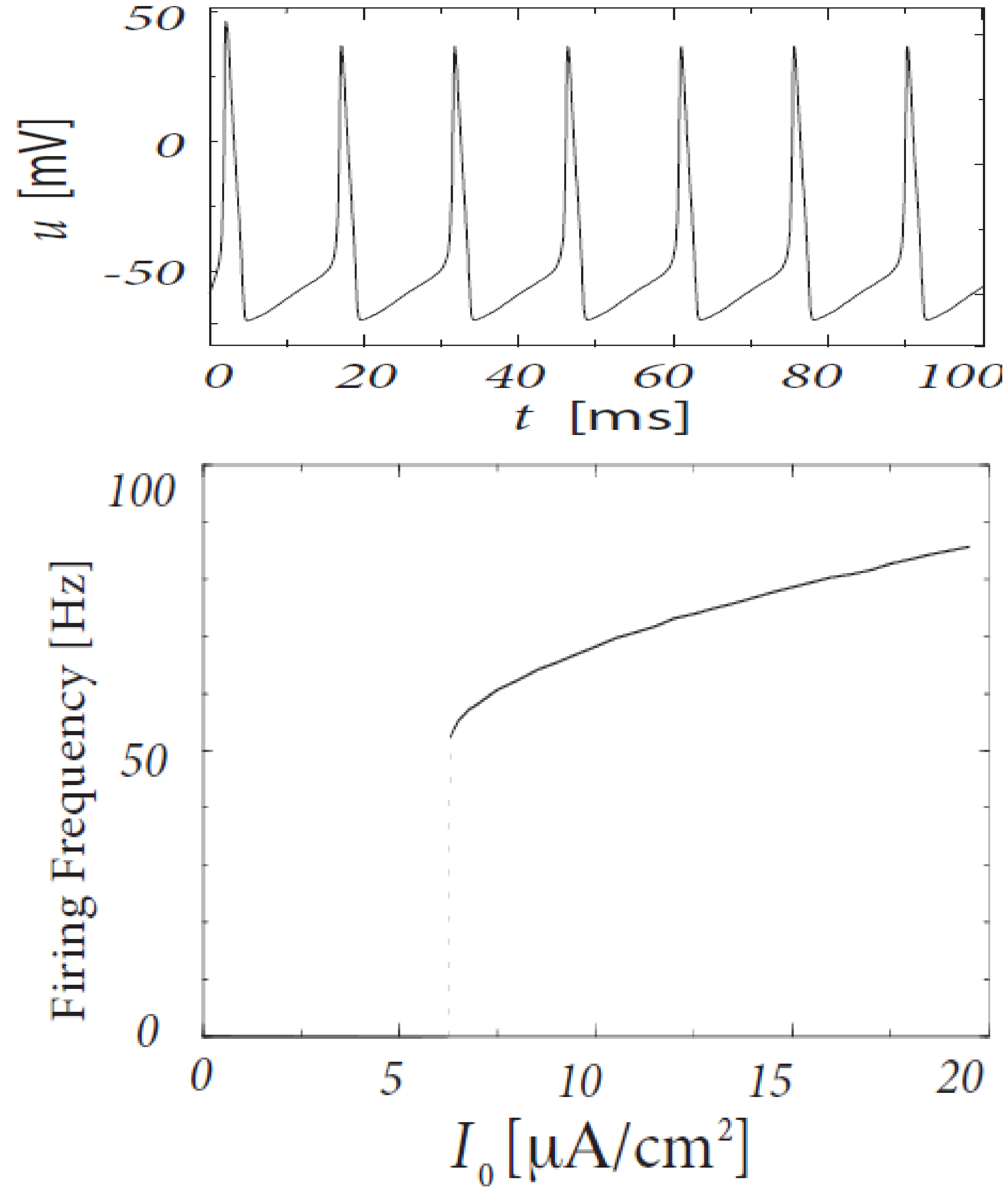




Neuronal Dynamics – 2.4. Threshold in HH model

Constant current input





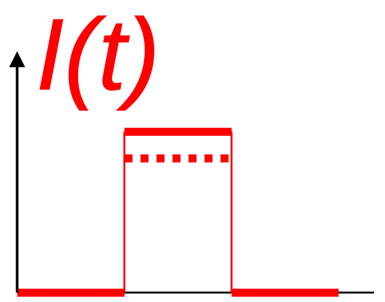
Threshold?

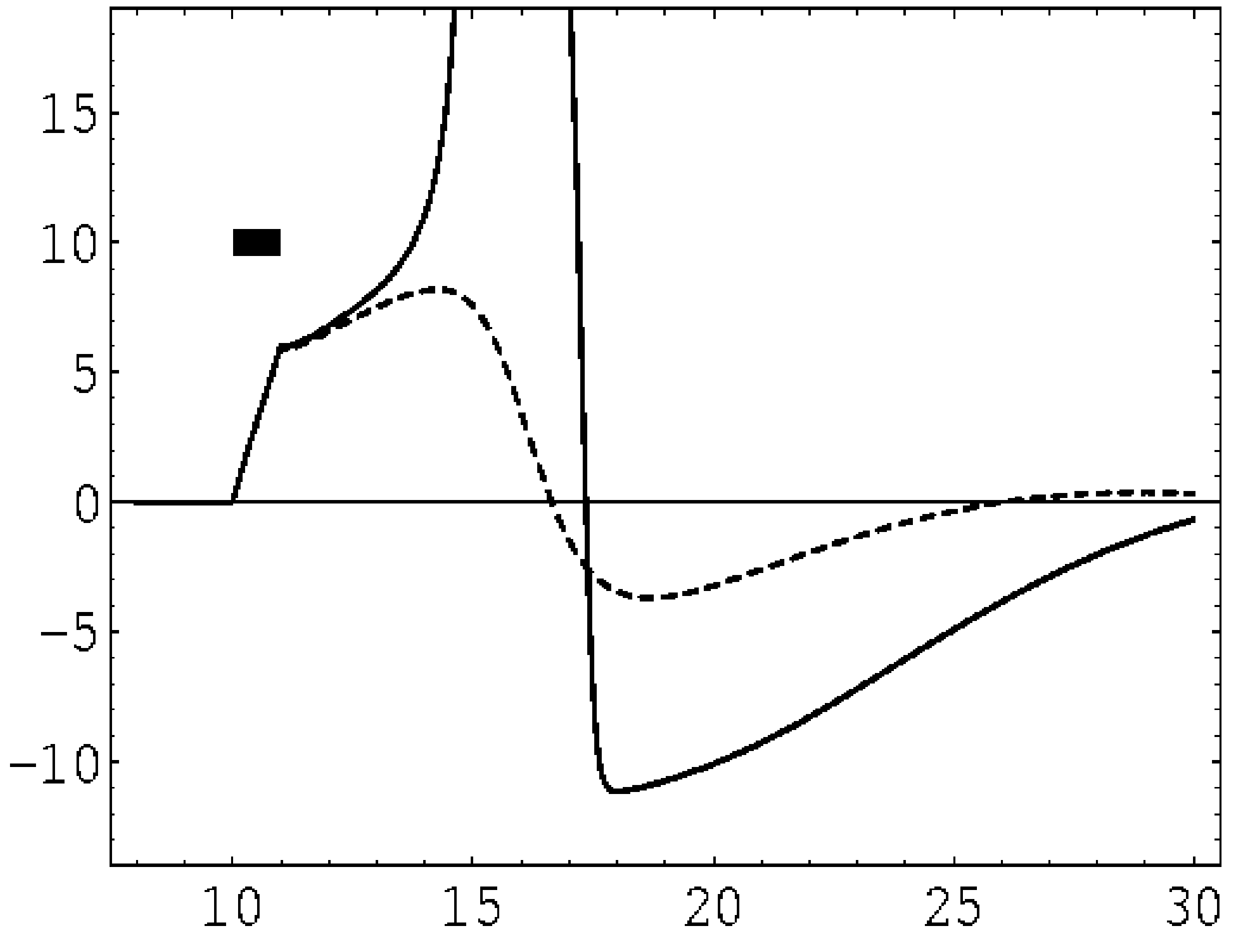
for repetitive firing

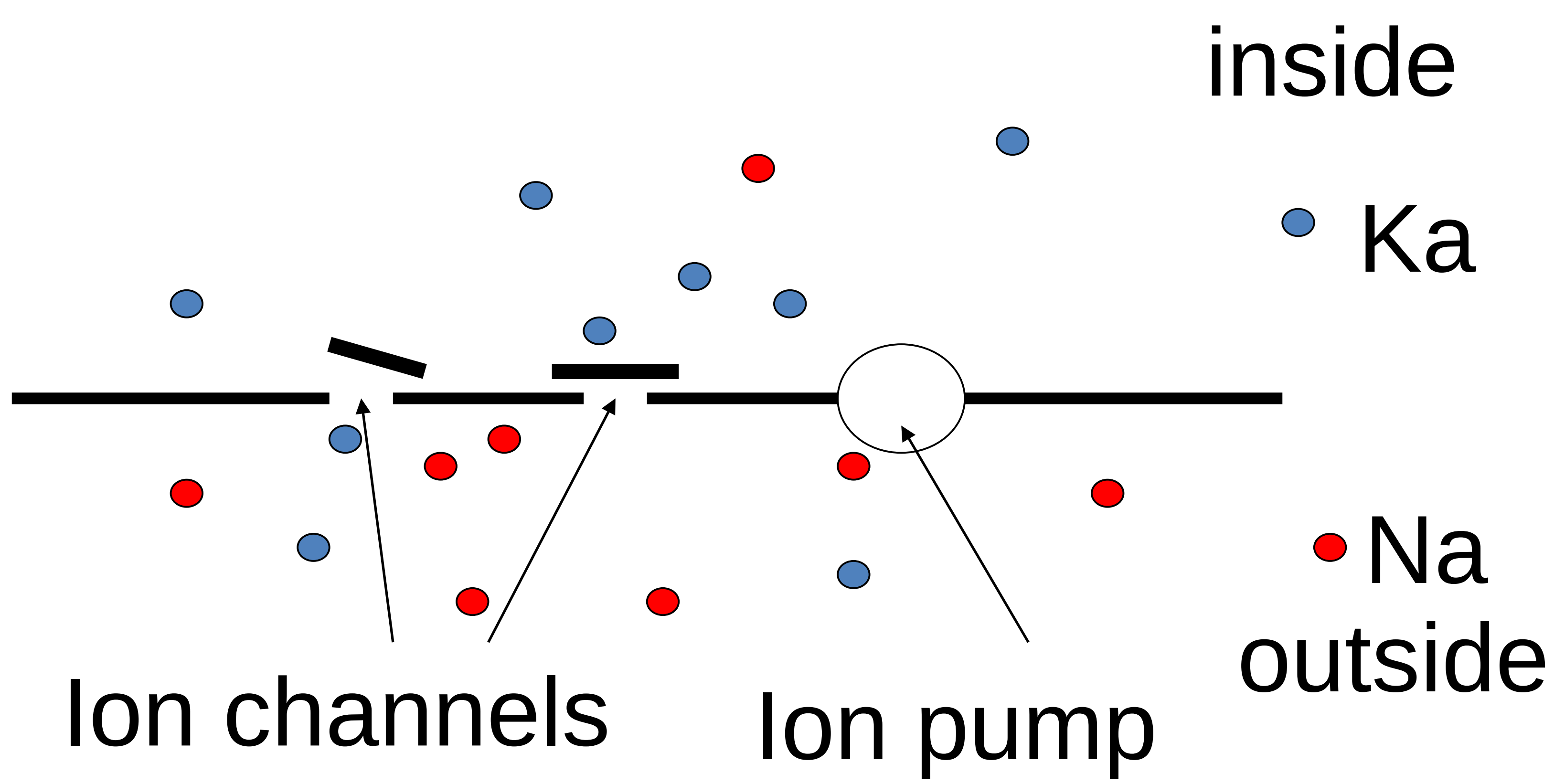
(***current*** threshold)

Neuronal Dynamics – 2.4. Threshold in HH model

pulse input







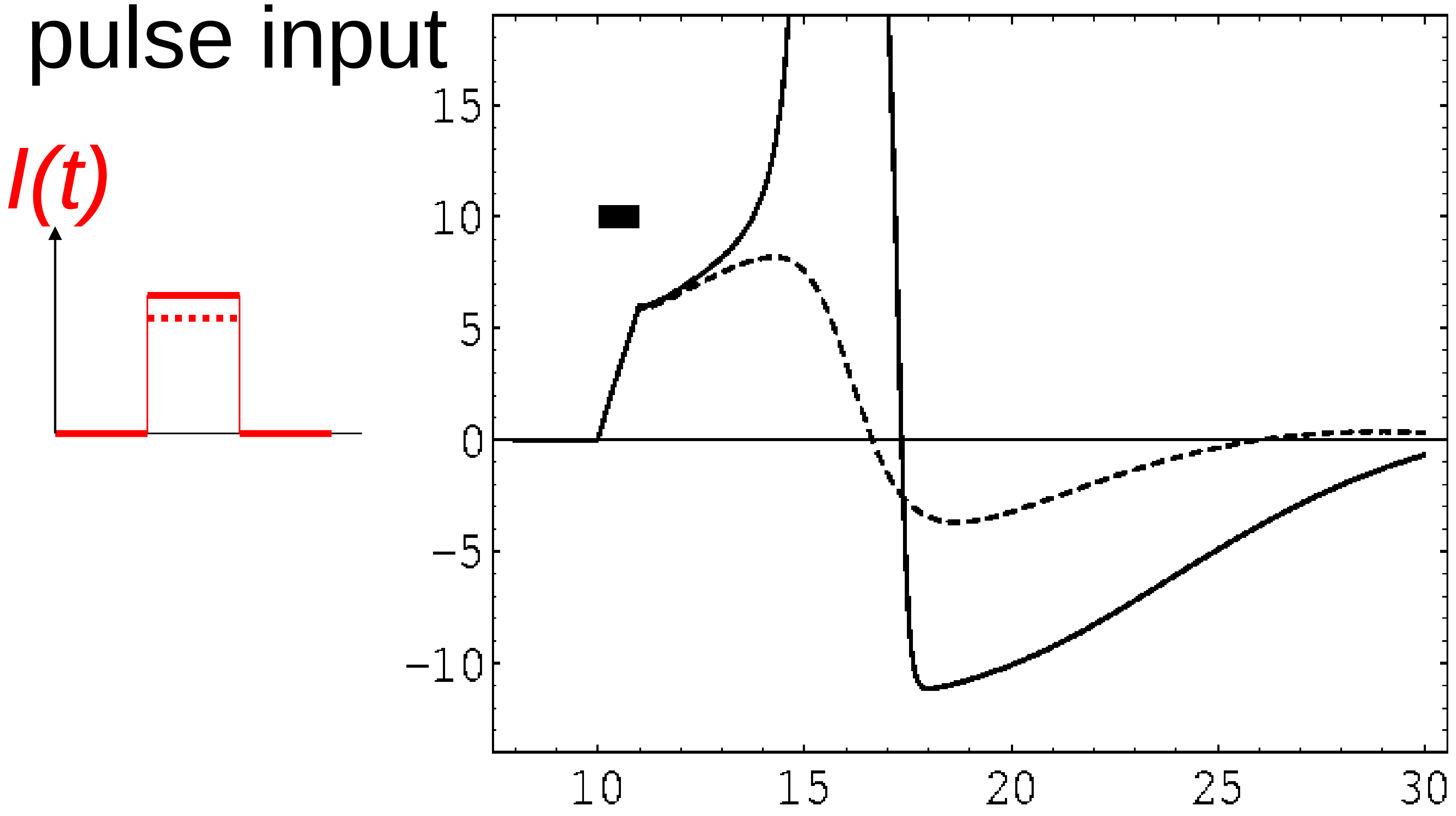
Threshold?

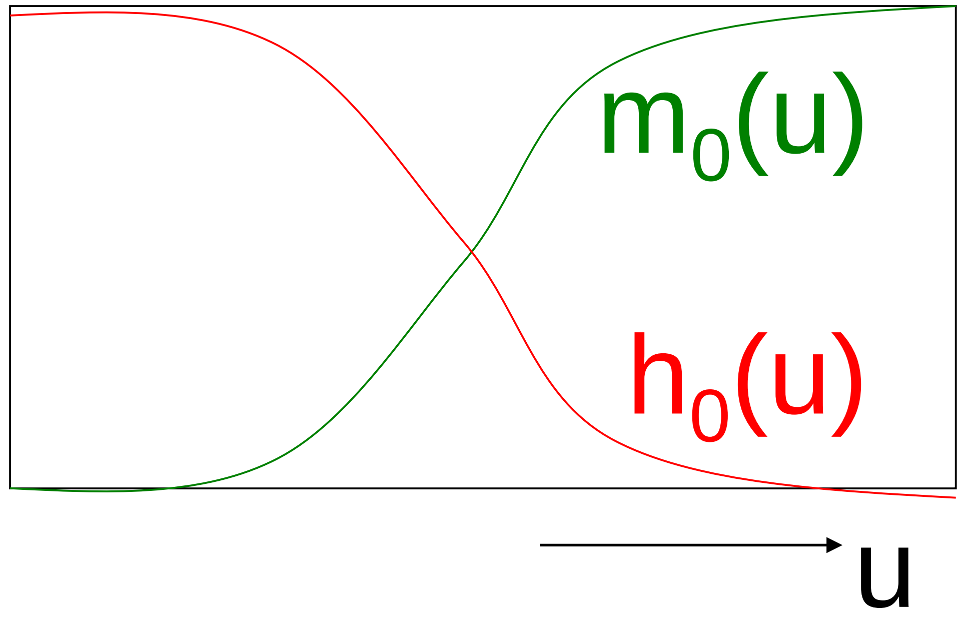
- AP if amplitude 7.0 units
- No AP if amplitude 6.9 units

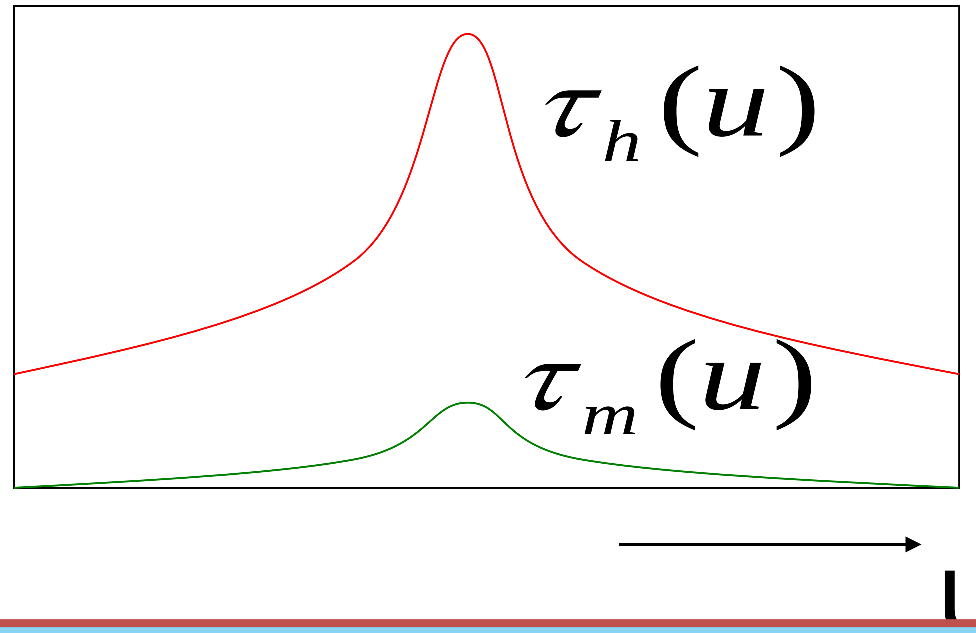
(pulse with 1ms duration)

(and pulse with 0.5 ms duration?)

Neuronal Dynamics – 2.4. Threshold in HH model







Mathematical explanation

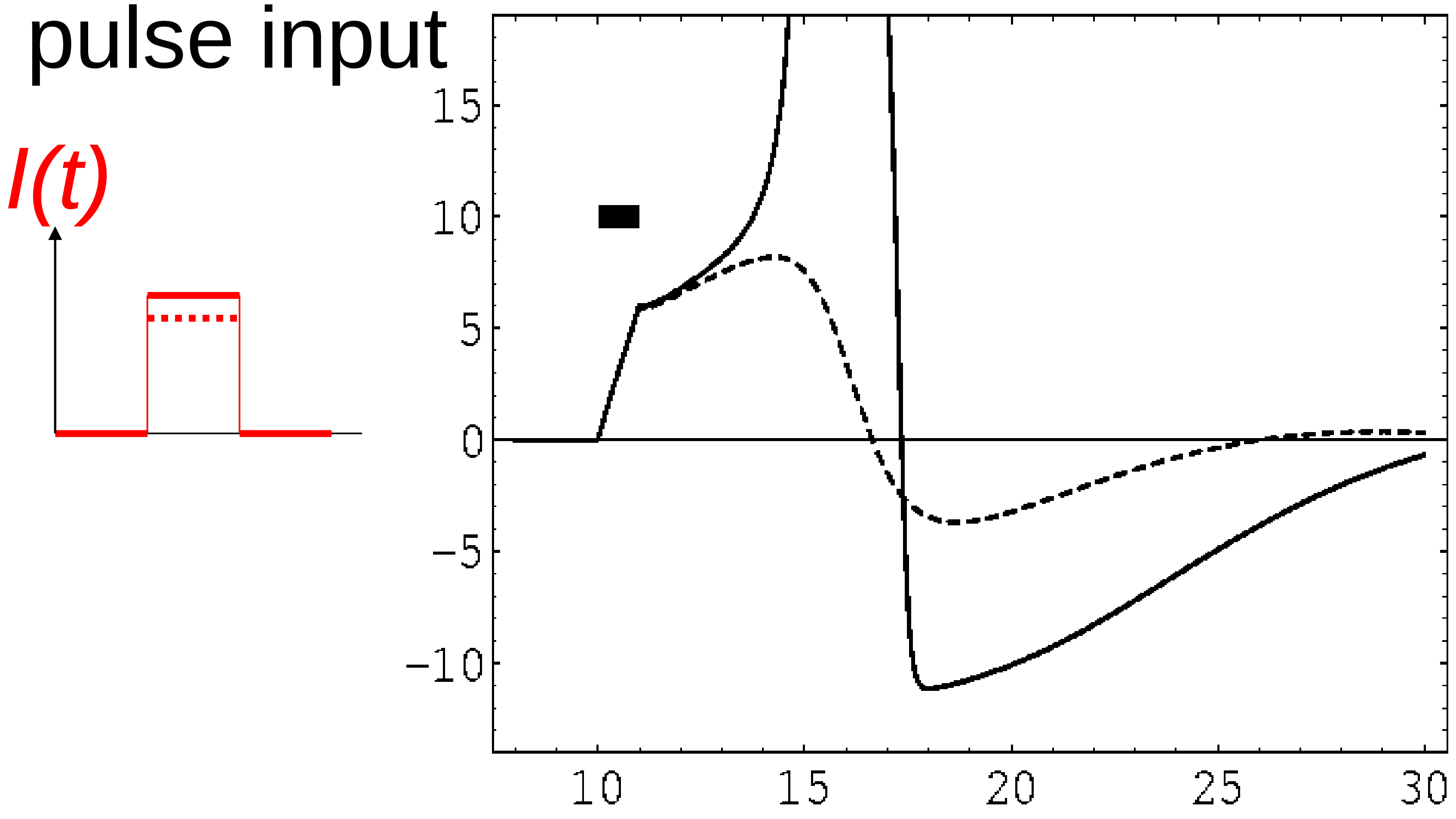
$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + \underbrace{I(t)}_{\text{Stim.}}$$

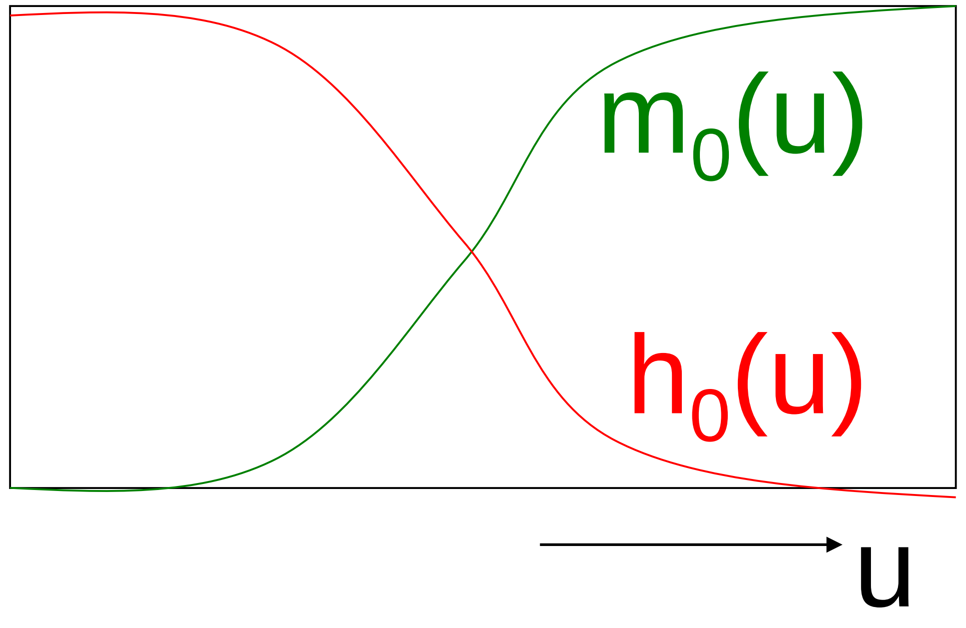
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

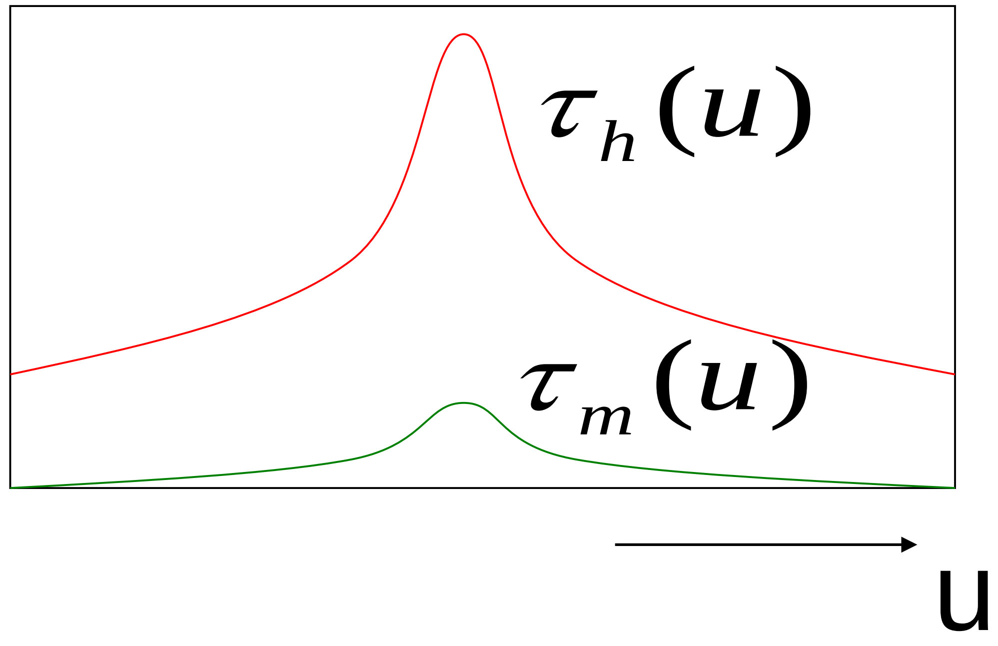
$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

Neuronal Dynamics – 2.4. Threshold in HH model

pulse input







Why start the explanation with *m* and not *h*?

What about *n*?

Where is the threshold?

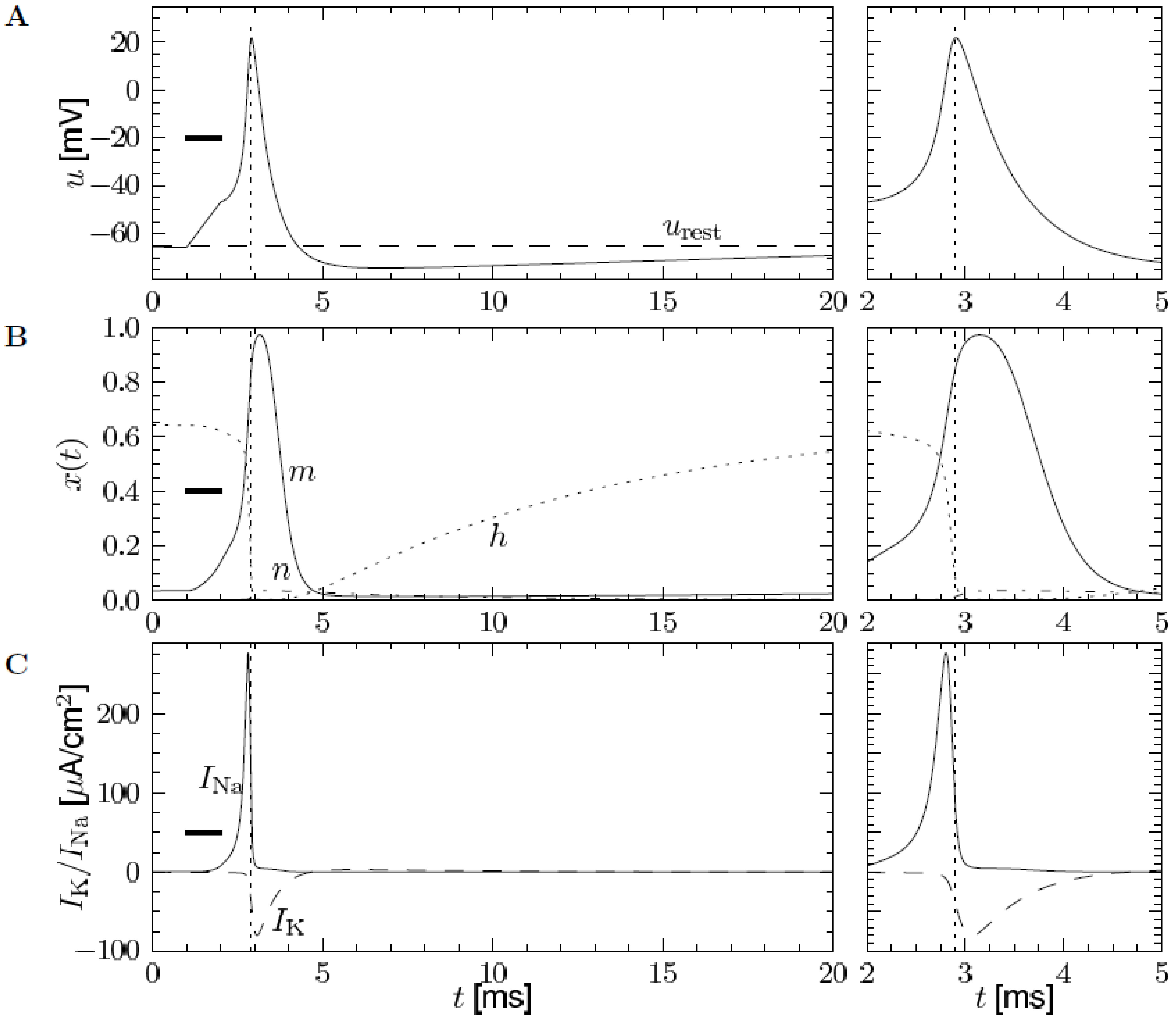
$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + \underbrace{I(t)}_{\text{Stim.}}$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$

Neuronal Dynamics – 2.4. Threshold in HH model



$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na})$$

$$-g_K n^4 (u-E_K)$$

$$-g_l(u-E_l)$$

$$+I(t)$$

First conclusion:

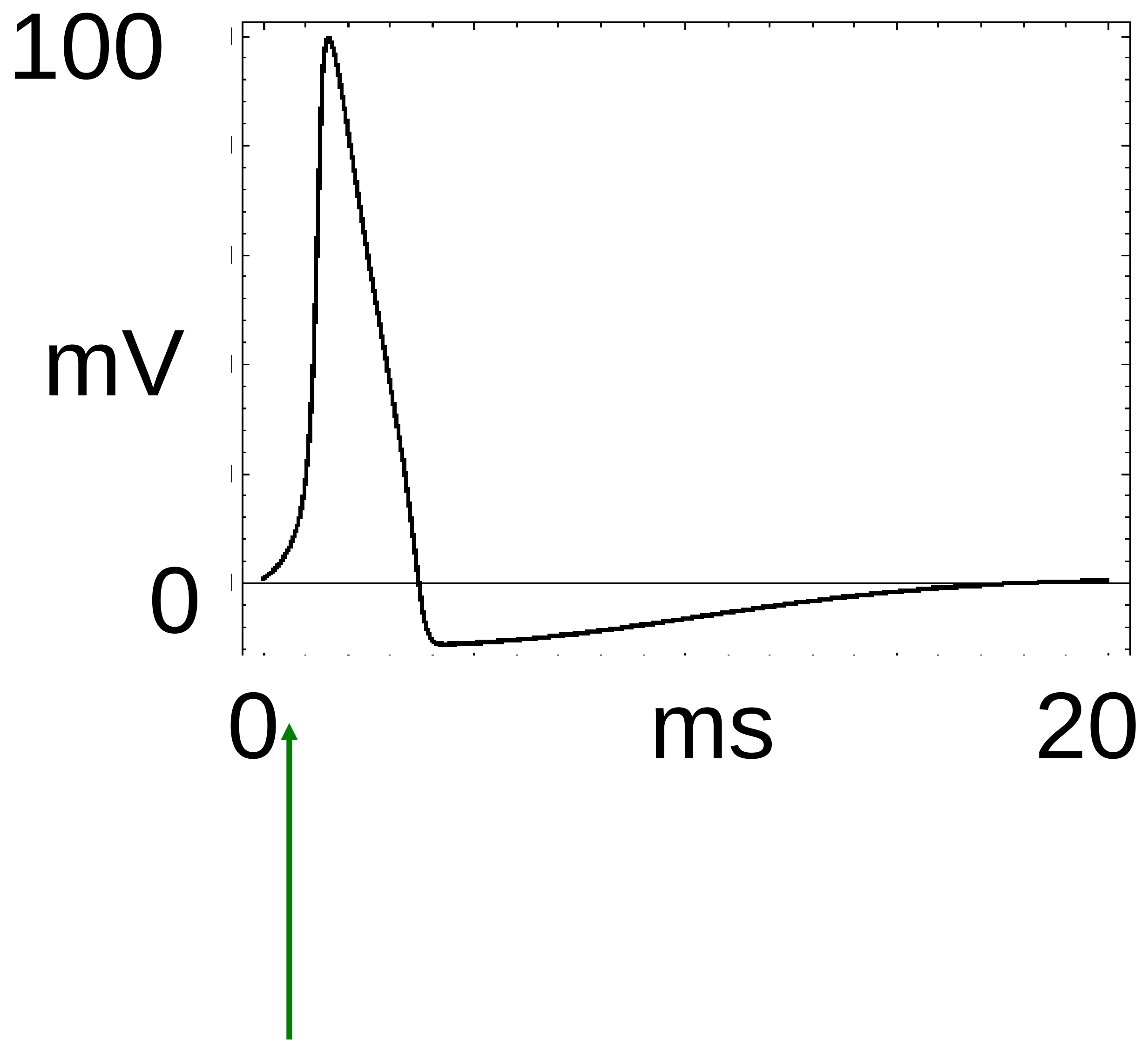
There is no strict threshold:

Coupled differential equations

'Effective' threshold
in simulations?

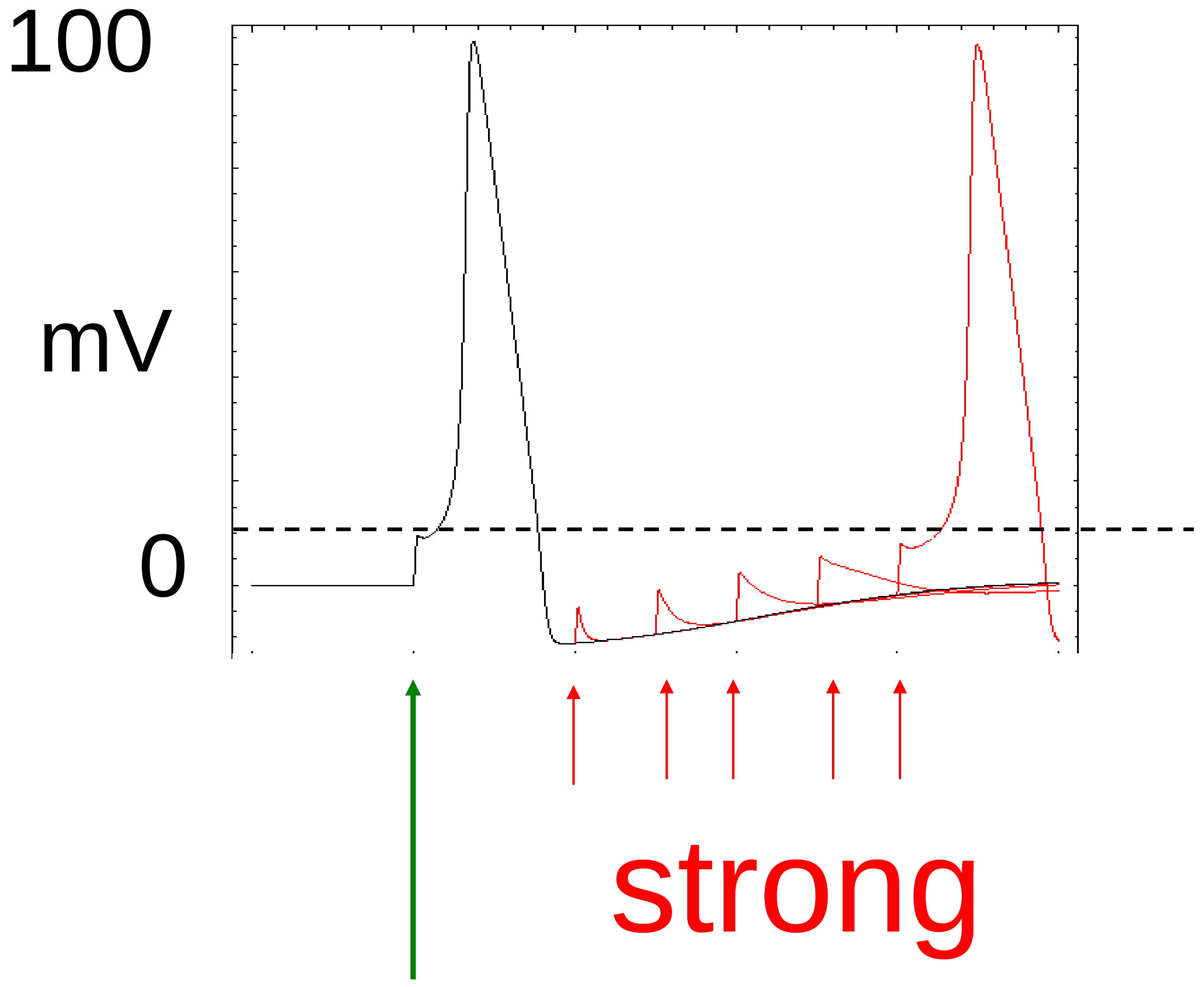
Where is the firing threshold?

Action potential



Strong stimulus

refractoriness

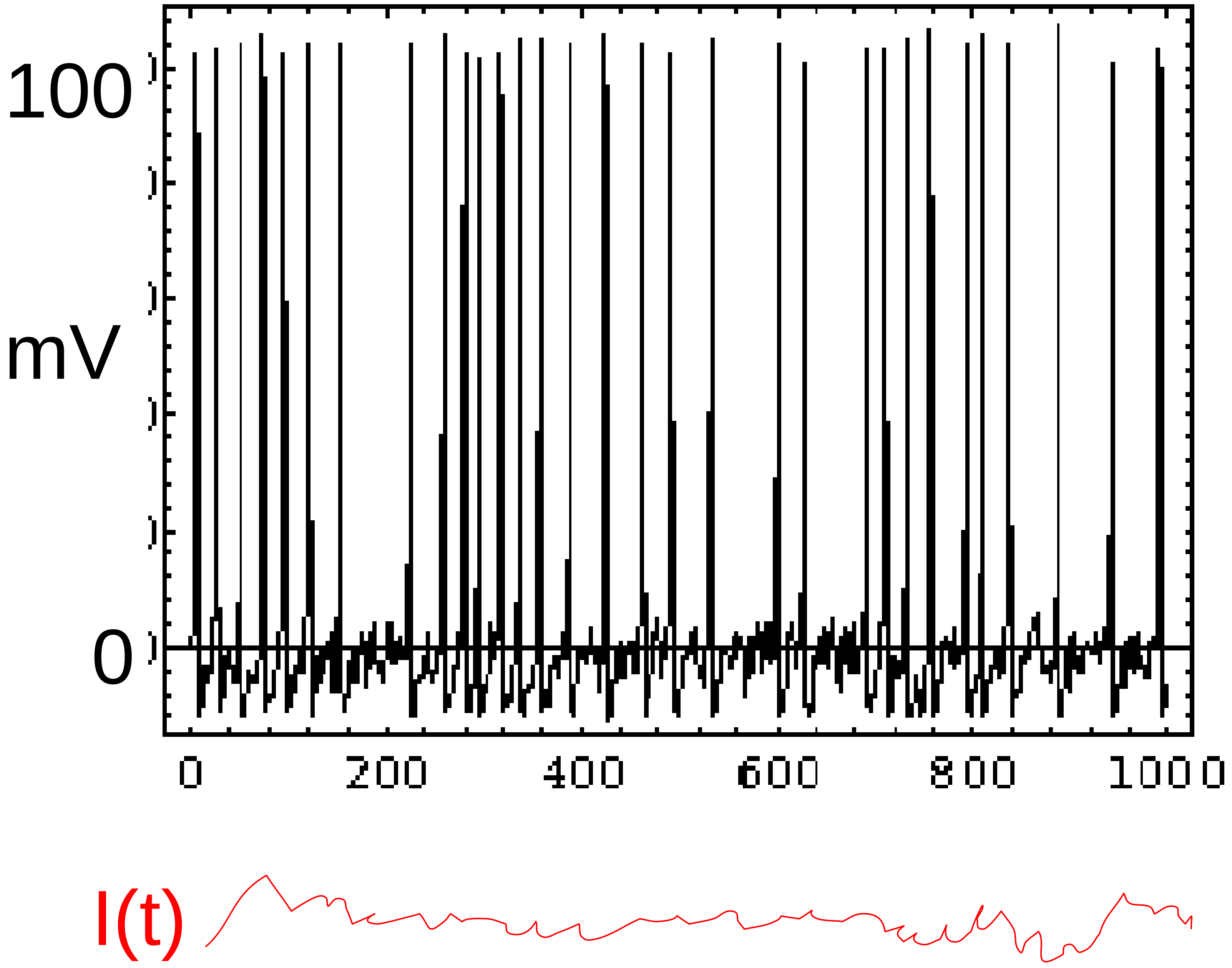


strong stimuli

Strong stimulus

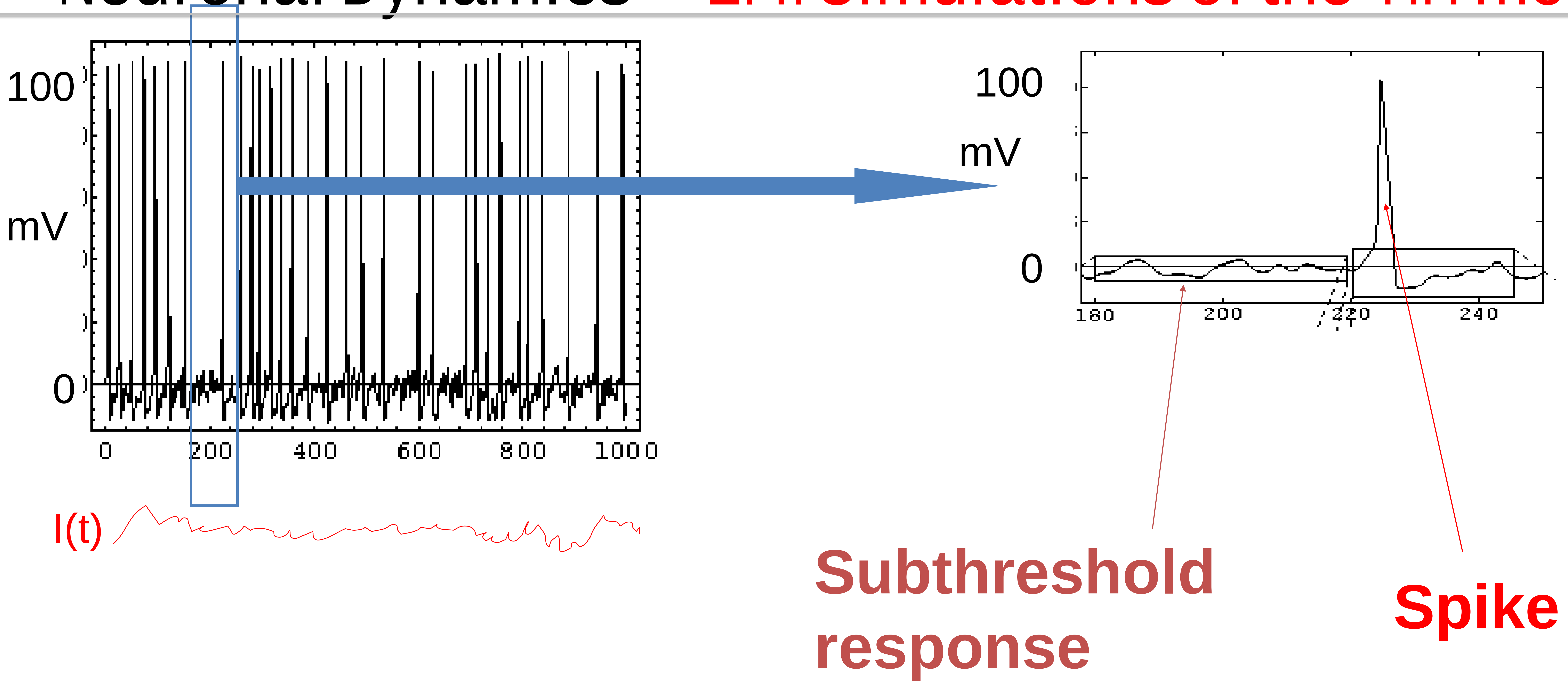
Refractoriness! Harder to elicit a second spike

Neuronal Dynamics – 2.4. Simulations of the HH model

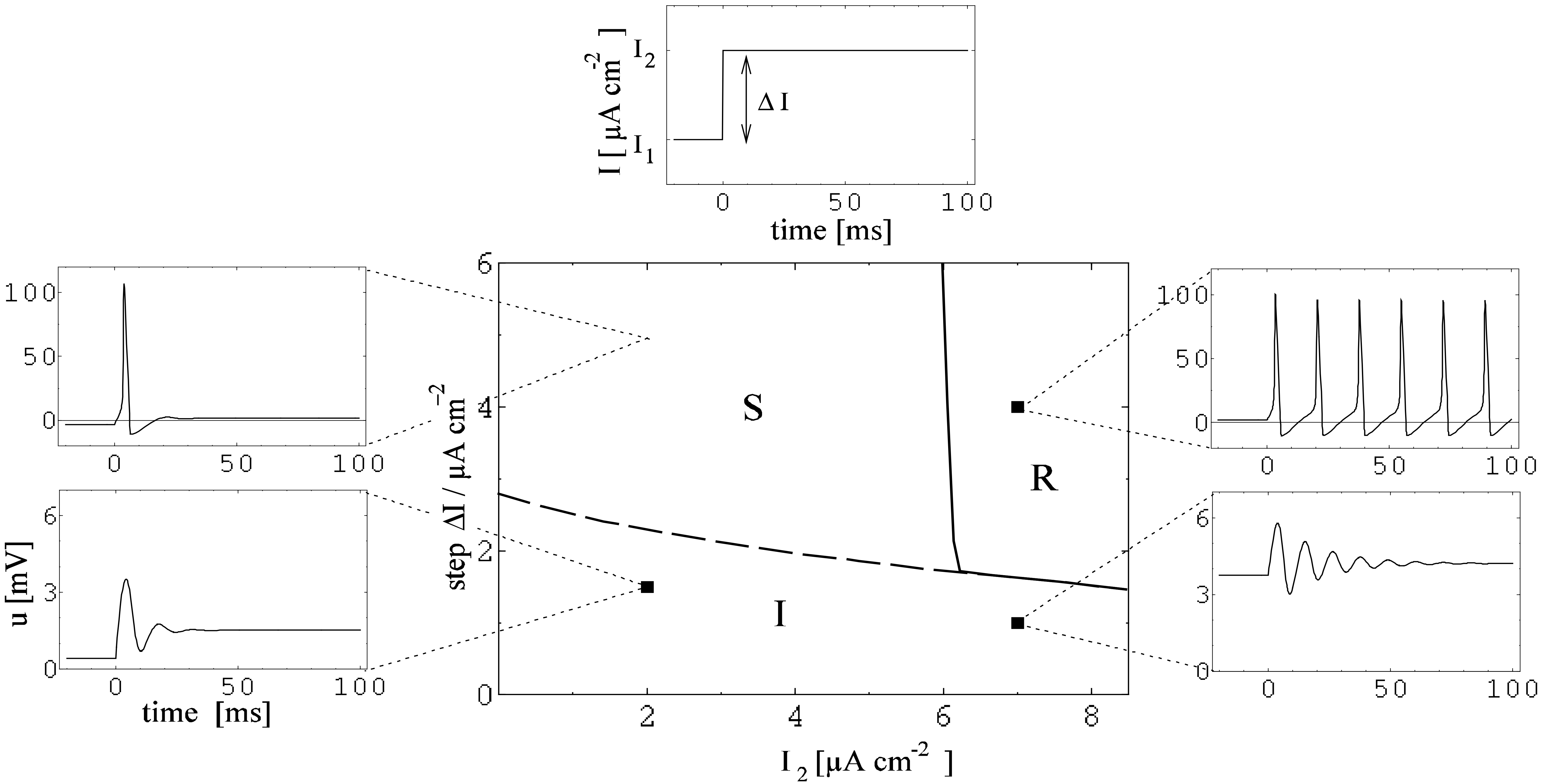


Stimulation with time-dependent input current

Neuronal Dynamics – 2.4. Simulations of the HH model



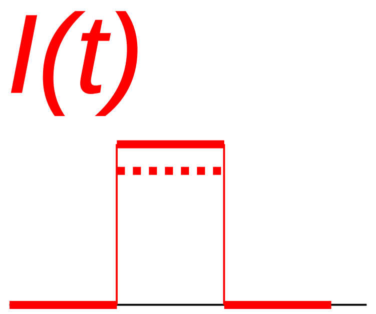
Step current input $\triangle I$ I_2



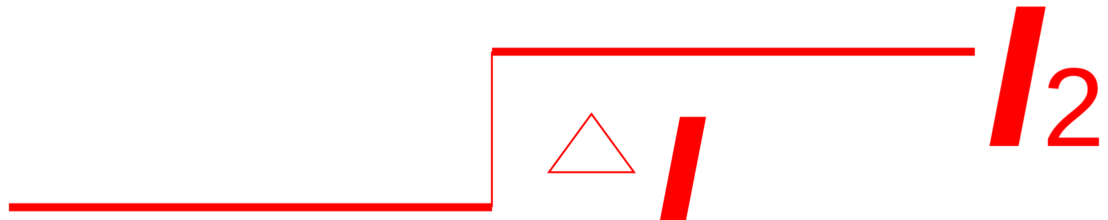
Neuronal Dynamics – 2.4. Threshold in HH model

Where is the firing threshold?

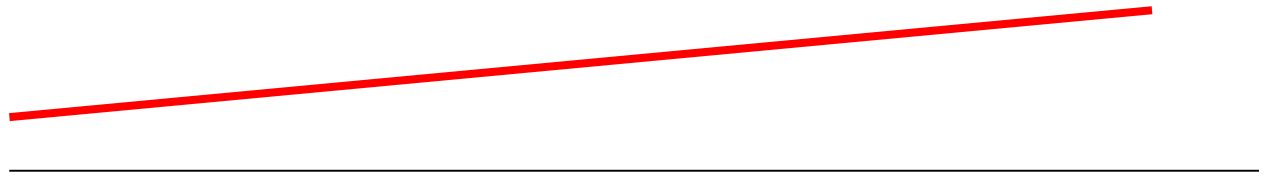
pulse input



step input



ramp input



There is no threshold

- no current threshold
- no voltage threshold

'effective' threshold

- depends on typical input

$$C\frac{du}{dt} = -g_{Na}m^3h(u - E_{Na}) - \ldots$$

Neuronal Dynamics – 2.4. Type I and Type II

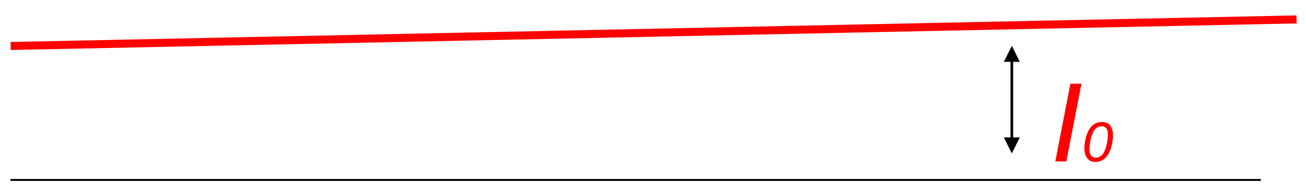
Hodgkin-Huxley model
with other parameters
(e.g. for cortical pyramidal
Neuron)

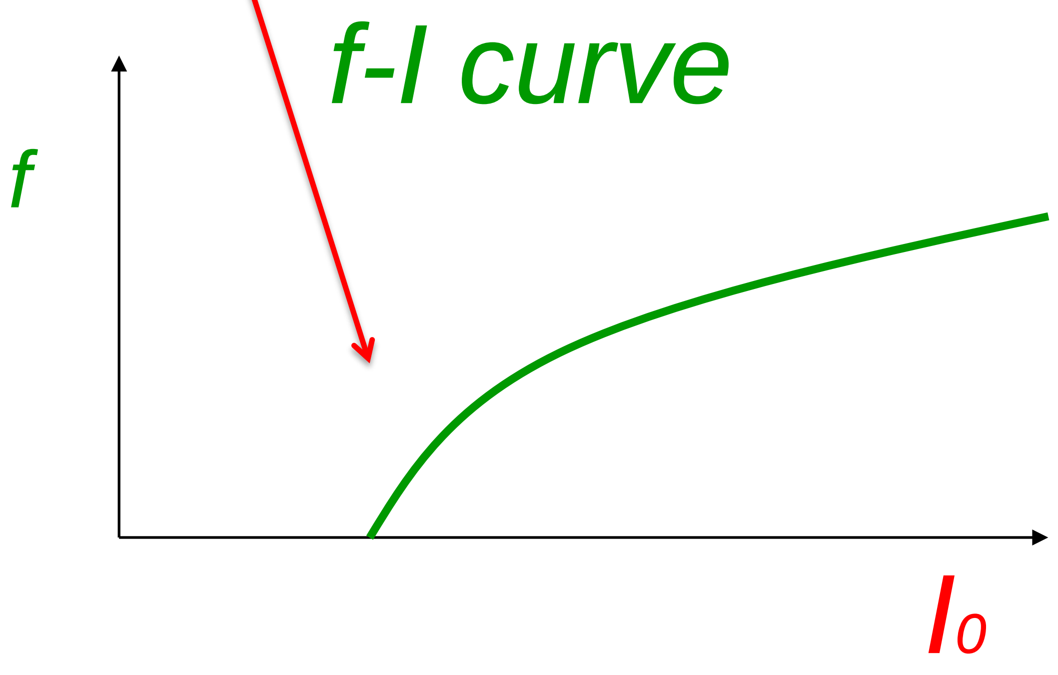
Hodgkin-Huxley model
with standard parameters
(giant axon of squid)

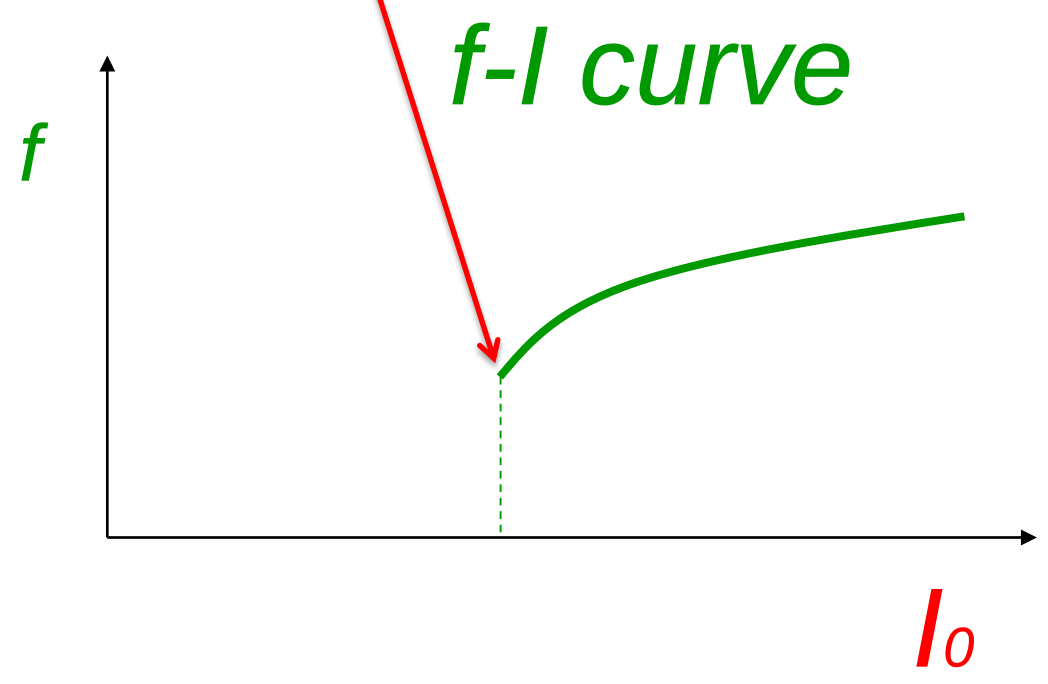
Response at firing threshold?

Type I	type II

ramp input/
constant input







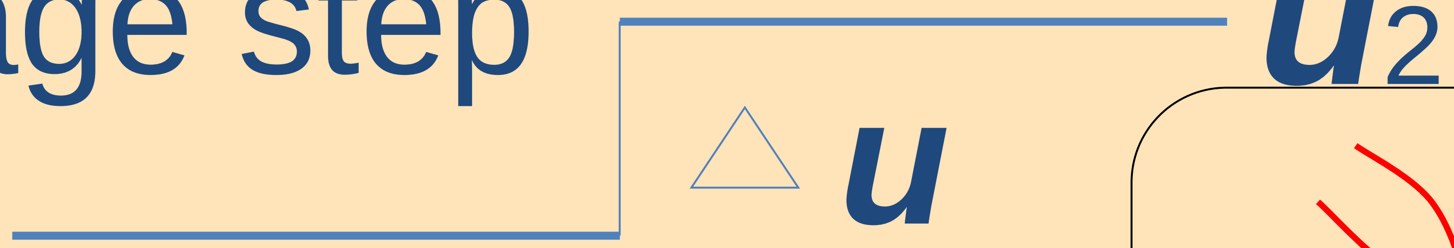
Neuronal Dynamics – 2.4. Hodgkin-Huxley model

-4 differential equations
-no explicit threshold
-effective threshold depends on stimulus
-BUT: voltage threshold good approximation

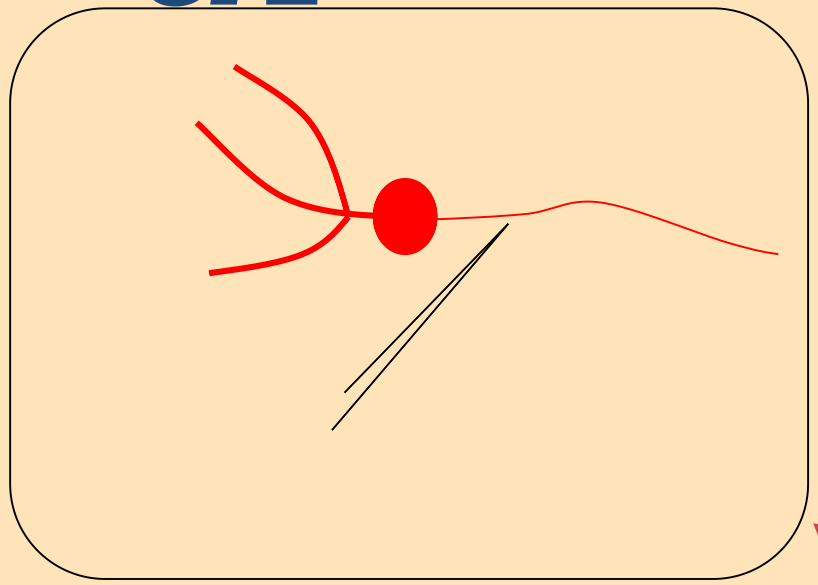
Giant axon of the squid
→ cortical neurons
-Change of parameters
-More ion channels
-Same framework

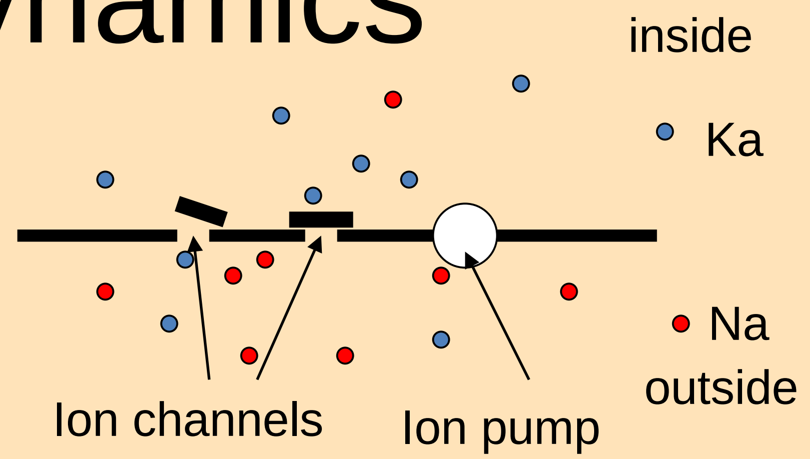
Exercise 3.1-3.3 – Hodgkin-Huxley – ion channel dynamics

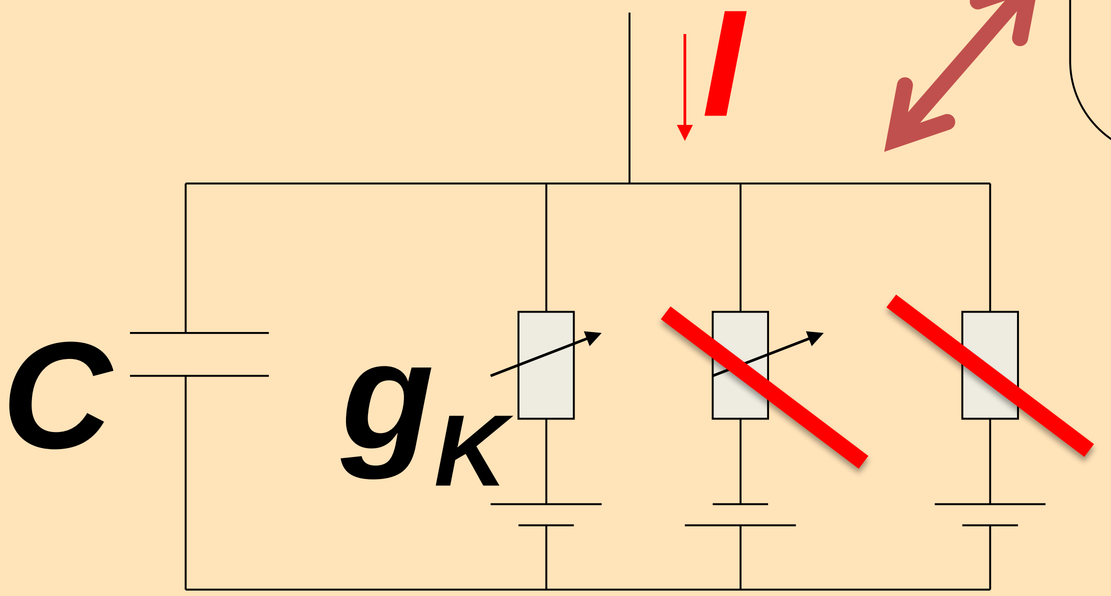
voltage step



Determine ion channel dynamics



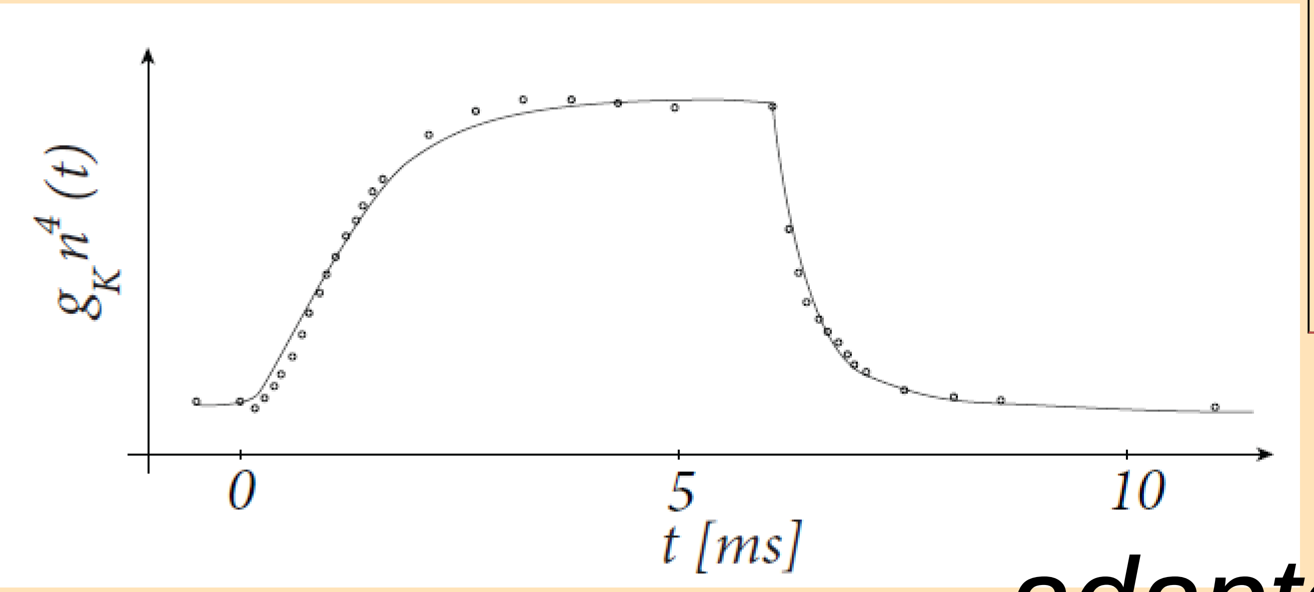


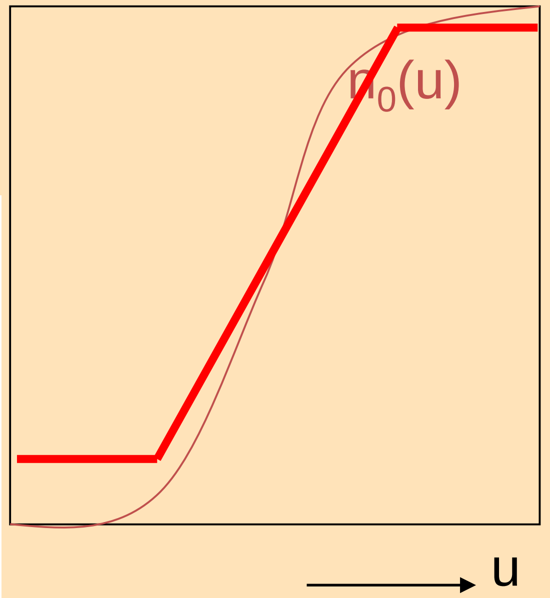


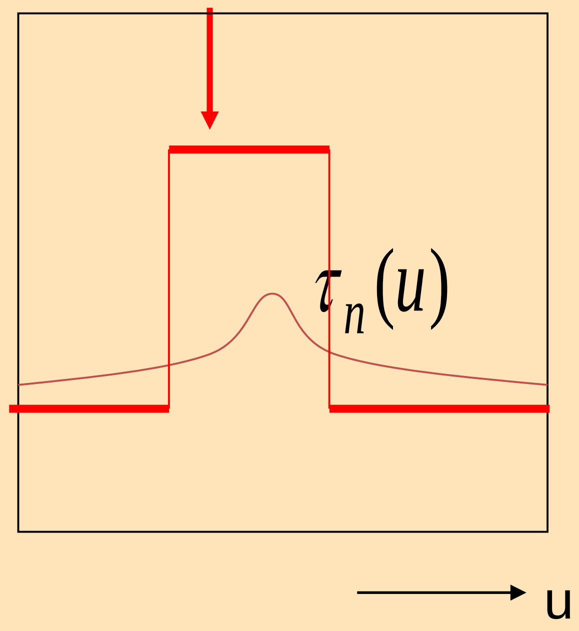
$$C\frac{du}{dt} = -g_K n^4 (u - E_K) + I(t)$$

stimulus

apply voltage step







Start Exercise 3 at 11:33
Next Lecture at:
11.48

adapted from Hodgkin&Huxley 1952

Week 2 – part 5: Detailed Biophysical Models

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

Week 2 – Biophysical modeling: The Hodgkin-Huxley model

Wulfram Gerstner

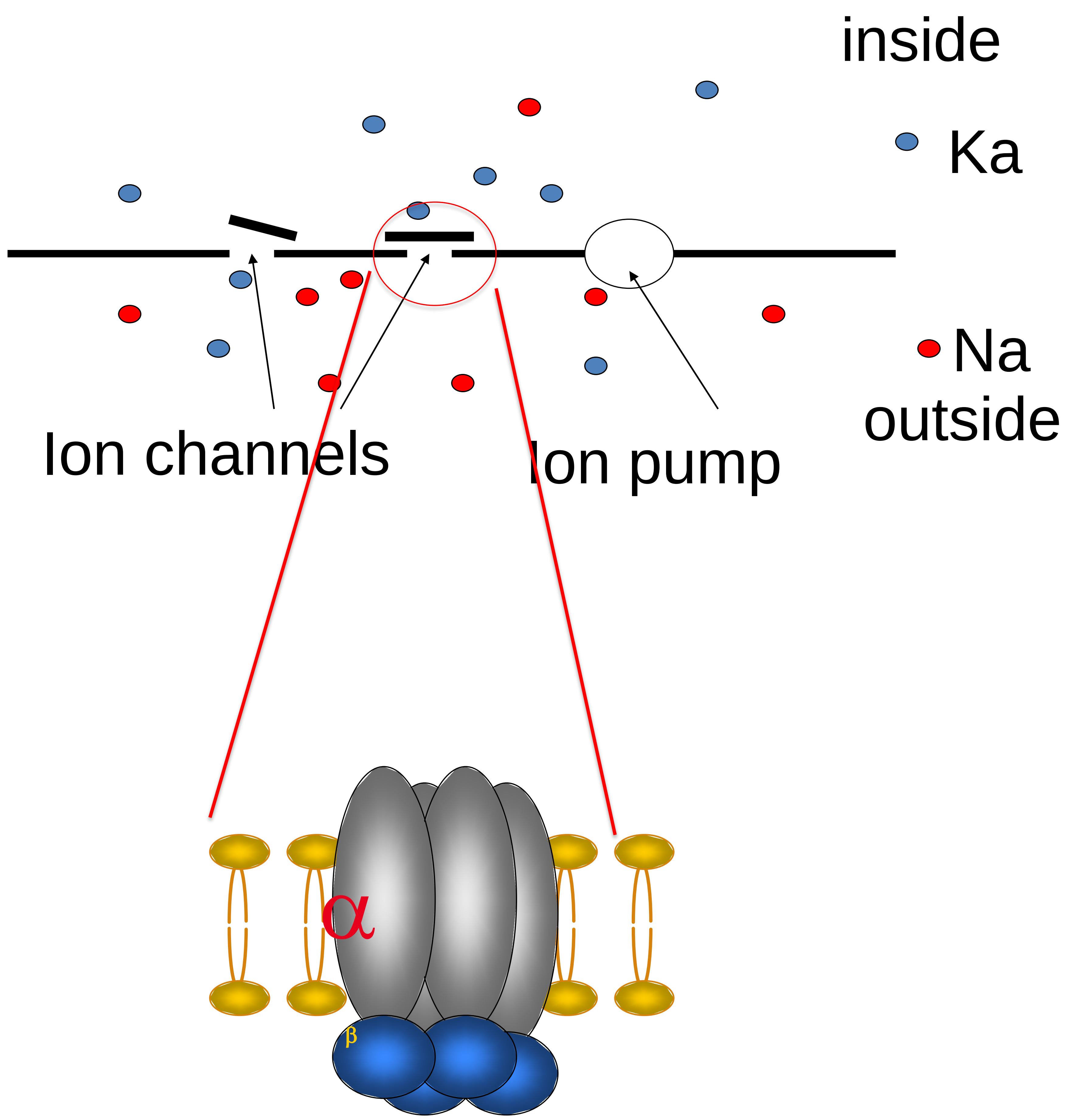
EPFL, Lausanne, Switzerland

Neuronal Dynamics – 2.5 Biophysical models

There are about 200 identified ion channels

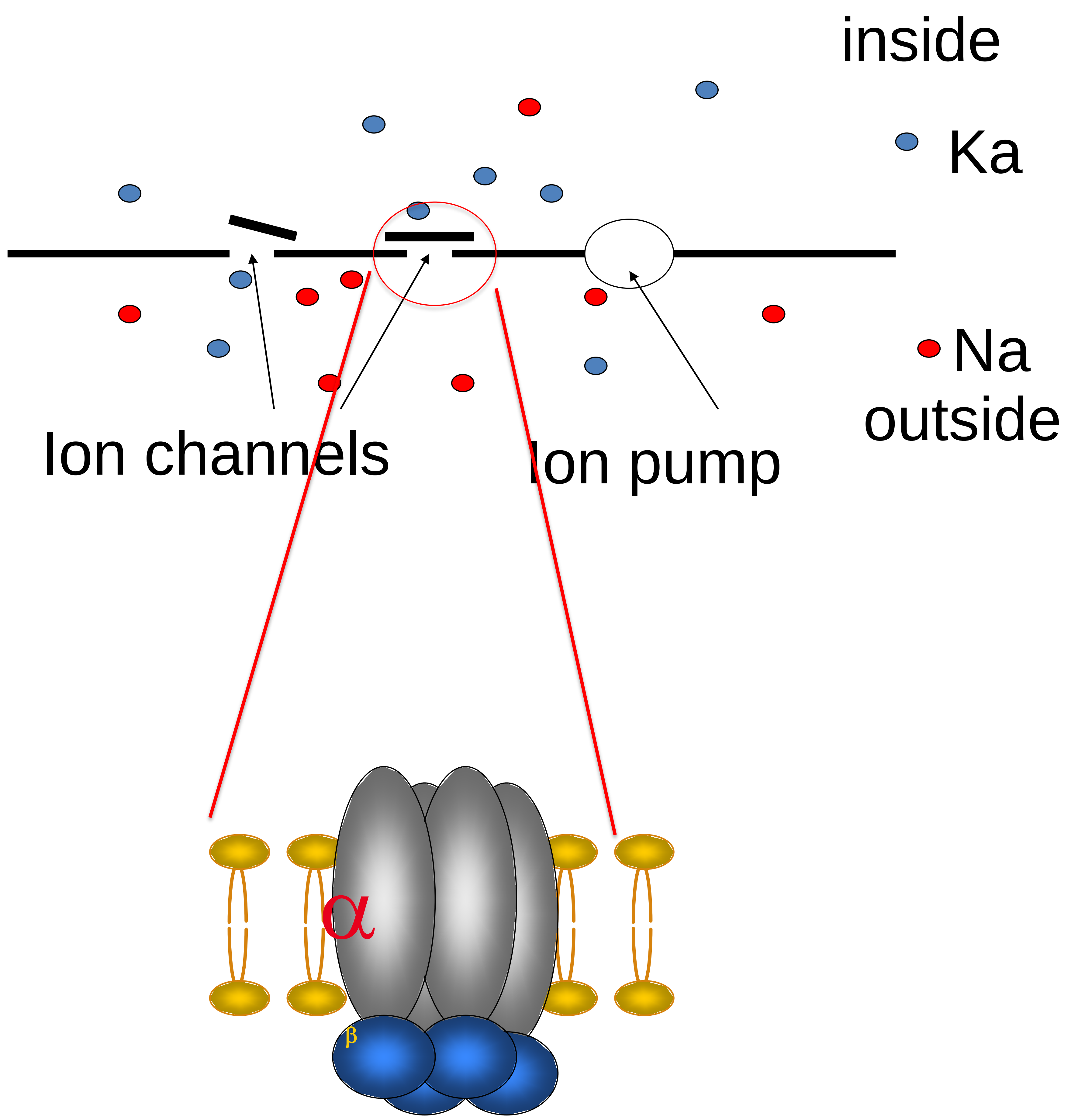
http://channelpedia.epfl.ch/

Hodgkin-Huxley model
Provides flexible framework

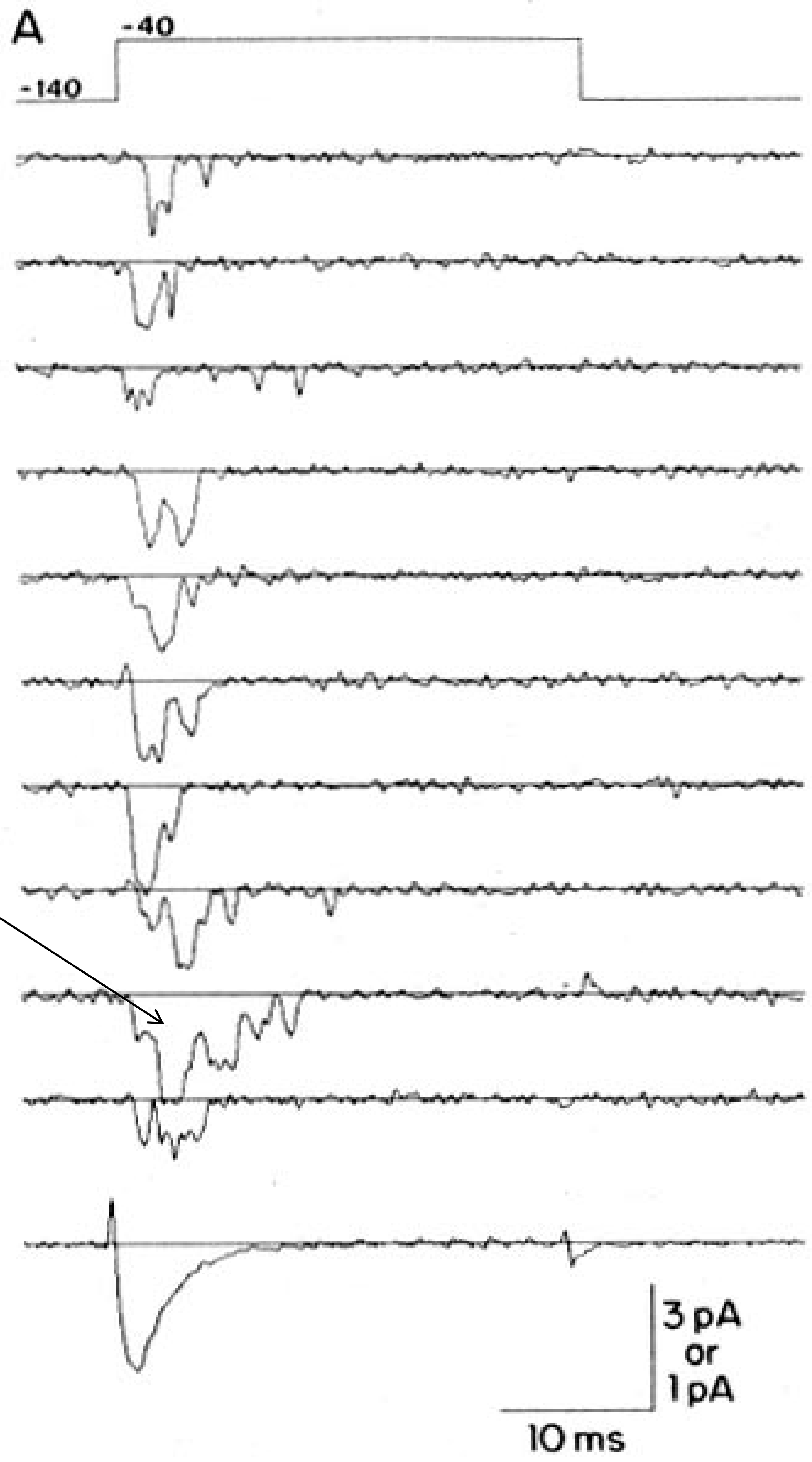


Individual ion channels can be measured.

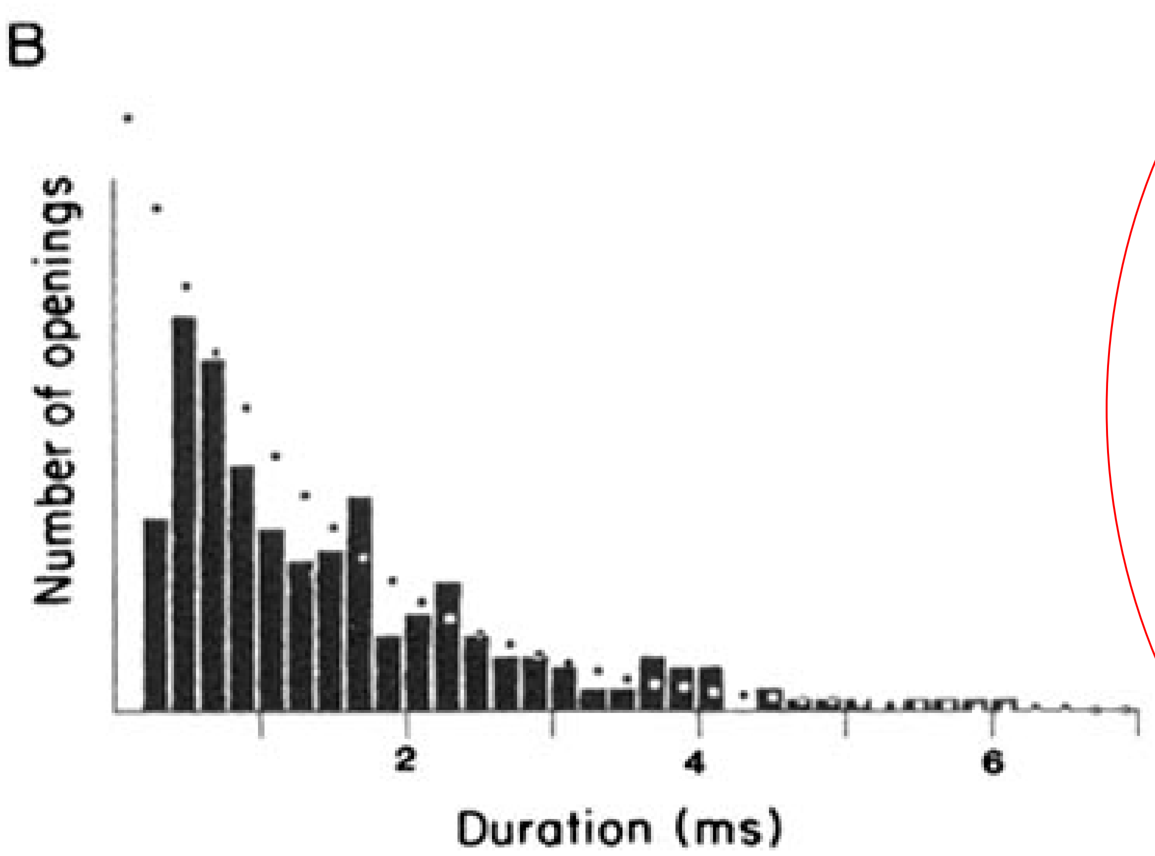
Opening and closing is stochastic

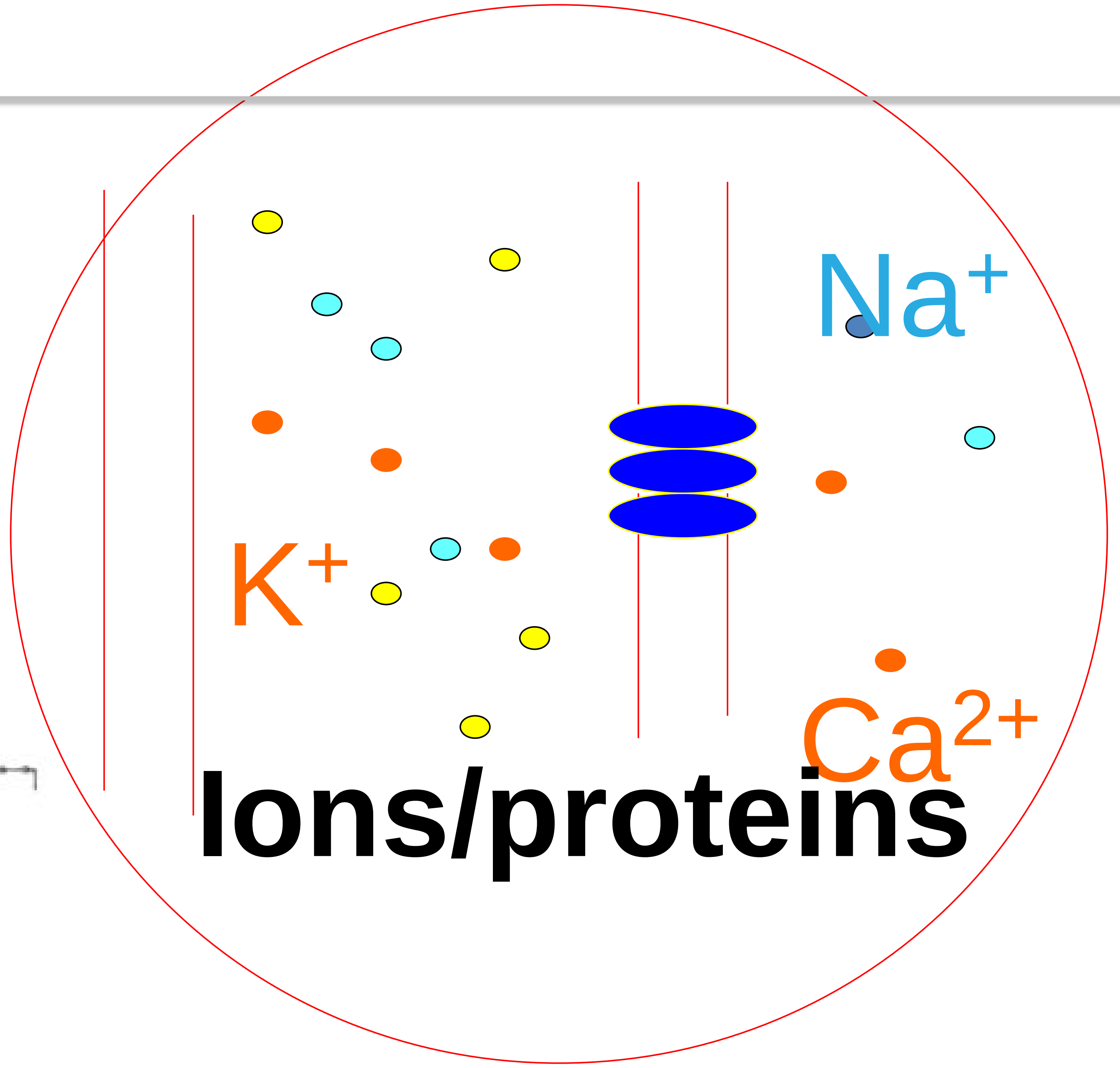


Neuronal Dynamics – 2.5 Ion channels



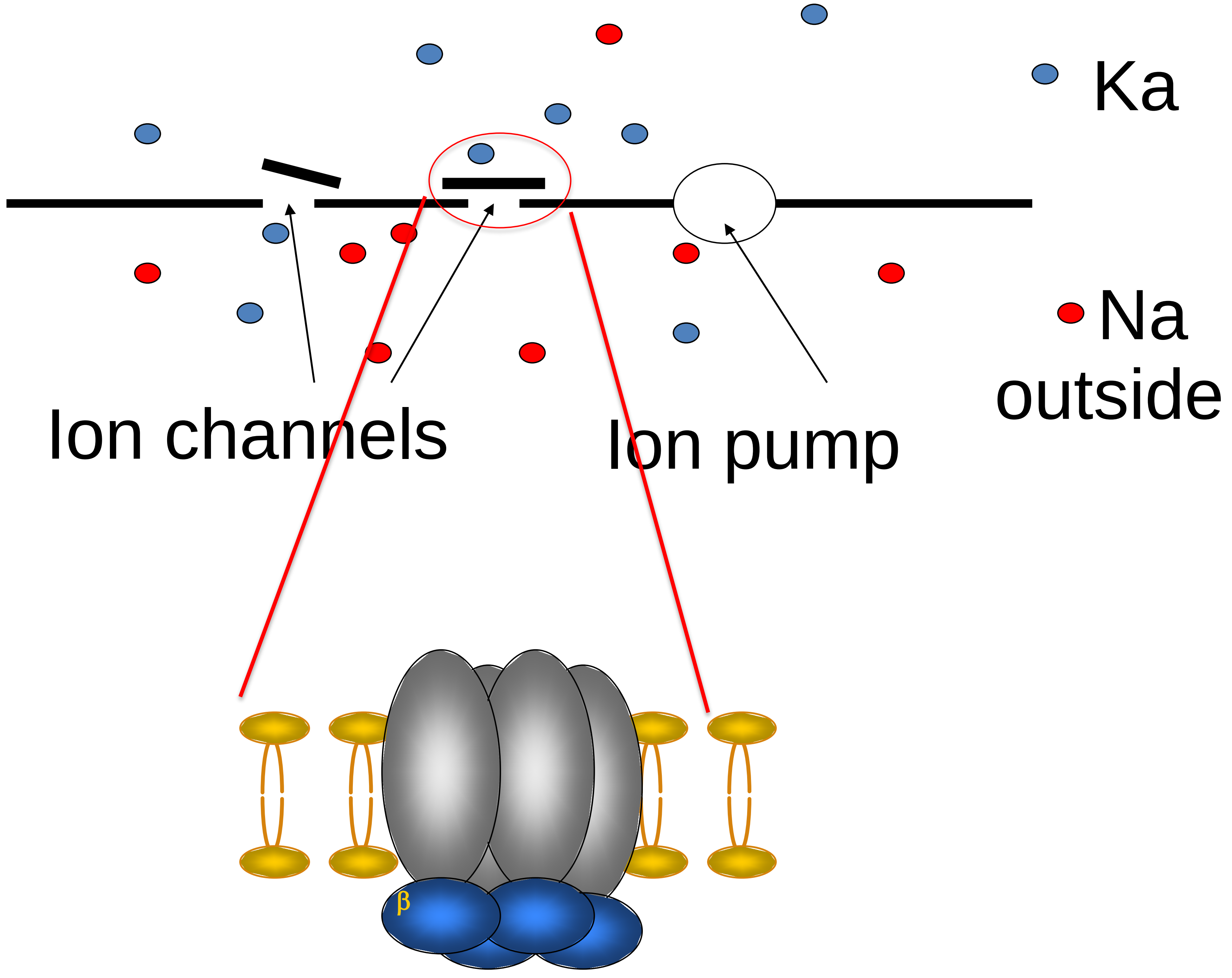
Steps:
Different number
of channels





Na+ channel from rat heart *(Patlak and Ortiz 1985)*
A traces from a patch containing several channels.
Bottom: average gives current time course.
B. Opening times of single channel events

Neuronal Dynamics – 2.5 Biophysical models



Hodgkin-Huxley:
-Cambridge lab
-Plymouth lab

Hodgkin-Huxley model provides flexible framework

Hodgkin&Huxley (1952)
Nobel Prize 1963

Exercise 4 – Hodgkin-Huxley model – gating dynamics

A) Often the gating dynamics is formulated as

$$\frac{dm}{dt} = \alpha_m(u)(1-m) - \beta_m(u)m \qquad \frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

Calculate $m_0(u) \quad and \quad \tau_m(u)$

B) Assume a form $\alpha_m(u) = \beta_m(u) = \dfrac{1}{1-\exp[-(u+a)/b]}$

How are *a* and *b* related to γ and θ in the equations

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$m_0(u) = 0.5\{1 + \tanh[\gamma(u-\theta)]\}$$

C) What is the time constant $\tau_m(u)$?

Biological Modeling of Neural Networks

TA in 2018:

Vasiliki Liakoni
Chiara Gastaldi
Bernd Illing

Now Computer Exercises:

Play with Hodgkin-Huxley model

The End

Week 2 – References and Suggested Reading

Reading: W. Gerstner, W.M. Kistler, R. Naud and L. Paninski,
Neuronal Dynamics: from single neurons to networks and models of cognition. Chapter 2: *The Hodgkin-Huxley Model,* Cambridge Univ. Press, 2014

- Hodgkin, A. L. and Huxley, A. F. (1952). *A quantitative description of membrane current and its application to conduction and excitation in nerve.* J Physiol, 117(4):500-544.
-Ranjan, R.,et al. (2011). *Channelpedia: an integrative and interactive database for ion channels*. Front Neuroinform, 5:36.
-Toledo-Rodriguez, M., Blumenfeld, B., Wu, C., Luo, J., Attali, B., Goodman, P., and Markram, H. (2004). *Correlation maps allow neuronal electrical properties to be predicted from single-cell gene expression profiles in rat neocortex*. Cerebral Cortex, 14:1310-1327.
-Yamada, W. M., Koch, C., and Adams, P. R. (1989). *Multiple channels and calcium dynamics*. In Koch, C. and Segev, I., editors, *Methods in neuronal modeling*, MIT Press.
- Aracri, P., et al. (2006). *Layer-specic properties of the persistent sodium current in sensorimotor cortex*. Journal of Neurophysiol., 95(6):3460-3468.