COM303: Digital Signal Processing

Lecture 17: Sampling and applications

overview

- raw sampling and aliasing
- DT processing of CT signals

Sinc Sampling

$$x[n] = \left\langle \operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right), x(t) \right\rangle$$

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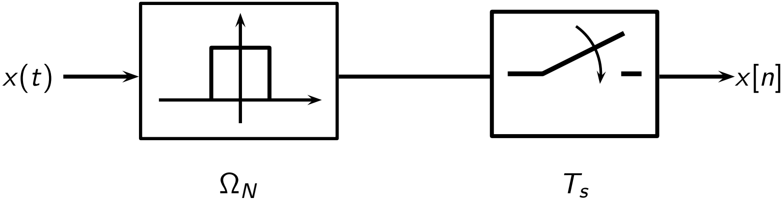
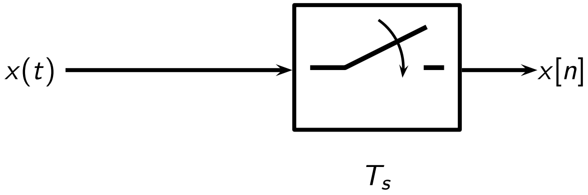


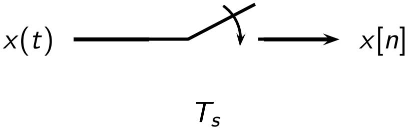
Sinc Sampling for Ω_N-BL signals

$$x[n] = (\mathrm{sinc}_{T_s} * x)(nT_s) = T_s\, x(nT_s)$$



“Raw” sampling - can we always do that?

$$x[n] = x(nT_s)$$



Remember the wagonwheel effect?

The continuous-time complex exponential

$$x(t) = e^{j\Omega_0 t} = e^{j2\pi F_0 t}$$

- always periodic, period $T_0 = 2\pi/\Omega_0 = 1/F_0$
- all angular speeds are allowed
- $\text{FT}\left\{e^{j\Omega_0 t}\right\} = 2\pi\delta(\Omega - \Omega_0)$
- bandlimited to Ω_0^+

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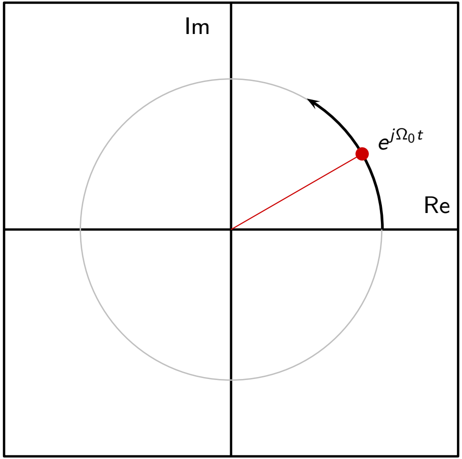

Raw samples of the continuous-time complex exponential

$$x[n] = e^{j\Omega_0 T_s n}$$

- raw samples are snapshots at regular intervals of the rotating point
- resulting digital frequency is $\omega_0 = \Omega_0 T_s = 2\pi(T_s/T_0)$

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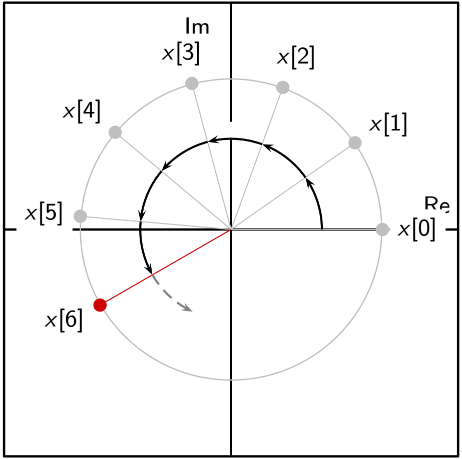
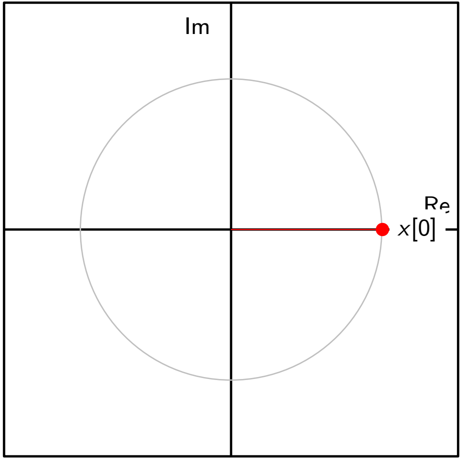
Easy: $T_s < T_0/2 \quad \Rightarrow \quad \omega_0 < \pi$



Tricky: $T_0/2 < T_s < T_0 \quad \Rightarrow \quad \pi < \omega_0 < 2\pi$



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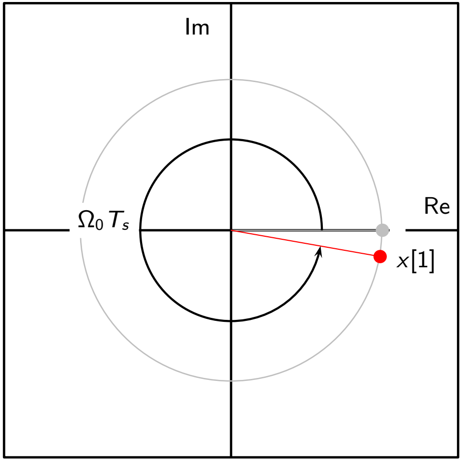

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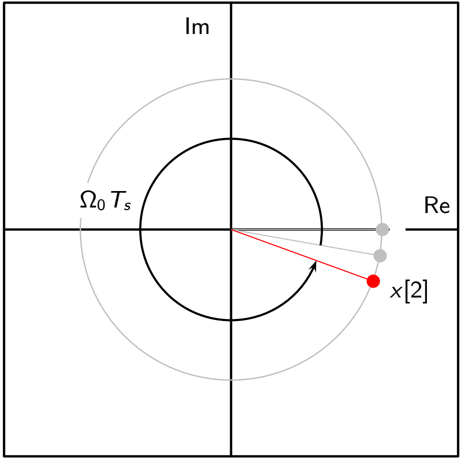

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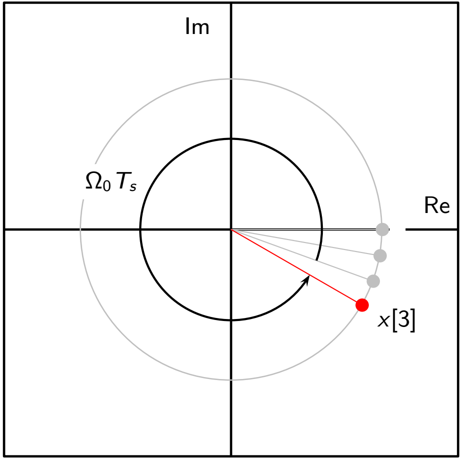

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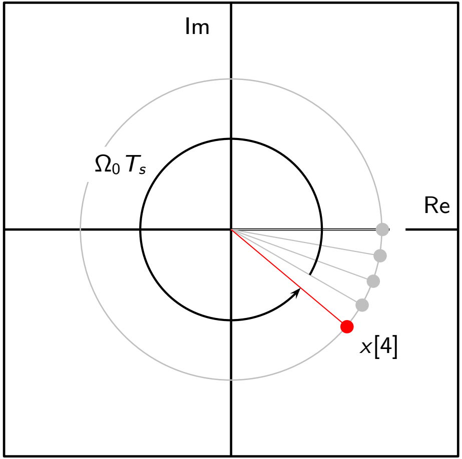

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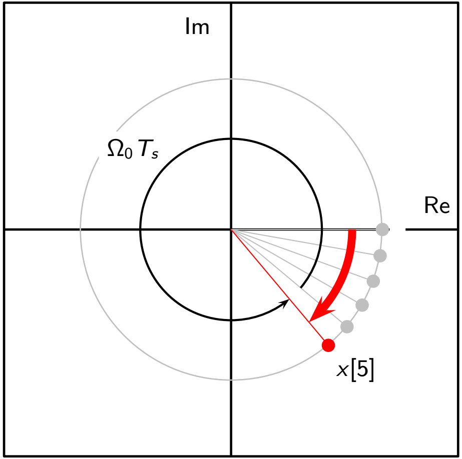
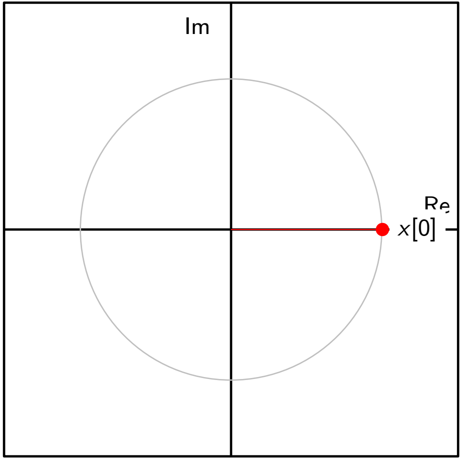


Trouble: $T_s > T_0 \quad \Rightarrow \quad \omega_0 > 2\pi$



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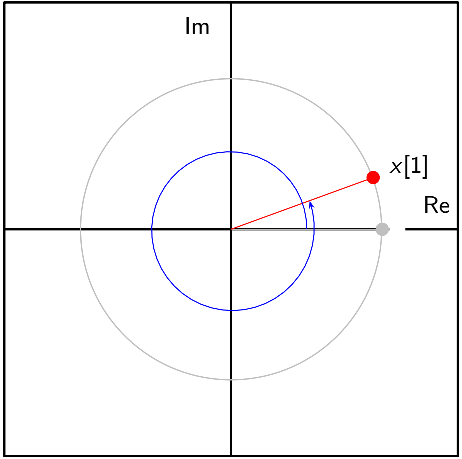

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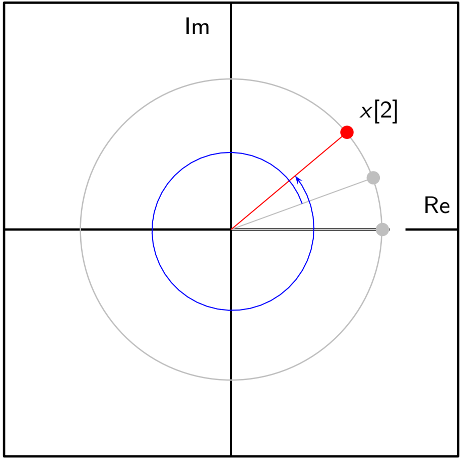

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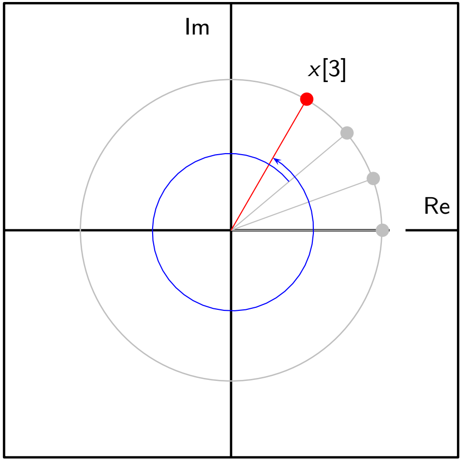

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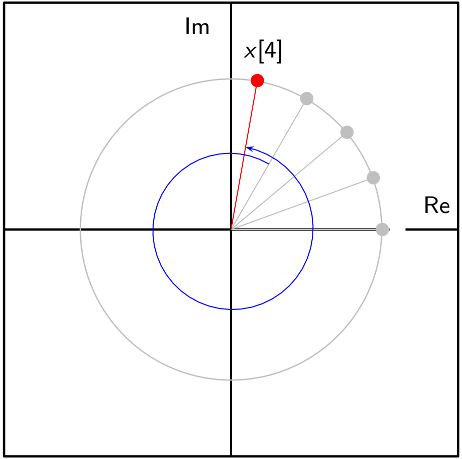

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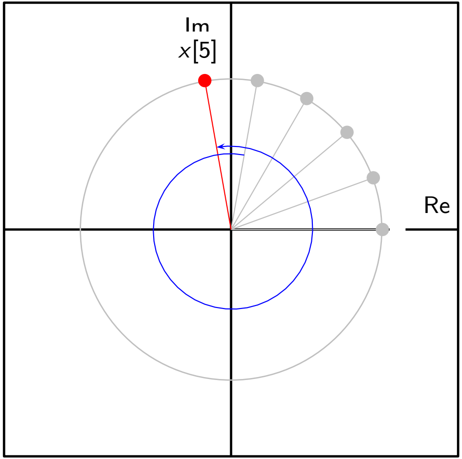
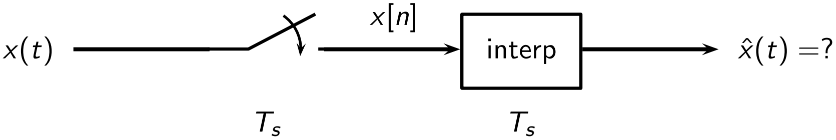


Aliasing



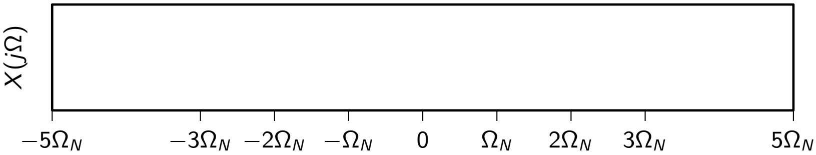
Aliasing

pick T_s: $\Omega_N = \pi / T_s$

input: $x(t) = e^{j\Omega_0 t}$

	digital frequency	$\hat{x}(t)$	
$\Omega_0 < \Omega_N$	$0 < \omega_0 < \pi$	$e^{j\Omega_0 t}$	
$\Omega_0 = \Omega_N$	$\omega_0 = \pi$	$e^{j\Omega_0 t}$	
$\Omega_N < \Omega_0 < 2\Omega_N$	$\pi < \omega_0 < 2\pi$	$e^{j\Omega_1 t}$,	$\Omega_1 = \Omega_0 - 2\Omega_N < 0$
$\Omega_0 > 2\Omega_N$	$\omega_0 > 2\pi$	$e^{j\Omega_2 t}$,	$\Omega_2 = \Omega_0 \mod 2\Omega_N$

Aliasing of sinusoids: increasing the input frequency



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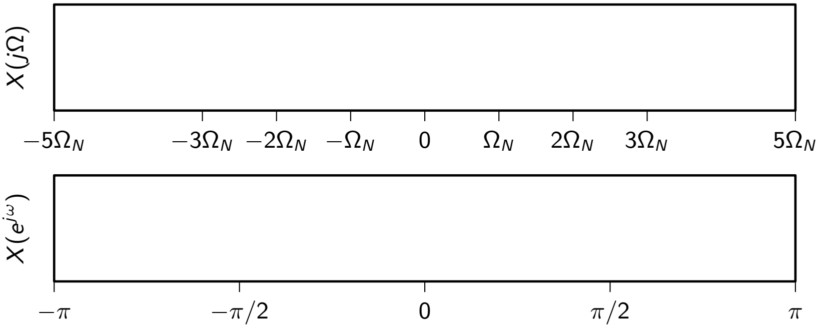

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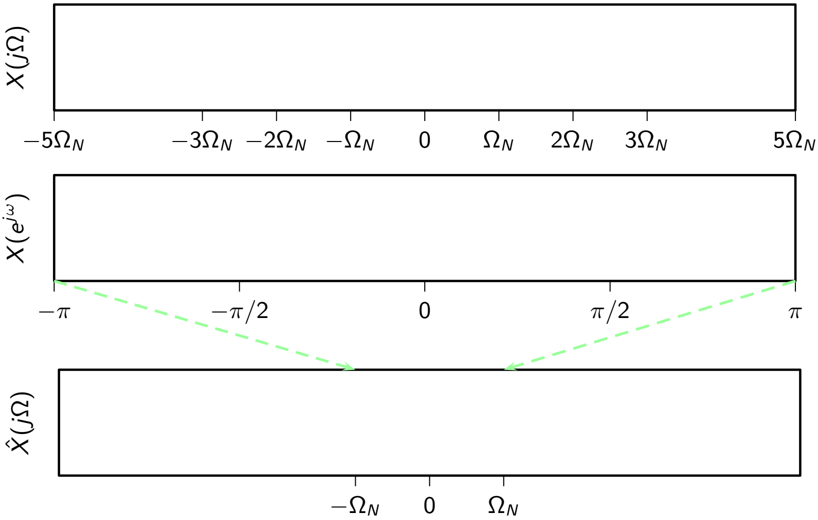

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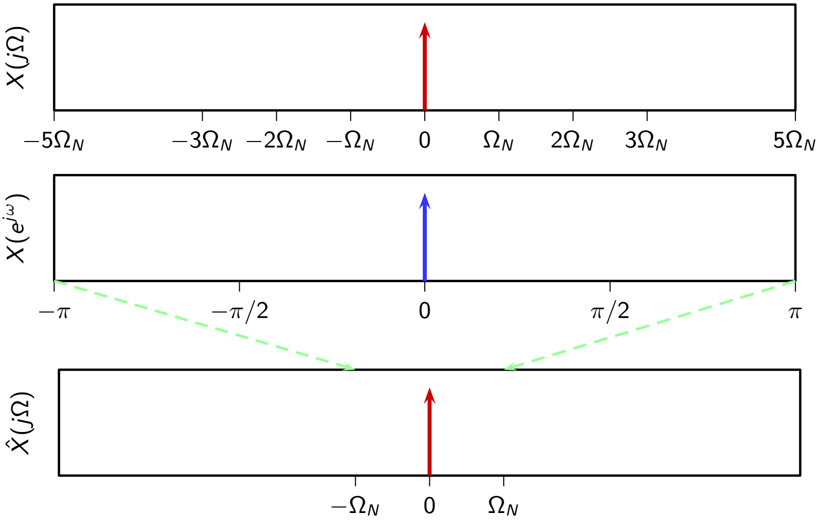

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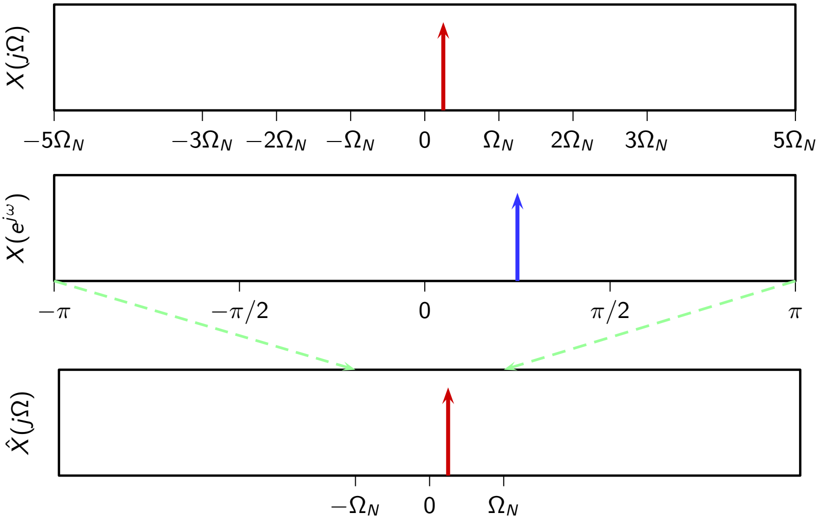

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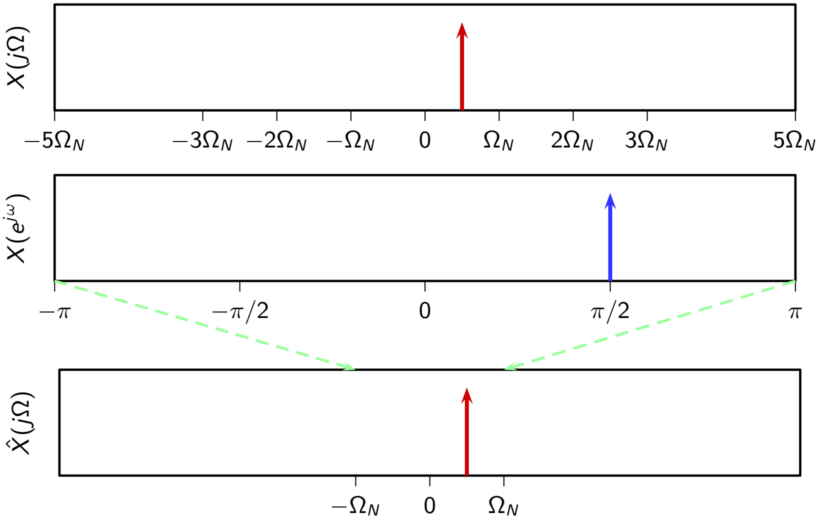

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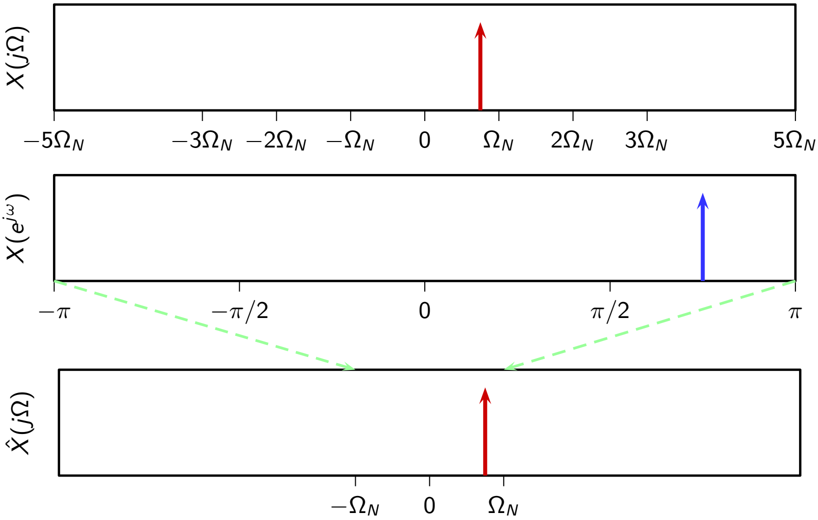

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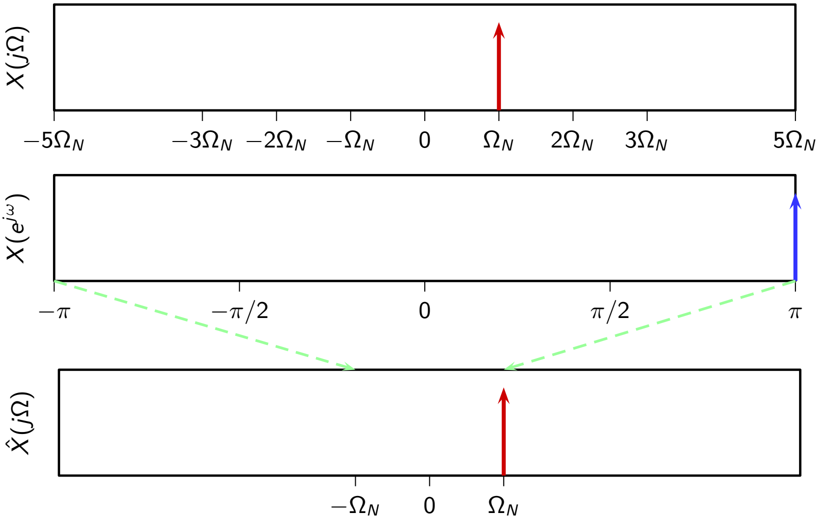

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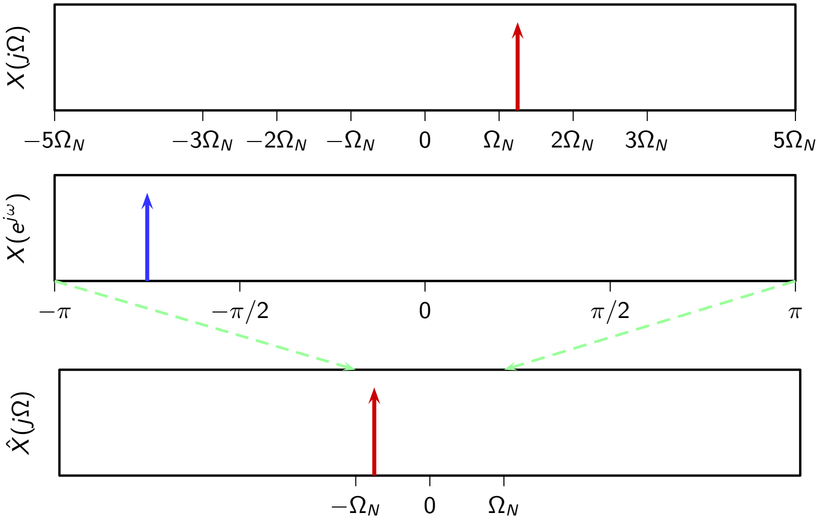

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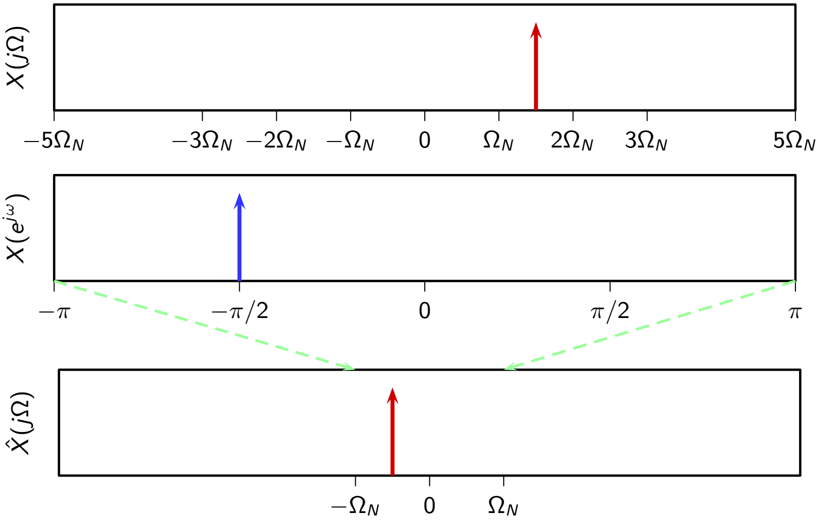

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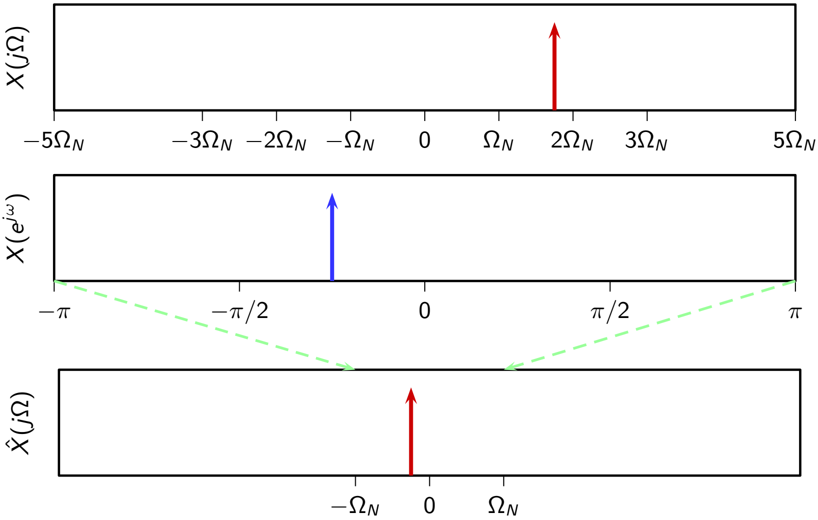

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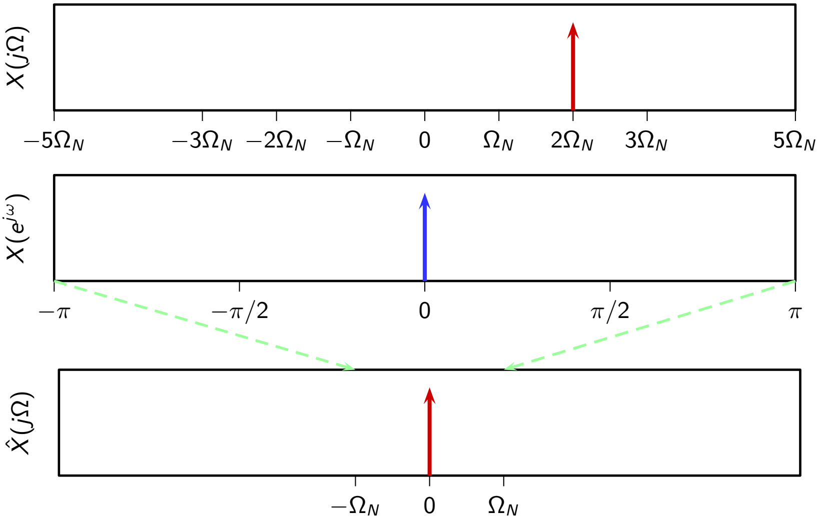

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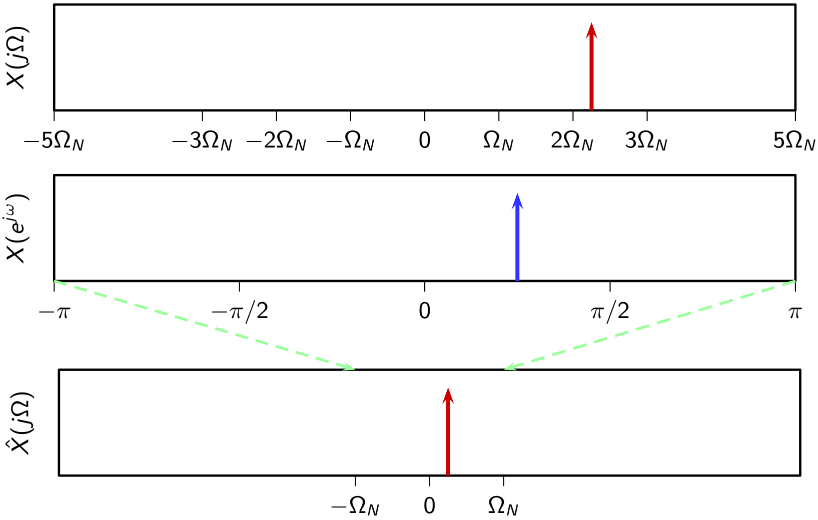

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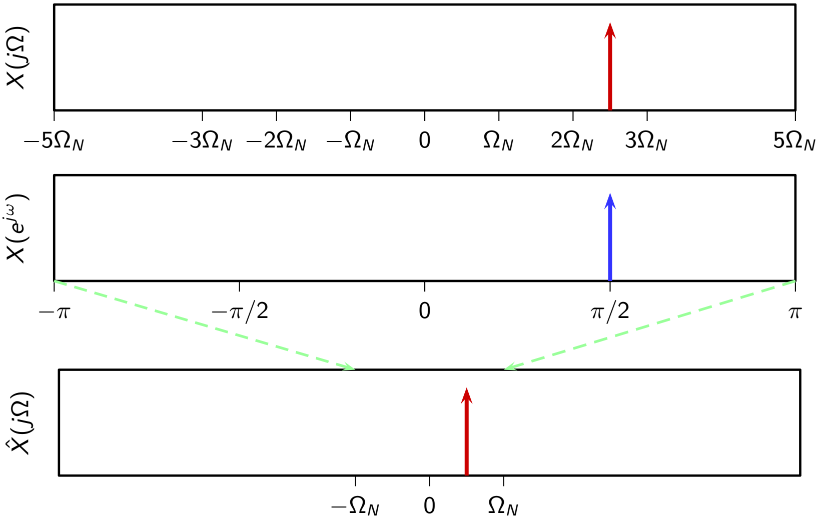

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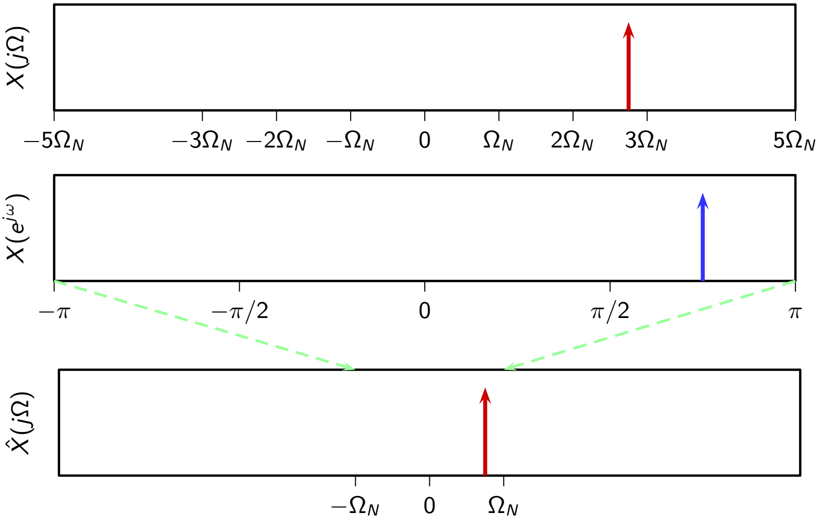

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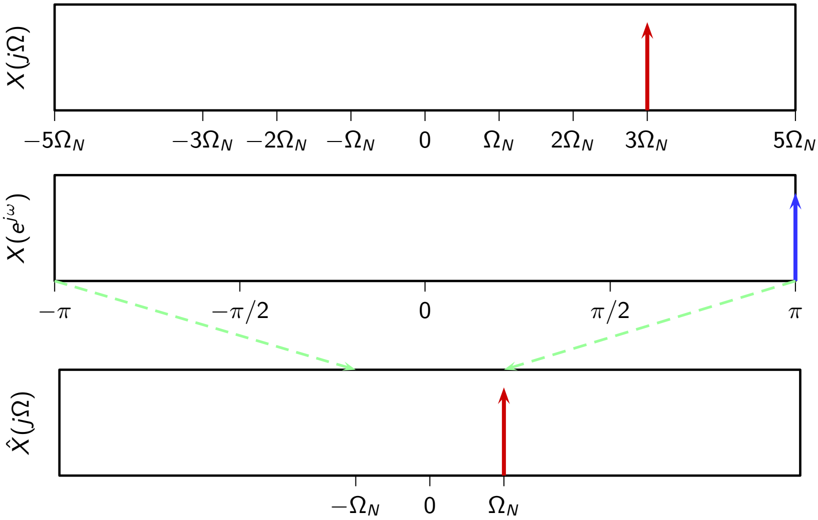

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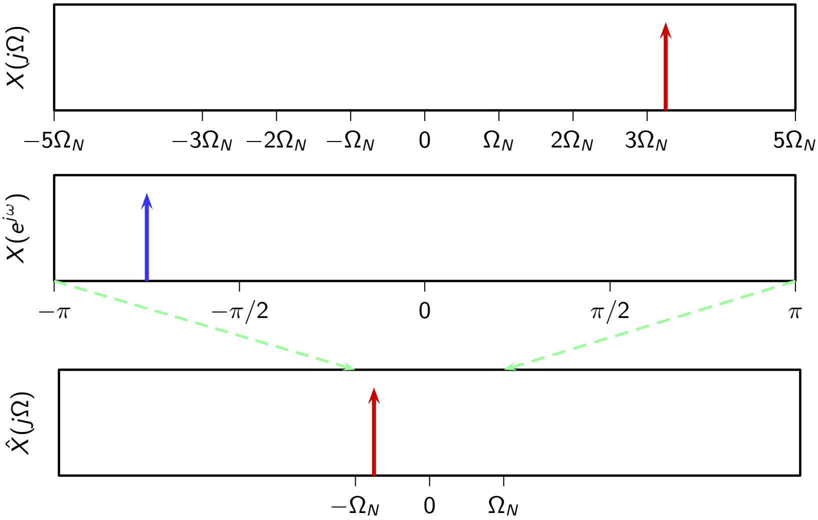

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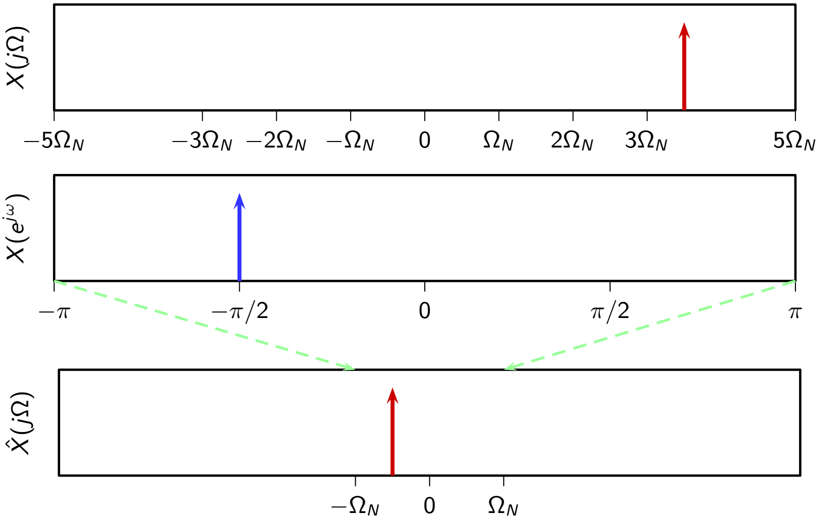

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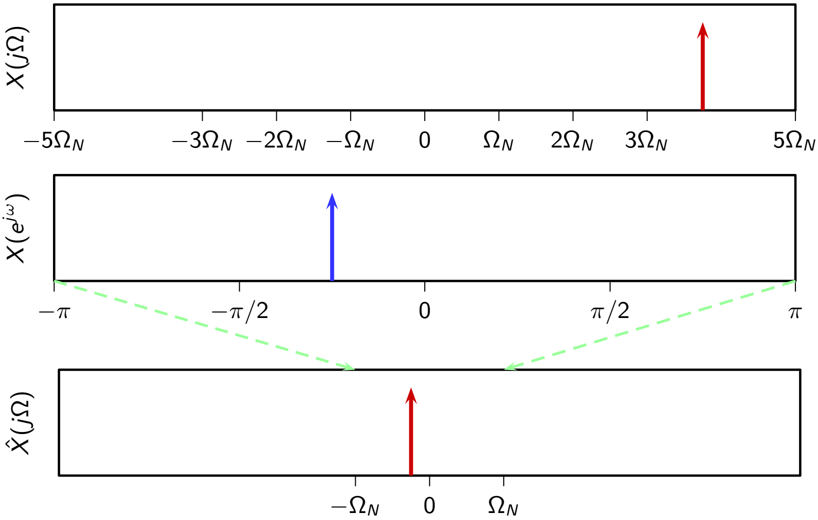

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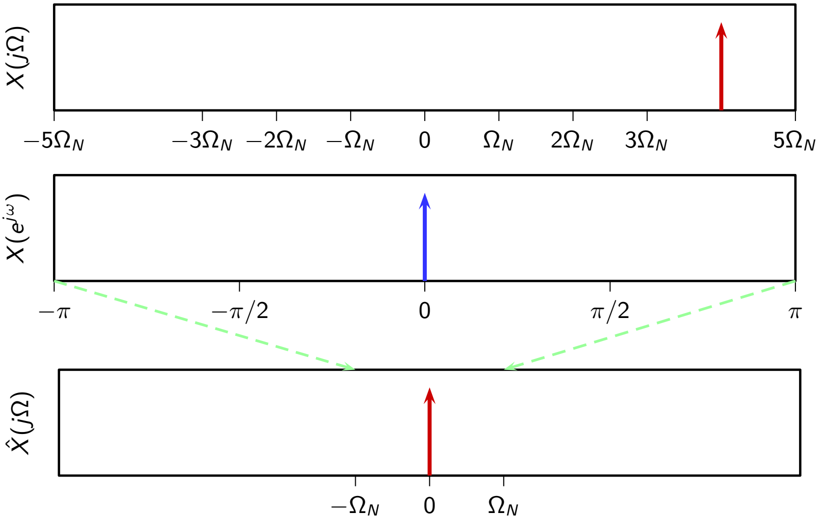

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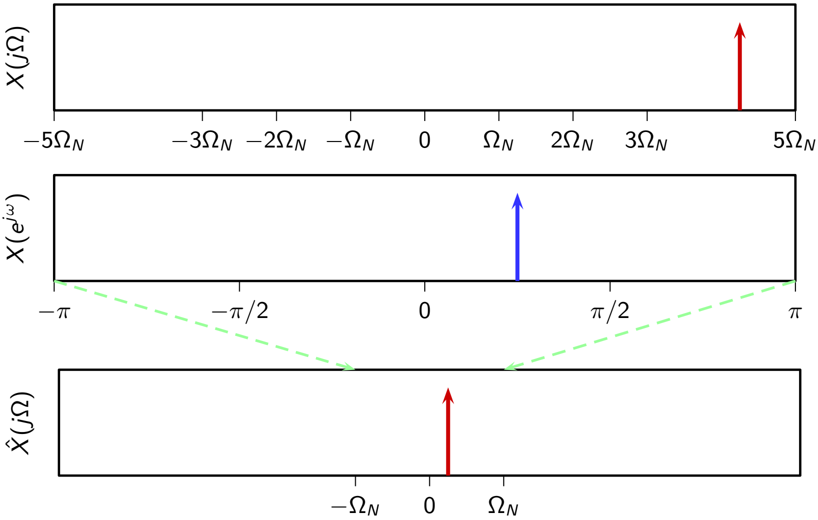
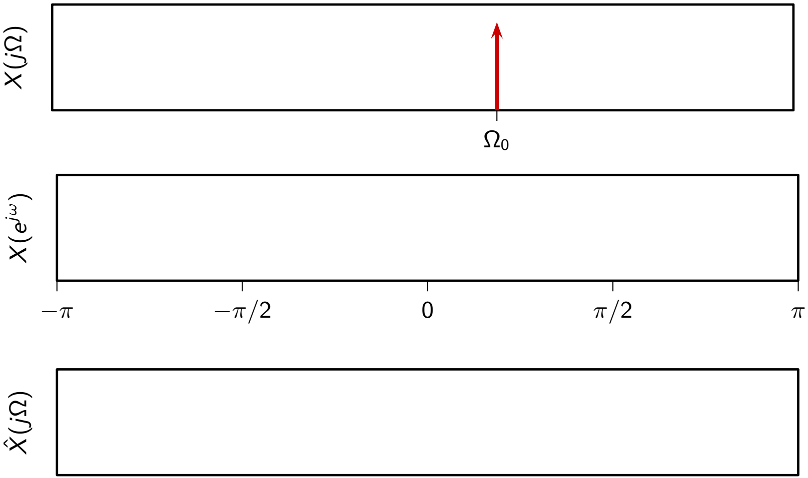


Aliasing of sinusoids: decreasing the sampling frequency



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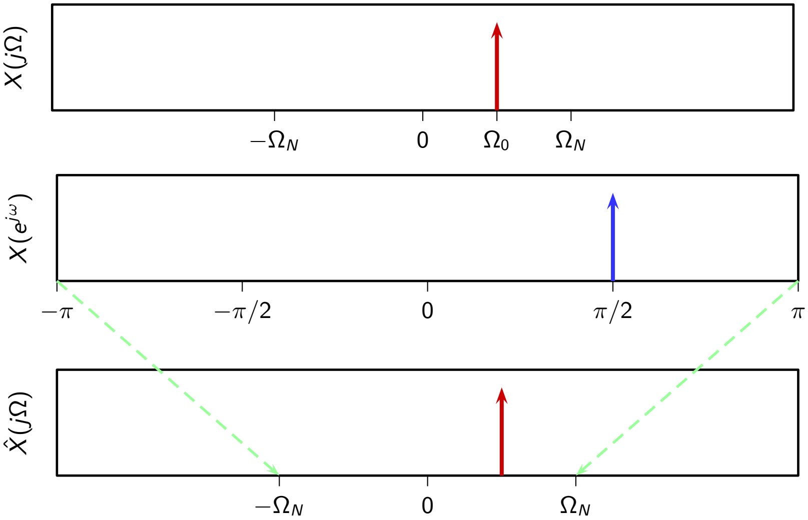

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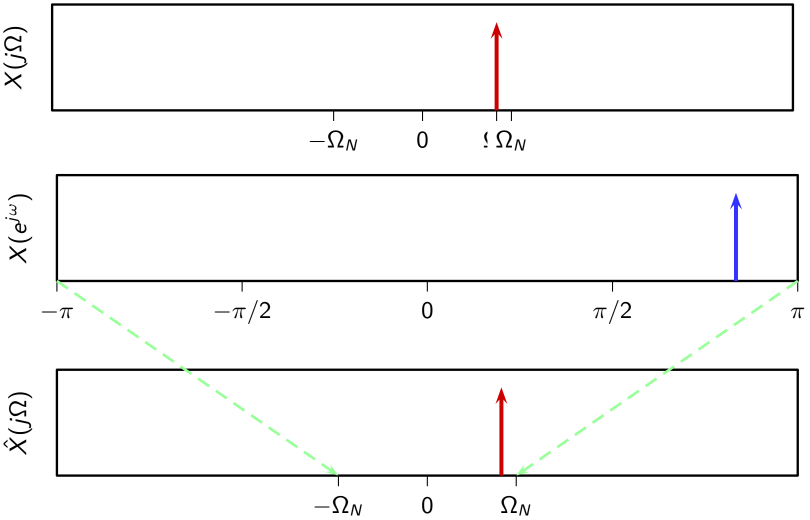

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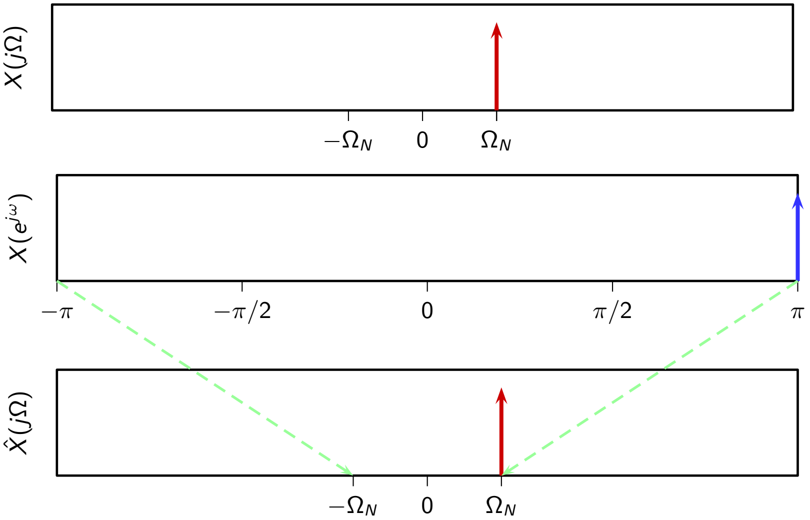

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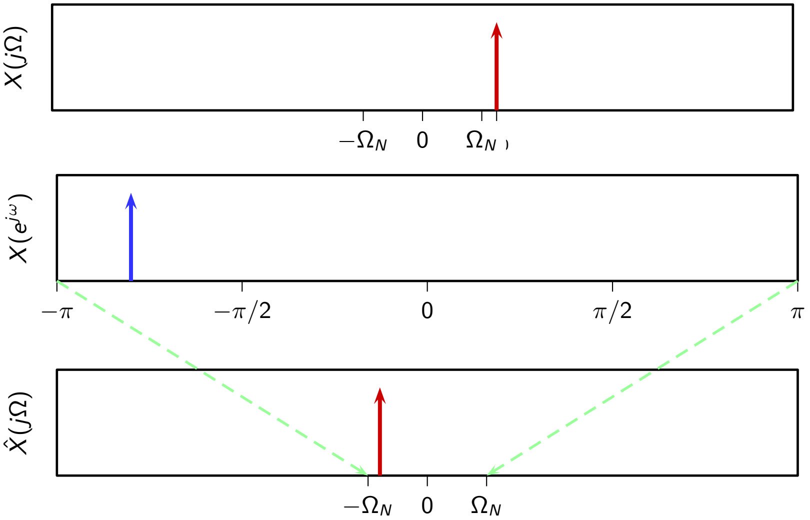

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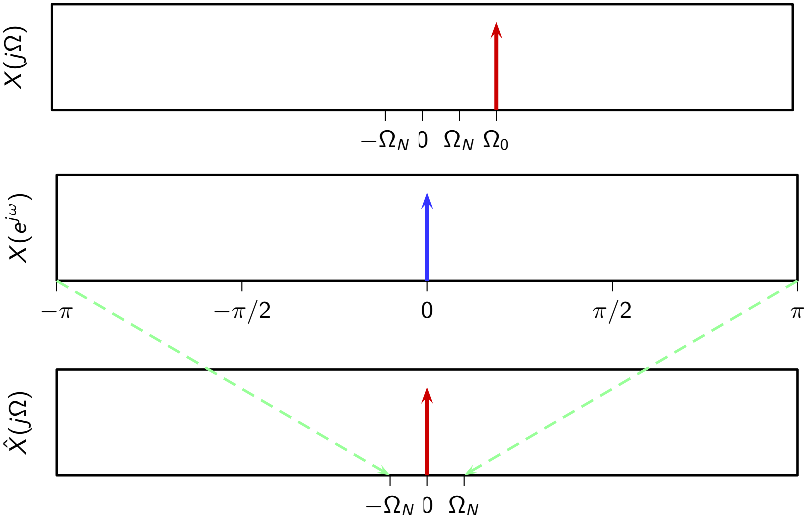

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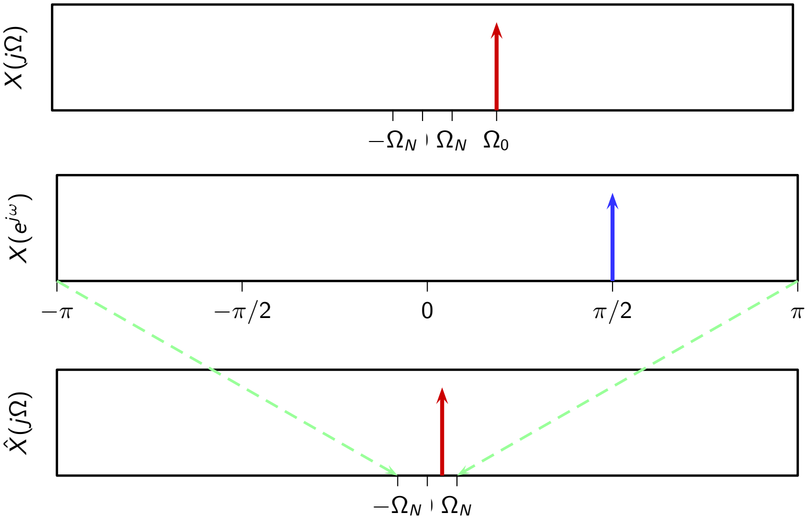
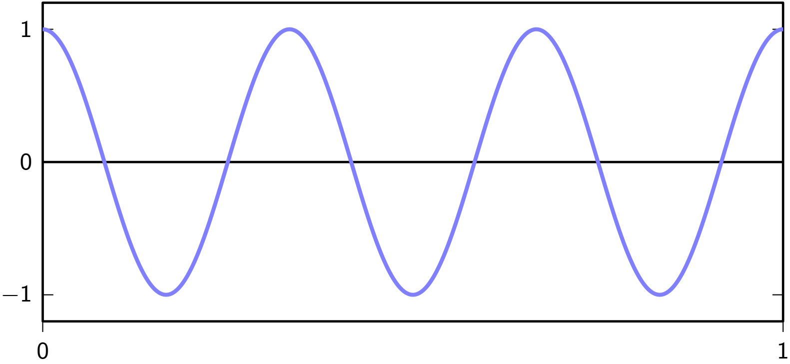


Sampling a Sinusoid

sampling frequency	digital frequency	interpolation
$F_s > 2F_0$	$0 < \omega_0 < \pi$	OK: $\hat{F}_0 = F_0$
$F_s = 2F_0$	$\omega_0 = \pi$	OK (max frequency $\hat{F}_0 = F_s$)
$F_0 < F_s < 2F_0$	$\pi < \omega_0 < 2\pi$	negative frequency: $\hat{F}_0 = F_0 - F_s$
$F_s < F_0$	$\omega_0 > 2\pi$	full aliasing: $\hat{F}_0 = F_0 \mod F_s$

Aliasing: Sampling a Sinusoid

$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



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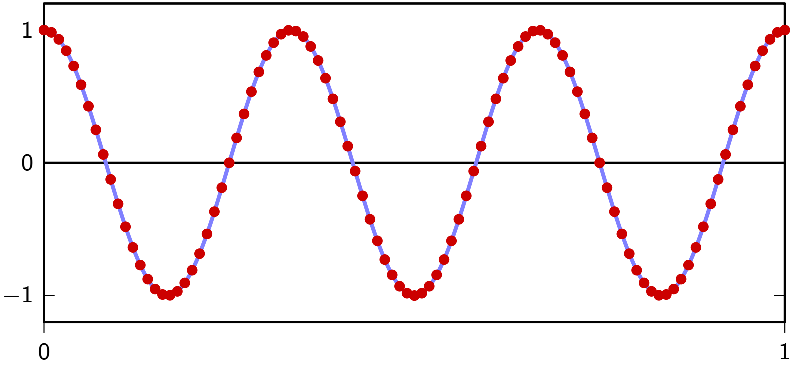
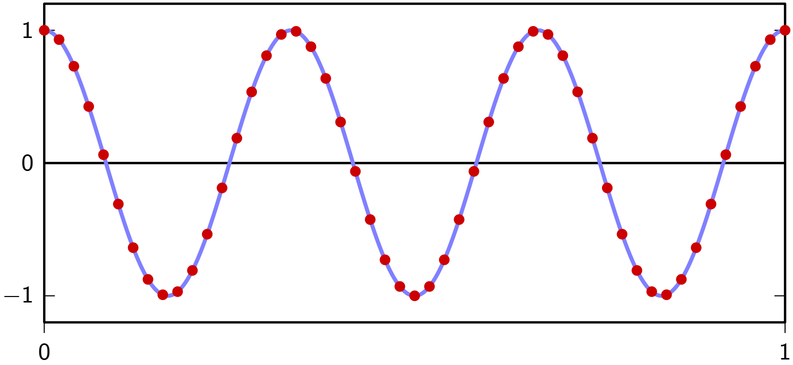


$F_s = 100\text{Hz}$

Aliasing: Sampling a Sinusoid

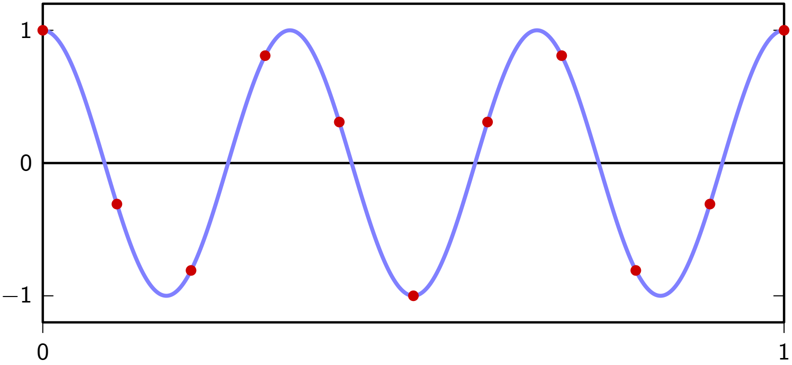
$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



$F_s = 50\text{Hz}$

Aliasing: Sampling a Sinusoid

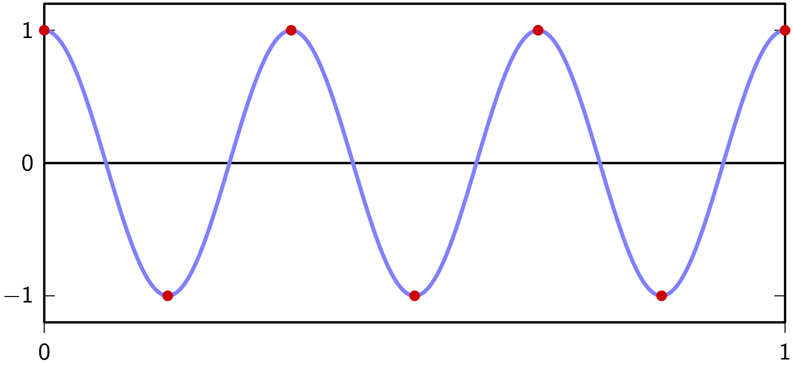
$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



$F_s = 10\text{Hz}$

Aliasing: Sampling a Sinusoid

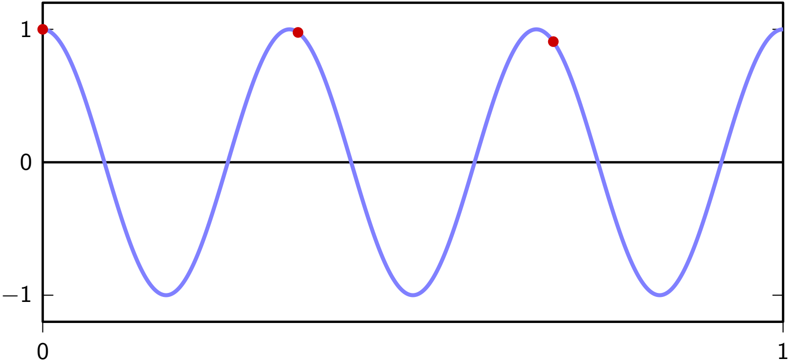
$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



$F_s = 6\text{Hz}$

Aliasing: Sampling a Sinusoid

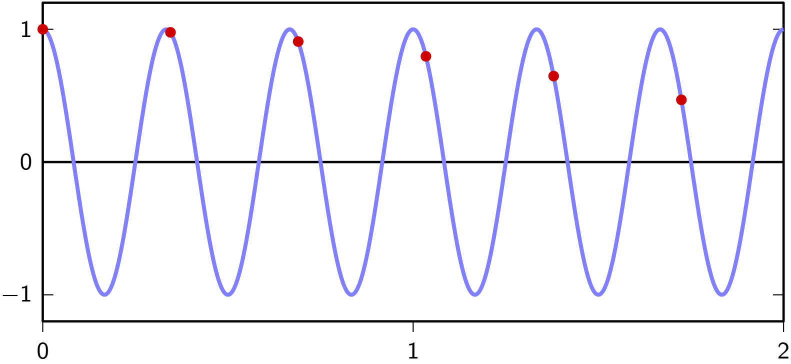
$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



$F_s = 2.9\text{Hz}$

Aliasing: Sampling a Sinusoid

$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



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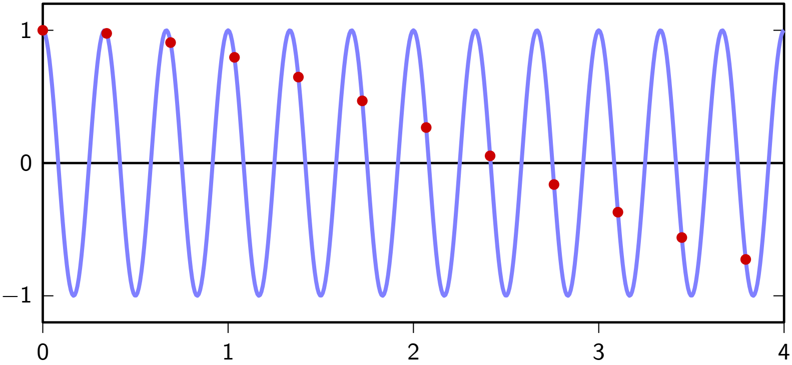

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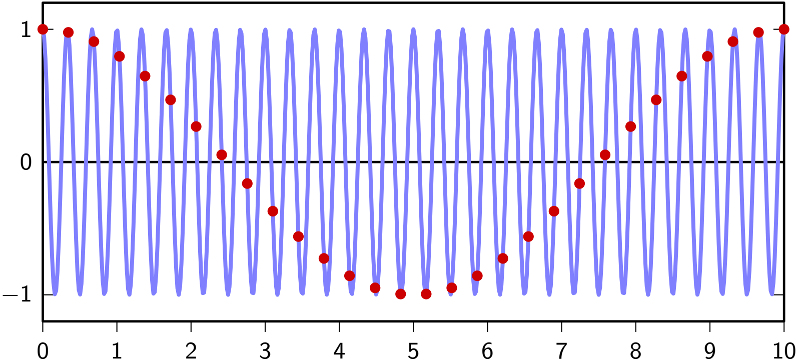
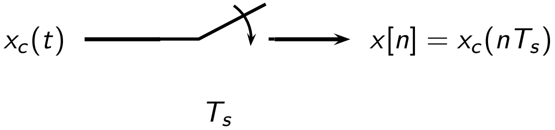
Aliasing: Sampling a Sinusoid

$$x(t) = \cos(6\pi t) \qquad (F_0 = 3\text{Hz})$$



$F_s = 2.9\text{Hz}$

Raw-sampling an arbitrary signal



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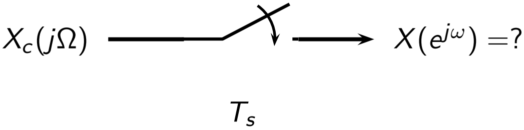
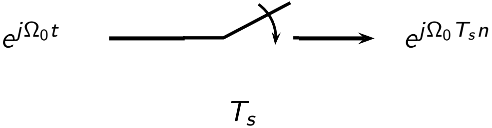


Key idea

- pick T_s (and set $\Omega_N = \pi / T_s$)
- pick $\Omega_0 < \Omega_N$



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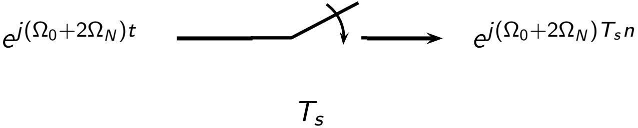

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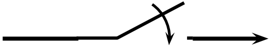
$e^{j(\Omega_0 + 2\Omega_N)t}$



$e^{j(\Omega_0 T_s n + 2\Omega_N T_s n)}$

T_s

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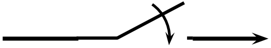

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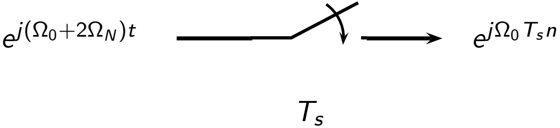

Key idea

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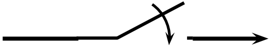
$$Ae^{j\Omega_0 t} + Be^{j(\Omega_0 + 2\Omega_N)t}$$



T_s

$$(A + B)e^{j\Omega_0 T_s n}$$

Spectrum of raw-sampled signals

outline: start with the inverse Fourier Transform

$$x[n] = x_c(nT_s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(j\Omega)e^{j\Omega\, nT_s}d\Omega$$

and manipulate the integral until it looks like

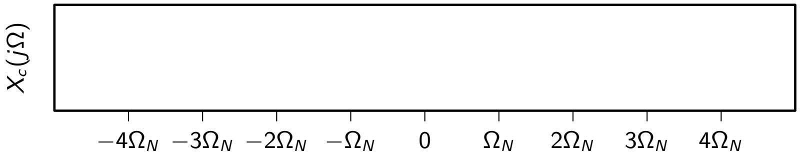
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\omega)e^{j\omega n}d\omega$$

Spectrum of raw-sampled signals

frequencies $2\Omega_N$ apart will be aliased, so split the integration interval

$$
\begin{aligned}
x[n] &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X_c(j\Omega)e^{j\Omega\, nT_s}d\Omega \\
&= \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\int_{(2k-1)\Omega_N}^{(2k+1)\Omega_N} X_c(j\Omega)e^{j\Omega\, nT_s}d\Omega
\end{aligned}
$$

Spectrum of raw-sampled signals



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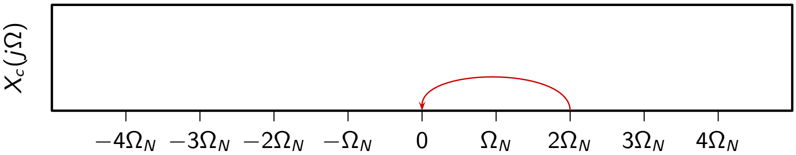

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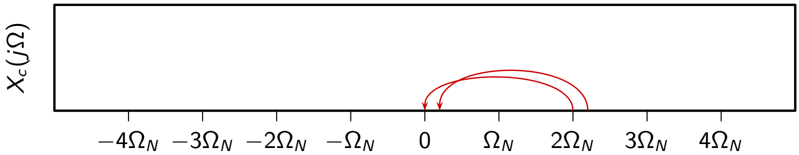

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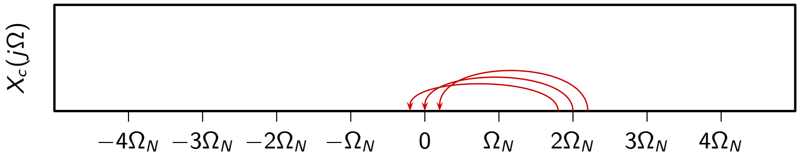

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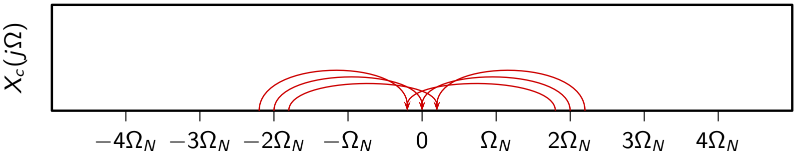

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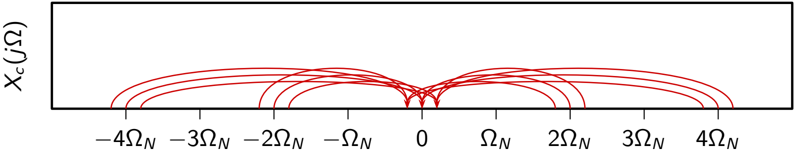


Spectrum of raw-sampled signals

$$x[n] = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \int_{(2k-1)\Omega_N}^{(2k+1)\Omega_N} X_c(j\Omega) e^{j\Omega\, nT_s} d\Omega$$

with a change of variable and using $e^{j(\Omega - 2k\Omega_N)T_s n} = e^{j\Omega T_s n}$:

$$x[n] = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \int_{-\Omega_N}^{\Omega_N} X_c(j(\Omega - 2k\Omega_N)) e^{j\Omega\, nT_s} d\Omega$$

$$= \frac{1}{2\pi} \int_{-\Omega_N}^{\Omega_N} \left[\sum_{k=-\infty}^{\infty} X_c(j(\Omega - 2k\Omega_N)) \right] e^{j\Omega\, nT_s} d\Omega$$

Spectrum of raw-sampled signals

periodization of the spectrum; define:

$$\tilde{X}_c(j\Omega) = \sum_{k=-\infty}^{\infty} X_c(j(\Omega - 2k\Omega_N))$$

so that:

$$x[n] = \frac{1}{2\pi} \int_{-\Omega_N}^{\Omega_N} \tilde{X}_c(j\Omega) e^{j\Omega\, T_s\, n} d\Omega$$

Spectrum of raw-sampled signals

set $\omega = \Omega T_s$:

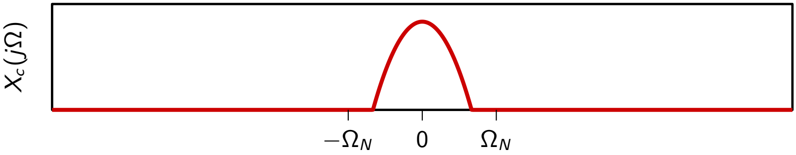
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{T_s}\tilde{X}_c\left(j\frac{\omega}{T_s}\right) e^{j\omega n} d\omega$$

$$= \mathrm{IDTFT}\left\{\frac{1}{T_s}\tilde{X}_c\left(j\frac{\omega}{T_s}\right)\right\}$$

$$X(e^{j\omega}) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X_c\left(j\frac{\omega}{T_s} - j\frac{2\pi k}{T_s}\right)$$

$$= \frac{\pi}{\Omega_N}\sum_{k=-\infty}^{\infty} X_c\left(j\frac{\omega\Omega_N}{\pi} - 2jk\Omega_N\right)$$

Example: signal bandlimited to Ω_0 and $\Omega_N > \Omega_0$



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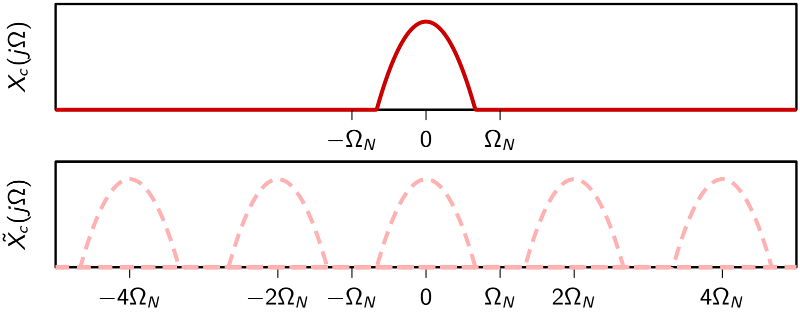

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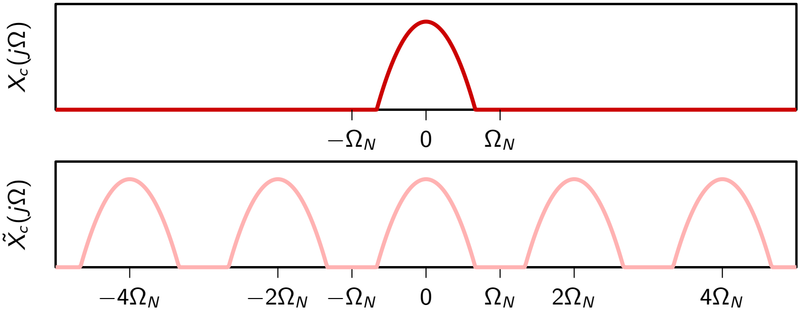

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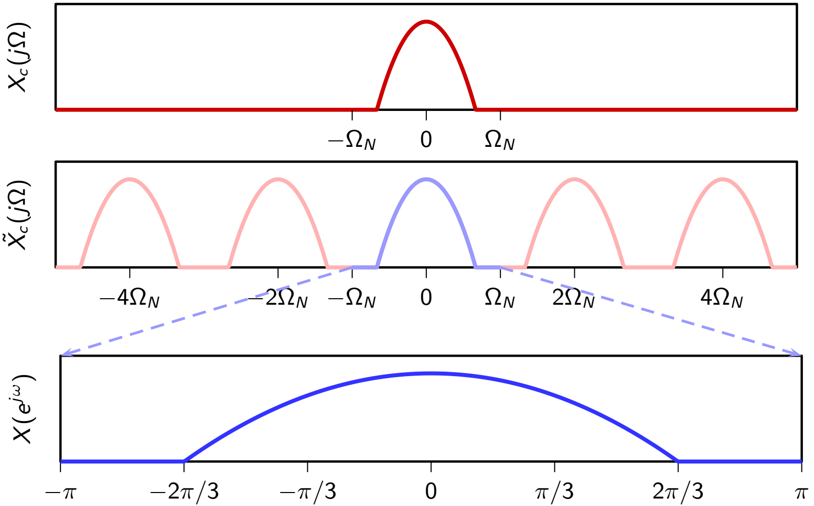
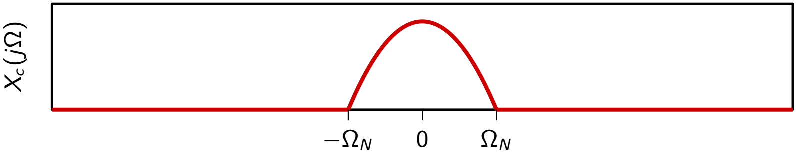


Example: signal bandlimited to Ω_0 and $\Omega_N = \Omega_0$



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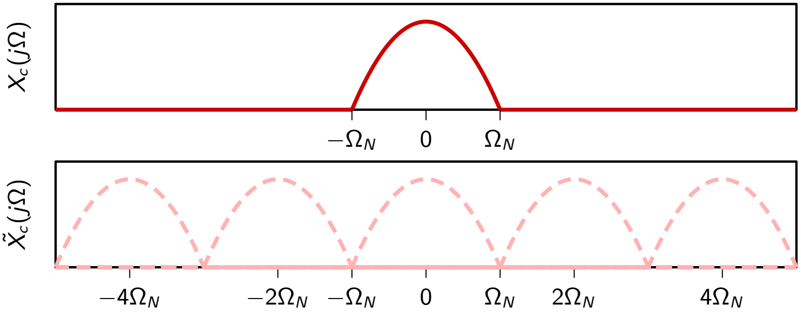

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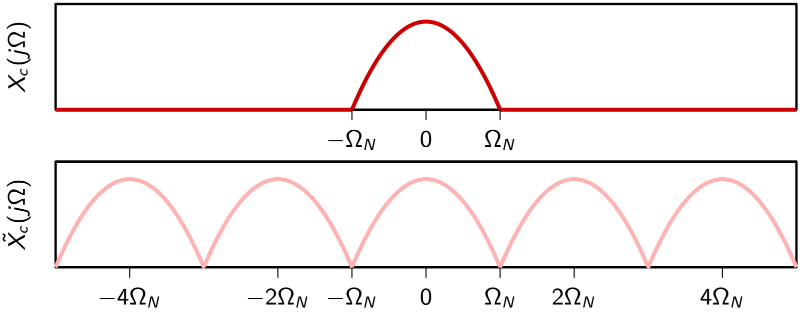

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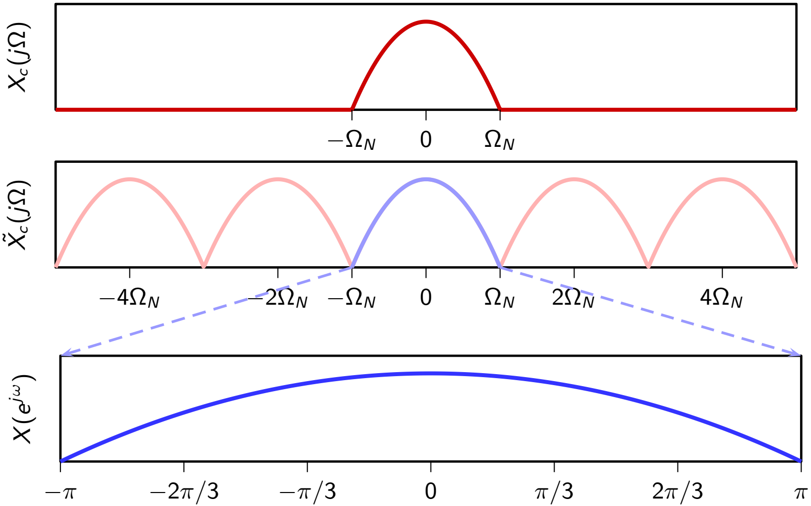

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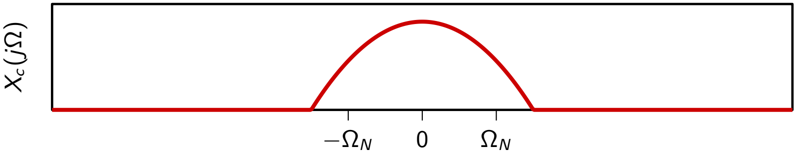

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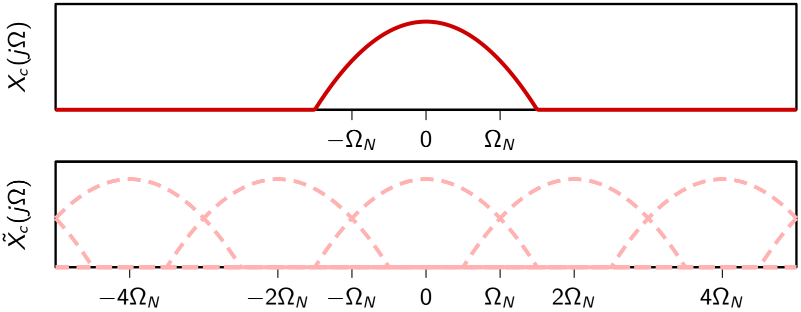

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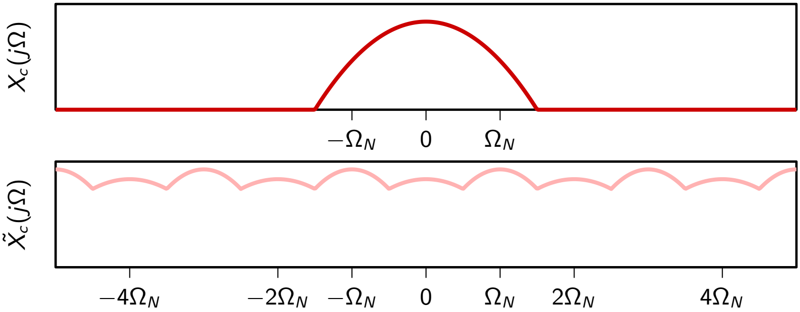

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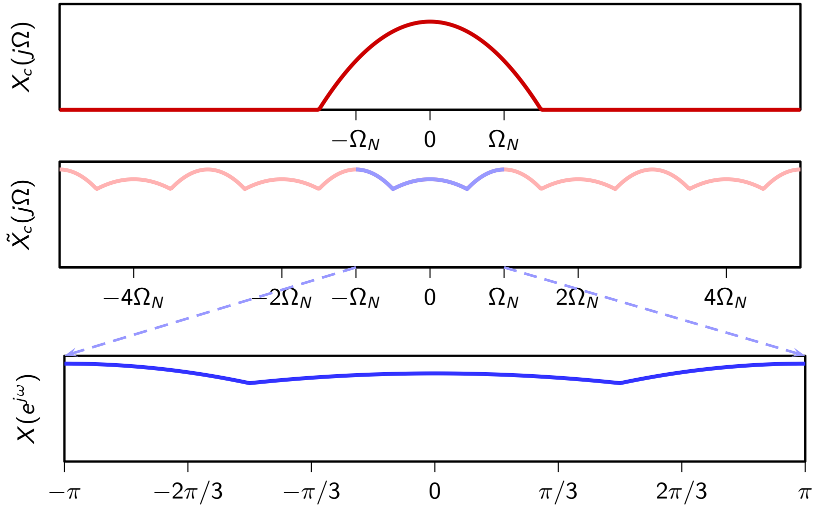
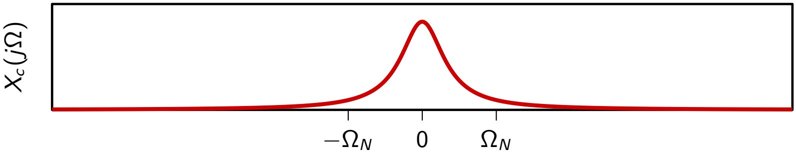


Example: non-bandlimited signal



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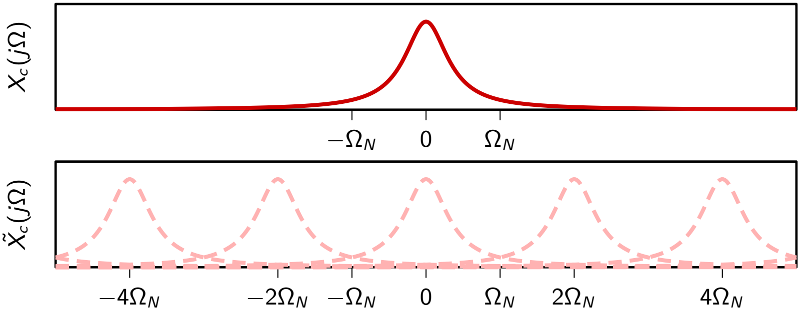

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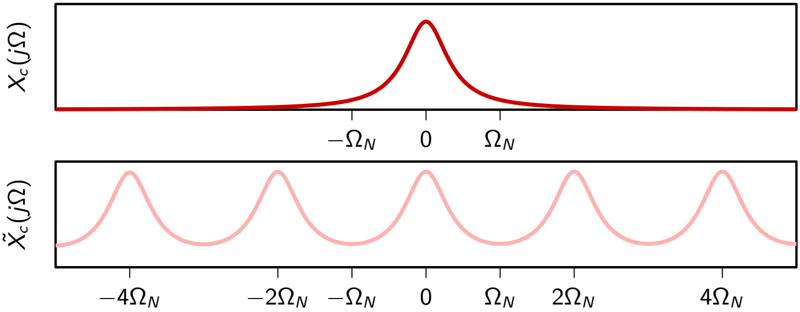

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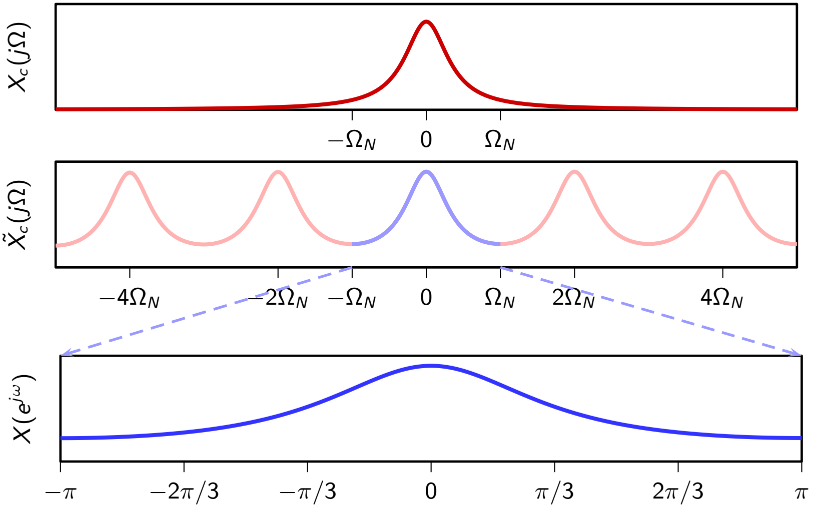

Sampling strategies

given a sampling period T_s

- if the signal is bandlimited to π/T_s or less, raw sampling is fine (i.e. equivalent to sinc sampling up to a scaling factor T_s)
- if the signal is not bandlimited, two choices:
  - bandlimit via a lowpass filter *in the continuous-time domain* before sampling (i.e. sinc sampling)
  - or, raw sample the signal and incur aliasing
- aliasing introduces errors we cannot control, so the sensible choice is to bandlimit in continuous time
- bandlimiting is also optimal wrt least squares approximation!

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Sinc Sampling and Interpolation

$$\hat{x}[n] = \left\langle \operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right), x(t) \right\rangle \;=\; (\operatorname{sinc}_{T_s} * x)(nT_s)$$

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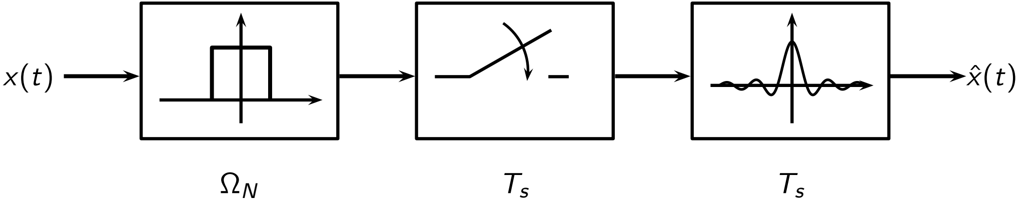
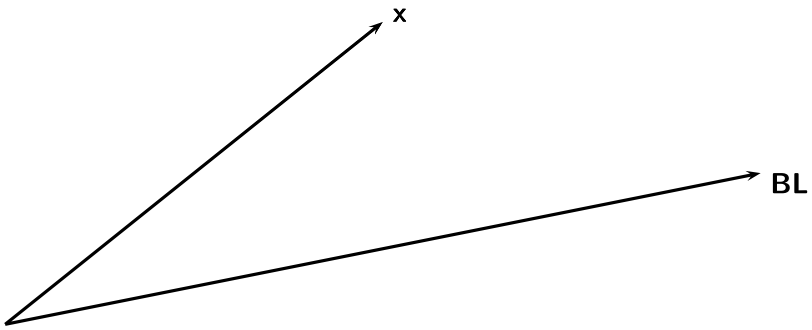


Least squares approximation with sinc sampling and interpolation



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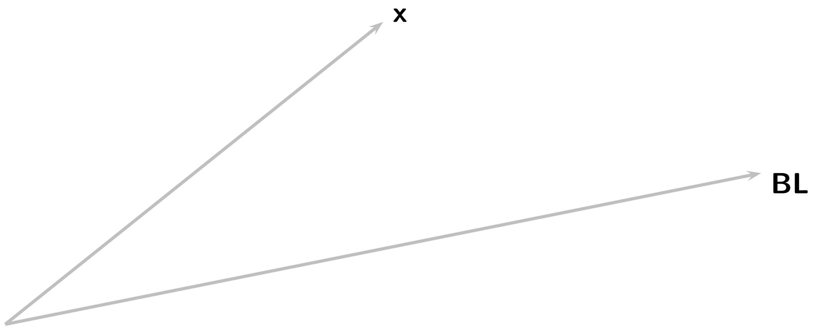

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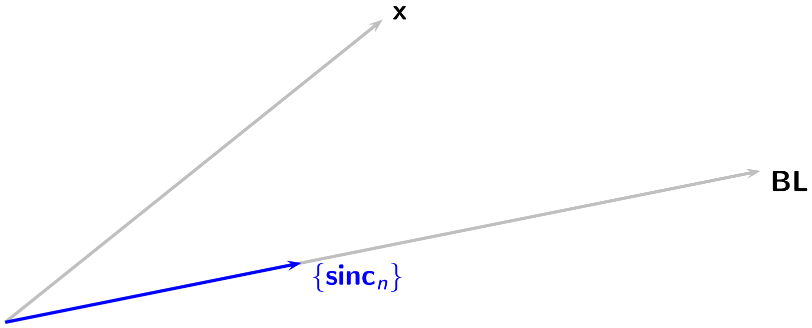

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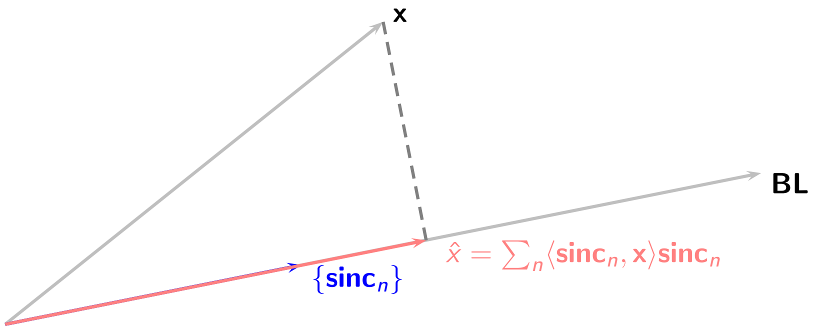

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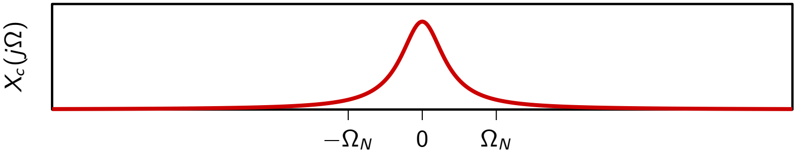

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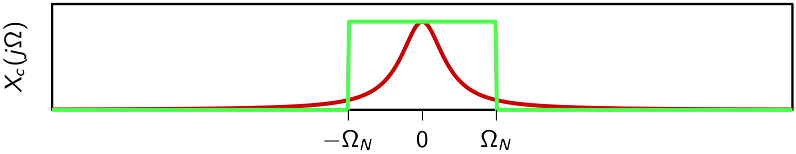

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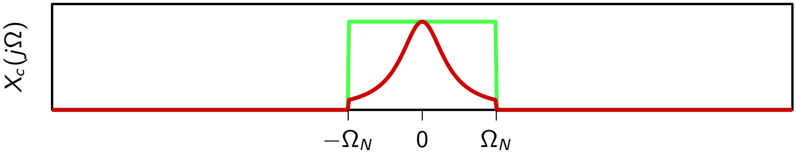

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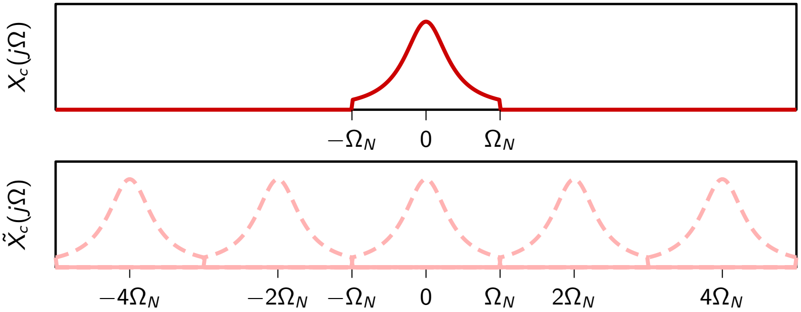

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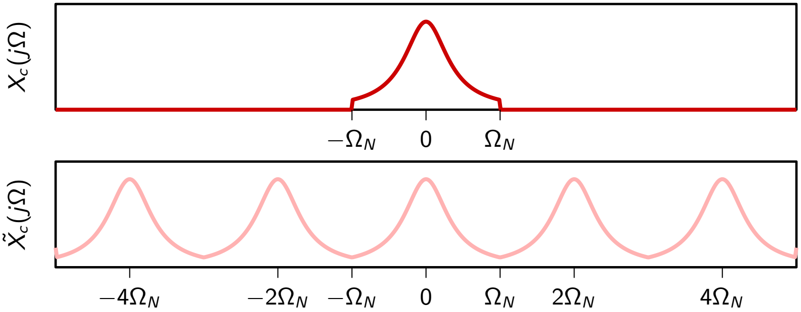

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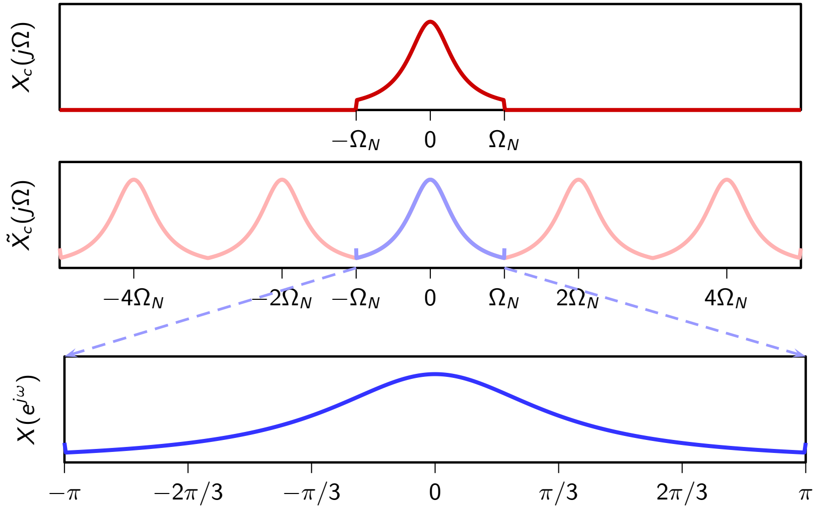

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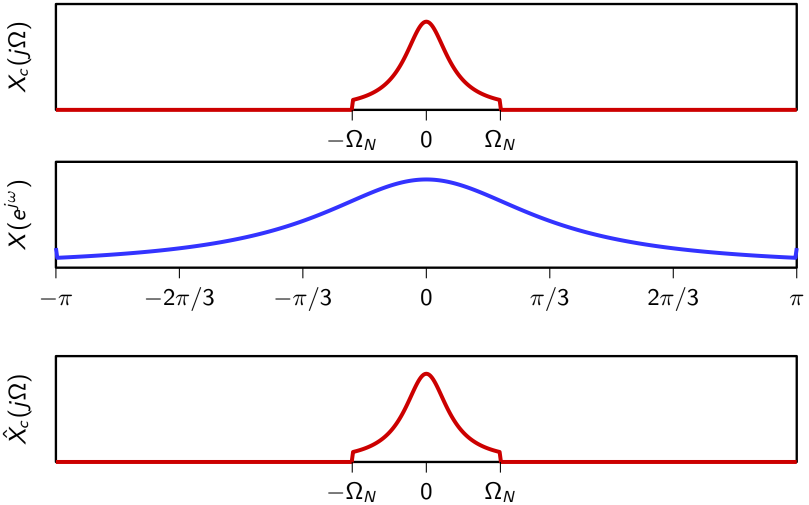

processing of analog signals

Overview:

- Impulse invariance
- Duality
- Examples

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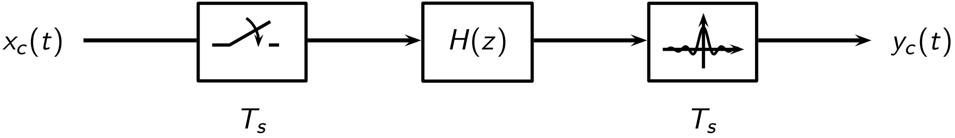
Basic setup



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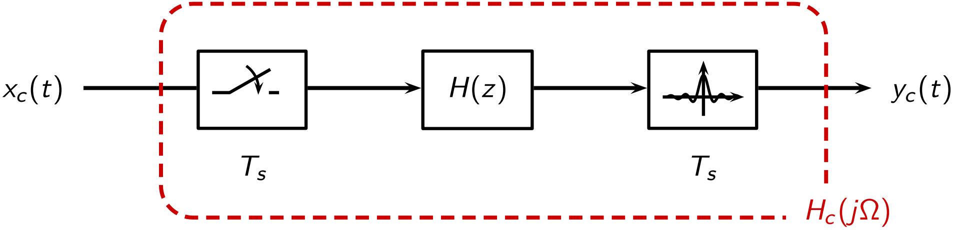

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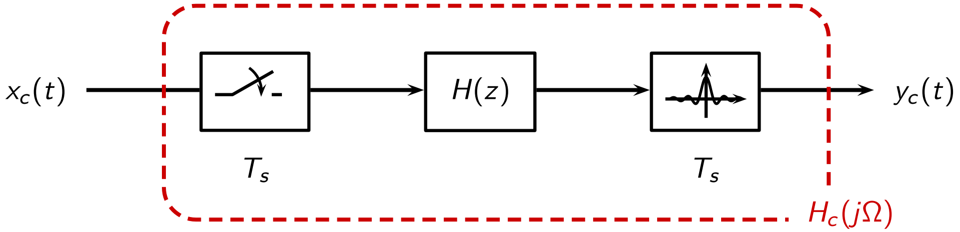

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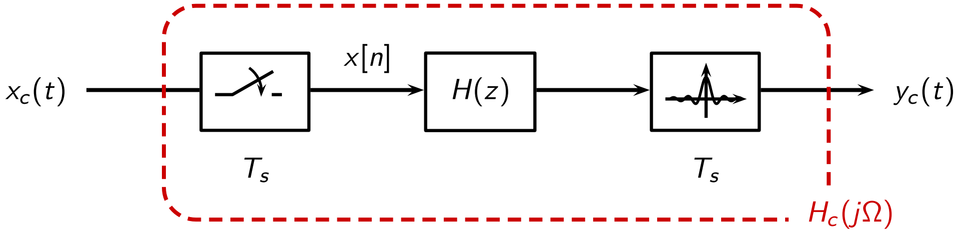

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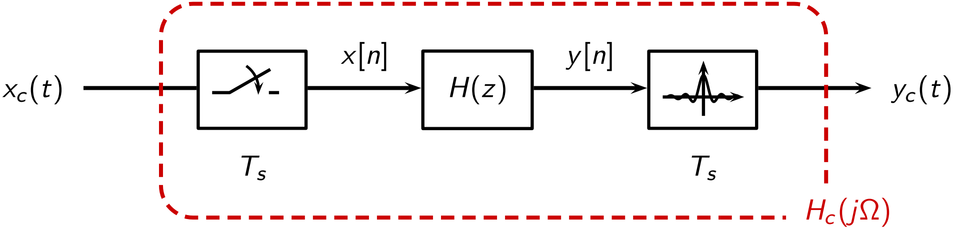
- $X(e^{j\omega}) = \frac{1}{T_s} X_c\left(j\frac{\omega}{T_s}\right)$

Basic setup



assume $x_c(t)$ is Ω_N-bandlimited:

- $X(e^{j\omega}) = \frac{1}{T_s} X_c\left(j\frac{\omega}{T_s}\right)$
- $Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$

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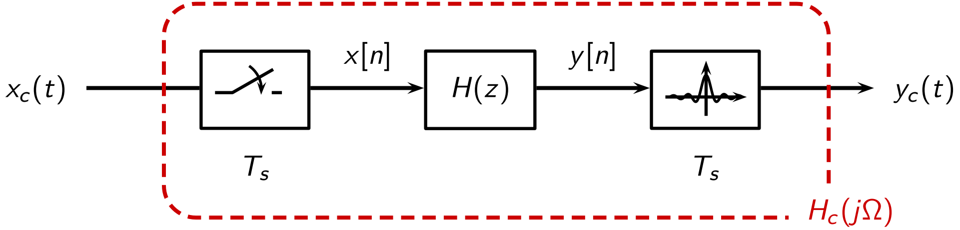

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- $Y_c(j\Omega) = T_s\, Y(e^{j\Omega T_s})$

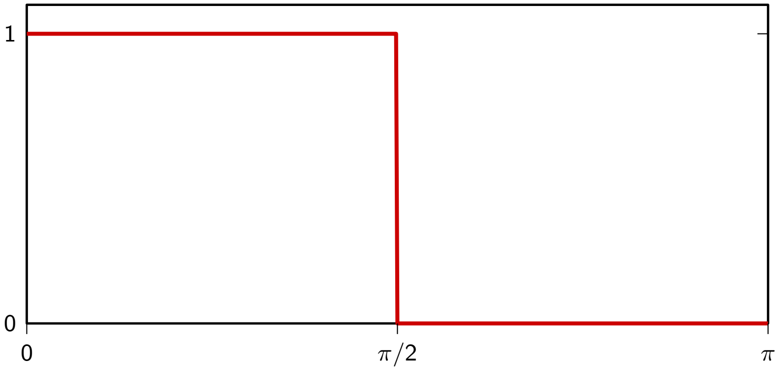
Impulse invariance

$$Y_c(j\Omega) = X_c(j\Omega)\, H(e^{j\pi\Omega/\Omega_N})$$

Impulse invariance

$$H_c(j\Omega) = H(e^{j\pi\Omega/\Omega_N})$$

Impulse invariance



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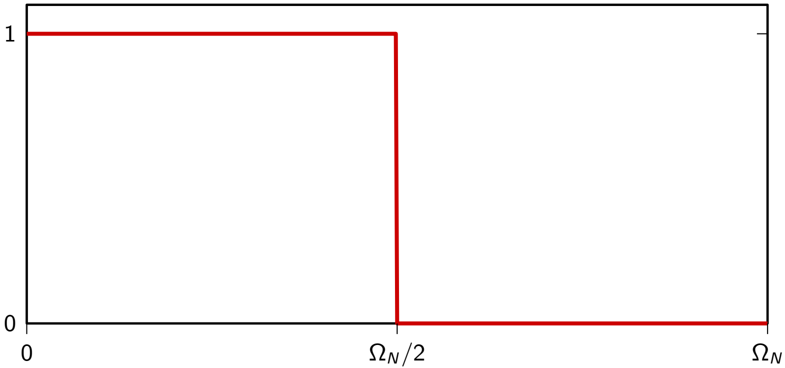
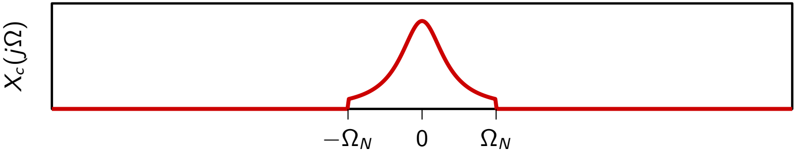


DT processing of CT signals



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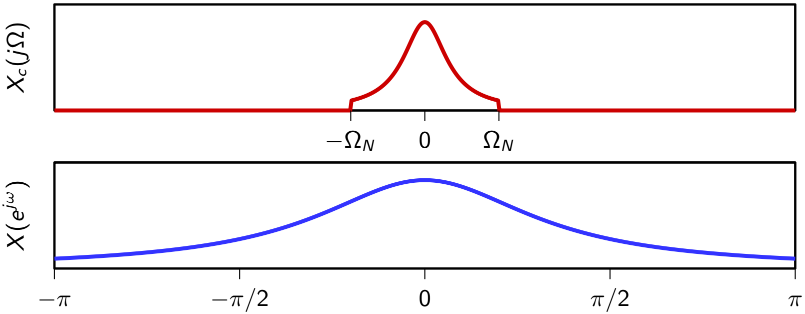

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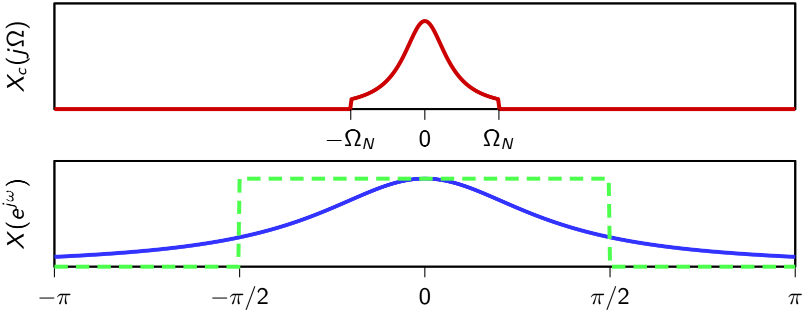

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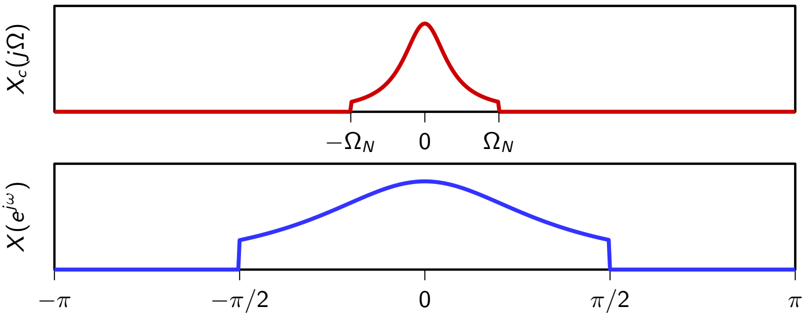

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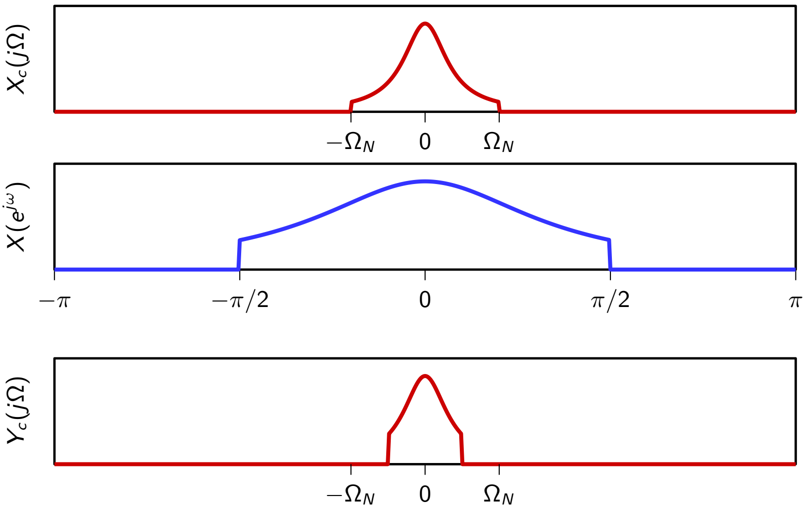

Example

design a discrete-time filter to isolate a band of frequencies between 4000 and 5000Hz; input signals are bandlimited to 7KHz.

Example

- 7KHz band limit $\Rightarrow$ we can use any sampling frequency above 14KHz
- pick $F_s = 16KHz$ so that $\Omega_N = 2\pi \cdot 8000$ rad/s
- we need a bandpass with a 1000Hz bandwidth
- start with a lowpass with cutoff 500Hz
- modulate it to center it around 4500Hz

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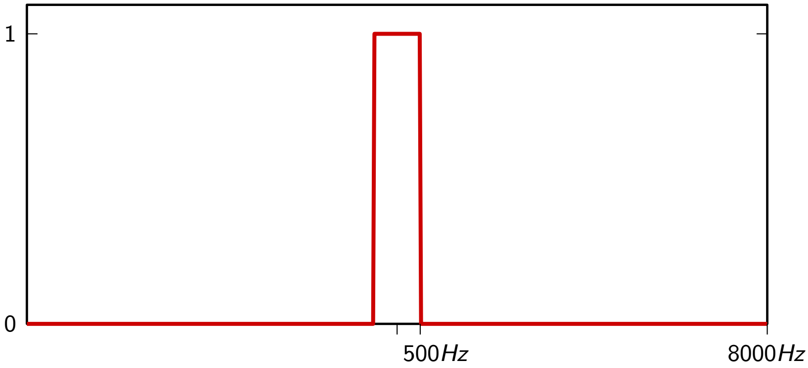

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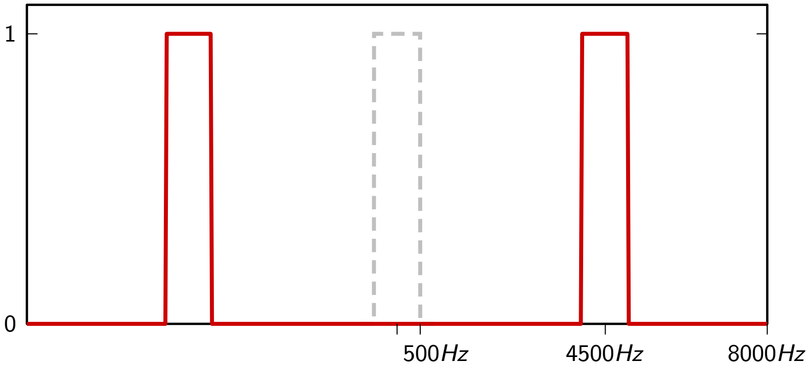

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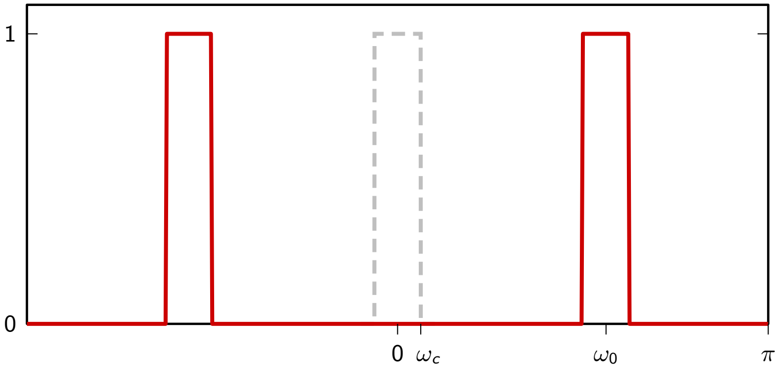

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- $\omega_c = \pi\frac{\Omega_c}{\Omega_N} = \pi\frac{500}{8000} = 0.0625\pi$
- $\omega_0 = \pi\frac{4500}{8000} = 0.5625\pi$
- design an FIR lowpass with cutoff ω_c using your favorite method
- multiply the impulse response by $2\cos\omega_0 n$

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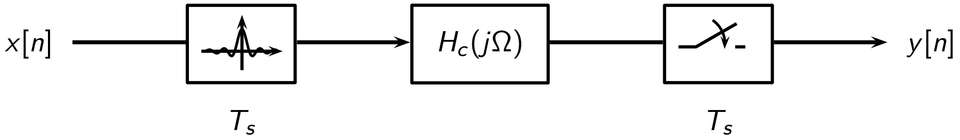
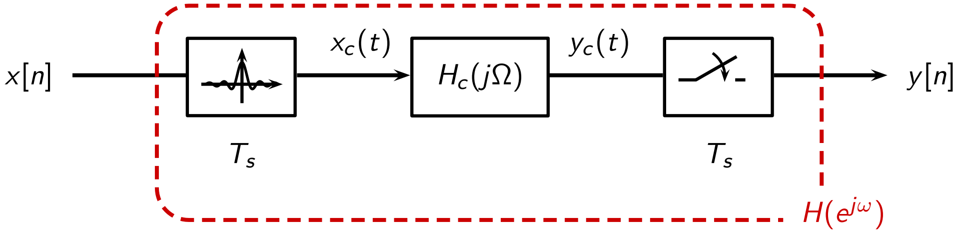
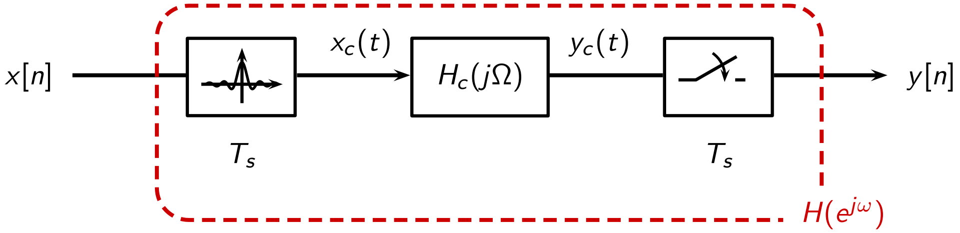
Duality



Duality

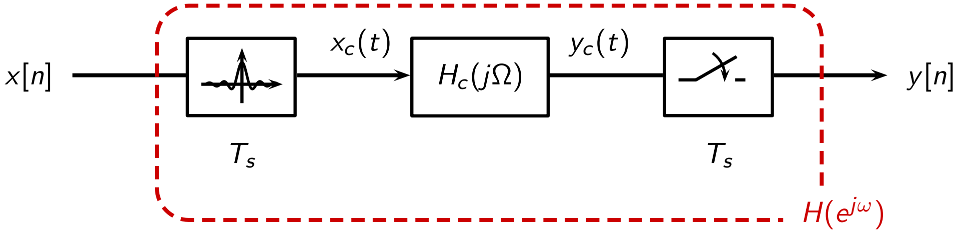


Duality



we can pick any T_s so pick $T_s = 1$:

Duality



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- $X_c(j\Omega) = X(e^{j\Omega})$

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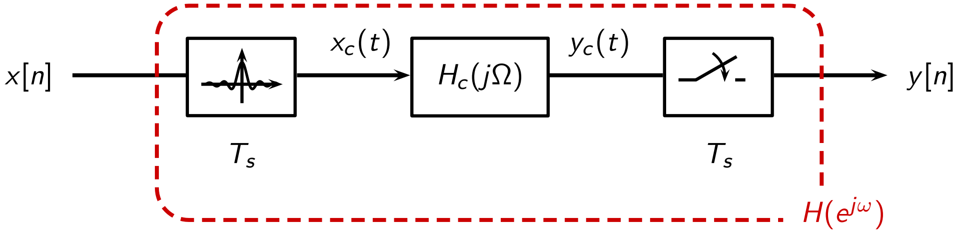

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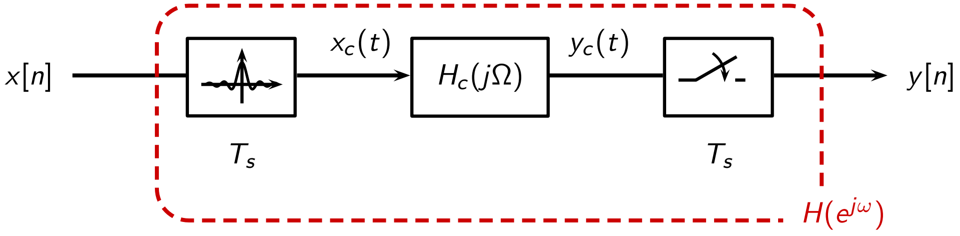

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- $X_c(j\Omega) = X(e^{j\Omega})$
- $Y_c(j\Omega) = X_c(j\Omega)H_c(j\Omega)$
- LTI systems cannot change the bandwidth $\Rightarrow Y(e^{j\omega}) = Y_c(j\omega)$

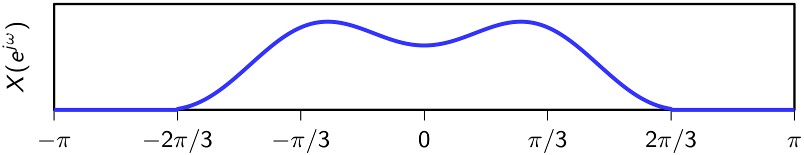
Duality

$$Y(e^{j\omega}) = X(e^{j\omega})\, H_c(j\omega)$$

Duality

$$H(e^{j\omega}) = H_c(j\omega)$$

CT processing of DT signals



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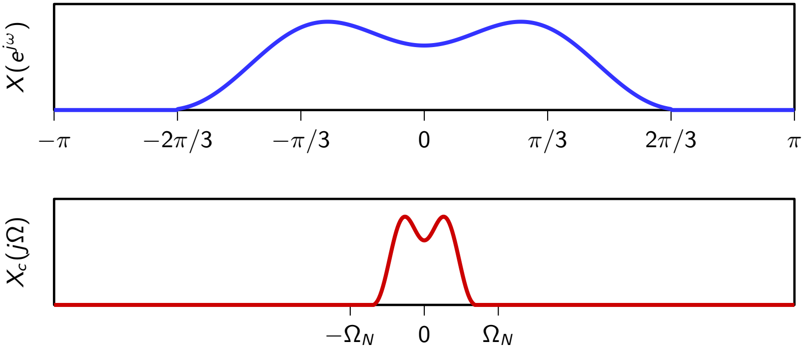

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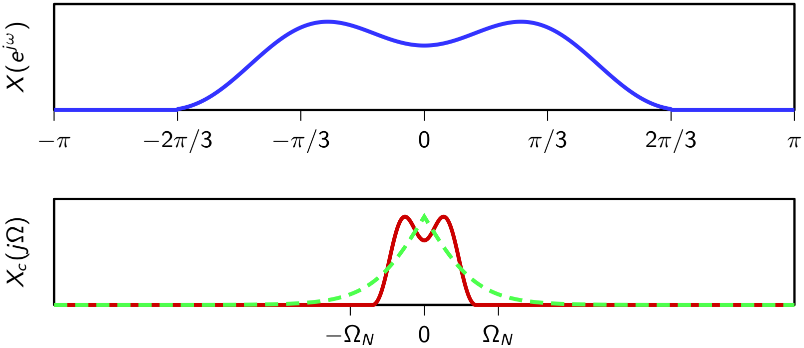

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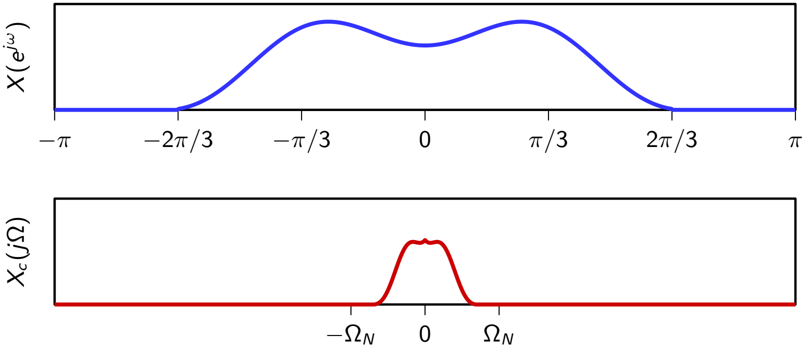

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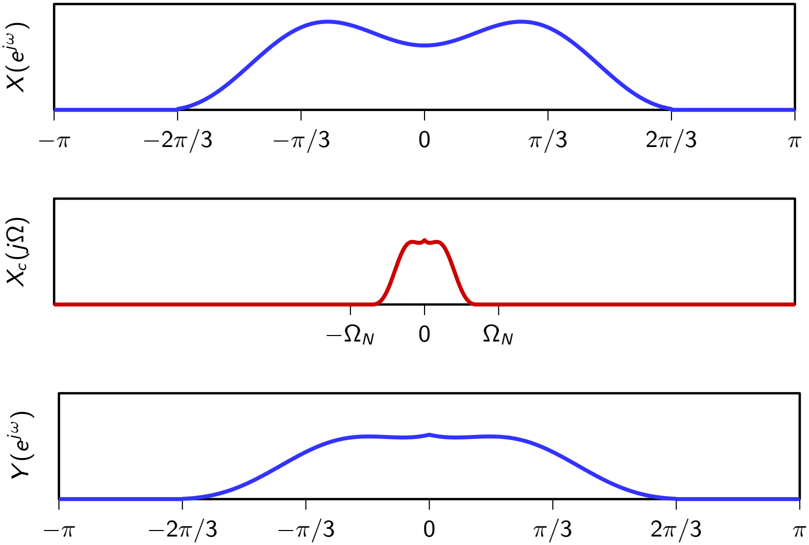
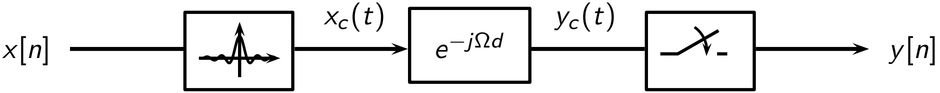


Example: fractional delay

$$H(e^{j\omega}) = e^{-j\omega d}$$

- if $d \in \mathbb{Z}$, simple delay
- if $d \notin \mathbb{Z}$, $h[n] = \mathrm{sinc}(n - d)$...

By duality



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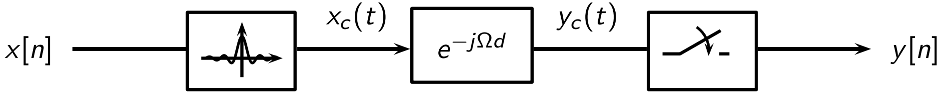
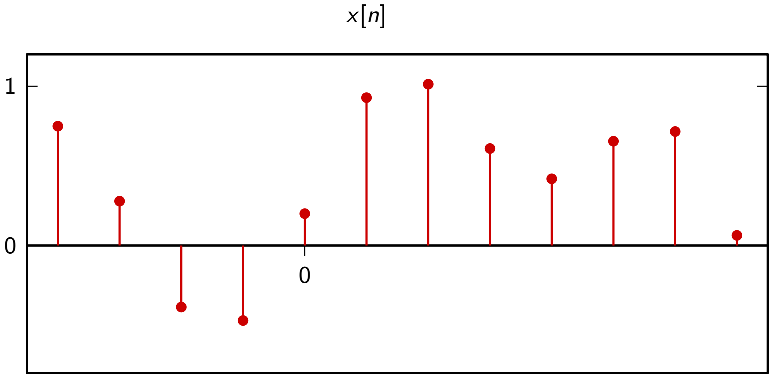


- $Y_c(j\Omega) = e^{-j\Omega d} X_c(j\Omega)$
- $y_c(t) = x_c(t - d)$
- $y[n]$ is the sampled interpolation of $x[n]$ delayed by d

Example: fractional delay



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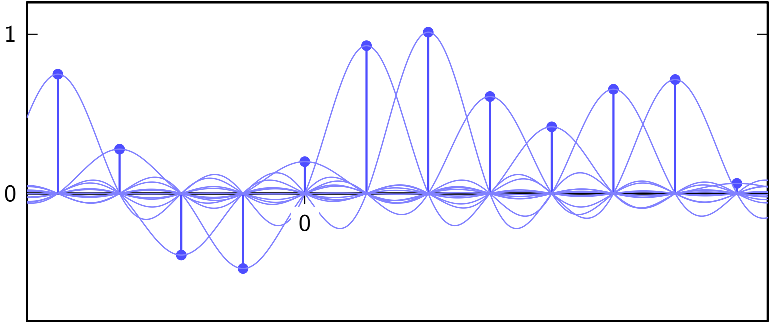

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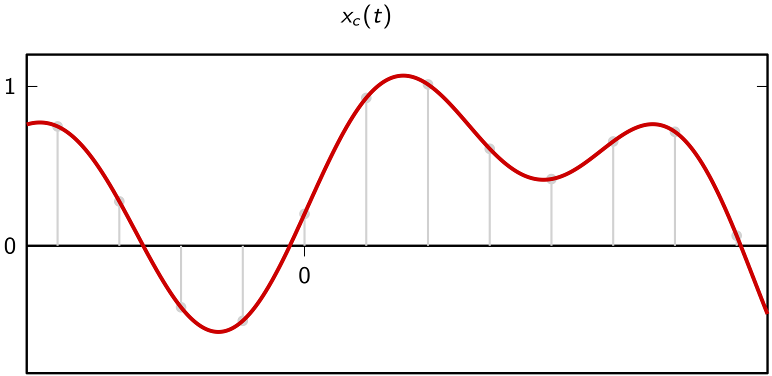

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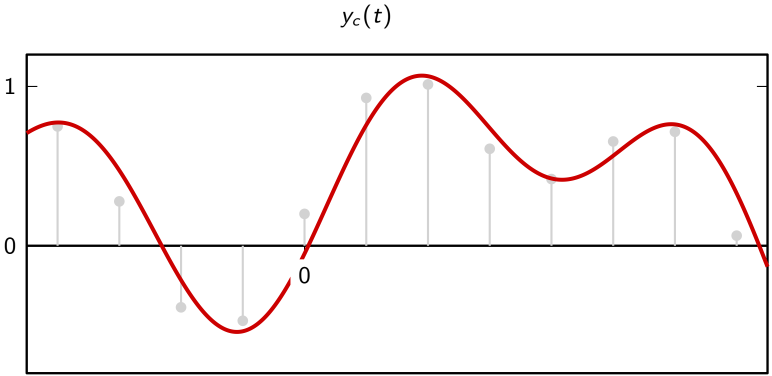

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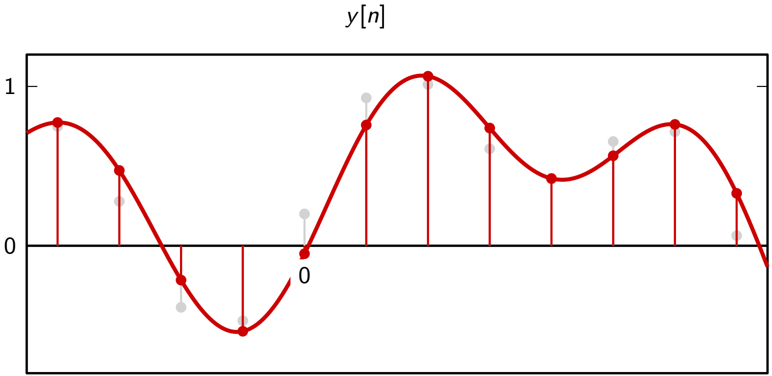

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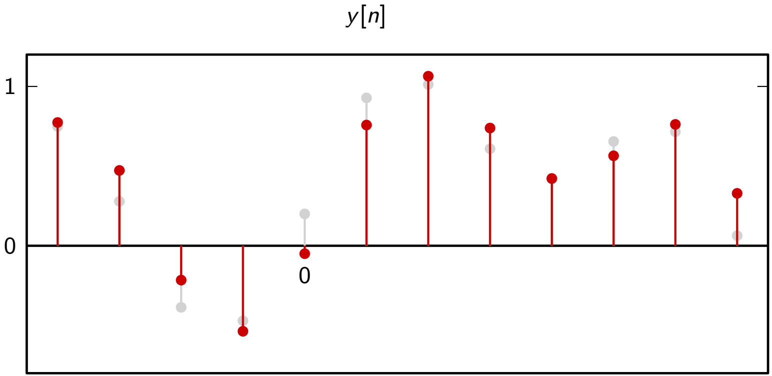


Example: fractional delay

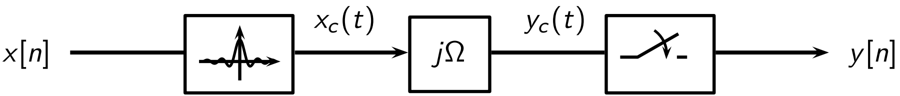
- to delay a discrete-time signal by a fraction of a sample we need an ideal filter!
- efficient approximations exist (e.g. cubic local interpolation)

Example: differentiator

$$H(e^{j\omega}) = j\omega$$

- in continuous time we know that $\mathrm{FT}\{x_c'(t)\} = j\Omega\, X_c(j\Omega)$
- in discrete time...

By duality



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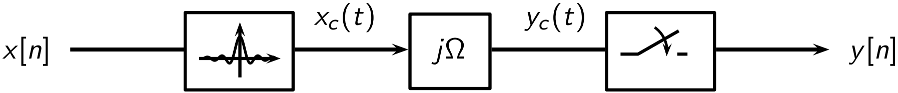
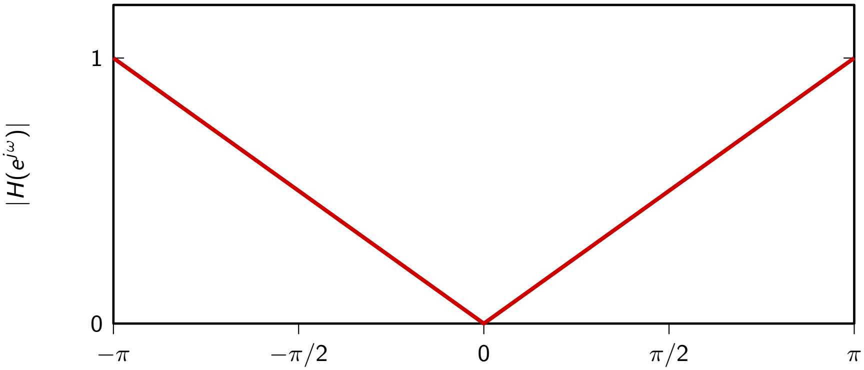


- $Y_c(j\Omega) = j\Omega X_c(j\Omega)$
- $y_c(t) = x_c'(t)$
- $y[n]$ is the sampled interpolation of $x[n]$, differentiated

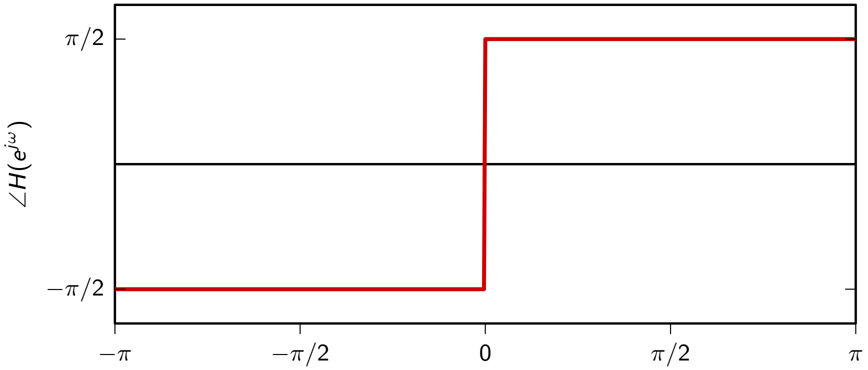
Digital differentiator, magnitude response

$$H(e^{j\omega}) = j\omega$$



Digital differentiator, phase response

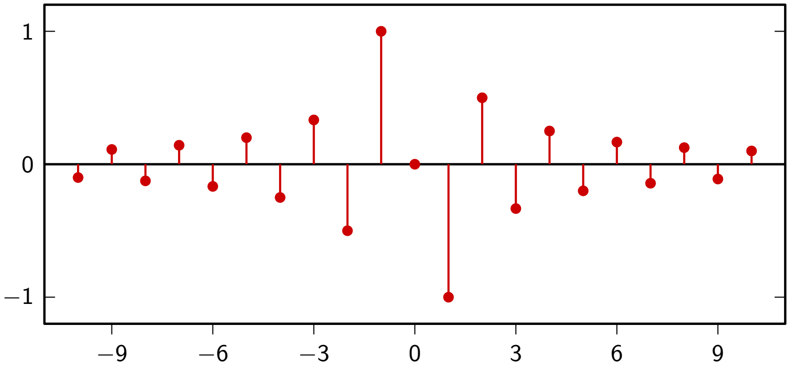
$$H(e^{j\omega}) = j\omega$$



Digital differentiator, impulse response

$$
\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} j\omega e^{j\omega n}\, d\omega \\
&= \ldots \textit{(integration by parts)} \ldots \\
&= \begin{cases} 0 & n = 0 \\ \dfrac{(-1)^n}{n} & n \neq 0 \end{cases}
\end{aligned}
$$

Digital differentiator, impulse response



Digital differentiator

- the digital differentiator is again an ideal filter!
- many approximations exist, with different properties

Wrap up

- Continuous-time processing of discrete-time sequences
- Discrete-time processing of continuous-time signals
- Jumping back and forth using sampling and interpolation
- In practice: Many applications of processing continuous-time signals in discrete time!