Assignment sheet 1

September 19, 2018

Assignment 1. A, B and C are events such that

$$\begin{aligned} P(A) &= 0.4, & P(B \cap C) &= 0.2, \\ P(B) &= 0.7, & P(C \cap (A \cup B)) &= 0.2, \\ P(C) &= 0.3, & P(B \cap (A \cup C)) &= 0.4, \quad \text{and} \\ & & P(A \cup B) &= 0.8. \end{aligned}$$

(a) Find the probability that exactly two of A, B and C occur.
(b) Find the probability that none of A, B and C occur.
(c) Find the probability that A and exactly one of B and C occur.

Assignment 2. Three students, A, B, C, have equal claims for an award. They decide that each will toss a coin, and that the man whose coin falls unlike the other two wins. (The 'odd man' wins.) If all three coins fall alike, they toss again.
(a) Describe a sample space for the result of the first toss of the three coins.
(b) Assign probabilities to the elements of the sample space.
(c) What is the probability that A wins on the first toss ? That B does ? That C does ?
(d) What is the probability that there is no winner on the first toss ?
(e) Given that there is a winner on the first toss, what is the probability that it is A ?

Assignment 3. During a television game show, a contestant is asked to pick one out of three doors. Behind one of the doors, there is a brand new car to be won, behind the other two doors, there is nothing. The contestant picks a door. Instead of opening them, the host of the show reveals that there is no car behind one of the other two doors and asks the contestant, whether he wants to alter his choice. Assume that the host would always reveal an empty door in order to make things more interesting irrespectively of whether the contestant is right or not in his initial choice. Now, based on contestant's behavior, his chance of winning a new car can be one of the following
(a) 1/3
(b) 1/2
(c) 2/3
Explain what behavior of the contestant can lead to every one of these three probabilities of winning. To do this properly, formulate an adequate probabilistic model by answering questions like where is the randomness coming from, i.e. what is the probability space and the studied random variable.

Assignment 4. A blood test for a particular medical condition turn out either 'positive' or 'negative'. 'Positive' indicates that the person tested has the disease in question, while 'negative' indicates that he does not have it. Suppose that such a test for this disease sometimes makes mistakes : 1 in 100 of those free of the disease have 'positive' test results, and 2 in 100 of those having the disease have 'negative' test results. The rest are correctly identified. The disease is also quite rare : one person in 1000 has the disease. Find the probability that a person with a 'positive' test result actually has the disease. Also, find the probability that a person with a 'negative' test result does not actually have the disease. Comment on your findings.

Assignment 5. (optional)
(a) Use the command
`set.seed(20092017)`
`runif(n,0,1)`
to generate $n = 1000$ i.i.d. samples from a Unif(0,1) distribution. Store the values as a vector X.
(b) Let $q_\lambda(x) = -log_e(1-x)/\lambda, x \in (0,1)$. Write a user-defined function called "quant" that will compute the value of $q_\lambda(x)$ for a given $x \in (0,1)$ and $\lambda \in (0,\infty)$.
(c) For each $\lambda \in \{1,2,4\}$, compute the vectors $q_\lambda(X)$ as $Y\lambda$.
(d) For each $\lambda \in \{1,2,4\}$, compute the empirical cdf of the sample $Y\lambda$ using the command `ecdf(Yλ)` and store it as $E\lambda$.
(e) Run the following command :

```
par(mfrow=c(1,3))
plot(E1)
plot(E2)
plot(E3)
```

Interpret what you see.
(f) Let

$$F_\lambda(x) = \begin{cases} 1 - exp(-\lambda x), & x \in [0,\infty) \\ 0, & x < 0 \end{cases}$$

denote the cdf of the Exp(λ) distribution. For each $\lambda \in \{1,2,4\}$, write a user-defined function called "cdfλ" that will compute the value of $F_\lambda(x)$ for a given $x \in (0,1)$.
(g) Run the following command :

```
par(mfrow=c(1,3))
curve(cdf1,-1,5)
curve(cdf2,-1,5)
curve(cdf3,-1,5)
```

Interpret what you see. Save the output as a pdf with file name "plot1.pdf".
(h) Run the following command :

```
par(mfrow=c(1,3))
curve(E1,-1,5)
curve(cdf1,-1,5,add=TRUE,col="red")
curve(E2,-1,5)
curve(cdf2,-1,5,add=TRUE,col="red")
curve(E3,-1,5)
curve(cdf3,-1,5,add=TRUE,col="red")
```

Save the output as a pdf with file name "plot2.pdf".
(i) Interpret what you see. What does it tell you about the sampling distribution of $Y\lambda$?
(The reason for what you see will be clear in due course of time).
(j) What is the relation between q_λ and F_λ?
(k) Will the observations made in above still hold if F_λ is replaced by any other continuous, strictly increasing cdf?
(l) Verify your assertion in (k) by replacing F_λ and q_λ with the in-built functions `pnorm` and `qnorm`, respectively. These are the cdf and the quantile function, respectively, of the N(0,1) distribution.
(Note : unlike F_λ and q_λ, the functions `pnorm` and `qnorm` have no closed form expressions).