COM-303 - Signal Processing for Communications

Homework #2

Exercise 1. Bases

Let $\{\mathbf{x}^{(k)}\}_{k=0,\dots,N-1}$ be a basis for a subspace S. Prove that any vector $\mathbf{z} \in S$ is *uniquely* represented in this basis.

Hint: remember that the vectors in a basis are linearly independent and use this to prove the thesis by contradiction.

Exercise 2. Plancherel-Parseval Equality

Let $x[n]$ and $y[n]$ be two complex valued sequences and $X[k]$ and $Y[k]$ their corresponding DFTs.

(a) Show that

$$\sum_{n=0}^{N-1} x[n]y^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]Y^*[k],$$

where the superscript $*$ denotes conjugation.

(b) What is the physical meaning of the above formula when $x[n] = y[n]$?

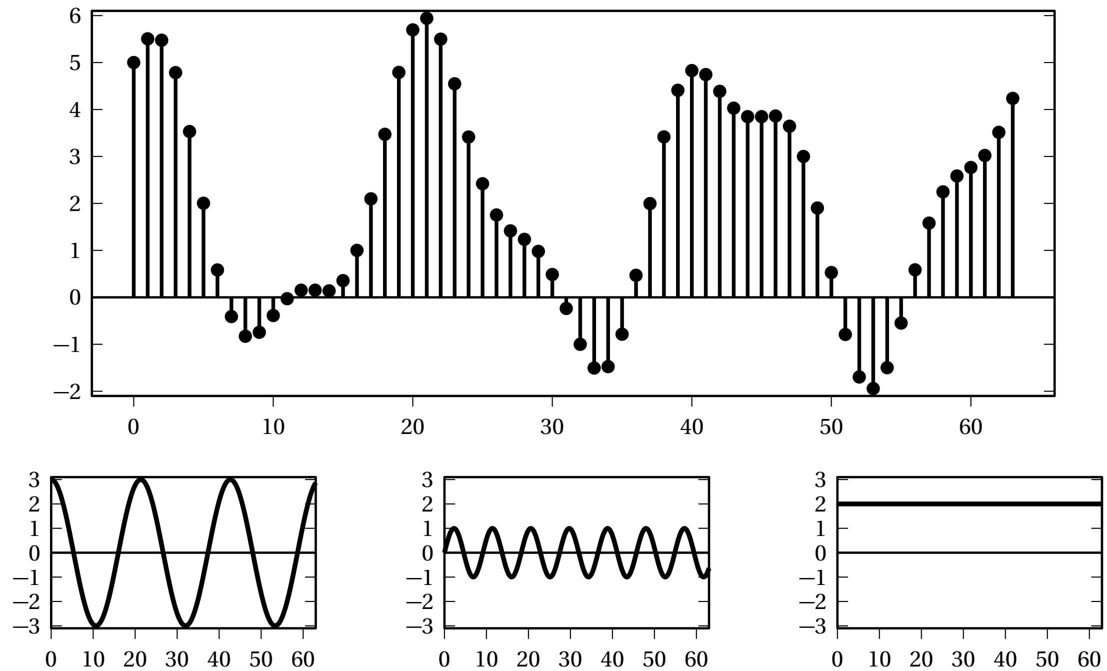
Exercise 3. DFT of elementary functions

Derive the formula for the DFT of the length-N signal

$$x[n] = \cos((2\pi/N)Ln + \phi).$$

Exercise 4. DFT

Consider the length-64 signal $x[n]$ in the figure below, which is the sum of the three 64-periodic signals plotted in the bottom panels of the figure. Compute the DFT coefficients $X[k], k = 0, 1, \dots, 63$.



Exercise 5. DFT computation

Compute the 8 DFT coefficients of the signal $\mathbf{x} = \begin{bmatrix}-1 & -1 & 1 & 1 & -1 & -1 & 1 & 1\end{bmatrix}^T$.

Exercise 6. Structure of DFT formulas

The DFT and IDFT formulas are similar, but not identical. Consider a length-N signal $x[n], N = 0, \dots, N-1$; what is the length-N signal $y[n]$ obtained as

$$y[n] = \text{DFT}\{\text{DFT}\{x[n]\}\}$$

(i.e. by applying the DFT algorithm twice in a row)?

Exercise 7. Signal repetitions

Consider a length-N signal $\mathbf{x} = [x[0]\, x[1] \dots x[N-1]]^T$ and its DFT $\mathbf{X} = [X[0]\, X[1] \dots X[N-1]]^T$.

Consider now the length-$2N$ vector obtained by duplicating each element of the original vector

$$\mathbf{x_2} = [x[0] \quad x[0] \quad x[1] \quad x[1] \quad x[2] \quad x[2] \quad \ldots \quad x[N-1] \quad x[N-1]]^T$$

and express its $2N$-point DFT in terms of the N original DFT coefficients $X[k]$.