COM-303 - Signal Processing for Communications

Homework #12

Exercise 1. Chanel capacity

Consider a 6 MHz-wide channel whose power constraint imposes a maximum SNR of 20 dB. What is the upper bound on the channel's capacity?

Exercise 2. QAM error rate

Assume that we are using a QAM signaling scheme to communicate over a certain channel. If we want to decrease the error rate, which of the following steps can we take? Select all that applies.

(a) Increase the constellation size M

(b) Decrease the constellation size M

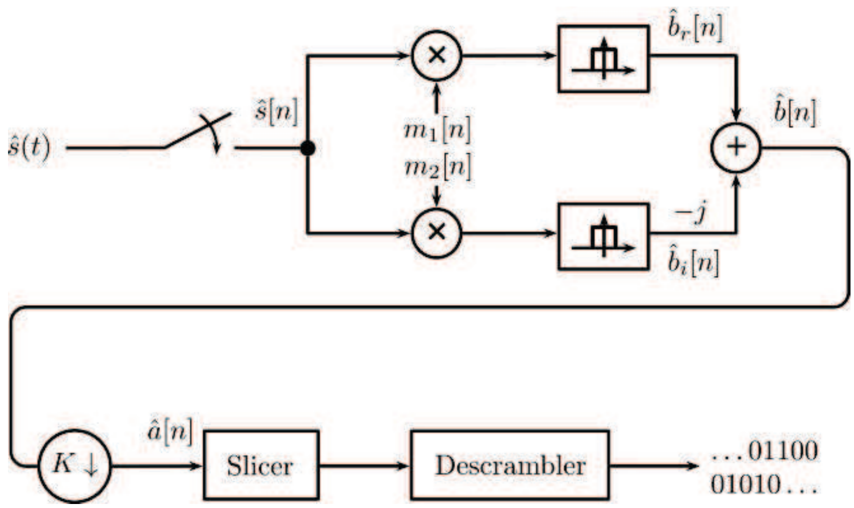
(c) Increase throughput

(d) Decrease throughput

(e) Decrease signal power

Exercise 3. QAM demodulation

Consider the QAM receiver shown below in which the discrete-time input signal $\hat{s}[n]$ is demodulated using two auxiliary signals $m_1[n]$ and $m_2[n]$.



Determine which of the following choices for $m_1[n]$ and $m_2[n]$ allow for a correct demodulation of the signal. Assume that ω_c is much larger than $2\omega_0$, the effective bandwidth of the input signal.

(a) $m_1[n] = \cos\omega_c n$, $m_2[n] = \sin\omega_c n$

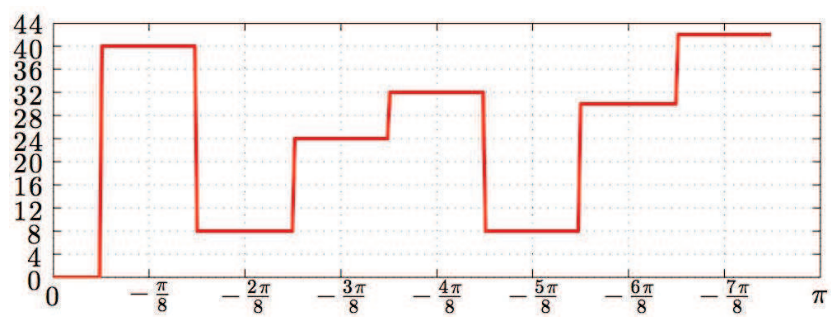
(b) $m_1[n] = 1 + \cos\omega_c n$, $m_2[n] = 1 + \sin\omega_c n$

(c) $m_1[n] = \cos\frac{\omega_c n}{2}\cos\frac{3\omega_c n}{2}$, $m_2[n] = \sin\frac{\omega_c n}{2}\cos\frac{3\omega_c n}{2}$

Exercise 4. ADSL

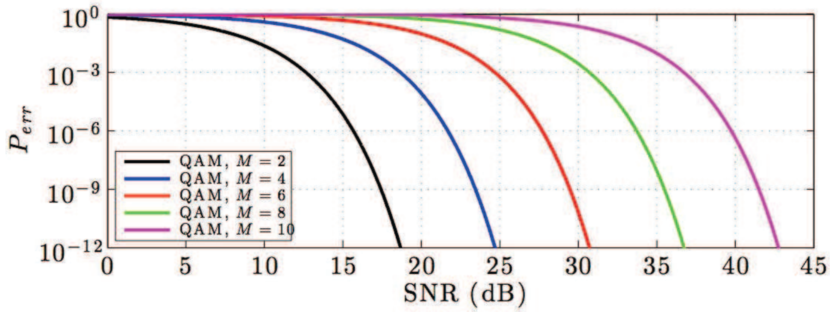
Consider a simplified ADSL transmission scheme with 8 sub-channels of equal width, CH_0 to CH_7; assume that the power constraint is the same for all sub-channels. Each sub-channel CH_i is centered at $\omega_i = \frac{i\pi}{N}$. Only on the sub-channels CH_2 to CH_7 are used for transmission.

QAM signaling is used on each of the allowed sub-channels, and the maximum achievable SNR's for each sub-channel are shown graphically here:



(a) Indicate the sub-channels with the lowest and highest throughput.

(b) Consider the following SNR curves for QAM signaling:



Based on the sub-channels SNR's shown in the first figure, determine the maximum throughput for channels CH_3, CH_4 and CH_7 when the maximum accepted probability of error for any sub-channel is $P_{err} = 10^{-6}$ and the sampling frequency of the system is $F_s = 2$ MHz.

Exercise 5. Bandwidth constraint

Consider the problem of designing a data communication system over an analog channel with a given bandwidth constraint:

(a) Assume the usable bandwidth extends from $F_{min} = 250$ MHz to $F_{max} = 500$ MHz. To meet the bandwidth constraint, the signal is upsampled by a factor $K = 4$ and interpolated at $F_s = 1$ GHz before D/A conversion. Determine the Baud rate (in symbols/s) and the throughput (in bits/s), assuming the alphabet $\mathscr{A}$ has 32 symbols and all symbols are equiprobable.

(b) Consider now a usable bandwidth extending from $F_{min} = 400$ MHz to $F_{max} = 600$ MHz. Choose among the possibilities below the combination of sampling frequency F_s and upsampling factor K that allows meeting the given bandwidth constraint in the analog domain:

   (a) $F_s = 1$ GHz, $K = 5$

   (b) $F_s = 2.4$ GHz, $K = 12$

   (c) $F_s = 1.5$ GHz, $K = 10$

   (d) $F_s = 1.5$ GHz, $K = 5$

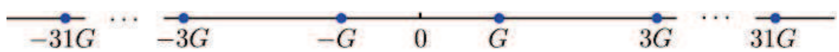
   (e) $F_s = 1.9$ GHz, $K = 9$

Exercise 6. Power constraint

Consider an analog channel whose power constraint results in a maximum achievable SNR of 30 dB. If the channel's bandwidth is 3 kHz, what is the maximum throughput R (in bits/s) that can be achieved by QAM signaling using a square constellation if we can accept a probability of error of 10^{-6}?

Exercise 7. Gain and probability of error

Consider a 32-PAM signaling scheme using the symbols shown in this diagram:



At the receiver, after demodulation, the symbols are affected by additive noise whose amplitude is uniformly distributed over the interval $[-100, 100]$.

Find the minimum value of G for which the error probability does not exceed 10^{-2}.
