Ball and Beam Control System

Introduction

In this exercise you will simulate an unstable system.

Consider the Ball and Beam system which is shown in Figure 1 (Appendix A), the purpose of the design is to control the position of the ball along the track by manipulating the torque of DC motor. To achieve this goal, the system will be examined and analysed with different steps of modelling, simulation and control. Finally the whole system will be controlled.

In this step, we will model the system. Assume that the control input is motor torque and the output is the position of the ball on the beam.

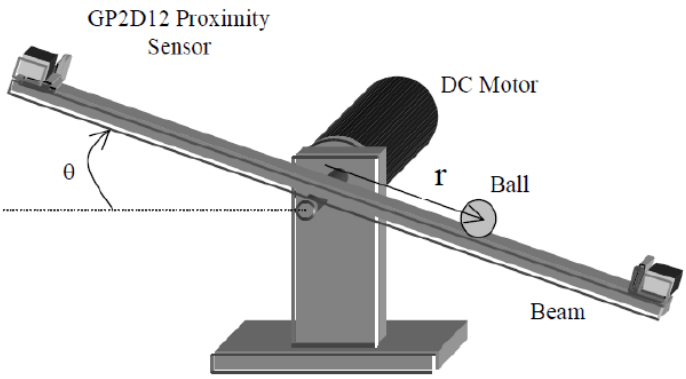


Figure 1-Ball and Beam Apparatus

Task 1: Simulate the Ball and Beam system

a) Where is the equilibrium point of the system?
b) The movement of this system can be expressed by how many independent variables? After selecting a coordinate system, specify these variables.
c) Calculate the Kinetic and potential forces involved in the whole set.
d) Find the dynamic equations of the system using the Lagrange method. (Appendix B)
e) Determine the state space equations of the system and linearize it around the equilibrium point. ($x_1 = r, x_2 = \dot{r}, x_3 = \theta, x_4 = \dot{\theta}$)
f) Simulate the system using nonlinear state-space equations and plot its step response (input is motor torque). **Analyze** the result and then apply sinusoidal torque input to the system so that the motion of the ball remains within the length of the beam.

m	mass of the ball	0.11 kg
R	radius of the ball	0.015 m
M	Mass of the beam	1 kg
g	gravitational acceleration	9.8 m/s^2
L	length of the beam	1.0 m
J_R	ball's moment of inertia	1e-5 kgm^2
J	beam's moment of inertia	2e-3
r	ball position coordinate	
θ	beam angle coordinate	

Task 2: Implemented controller for the simulated model

a) For the linearized model of the Ball and Beam system which is obtained in the Task 1:
   a. Define the matrix transfer function of the system, then calculate the zero input response of the system for the initial condition $x_0^T = [0.2 \;\; 0.05 \;\; 1 \;\; 0.01]$
   b. Simulate the system for the above initial condition and then check the correctness of your answer in the previous section by observing the state response. (For the first second)

b) Check the controllability and observability of the system:
   a. Investigate the controllability of the system. Identify uncontrollable mode, if any.
   b. Investigate the observability of the system for different measurement condition
      i. Outputs r, θ
      ii. Outputs $\dot{r}, \dot{\theta}$
      iii. Outputs r
      iv. Outputs θ
      v. Outputs $r, \dot{\theta}$
      vi. Outputs $\dot{r}, \theta$

      Identify the unobservable mode, if any.

c) In the first design step, we define a performance criterion for the system. This criterion can be a combination of physical limitation and system behaviour (including the transient response, disturbances, noise, uncertainty, etc).

   In the Catalogue of the experimental apparatus of the Ball and Beam system, the maximum motor torque $|\tau| \le 1$ N.m and the maximum angle to the horizon $|\theta| \le 80°$ are stated.

   In discussing in the behaviour of the system, we can refer to the smooth movement of the ball and the beam. (Smoothness)

   Setting time for the r and θ states in changing position are about 3 seconds and overshoot is about 20 %.

   To start the design, move the poles of the system around the point (-7, -7, 0.5±0.5j) and with the assumption of the initial condition $x_0^T = [0.5 \;\; 0.2 \;\; 1 \;\; 0.3]$, simulate the system and analyze the system behaviour according to the performance criteria. Also compare the results of the nonlinear and linear models.

   Then, in order to achieve the performance criteria, move the poles to the appropriate points with purposeful trial and error, and perform the simulations again base on the mentioned cases, and compare the results with the previous case.

d) We want to design an optimal controller that, in addition to stabilizing the system, also meets the desired performance criteria. Assuming the following performance index:

$$J = \int_0^\infty (\Delta x^T Q \Delta x + \Delta u^T R \Delta u) dt.$$

First select the R and Q matrices to use normalized variables in the objective function. Then change the weight matrices with purposeful trial and error in order to achieve the desired performance criteria. Note that these matrices can be non-diagonal. Repeat the simulations based on the items mentioned in the previous paragraph.

e) Make accurate comparisons between the best results in sections c and d. For this purpose, compile a table in which the 2 norm and infinity norm of the error signal, the 2 norm and infinity norm of the control signal, the setting time, the overshoot, and the amount of calculation are considered. (only for the nonlinear model) In this section you can use the norm command in Matlab.
In general, which of the two control methods used in this project do you think is better?

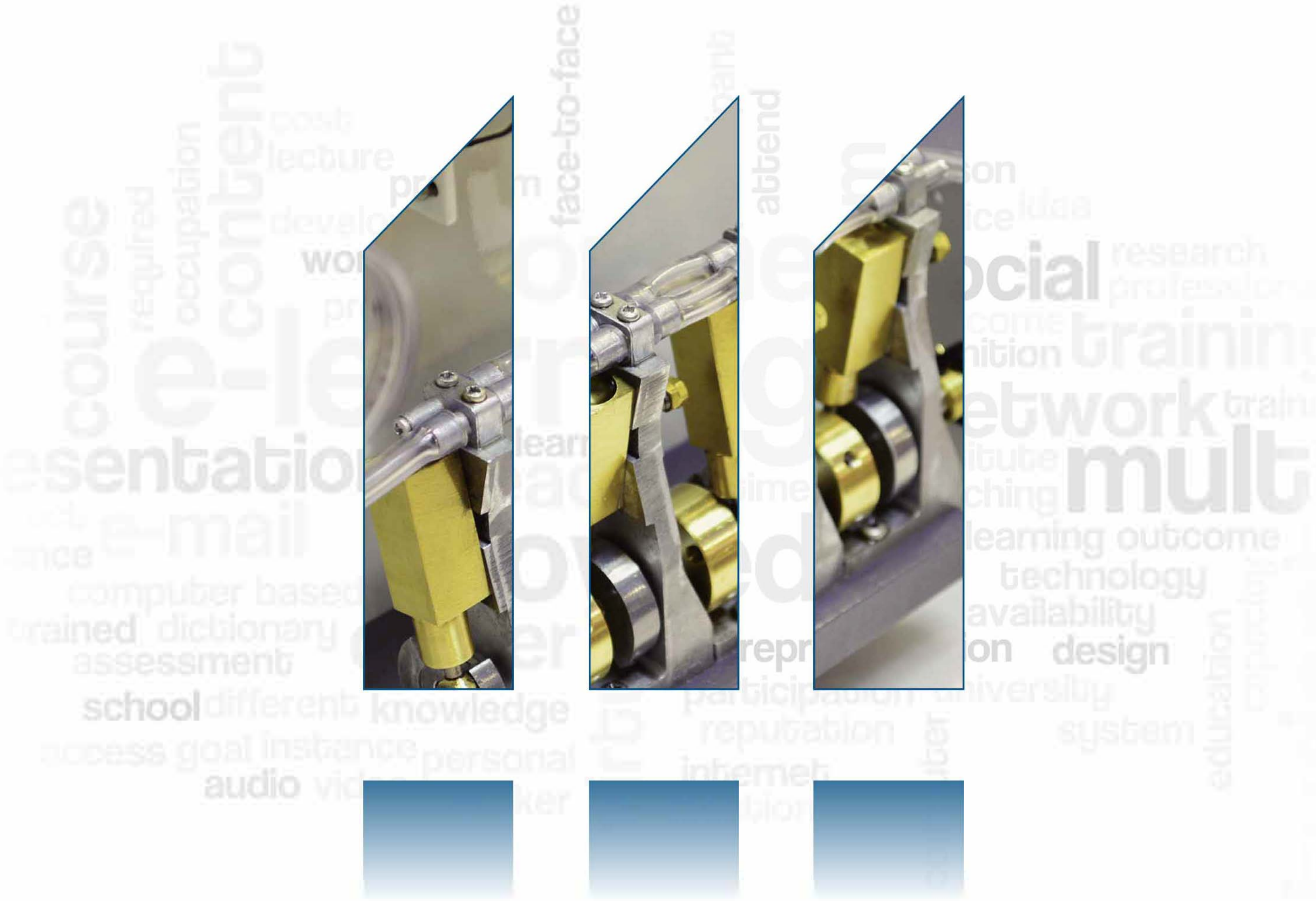
f) In practice, due to the measurement error, the size of the length and the mass of beam have a small error, which has the greatest effect on the moment of inertia of the beam. So assuming an uncertainty of 15% in the moment of inertia of the beam, compare the simulation results for the optimal states and the pole placement. (your own suggested points and do this for linear and nonlinear models)

Report

Write the report based on your observation. Each figure or plot must have a caption and title. Don't forget to comment on your code.

Appendix A

In this appendix you will find information on Ball and Beam apparatus.



Control Engineering

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CE106 Ball and Beam Apparatus

Compact, self-contained, bench-mounting apparatus to study basic and advanced principles of control, including control of naturally unstable systems

- Naturally unstable mechanical control system
- Self-contained, compact and bench-mounting unit
- Ideal for classroom demonstrations and student project work
- Highly visual apparatus, with moving ball and front panel mimic diagram of the process – students can clearly see what they are controlling
- All inputs and outputs buffered for connection to TecQuipment's optional controllers or other suitable controllers
- For basic and advanced experiments with angle, velocity and position control
- Mimics real control problems in unstable systems, such as missile or rocket take-off

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CE106 Ball and Beam Apparatus

Description

The Ball and Beam Apparatus shows the control problems of unstable systems, for example a rocket or missile during launch, which needs active control to prevent the missile going unstable and toppling over.

The apparatus has a steel ball which is free to roll on two parallel tensioned wires. The wires are on a beam that pivots at its centre. A servo motor controls the beam angle and sensors measure the beam angle and ball position. The basic control problem is to vary the beam angle to control the ball position. The system is a double integrator, so it is naturally unstable. It needs active feedback control using phase-advance methods.

The CE106 comes with a user guide which includes full details of how to use the equipment and typical experiments.

It also includes a set of cables and connectors for connection to other equipment. All control connections work with 0 to 10 VDC signals.

Note: You must use the CE106 with TecQuipment's optional CE120 Controller, the optional CE122 Digital Interface or other suitable controllers with 10 V inputs and outputs. Details of the CE120 and CE122 are on separate datasheets.

The CE106 includes a set of cables and connectors for connection to other equipment.

All control connections work with 0 to 10 VDC signals.

Essential Base Unit

- Controller (CE120) – A controller with analogue and digital controls and instruments

  or
- Digital Interface (CE122) – An interface which connects between most products in the Control Engineering range and a suitable computer (not included)

  or
- Other suitable controller with 10 V inputs and outputs

Both the CE120 and the CE122 include TecQuipment's CE2000 Control Software with editable, pre-made control experiments for use with the CE106.

Standard Features

- Supplied with comprehensive user guide
- Five-year warranty
- Made in accordance with the latest European Union directives

Experiments

- Measurement of system dynamics by transient and closed-loop methods
- Design of analogue phase-advance compensators
- Design of state reconstructors to obtain estimates of ball velocity and position

The flexible design of the equipment allows the user to develop many other analysis and control exercises to suit their needs. It is good for extended or advanced control experiments, and is ideal for student project work.

Essential Services

Electrical supply:
240/110 VAC, 1 A, 50/60 Hz, with earth

Other voltages and frequencies available to special order

Bench space needed:
1.5 m x 750 mm

Operating Conditions

Operating environment:
Laboratory

Storage temperature range:
–25°C to +55°C (when packed for transport)

Operating temperature range:
+5°C to +40°C

Operating relative humidity range:
80% at temperatures < 31°C decreasing linearly to 50% at 40°C

Sound Levels

Less than 70 dB(A)

Specifications

Nett dimensions and weight:
1070 mm x 330 x 420 mm, 18 kg

Packed dimension and weight:
0.64 m^3, 52 kg (approx – packed for export)

Input: 0 to 10 VDC
- Motor

Outputs: 0 to +/– 10 VDC
- Ball position
- Beam angle

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Appendix B

In this appendix you will find information on Lagrange equations.

Lagrange equations

The total kinetic energy E_c of a mechanical system is equal to the sum of all the kinetic energies of translation and rotation of its parts. In general, it is a function that can depend on all the generalized coordinates and velocities and time:

$$E_\mathrm{c}(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n, t)$$

The motion of a system is usually not completely free, but subject to some constraints; there are forces of constraint resulting from those constraints. For example, in a car moving on a road, the normal force exerted by the road on the tires is the force of constraint that ensures that the trajectory of the car stays over the surface of the road. The static friction on a wheel with traction is also a force of constraint, which ensures that the wheel rotates without sliding on the surface. The fact that the car remains in contact with the road surface as it moves and the friction on the tires prevents them from moving sideways, reduces the three coordinates of position to a single degree of freedom: the displacement along the road. When the wheels rotate without skidding, the angular velocity of the wheels depends on the speed of the car on the road. That dependence also implies a dependence between the angle of rotation of the wheels and the displacement of the car along the road; thus, only one of those two variables is enough to describe the motion of the car and the rotation of the wheels.

Each constraint on a system's motion reduces its number of degrees of freedom. A constraint that can be written as an equation depending on the generalized coordinates of the system is called holonomic. When a system is holonomic, namely all constraints to its motion are holonomic, Newton's second law leads to the following equations (Proof is given in Appendix C):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_\mathrm{c}}{\partial \dot{q}_j}\right) - \frac{\partial E_\mathrm{c}}{\partial q_j} = Q_j \qquad j = 1, \dots n,$$

where Q_j is the j component of the generalized force, defined as:

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j},$$

where all forces $\vec{F}_i$ (internal or external) are summed and $\vec{r}_i$ is the position of the point force is $\vec{F}_i$ applied. However, some of the forces do not contribute to the generalized force Q_j. For example, normal forces and static friction can be neglected because they are applied in a fixed point $\vec{r}_i$ and therefore, $\vec{F}_i \cdot d\vec{r}_i$ =0. The tension in a string with constant length can also be ignored because it acts in opposite directions at both ends of the string and the sum of $\vec{F}_i \cdot d\vec{r}_i$ at both ends gives zero.

Appendix C

In this appendix you will find information on proof of Lagrange equations.

Lagrange equations

This appendix shows the derivation of Lagrange equations from Newton's second law. Let us consider a system formed by m rigid bodies whose center of mass position vectors are: $\vec{r}_1$, $\vec{r}_2$, ..., $\vec{r}_m$. Thus, to determine the configuration of the whole system at a given instant, it is necessary to know the values of $3\,m$ coordinates, which could be a combination of distances and angles.

The system is called holonomic, if some of those $3\,m$ are related, making it possible to reduce the number of independent coordinates to n generalized coordinates ($n < 3\,m$):

$$q_1(t), q_2(t), \ldots, q_n(t)$$

Each position vector $\vec{r}_i$ can depend on some of the generalized coordinates and on time:

$$\vec{r}_i(q_1, q_2, \ldots, q_n, t)$$

and the velocity vector of the i th body is then

$$\vec{v}_i = \frac{\mathrm{d}\,\vec{r}_i}{\mathrm{d}\,t} = \frac{\partial\vec{r}_i}{\partial t} + \sum_{k=1}^{n} \frac{\partial\vec{r}_i}{\partial q_k}\,\dot{q}_k$$

Therefore, $\vec{v}_i$ depends also on time, the generalized coordinates, and the generalized velocities $\dot{q}_i$:

$$\vec{v}_i(q_1, q_2, \ldots, q_n, \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n, t)$$

and the partial derivatives of $\vec{v}_i$ are then

$$\frac{\partial\vec{v}_i}{\partial\dot{q}_j} = \frac{\partial\vec{r}_i}{\partial q_j} \qquad\qquad \frac{\partial\vec{v}_i}{\partial q_j} = \frac{\partial\vec{r}_i}{\partial q_j\partial t} + \sum_{k=1}^{n} \frac{\partial^2\vec{r}_i}{\partial q_j\partial q_k}\,\dot{q}_k \tag{C.1}$$

The acceleration vector of the i th rigid body is:

$$\vec{a}_i = \frac{\mathrm{d}\,\vec{v}_i}{\mathrm{d}\,t} \tag{C.2}$$

If at a given instant the value of each coordinate q_j is changed to $q_j + \delta q_j$, where the virtual displacements δq_j are consistent with the constraints, each position vector will undergo a displacement:

$$\delta\,\vec{r}_i = \sum_{j=1}^{n} \frac{\partial\vec{r}_i}{\partial q_j}\,\delta q_j \tag{C.3}$$

Taking the dot product of each side of this equation times the sides of equation C.2, we obtain

$$\vec{a}_i \cdot \delta\,\vec{r}_i = \sum_{j=1}^{n} \frac{\mathrm{d}\,\vec{v}_i}{\mathrm{d}\,t} \cdot \frac{\partial\vec{r}_i}{\partial q_j}\,\delta q_j \tag{C.4}$$

since the derivative of the product $\vec{v}_i \cdot \partial\vec{r}_i/\partial q_j$ is,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\vec{v}_i \cdot \frac{\partial\vec{r}_i}{\partial q_j}\right) &= \frac{\mathrm{d}\,\vec{v}_i}{\mathrm{d}\,t} \cdot \frac{\partial\vec{r}_i}{\partial q_j} + \vec{v}_i \cdot \frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{\partial\vec{r}_i}{\partial q_j}\right) \\ &= \frac{\mathrm{d}\,\vec{v}_i}{\mathrm{d}\,t} \cdot \frac{\partial\vec{r}_i}{\partial q_j} + \vec{v}_i \cdot \left(\frac{\partial\vec{r}_i}{\partial q_j\partial t} + \sum_{k=1}^{n} \frac{\partial^2\vec{r}_i}{\partial q_j\partial q_k}\,\dot{q}_k\right)\end{aligned}$$

According to equations C.1, the derivative $\partial\vec{r}_i/\partial q_j$ and the term inside the parenthesis on the right-hand side of the equation are the partial derivatives of $\vec{v}_i$, with respect to $\dot{q}_j$ and q_j . Therefore, the following result is obtained

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\vec{v}_i \cdot \frac{\partial\vec{v}_i}{\partial\dot{q}_j}\right) = \frac{\mathrm{d}\,\vec{v}_i}{\mathrm{d}\,t} \cdot \frac{\partial\vec{r}_i}{\partial q_j} + \vec{v}_i \cdot \frac{\partial\vec{v}_i}{\partial q_j}$$

Equation C.4 can then be written as,

$$\vec{a}_i \cdot \delta\,\vec{r}_i = \sum_{j=1}^{n} \left[\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\vec{v}_i \cdot \frac{\partial\vec{v}_i}{\partial\dot{q}_j}\right) - \vec{v}_i \cdot \frac{\partial\vec{v}_i}{\partial q_j}\right]\delta q_j \tag{C.5}$$

Furthermore, notice that the partial derivatives of v_i^2 with respect to the generalized coordinates and velocities are:

$$\frac{\partial v_i^2}{\partial q_j} = \frac{\partial(\vec{v}_i \cdot \vec{v}_i)}{\partial q_j} = 2\,\vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial q_j}$$
$$\frac{\partial v_i^2}{\partial \dot{q}_j} = \frac{\partial(\vec{v}_i \cdot \vec{v}_i)}{\partial \dot{q}_j} = 2\,\vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j}$$

substituting these two expressions in equation C.5 and multiplying both sides of the equation by the mass m_i of the i th body we get,

$$\begin{aligned} m_i\,\vec{a}_i \cdot \delta\,\vec{r}_i &= \sum_{j=1}^{n} \left[\frac{\mathrm{d}}{\mathrm{d}\,t} \left(\frac{m_i}{2} \frac{\partial v_i^2}{\partial \dot{q}_j} \right) - \frac{m_i}{2} \frac{\partial v_i^2}{\partial q_j} \right] \delta q_j \\ &= \sum_{j=1}^{n} \left[\frac{\mathrm{d}}{\mathrm{d}\,t} \left(\frac{\partial E_{\mathrm{c}i}}{\partial \dot{q}_j} \right) - \frac{\partial E_{\mathrm{c}i}}{\partial q_j} \right] \delta q_j \end{aligned}$$

where $E_{\mathrm{c}i}$ is the kinetic energy of the i th body. According to Newton's second law, $m_i\,\vec{a}_i$is the total force acting on the ith body. Using the expression C.3 and summing over all the i th body, we obtain

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j}\,\delta q_j = \sum_{j=1}^{n} \left[\frac{\mathrm{d}}{\mathrm{d}\,t} \left(\frac{\partial E_{\mathrm{c}}}{\partial \dot{q}_j} \right) - \frac{\partial E_{\mathrm{c}}}{\partial q_j} \right] \delta q_j$$

which leads to Lagrange equations:

$$\frac{\mathrm{d}}{\mathrm{d}\,t} \left(\frac{\partial E_{\mathrm{c}}}{\partial \dot{q}_j} \right) - \frac{\partial E_{\mathrm{c}}}{\partial q_j} = Q_j \qquad j = 1, \ldots n \tag{C.6}$$

where E_{c} is the total kinetic energy of the system and the generalized force Q_j has been defined as

$$Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} \tag{C.7}$$