# Coupled Drives

## Introduction

In this exercise you will simulate a system. Campus students do the same but with measurements on a real system.

The Coupled Drives Apparatus has been designed to enable practical investigations into the recurring industrial problem of controlling the tension and sped of material in a continuous process. For example textile fibre manufacturing process, yarn is wound from one spool to another at high speed. Between the two spools the yarn is processed in some way which may require the yarn speed and tension to be controlled within defined limits. A layout of the laboratory equipment is shown in Figure 1.

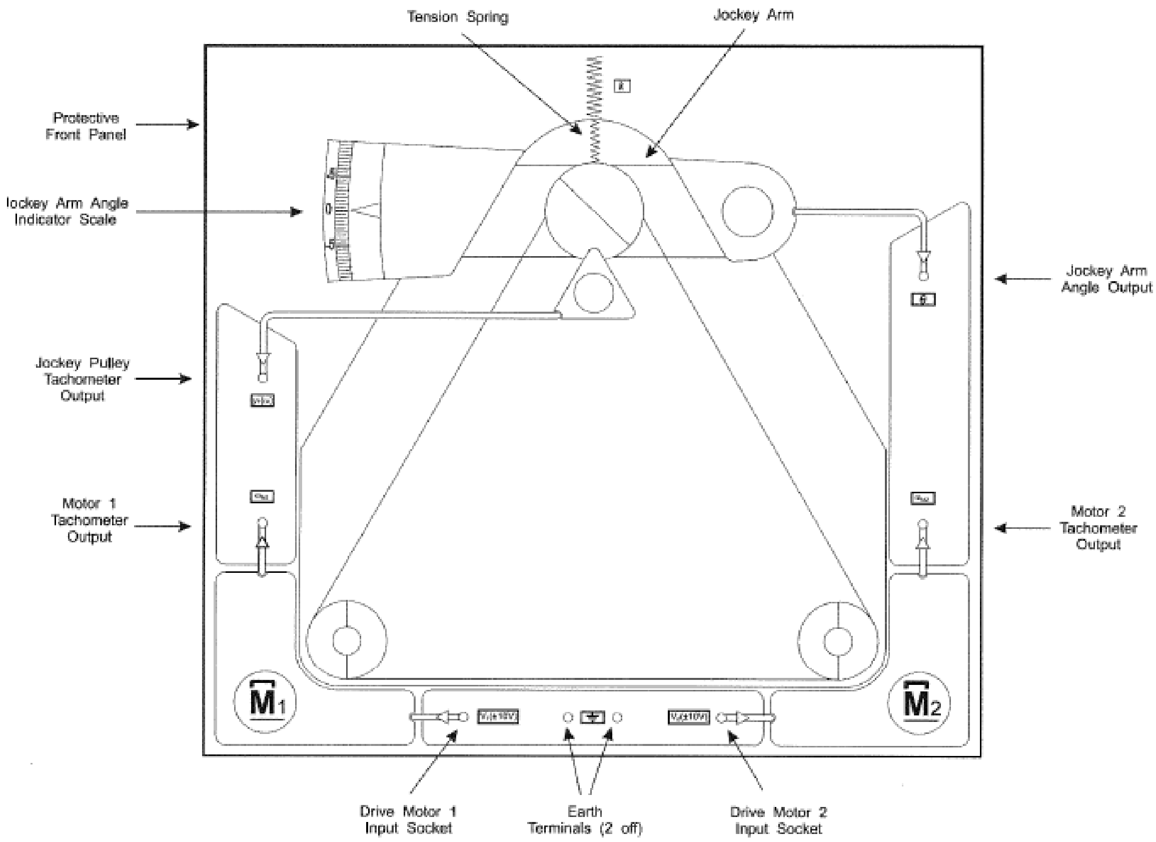



**Figure 1: The laboratory equipment.**

The Coupled drives can be modelled as in Appendix A. It is a system with two input signals, inputs $v_1$ and $v_2$ to the motors $M_1$ and $M_2$, and two output signals, tachometer $y_1$ and tension transducer $y_2$. The system is coupled and thus, we would like to decouple the system. That is described in Appendix B.

Your task will be to study the behavior of the system without and with decoupling the system. What we want to accomplish is to be able to control speed and tension separately. That is, we want to, for example, change the speed without affecting the tension.

## The simulation model

In order to understand the simulation model you are encouraged to study Appendix A. The simulation model is written in Simulink and you can find it in the file Coupled_drives.slx.

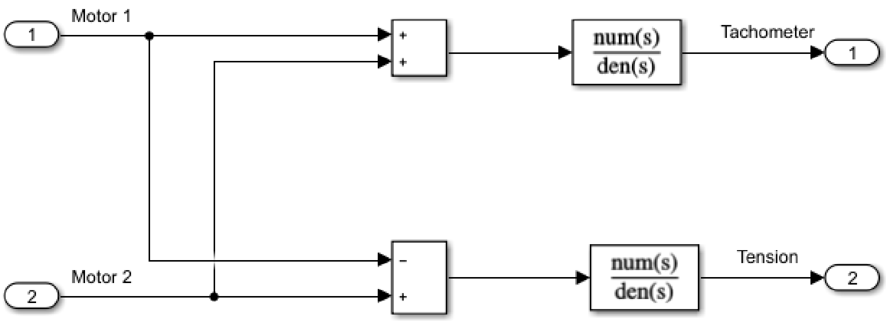



Figure 2: Description of the simulation model

Input signals are $v_1$ and $v_2$ to Motor 1, $M_1$ and Motor 2, $M_2$. The outputs are the signals from the tachometer, $y_1$, and the tension transducer, $y_2$.

## Task 1: Study the step response without decoupling

Replace the output with a Scope. You can have one scope with two input or two separate scopes. Let one of the motor control signals have a constant value (e.g. 1) and one have a pulse generator as input. The Pulse generator shall have the following settings: Amplitude=1, Period= 50 sec, Pulse width = 50 %.

## Task 2: Study the step response for a decoupled system

Make a decoupling according to Appendix B. Then study step responses in the same way as in task 1. In this task you need to study two different cases: (i) The setpoint for the speed is kept constant and the setpoint of the tension is a pulsed signal and (ii) the setpoint of the tension is kept constant and the setpoint of the speed is a pulsed signal.

## Task 3: Study the decoupled system with feedback control

Add two PID-controllers (or any other controller you would like to use), one for the speed control and one for the tension control. As before let one setpoint be constant and the other pulsed. Elaborate with different control parameters and try to find a suitable setting.

## Report

Write a report based on your observations. Keep in mind that we want to accomplish is to be able to control speed and tension separately.

# Appendix A

This appendix describes how to do physical modelling of the system. You will not need to do the modelling, but reading the appendix will increase the understanding of physical modelling in general and the system in particular.

## SECTION 2.0 CONTROL THEORY

The behaviour of the drives apparatus corresponds closely to the dynamical responses found in many material handling systems. Moreover, the dynamical behaviour can be rather complex. For this reason the first part (Section 2.1) considers in detail the dynamical modelling of the coupled drives. It is possible to step over this at a first reading and go to Section 2.3 where a simplified model is discussed.

### *2.1 Modelling*

The coupled electric drives can be represented for modelling purposes by the schematic of Figure 8.

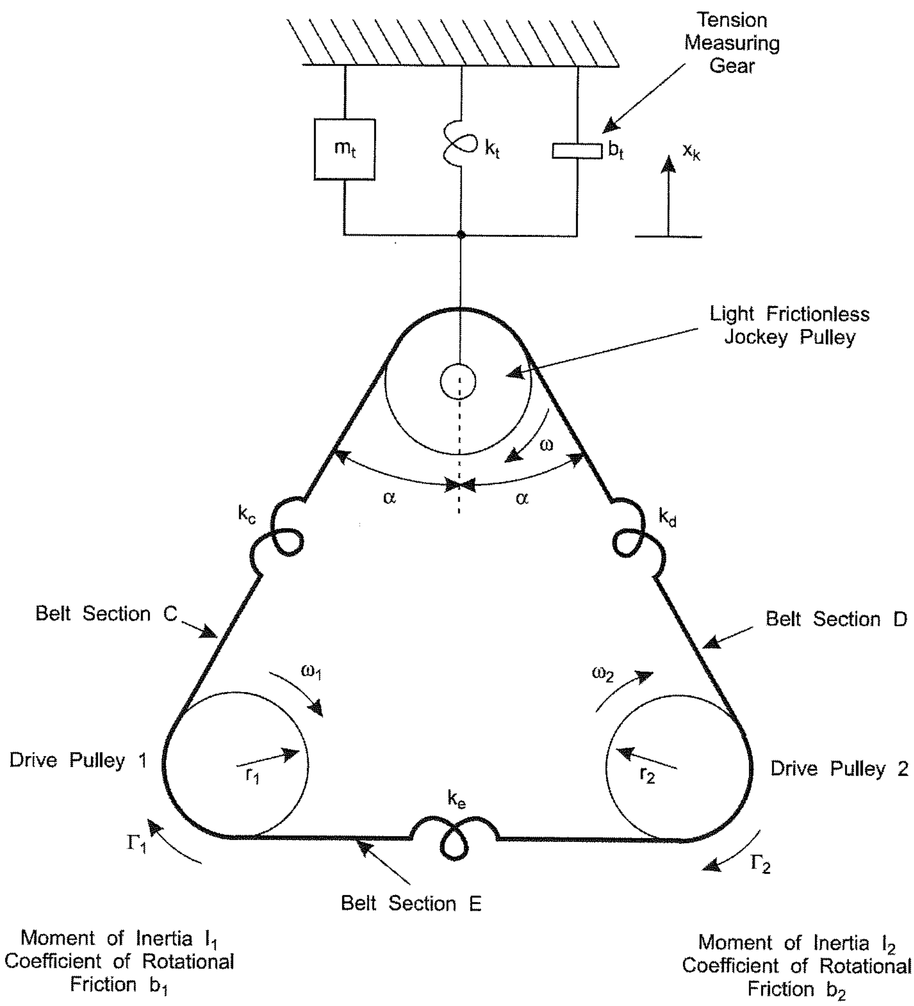



*Figure 8*

In this figure the jockey arm and tension measuring equipment are represented by a mass, spring, dissipater combination. For simplicity, both sections of the flexible belt are assumed to be at an angle $\alpha$ to the line of motion of the pulley.

By the same token, the angular deflection of the jockey arm is assumed to be small, such that the pulley can be assumed to translate a distance $x_k$ as indicated on the figure.

The sections of belt are assumed to be linear springs of stiffness $k_c$, $k_d$ and $k_e$, while the drive pulleys 1 and 2 are assumed to have moments of inertia and rotational friction coefficients $I_1$, $b_1$ and $I_2$, $b_2$ respectively. Transitional velocities of the belt sections are denoted by v, with an appropriate subscript. Likewise, F denotes force as usual.

To obtain a dynamical model of the system, we begin by considering each sub-system in the free body diagram of Figure 9.

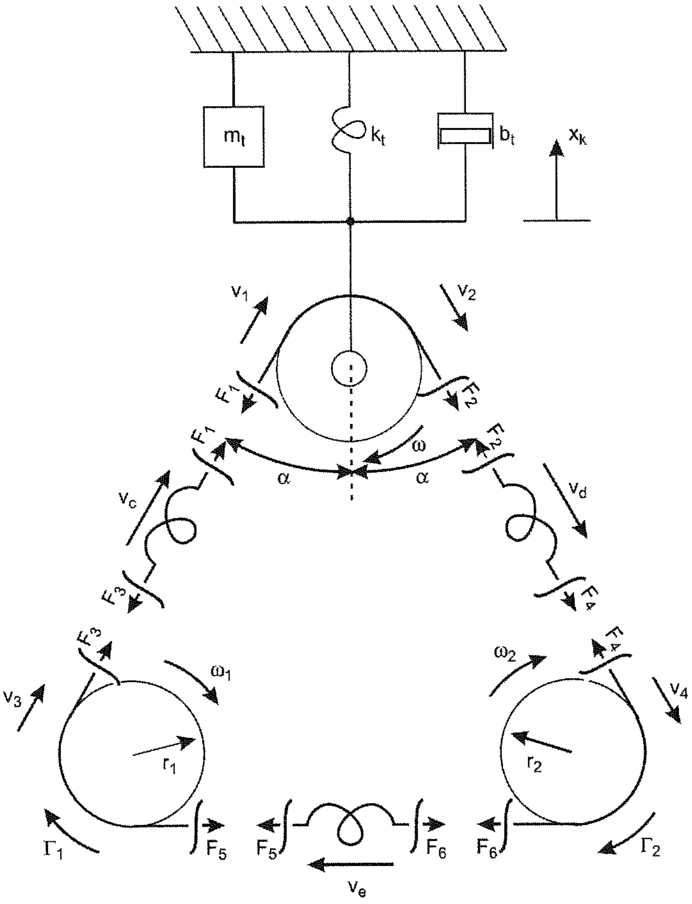



*Figure 9*

### The Jockey Pulley Assembly

The pulley is assumed to be light and rotates without friction, such that,

$$F_1 = F_2 = F$$

Resolving forces vertically gives,

$$F_k = 2F\cos\alpha \tag{4}$$

but for conservation of power,

$$\dot{x}_k F_k = F(v_1 - v_2) \tag{5a}$$

Hence,

$$\dot{x}_k(2\cos\alpha) = v_1 - v_2 \tag{5b}$$

The force $F_k$ is given by a force balance on the tension measuring assembly,

$$F_k = \dot{p} + k_t x_k + b_t \dot{x}_k \tag{5c}$$

where p is the momentum of mass $m_t$.

### The Drive Pulleys 1 and 2

A torque balance on the drive pulleys gives,

$$\Gamma_1 + F_3 r_1 - F_5 r_1 = \dot{h}_1 + b_1 \omega_1 \tag{6}$$

$$\Gamma_2 + F_6 r_2 - F_4 r_2 = \dot{h}_2 + b_2 \omega_2 \tag{7}$$

where $h_1$ and $h_2$ are the drive pulley/motor angular momenta.

$$h_1 = I_1 \omega_1, \quad h_2 = I_2 \omega_2 \tag{8}$$

In addition, the angular velocities $\omega_1$ and $\omega_2$ are given by,

$$v_3 = \omega_1 \Gamma_1, \quad v_4 = \omega_2 \Gamma_2 \tag{9}$$

### The Belt Sections

A force balance on each belt section gives,

$$F = k_c x_c = k_d x_d \tag{10a}$$

$$F' = k_e x_e \tag{10b}$$

where,

$$F' = F_5 = F_6$$

and

$$F = F_1 = F_2 = F_3 = F_4$$

and $x_c$, $x_d$ and $x_e$ are the respective extensions of the belt section c, d, and e.

Now the state space description of the system can be determined by eliminating variables in Equations 4 to 10b.

If the system states are taken as $h_1$, $h_2$, $x_c$, $x_e$, $x_k$ and p, then we have,

$$\dot{h}_1 = \left[\frac{-b_1}{I_1}\right] h_1 + r_1 k_c x_c - r_1 k_e x_e + \Gamma_1 \tag{11a}$$

$$\dot{h}_2 = \left[\frac{-b_2}{I_2}\right] h_2 + r_2 k_c x_c - r_2 k_e x_e + \Gamma_2 \tag{11b}$$

$$\dot{x}_c = \left[1 + \frac{k_c}{k_d}\right]^{-1} \left[-h_1 \frac{r_1}{I_1} + h_2 \frac{r_2}{I_2} + p \frac{2\cos\alpha}{m_t}\right] \tag{11c}$$

$$\dot{x}_e = \frac{r_1}{I_1} h_1 - \frac{r_2}{I_2} h_2 \tag{11d}$$

$$\dot{x}_k = \frac{p}{m_t} \tag{11e}$$

$$\dot{p} = 2(\cos\alpha)k_c x_c - k_t x_k - \frac{b_t}{m_t}p \tag{11f}$$

The state equation set (11a–11f), provides the source from which all relations and transfer functions concerning the coupled drives can be found. The equations can be simplified by using known relationships between the physical constants in the coupled drives apparatus. To be specific, the drive motors are the same; it is, therefore, reasonable to assume that,

$$I_1 = I_2 = I \tag{12a}$$

$$b_1 = b_2 = b \tag{12b}$$

By the same token, the pulleys all have the same radius and the belt sections have the same length. Thus,

$$r_1 = r_2 = r \tag{13a}$$

$$k_c = k_d = k_e = k \tag{13b}$$

Using these relations in the state equations gives, by addition of the angular momentum equations,

$$\dot{h}_1 + \dot{h}_2 = -\frac{b}{I}(h_1 + h_2) + \Gamma_1 + \Gamma_2 \tag{14}$$

This provides the first transfer function relation,

$$\omega_1(s) + \omega_2(s) = \frac{1}{sI + b}[\Gamma_1(s) + \Gamma_2(s)] \tag{15}$$

where s is the complex frequency variable.

By manipulation, the velocity $v_c$ of belt section 2 is obtained in terms of the drive pulley velocities as,

$$v_c(s) = \left[\frac{s^2 m_t + sb_t + k_t}{\left(1 + \frac{k_c}{k_d}\right)(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k_c}\right] r[\omega_2(s) - \omega_2(s)]$$

$$v_c(s) = \left[\frac{s^2 m_t + sb_t + k_t}{2(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k}\right] r[\omega_2(s) - \omega_1(s)] \tag{16}$$

The angular velocity of the drive pulleys can similarly be expressed in terms of the input torques. Thus,

$$\omega_1(s) = \left[\frac{Is^2 + bs + k^1(s)}{(Is + b)(Is^2 + bs + 2k^1(s))}\right]\Gamma_1 s + \left[\frac{k^1(s)}{(Is + b)(Is^2 + bs + 2k^1(s))}\right]\Gamma_2 s \tag{17}$$

$$\omega_2(s) = \left[\frac{Is^2 + bs + k^1(s)}{(Is + b)(Is^2 + bs + 2k^1(s))}\right]\Gamma_2 s + \left[\frac{k^1(s)}{(Is + b)(Is^2 + bs + 2k^1(s))}\right]\Gamma_1 s \tag{18}$$

Where,

$$k^1(s) = r_2(k_c G_1(s) + k_e) = r^2 k(G_1(s) + 1) \tag{19}$$

and, from Equation 16, $G_1(s)$ is defined as,

$$G_1(s) = \frac{s^2 m_t + sb_t + k_t}{2(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k} \tag{20}$$

Subtracting Equation 17 from Equation 18 yields,

$$\omega_2(s) - \omega_1(s) = \frac{s}{Is^2 + bs + 2k^1(s)}[\Gamma_2(s) - \Gamma_1(s)] \tag{21a}$$

The angular velocity $\omega$ of the jockey pulley can be expressed as a function of the drive pulley angular velocities, thus,

Consider the tangential velocity $v_1$ at the pulley. This is the sum of two components:

$v_1$ = (translational velocity due to rotation) + (resolved translational velocity of centre of rotation)

The jockey pulley has a velocity in the vertical direction of $\dot{x}_k$, with a component $\dot{x}_k \cos\alpha$ parallel to the belt section (c). Hence if $\hat{v}_1$ is the velocity due to rotation:

$$v_1 = \hat{v}_1 + \dot{x}_k \cos\alpha$$

and similarly

$$v_2 = \hat{v}_2 + \dot{x}_k \cos\alpha$$

Note that the translational velocities due to rotation alone will be equal (ie $\hat{v}_1 = \hat{v}_2$).

By definition then:

$$\omega = \frac{\hat{v}_1}{r} = \frac{v_1 - \dot{x}_k \cos\alpha}{r}$$

Also:

$$v_c = v_1 - v_3$$

Combining these two equations gives:

$$\omega = \frac{v_c + v_3 - \dot{x}_k \cos\alpha}{r} = \omega_1 + \frac{v_c - \dot{x}_k \cos\alpha}{r}$$

Taking Laplace transforms and substituting for $v_c(s)$ from Equation 16 gives:

$$\omega(s) = \omega_1(s) + G_1(s)[\omega_2(s) - \omega_1(s)] - \frac{sx_k(s)\cos\alpha}{r} \tag{21b}$$

Similarly, by writing $\omega$ in terms of $\hat{v}_2$ gives:

$$\omega(s) = \omega_2(s) + G_1(s)(\omega_2(s) - \omega_1(s)) + \frac{sx_k(s)\cos\alpha}{r}$$

Addition of these two expressions for $\omega(s)$ gives:

$$\omega(s) = \tfrac{1}{2}(\omega_1(s) + \omega_2(s))$$

or, alternatively, in terms of the drive torques:

$$\omega(s) = \frac{\Gamma_1(s) + \Gamma_2(s)}{2(sI + b)} \tag{21c}$$

The transfer function relationships for angular velocities in the system (Equations 15, 17, 18, 21a and 21b) are summarised in Figure 10.

Notice the points concerning these transfer function relations:

(i) The term $Is^2 + bs + k^1(s)$ occurs in almost all the transfer function denominators and is the contribution of the drive system dynamics together with the jockey arm dynamics. The latter is embedded in the term $k_1(s)$, which can be written in full by combining Equations 19 and 20. Thus,

$$k^1(s) = kr^2\left(1 + \left[\frac{s^2 m_t + sb_t + k_t}{2(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k}\right]\right) \tag{22}$$

If the tension arm is clamped using the locking bar provided, then $k_1(s)$ reduces to a constant,

$$k^1(s) \rightarrow k^1 = r^2\left[k_e + \frac{k_c}{1 + \frac{k_c}{k_d}}\right] \tag{23a}$$

and, because $k_c = k_d = k_e$, then,

$$k^1 = \frac{3}{2}kr^2 \tag{23b}$$

where the right-hand side is the effective torsional stiffness of the belt.

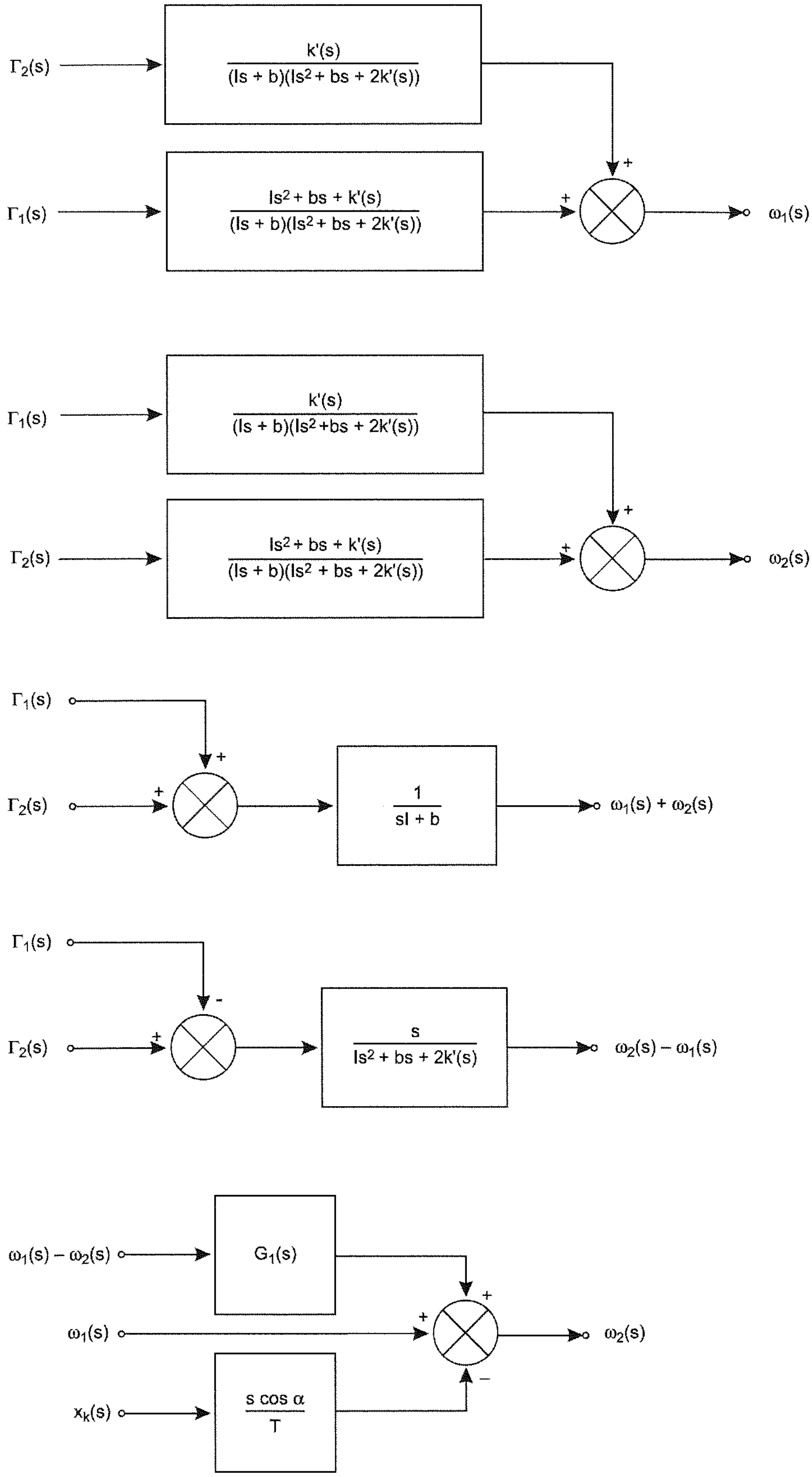
(ii) The sum and difference transfer functions (Equations 15 and 21a) provide the means whereby the dynamical characteristics of the terms $(Is + b)$ and $(Is^2 + bs + 2k^1(s))$ may be separately determined.

The transfer function which relates the displacement of the jockey arm to the input drive torques is obtained from the state space equations (Equations 11a–11f).

$$x_k(s) = \frac{2rk_c\cos\alpha(\Gamma_2(s) - \Gamma_1(s))}{\left[\left(1 + \frac{k_c}{k_d}\right)(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k_c\right][Is^2 + bs + 2k^1(s)]}$$

which simplifies to,

$$x_k(s) = \frac{2rk\cos\alpha(\Gamma_2(s) - \Gamma_1(s))}{[2(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k][Is^2 + bs + 2k^1(s)]} \tag{24}$$




(a)

Equation 17

(b)

Equation 18

(c)

Equation 15

(d)

Equation 21a

(e)

Equation 21b

*Figure 10*

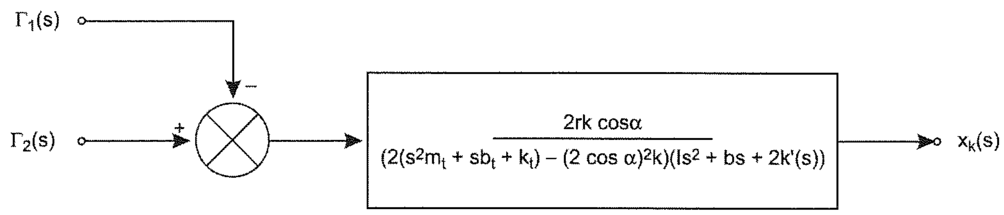



*Figure 11*

This transfer function, which is illustrated in Figure 11, relates the displacement of the jockey arm and hence the belt tension to the **difference** between the drive motor torques.

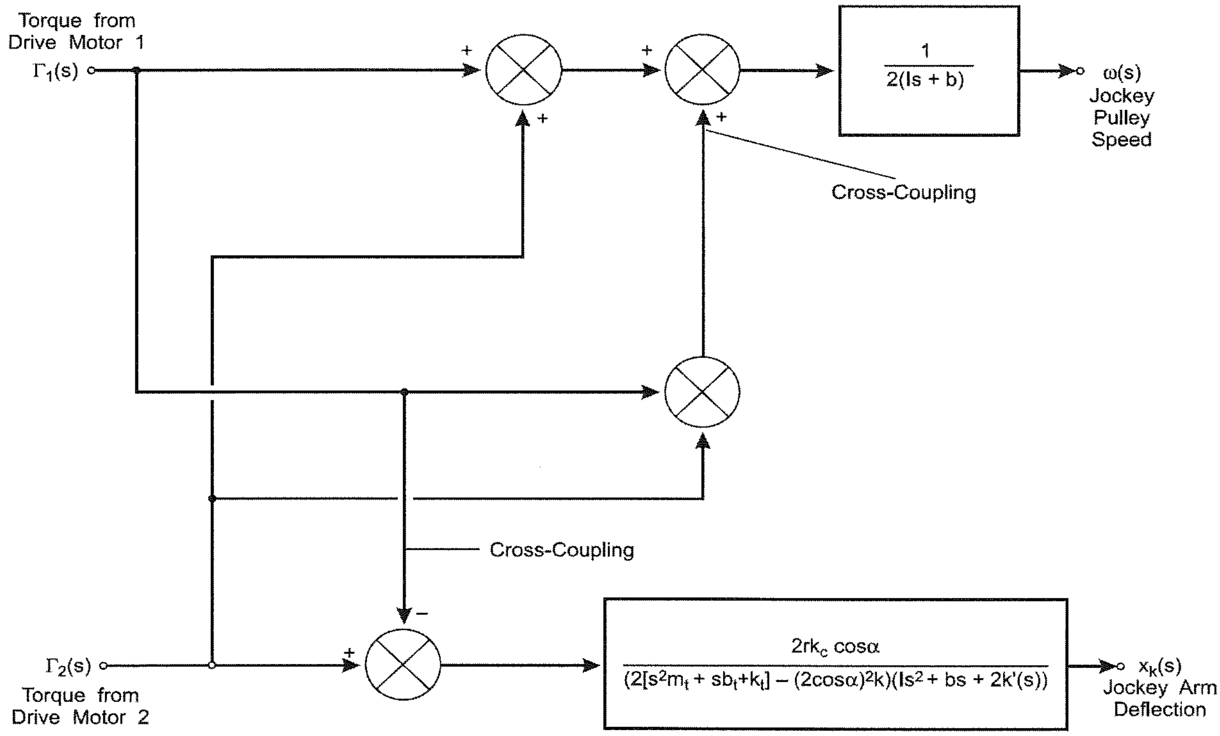



*Figure 12*

One would expect this from physical reasoning and we will exploit the fact later when we consider the independent control to belt tension (as measured by the jockey arm deflection $x_k$) and jockey pulley angular velocity, ω.

The coupling between the outputs ω(s), $x_k(s)$ and the inputs $\Gamma_1(s)$, $\Gamma_2(s)$ is exemplified by a block diagram representation of Equations 21c and 24 in Figure 12.

To summarise, the coupled electric drives apparatus can be represented by the state space equations 11a–11f. The angular velocity of the jockey arm is given by Equation 21b, while the drive motor velocities are related to the drive motor torques by Equations 17 and 18. The deflection of the jockey arm is given by Equation 24, and the input/output block diagram (Figure 12) illustrates the dynamical interaction between the system variables.

## 2.2 *The Overall System Model (Including Actuators and Transducers)*

The block diagram shown in Figure 12 relates the theoretical inputs (the motor torque's $\Gamma_1$ and $\Gamma_2$) to the physical variables ($\omega(s)$ and $x_k(s)$) which form the system outputs.

However, as indicated in Section 1, the apparatus is actuated by the voltages $\upsilon_1$ and $\upsilon_2$ applied to the drive motor amplifiers and the system outputs $\omega$ and $x_k$ are obtained in terms of the jockey arm tachogenerator output $y_1$ and the tension transducer output $y_2$, respectively.

Also from Equation 1, the jockey arm angle $\theta$ and the $x_k$ are related by:

$$\theta = \frac{x_k}{L} \qquad \text{(1 repeated)}$$

Moreover, it is the jockey arm deflection $\theta$ that is measured by the potentiometer which forms the tension transducer output $y_2$.

These signals are related by constants of proportionality, thus:

$$\begin{aligned} \Gamma_1 &= g_a v_1 \\ \Gamma_2 &= g_b v_2 \\ y_1 &= g_1 \omega \\ y_2 &= g_2 \theta \end{aligned} \qquad (25)$$

such that the block diagram can be re drawn, as shown in Figure 13, in terms of electrical input signals, $v_1$, $v_2$ and the measurable outputs.

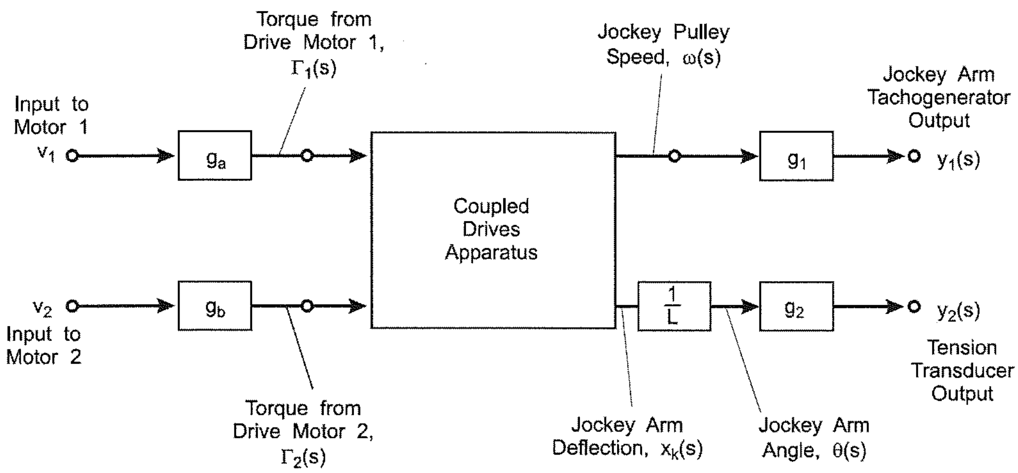



*Figure 13*

The constants $g_a$ and $g_b$ are determined by the motor and amplifier characteristics; while $g_1$ and $g_2$ are functions of the tachogenerator and tension transducer construction.

# Appendix B
In this appendix you will find information on decoupling

## 2.3 Transfer Function Simplification

### Reduction of Interaction

The central problem is applying feedback control to a multi-input/multi-output system is the coupling between system outputs.

For example, we may wish to independently control the value of two output variables ($y_1(s)$ and $y_2(s)$ in the coupled drives system) by manipulating two input variables ($v_1(s)$ and $v_2(s)$ in the coupled drives system).

However, because of the physical nature of the system both inputs influence both outputs such that the design of two **independent** feedback loops is not easy.

This interdependence is illustrated in the case of the coupled drives apparatus by Figure 12, where the cross-coupling terms between the transmission paths,

$$\Gamma_1(s) \rightarrow \omega(s)$$

and,

$$\Gamma_2(s) \rightarrow x_k(s)$$

are indicated.

There are a number of solutions to the problem of interaction between control loops. The most practical procedure is to reconsider the system model and ask:

***"Can combinations of the input be formed and used to substantially reduce the coupling between the outputs?"***

In the specific case of the coupled drives this would involve forming new input variables $u_1(s)$ and $u_2(s)$ from which $v_1(s)$ and $v_2(s)$ are constructed such that the transfer function, $u_1(s) \rightarrow y_1(s)$ and $u_2(s) \rightarrow y_2(s)$ are not coupled.

Such a scheme would give rise to a **DECOUPLING PRE-COMPENSATOR** with inputs $u_1(s)$, $u_2(s)$ and outputs $v_1(s)$ and $v_2(s)$.

For the coupled drives apparatus, a suitable pre-compensator can be designed from practical consideration of Figure 12 as follows.

If the same signal $u_1$ is applied to **both** drive motors, it is reasonable to expect that, if the motors are identical, the speed $\omega(s)$ will alter, while the tension (as sensed by $x_k(s)$) will be nominally unaltered.

By the same token, if a signal $u_2$ is applied to one drive and its negative, $-u_2$, to the other drive, then speed $\omega(s)$ should be unaltered while $x_k(s)$ will change accordingly. Reasoning of this kind leads to the formulation of the new inputs $u_1(s)$ and $u_2(s)$ (see Figure 14) defined by,

$$v_1(s) = u_1(s) - u_2(s) \qquad (26a)$$

$$v_2(s) = u_1(s) + u_2(s) \qquad (26b)$$

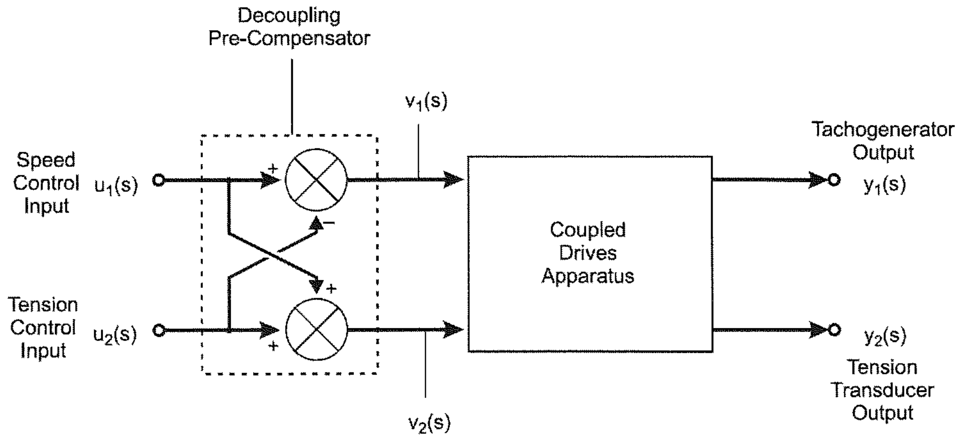



*Figure 14*

Using Equations 26a and 26b in Equations 21a–21c, 24 and 25 gives the following overall transfer function (see Figure 15),

$$\omega(s) = \frac{g_m u_1(s)}{Is + b} \tag{27a}$$

$$x_k(s) = \frac{(4 g_m r k \cos\alpha) u_2(s)}{(Is^2 + bs + 2k^1(s))(2(s^2 m_t + sb_t + k_t) - (2\cos\alpha)^2 k)} \tag{27b}$$

Where the motor and amplifier characteristics are assumed identical such that,

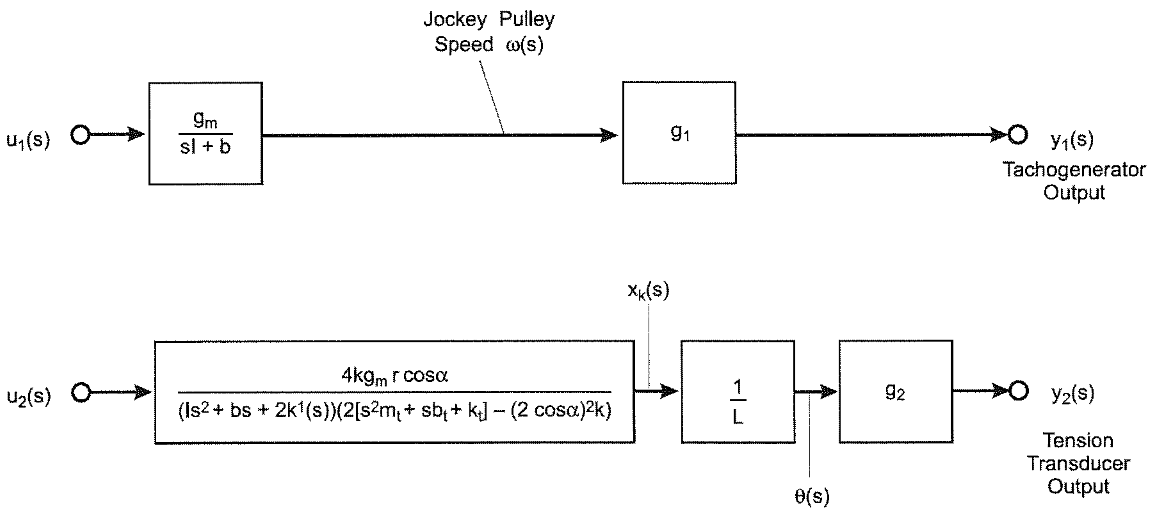



*Figure 15*

Note that $x_k(s)$ is now dependent only upon $u_2(s)$, and that $\omega(s)$ is only dependent upon $u_1(s)$. Thus, as shown in Figure 15, the two outputs are now completely decoupled.