PSTAT 222C Spring 2025 – Asn 2

Note. **Hand in all Code used for all computations .**

Problem 1: Implicit vs Crank-Nicolson FD PDE Solvers

Consider the **corridor** option from Asn 1 with payoff 1 if $X_t \in [L_1, L_2]$ for all $t \le T$ and payoff zero otherwise. Thus, the asset price is required to remain between L_1 and L_2 the entire time until maturity. Our goal is to find the option price $V(t, x)$.

We use the CEV model

$$dX_t = rX_t dt + \sigma X_t^\gamma dW_t$$

where we select $\gamma = 0.8$ with parameters $X_0 = 20, \sigma = 0.4, r = 0.05, T = 1/2$ and $L_1 = 15, L_2 = 25$.

(a) Implement the Implicit FD Solver for the above. Be sure to write down the PDE you're solving and all the boundary conditions. Use $\Delta t = 0.01$ (50 time steps), and spatial domain $X_{min} = 15, X_{max} = 25$ with either $\Delta X = 0.1$ or $\Delta X = 0.02$ (101, and 501 spatial grid points). Report the solution at $t = 0, X_0 = 20$ and the plot $X_0 \mapsto V(0, X_0)$.

(b) Implement the Crank-Nicolson solver for the above with the same Δt and ΔX's. Compare the option price at $t = 0, X_0 = 20$: does C-N scheme appear to be more accurate than Implicit (you should also compare to the MC solution on Asn 1)?

(c) Repeat the analysis of parts a-b for a Compound option: the right to buy a Call with maturity $T_2 = 1/2$ and strike $K = 20$ at time $T_1 = 1/4$ for \$2 (this is the strike of the compound option). Be sure to specify the boundary conditions and the range of your X-grid.

Hint: for debugging purposes, you can set $\gamma = 1$ and use a Put payoff to compare to the exact B-S formula.

Problem 2: Explicit scheme for the Heston model

Consider the Heston model from Asn 1: this is a stochastic volatility model with the dynamics

$$dS_t = rS_t dt + S_t \sqrt{V_t} dW_t^1 \tag{1}$$

$$dV_t = \kappa(\theta - V_t)\, dt + \eta \sqrt{V_t} dW_t^2. \tag{2}$$

Above V_t is the volatility process and (S_t) is the asset price; the driving Wiener processes W^1, W^2 have constant correlation ρ. The model parameters are: $r = 0.05; \kappa = 1; \theta = 0.2; \eta = 0.5; \rho = -0.4$ with initial conditions $S_0 = 100, V_0 = 0.25$ and option parameters $T = 1, K = 100$.

(a) Implement an explicit FD solver (as a function that can be called) for the above problem and the Put payoff $\Phi(S_T) = e^{-rT}(K - S_T)_+$. Specify your boundary conditions and the PDE being solved.

(b) Do the above for a range of Δt and $\Delta S, \Delta V$ values and discuss the observed convergence (make sure your schemes are stable!) of the option value at (S_0, V_0). Compare to the answer you had on Asn 1.

Problem 3: GP option pricing for the Heston Model

Using the same model as in Problem 2, our goal is to learn the pricing surface as a function of (S_0, V_0). To this end, select 100 pairs of initial share prices and volatilities (the latter should be in the range $[0.05, 0.6]$). Use Monte Carlo (with a Milstein scheme for (V_t) as in Asn 1) and $\Delta t = 0.01$ with $M = 1000$ simulations to obtain approximate Put prices for each of those initial conditions.

(a) Fit a GP surrogate with a Squared Exponential kernel with pre-specified lengthscales $\ell_S = 10, \ell_V = 0.1$ and process stdev $\eta = 10$. Watch out for the observation noise – make sure you're not over- or under-fitting.

(b) Fit a GP surrogate with a Matern-5/2 kernel and optimized lengthscales (via MLE optimization).

(c) Compare the predicted option surfaces among themselves, as well as to the (linearly) interpolated surface from the FD solver in Q2. Discuss what you observe.