RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE532
Course Title	Signals and Systems I
Semester/Year	F2022

Instructor	Dr. Androutsos

ASSIGNMENT No. 2

Assignment Title	Continuous-Time Convolution

Submission Date	2022-10-29
Due Date	2022-10-31

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A. Impulse Response

A1.

```
% CH2MP1.m : Chapter 2, MATLAB Program 1
% Script M-file determines characteristic roots of op-amp circuit.
% Set component values:
R = [1e4, 1e4, 1e4]; C = [1e-6, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);

p = poly(lambda);

p
lambda
```

```
>> CH2MP1

p =

   1.0e+04 *

    0.0001    0.0300    1.0000


lambda =

 -261.8034
  -38.1966
```

A2.

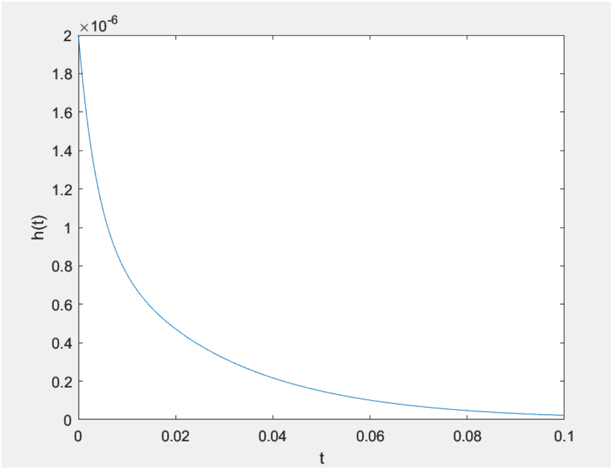
```
% CH2MP1.m : Chapter 2, MATLAB Program 1
% Script M-file determines characteristic roots of op-amp circuit.
% Set component values:
R = [1e4, 1e4, 1e4]; C = [1e-6, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);

p = poly(lambda);

p;
lambda;

t = [0:0.0005:0.1];
u = @(t) 1.0.*(t>=0);
h = @(t) (C(1).*exp(lambda(1).*t)+ (C(2).*exp(lambda(2).*t)).*u(t));

plot(t,h(t))
xlabel('t');
ylabel('h(t)');
```



A3.

```matlab
function [lambda] = CH2MP2(R,C)
% CH2MP2.m : Chapter 2, MATLAB Program 2
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS: R = length-3 vector of resistances
% C = length-2 vector of capacitances
% OUTPUTS: lambda = characteristic roots
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A)

%lambda = CH2MP2([1e4, 1e4, 1e4],[1e-9, 1e-6]); -> enter this line into
%                                                   command window or
%                                                   uncomment it and end
%                                                   execution

```

```
>> lambda = CH2MP2([1e4, 1e4, 1e4],[1e-9, 1e-6])

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i
```

B. Convolution

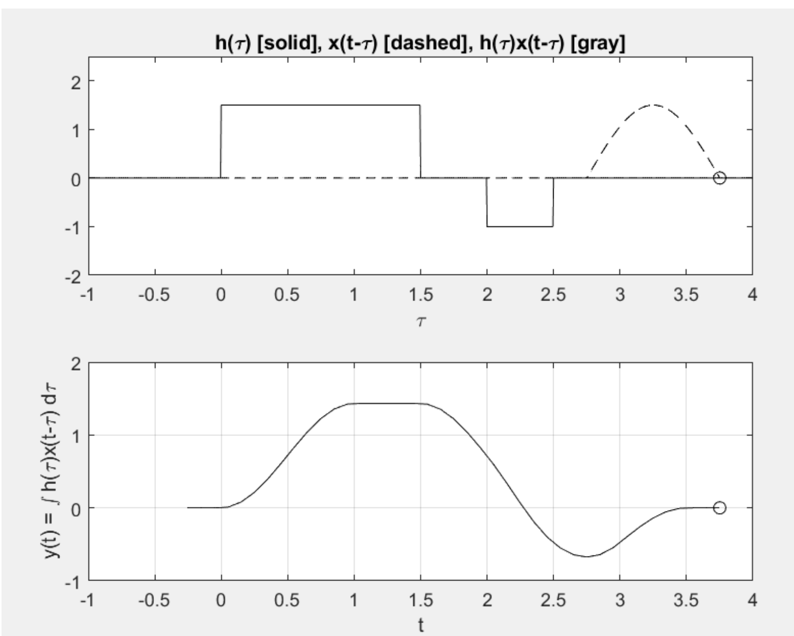
B1.

```matlab
% CH2MP4.m : Chapter 2, MATLAB Program 4
% Script M-file graphically demonstrates the convolution process.
figure(1) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;

end
```



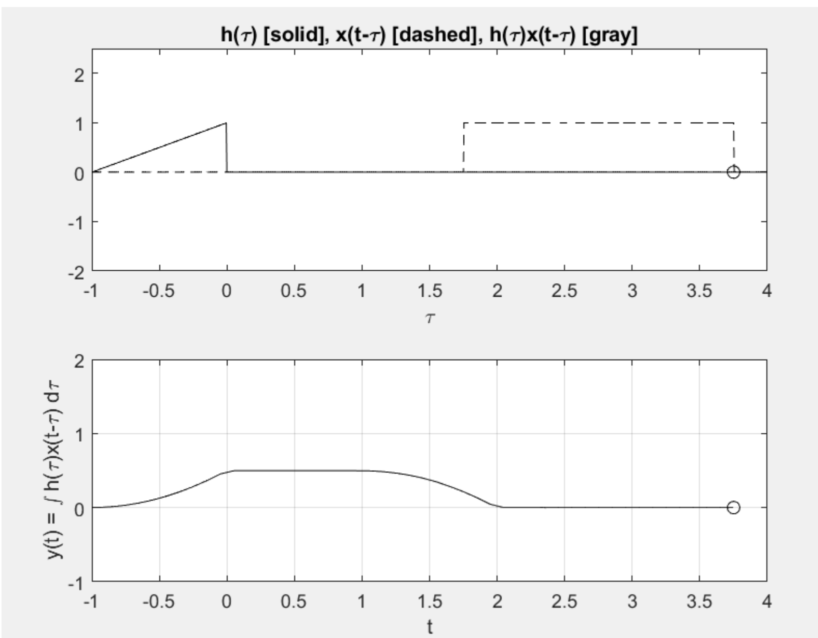
B2.

```
figure(1) % Create figure window and make visible on screen
x = @(t) u(t)-u(t-2);
h = @(t) (t+1).*(u(t+1)-u(t));
y = @(t) x(t).*h(t);
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -1.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]);
grid;
drawnow;

end
```



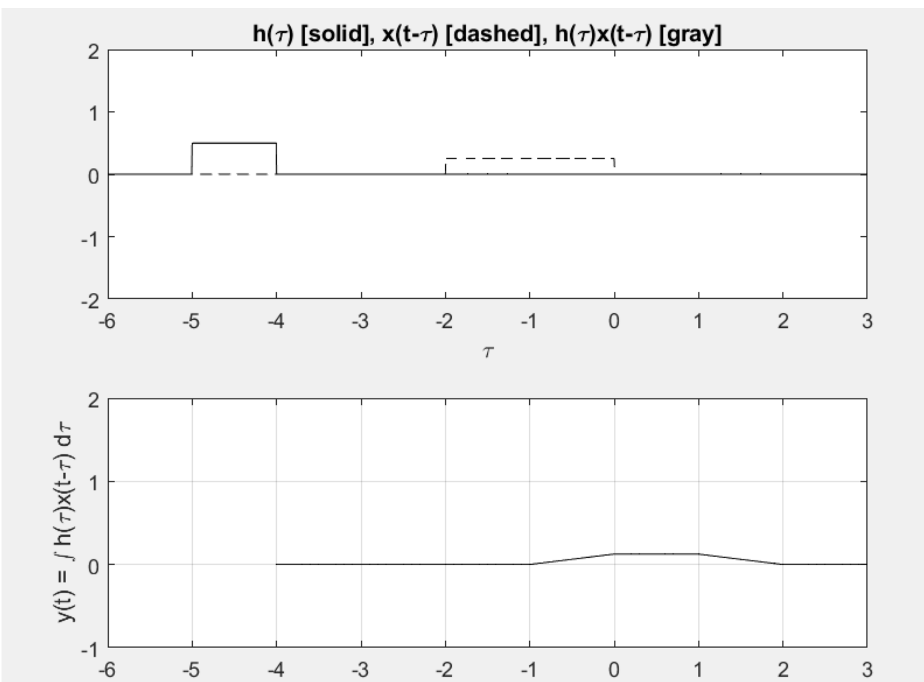
B3a.

```
figure(1) % Create figure window and make visible on screen
A = 0.25; %example random value taken
B = 0.5;
u = @(t) 1.0.*(t>=0);
x = @(t) A.*(u(t-4)-u(t-6));
h = @(t) B.*(u(t+5)-u(t+4));
dtau = 0.005;
tau = -6:dtau:3;
ti = 0;
tvec = -4:.1:4;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;

end
```



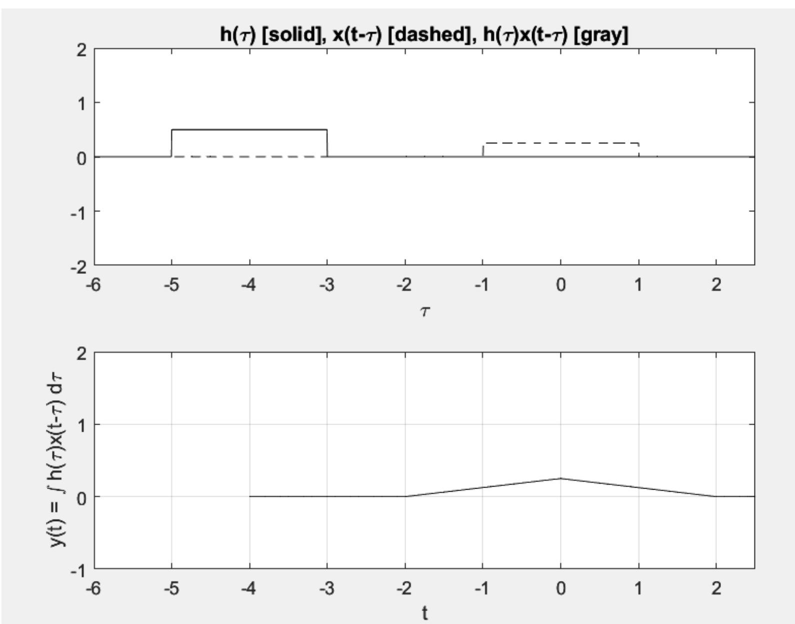
B3b.

```
figure(1) % Create figure window and make visible on screen
A = 0.25; %example random value taken
B = 0.5;
u = @(t) 1.0.*(t>=0);
x = @(t) A.*(u(t-3)-u(t-5));
h = @(t) B.*(u(t+5)-u(t+3));
dtau = 0.005;
tau = -6:dtau:2.5;
ti = 0;
tvec = -4:.1:4;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;

end
```



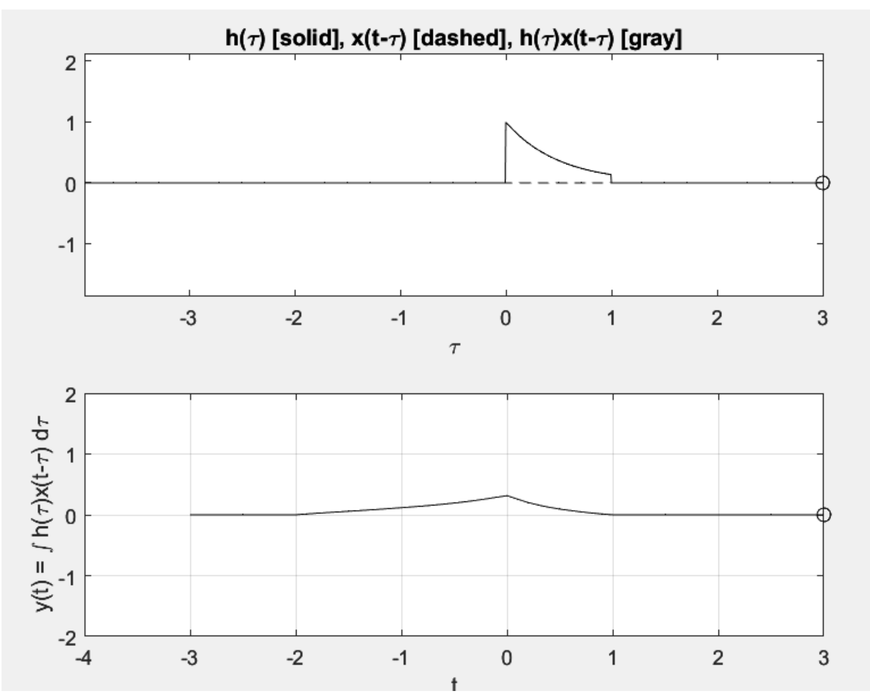
B3h.

```
figure(1) % Create figure window and make visible on screen
u = @(t) 1.0.*(t>=(0));
x = @(t) exp(t).*(u(t+2)-u(t));
x2 = @(t) exp(-2.*t).*(u(t)-u(t-1));
dtau = 0.005;
tau = -4:dtau:3;
ti = 0;
tvec = -3:.1:3;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*x2(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,x2(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau');
title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children');
set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t');
ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -2.0 2.0]);
grid;
drawnow;

end
```



C. System Behaviour and Stability

C1.

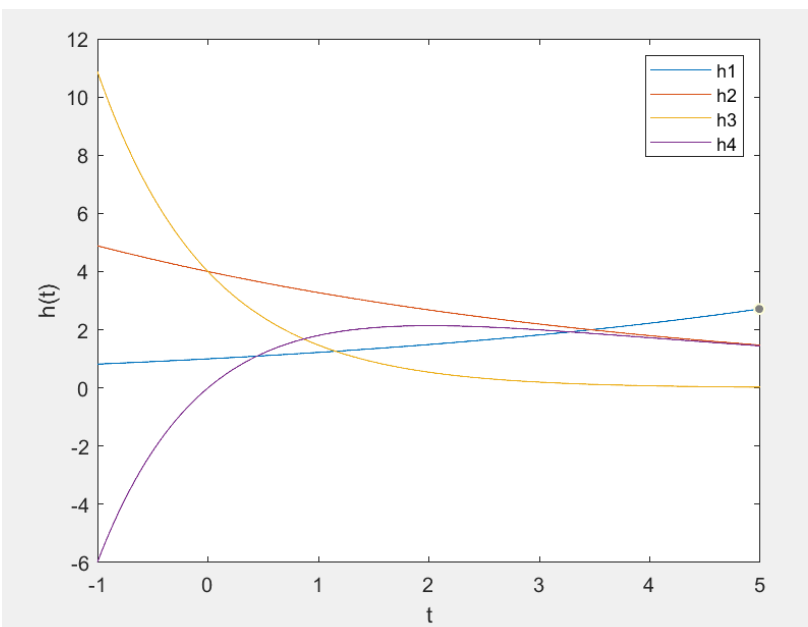
```
t = [-1:0.001:5];

u = @(t) 1.0.*(t >= -1);
h1 = @(t) exp(t./5).*u(t);
h2 = @(t) 4.*exp(-t./5).*u(t);
h3 = @(t) 4.*exp(-t).*u(t);
h4 = @(t) 4.*(exp(-t./5) - exp(-t)).*u(t);

plot(t, h1(t));
xlabel('t');
ylabel('h(t)');
hold on;
plot(t, h2(t));
plot(t, h3(t));
plot(t, h4(t));

legend('h1', 'h2', 'h3', 'h4');
hold off

```



C2.

```
%Eigenvalues/characteristic values of functions in C1:

%h1: 1/5
%h2: -1/5
%h3: -1
%h4: -1 AND -1/5
```

C3.1.

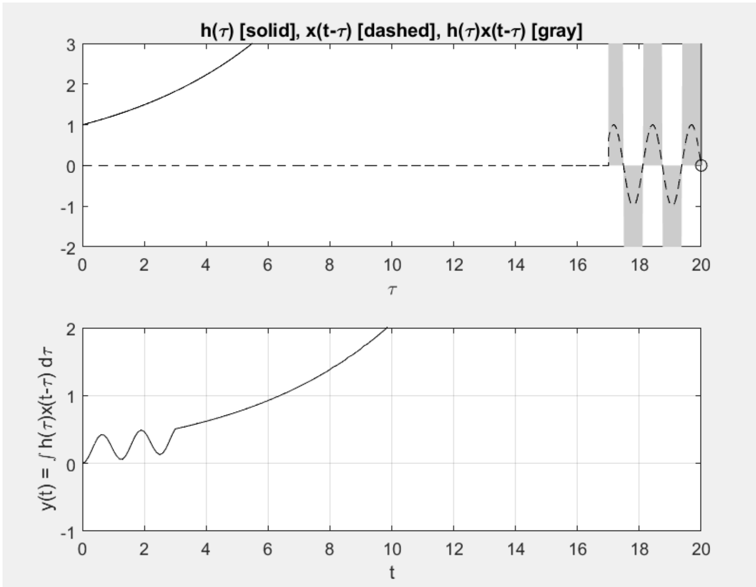
```
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) exp(t/5).*(u(t)-u(t-20));

dtau = 0.005;
tau = 0:dtau:20;
ti = 0;
tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 3.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau');
title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children');
set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]);
grid;
drawnow;

end
```



C3.2.

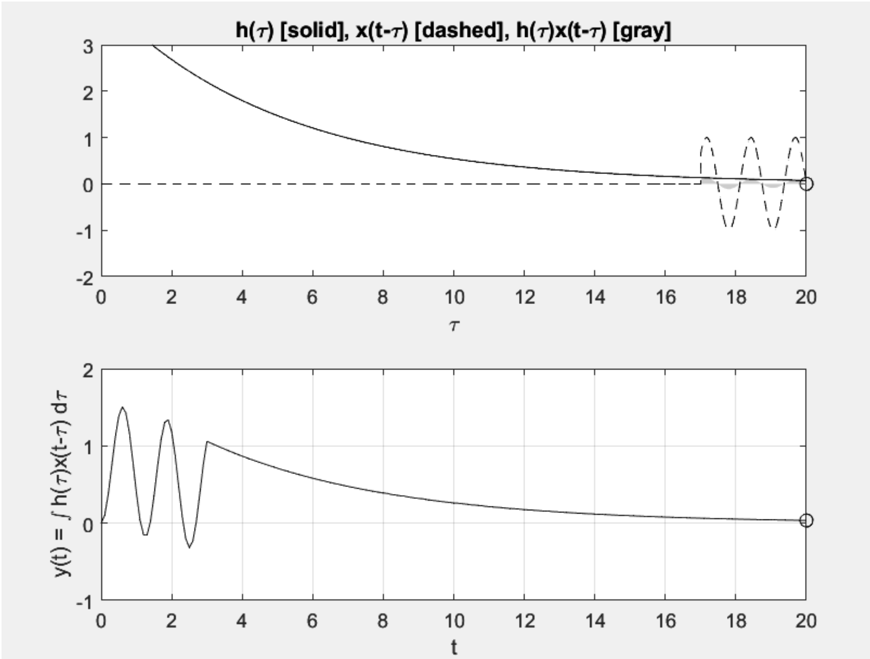
```
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4.*exp(-t./5).*u(t);

dtau = 0.005;
tau = 0:dtau:20;
ti = 0;
tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 3.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau');
title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children');
set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]);
grid;
drawnow;

end
```



C3.3.

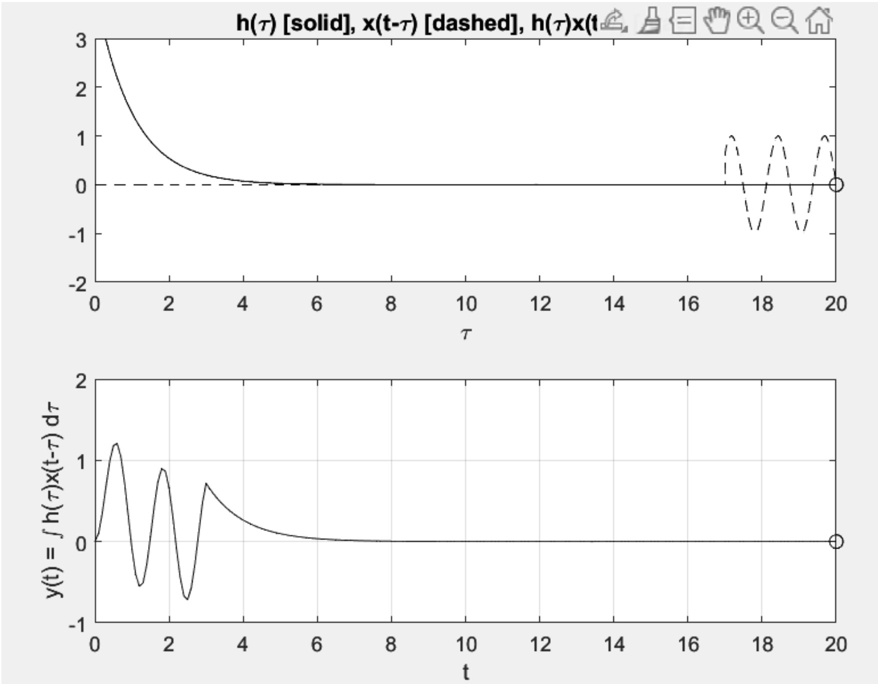
```
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4.*exp(-t).*u(t);

dtau = 0.005;
tau = 0:dtau:20;
ti = 0;
tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 3.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau');
title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children');
set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]);
grid;
drawnow;

end
```



C3.4.

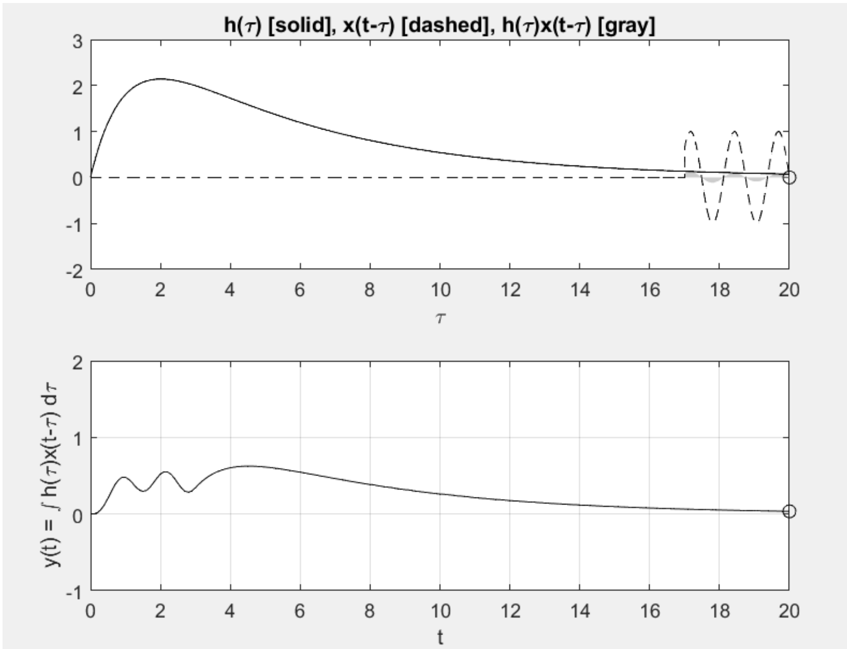
```
u = @(t) 1.0*(t>=0);
x = @(t) sin(5*t).*(u(t)-u(t-3));
h = @(t) 4.*(exp(-t./5) - exp(-t)).*u(t);

dtau = 0.005;
tau = 0:dtau:20;
ti = 0;
tvec = 0:.1:20;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory

for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 3.0]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau');
title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children');
set(gca,'children',[c(2);c(3);c(4);c(1)]);

subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]);
grid;
drawnow;

end
```



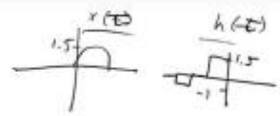
Similarity between the graphs:

```
% Each of the convolutions done in section C3 begins in a sinusoidal
% appearance, however, it begins merging and turning into the exponential
% line as time continues. The convolution of h2 and h3 look quite similar,
% with h4 being close, but trending upwards instead.
```

D. Discussion

D1.

B1 manual calculation (CH2MP4.m)



Region 1

$\boxed{t<0}$ $\boxed{y(t)=0}$

Region 2

$\boxed{0\le t<1}$

$$y(t)=\int_0^t (1.5)(1.5\sin(\pi\tau))\,d\tau = \int_0^t 2.25\sin(\pi\tau)\,d\tau = -\frac{2.25\cos(\pi\tau)}{\pi}\Big]_0^t$$

$$\boxed{=\frac{2.25}{\pi}(-\cos(\pi t)+1)}$$

Region 3

$$\left.\begin{matrix} t-1.5<0 \\ t>1 \end{matrix}\right\} \boxed{1\le t<1.5}$$

$$y(t)=\int_0^1 (1.5)(1.5\sin(\pi\tau))\,d\tau = 2.25\cdot\frac{1}{\pi}\left[-\cos(u)\right]_0^{\pi} = \frac{+2.25}{\pi}\cdot 2$$

$$\boxed{=\frac{4.5}{\pi}}$$

Region 4

$$\underbrace{0\le t-1.5<0.5}_{\boxed{1.5\le t<2}}$$

$$y(t)=\int_{t-1.5}^{1} (1.5)(1.5\sin(\pi\tau))\,d\tau = \frac{-2.25}{\pi}\cos(\pi\tau)\Big]_{t-1.5}^{1}$$

$$\boxed{=\frac{2.25}{\pi}\left[\cos(\pi(t-1.5))+1\right]}$$

Region 5

$$\left.\begin{matrix} 0\le t-2 \\ t-1.5<1 \end{matrix}\right\} \Rightarrow \boxed{2\le t<2.5}$$

$$y(t)=\underbrace{\int_0^{t-2} (-1)(1.5\sin(\pi\tau))\,d\tau}_{\text{do this here}} + \overbrace{\int_{t-1.5}^{1} (1.5)(1.5\sin(\pi\tau))\,d\tau}^{\text{Same as region 4}}$$

$$\boxed{=\frac{1.5}{\pi}\left[\cos(\pi(t-2))-1\right]+\frac{2.25}{\pi}\left[\cos(\pi(t-1.5))+1\right]}$$

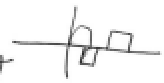
Region 6

$$\left.\begin{array}{l} 0 \le t-2.5 \\ t-2 \le 1 \end{array}\right\} \boxed{2.5 \le t < 3}$$

$$y(t) = \int_{t-2.5}^{t-2} (-1)(1.5\sin(\pi\tau))\,d\tau = \frac{1.5}{\pi}\cos(\pi\tau)\Big]_{t-2.5}^{t-2}$$

$$\boxed{= \frac{1.5}{\pi}\left[\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right]}$$

Region 7



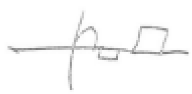
$$\left.\begin{array}{l} t-1.5 \le 1 \\ 1 \le t-2 \end{array}\right\} \Rightarrow \boxed{3 \le t < 3.5}$$

$$y(t) = \int_{t-2.5}^{1} (-1)(1.5\sin(\pi\tau))\,d\tau = \frac{1.5}{\pi}\cos(\pi\tau)\Big]_{t-2.5}^{1}$$

$$\boxed{= \frac{1.5}{\pi}\left[-\cos(\pi(t-2.5)) - 1\right]}$$

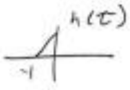
Region 8

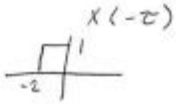
$\boxed{t \ge 3.5}$ $\boxed{y(t) = 0}$



$$y(t) = \begin{cases} 0 & , t < 0 \\ \frac{2.25}{\pi}(-\cos(\pi t) + 1) & , 0 \le t < 1 \\ 4.5/\pi & , 1 \le t < 1.5 \\ \frac{2.25}{\pi}\left[\cos(\pi(t-1.5)) + 1\right] & , 1.5 \le t < 2 \\ 1.5/\pi\left[\cos(\pi(t-2)) - 1\right] + \frac{2.25}{\pi}\left[\cos(\pi(t-1.5)) + 1\right] & , 2 \le t < 2.5 \\ 1.5/\pi\left[\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right] & , 2.5 \le t < 3 \\ 1.5/\pi\left[-\cos(\pi(t-2.5)) - 1\right] & , 3 \le t < 3.5 \\ 0 & , 3.5 \le t \end{cases}$$

B2 manual calculation



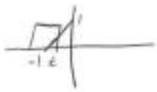


Region 1

$\boxed{t<1}$ $\boxed{y(t)=0}$

Region 2

$\boxed{-1\le t<0}$



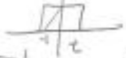
$$y(t)=\int_{-1}^{t}(\tau+1)(1)\,d\tau=\int_{-1}^{t}\tau\,d\tau+\int_{-1}^{t}1\,d\tau$$

$$=\frac{\tau^2}{2}\Big]_{-1}^{t}+\tau\Big]_{-1}^{t}=\frac{t^2}{2}-\frac{1}{2}+t-1$$

$$\boxed{=\frac{t^2}{2}+t+\frac{1}{2}}$$

Region 3

$\left.\begin{array}{l}t<1\\ t\ge 0\end{array}\right\}\Rightarrow\boxed{0\le t<1}$

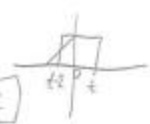


$$y(t)=\int_{-1}^{0}(1)(\tau+1)\,d\tau=\frac{\tau^2}{2}+\tau\Big|_{-1}^{0}$$

$$=\frac{1}{2}-1=\boxed{-0.5}$$

Region 4

$\left.\begin{array}{l}t-2<0\\ -1\le t-2\end{array}\right\}\boxed{1\le t<2}$



$$y(t)=\int_{t-2}^{0}(1)(\tau+1)\,d\tau=\frac{\tau^2}{2}+\tau\Big|_{t-2}^{0}$$

$$\boxed{=-\left[\frac{(t-2)^2}{2}+(t-2)\right]}$$

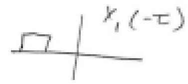
Region 5

$\boxed{t\ge 2}$

$\boxed{y(t)=0}$

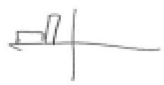
$$y(t)=\begin{cases}0, & t<1\\ \frac{t^2}{2}+t+\frac{1}{2}, & -1\le t<0\\ -0.5, & 0\le t<1\\ -\left[\frac{(t-2)^2}{2}+(t-2)\right], & 1\le t<2\\ 0, & t\ge 2\end{cases}$$

B3 a) manual calculation



$A = 0.25$, $B = 0.5$

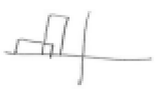
Region 1



$t-4 < -5$

$\boxed{t < -1}$ $\boxed{y(t) = 0}$

Region 2



$-5 \le t-4 < -4 \rightarrow \boxed{-1 \le t < 0}$

$$y(t) = \int_{-5}^{t-4} (0.5)(0.125)\,d\tau = 0.125\,\tau \Big|_{-5}^{t-4} = 0.125(t-4) + 0.125(5)$$

$$= 0.125(t) - (0.125 \cdot 4) + 0.125(5)$$

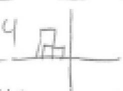
$$= \boxed{0.125t + 0.125}$$

Region 3

$t-4 \ge 4$, $t-6 < 5$ $\boxed{0 \le t < 1}$

$$y(t) = \int_{-5}^{-4} 0.125\,d\tau = 0.125(-4) - 0.125(-5) = \boxed{0.125}$$

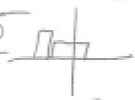
Region 4



$t-6 < -4$, $t-6 \ge -5$ $\boxed{1 \le t < 2}$

$$y(t) = \int_{t-6}^{-4} 0.125\,d\tau = 0.125(-4) - 0.125(t-6) = \boxed{-0.125t + 0.25}$$

Region 5



$\boxed{t \ge 2}$ $\boxed{y(t) = 0}$

$$y(t) = \begin{cases} 0, & t < -1 \\ 0.125t + 0.125, & -1 \le t < 0 \\ 0.125, & 0 \le t < 1 \\ -0.125t + 0.25, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

B3 b) manual calculations

$x_1(-\tau)$

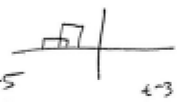
$x_2(\tau)$

$A=0.25$, $B=0.5$

Region 1

$t-3<-5 \rightarrow \boxed{t<-2}$ $\boxed{y(t)=0}$

Region 2

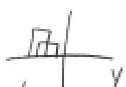


$\underbrace{t-3<-3,\ t-3\geq -5}$

$\boxed{-2\leq t<0}$

$$y(t)=\int_{-5}^{t-3} 0.125\,d\tau = 0.125(t-3)-0.125(-5)$$
$$=\boxed{0.125t+0.25}$$

Region 3



$\left.\begin{matrix} t-5\geq -5 \\ t-5<3 \end{matrix}\right\} \boxed{0\leq t<2}$

$$y(t)=\int_{t-5}^{-3} 0.125\,d\tau = 0.125(-3)-0.125(t-5)$$
$$=\boxed{-0.125t+0.25}$$

Region 4

$\boxed{t\geq 2}$ $\boxed{y(t)=0}$

$$y(t)=\begin{cases} 0 & , t<-2 \\ 0.125t+0.25 & , -2\leq t<0 \\ -0.125t+0.25 & , 0\leq t<2 \\ 0 & , t\geq 2 \end{cases}$$

B3 h) manual calculations

$x_1(t)$ $\quad$ $x_2(-\tau)$ $\quad$ $e^{-2(t-\tau)}$

Region 1

$\boxed{t \le -2}$ $\boxed{y(t)=0}$

Region 2

$t > -2$, $t-1 \le -2$ $\}$ $\boxed{-2 \le t < -1}$

$$y(t) = \int_{-2}^{t} e^{\tau} e^{-2(t-\tau)} d\tau = e^{-2t} \int_{-2}^{t} e^{\tau} e^{2\tau} d\tau$$

$$= e^{-2t} \left[\frac{1}{3} e^{3\tau}\right]_{-2}^{t} = \frac{1}{3} e^{-2t} \left[e^{3t} - e^{3(-2)}\right]$$

$$= y(t) = \boxed{\frac{1}{3}\left[e^{t} - e^{-2(t+3)}\right]}$$

Region 3

$t-1 \ge -2$, $t < 0 \rightarrow \boxed{-1 \le t < 0}$

$$y(t) = \int_{t-1}^{t} e^{\tau} e^{-2(t-\tau)} d\tau$$

$$= \int_{t-1}^{t} e^{\tau} e^{-2t} e^{2\tau} d\tau = e^{-2t} \int_{t-1}^{t} e^{3\tau} d\tau$$

$$= e^{-2t}\left[\frac{1}{3} e^{3\tau}\right]_{t-1}^{t} = \boxed{\frac{1}{3}\left[e^{t} - e^{t-3}\right]}$$

Region 4

$t-1 \le 0$, $t \ge 0$ $\}$ $\boxed{0 \le t < 1}$

$$y(t) = \int_{t-1}^{0} e^{\tau} e^{-2(t-\tau)} d\tau = e^{-2t} \int_{t-1}^{0} e^{3\tau} d\tau = e^{-2t}\left[\frac{1}{3} e^{3\tau}\right]_{t-1}^{0}$$

$$= \boxed{\frac{1}{3}\left[e^{-2t} - e^{t-3}\right]}$$

Region 5

$t-1 \ge 0 \rightarrow \boxed{t \ge 1}$

$\boxed{y(t) = 0}$

$$y(t) = \begin{cases} 0, & t \le -2 \\ \frac{1}{3}\left[e^{t} - e^{-2(t+3)}\right], & -2 \le t < -1 \\ \frac{1}{3}\left[e^{t} - e^{t-3}\right], & -1 \le t < 0 \\ \frac{1}{3}\left[e^{-2t} - e^{t-3}\right], & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

D2.

We can see that if the duration of two signals were T1 and T2, respectively, then the duration of the signal that results from the convolution would have a duration of T = T1+T2.