# Week 7 Spatiotemporal models

- Monitoring network such as LTERs
- General modeling

$$Y(s,t) = \mu(s,t) + \epsilon(s,t)$$

$$\mu(s,t) = x(s,t)\beta(s,t)$$

$$\epsilon = w(s,t) + e(s,t)$$

Spatiotemporal process

White noise

$$Y|\mu w \sim Normal(\mu + w, e)$$

model 1:

Additive $e(s,t) = \alpha(t) + w(s) + \epsilon(s,t)$

model 2:

Temporal evolution at s $e(s,t) = \alpha_s(t) + \epsilon(s,t)$

model 3:

Spatial structure at t $e(s,t) = w_t(s) + \epsilon(s,t)$

# To illustrate a specific problem…

Environ Ecol Stat (2008) 15:59–70
DOI 10.1007/s10651-007-0040-1

**A hierarchical Bayesian non-linear spatio-temporal model for the spread of invasive species with application to the Eurasian Collared-Dove**

**Mevin B. Hooten · Christopher K. Wikle**

# Model

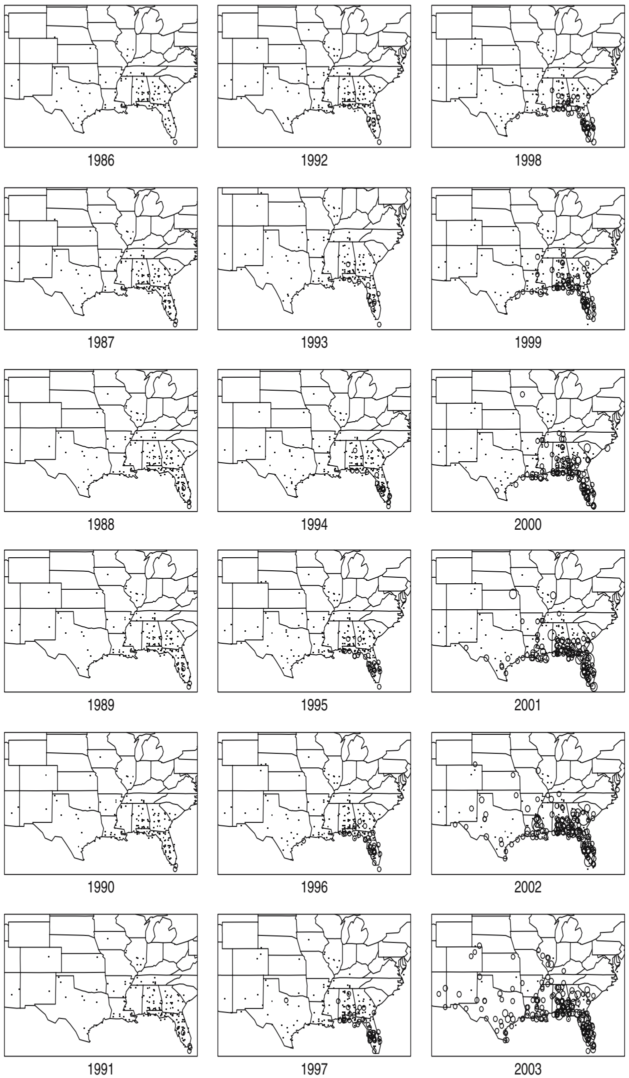



**Fig. 1** Spread of ECD throughout the United States from 1986 through 2003 (points represent zero counts at sampled location, while circle size corresponds to non-zero count magnitude)

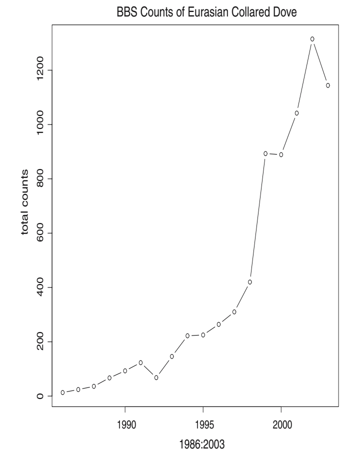



**Fig. 2** Population growth of ECD in the United States from 1986 through 2003 (total counts over time)

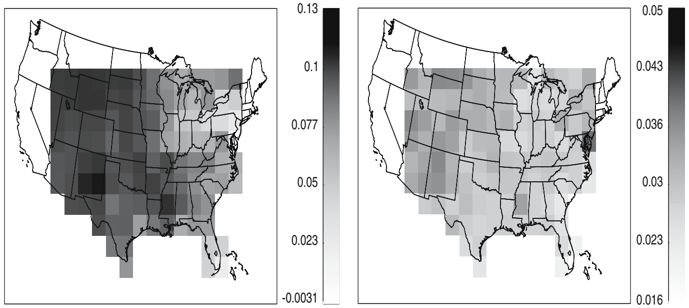
Data model $$n_{i,t}|\lambda_{i,t} \sim Pois(\lambda_{i,t}), \quad i = 1, \dots, m \quad t = 1, \dots, T.$$

Process Model $$\log(\boldsymbol{\lambda}_t) = \mathbf{K}_t \mathbf{u}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \sigma_\varepsilon^2 \mathbf{I}), \quad t = 1, \dots, T.$$

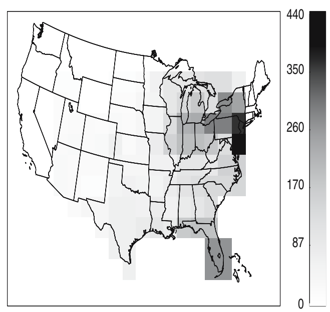
reaction-diffusion equation $$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\delta(x, y)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\delta(x, y)\frac{\partial u}{\partial y}\right) + \gamma_0 u\left(1 - \frac{u}{\gamma_1}\right),$$

Parameter model $$\boldsymbol{\delta} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}, \quad \boldsymbol{\xi} \sim N(\mathbf{0}, \sigma_\delta^2 \mathbf{R}(\theta)). \quad R(\theta, d) = exp(-\theta||d||).$$

# Results

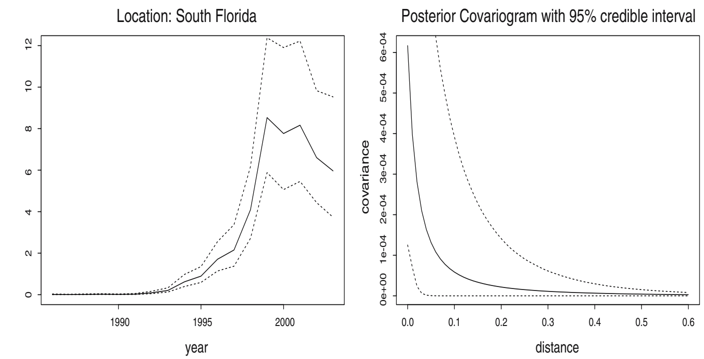



**(a)** Posterior mean of δ **(b)** Posterior standard dev. of δ




**(c)** Human population covariate map (i.e., 2nd column of **X**).

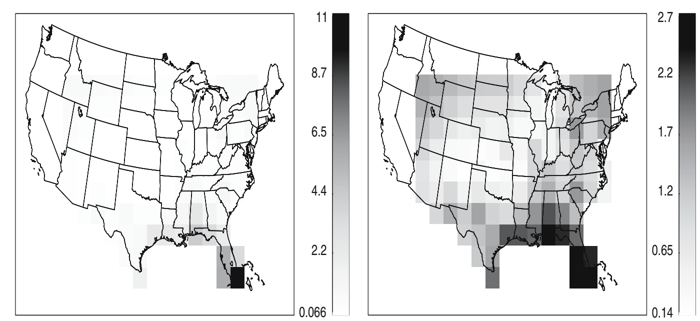
**Fig. 4** Posterior summary of $\delta$ and covariate. (**a**) Posterior mean of $\delta$. (**b**) Posterior standard dev. of $\delta$. (**c**) Human population covariate map (i.e., 2nd column of **X**)




**(a)** Posterior mean (and 95% credible interval) of the Poisson intensity process at a South Florida location through time (represents ECD population growth).

**(b)** Posterior covariogram with 95% credible intervals (i.e., covariance structure of $\boldsymbol{\xi}$). Maximum distance in the covariogram equals maximum map distance.

**Fig. 5** Posterior summary of $\boldsymbol{\lambda}$ and $\boldsymbol{\xi}$. (**a**) Posterior mean (and 95% credible interval) of the Poisson intensity process at a South Florida location through time (represents ECD population growth). (**b**) Posterior covariogram with 95% credible intervals (i.e., covariance structure of $\boldsymbol{\xi}$). Maximum distance in the covariogram equals maximum map distance




**(a)** Posterior mean for Poisson intensity prediction.

**(b)** Posterior standard deviation for Poisson intensity prediction.

**Fig. 7** Posterior prediction of $\boldsymbol{\lambda}$ for 2004. (**a**) Posterior mean for Poisson intensity prediction. (**b**) Posterior standard deviation for Poisson intensity prediction