Problem Set 8: Complex Numbers

Goal: Become familiar with math operations using complex numbers; see how complex numbers can be used to show the frequency response of an RC circuit.

Note: This PSet will be much easier if you have already watched the lectures on complex numbers.

Deliverable: This worksheet and two plots.

$z_1 = 1 + j$ E: $\sqrt{2}e^{0.79}$

$r = \sqrt{2}$ $\theta = \tan^{-1}\left(\frac{1}{1}\right)$

P: $(\sqrt{2}, 0.79)$

Part I: Basic Operations with complex numbers

For the following, take $z_1 = 1 + j$ and $z_2 = -3 + 4j$.

1. Convert z_1 and z_2 to polar and exponential notation (find r, θ).

$z_2 = -3 + 4j$

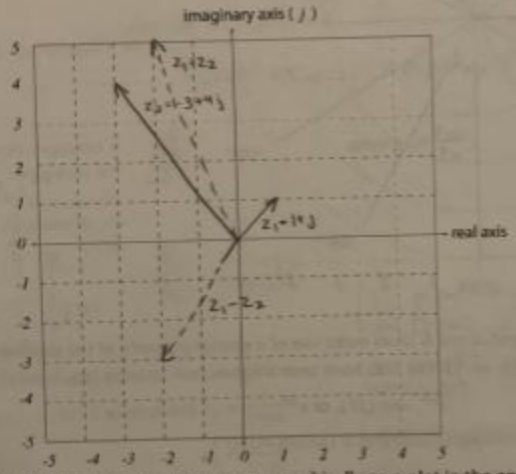
$r = \sqrt{(-3)^2 + (4)^2}$ $\theta = \tan^{-1}\left(\frac{4}{-3}\right)$

$r = 5$

P: $(2.24, 2.93)$

E: $2.24e^{0.93}$

2. Plot z_1 and z_2 on the complex plane below.



$1+j-3+4j$

$-2+5j$

3. Compute $z_1 + z_2$. Show $z_1 + z_2$ graphically on a plot in the complex plane from 2.

$5j - 2$

$1+j-3-4j$

$-2-3j$

4. Compute $z_1 - z_2$. Show $z_1 - z_2$ graphically on a plot in the complex plan from 2.

$-2-3j$

5. Compute $z_1 z_2$. Repeat the computation using a different notation. $(1+j)(-3+4j)$

$-3+4j-3j+4j^2$

-1

$j-7$

6. Compute z_1/z_2 using complex notation. Compute z_2/z_1 and compare.

$-3+j+4$

$\frac{j+1}{4j-3}\frac{(4j+3)}{(4j+3)}$ $\frac{4j^2+3j+4j+3}{16j^2+12j-12j-9}$ $\frac{-4+7j+3}{-16+9}$

$-3+j$

$j-7$

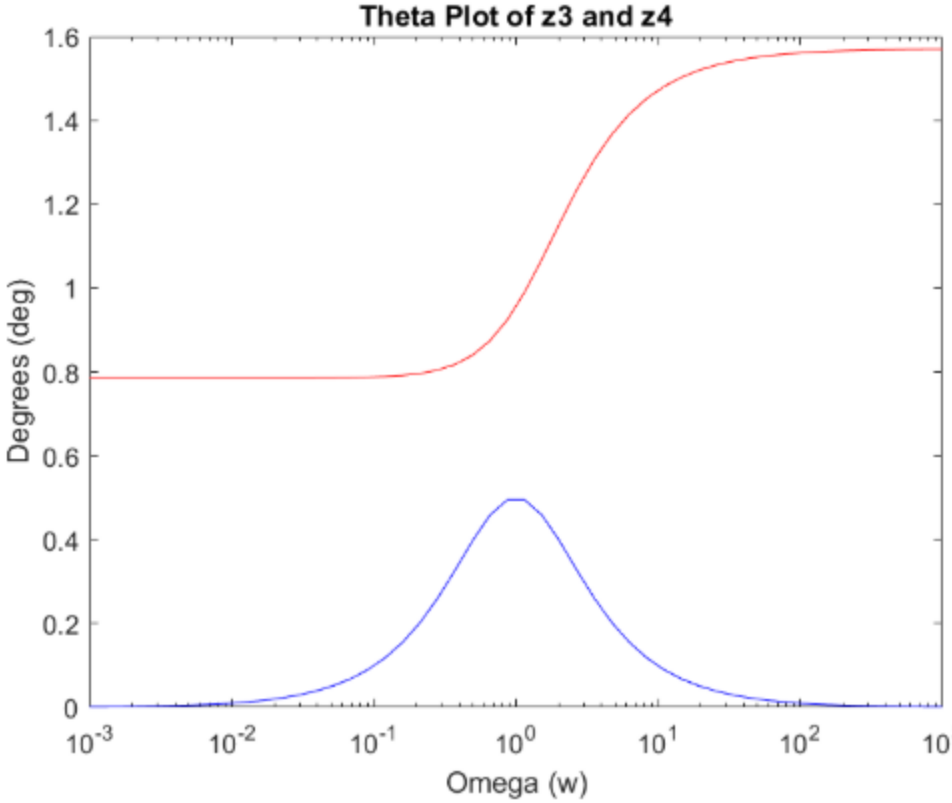
7. Compute z_1^4

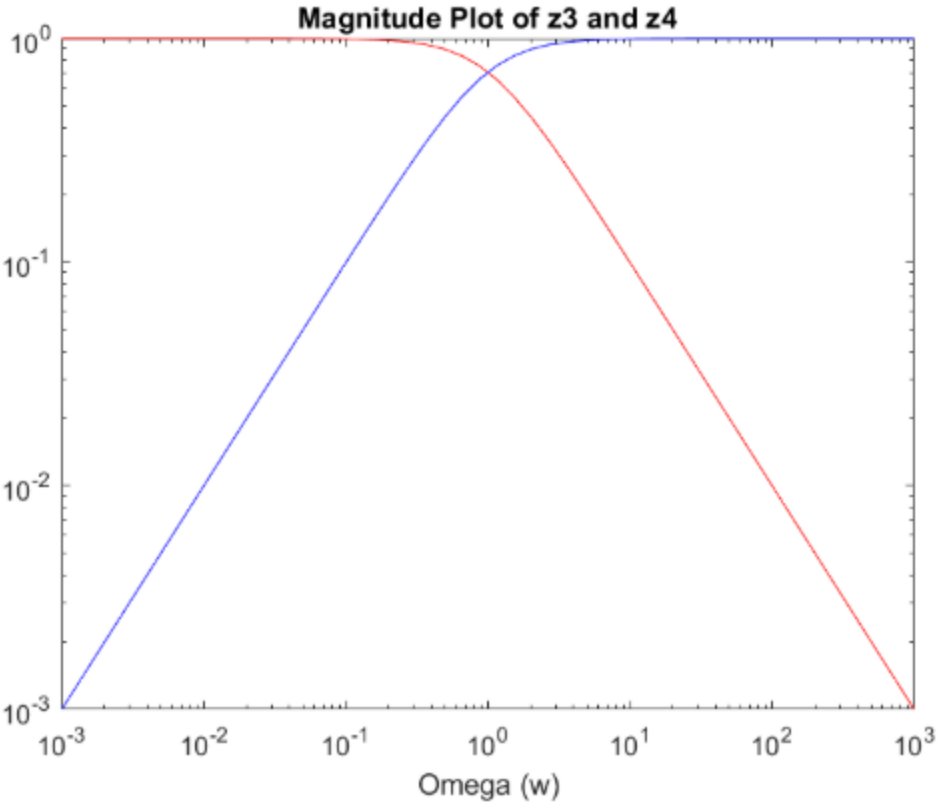
$(1+j)(1+j)$

$1+2j+j^2$ $(2j)(2j)$

$4j^2 = -4$

$\frac{7j+1}{-25}$ $\frac{-7j}{25} + \frac{1}{25}$


Theta Plot of z3 and z4
Degrees (deg)
Omega (w)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10-3
10-2
10-1
100
101
102
103


Magnitude Plot of z3 and z4
10^0
10^-1
10^-2
10^-3
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
Omega (w)