CS 3200: Introduction to Scientific Computing

In-class Activity 2 : Numerical Integration

Problem: You need to perform integration on a function and have completely forgotten your calculus training. Use the midpoint rule to determine the integral of the function.

$$I = \int_a^b f(x)\,dx \text{, where } a = 0, b = 1, f(x) = x^2$$

(a) Verify that the solution is 1/3

$$\int_a^b x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = 1/3$$

(b) Use the midpoint rue with one point and h = 1 to estimate the integral

$I_h =$ $1 \times 0.5^2 = 0.25 = \frac{1}{4} = \frac{4}{16}$

(c) Use the midpoint rule with two points and so h = 0.5 to estimate the integral

$I_{h/2} =$ $0.5 \times (0.25^2 + 0.75^2) = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16}\right) = \frac{5}{16}$

(c) The error of the midpoint rule on one interval is given by $I - I_h = \frac{h^3}{24}\frac{d^2 f}{dx^2}(\varsigma)$ use this form to derive the

equation $I_{h/2} - I_h = \frac{3}{4}\frac{h^3}{24}\frac{d^2 f}{dx^2}(\varsigma)$ and estimate the errors in I_h and $I_{h/2}$

$I - I_h = \frac{h^3}{24}\frac{d^2 f}{dx^2}(\xi_0)$

$I - I_{h/2} = \frac{1}{8}\frac{h^3}{24}\frac{d^2 f(\xi_1)}{dx^2} + \frac{1}{8}\frac{h^3}{24}\frac{d^2 f(\xi_2)}{dx^2} \approx \frac{1}{4}\frac{h^3}{24}\frac{d^2 f}{dx^2}(\xi^*)$

subtract bottom from top

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Assume ξ^* close to ξ_0

$$I_{h/2} - I = \frac{3}{4} \frac{h^3}{24} \frac{d^2 f}{dx^2}(\xi^*)$$

(d) Hence calculate the estimate of the error in I_h and $I_{h/2}$ and use the exact solution ,I, to verify if these estimates work

$$I_{h/2} - I_h = 1/16$$

estimated error in $I_h = \frac{4}{3} \times \frac{1}{16} = \frac{1}{12}$

estimated error in $I_{h/2} = \frac{1}{3} \times \frac{1}{16} = \frac{1}{48}$

The estimated errors match the actual errors in this case.

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