Assigned: 4-01-20
Due Date: 4-15-18

CS 3200: Introduction to Scientific Computing

Assignment 5 Gradient Descent and Eigenvalues

Note: Please do your programming in Matlab and document thoroughly! The code must compile on one of the lab machines with your instructions.

Instructions

1. This question requires you to use the supplied Gradient Descent program to compute a least squares approximation for the US Census data on page 142 of the Moler book . The data is given by the years
   x=[1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000]'
   and population y
   y=[75.995 91.972 105.711 123.203 131.669 150.697 179.323 203.212 226.505 249.633 281.422]'

   (a) Verify that the problem as stated fails with the existing code.
   (b) Scale the year values by s = (year-1950)/50 so that hey are in the range [-1,1] and show that the problems can then be solved.
   (c) With a tolerance of 1e-5 vary the degree of polynomial 2,3,4,5. Given that we don't know the exact solution compare how small the least squares residual may be in each case.
   (d) How well do your polynomials predict the US population in 2010 of 308.745M and in 2019 of 328.239M? On the basis of this and the least squares residual what is the best polynomial to use?
   (e) Vary the values of α with this best polynomial to see what the optimal value of α is in terms of reducing the work.

2. This question is concerned with eigenvalues and eigenvectors
   (a) Use the PowerMethod matlab code supplied to find the largest eigenvalue and corresponding eigenvalues for the matrix
   A = [1 2 3; 4 5 6; 7 8 9] with a starting vector of your choice. Verify your result with the Matlab eig function.
   (b) Repeat part (a) for the matrix B=[2 3 2; 1 0 -2 -1 -3 -1] using the initial guess [2; 3; 2] what happens and why?
   (c) What happens if you use the starting vector [1;-1;1] ? Can you explain why?
   (d) Apply the power method to finding the smallest eigenvalue of A by calculating the largest eigenvalue of A inverse. Do not generate the inverse matrix using matlab but linear solves with the matlab noting that if y = A_inverse x then Ay = x. Note only
   A single character needs to change in the matlab function PowerMethod

(e) Consider the biology example with birth rates $b_1 = 0.3, b_2 = 0.3, b_3 = 0.3, b_4 = 0.1$ and death rates $d_1 = 0.1, d_2 = 0.2, d_3 = 0.5, d_4 = 0.9$

(i) What is the largest eigenvalue of the resulting matrix?

(ii) Based upon this eigenvalues what do you expect to happen to the population eventually?

(iii) Suppose at year zero the starting vector is $P_1^0 = 100, P_2^0 = 200, P_3^0 = 150, P_4^0 = 75$ then what is the population at year 1000. How does this match your answer to part (ii)?

(iv) Suppose the problem is changed so that the death rate P_4 is $d_4 = 0.01$ instead of 0.9. How does this change the long term dynamics of population?

3. What to turn in

For these assignments, we expect both ***SOURCE CODES*** and a written ***REPORT*** be uploaded as a zip or tarball file to Canvas.

- Source code for all programs that you write, thoroughly documented.
  - Include a README file describing how to compile and run your code.
- Your report should be in PDF format and should stand on its own.
  - It should describe the methods used.
  - It should explain your results and contain figures.
  - It should also answer any questions asked above.
  - It should cite any sources used for information, including source code.
  - It should list all of your collaborators.

This homework is due on April 14th by 11:59 pm. If you don't understand these directions, please send questions to the instructoir or the TAs or raise questions at office hours or with the TAs on Dischord..