CS 3200: Introduction to Scientific Computing

In-class Activity 3 : Numerical Integration 2

Problem:. Use various numerical integration methods to determine the integral of the function. (For all methods, $N = 3$)

$\int_a^b f(x)\,dx$, where $a = 1, b = 4, f(x) = 2x^2 + x + 1$ (exact solution= 52.5)

Use these values of f(1) in solving the problems:

$f(1) = 4$	$f\left(-\frac{3}{2}\sqrt{\frac{3}{5}}+\frac{5}{2}\right) = f(1.34) = 5.9$	$f\left(\frac{3}{2}\right) = 7$	$f(2) = 11$	$f\left(\frac{5}{2}\right) = 16$
$f(3) = 22$	$f\left(\frac{3}{2}\sqrt{\frac{3}{5}}+\frac{5}{2}\right) = f(3.66) = 31.5$		$f\left(\frac{7}{2}\right) = 29$	$f(4) = 37$

1. **Solve** the problem using the Composite Trapezoid Method with h= 0.5

Remember:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{N} w_i f(x_i)$$

$\Delta x = \frac{b-a}{N-1}$

$x_i = a + (i-1)\Delta x$

$w_i = \begin{cases} \frac{\Delta x}{2}, & i = 1, N \\ \Delta x, & i = 2, \ldots, N-1 \end{cases}$

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2. **Solve** the problem using the Composite Simpson Rule with h= 0.5

Remember:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{N} \int_a^b g(x)dx = \sum_{i=1}^{2N+1} w_i f(x_i)$$

$$\Delta x = \frac{b-a}{2N}$$

$$w_i = \begin{cases} \frac{\Delta x}{3} & : \quad i = 1, 2N+1 \\ \frac{4\Delta x}{3} & : \quad i = 2, \ldots, 2N \quad (i \text{ even}) \\ \frac{2\Delta x}{3} & : \quad i = 3, \ldots, 2N-1 \quad (i \text{ odd}) \end{cases}$$

$$x_i = a + (i-1)\Delta x$$

3. **Solve** the problem using the Gauss-Legendre Rule with N=2

Remember:

$$\int_a^b f(x)dx \approx \frac{b-a}{2} \sum_{i=1}^{N} w_i f\left(\frac{b-a}{2} x_i + \frac{a+b}{2}\right)$$

N	x_i	w_i
1	0	2
2	$\pm\sqrt{\frac{1}{3}}$	1
3	0	$\frac{8}{9}$
	$\pm\sqrt{\frac{3}{5}}$	$\frac{5}{9}$

$f((5-\sqrt{3})/2) = 7.973720558, \;\; f((5+\sqrt{3})/2) = 27.0262793$

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