Quantum Docs

Dan Sehayek

September 1, 2016

Welcome to Quantum Docs! I recently took notes from an archived course on edX called Quantum Mechanics and Quantum Computation. This course was offered by Berkeley University. This document serves as a summary of my notes from the course.

1 Principles of Quantum Mechanics

Quantum mechanics is very mysterious and counter intuitive in that it completely contradicts our understanding of what happens in the macroscopic world. Many of the concepts that we discuss in quantum mechanics lead to absurd implications that completely defy any logic. While its counter intuitiveness makes it difficult for any one to accept: It has many wonderful applications that are constantly being developed and studied such as quantum computing and quantum optics. Features of quantum mechanics include:

One cannot have complete knowledge of the state of a system.

Any measurement that it made on a particle will disturb is state.

Quantum objects behave like a particle in some ways and like a wave in other ways.

2 The Double Slit Experiment

It has been established that light is an electromagnetic wave propagating through space. Yet it has also been established light is composed of individual particles called photons. What is the reason for this wave particle duality?

Imagine randomly firing marbles toward a barrier. Certain marbles would pass through Slit 1 while other marbles would pass through Slit 2. And some would be blocked by the barrier. Any marble that passes the barrier lands and is traced on the observing screen behind the barrier.

We would imagine that if we closed slit 2: The marbles would only be able to travel through Slit 1. We would thus have a small peak behind Slit 1. A graph showing the number of times marbles hit a position y on the observing screen would produce a normal distribution behind Slit 1. Note that while some would land directly behind Slit 1: Others would land with a certain offset due to the random initial directions and collisions with the edges of the slit. A similar thing would happen if we closed Slit 1 and opened Slit 2.

If analysed the distribution pattern based on position y with both slits open: We would see that sum of the graph/normal distribution from Slit 1 and the graph/normal distribution from Slit 2. These graphs can also be thought of as representing the probability that a bullet will land at a particular spot y on the screen. We let $P_1(y))$ represent the probability that a marble will land at position y when only Slit 1 is open and let $P_2(y))$ represent the probability that a marble will land at position y when only Slit 2 is open. Additionally: We let $P_{12}(y)$ represent the probability that the marble lands at position y when both slits are open. Then it would follow that $P_{12}(y) = P_1(y) + P_2(y)$.

Let us now consider what would happen if we allowed waves to travel through the slits. When the waves pass through the slits and diffract: An interference pattern occurs. This can be seen by plotting the intensity or the amount of energy carried by the waves at each position y on the observing screen. The dark spots of the interference pattern occur when the wave from Slit 1 arrives completely out of phase with the wave from Slit 2. That is that the path difference is a half integer multiple of the wavelength ($PD = n\lambda \setminus 2$ where n is any odd integer). The bright spots occur when the wave from Slit 1 arrives completely in phase with the wave from Slit 2. That is that the path difference is an integer multiple of the wavelength ($PD = n\lambda$ where n is any integer multiple). The bright spot in the middle is bright because the path length difference between the source at Slit 1 and the source at Slit 2 is equal to 0 (the two waves travel the exact same distance). The first dark spots occur when the path length difference is half of a wavelength (one wave travels a distance of half a wavelength more than the other). Note that it is not the intensities/energies of the waves that add but the heights.

We therefore have $I_{12}(y) \neq I_1(y) + I_2(y)$ and $h_{12}(y) = h_1(y) + h_2(y)$ where $I_{12}(y) = h(y)^2$.

We understand that lowering the intensity of the marbles means lowering the rate at which the marbles are fired. In other words: It is the frequency with which the marbles hit the screen and not the energy that it transferred to the screen after each collision that is lowered.

Lowering the intensity of waves means decreasing the amplitudes. In this case: It is the energy that is transferred to the screen and not the frequency of the waves hitting the screen that is lowered.

When Young performed this experiment with light: An interference pattern was produced. This would suggest to us that light is a wave. But let us imagine that the observing screen consisted of thousands of tiny photo detectors. High intensities of light would cause the photo detectors to absorb a lot of energy while low intensities would cause the photo detectors to absorb less energy. If we produce a graph that represents the intensity at each position y on the screen: We absorb the same interference pattern as described earlier.

Now imagine that we turn down the intensity of the light. To begin with: The intensity amplitudes on the distribution graph would gradually decrease as expected. However: If we lower the energy enough: We will reach a point when all of the photo detectors will record the same minimum energy E_0 but at different rates. This energy E_0 corresponds to the energy carried by a single photon.

Recall that we established that marble hitting a screen would always have the same energy but different frequencies. Photo detectors in bright spots of the interference pattern would record energy E_0 very frequently while dark spots would record energy E_0 a lot less frequently. The behaviour resembles the behaviour of marbles or particles rather than waves. At this point: Photons seems to have behaviour that resembles that of a wave and behaviour that resembles that of a particle. This is the wave particle duality phenomenon that was described earlier.

Let us try something different. We will lower the intensity to the point where only one photo detector will record something each second. This means that the source only sends one photon at a time. We plot a graph that resembles the number of photons that were absorbed by each photo detector. The resulting distribution will represent the probability that a single photon will land at a particular position y.

We would expect that the photon would go through either one slit or the other. Then similar to the marbles: The probability that the photon would land on position y would be $P_{12}(y) = P_1(y) + P_2(y)$. This would correspond to a two peaked distribution where each corresponds to either Slit 1 or Slit 2. However: This is not what we see! Instead we see the same interference pattern as before.

We understand that in order to have an interference pattern: Light from one slit must interference with light from the other slit. However: There is only photon passing the barrier each time. The current explanation is that the photon travels through both slits and then interferes with itself. We establish that the probability $P(y)$ that a photon will be recorded at position y is proportional to the square of the probability amplitude $a(y)$. We have that $a_{12}(y) = a_1(y) + a_2(y)$ and that $P_{12}(y) = |a_{12}(y)|^2 = P_1(y) + P_2(y)$.

It would make sense for us to try to determine which slit each photon went through. We may do this by placing a photo detector at each slit so that each time a photon travelled through the barrier: We would be able to record which slit it went through and where it hit the screen. The amazing thing is that if we perform this experiment: There is no longer an interference pattern! The very act of observation causes a particle like behaviour rather than a wave like behaviour. This is an example of how measurement and observation alters a quantum system.

This is referred to wave particle duality. Amazingly: Performing the same experiment with electrons would produce the same result.

3 Axioms of Quantum Mechanics

The superposition principle establishes that a particle can be in two different states at the same time.

The measurement principle establishes that the act of measuring a particle influences its state.

The unitary evolution axiom describes how the state of a quantum system evolves with time.

4 Schrödinger's Cat

Schrödinger's Cat experiment was a thought experiment designed by Schrdinger with the purpose of helping to establish the absurd and counter intuitive implications of quantum mechanics. We recently established the axioms of quantum mechanics. The superposition principle essentially states that while you are not looking: An object can be in more than one state at once. The measurement rule essentially states that when we measure the related property such as position: The object has to choose to be in just one state.

In Young's Double Slit Experiment: We established that the photon or electron passes through both slits at the same time and therefore is in more than one state at once. This is the superposition principle. We also established that the photon or electron immediately begins passing through one slit each time and therefore only occupies one state as soon as we start looking. This is the measurement rule.

The setup for this experiment is as follows. Suppose we have an atom that has a 50 percent of radioactively decaying within the next minute. We then set up a Geiger counter that will fire if the atom decays and will not fire if the atoms does not decay. If the Geiger counter fires: Poisonous gas will be released and the cat will be dead. If the Geiger counter does not fire: Poisonous gas will not be released and the cat will be alive.

We place this setup in a box with a cat. The box is sealed shut for one minute so that there is no way of observing what is happening inside the box. This means that we cannot see what happened to the atom. The atom is either decayed or not decayed each with a 50 percent chance of occurring. The superposition principle of quantum mechanics establishes that while the box is closed: The atom is in all possible states at the same time. That is that the atom is both decayed and not decayed. The Geiger counter is therefore both fired and not fired. The poisonous gas is both released and not released. The cat is both dead and alive.

When you open the box: We are measuring the state of the atom and that cat therefore must be either dead or alive. This is the measurement principle in effect. But how could the cat have been both dead and alive at the same time before we opened the box?

Now suppose that we were able to complete the same exact experiment with a time span of 8 hours instead of one minute. If we find that the cat is alive: The cat would be hungry considering that it has not eaten for 8 hours. If we find that the cat is dead: An examination by a veterinary forensic pathologist would determine that the cat died eight hours ago. Our observation not only determines the current reality but also creates the history appropriate to that reality.