Сосонов

№ 10.3

$$y'' + y' = \frac{1}{1+e^x}$$

$k^2 + k = 0$

$k(k+1) = 0, \quad k_1 = 0$

$k_2 = -1$

$y_0 = C_1 + C_2 e^{-x}, \quad y_{o.н} = y^0 + \overline{y}$

$\left.\begin{array}{l} y_1 = e^0 = 1 \\ y_2 = e^{-x} \end{array}\right\}$ независимые решения

$U = C_1(x) + C_2(x) e^{-x}$

$$\begin{cases} C_1'(x) + C_2'(x) e^{-x} = 0 \\ C_1'(x) \cdot 0 - C_2'(x) e^{-x} = \dfrac{1}{1+e^x} \end{cases}$$

Для данной системы:

$$W[1, e^{-x}] = \begin{vmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{vmatrix} = -e^{-x}$$

Ответ:

$$\Delta_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{1}{1+e^x} & -e^{-x} \end{vmatrix} = \frac{-e^{-x}}{1+e^x}$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & \frac{1}{1+e^x} \end{vmatrix} = \frac{1}{1+e^x}$$

$$C_1'(x) = \frac{\Delta_1}{W} = -\frac{e^{-x}}{1+e^x} \cdot \frac{1}{-e^{-x}} = \frac{1}{1+e^x}$$

$$C_1(x) = \int \frac{1}{1+e^x}\,dx = \ln|1+e^x|$$

$$C_2'(x) = \frac{\Delta_2}{W} = \frac{1}{(1+e^x)} \cdot \frac{1}{-e^{-x}} = -\frac{e^x}{1+e^x}$$

$$C_2(x) = -\int \frac{e^x}{1+e^x}\,dx = \left\{ \begin{aligned} &u = e^x & dx = \frac{du}{e^x} \\ &du = e^x dx, & \end{aligned} \right\} =$$

$$= -\int \frac{1}{1+u}\,du = -\ln|1+u| = -\ln|1+e^x|$$

$$u = \ln|1+e^x| - e^{-x}\ln|1+e^x|$$

Ответ:

$$y = C_1 + C_2 e^{-x} + \ln|1+e^x| - \ln|1+e^x| \cdot e^{-x}$$

Ответ:

$$\Delta_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{1}{1+e^x} & -e^{-x} \end{vmatrix} = \frac{-e^{-x}}{1+e^x}$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & \frac{1}{1+e^x} \end{vmatrix} = \frac{1}{1+e^x}$$

$$C_1'(x) = \frac{\Delta_1}{W} = -\frac{e^{-x}}{1+e^x} \cdot \frac{1}{-e^{-x}} = \frac{1}{1+e^x}$$

$$C_1(x) = \int \frac{1}{1+e^x}\,dx = \ln|1+e^x|$$

$$C_2'(x) = \frac{\Delta_2}{W} = \frac{1}{(1+e^x)} \cdot \frac{1}{-e^{-x}} = -\frac{e^x}{1+e^x}$$

$$C_2(x) = -\int \frac{e^x}{1+e^x}\,dx = \left\{ \begin{matrix} u = e^x & dx = \frac{du}{e^x} \\ du = e^x dx, & \end{matrix} \right\} =$$

$$= -\int \frac{1}{1+u}\,du = -\ln|1+u| = -\ln|1+e^x|$$

$$u = \ln|1+e^x| - e^{-x}\ln|1+e^x|$$

Ответ:

$$y = C_1 + C_2 e^{-x} + \ln|1+e^x| - \ln|1+e^x| \cdot e^{-x}$$