Домашнее задание N 4

N 16.5

Соколов

$$y' = \frac{1}{1+x^2}$$

$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + C_1$$

$$y = \int (\operatorname{arctg} x + C_1)\,dx$$

$$\int \operatorname{arctg} x\,dx = x \operatorname{arctg} x - \int \frac{x}{1+x^2}\,dx =$$

$$= x \operatorname{arctg} x - \frac{\ln(x^2+1)}{2} + C$$

$$y = x \operatorname{arctg} x - \frac{\ln(x^2+1)}{2} + C_1 x + C_2$$

Ответ:

$$x \operatorname{arctg} x - \frac{\ln(x^2+1)}{2} + C_1 x + C_2$$

№ 16.6 Самой

$$y'' = \frac{1}{\cos^2 x}$$

$$y' = \int \frac{1}{\cos^2 x} dx = tg\, x + C_1$$

$$y = \int (tg\, x + C_1)\, dx = -\ln|\cos x| + C_1 x + C_2$$

$$\int tg\, x\, dx = -\ln|\cos x| + C$$

$$y\left(\frac{\pi}{4}\right) = \frac{\ln 2}{2}, \quad y'\left(\frac{\pi}{4}\right) = 1$$

$$y'\left(\frac{\pi}{4}\right) = tg\frac{\pi}{4} + C_1 = 1, \quad C_1 = 0$$

$$y\left(\frac{\pi}{4}\right) = \frac{\ln 2}{2}, \quad -\ln\left|\cos\frac{\pi}{4}\right| + C_1\frac{\pi}{4} + C_2 = \frac{\ln 2}{2}$$

$$-\ln\frac{\sqrt{2}}{2} + C_2 = \frac{\ln 2}{2}$$

$$C_2 = \frac{\ln 2 + 2\ln\frac{\sqrt{2}}{2}}{2} = \frac{\ln\frac{4}{4}}{2} = 0$$

$$C_1 = C_2 = 0$$

Ответ: $-\ln|\cos x| = y$

№ 16.7 Смоленск

$$xy'' = y' \ln\frac{y'}{x} \quad /:x$$

$$y'' = \frac{y'}{x} \ln\frac{y'}{x}$$

$$z = y'$$

$$z' = \frac{z}{x} \ln\frac{z}{x}, \qquad u = \frac{z}{x}, \quad z = xu$$

$$z' = xu' + u$$

$$\frac{dz}{dx} = \frac{x\,du}{dx} + u$$

$$\frac{dz}{dx} = u\ln u = \frac{x\,du}{dx} + u$$

$$\frac{x\,du}{dx} = u\ln u - u$$

$$\frac{x\,du}{dx} = (\ln u - 1)u$$

$$\frac{1}{(\ln u - 1)u}\,du = \frac{1}{x}\,dx$$

$$\int \frac{1}{(\ln u - 1)u} du = \int \frac{1}{x} dx$$

Согласие

$$\int \frac{1}{(\ln u - 1)u} du = \left\{ \begin{array}{ll} t = \ln u & du = u\,dt \\ dt = \frac{1}{u} du, & \end{array} \right\} =$$

$$= \int \frac{1}{t-1} dt = \ln|t-1| + C$$

$$\ln|\ln u - 1| = \ln x + \cancel{\ln e_2} + \ln C_1$$

$$\ln u - 1 = x C_1$$

$$\ln u = x C_1 + 1$$

$$u = e^{xC_1+1}$$

$$\frac{z}{x} = e^{xC_1+1}, \quad z = x e^{xC_1+1}, \quad y' = x e^{xC_1+1}$$

$$\frac{dy}{dx} = x e^{xC_1+1}, \quad \int dy = \int x e^{xC_1+1} dx$$

$$\int x e^{xC_1+1} dx = \frac{1}{C_1} x e^{xC_1+1} - \frac{1}{C_1} \int e^{xC_1+1} dx =$$

$$= \boxed{\frac{1}{C_1}\left(x e^{xC_1+1} - \frac{1}{C_1} e^{xC_1+1}\right) + C_2 = y}$$ – Ответ

№16.8 Соколов

$$y'' + \frac{2}{1-y}(y')^2 = 0$$

$$y' = z(y)$$

$$y'' = z'(y)\,z$$

$$z'z + \frac{2}{(1-y)}z^2 = 0 \quad /:z$$

$$z' = -\frac{2}{1-y}z$$

$$\frac{dz}{dy} = -\frac{2}{1-y}z$$

$$\int \frac{1}{z}\,dz = -2\int \frac{1}{(1-y)}\,dy$$

$$\ln z = -2\ln|1-y| + \ln C_1$$

$$\ln z = \ln\left|\frac{C_1}{(1-y)^2}\right|$$

$$z = \frac{C_1}{(1-y)^2}$$

$$y' = \frac{C_1}{(1-y)^2}$$

$$\frac{dy}{dx} = \frac{C_1}{(1-y)^2}$$

$$\int (1-y)^2 dy = \int C_1 dx$$

$$\int (1-2y+y^2)\,dy = y - 2y^2 + \frac{y^3}{3}$$

$$\frac{y^3}{3} - 2y^2 + y = C_1 x + C_2$$

$$y(y^2 - 6y + 3) = 3C_1 x + 3C_2$$

$$y = \frac{3C_1 x + 3C_2}{y^2 - 6y + 3}$$

Ответ: $y = \dfrac{3C_1 x + 3C_2}{y^2 - 6y + 3}$

№ 16.9 Решение

$y'' = x \sin x$

$y' = \int x \sin x \, dx = -x \cos x - \int -\cos x \, dx = -x \cos x + \sin x + C$

$y = -\int x \cos x \, dx + \int \sin x \, dx + \int C_1 dx$

$+\int x \cos x \, dx = x \sin x + \int \sin x \, dx = x \sin x + \cos x + C$

$y = -x \sin x - \cos x - \cos x + C_1 x + C_2$

Ответ:

$y = -x \sin x - 2\cos x + C_1 x + C_2$

№ 16.10 Решение

$y''' = \frac{6}{x^3}, \quad y(1) = 2, \quad y'(1) = 1, \quad y''(1) = 3$

$y'' = \int \frac{6}{x^3} dx = 6 \int x^{-3} dx = 6 \cdot \frac{x^{-2}}{-2} + C_1 =$

$= -3x^{-2} + C_1$

$$y' = \int (-3x^{-2} + C_1)\,dx = 3x^{-1} + C_1 x + C_2$$

$$y = \int (3x^{-1} + C_1 x + C_2)\,dx = 3\ln|x| + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$y''(1) = 3$, $\quad -3 + C_1 = 3$, $\quad C_1 = 6$

$y'(1) = 1$, $\quad 3 + 6 + C_2 = 1$, $\quad C_2 = -8$

$y(1) = 2$, $\quad 3 \cdot 0 + 3 + (-8) + C_3 = 2$

$-6 + C_3 = 2$, $\quad C_3 = 8$

Ответ:

$y = 3\ln x + 3x^2 - 8x + 8$

№ 16.11 Соколов

$y'' = y' + x$

~~ДУ~~

$y'' + py' + qy = s$

$p = -1, \; q = 0, \quad s = x$

$y'' - y' = x$

$y' = z$

$z' - z = x$

$z = uv, \quad z' = u'v + uv'$

$u'v + uv' - uv = x$

$u'v + u(v' - v) = x$

$$\begin{cases} v' - v = 0 \\ u'v = x \end{cases}$$

$v' - v = 0 \quad / : v$

$\frac{v'}{v} - 1 = 0$

$$\int \frac{dv}{v} = \int 1\,dx, \qquad \ln|v| = x + C_1$$

$$v = e^{x+C_1}$$

$$u'v = \cancel{x}, \qquad u' \cdot e^{x+C_1} = x$$

$$\int du = \int x e^{-x-C_1}\,dx$$

$$\int x e^{-x-C_1}\,dx = -x e^{-x-C_1} + e^{-x-C_1} + C_2$$

$$u = -xe^{-x-C_1} + e^{-x-C_1} + C_2$$

$$z = uv = e^{x+C_1}\left(-xe^{-x-C_1} + e^{-x-C_1} + C_2\right) \neq \cancel{-x + C_2 + 1}$$

$$\cancel{z' - z = x \Rightarrow -1 + x - 1 \neq x}$$

$$\cancel{-1 + x \neq x}$$

$$z = -x + e^{x+C_1} C_2 + 1 \qquad z' - z = x$$

$$z' = -1 + C_2 e^{x+C_1} \qquad -1 + C_1 e^{x+C_1} + x - e^{x+C_1} C_1 + 1 =$$

$$= x$$

$$y' = -x + e^{x+C_1} C_2 + 1$$

$$y = \int (-x + e^{x+C_1} C_2 + 1)\,dx$$

Ответ:

$$y = C_2 e^{x+C_1} - \frac{x^2}{2} + x + C_3$$

№ 16.12 Соломов

$y y'' = (y')^2 \quad | : y$

$$y'' = \frac{(y')^2}{y}$$

$$\frac{y''}{(y')^2} = \frac{1}{y} \quad | \cdot dx$$

$$\frac{y''\,dx}{(y')^2} = \frac{dx}{y}$$

$$y' = \frac{dy}{dx}, \quad dx = \frac{dy}{y'}$$

$$\int \frac{y''\,dx}{y'} = \int \frac{1}{y}\,dy$$

$$\ln(y') = \ln y + C_1$$

$$y' = y e^{C_1}$$

$$\frac{dy}{dx} = y e^{C_1}, \quad x = \int \frac{1}{y e^{C_1}}\,dy \cancel{\pm C_2 \pm e^{-C_1} \ln y}$$

$$x = \int \frac{1}{ye^{c_1}}dy = \ln|ye^{c_1}| + C_2$$

$$x = \ln|ye^{c_1}| + C_2$$

$\cancel{\ln|ye^{c_1}=}$

$$\ln|y| = \frac{\ln e^{x}}{\ln(e^{c_1+c_2})}$$

$\cancel{y}=1$

$$\ln|ye^{c_1}e^{c_2}| = \ln e^{x}$$

$$ye^{c_1}e^{c_2} = e^{x}, \quad y = e^{x-(c_1+c_2)}$$

Ответ: $y = e^{x-(c_1+c_2)}$