Домашняя работа №11 Соснова

№ 7

Вычислить $\iint\limits_D (x^2+y^2)\,dx\,dy$, если область D ограничена линиями

$y=x$, $x=0$, $y=1$, $y=2$

Решение:

$$\iint\limits_D (x^2+y^2)\,dx\,dy = \int_1^2 dy \int_0^y (x^2+y^2)\,dx =$$

$$= \int_1^2 \left[\frac{x^3}{3} + y^2 x\right]_0^y dy = \int_1^2 \left(\frac{4y^3}{3}\right) dy =$$

$$= \left(\frac{4}{3}\cdot\frac{y^4}{4}\right)\Bigg|_1^2 = \frac{4\cdot 16}{3\cdot 4} - \frac{4}{3}\cdot\frac{1}{4} = \frac{15}{3} = 5$$

Ответ: 5

№3 Соколов

Вычислить $\iint\limits_D (3x^2-2xy+y)\,dx\,dy$,

если область D ограничена линиями

$x=0$, $x=y^2$, $y=2$, $y=0$

Решение:

$$\iint\limits_D (3x^2-2xy+y)\,dx\,dy = \int_0^2 dy \int_0^{y^2} (3x^2-2xy+y)\,dx =$$

$$= \int_0^2 \left[x^3 - yx^2 + yx\right]\Big|_0^{y^2} dy =$$

$$= \int_0^2 (y^6 - y^5 + y^3)\,dy = \left[\frac{y^7}{7} - \frac{y^6}{6} + \frac{y^4}{4}\right]\Big|_0^2 =$$

$$= \frac{128}{7} - \frac{64}{6} + \frac{16}{4} = \frac{128}{7}^{\llcorner 3} - \frac{32}{3}^{\llcorner 7} + 4^{\llcorner 21} =$$

$$= \frac{384 - 224 + 84}{21} = \frac{224}{21}$$

Ответ: $\frac{224}{21}$

№41 Соломов

$$\begin{cases} x = y^2 - 2y \\ x + y = 0 \end{cases}$$

$$x - x = y^2 - 2y + \cancel{y}$$

$$y^2 - y = 0$$

$$y(y-1) = 0, \quad y = 0$$
$$y = 1$$

$A(0;0)$, $B(-1;1)$

$$\iint\limits_D dx\,dy = \int_0^1 dy \int_{y^2-2y}^{-y} dx =$$

$$= \int_0^1 x\Big|_{y^2-2y}^{-y} dy = \int_0^1 (y - y^2)\,dy =$$

$$= \left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

Ответ: $\frac{1}{6}$ (кв. ед.)

№ 43 Сазонов

$$\begin{cases} y^2 = 4x - x^2, & y = \sqrt{4x-x^2} \\ y^2 = 2x & y = \sqrt{2x} \end{cases}$$

$$y^2 - y^2 = 4x - x^2 - 2x$$

$$x(2-x) = 0 \qquad x = 0, \; y = 0$$

$$x = 2, \; y = 2$$

$$\iint\limits_{0} dx\,dy = \int_0^2 dx \int_{\sqrt{2x}}^{\sqrt{4x-x^2}} dy =$$

$$= \int_0^2 y \Big|_{\sqrt{2x}}^{\sqrt{4x-x^2}} dx = \int_0^2 \left(\sqrt{4x-x^2} - \sqrt{2x}\right) dx =$$

$$= \left[\sqrt{1 - \frac{(x-2)^2}{4}}\,(x-2) + 2\arcsin\left(\frac{x-2}{2}\right) - \frac{2\sqrt{2}\,x^{\frac{3}{2}}}{3}\right]_0^2$$

$$= -\frac{8}{3} - \left(-\frac{2\pi}{2}\right) = \pi - \frac{8}{3}$$

Ответ: $\left(\pi - \frac{8}{3}\right)$ (кв. ед.)

$$\int \sqrt{4x - x^2}\, dx = \int \sqrt{4 - (x-2)^2}\, dx =$$

$$= \left\{ \begin{array}{l} u = x - 2 \\ du = dx \end{array} \right\} = \int \sqrt{4 - u^2}\, du =$$

$$= \left\{ \begin{array}{l} u = 2\sin(v), \quad v = \arcsin \frac{u}{2}, \\ du = 2\cos(v)\, dv \end{array} \right\} =$$

$$= \int 2\cos v \sqrt{4 - 4\sin^2(v)}\, dv$$

$$4 - 4\sin^2(v) = 4\cos^2(v)$$

$$\int 2\cos v \sqrt{4 - 4\sin^2 v}\, dv = \int 2\cos v \cdot 2\cos v\, dv =$$

$$= 4\int \cos^2(v)\, dv ;$$

$$\int \cos^n(v)\, dv = \frac{n-1}{n} \int \cos^{n-2}(v)\, dv +$$

$$+ \frac{\cos^{n-1}(v)\sin(v)}{n}$$

Сегодня

при $n = 2$.

$$\int \cos^2 v \, dv = \frac{\cos(v)\sin(v)}{2} + \frac{1}{2}\int dv =$$

$$= \frac{\cos(v)\sin(v) + v}{2}; \quad \cos(\arcsin \frac{u}{2}) = \sqrt{1 - \frac{u^2}{4}}$$

$$4\int \cos^2(v)\, dv = 2\cos(v)\sin(v) + 2v =$$

$$= 2\cos(\arcsin \frac{u}{2})\sin(\arcsin \frac{u}{2}) + 2\arcsin \frac{u}{2} =$$

$$= u\sqrt{1 - \frac{u^2}{4}} + 2\arcsin \frac{u}{2} = \{u = x - 2\} =$$

$$= \sqrt{1 - \frac{(x-2)^2}{4}}\,(x-2) + 2\arcsin\left(\frac{x-2}{2}\right)$$

$$\int \sqrt{2x}\, dx = \frac{\sqrt{2} \cdot 2 \cdot x^{\frac{3}{2}}}{3} + C$$