Homework 2

Dan Wang (*daw1230*@uw.edu)

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1 Writing assignments

1.1 Kalman Gain

For the predicted state, we have $p_1 = \frac{1}{\bar{\sigma}\sqrt{2\pi}} \exp\left(-\frac{(x-\bar{\mu})^2}{2\bar{\sigma}^2}\right)$, that is, $x \sim \mathcal{N}(\bar{\mu}, \bar{\sigma})$.

For the observation, we have $p_2 = \frac{1}{\sigma_{obs}\sqrt{2\pi}} \exp\left(-\frac{(z-x)^2}{2\sigma_{obs}^2}\right) = \frac{1}{\sigma_{obs}\sqrt{2\pi}} \exp\left(-\frac{(x-z)^2}{2\sigma_{obs}^2}\right)$, that is, $x \sim \mathcal{N}(z, \sigma_{obs})$.

As a multiplication of the obove two, the Kalman filter correction has mean given by

$$\mu = \frac{\sigma_{obs}^2}{\overline{\sigma}^2 + \sigma_{obs}^2}\bar{\mu} + \frac{\overline{\sigma}^2}{\overline{\sigma}^2 + \sigma_{obs}^2} z = (1-K)\bar{\mu} + Kz = \bar{\mu} + K(z-\bar{\mu}),$$

where $K = \frac{\overline{\sigma}^2}{\overline{\sigma}^2 + \sigma_{obs}^2}$. For variance, we have

$$\sigma^2 = \frac{1}{\overline{\sigma}^{-2} + \sigma_{obs}^{-2}} = \frac{\sigma_{obs}^2}{\overline{\sigma}^2 + \sigma_{obs}^2}\overline{\sigma}^2 = (1-K)\overline{\sigma}^2.$$

1.2 Motion Model Jacobian

$G = \begin{pmatrix} 1 & 0 & -\delta_{trans} sin(\theta + \delta_{rot1}) \\ 0 & 1 & \delta_{trans} cos(\theta + \delta_{rot1}) \\ 0 & 0 & 1 \end{pmatrix}$ and

$V = \begin{pmatrix} -\delta_{trans} sin(\theta + \delta_{rot1}) & cos(\theta + \delta_{rot1}) & 0 \\ \delta_{trans} cos(\theta + \delta_{rot1}) & sin(\theta + \delta_{rot1}) & 0 \\ 1 & 0 & 1 \end{pmatrix}$.

2 Programming assignments

2.1 EKF implementation

1. See Fig. 1.
2. See Fig. 2. The mean position error increases as the data factor and filter factor increase.
3. See Fig. 3. The mean position error remains small and is not affected by the filter factor, while the ANEES decreases as the filter factor increases.

2.2 PF implementation

1. See Fig. 4.
2. See Fig. 5. The mean position error increases as the data factor and filter factor increase.
3. See Fig. 6. The mean position changes parabolically with the filter factor. The ANEES increases dramatically as the filter factor decreases.
4. See Fig. 7. Note that the y-axes are in different scales. With particle numbers increasing, the ANEES decreases very obviously. The mean position error also decreases, but the effect of increasing particle number on mean position error is not as much as on ANEES.

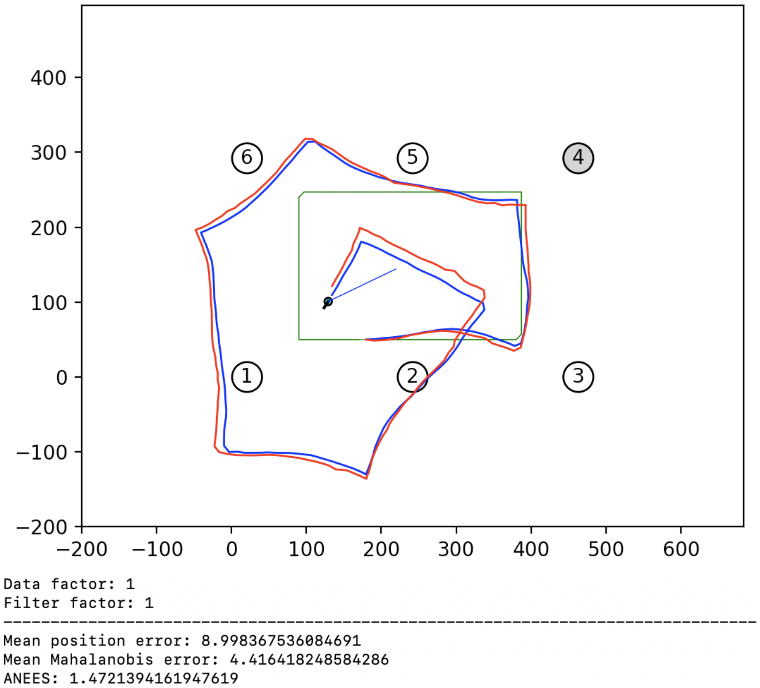

Figure 1: EKF: Real robot path and the filter path under the default parameters

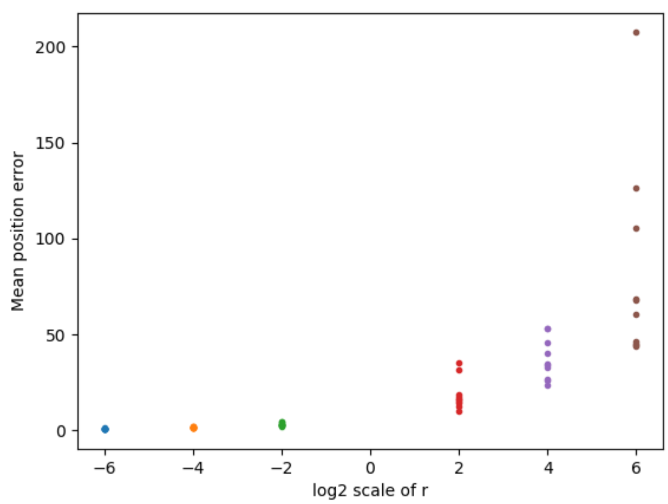

Figure 2: EKF: Mean position error with regard to log2 scale of r

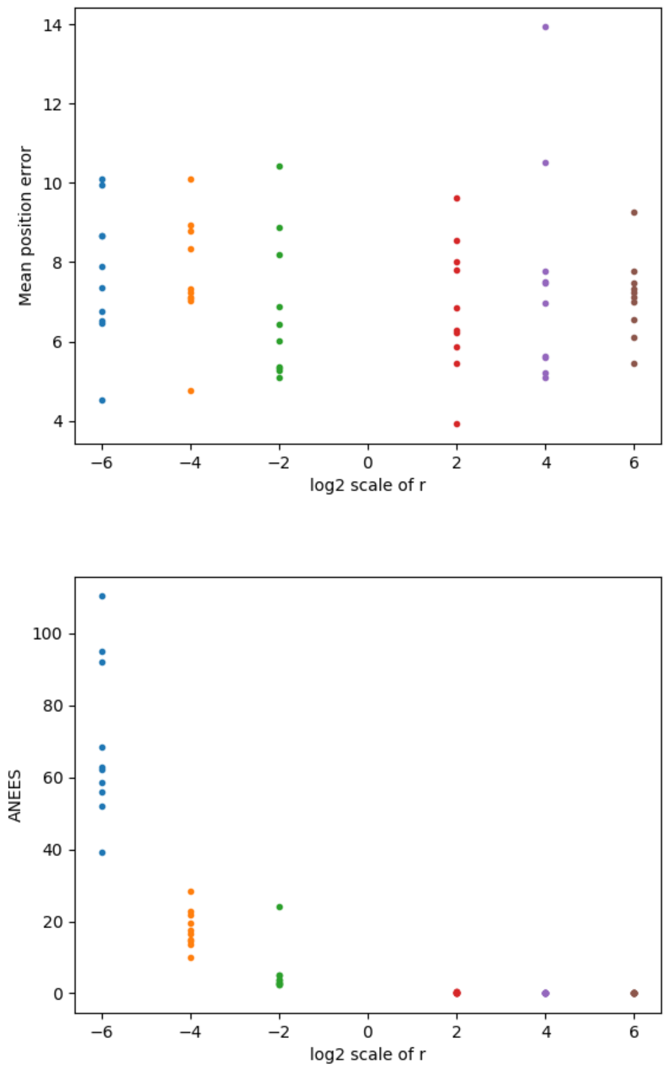

Figure 3: EKF: Mean position error and ANEES with regard to log2 scale of r

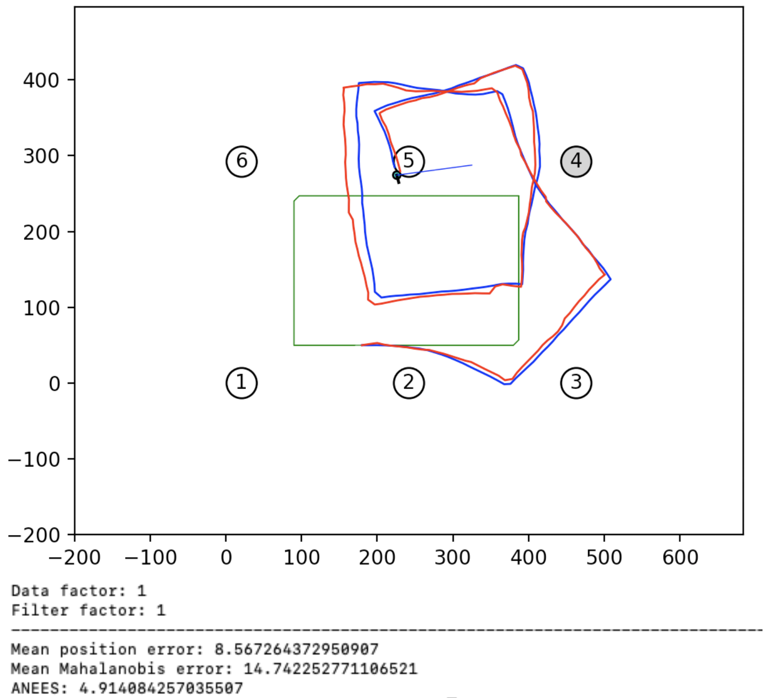

Figure 4: PF: Real robot path and the filter path under the default parameters

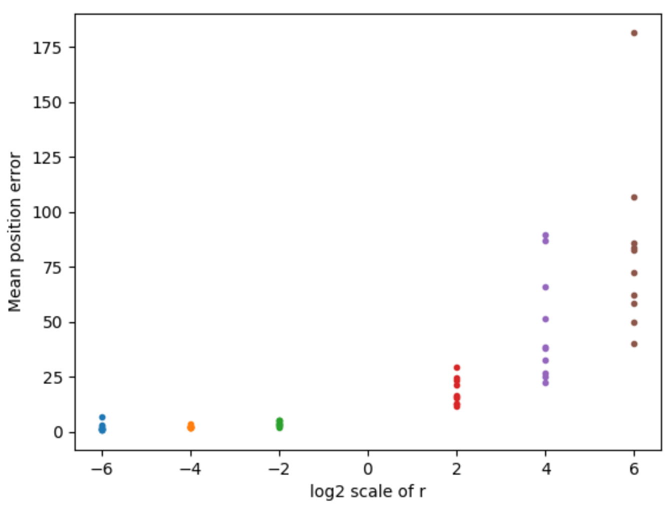

Figure 5: PF: Mean position error with regard to log2 scale of r

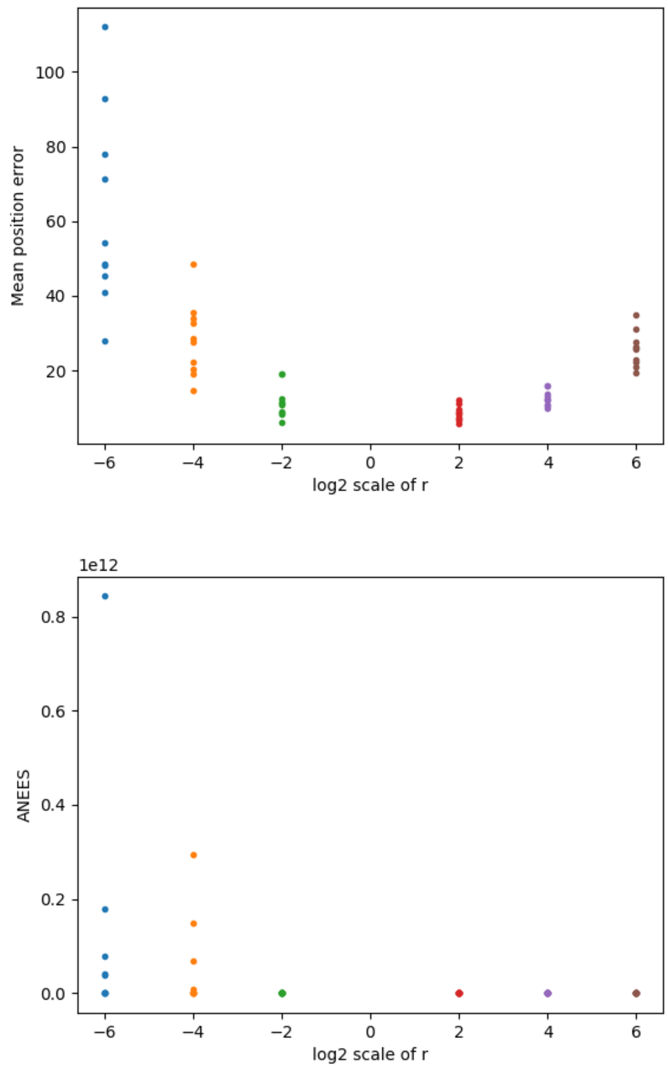

Figure 6: PF: Mean position error and ANEES with regard to log2 scale of r

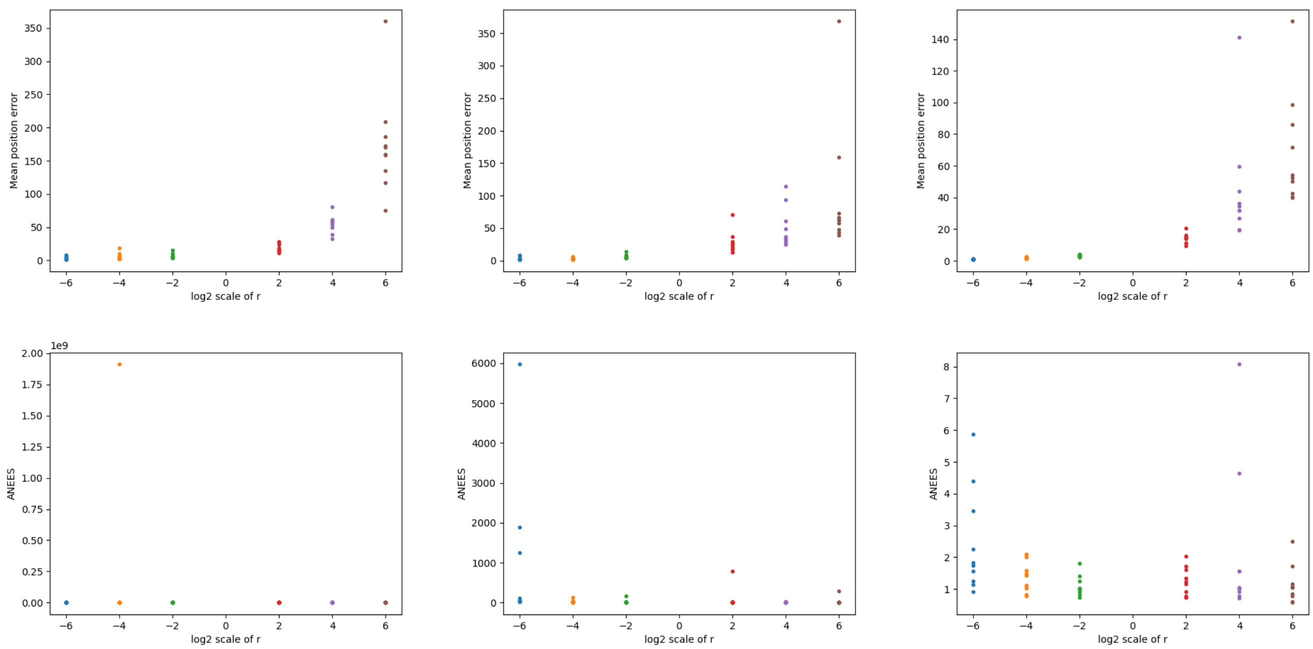

Figure 7: PF: Mean position error (top three plots with 20, 50, and 500 particles, respectively) and ANEES (bottom three plots with 20, 50, and 500 particles, respectively) with regard to log2 scale of r