Homework 3

Dan Wang

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1 A* Implementation

Evaluation upon different ϵ values and visualize the final path and state visited:

1. $\epsilon = 1$
Cost of path: 251.450793
State expanded: 1781

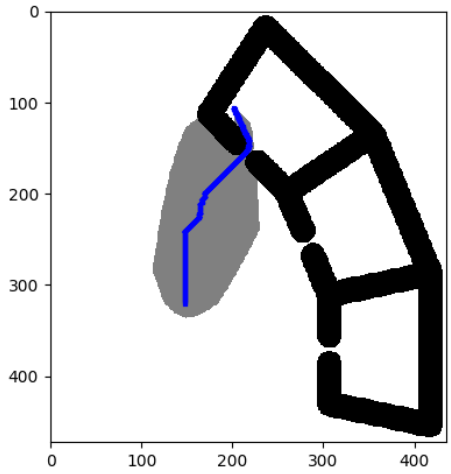


Figure 1: $\epsilon = 1$

2. $\epsilon = 10$
Cost of path: 257.308658
State expanded: 1224

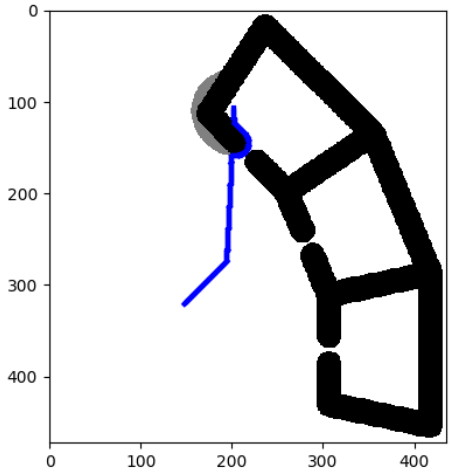


Figure 2: $\epsilon = 10$

3. $\epsilon = 20$
Cost of path: 256.722871
State expanded: 1591

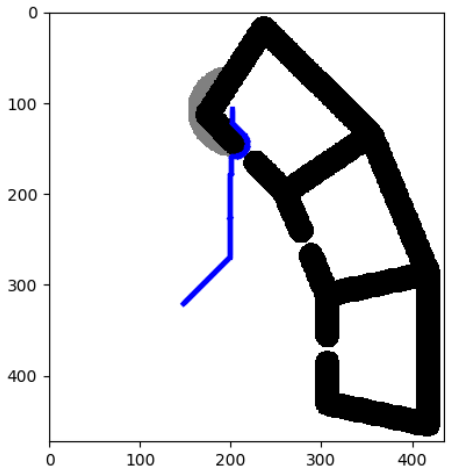


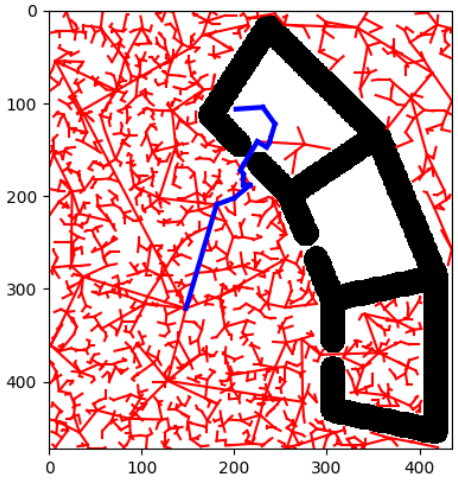
Figure 3: $\epsilon = 20$

Discussions: Vanilla A* is guaranteed to find the lowest cost of path, that is, the shortest path. Weighted A* can reduce the number of expanded states and thus improve the efficiency, at the cost of deviating somewhat from the optimal path. Also, as ϵ getting larger (like 20 in this case), the effect of reducing the expanded-state number can be compromised.

2 RRT Implementation

Providing mean and standard deviation of cost and plan time over 10 runs, this section reports results of the three cases: (5% probability, $\eta = 1$), (20% probability, $\eta = 1$), and (20% probability, $\eta = 0.5$). We can compare the first two cases for problem 1 and the last two cases for problem 2.

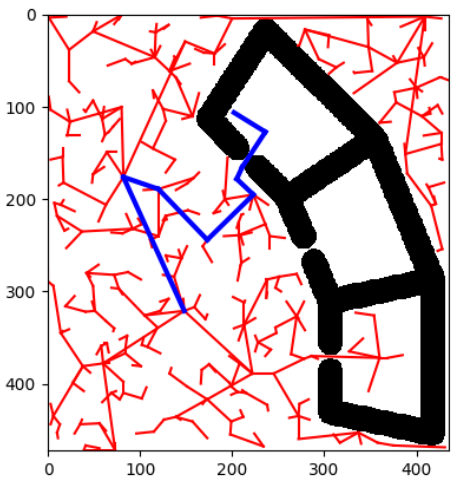
Case 1: Bias the sampling to pick the goal with 5% probability (and $\eta = 1$)
Mean of cost: 372.88
Standard deviation of cost: 50.85
Mean of plan time: 39.34 s
Standard deviation of plan time: 60.00 s
An example of final state is shown below:




```
Cost: 339.049318
Planning Time: 10s
Cost: 341.922868
Planning Time: 208s
Cost: 371.109690
Planning Time: 64s
Cost: 436.614827
Planning Time: 45s
Cost: 363.789900
Planning Time: 0s
Cost: 263.290543
Planning Time: 5s
Cost: 353.312149
Planning Time: 15s
Cost: 416.225835
Planning Time: 38s
Cost: 405.005779
Planning Time: 2s
Cost: 438.444776
Planning Time: 2s
```

Figure 4: RRT: Final state of the tree with cost of 308.540530 and plan time of 22 s as well as statistical results of 10 runs

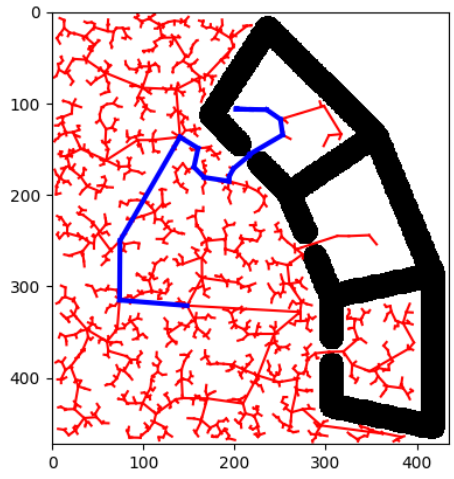
Case 2: Bias the sampling to pick the goal with 20% probability (and $\eta = 1$)
Mean of cost: 395.23
Standard deviation of cost: 100.00
Mean of plan time: 19.27 s
Standard deviation of plan time: 22.94 s
An example of final state is shown below:




```
Cost: 317.480458
Planning Time: 1s
Cost: 335.400195
Planning Time: 4s
Cost: 353.580807
Planning Time: 5s
Cost: 403.381393
Planning Time: 80s
Cost: 518.311291
Planning Time: 26s
Cost: 345.377703
Planning Time: 6s
Cost: 645.571273
Planning Time: 36s
Cost: 325.969468
Planning Time: 3s
Cost: 354.943424
Planning Time: 13s
Cost: 352.278868
Planning Time: 15s
```

Figure 5: RRT: Final state of the tree with cost of 470.389497 and plan time of 2 s as well as statistical results of 10 runs

Case 3: Bias the sampling to pick the goal with 20% probability (and $\eta = 0.5$)
Mean of cost: 308.55
Standard deviation of cost: 60.74
Mean of plan time: 4.51 s
Standard deviation of plan time: 5.51 s
An example of final state is shown below:




```
Cost: 257.153962
Planning Time: 0s
Cost: 258.062075
Planning Time: 12s
Cost: 315.405578
Planning Time: 2s
Cost: 255.716008
Planning Time: 0s
Cost: 423.218747
Planning Time: 17s
Cost: 285.861365
Planning Time: 2s
Cost: 272.669649
Planning Time: 2s
Cost: 300.897152
Planning Time: 5s
Cost: 425.017977
Planning Time: 0s
Cost: 291.486521
Planning Time: 0s
```

Figure 6: RRT: Final state of the tree with cost of 508.04 and plan time of 21 s as well as statistical results of 10 runs

Discussions:

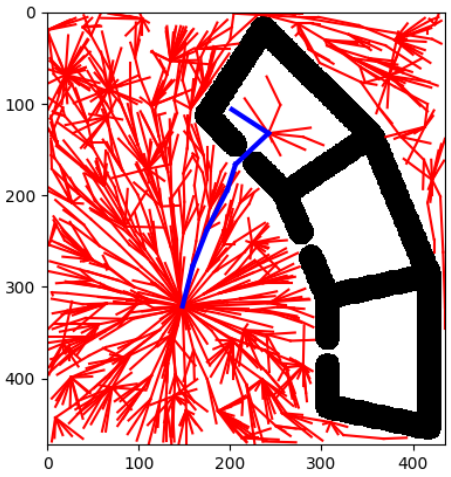
1. Comparing Case 1 and Case 2, we can tell increasing the probability can reduce the plan time.

2. Comparing Case 2 and Case 3, we can tell decreasing η can reduce both the plan time and mean of cost.

3 RRT* Implementation

Using the same evaluation scheme as in Section 2.

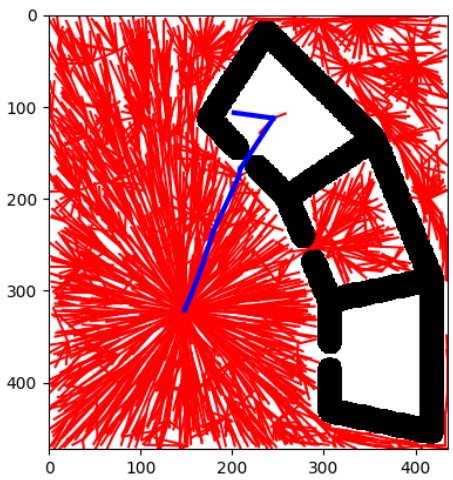
Case 1: Bias the sampling to pick the goal with 5% probability (and $\eta = 1$)
Mean of cost: 279.26
Standard deviation of cost: 20.84
Mean of plan time: 223.11 s
Standard deviation of plan time: 402.04 s
An example of final state is shown below:




```
Cost: 313.975512
Planning Time: 112s
Cost: 267.382084
Planning Time: 19s
Cost: 268.258095
Planning Time: 156s
Cost: 297.763051
Planning Time: 45s
Cost: 256.244285
Planning Time: 71s
Cost: 264.867889
Planning Time: 1408s
Cost: 245.714513
Planning Time: 131s
Cost: 294.707703
Planning Time: 9s
Cost: 299.480076
Planning Time: 16s
Cost: 284.215580
Planning Time: 258s
```

Figure 7: RRT*: Final state of the tree with cost of 263.96 and plan time of 66 s as well as statistical results of 10 runs

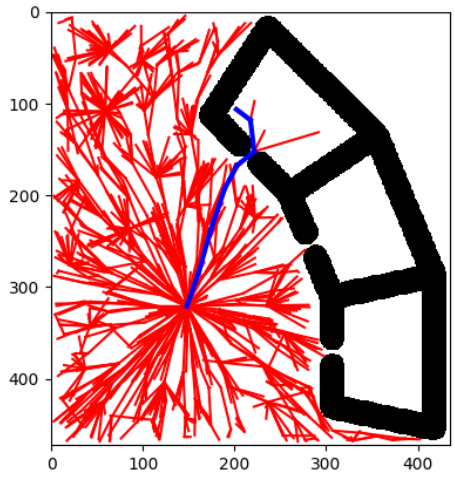
Case 2: Bias the sampling to pick the goal with 20% probability (and $\eta = 1$)
Mean of cost: 329.17
Standard deviation of cost: 92.14
Mean of plan time: 75.1 s
Standard deviation of plan time: 114.84 s
An example of final state is shown below:




```
Cost: 284.831428
Planning Time: 21s
Cost: 287.248366
Planning Time: 43s
Cost: 305.514532
Planning Time: 62s
Cost: 588.270122
Planning Time: 8s
Cost: 271.623631
Planning Time: 56s
Cost: 326.238173
Planning Time: 25s
Cost: 318.260969
Planning Time: 414s
Cost: 256.753261
Planning Time: 75s
Cost: 375.339169
Planning Time: 19s
Cost: 277.577667
Planning Time: 24s
```

Figure 8: RRT*: Final state of the tree with cost of 275.51 and plan time of 403 s as well as statistical results of 10 runs

Case 3: Bias the sampling to pick the goal with 20% probability (and $\eta = 0.5$)
Mean of cost: 255.54
Standard deviation of cost: 23.61
Mean of plan time: 553.17 s
Standard deviation of plan time: 1148.05 s
An example of final state is shown below:




```
Cost: 274.312924
Planning Time: 33s
Cost: 247.459560
Planning Time: 125s
Cost: 250.566362
Planning Time: 363s
Cost: 255.860503
Planning Time: 165s
Cost: 241.091283
Planning Time: 25s
Cost: 237.982585
Planning Time: 124s
Cost: 248.911513
Planning Time: 63s
Cost: 239.832150
Planning Time: 3949s
Cost: 319.431285
Planning Time: 11s
Cost: 239.985800
Planning Time: 668s
```

Figure 9: RRT*: Final state of the tree with cost of 242.16 and plan time of 84 s as well as statistical results of 10 runs

Discussions: Compared with the RRT results in Section 2, RRT* algorithm

needs more plan time (i.e., large mean of plan time with large standard deviation) but achieves lower cost. Also, we can reach the same conclusions as in Section 2 regarding changing η and bias probability.

4 Nonholonomic Car: RRT Implementation

Providing mean and standard deviation of cost and plan time over 10 runs:

1. Case 1: Bias the sampling to pick the goal with 5% probability
   Mean of cost: 493.2 s
   Standard deviation of cost: 58.14 s
   Mean of plan time: 25.35 s
   Standard deviation of plan time: 18.14 s
   An example of final state is shown below:

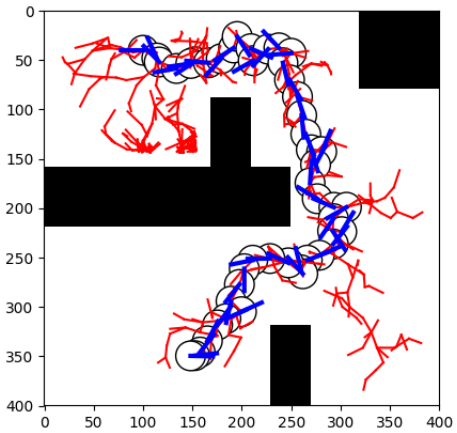


Figure 10: Nonholonomic Car: Final state [349.46, 148.05, 1.55] reached with cost of 469 s and plan time of 10 s (Goal state is [350, 150, 1.57])

```
Goal reached! State: [349.25757456 146.95168964   1.5418329 ], Goal state: [350.   150.     1.57]
xy_diff: 3.137, ang_diff: 1.614
Cost: 428.000000
Planning Time: 32s
Goal reached! State: [350.64427825 147.34808629   1.54164984], Goal state: [350.   150.     1.57]
xy_diff: 2.729, ang_diff: 1.624
Cost: 551.000000
Planning Time: 33s
Goal reached! State: [346.66507237 142.27264453   1.55920958], Goal state: [350.   150.     1.57]
xy_diff: 8.416, ang_diff: 0.618
Cost: 566.000000
Planning Time: 19s
Goal reached! State: [348.24994211 141.4770815    1.59176042], Goal state: [350.   150.     1.57]
xy_diff: 8.701, ang_diff: 1.247
Cost: 525.000000
Planning Time: 20s
Goal reached! State: [349.86210257 148.63165997   1.53993272], Goal state: [350.   150.     1.57]
xy_diff: 1.375, ang_diff: 1.723
Cost: 418.000000
Planning Time: 8s
Goal reached! State: [349.33022658 144.57276557   1.62477351], Goal state: [350.   150.     1.57]
xy_diff: 5.468, ang_diff: 3.138
Cost: 415.000000
Planning Time: 7s
Goal reached! State: [350.12897889 146.58813323   1.53078063], Goal state: [350.   150.     1.57]
xy_diff: 3.414, ang_diff: 2.247
Cost: 450.000000
Planning Time: 12s
Goal reached! State: [351.45940852 153.63213965   1.49798808], Goal state: [350.   150.     1.57]
xy_diff: 3.914, ang_diff: 4.126
Cost: 516.000000
Planning Time: 70s
Goal reached! State: [354.65586178 141.91649293   1.50095394], Goal state: [350.   150.     1.57]
xy_diff: 9.328, ang_diff: 3.956
Cost: 494.000000
Planning Time: 37s
Goal reached! State: [349.63900659 152.77842354   1.5515955 ], Goal state: [350.   150.     1.57]
xy_diff: 2.802, ang_diff: 1.055
Cost: 569.000000
Planning Time: 10s
```

Figure 11: Nonholonomic Car: Statistical results of 10 runs

Case 2: Bias the sampling to pick the goal with 20% probability
Mean of cost: 503.2 s
Standard deviation of cost: 43.22 s
Mean of plan time: 21.23 s
Standard deviation of plan time: 18.01 s
An example of final state is shown below:

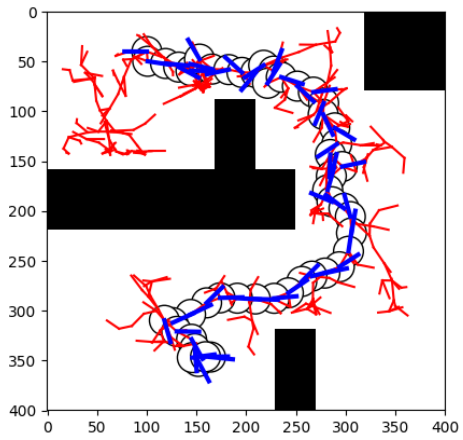


Figure 12: Nonholonomic Car: Final state [346.05, 159.0, 1.58] reached with cost of 484 s and plan time of 9 s (Goal state is [350, 150, 1.57])

```
Goal reached! State: [349.29872578 153.42964163   1.48865564], Goal state: [350.   150.     1.57]
xy_diff: 3.501, ang_diff: 4.661
Cost: 542.000000
Planning Time: 13s
Goal reached! State: [350.4690014  144.57404518   1.61468966], Goal state: [350.   150.     1.57]
xy_diff: 5.446, ang_diff: 2.561
Cost: 509.000000
Planning Time: 11s
Goal reached! State: [354.63545911 141.6687515    1.5669464 ], Goal state: [350.   150.     1.57]
xy_diff: 9.534, ang_diff: 0.175
Cost: 549.000000
Planning Time: 56s
Goal reached! State: [346.15856367 158.91260873   1.63022911], Goal state: [350.   150.     1.57]
xy_diff: 9.705, ang_diff: 3.451
Cost: 475.000000
Planning Time: 4s
Goal reached! State: [349.24740733 154.36983829   1.55400978], Goal state: [350.   150.     1.57]
xy_diff: 4.434, ang_diff: 0.916
Cost: 437.000000
Planning Time: 6s
Goal reached! State: [348.19884919 145.80059523   1.56438899], Goal state: [350.   150.     1.57]
xy_diff: 4.569, ang_diff: 0.321
Cost: 461.000000
Planning Time: 13s
Goal reached! State: [344.9491365  141.92088217   1.6561591 ], Goal state: [350.   150.     1.57]
xy_diff: 9.528, ang_diff: 4.937
Cost: 498.000000
Planning Time: 25s
Goal reached! State: [351.43109899 140.42419608   1.5943847 ], Goal state: [350.   150.     1.57]
xy_diff: 9.682, ang_diff: 1.397
Cost: 471.000000
Planning Time: 12s
Goal reached! State: [348.17069024 155.17819498   1.59214752], Goal state: [350.   150.     1.57]
xy_diff: 5.492, ang_diff: 1.269
Cost: 503.000000
Planning Time: 13s
Goal reached! State: [349.97434122 150.37742576   1.5666167 ], Goal state: [350.   150.     1.57]
xy_diff: 0.378, ang_diff: 0.194
Cost: 587.000000
Planning Time: 54s
```

Figure 13: Nonholonomic Car: Statistical results of 10 runs

2. Biasing the sampling to pick the goal with 5% probability and providing

mean and standard deviation of cost and plan time over 10 runs (from seed(0) to seed(9)):

Using the new distance function:

Mean of cost: 506.57 s

Standard deviation of cost: 51.64 s

Mean of plan time: larger than 101.46 s

Standard deviation of plan time: 124.15 s

Note seeds 5, 6, and 8 are very time-consuming and thus omitted here.

An example of final state using seed(0) is shown below:

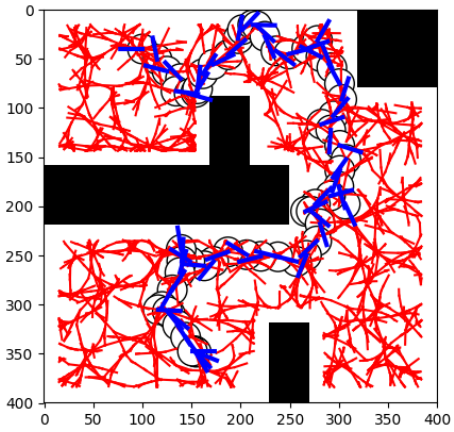


Figure 14: Nonholonomic Car: New distance: Final state [347.36, 150.88, 1.57] reached with cost of 590 s and plan time of 366 s (Goal state is [350, 150, 1.57])

```
Goal reached! State: [347.36065228 150.88351036   1.56961649], Goal state: [350.   150.     1.57]
xy_diff: 2.783, ang_diff: 0.022
Cost: 590.000000
Planning Time: 353s
Goal reached! State: [344.60828097 145.49734818   1.51646558], Goal state: [350.   150.     1.57]
xy_diff: 7.025, ang_diff: 3.067
Cost: 436.000000
Planning Time: 7s
Goal reached! State: [347.39916538 143.49184855   1.62929819], Goal state: [350.   150.     1.57]
xy_diff: 7.009, ang_diff: 3.398
Cost: 557.000000
Planning Time: 65s
Goal reached! State: [355.21870252 142.01437254   1.64030982], Goal state: [350.   150.     1.57]
xy_diff: 9.540, ang_diff: 4.028
Cost: 471.000000
Planning Time: 14s
Goal reached! State: [341.51602764 151.56973533   1.53451817], Goal state: [350.   150.     1.57]
xy_diff: 8.628, ang_diff: 2.033
Cost: 530.000000
Planning Time: 29s
Goal reached! State: [344.8276028  148.25259452   1.55075258], Goal state: [350.   150.     1.57]
xy_diff: 5.460, ang_diff: 1.103
Cost: 504.000000
Planning Time: 222s
Goal reached! State: [359.80400038 149.71336843   1.57291377], Goal state: [350.   150.     1.57]
xy_diff: 9.808, ang_diff: 0.167
Cost: 458.000000
Planning Time: 16s
```

Figure 15: Nonholonomic Car: Provided distance: Statistical results from seeds (0)–(4), (7), and (9)

Using the provided (old) distance function:
Mean of cost: 474.9 s
Standard deviation of cost: 44.89 s
Mean of plan time: 39.91 s
Standard deviation of plan time: 67.18 s
An example of final state using seed(0) is shown below:

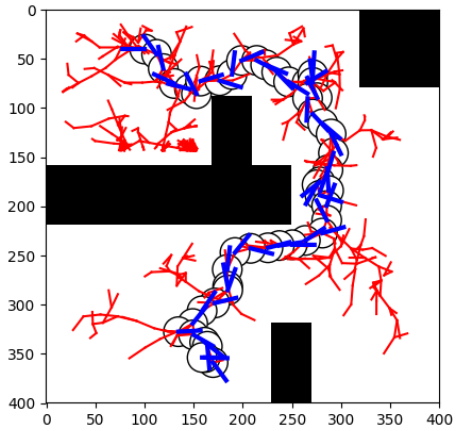


Figure 16: Nonholonomic Car: Provided distance: Final state [353.63 158.56 1.57] reached with cost of 475 s and plan time of 14 s (Goal state is [350, 150, 1.57])

```
Goal reached! State: [353.62899939 158.55538959   1.5660196 ], Goal state: [350.   150.     1.57]
xy_diff: 9.293, ang_diff: 0.228
Cost: 475.000000
Planning Time: 14s
Goal reached! State: [343.53622775 144.70257112   1.58230843], Goal state: [350.   150.     1.57]
xy_diff: 8.357, ang_diff: 0.705
Cost: 563.000000
Planning Time: 240s
Goal reached! State: [350.70875738 147.15312443   1.54204744], Goal state: [350.   150.     1.57]
xy_diff: 2.934, ang_diff: 1.602
Cost: 440.000000
Planning Time: 9s
Goal reached! State: [350.54405561 153.54556078   1.53134729], Goal state: [350.   150.     1.57]
xy_diff: 3.587, ang_diff: 2.215
Cost: 470.000000
Planning Time: 11s
Goal reached! State: [350.03388637 150.41380858   1.56810979], Goal state: [350.   150.     1.57]
xy_diff: 0.415, ang_diff: 0.108
Cost: 489.000000
Planning Time: 17s
Goal reached! State: [346.79322944 141.88555303   1.57950928], Goal state: [350.   150.     1.57]
xy_diff: 8.725, ang_diff: 0.545
Cost: 470.000000
Planning Time: 23s
Goal reached! State: [348.48595436 152.9264989    1.55695551], Goal state: [350.   150.     1.57]
xy_diff: 3.295, ang_diff: 0.747
Cost: 493.000000
Planning Time: 11s
Goal reached! State: [354.31895113 158.6390748    1.57598183], Goal state: [350.   150.     1.57]
xy_diff: 9.659, ang_diff: 0.343
Cost: 393.000000
Planning Time: 36s
Goal reached! State: [349.25761995 145.16448238   1.56331018], Goal state: [350.   150.     1.57]
xy_diff: 4.892, ang_diff: 0.383
Cost: 522.000000
Planning Time: 12s
Goal reached! State: [350.52885488 152.79205494   1.62251709], Goal state: [350.   150.     1.57]
xy_diff: 2.842, ang_diff: 3.009
Cost: 434.000000
Planning Time: 21s
```

Figure 17: Nonholonomic Car: Provided distance: Statistical results from seed(0) to seed(9)

Discussions: The new distance function doesn't consider the practical way that the car moves. Compared with the provided distance function, the new distance function yields larger cost and plan time.