The projective plane

Each *point* (x,y) on the plane (at z=1) is represented by a *ray* (sx,sy,s)

All points on the ray are equivalent:

$(x, y, 1) \cong (sx, sy, s)$

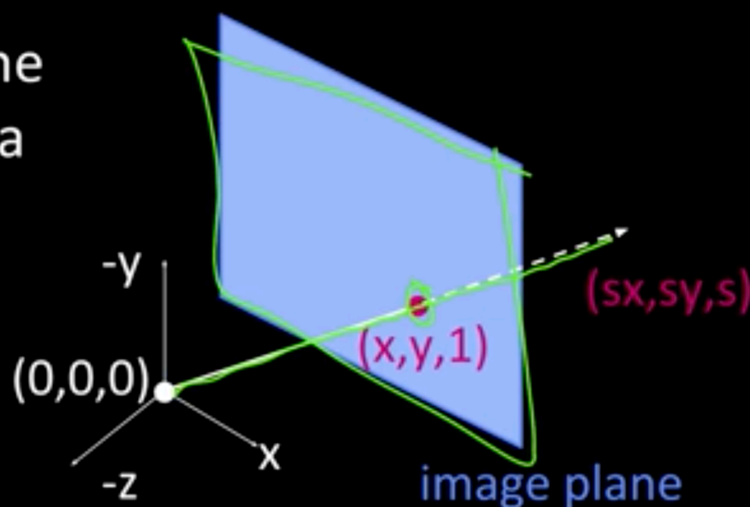
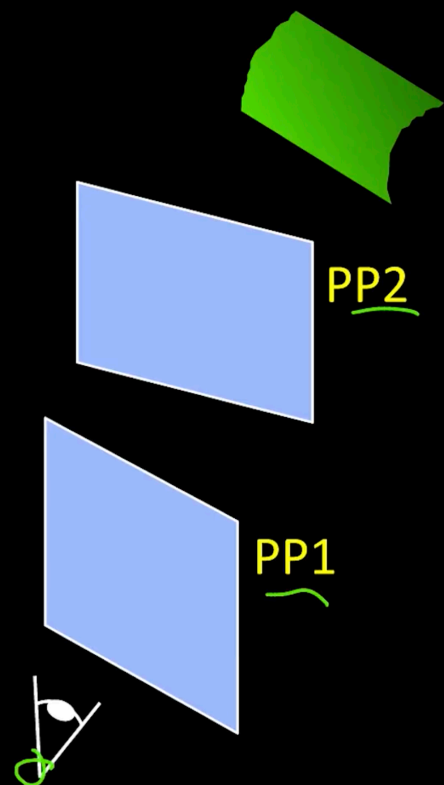


Image reprojection

Basic question:

- How to relate two images from the same camera center?
- How to map a pixel from projective plane PP1 to PP2?

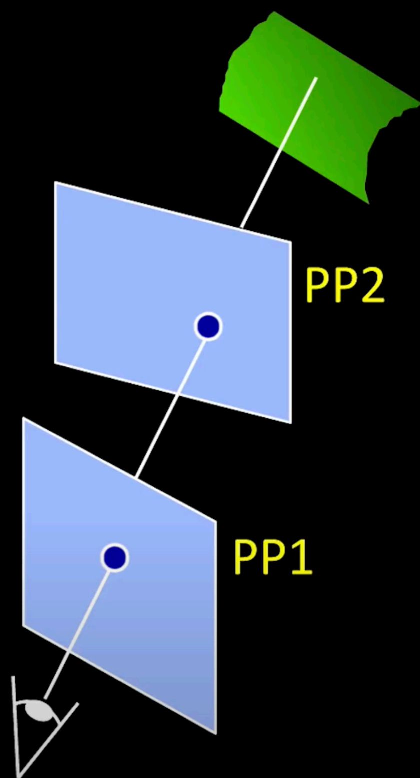


Source: Alyosha Efros

Image reprojection

Answer

- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2



Source: Alyosha Efros

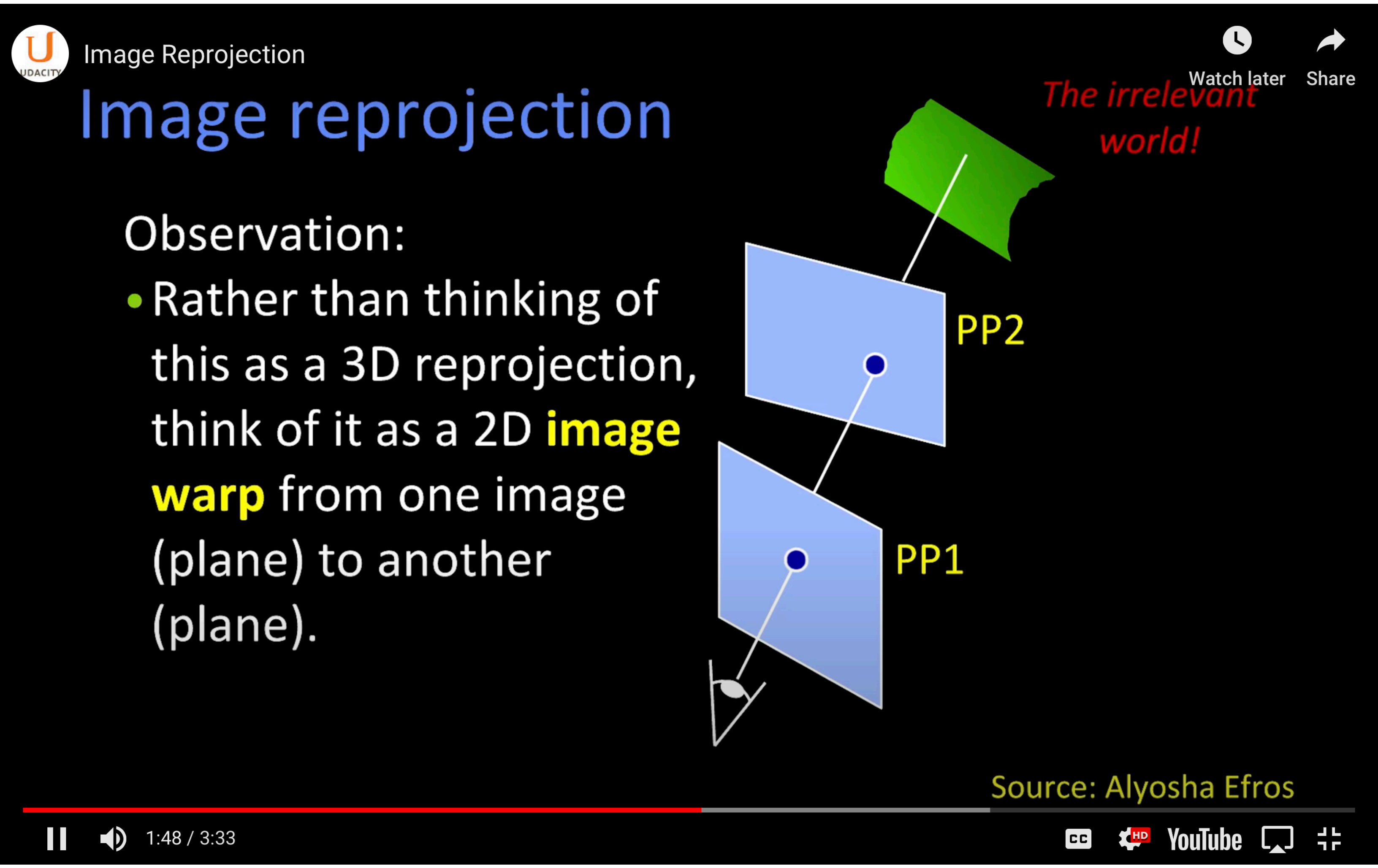


Image Reprojection
Watch later
Share
Image reprojection
The irrelevant world!
Observation:
• Rather than thinking of this as a 3D reprojection, think of it as a 2D image warp from one image (plane) to another (plane).
PP2
PP1
Source: Alyosha Efros
1:48 / 3:33
YouTube

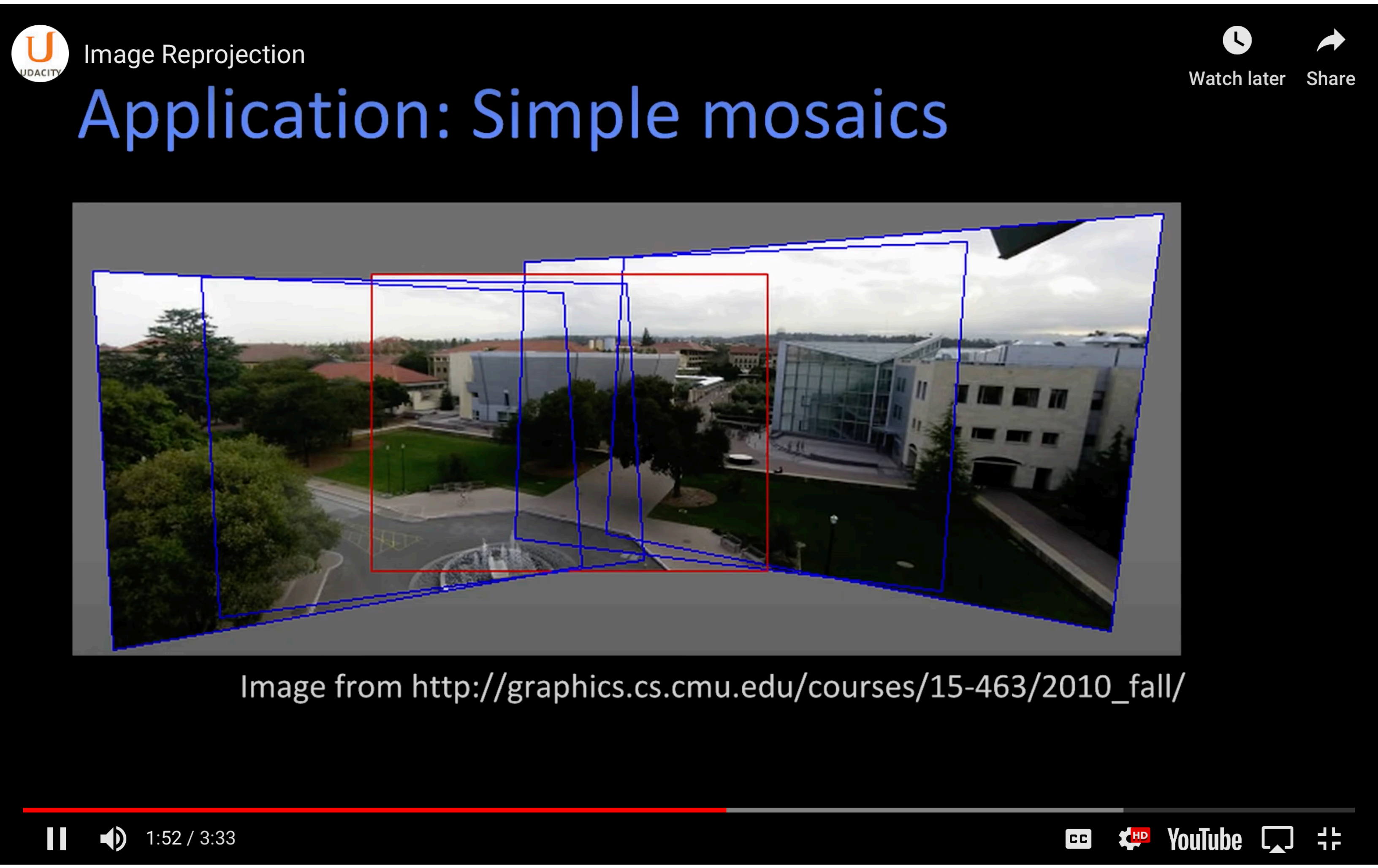


Image Reprojection
Watch later
Share
Application: Simple mosaics
Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/
1:52 / 3:33
YouTube

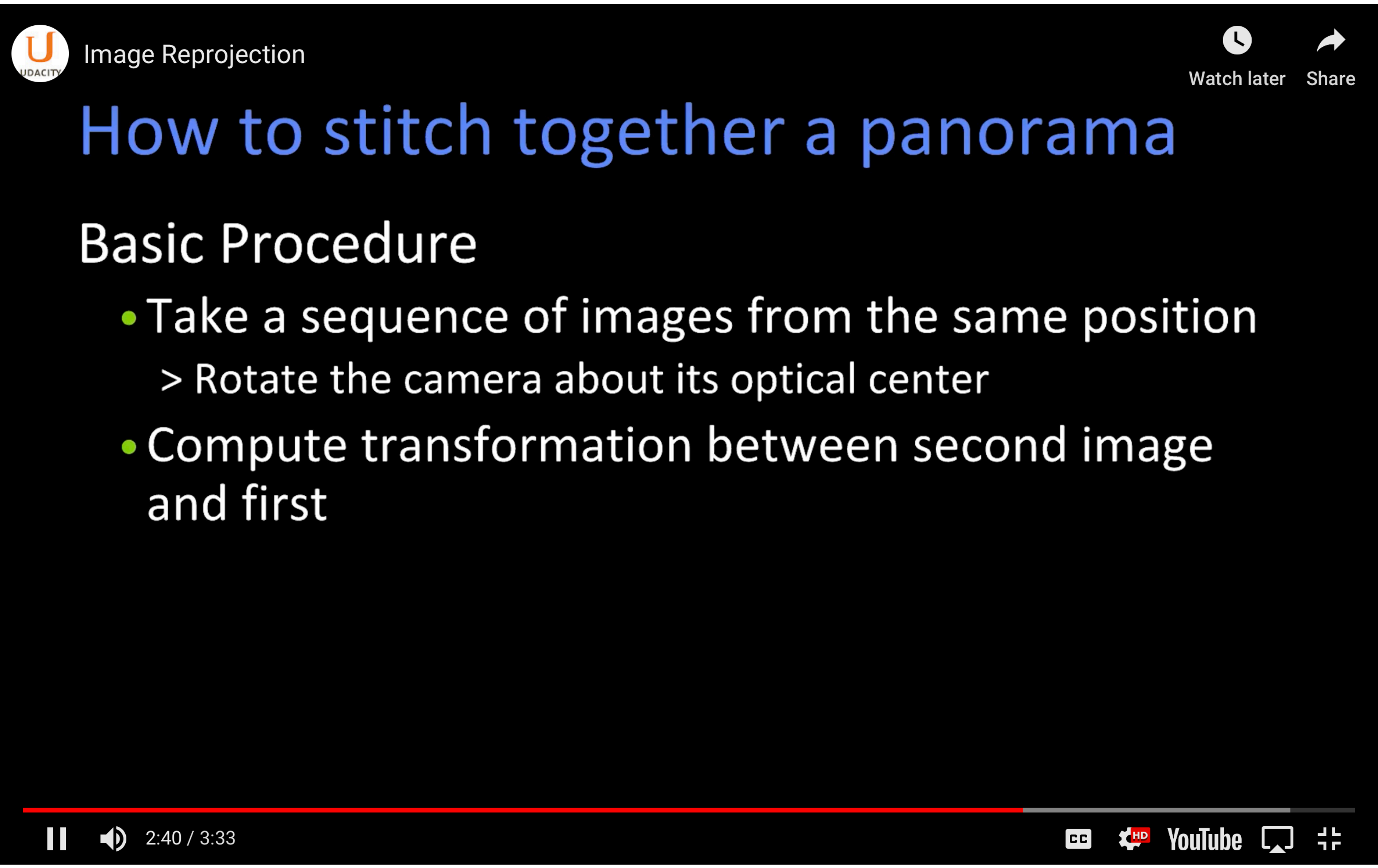
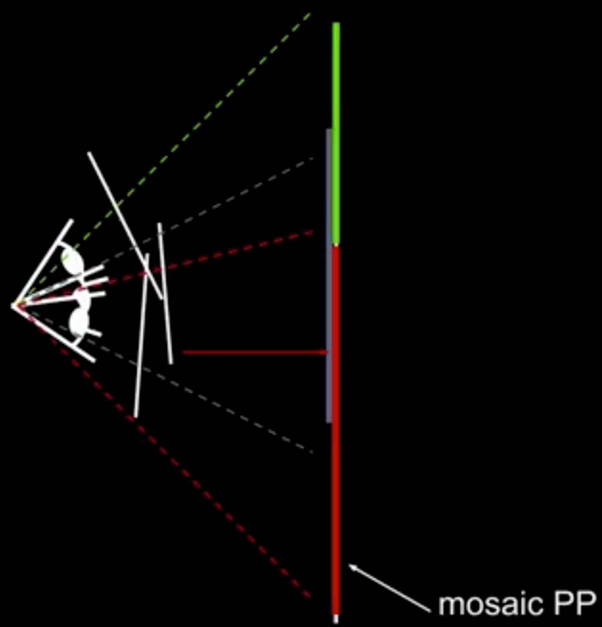


Image Reprojection
Watch later
Share
How to stitch together a panorama
Basic Procedure
• Take a sequence of images from the same position
> Rotate the camera about its optical center
• Compute transformation between second image and first
2:40 / 3:33

How to stitch together a panorama

- Transform the second image to overlap with the first
- Blend the two together to create a mosaic
- (If there are more images, repeat)

Image reprojection

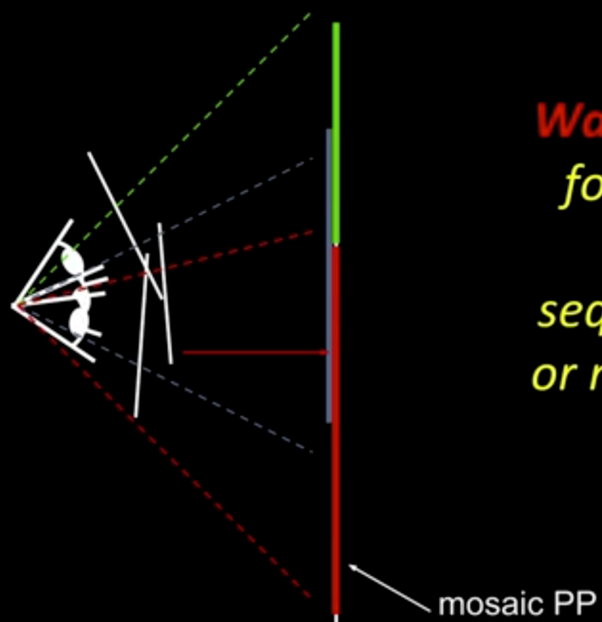


The mosaic has a natural interpretation in 3D:

- The images are *reprojected* onto a common plane
- The mosaic is formed on this plane.

Source: Steve Seitz

Image reprojection



Warning: *This model only holds for angular views up to 180°. Beyond that need to use sequence that "bends the rays" or map onto a different surface, say, a cylinder.*

Source: Steve Seitz

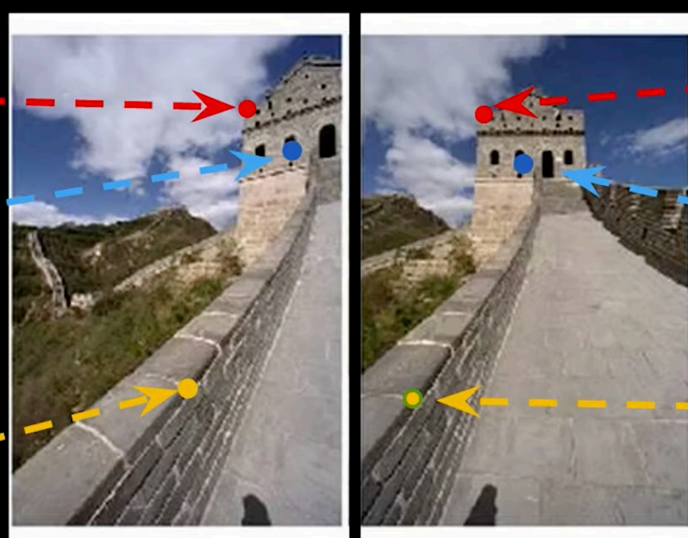
Homography

(x_1, y_1) (x_1', y_1')

(x_2, y_2) (x_2', y_2')

$\vdots$ $\vdots$

(x_N, y_N) (x_N', y_N')

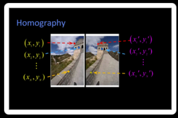


Homographies

Watch later Share

Solving for homographies

$$\mathbf{p'} = \mathbf{Hp} \qquad \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



1:09 / 3:53

YouTube

Solving for homographies – non-homogeneous

Since 8 unknowns, can set scale factor i=1.

Set up a system of linear equations $\mathbf{Ah} = \mathbf{b}$ where vector of unknowns

$$h = [a, b, c, d, e, f, g, h]^T$$

Need at least 4 points for 8 eqs, but the more the better...

Solve for h by $\min \|Ah - b\|^2$ using least-squares

Homographies

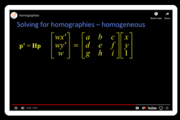
Solving for homographies – homogeneous

$$\mathbf{p'} = \mathbf{Hp} \qquad \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Solving for homographies – homogeneous

Just like we did for the extrinsics, multiply through, and divide out by w.
Gives two homogeneous equations per point.

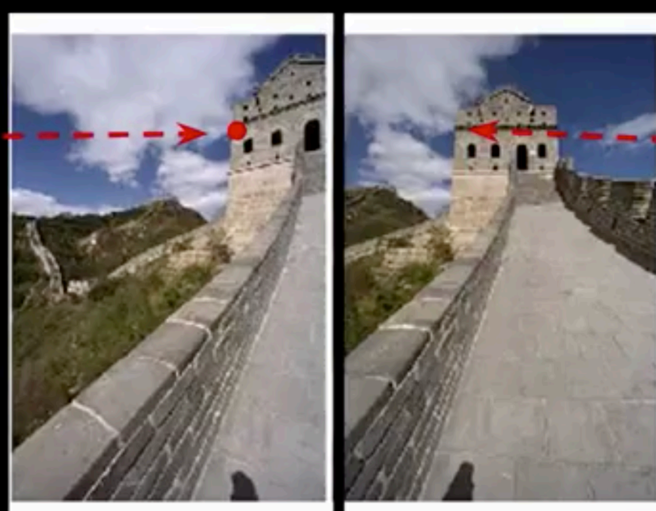
Solve using SVD just like before. This is the cool way.



Apply the Homography

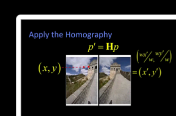
$$p' = \mathbf{H}p$$

$$(x, y)$$

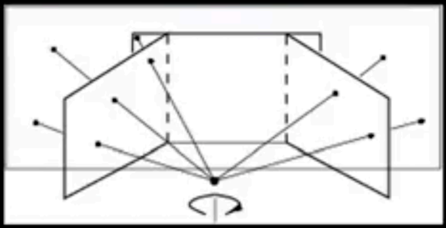


$$\left({}^{wx'}\!/_{w}, {}^{wy'}\!/_{w} \right)$$

$$= (x', y')$$



Mosaics



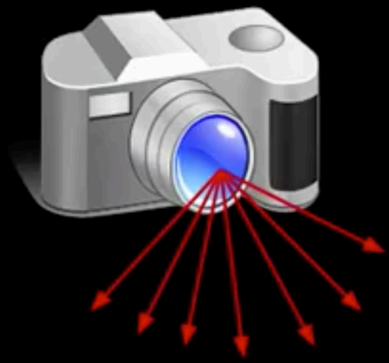


image from S. Seitz

Homographies and 3D planes

Remember this:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \simeq \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Homographies and 3D Planes

Homographies and 3D planes

- Suppose the 3D points are on a plane:

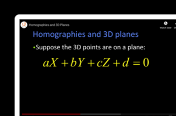
$$aX + bY + cZ + d = 0$$

1:12 / 3:16

Homographies and 3D planes

- On the plane *[a b c d]* can replace Z:

$$
\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \simeq \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X \\ Y \\ (aX+bY+d)/(-c) \\ 1 \end{bmatrix}
$$



Homographies and 3D planes

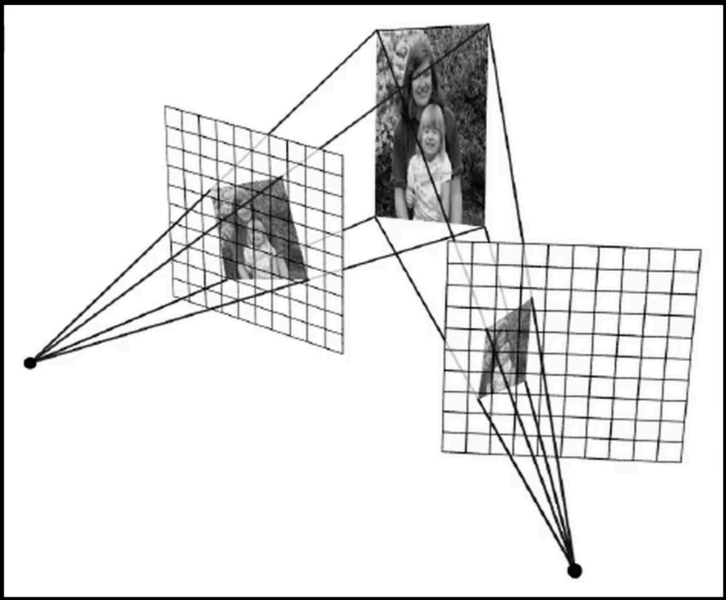
- So, can put the Z coefficients into the others:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \simeq \begin{bmatrix} m'_{00} & m'_{01} & 0 & m'_{03} \\ m'_{10} & m'_{11} & 0 & m'_{13} \\ m'_{20} & m'_{21} & 0 & m'_{23} \end{bmatrix} \begin{bmatrix} X \\ Y \\ (aX + bY + d)/(-c) \\ 1 \end{bmatrix}$$

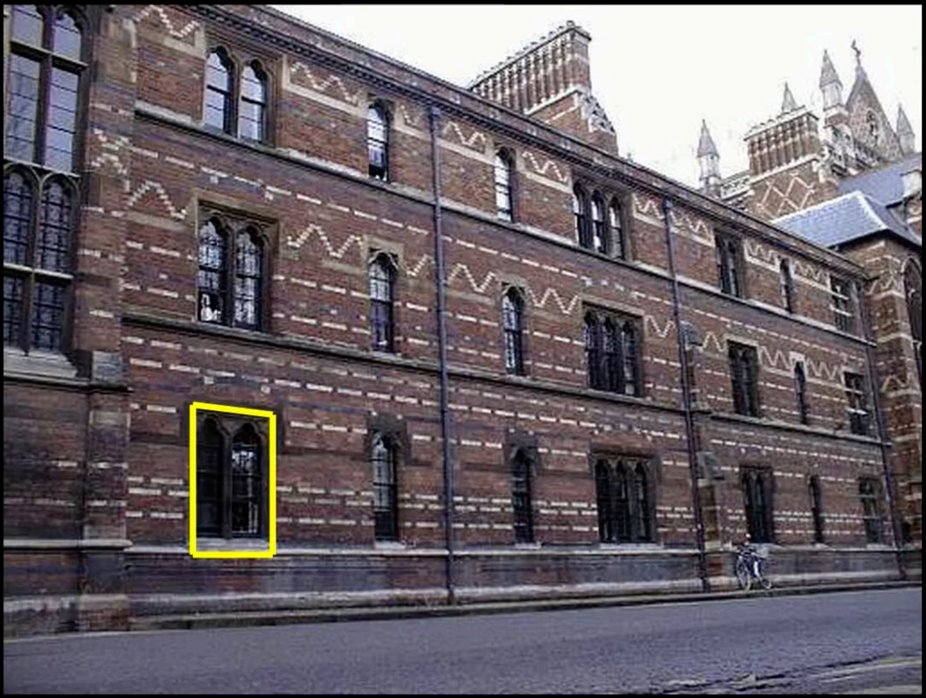
3x3 Homography!

Image reprojection

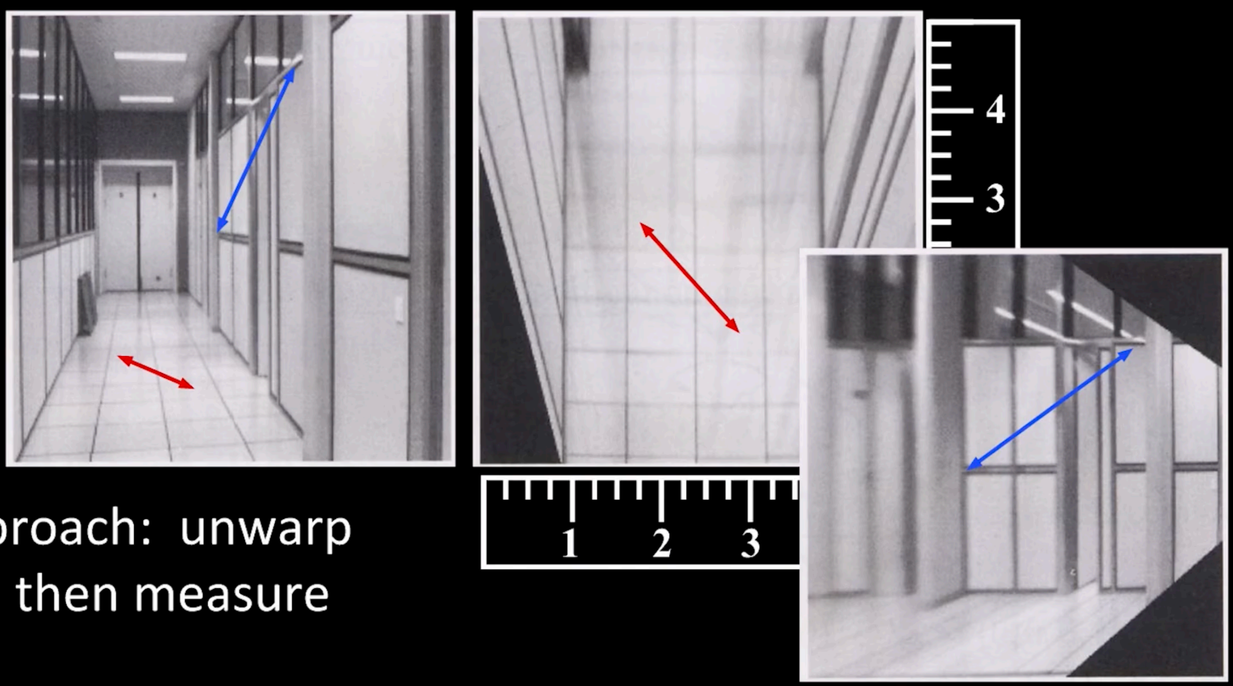
- Mapping between planes is a homography.
- Whether a plane in the world to the image or between image planes.



Rectifying slanted views



Measurements on planes



Approach: unwarp
then measure

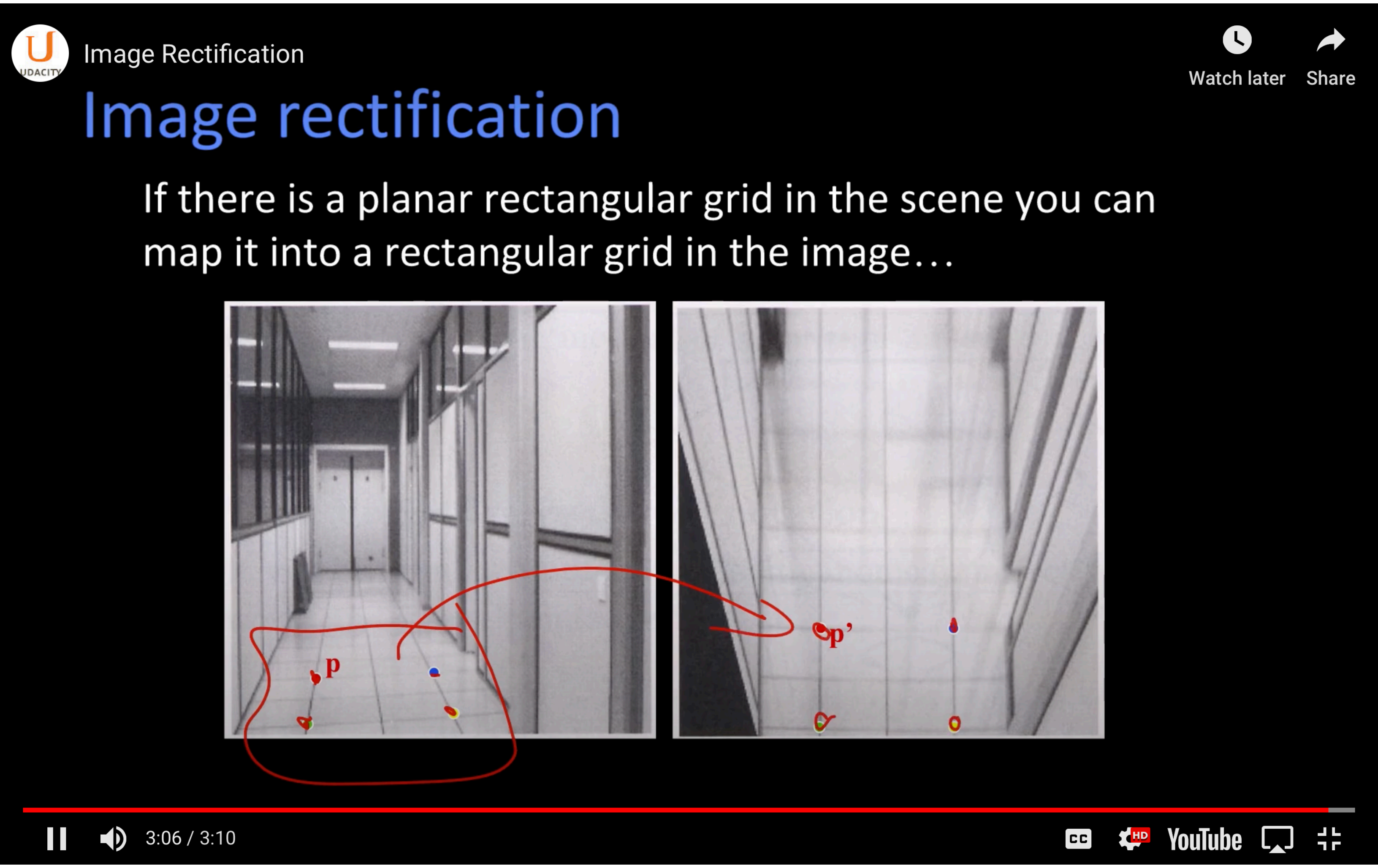
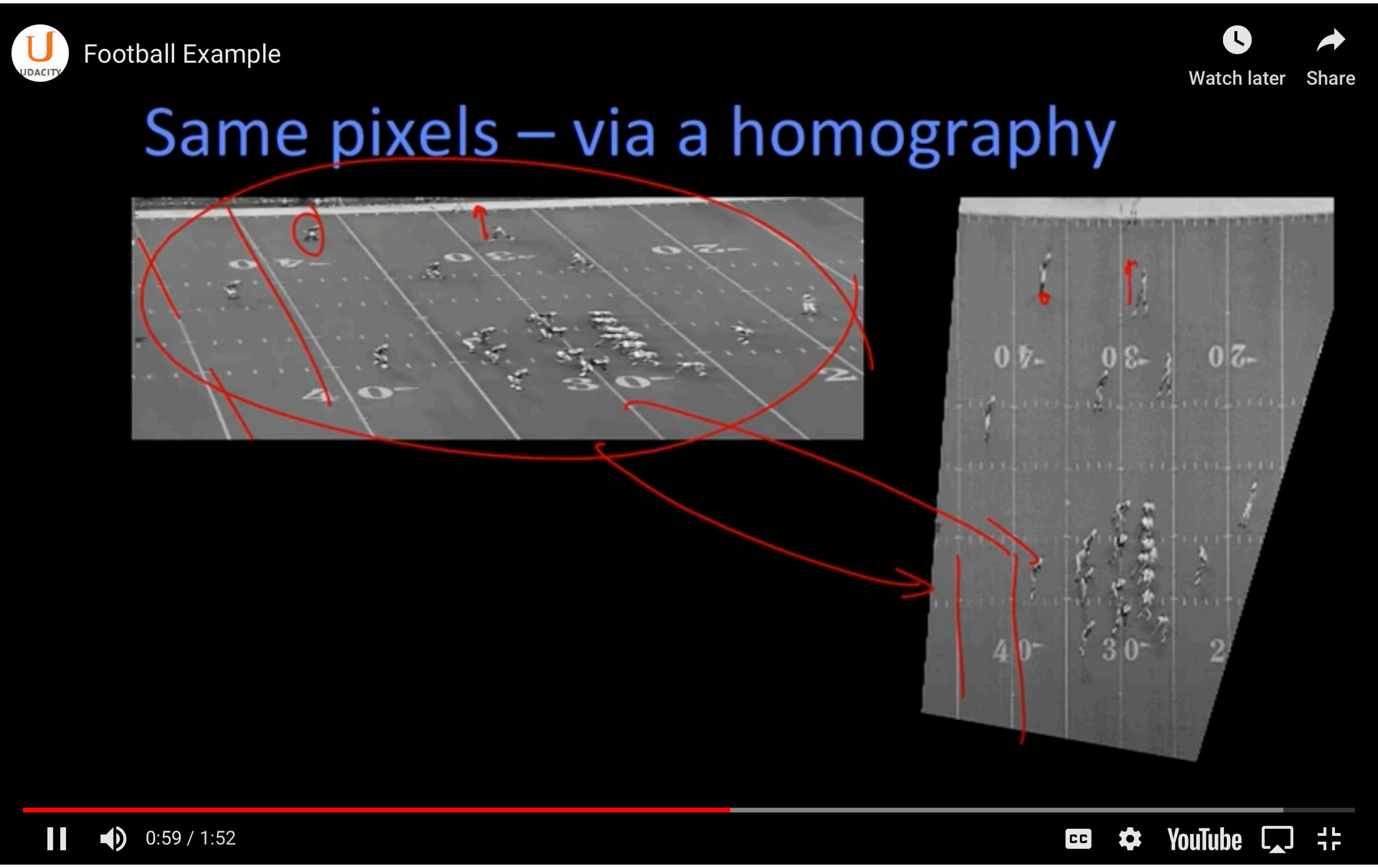


Image Rectification
Watch later
Share
Image rectification
If there is a planar rectangular grid in the scene you can map it into a rectangular grid in the image…
p
p'
3:06 / 3:10


Football Example
Watch later
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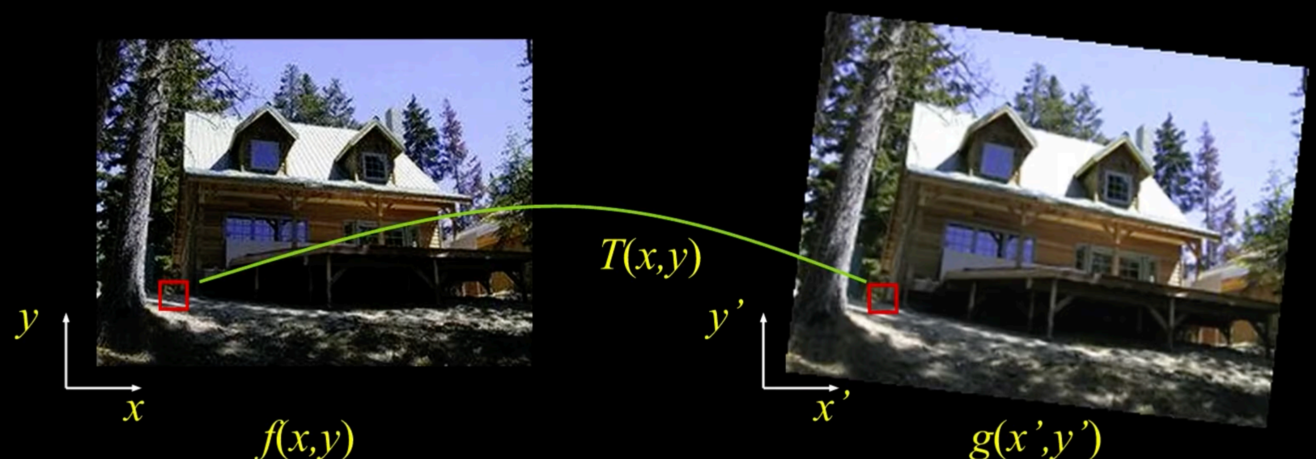
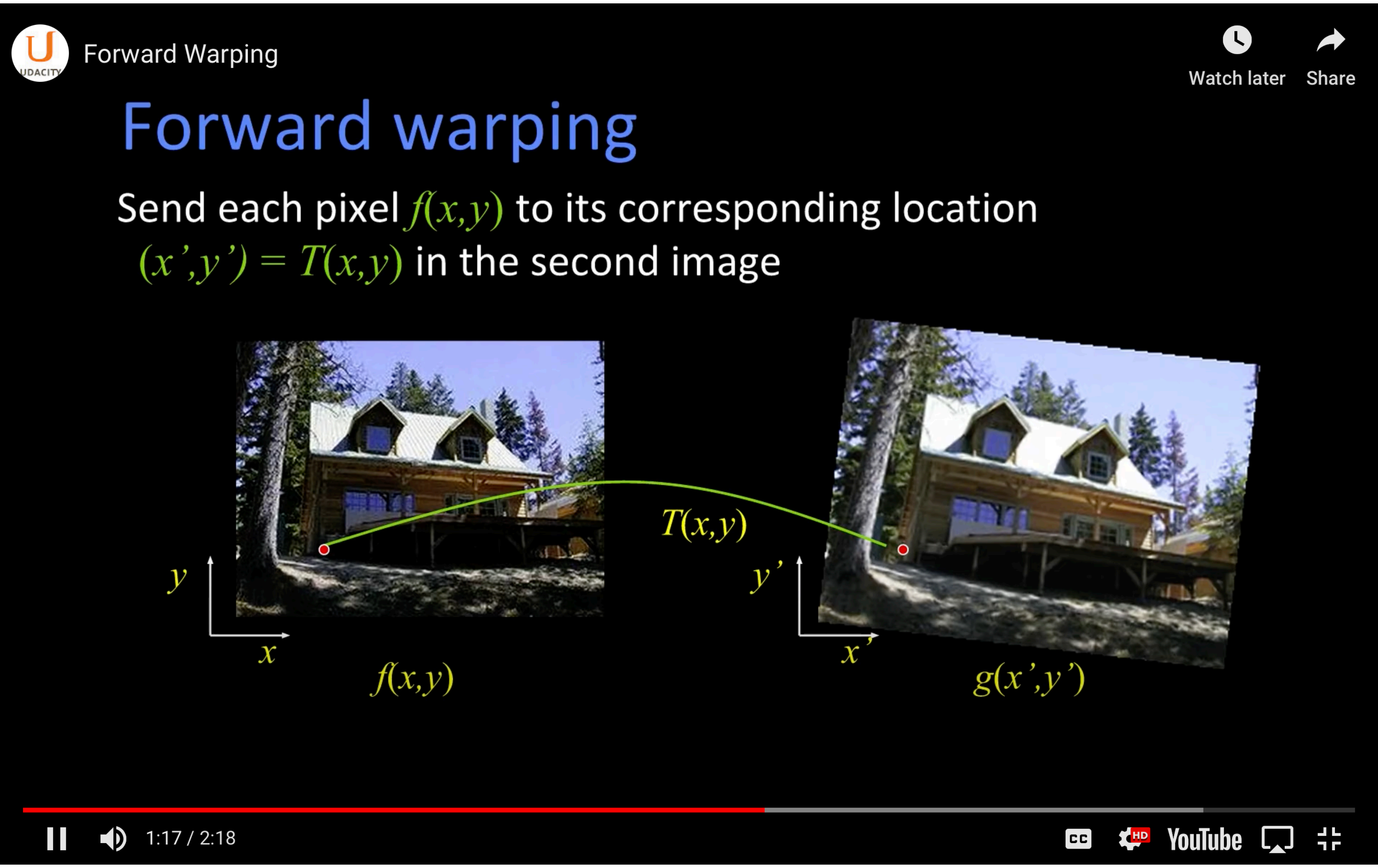
Same pixels – via a homography
0:59 / 1:52

Image warping

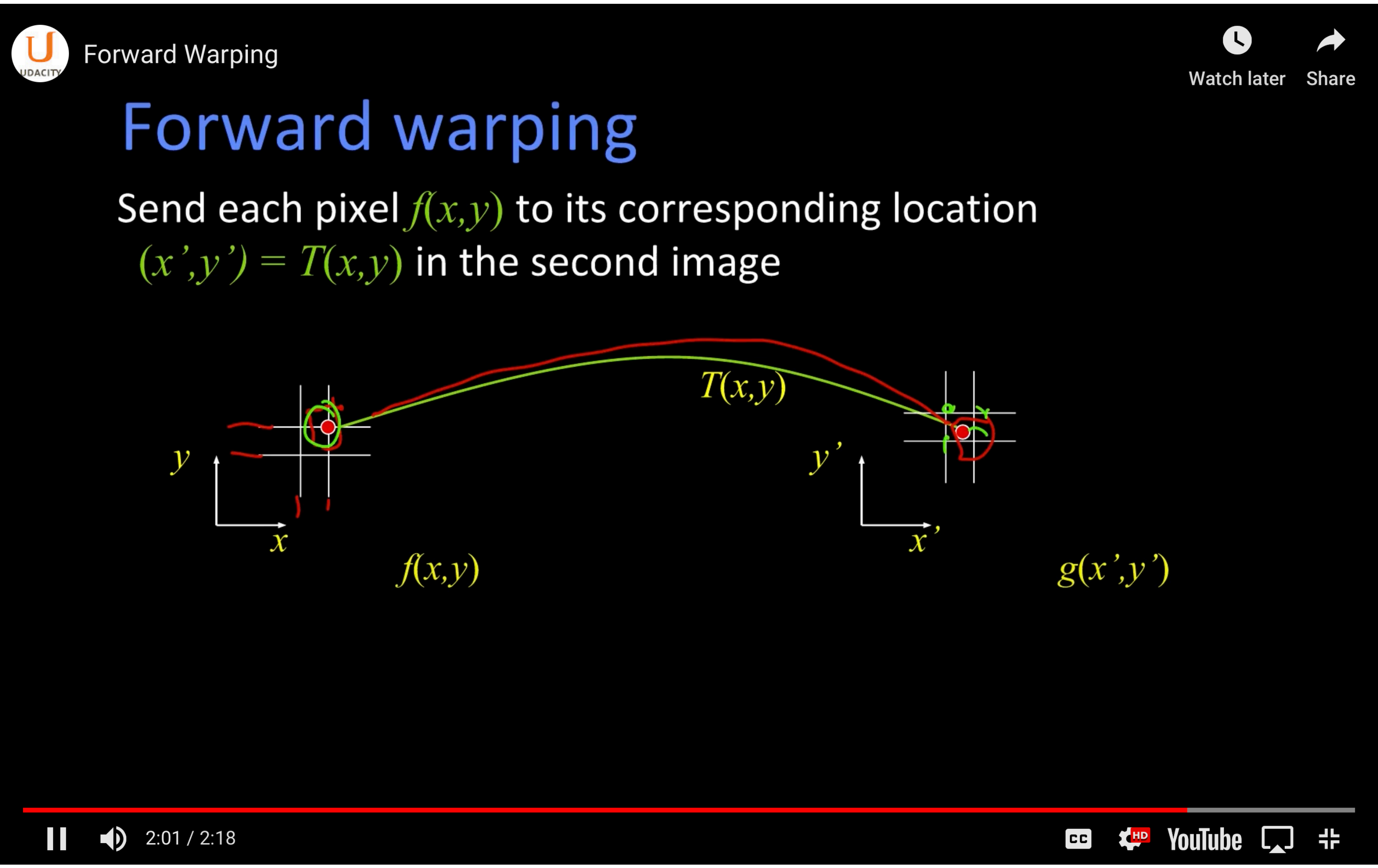
Given a coordinate transform and a source image $f(x,y)$,
how do we compute a transformed image $g(x',y')$?



Slide from Alyosha Efros,

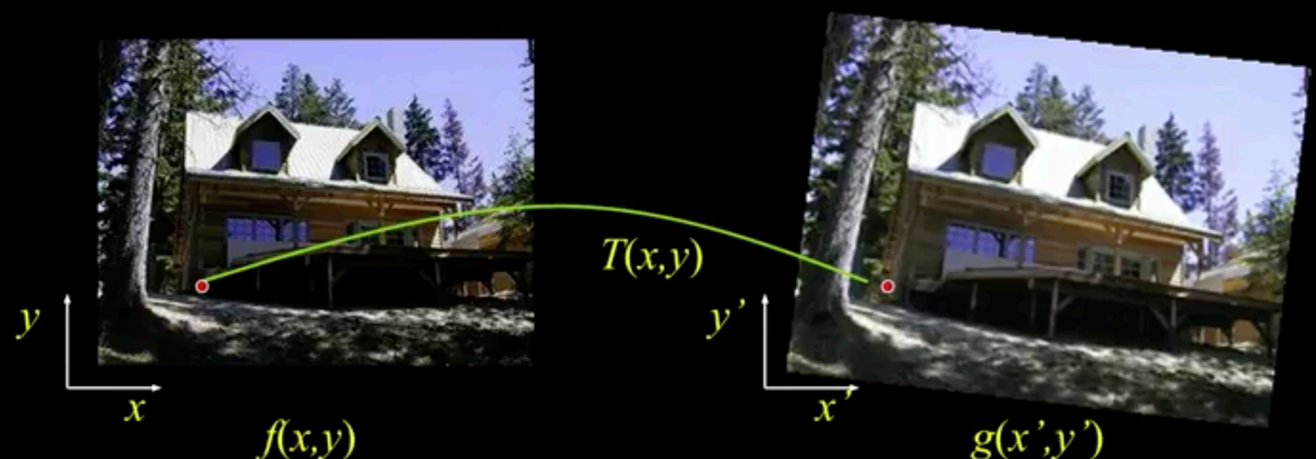


Forward Warping
Watch later
Share
Forward warping
Send each pixel f(x,y) to its corresponding location
(x',y') = T(x,y) in the second image
T(x,y)
y
x
f(x,y)
y'
x'
g(x',y')
1:17 / 2:18
YouTube


Forward Warping
Watch later
Share
Forward warping
Send each pixel f(x,y) to its corresponding location
(x',y') = T(x,y) in the second image
T(x,y)
y
x
y'
x'
f(x,y)
g(x',y')
2:01 / 2:18

Inverse warping

Get each pixel $g(x',y')$ from its corresponding location $(x,y) = T^{-1}(x',y')$ in the first image



Inverse Warping

Bilinear interpolation

$$
\begin{aligned}
f(x,y) = \; & (1-a)(1-b) && f[i,j] \\
& +a(1-b) && f[i+1,j] \\
& +ab && f[i+1,j+1] \\
& +(1-a)b && f[i,j+1]
\end{aligned}
$$