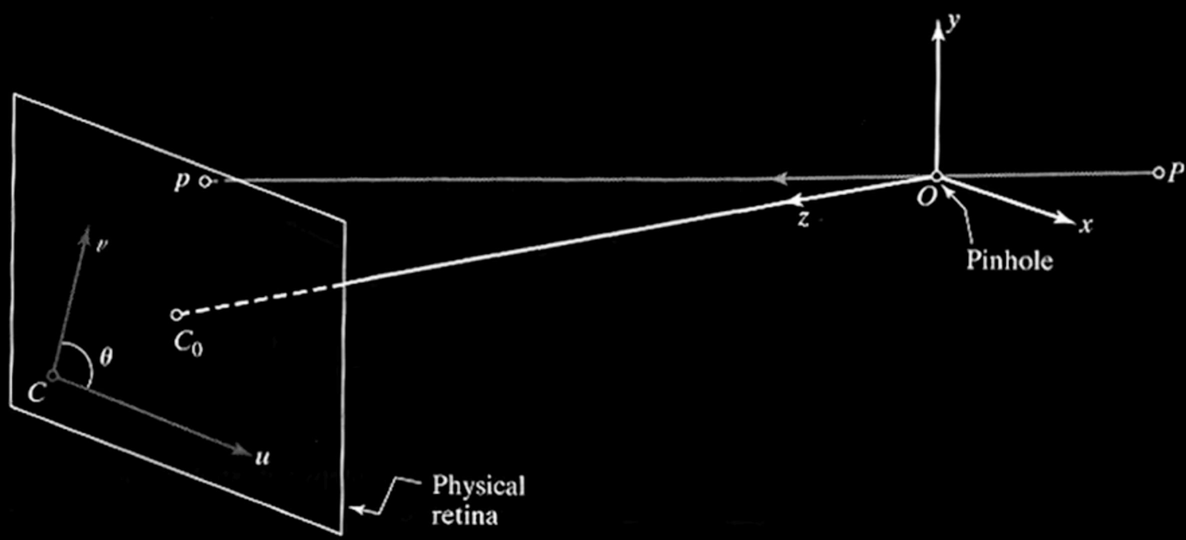
Ideal intrinsic parameters

Ideal Perspective projection:

$$u = f\frac{x}{z}$$

$$v = f\frac{y}{z}$$

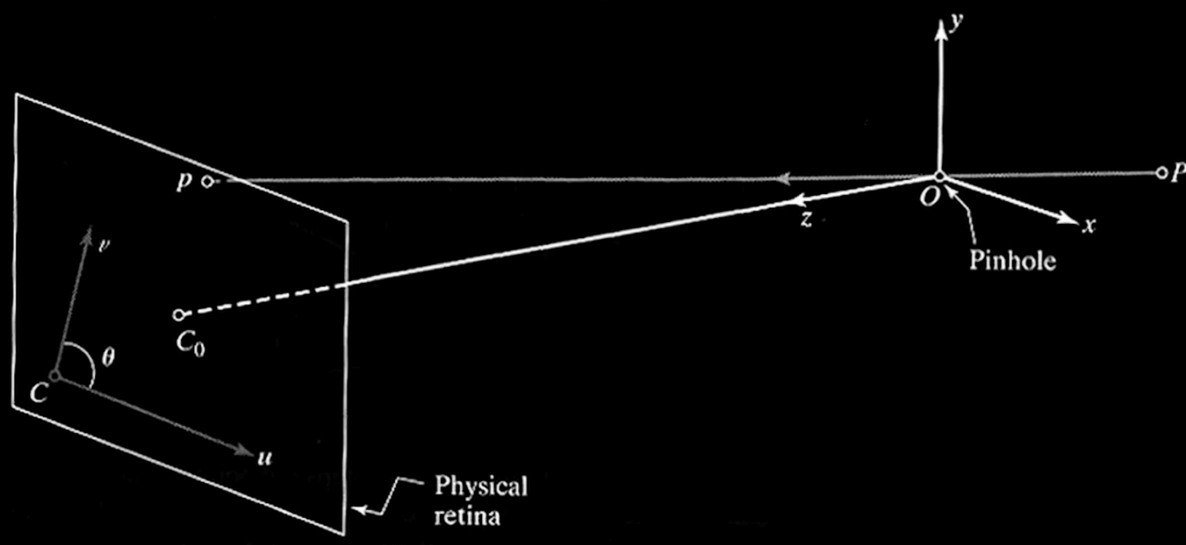


Real intrinsic parameters (1)

But "pixels" are in some arbitrary spatial units

$$u = \alpha \frac{x}{z}$$

$$v = \alpha \frac{y}{z}$$

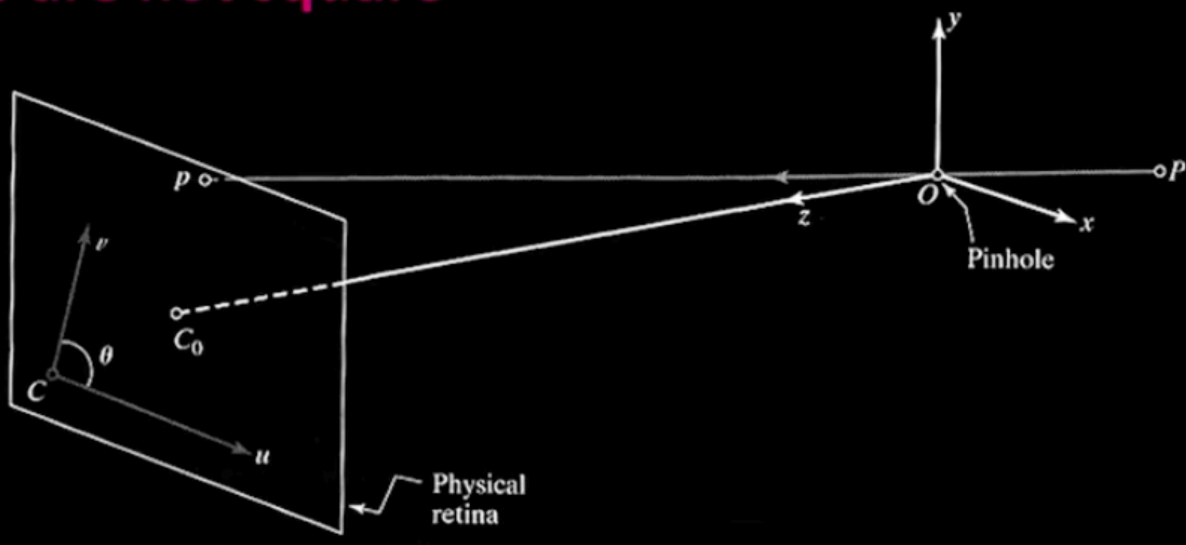


Real intrinsic parameters (2)

Maybe pixels are not square

$$u = \alpha \frac{x}{z}$$

$$v = \beta \frac{y}{z}$$

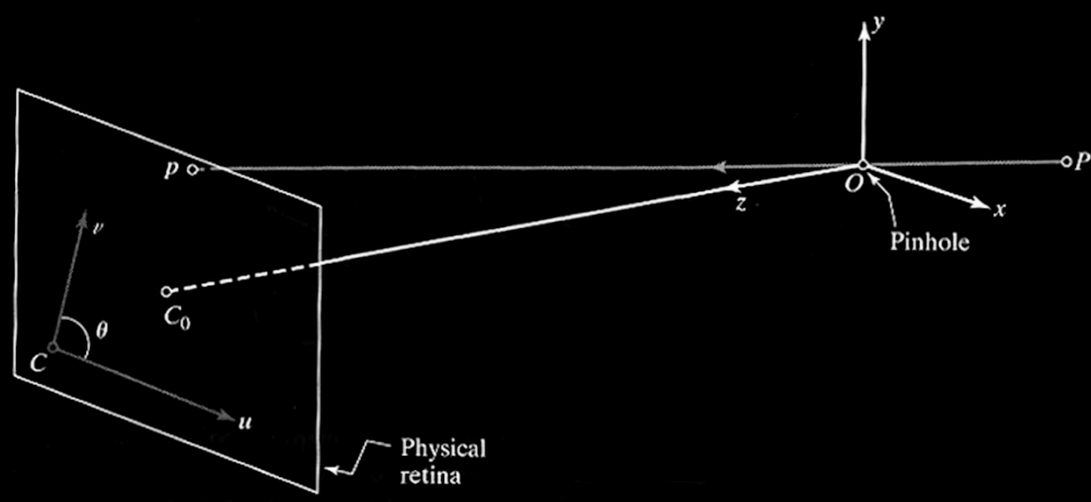


Real intrinsic parameters (3)

We don't know the origin of our camera pixel coordinates

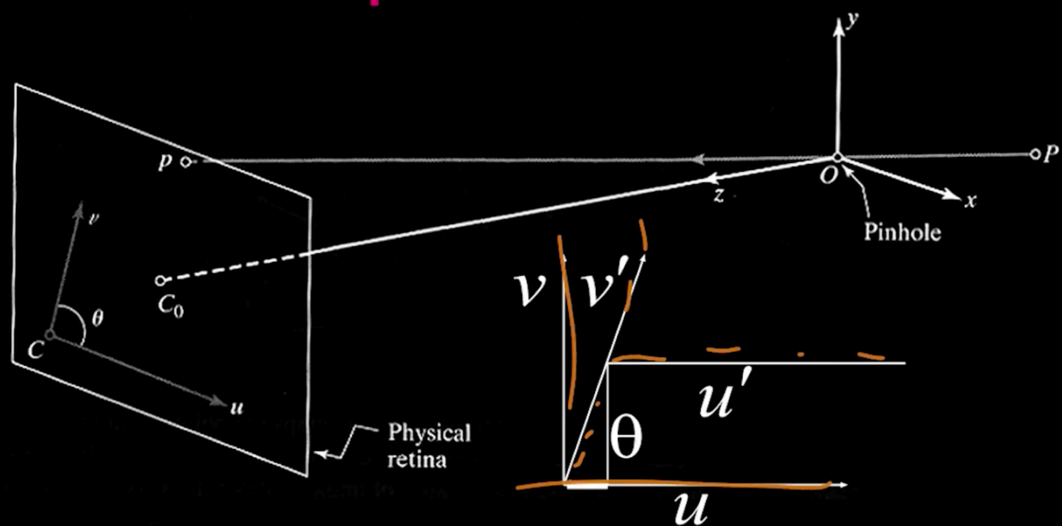
$$u = \alpha \frac{x}{z} + u_0$$

$$v = \beta \frac{y}{z} + v_0$$



Really ugly intrinsic parameters (4)

May be skew between camera pixel axes



$$v' \sin(\theta) = v$$

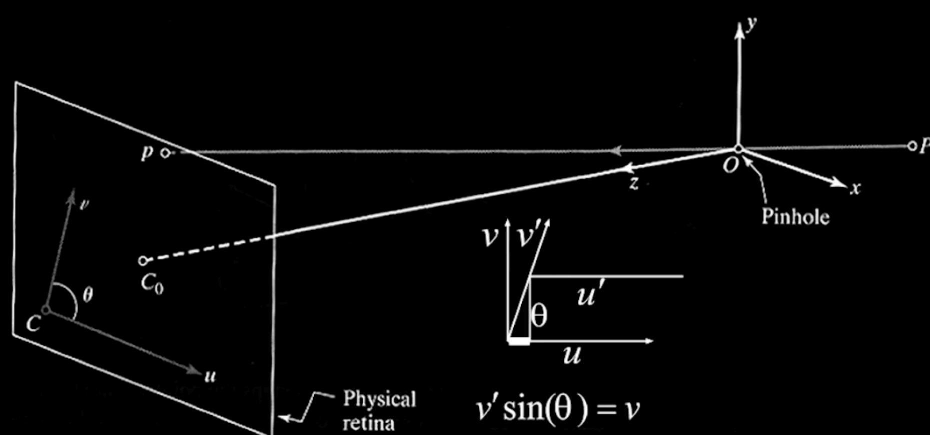
$$u' = u - \cos(\theta) v' = u - \cot(\theta) v$$

Really ugly intrinsic parameters (4)

May be skew between camera pixel axes

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$



$$v' \sin(\theta) = v$$

$$u' = u - \cos(\theta) v' = u - \cot(\theta) v$$

Intrinsic parameters, non-homogeneous coordinates

Notice division by z

$$u = \alpha \frac{x}{z} - \alpha \cot(\theta) \frac{y}{z} + u_0$$

$$v = \frac{\beta}{\sin(\theta)} \frac{y}{z} + v_0$$

$$\begin{pmatrix} z*u \\ z*v \\ z \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha \cot(\theta) & u_0 & 0 \\ 0 & \dfrac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} z*u \\ z*v \\ z \end{pmatrix} = \begin{pmatrix} \alpha & -\alpha\cot(\theta) & u_0 & 0 \\ 0 & \dfrac{\beta}{\sin(\theta)} & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$\overrightarrow{\mathrm{p}'} = \mathbf{K} \; {}^{C}\overrightarrow{p}$$

In homogeneous pixels

Intrinsic matrix

In camera-based 3D coords

Kinder, gentler intrinsics

- Can use simpler notation for intrinsics – remove last column which is zero:

$$K = \begin{bmatrix} f & s & c_x \\ 0 & a\,f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

f –focal length
s – skew
a – aspect ratio
c_x, c_y - offset

(5 DOF)

Kinder, gentler intrinsics

- If square pixels, no skew, and optical center is in the center (assume origin in the middle):

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In this case only one DOF, focal length f

Combining extrinsic & intrinsic calibration parameters

Pixels

$$\vec{p}\,' = \mathrm{K}\ {}^{C}\vec{p}$$

Intrinsic

World 3D coordinates

Camera 3D coordinates

$$\begin{pmatrix} \\ {}^{C}\vec{p} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} - & - & - & | \\ - & {}^{C}_{W}R & - & {}^{C}_{W}\vec{t} \\ - & - & - & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \\ {}^{W}\vec{p} \\ \\ \\ \end{pmatrix}$$

Extrinsic

Combining extrinsic & intrinsic calibration parameters

$$\vec{p}\,' = \underset{\boxed{\begin{matrix}\text{K}\\ 3\text{x}3\end{matrix}}}{K} \underbrace{\left({}^{C}_{W}R \quad {}^{C}_{W}\vec{t} \right)}_{\boxed{3\text{x}4}} \; {}^{W}\vec{p}$$

$$\vec{p}\,' = M \; {}^{W}\vec{p}$$

Other ways to write the same equation

pixel coordinates

world coordinates

$$\vec{p}\,' = M \;\; {}^{W}\vec{p}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \simeq \begin{pmatrix} s*u \\ s*v \\ s \end{pmatrix} = \begin{pmatrix} . & m_1^T & . & . \\ . & m_2^T & . & . \\ . & m_3^T & . & . \end{pmatrix} \begin{pmatrix} {}^{W}p_x \\ {}^{W}p_y \\ {}^{W}p_z \\ 1 \end{pmatrix}$$

projectively similar

Conversion back from homogeneous coordinates

$$u = \frac{m_1 \cdot \vec{P}}{m_3 \cdot \vec{P}}$$

$$v = \frac{m_2 \cdot \vec{P}}{m_3 \cdot \vec{P}}$$

Finally: Camera parameters

- A camera (and its matrix) M (or Π) is described by several parameters
  - Translation **T** of the optical center from the origin of world coordinates
  - Rotation **R** of the camera system
  - focal length and aspect **(f, a)** [or pixel size **(s_x, s_y)**] , principle point **(x'_c, y'_c)**, and skew **(s)**
  - **blue** parameters are called "**extrinsics**," red are "intrinsics"

Finally: Camera parameters

- Projection equation – the cumulative effect of all parameters:

$$\underset{(3x4)}{\mathbf{M}} = \begin{bmatrix} f & s & x'_c \\ 0 & af & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}$$

intrinsics projection rotation translation

Finally: Camera parameters

- Projection equation – the cumulative effect of all parameters:

$$\underset{(3\text{x}4)}{\mathbf{M}} = \underbrace{\begin{bmatrix} f & s & x'_c \\ 0 & af & y'_c \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}}_{\text{translation}}$$

$$\boxed{\textit{DoFs: } 5+0+3+3 = 11}$$