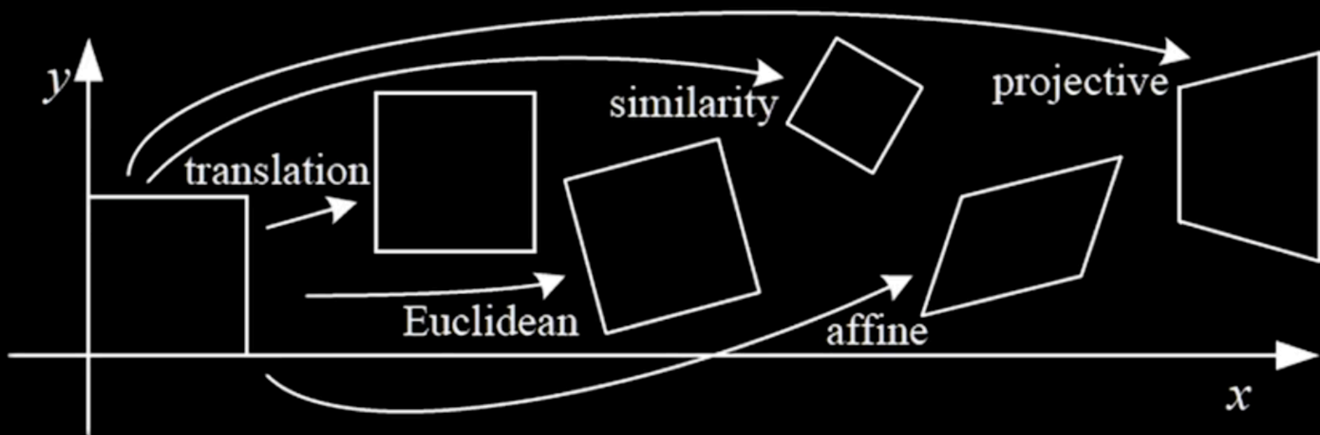



*Homogenous vector*

**Example: translation**

$$x' = x + \vec{t} \qquad x' = \begin{bmatrix} I & \vec{t} \end{bmatrix} \bar{x} \qquad \bar{x}' = \begin{bmatrix} I & \vec{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \bar{x}$$

$$\begin{bmatrix} \ \\ \ \end{bmatrix} = \begin{bmatrix} \ \\ \ \end{bmatrix} + \begin{bmatrix} \mathbf{tx} \\ \mathrm{ty} \end{bmatrix} \qquad \begin{bmatrix} \ \\ \ \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathrm{tx} \\ 0 & 1 & \mathrm{ty} \end{bmatrix} \cdot \begin{bmatrix} \ \\ \ \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \ \\ \ \\ \ \end{bmatrix} = \begin{bmatrix} 1 & 0 & \mathrm{tx} \\ 0 & 1 & \mathrm{ty} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \ \\ \ \\ \ \end{bmatrix}$$

# Projective Transformations

- *Projective* transformations: for 2D images it's a 3x3 matrix applied to homogenous coordinates

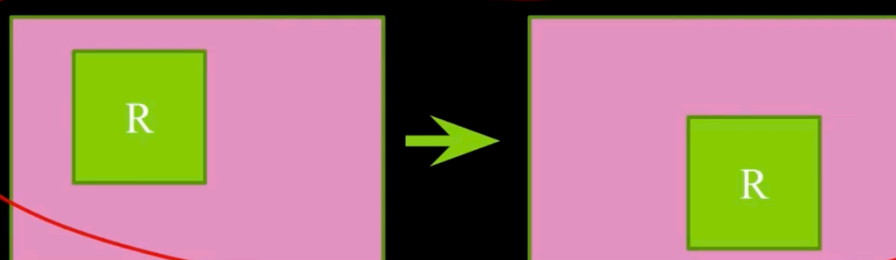
$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

# Special Projective Transformations

- Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lengths/Areas
  - Angles
  - Orientation
  - Lines

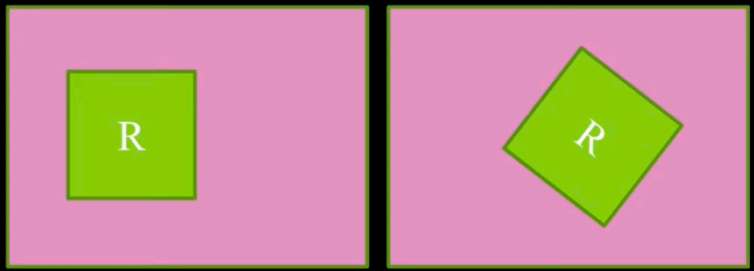


# Special Projective Transformations

- Euclidean (Rigid body)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_x \\ \sin(\theta) & \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lengths/Areas
  - Angles
  - Lines

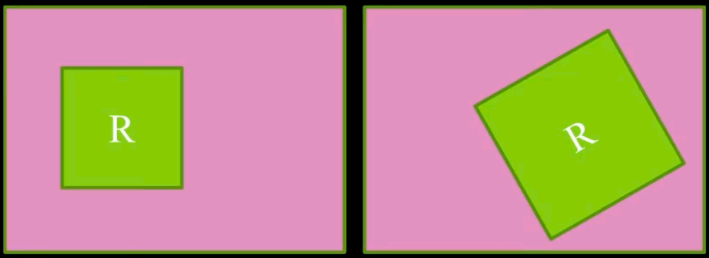


# Special Projective Transformations

- Similarity (trans, rot, scale) transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a\cos(\theta) & -a\sin(\theta) & t_x \\ a\sin(\theta) & a\cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lengths/Areas
  - Angles
  - Lines

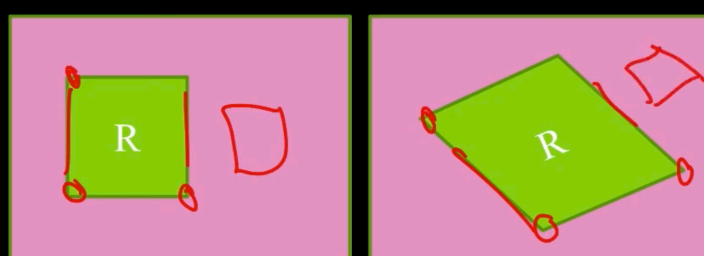


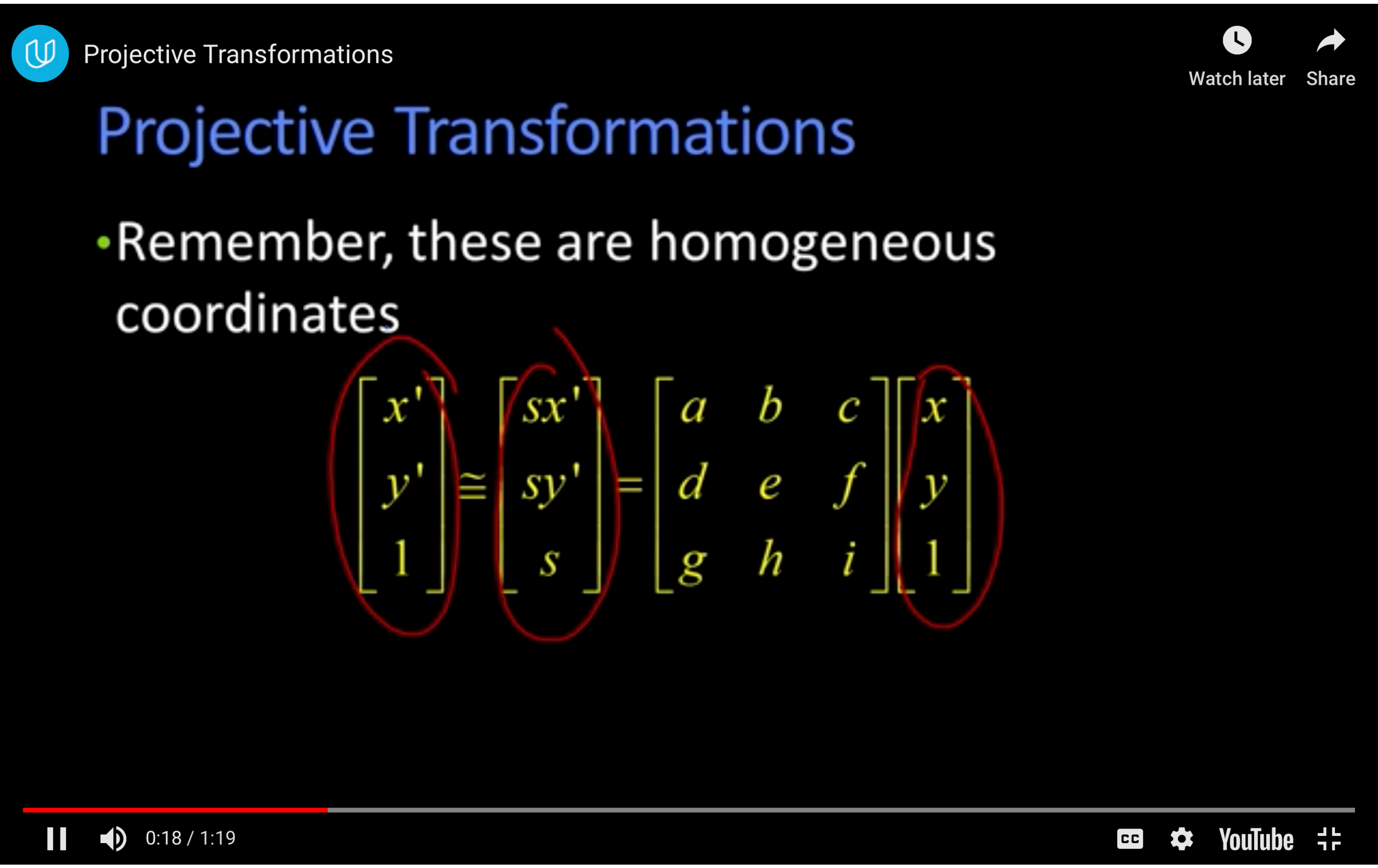
# Special Projective Transformations

- Affine transform

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Parallel Lines
  - Ratio of Areas
  - Lines





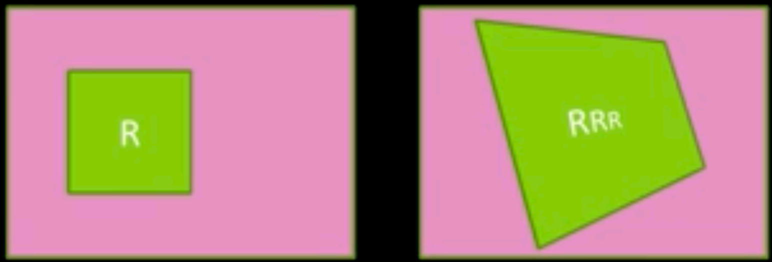
Projective Transformations
Watch later
Share
Projective Transformations
• Remember, these are homogeneous coordinates
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} sx' \\ sy' \\ s \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
0:18 / 1:19
YouTube

# Projective Transformations

- General projective transform (or Homography)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Preserves:
  - Lines
  - Also cross ratios (maybe later)

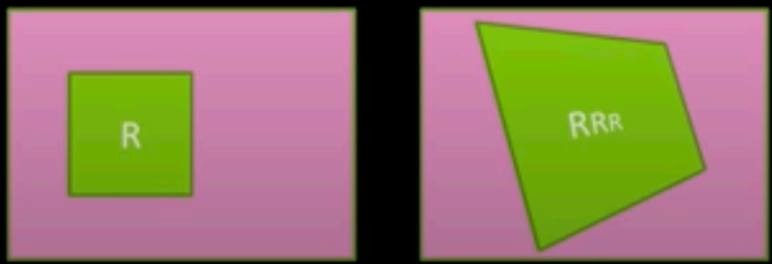


# Projective Transformations

- General projective transform (or Homography)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \cong \begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

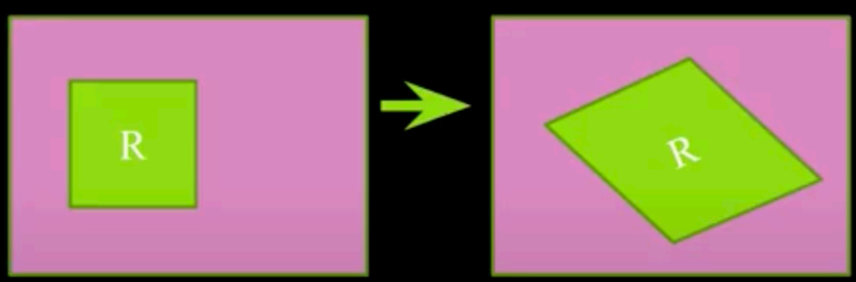
- Preserves:
  - Lines
  - Also cross ratios (maybe later)



# Quiz 2 - answer

- Affine transform: a 3 point transformation
  - 6 unknowns – each point pair gives two equations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

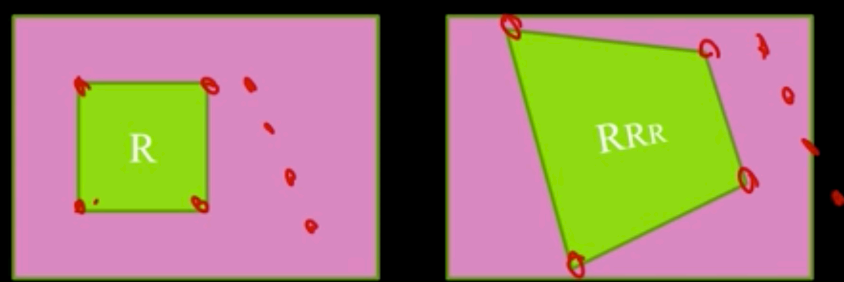


Transformation Solution 3

Watch later Share

# Quiz 3 -answer

- Homography




$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \simeq \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

1:30 / 1:48