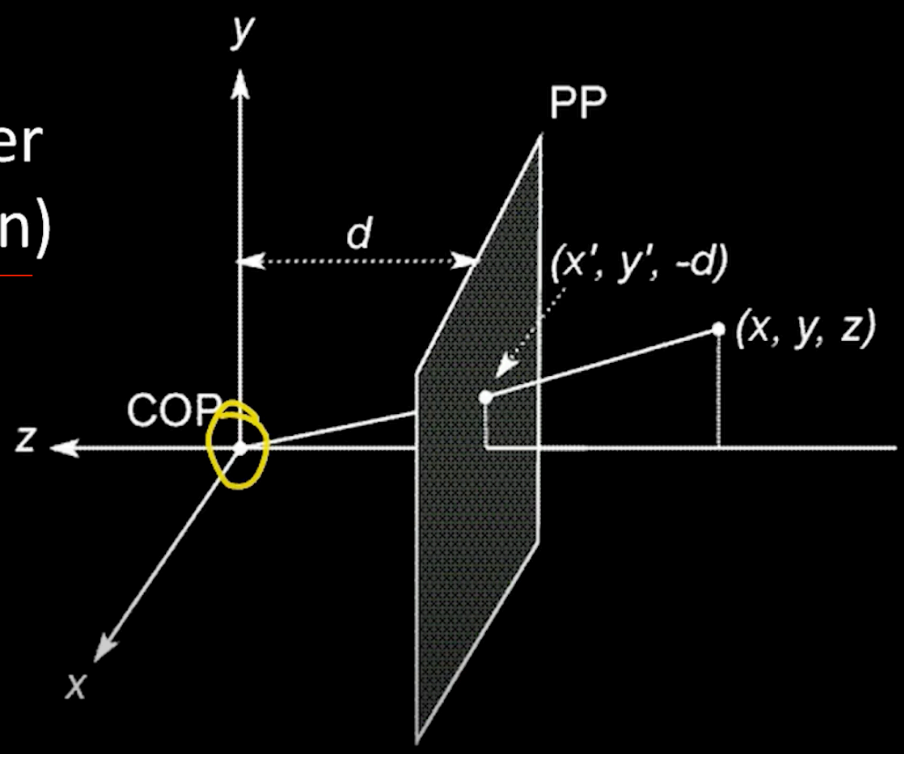
Modeling Projection - Coordinate System

- Put the optical center (Center Of Projection) at the origin

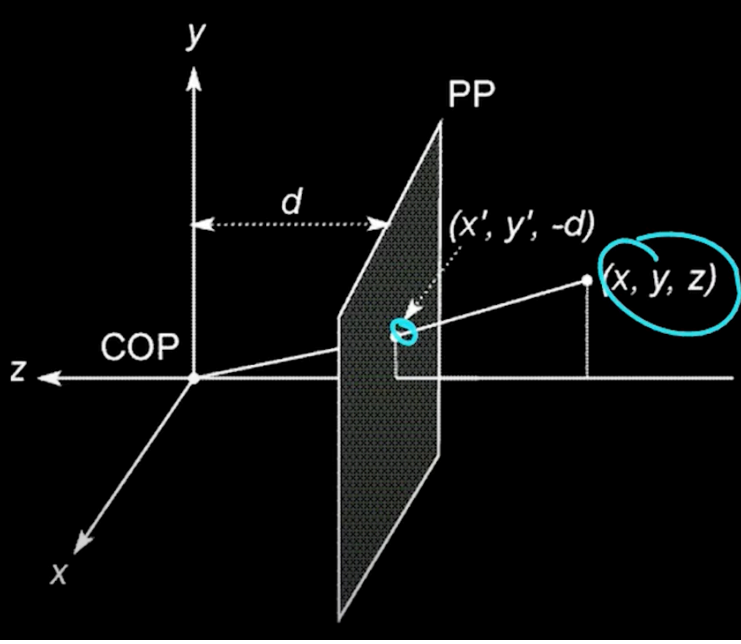


Modeling Projection

Projection equations

- Compute intersection with Perspective Projection of ray from (x,y,z) to COP
- Derived using similar triangles

$$(X,Y,Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$



Modeling Projection

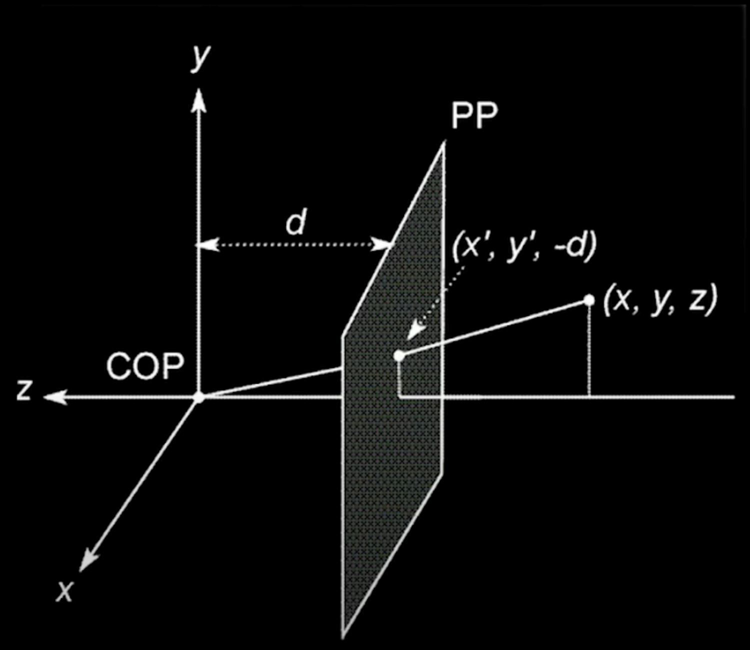
Projection equations

$$(X,Y,Z) \rightarrow (-d\frac{X}{Z}, -d\frac{Y}{Z}, -d)$$

We get the projection by throwing out the last coordinate:

$$(x', y') = (-d\frac{X}{Z}, -d\frac{Y}{Z})$$

Distant objects are smaller



Homogeneous Coordinates

Is this a linear transformation?

No – division by the (not constant) Z is non-linear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image (2D) coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene (3D) coordinates

Homogeneous Coordinates

Converting *from* homogeneous coordinates:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

(this makes homogenous coordinates invariant under scale)

Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

$$\Rightarrow (u, v)$$

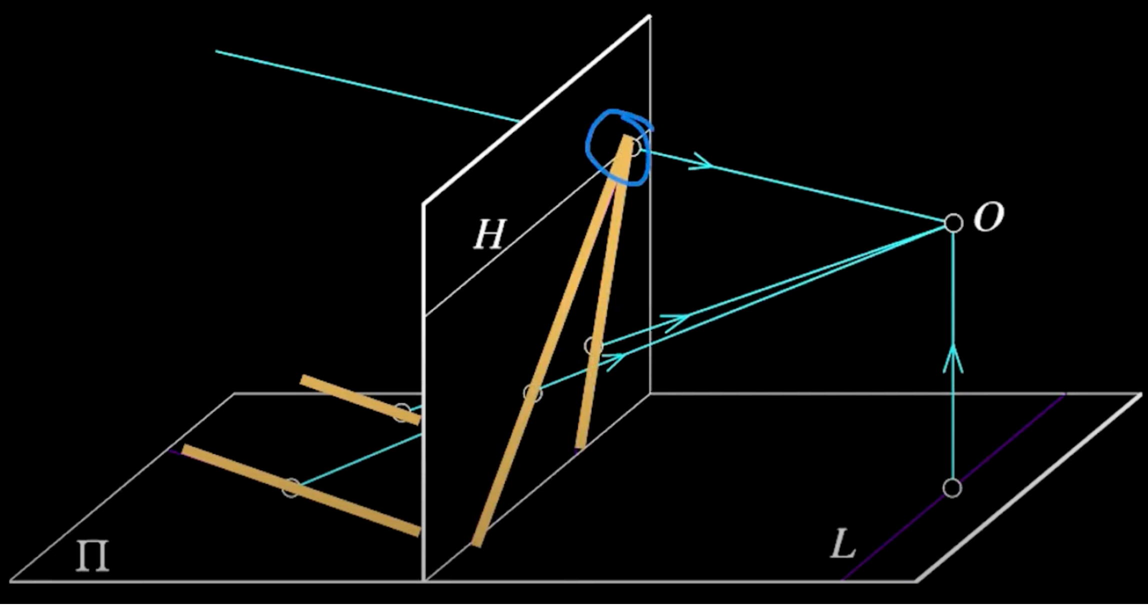
Perspective Projection

- How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

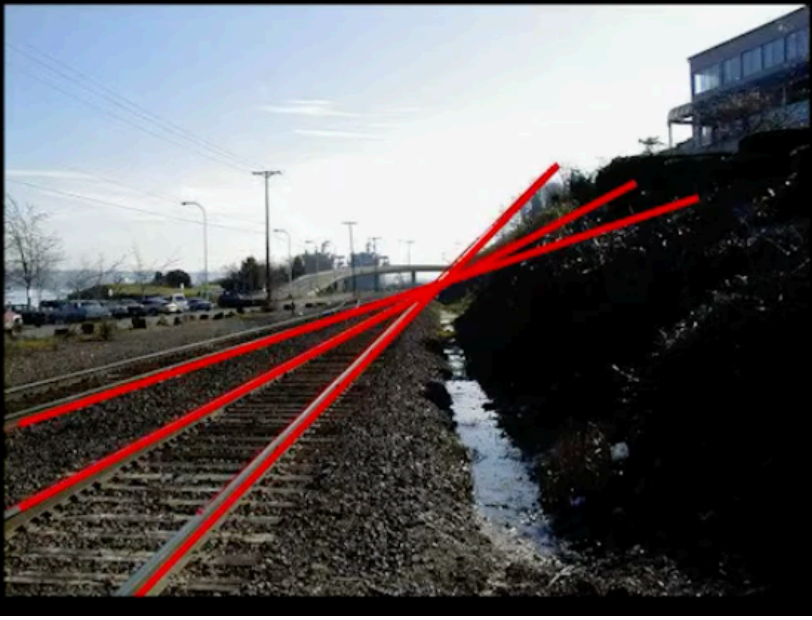
Parallel Lines in the World Meet in the Image

"**Vanishing**" point



Parallel Lines in the World Meet in the Image

"Vanishing" point



Parallel lines converge in math too…

Line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Perspective projection of the line

$$x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct}$$

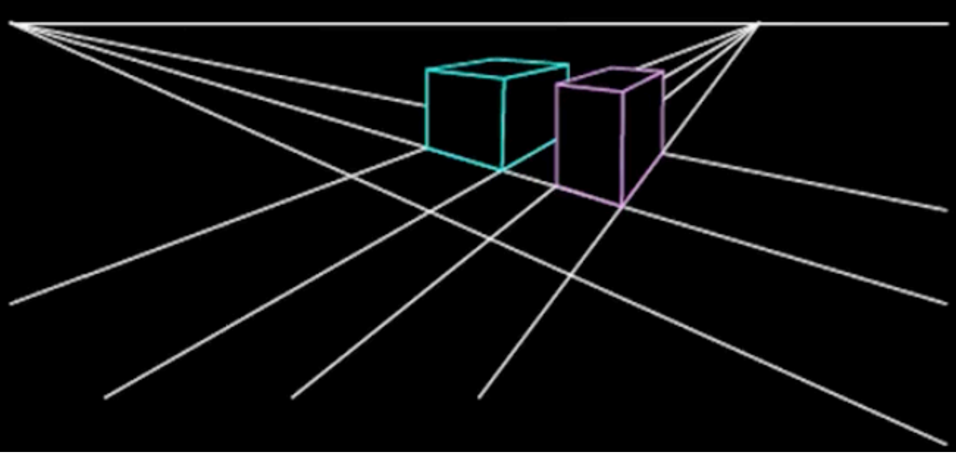
$$y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct}$$

In the limit as $t \to \pm\infty$ we have (for $c \neq 0$):

$$x'(t) \to \frac{fa}{c}, \quad y'(t) \to \frac{fb}{c}$$

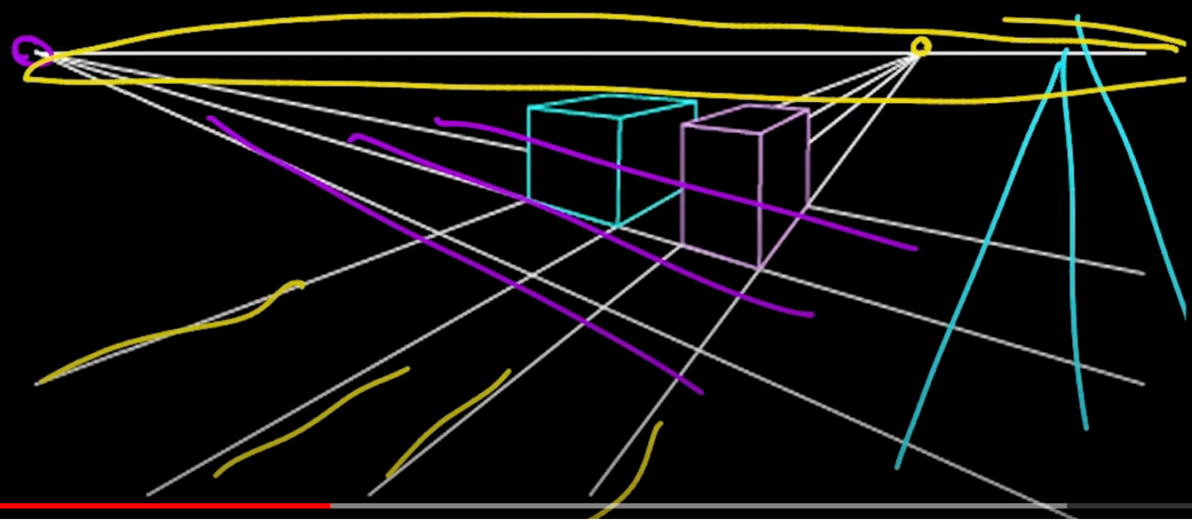
Vanishing points

- Each set of parallel lines (=direction) meets at a different point



Vanishing points

- Sets of parallel lines on the same plane lead to collinear vanishing points.
  - The line is called the horizon for that plane



Vanishing points