

Sensor Fusion in Mobile Robotics

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Kalman filter

KALMAN FILTER PREDICTION

$$x' = Fx + \nu$$

$$P' = FPF^T + Q$$

KALMAN FILTER UPDATE

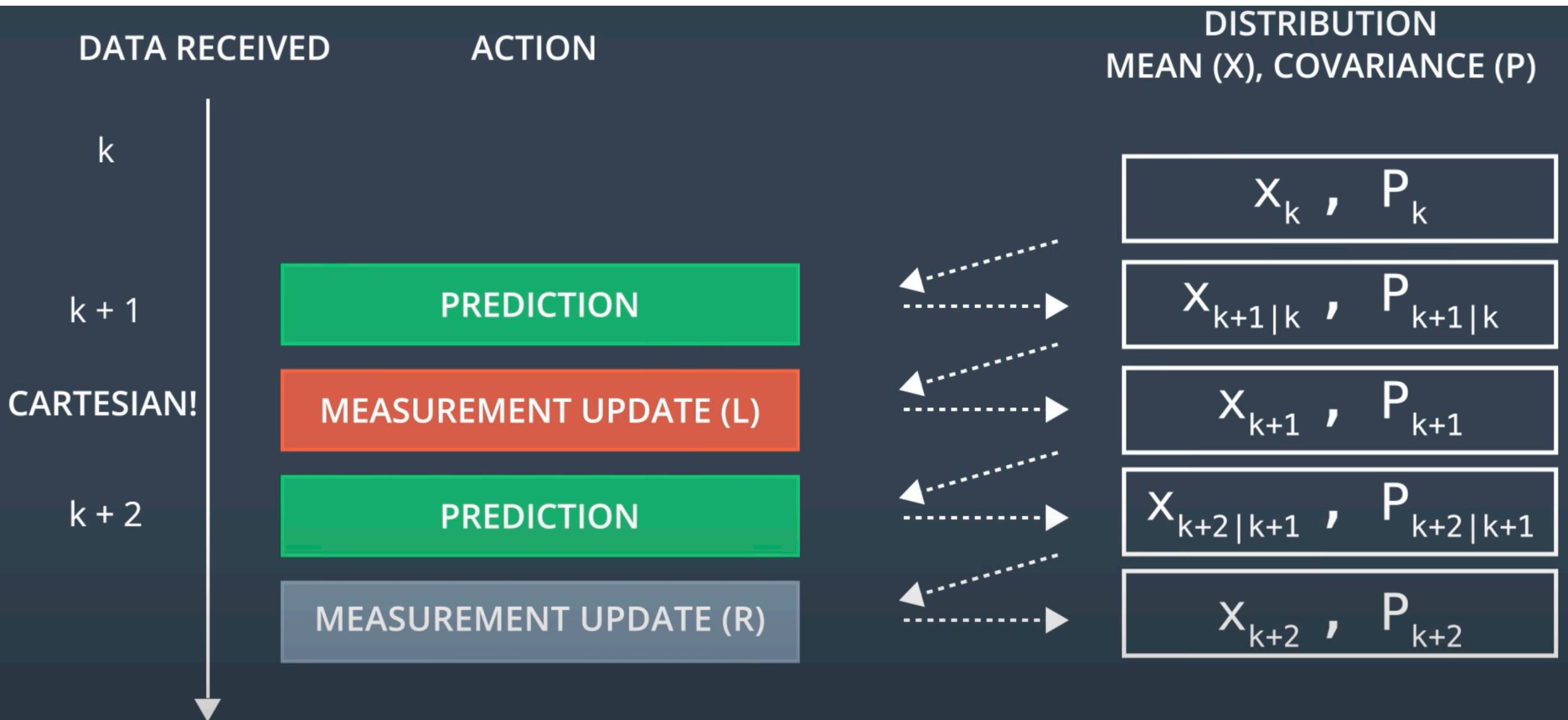
$$y = z - Hx'$$

$$S = HP'H^T + R$$

$$K = P'H^TS^{-1}$$

$$x = x' + Ky$$

$$P = (I - KH)P'$$



Prediction – Bicycle

Predicting the motion of a bicycle around the mobile robot

KALMAN FILTER PREDICTION

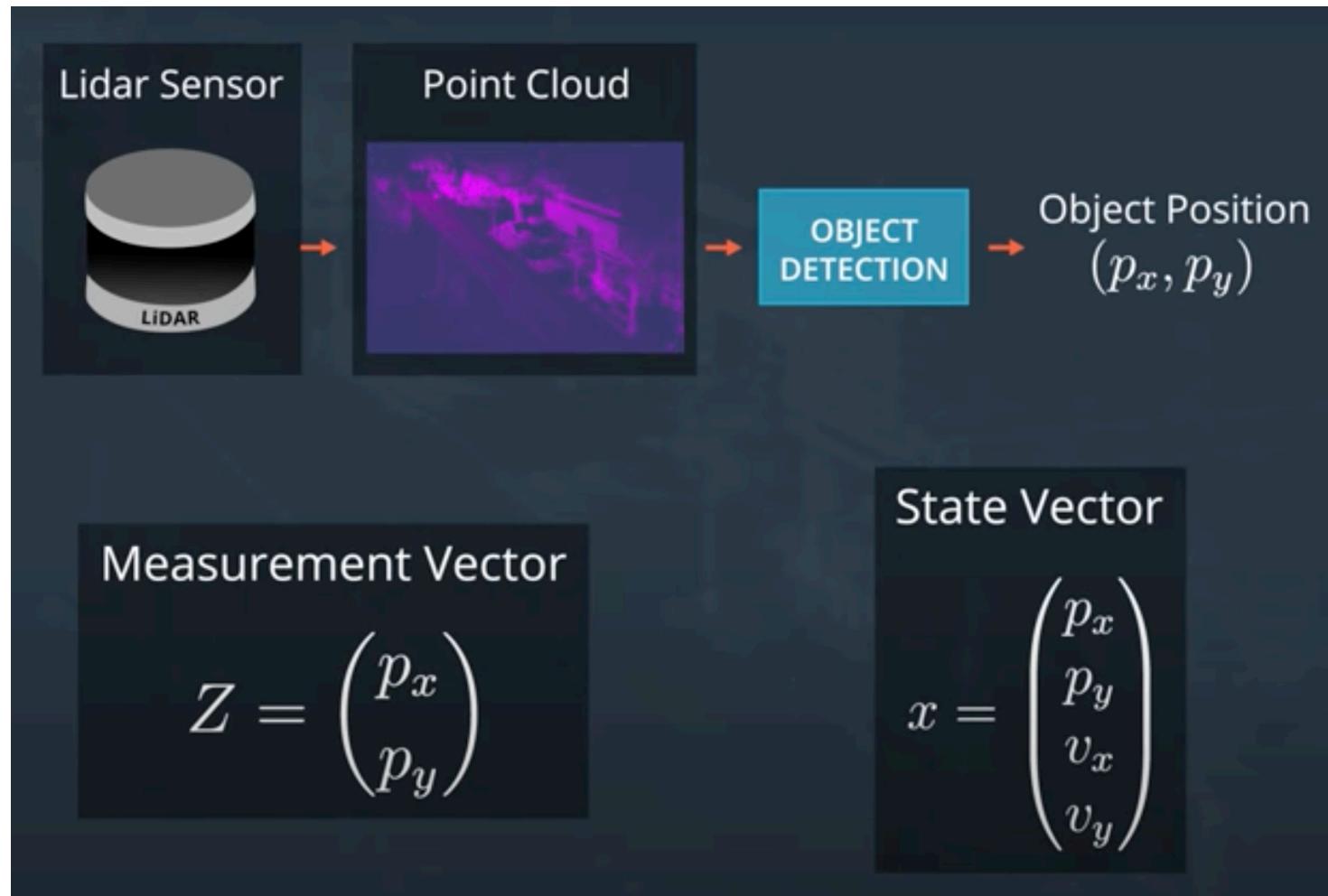
$$x' = Fx + \nu$$

$$P' = FPF^T + Q$$

$$\begin{pmatrix} p'_x \\ p'_y \\ v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ v_x \\ v_y \end{pmatrix} + \begin{pmatrix} \nu_{px} \\ \nu_{py} \\ \nu_{vx} \\ \nu_{vy} \end{pmatrix}$$

$$Q = GQ\nu G^T = \begin{pmatrix} \frac{\Delta t^4}{4}\sigma_{ax}^2 & 0 & \frac{\Delta t^3}{2}\sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^4}{4}\sigma_{ay}^2 & 0 & \frac{\Delta t^3}{2}\sigma_{ay}^2 \\ \frac{\Delta t^3}{2}\sigma_{ax}^2 & 0 & \Delta t^2\sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^3}{2}\sigma_{ay}^2 & 0 & \Delta t^2\sigma_{ay}^2 \end{pmatrix}$$

Measurement Update – Lidar



$$H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$R = E[\omega\omega^T] = \begin{pmatrix} \sigma_{px}^2 & 0 \\ 0 & \sigma_{py}^2 \end{pmatrix}$$

Measurement Update – Radar

RANGE: ρ (rho)

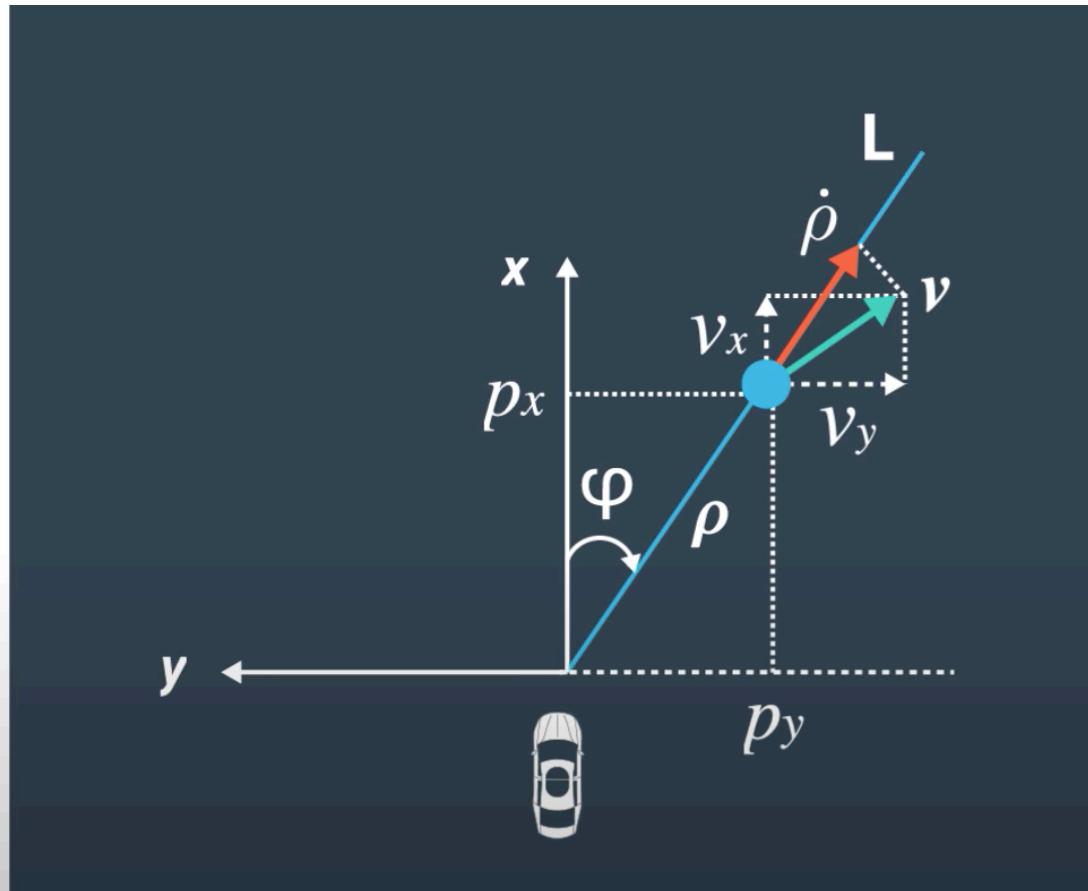
radial distance from origin

BEARING: φ (phi)

angle between ρ and x

RADIAL VELOCITY: $\dot{\rho}$ (rho dot)

change of ρ (range rate)



EKF

Kalman Filter

Prediction

$$x' = Fx + u$$

$$P' = FPF^T + Q$$

Measurement update

$$y = z - Hx'$$

$$S = HP'H^T + R$$

$$K = P'H^TS^{-1}$$

$$x = x' + Ky$$

$$P = (I - KH)P'$$

Extended Kalman Filter

$$x' = f(x, u)$$

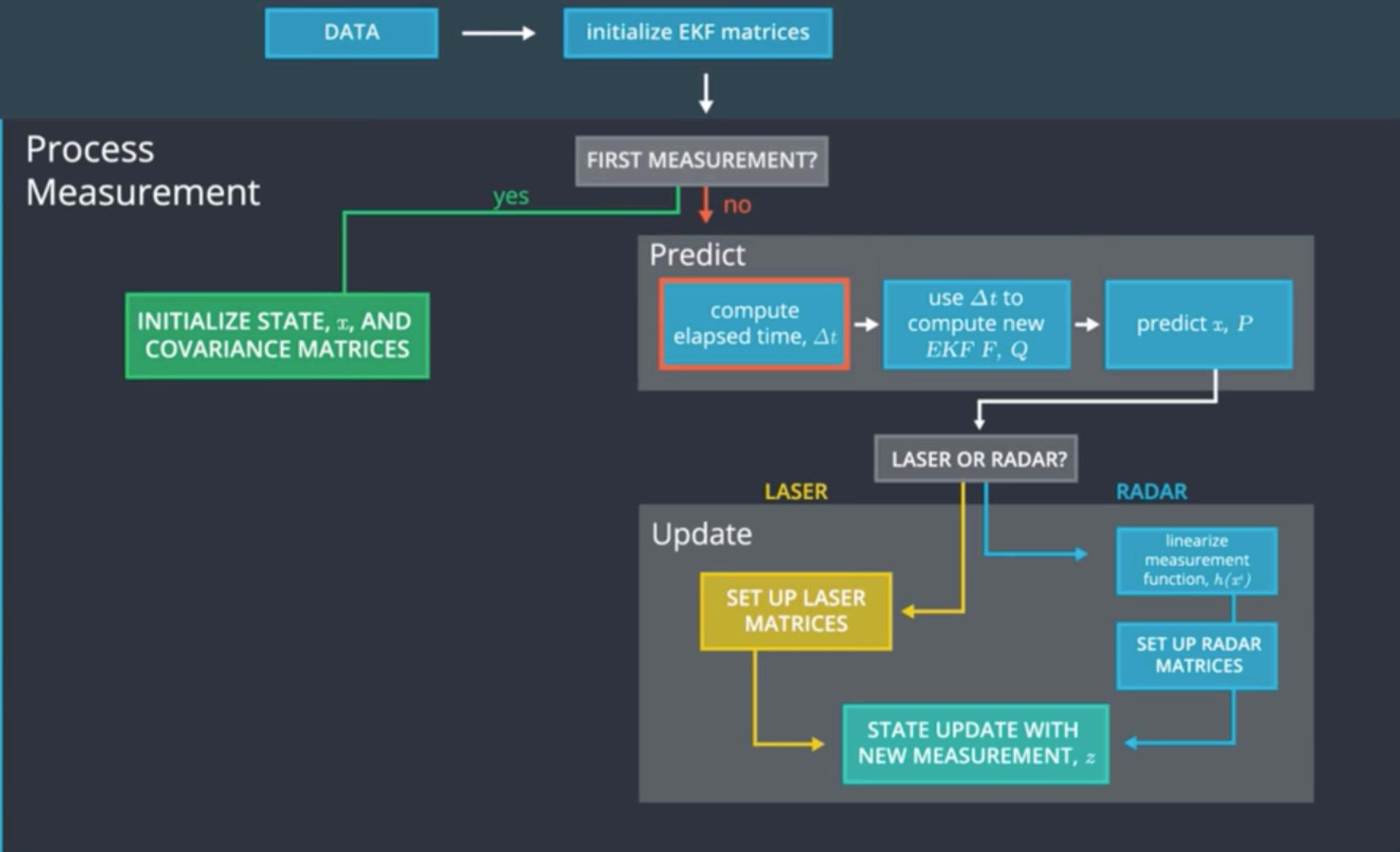
use F_j instead of F

$$y = z - h(x')$$

use H_j instead of H

$$h(x') = \begin{pmatrix} \rho \\ \phi \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} \sqrt{p_x'^2 + p_y'^2} \\ \arctan(p_y'/p_x') \\ \frac{p_x' v_x' + p_y' v_y'}{\sqrt{p_x'^2 + p_y'^2}} \end{pmatrix}$$

$$H_j = \begin{bmatrix} \frac{p_x}{\sqrt{p_x^2 + p_y^2}} & \frac{p_y}{\sqrt{p_x^2 + p_y^2}} & 0 & 0 \\ -\frac{p_y}{p_x^2 + p_y^2} & \frac{p_x}{p_x^2 + p_y^2} & 0 & 0 \\ \frac{p_y(v_x p_y - v_y p_x)}{(p_x^2 + p_y^2)^{3/2}} & \frac{p_x(v_y p_x - v_x p_y)}{(p_x^2 + p_y^2)^{3/2}} & \frac{p_x}{\sqrt{p_x^2 + p_y^2}} & \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \end{bmatrix}$$



$$x = \begin{pmatrix} p_x \\ p_y \\ v_x \\ v_y \end{pmatrix}$$

Raw Data

L	3.122427e-01	5.803398e-01	1477010443000000	6.000000e-01	6.000000e-01	5.199937e+00	0	0	6.911322e-03		
R	1.014892e+00	5.543292e-01	4.892807e+00	1477010443050000	8.599968e-01	6.000449e-01	5.199747e+00	1.796856e-03	3.455661e-04	1.382155e-02	
L	1.173848e+00	4.810729e-01	1477010443100000	1.119984e+00	6.002246e-01	5.199429e+00	5.389957e-03	1.036644e-03	2.072960e-02		
R	1.047505e+00	3.892401e-01	4.511325e+00	1477010443150000	1.379955e+00	6.006288e-01	5.198979e+00	1.077814e-02	2.073124e-03	2.763437e-02	
L	1.650626e+00	6.246904e-01	1477010443200000	1.639904e+00	6.013473e-01	5.198392e+00	1.795970e-02	3.454842e-03	3.453479e-02		
R	1.698300e+00	2.982801e-01	5.209986e+00	1477010443250000	1.8999823e+00	6.024697e-01	5.197661e+00	2.693234e-02	5.181582e-03	4.142974e-02	
L	2.188824e+00	6.487392e-01	1477010443300000	2.159704e+00	6.040855e-01	5.196776e+00	3.769324e-02	7.253069e-03	4.831816e-02		
R	2.044382e+00	2.760018e-01	5.043867e+00	1477010443350000	2.419540e+00	6.062840e-01	5.195728e+00	5.023894e-02	9.668977e-03	5.519894e-02	
L	2.655256e+00	6.659798e-01	1477010443400000	2.679323e+00	6.091545e-01	5.194504e+00	6.456542e-02	1.242892e-02	6.207101e-02		
R	2.990916e+00	2.176679e-01	5.191807e+00	1477010443450000	2.939043e+00	6.127858e-01	5.193090e+00	8.066803e-02	1.553247e-02	6.893328e-02	
L	3.012223e+00	6.370455e-01	1477010443500000	3.198690e+00	6.172666e-01	5.191470e+00	9.854147e-02	1.897914e-02	7.578466e-02		
R	3.593878e+00	1.354522e-01	5.161753e+00	1477010443550000	3.458253e+00	6.226855e-01	5.189627e+00	1.181798e-01	2.276837e-02	8.262407e-02	
L	3.893650e+00	3.117930e-01	1477010443600000	3.717722e+00	6.291305e-01	5.187542e+00	1.395764e-01	2.689958e-02	8.945044e-02		
R	4.255547e+00	1.648397e-01	5.433327e+00	1477010443650000	3.977082e+00	6.366893e-01	5.185194e+00	1.627238e-01	3.137210e-02	9.626268e-02	
L	4.309346e+00	5.785637e-01	1477010443700000	4.236322e+00	6.454494e-01	5.182560e+00	1.876140e-01	3.618523e-02	1.030597e-01		
R	4.670263e+00	1.481801e-01	5.120847e+00	1477010443750000	4.495424e+00	6.554977e-01	5.179618e+00	2.142382e-01	4.133822e-02	1.098405e-01	
L	4.351431e+00	8.991741e-01	1477010443800000	4.754374e+00	6.669207e-01	5.176340e+00	2.425866e-01	4.683024e-02	1.166039e-01		
R	5.251417e+00	1.271635e-01	4.825914e+00	1477010443850000	5.013155e+00	6.798044e-01	5.172700e+00	2.726487e-01	5.266044e-02	1.233489e-01	
L	5.518935e+00	6.482327e-01	1477010443900000	5.271746e+00	6.942343e-01	5.168671e+00	3.044132e-01	5.882788e-02	1.300744e-01		
R	5.267293e+00	1.216834e-01	5.423506e+00	1477010443950000	5.530128e+00	7.102953e-01	5.164221e+00	3.378677e-01	6.533161e-02	1.367794e-01	
L	6.022003e+00	7.086193e-01	1477010444000000	5.788279e+00	7.280715e-01	5.159319e+00	3.729989e-01	7.217058e-02	1.434628e-01		
R	5.905749e+00	6.329996e-02	4.879680e+00	1477010444050000	6.046176e+00	7.476465e-01	5.153933e+00	4.097925e-01	7.934372e-02	1.501236e-01	
L	6.342486e+00	9.488326e-01	1477010444100000	6.303794e+00	7.691030e-01	5.148029e+00	4.482333e-01	8.684990e-02	1.567606e-01		
R	6.673922e+00	1.256145e-01	5.006870e+00	1477010444150000	6.561105e+00	7.925232e-01	5.141571e+00	4.883049e-01	9.468793e-02	1.633729e-01	
L	6.782143e+00	7.140359e-01	1477010444200000	6.818081e+00	8.179882e-01	5.134523e+00	5.299897e-01	1.028566e-01	1.699593e-01		
R	7.318441e+00	8.629228e-02	4.649107e+00	1477010444250000	7.074691e+00	8.455782e-01	5.126847e+00	5.732691e-01	1.113545e-01	1.765190e-01	
L	7.137350e+00	9.572165e-01	1477010444300000	7.330903e+00	8.753725e-01	5.118505e+00	6.181232e-01	1.201805e-01	1.830507e-01		