Discrete gradient

For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1,y)-f(x,y)}{1}$$
$$\approx f(x+1,y)-f(x,y)$$

"right derivative" But is it???

Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$

-1	1

$$\frac{\partial f(x,y)}{\partial y}$$

-1
1

? or

1
-1

(correlation filters)

The discrete gradient

0	0
-1	+1
0	0

H

Not symmetric around image point; which is "middle" pixel?

0	0	0
-1/2	0	+1/2
0	0	0

H

Average of "left" and "right" derivative . See?

-1 +1

-1 +1

-1 0 +1

1/2 -1/2 0 +1/2

Example: Sobel operator

$$\frac{1}{8} * \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{8} * \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

s_x $\qquad$ s_y

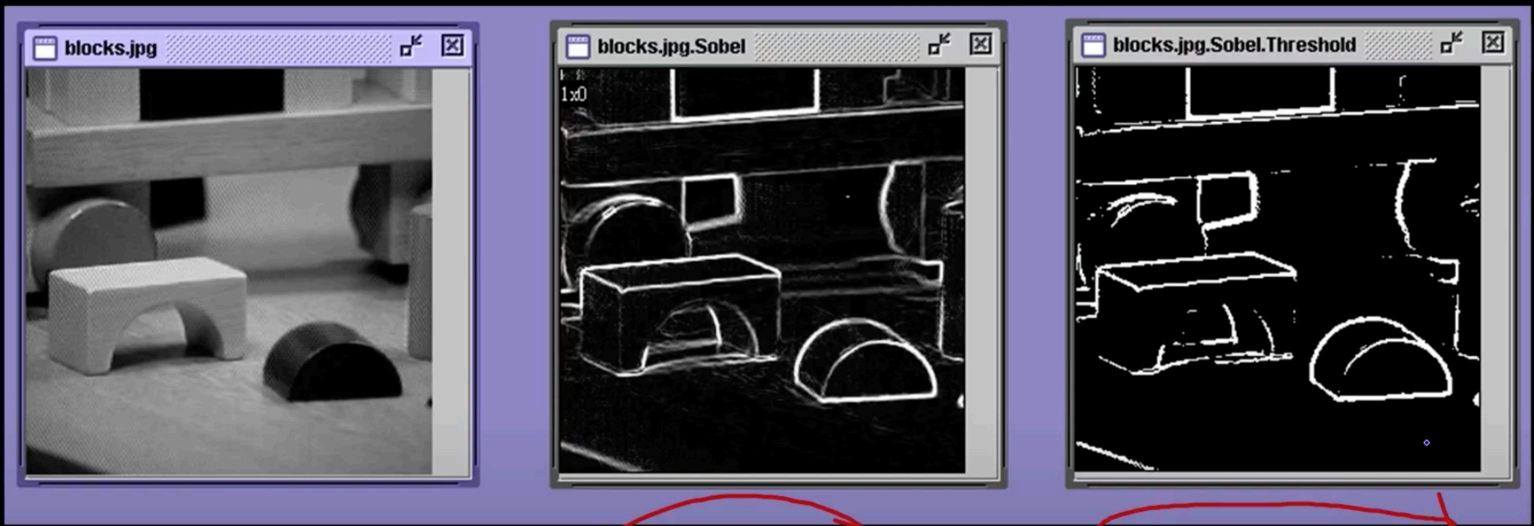
(here positive y is up)

(Sobel) Gradient is $\nabla \mathbf{I} = [\mathbf{g_x} \;\; \mathbf{g_y}]^T$

$g = (g_x{}^2 + g_y{}^2)^{1/2}$ is the gradient magnitude.

$\theta = \text{atan2}(g_y , g_x)$ is the gradient direction.

Sobel Operator on Blocks Image



original image

gradient magnitude

thresholded gradient magnitude

Some Well-Known Gradients Masks

- **Sobel:**

Sx

-1	0	1
-2	0	2
-1	0	1

Sy

1	2	1
0	0	0
-1	-2	-1

- **Prewitt:**

Sx

-1	0	1
-1	0	1
-1	0	1

Sy

1	1	1
0	0	0
-1	-1	-1

- **Roberts:**

Sx

0	1
-1	0

Sy

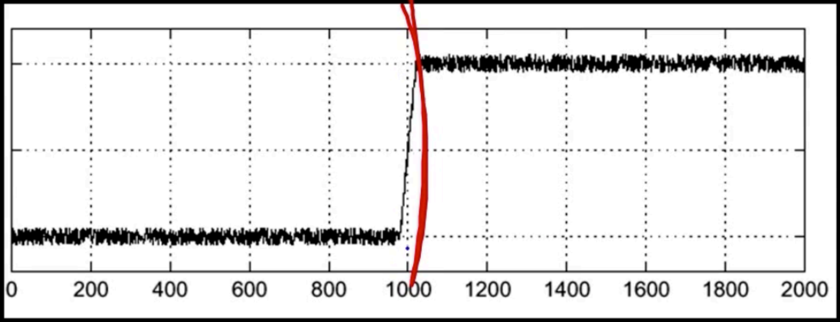
1	0
0	-1

But in the real world...

Consider a single row or column of the image

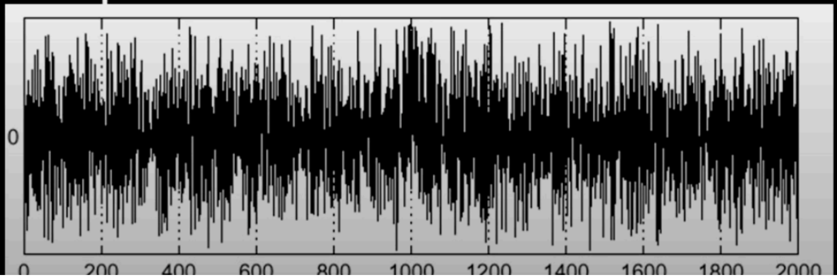
(plotting intensity as a function of *x*)

$f(x)$



Apply derivative operator....

$\frac{d}{dx} f(x)$



Uh, where's the edge?

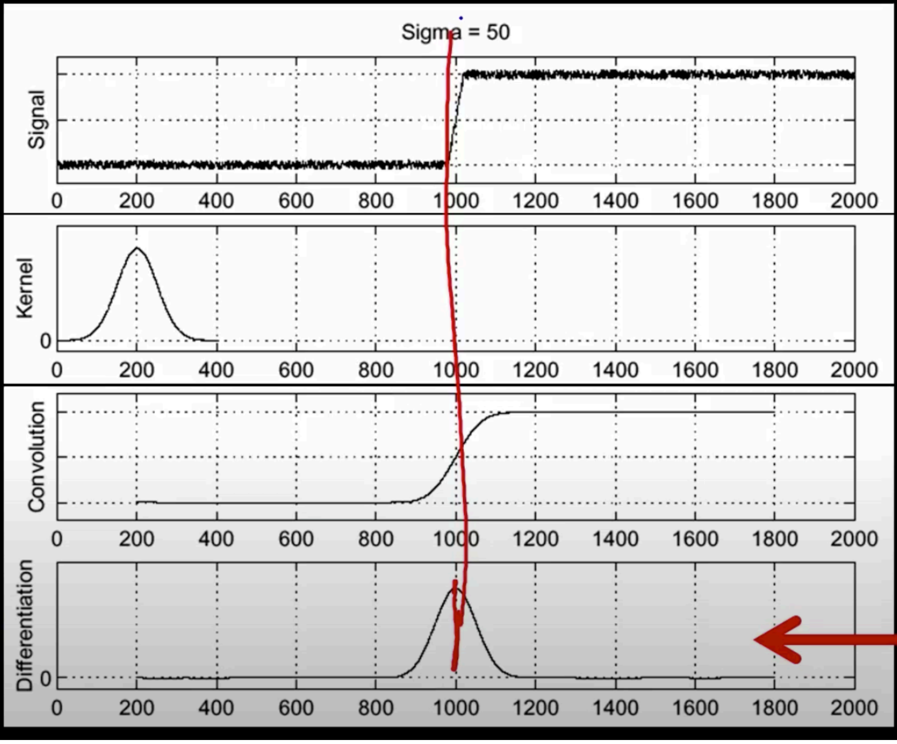
Solution: smooth first

f

h

$h * f$

$\frac{\partial}{\partial x}(h * f)$



Where is the edge?

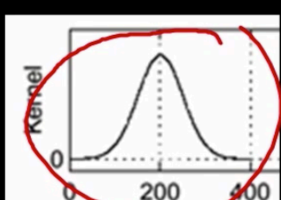
Look for peaks

Derivative theorem of convolution

This saves us one operation: $\frac{\partial}{\partial x}(h * f) = (\frac{\partial}{\partial x} h) * f$

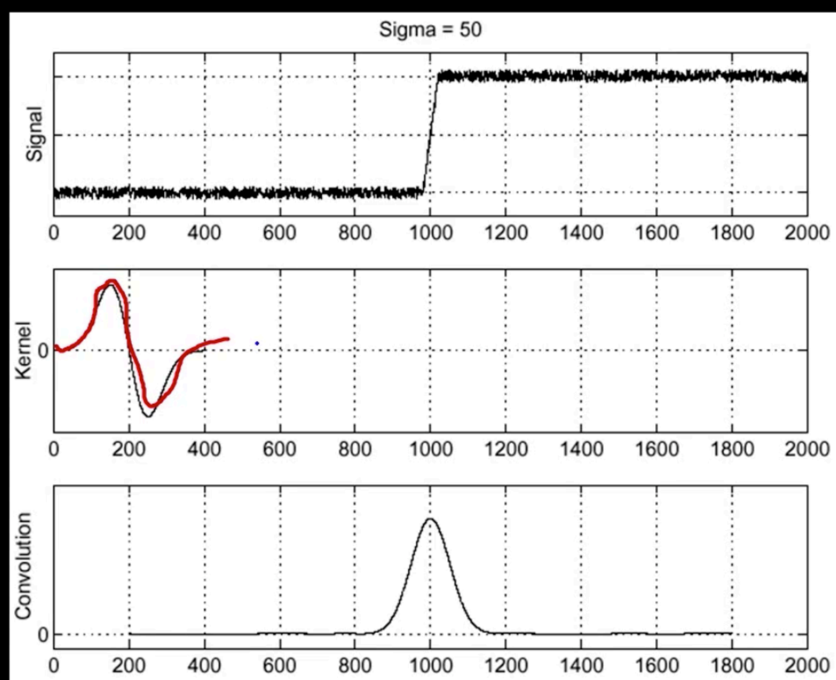
f

h



$\frac{\partial}{\partial x} h$

$(\frac{\partial}{\partial x} h) * f$

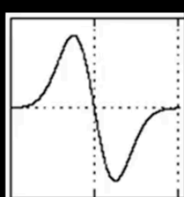


2^{nd} derivative of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h * f)$

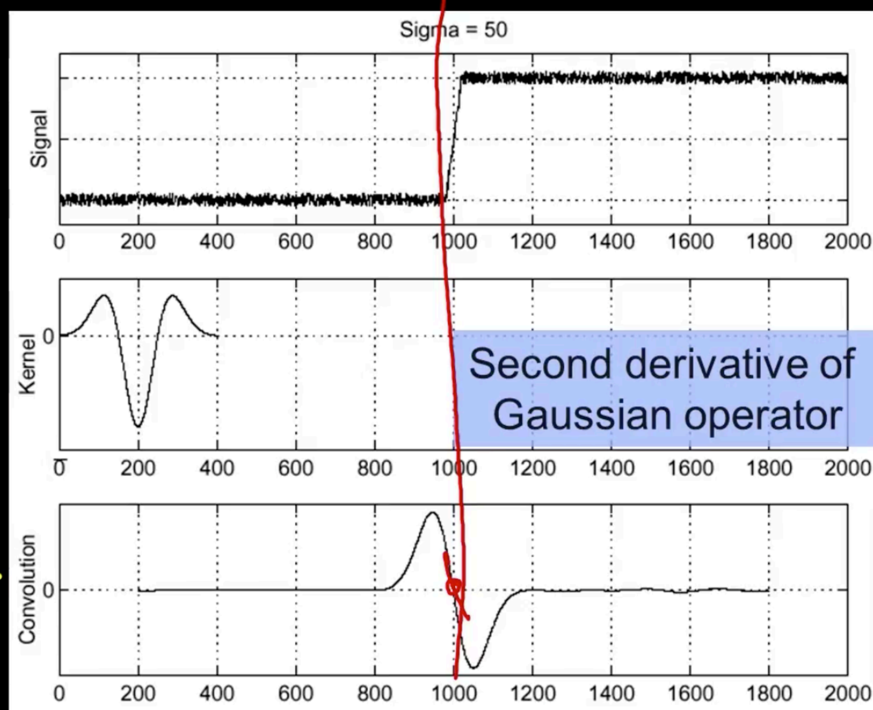
f

$\boxed{\frac{\partial}{\partial x} h}$

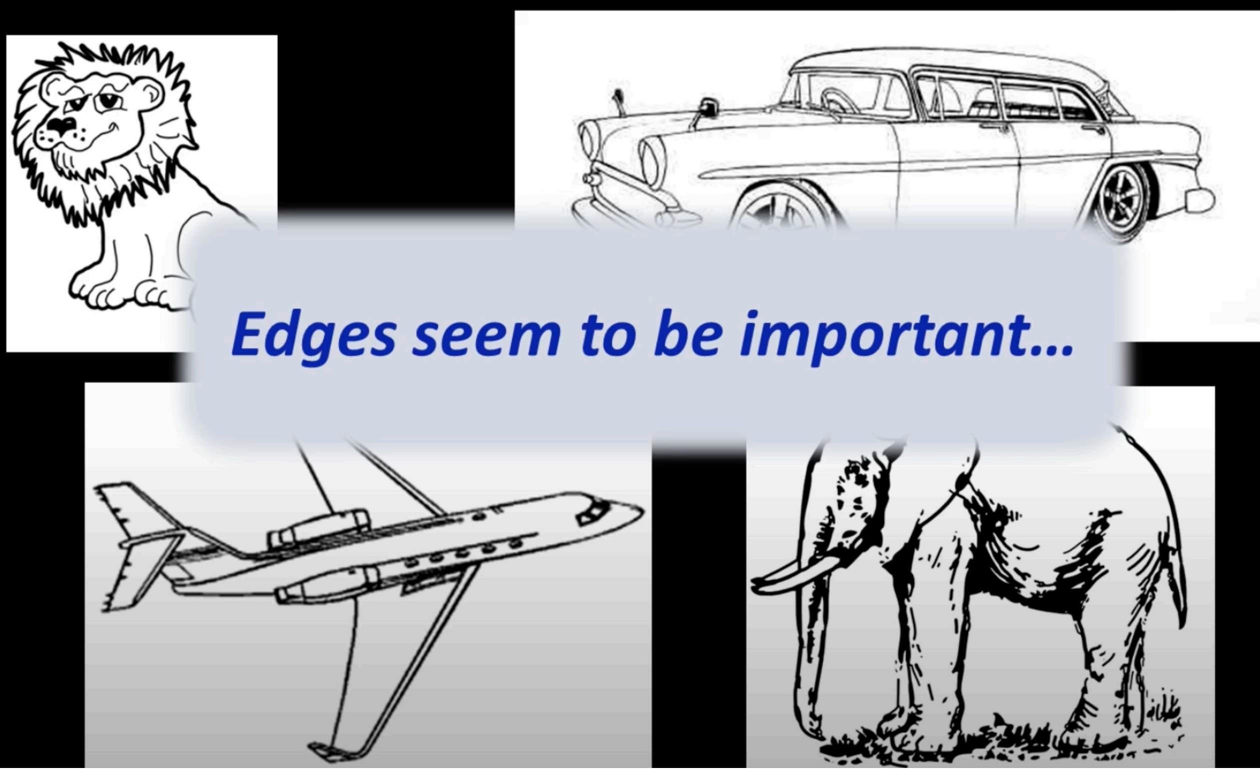


$\frac{\partial^2}{\partial x^2} h$

$(\frac{\partial^2}{\partial x^2} h) * f$



Reduced images

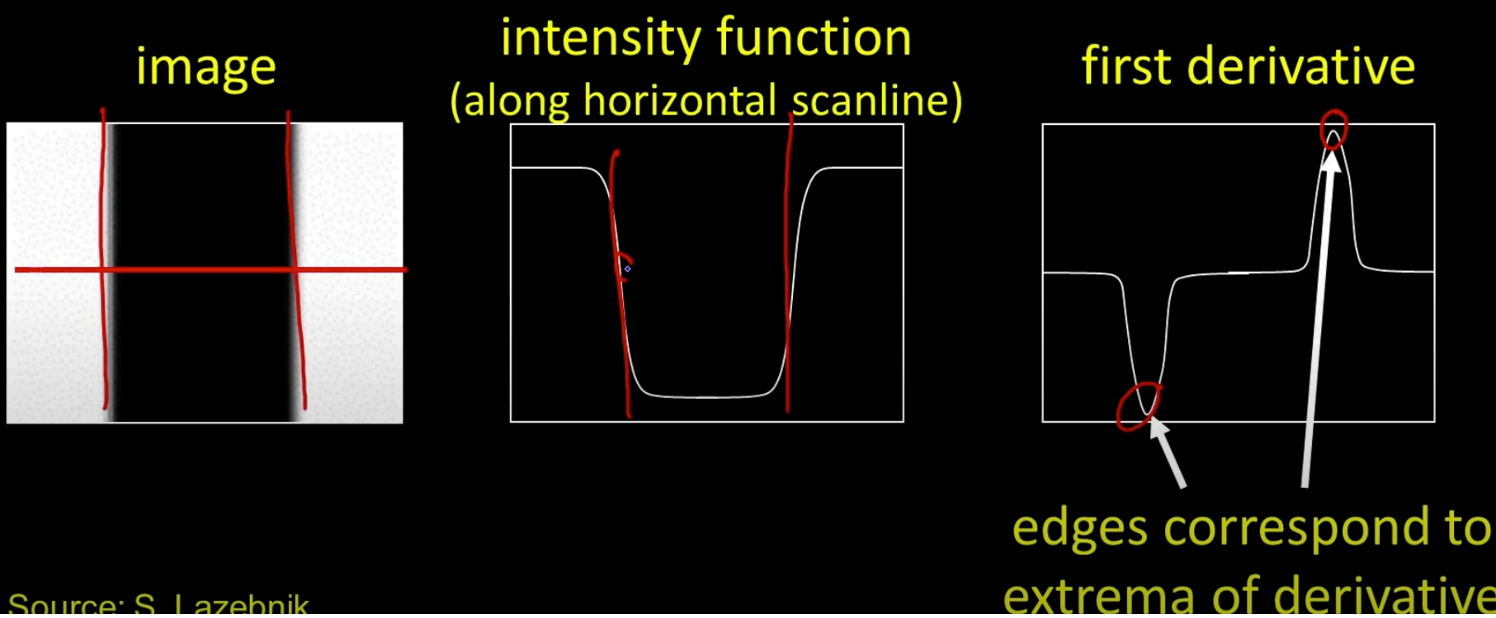


Edges seem to be important…

Edge detection

Derivatives and edges

An edge is a place of rapid change in the image intensity function.



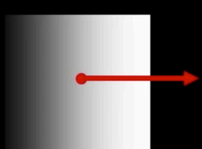
Source: S. Lazebnik

Differential Operators

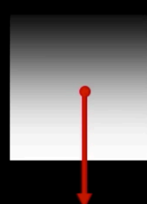
- Differential operators –when applied to the image returns some derivatives.
- Model these “operators” as masks/kernels that compute the image gradient function.
- Threshold the this gradient function to select the edge pixels.

Image gradient

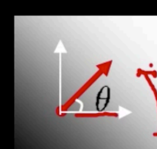
The gradient of an image: $$\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$$



$$\nabla f = [\frac{\partial f}{\partial x}, 0]$$



$$\nabla f = [0, \frac{\partial f}{\partial y}]$$



$$\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$$

The gradient points in the direction of most rapid increase in intensity

Image gradient

The gradient of an image: $\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$

The gradient direction is given by: $\theta = \tan^{-1}(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x})$

The *amount of change* is given by the gradient magnitude: $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$