Parametric model

- A ***parametric*** model can represent a class of instances where each is defined by a value of the parameters.
- Examples include lines, or circles, or even a parameterized template.

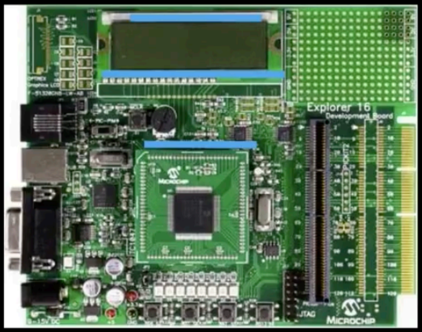
Fitting a parametric model

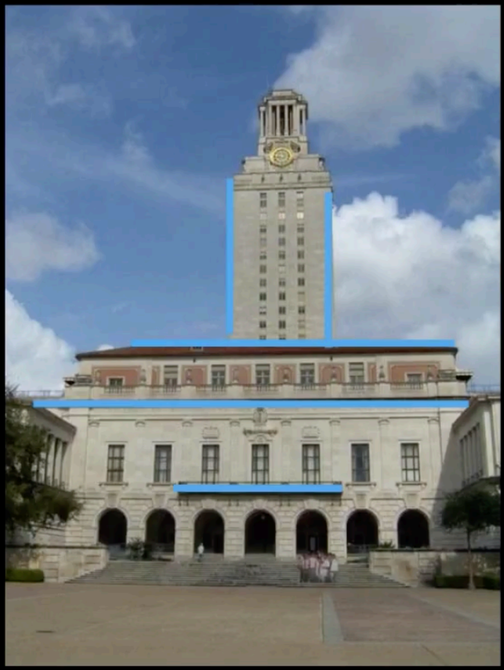
- Choose a parametric model to represent a set of features
- Membership criterion is not local:

  Can't tell whether a point in the image belongs to a given model just by looking at that point
- Computational complexity is important

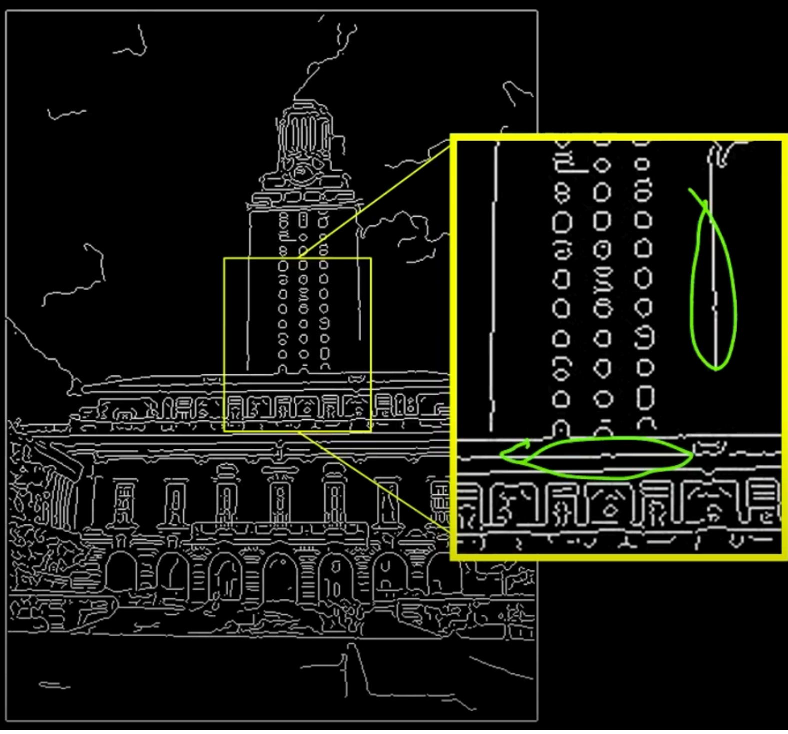
  Not feasible to examine possible parameter setting

Example: Line fitting





Difficulty of line fitting



- Extra edge points (clutter), multiple models.
- Only some parts of each line detected, and some parts are missing.
- Noise in measured edge points, orientations.

Voting

Voting is a general technique where we let the features vote for all models that are compatible with it.

1. Cycle through features, each casting votes for model parameters.
2. Look for model parameters that receive a lot of votes.

Voting – why it works

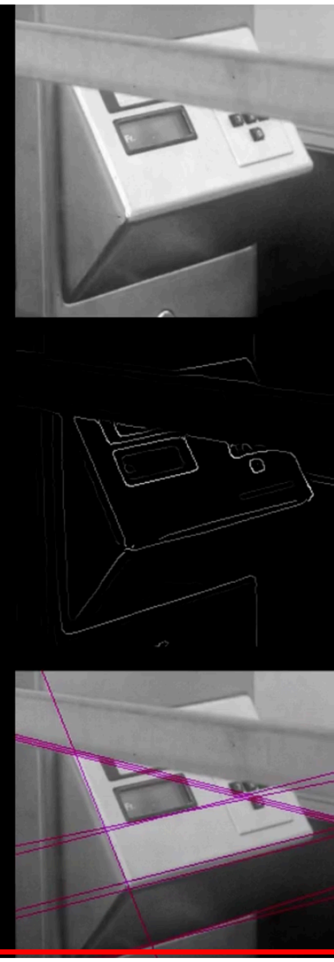
- Noise & clutter features will cast votes too, but typically their votes should be inconsistent with the majority of “good” features.
- Ok if some features not observed, as model can span multiple fragments.

Fitting lines

Hough Transform is a voting technique that can be used to answer all of these

Main idea

1. Each edge point votes for compatible lines.
2. Look for lines that get many votes.



Hough space

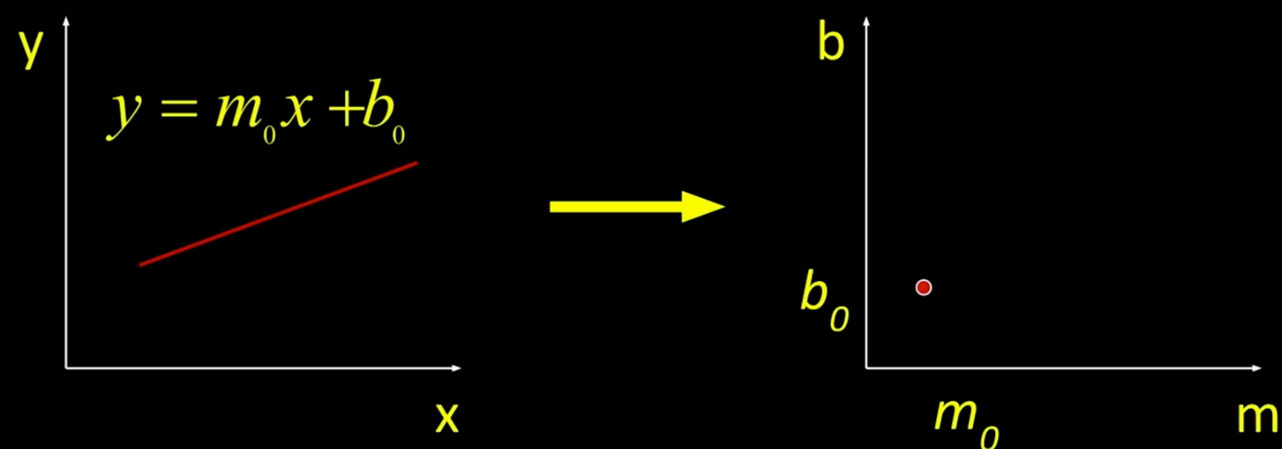


image space

Hough (parameter) space

A line in the image corresponds to a point in Hough space

Hough space

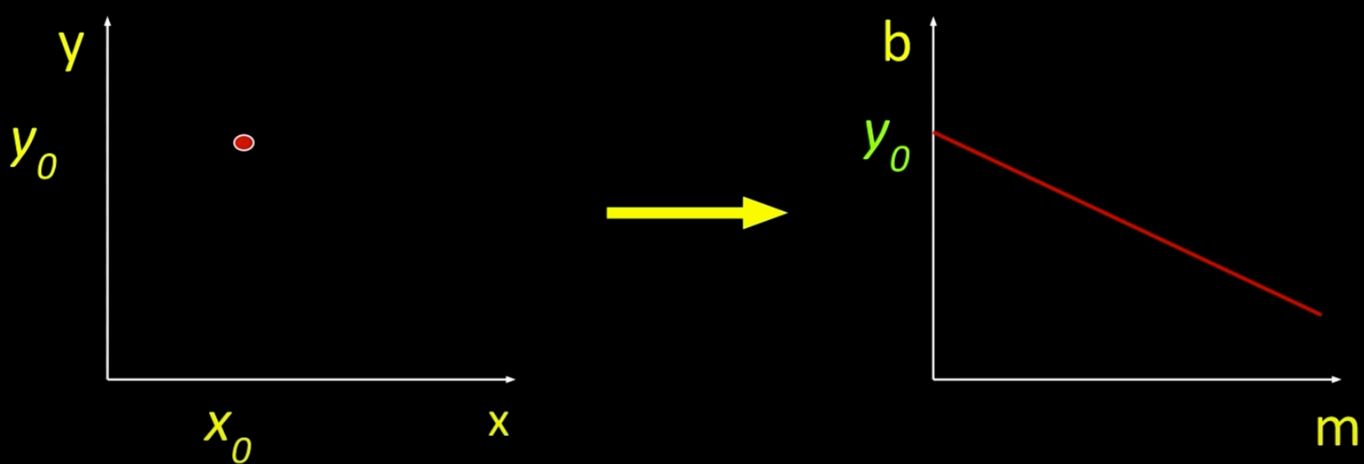
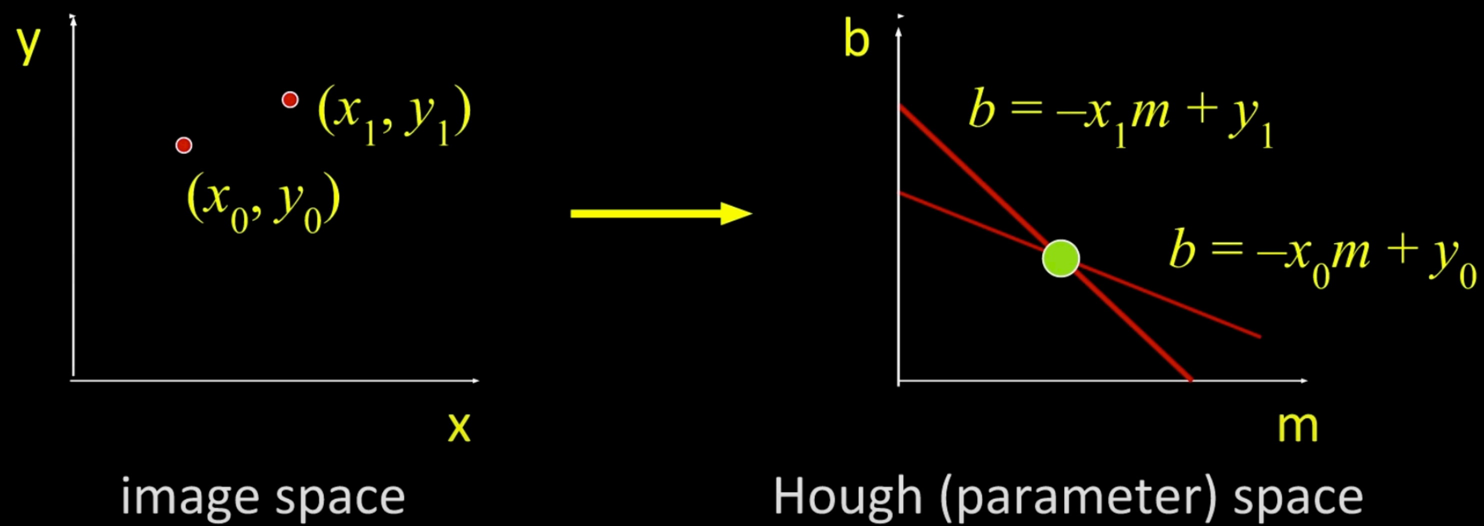


image space

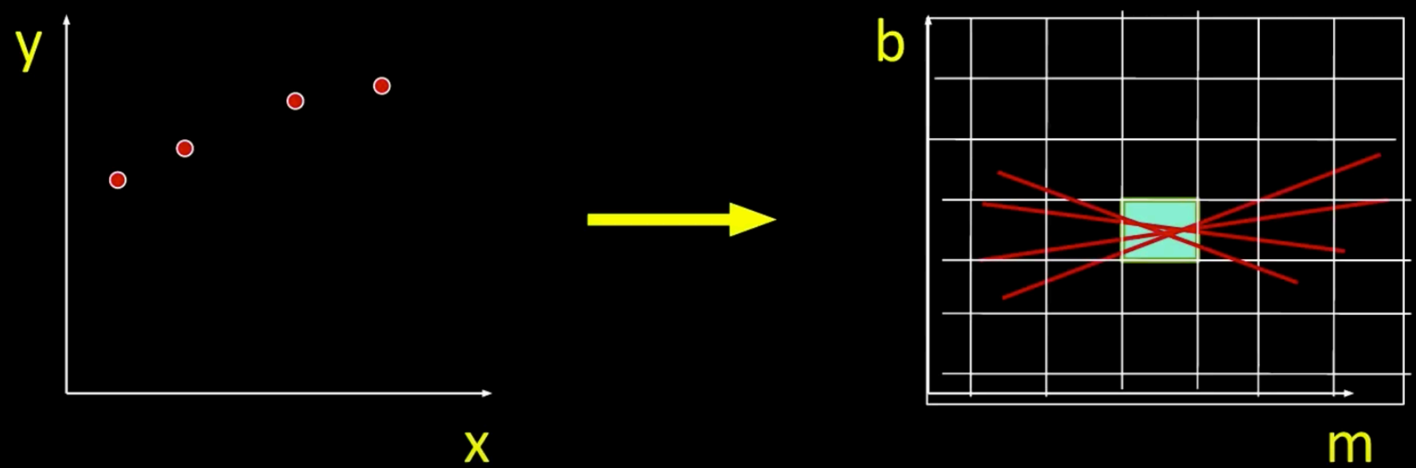
Hough (parameter) space

$$y_0 = m x_0 + b \longrightarrow b = -x_0 m + y_0$$

Hough space

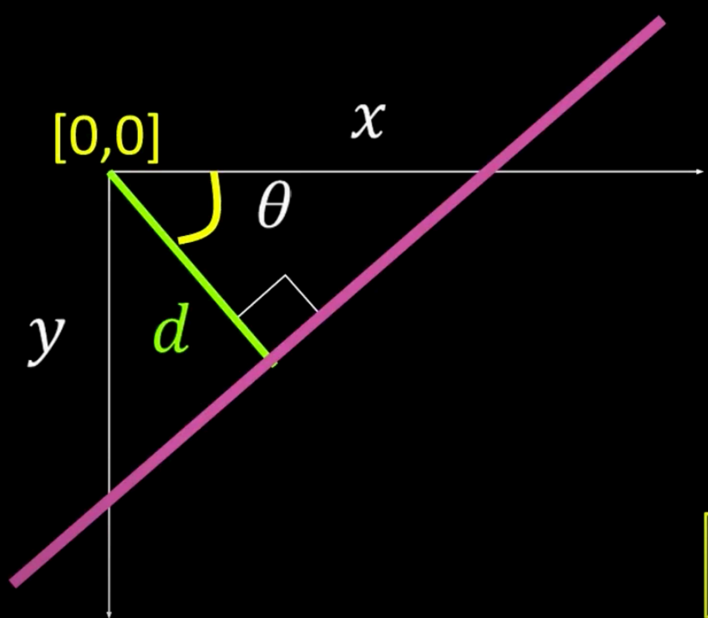

y
(x_1, y_1)
(x_0, y_0)
x
image space
b
$b = -x_1 m + y_1$
$b = -x_0 m + y_0$
m
Hough (parameter) space

Hough algorithm



- Let each edge point in image space vote for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

Polar representation for lines



d: perpendicular distance from line to origin

θ: angle the perpendicular makes with the x-axis

$$\boxed{x\cos\theta + y\sin\theta = d}$$

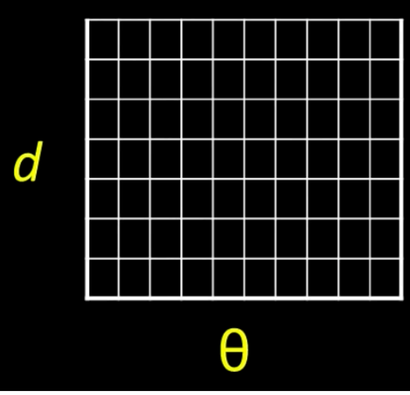
Point in image space is now sinusoid segment in Hough space

Hough transform algorithm

Using the polar parameterization:

$$x\cos\theta - y\sin\theta = d$$

And a *Hough Accumulator Array (keeps the votes)*



Source: Steve Seitz

Basic Hough transform algorithm

1. Initialize H[d, θ]=0
2. For each ***edge*** point in $E(x, y)$ in the image

    for θ = 0 to 180 // some quantization; why not 2pi?

    $d = x\cos\theta - y\sin\theta$ // maybe negative

    H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given

    by $d = x\cos\theta - y\sin\theta$

Complexity of the Hough transform

Space complexity?

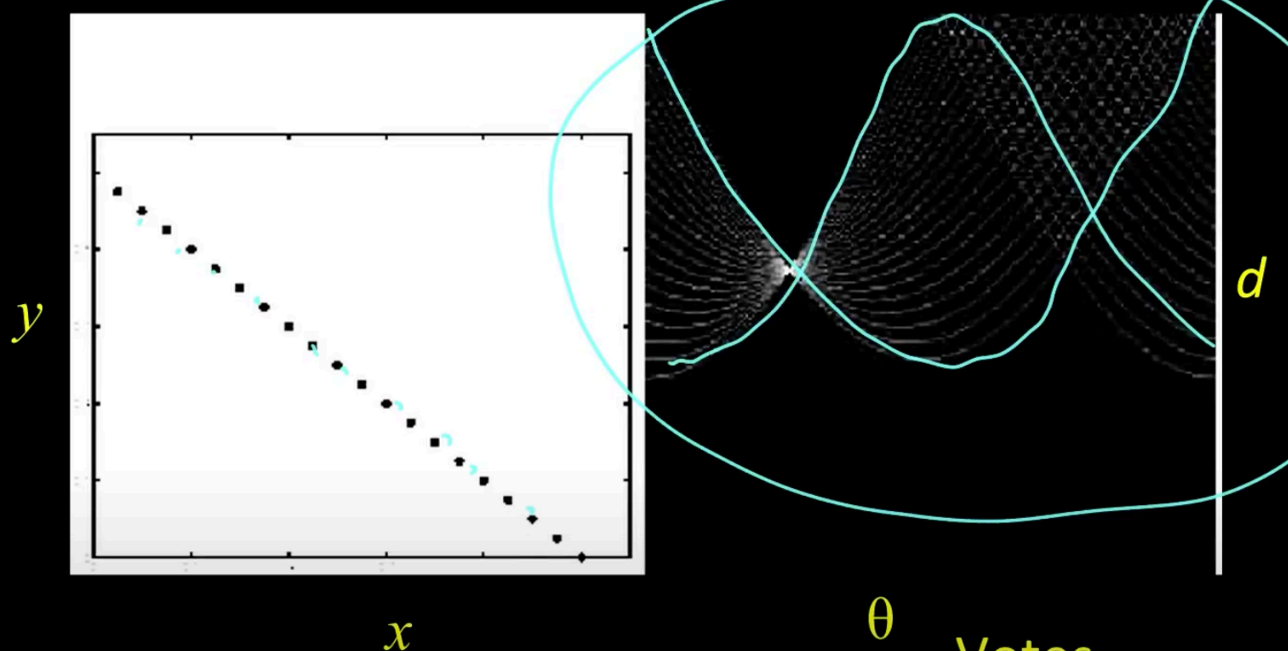
for exm.
two dimension

k^n (n dimensions, k bins each)

Time complexity (in terms of number of voting elements)?

Constant

Hough example



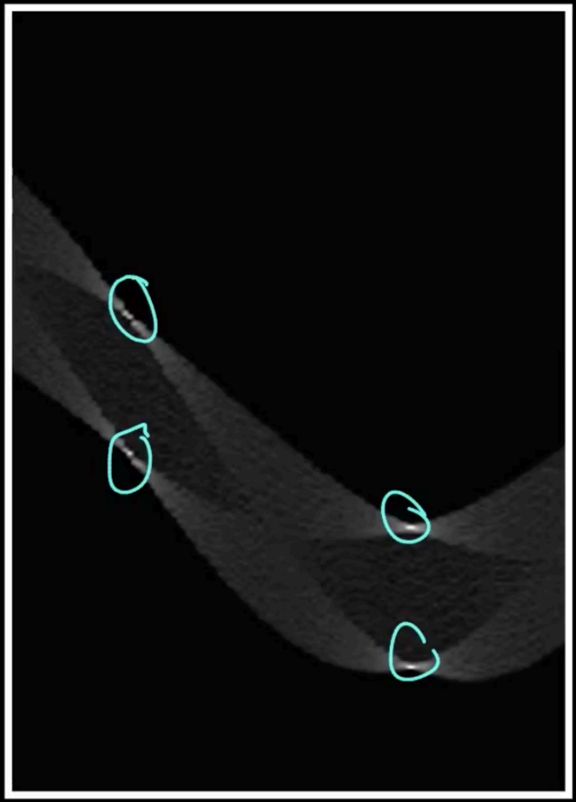
Votes

Image space
edge coordinates

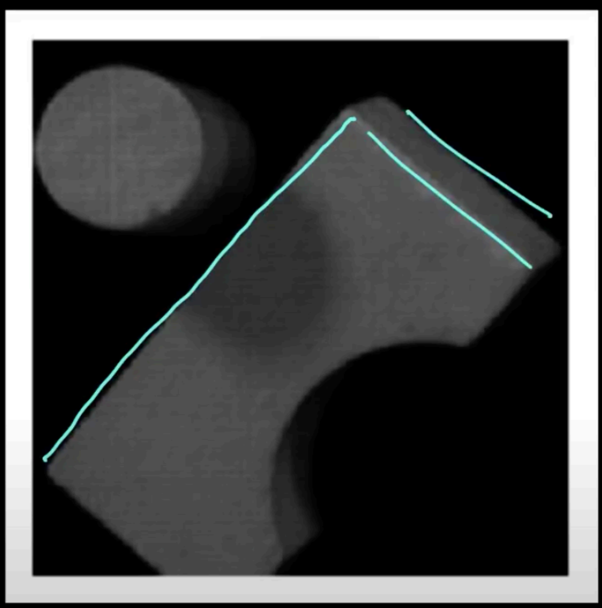
Bright value = high vote count
Black = no votes

Example: Hough transform of a square

Square :

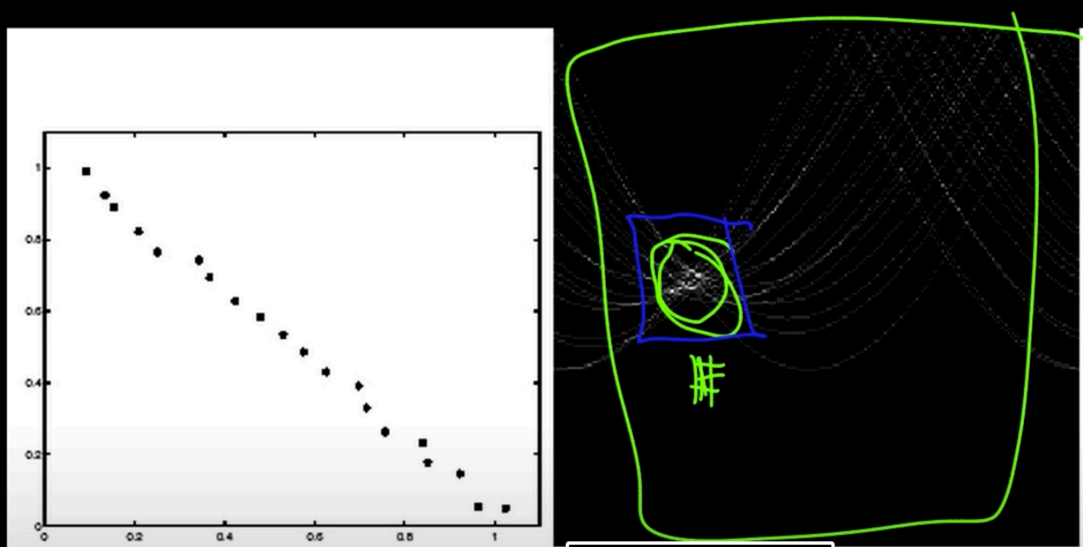


Hough transform of blocks scene



Impact of noise on Hough

y

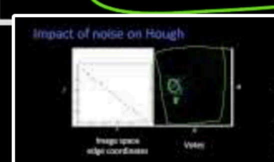


d

x

θ

Image space
edge coordinates



tes

1:14

Impact of more noise on Hough

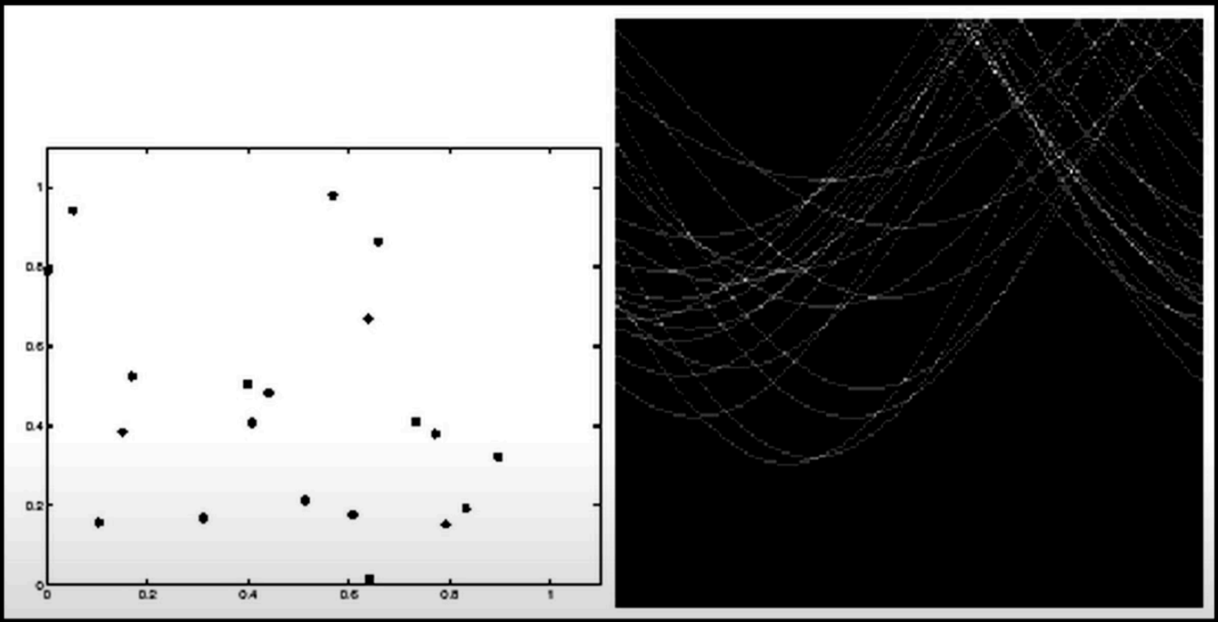


Image space
edge coordinates

Votes

Extensions – using the gradient

1. Initialize H[d, θ]=0
2. For each ***edge*** point in $E(x, y)$ in the image
   θ = gradient at (x,y)
   $d = x\cos\theta - y\sin\theta$
   H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given
   by $d = x\cos\theta - y\sin\theta$

$$\nabla f = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]$$

$$\theta = \tan^{-1}(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x})$$

Extensions

Extension 2

Give more votes for stronger edges

Extension 3

change the sampling of (d, θ) to give more/less resolution

Extension 4

The same procedure can be used with circles, squares, or any other shape