

# Visual Depth Perception: Stereopsis

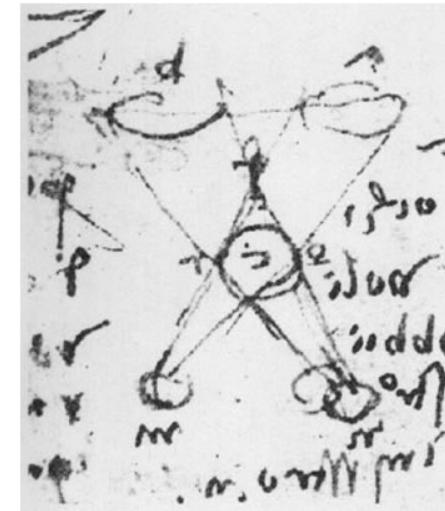
Course 3, Module 1, Lesson 3 – Part 1



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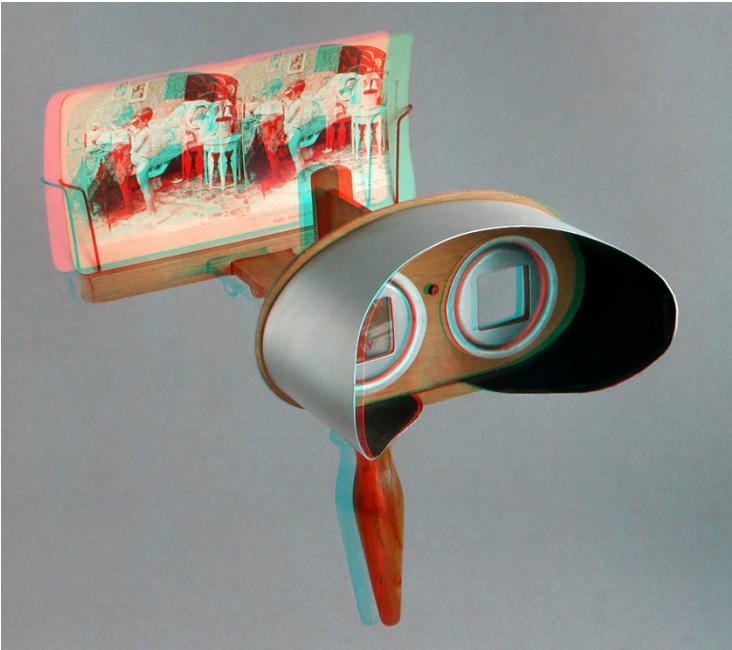
# Stereopsis

- Charles Wheatstone (1838): “... the mind perceives an object of three dimensions by means of the two dissimilar pictures projected by it on the two retinæ ...”
- Pre 19th century: Leonardo da Vinci



Peter Hohenstatt : Leonardo da Vinci, 1452-1519, Knemann Verlag, 1998

# Stereoscopes: A 19th Century Pastime

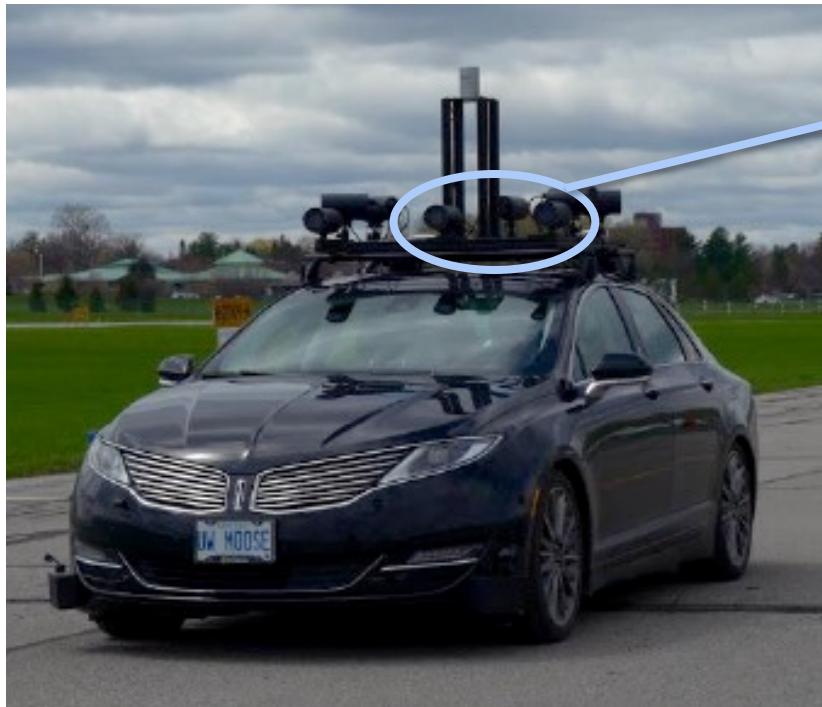


[https://commons.wikimedia.org/wiki/File:Holmes\\_stereoscope\\_anaglyph\\_01.jpg](https://commons.wikimedia.org/wiki/File:Holmes_stereoscope_anaglyph_01.jpg)

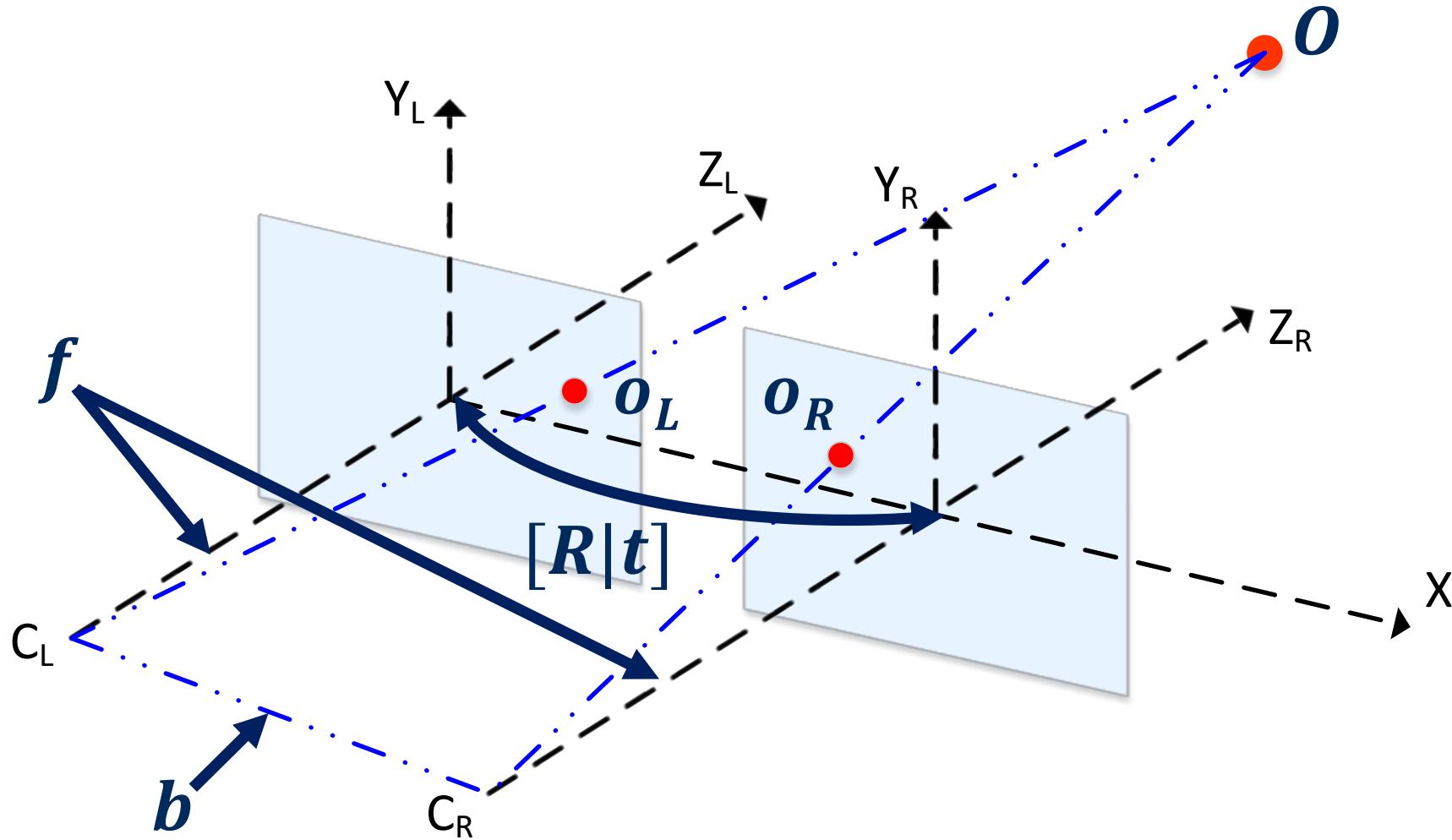


[https://commons.wikimedia.org/wiki/File:Lincoln\\_stereoscope.jpg](https://commons.wikimedia.org/wiki/File:Lincoln_stereoscope.jpg)

# Stereo Cameras



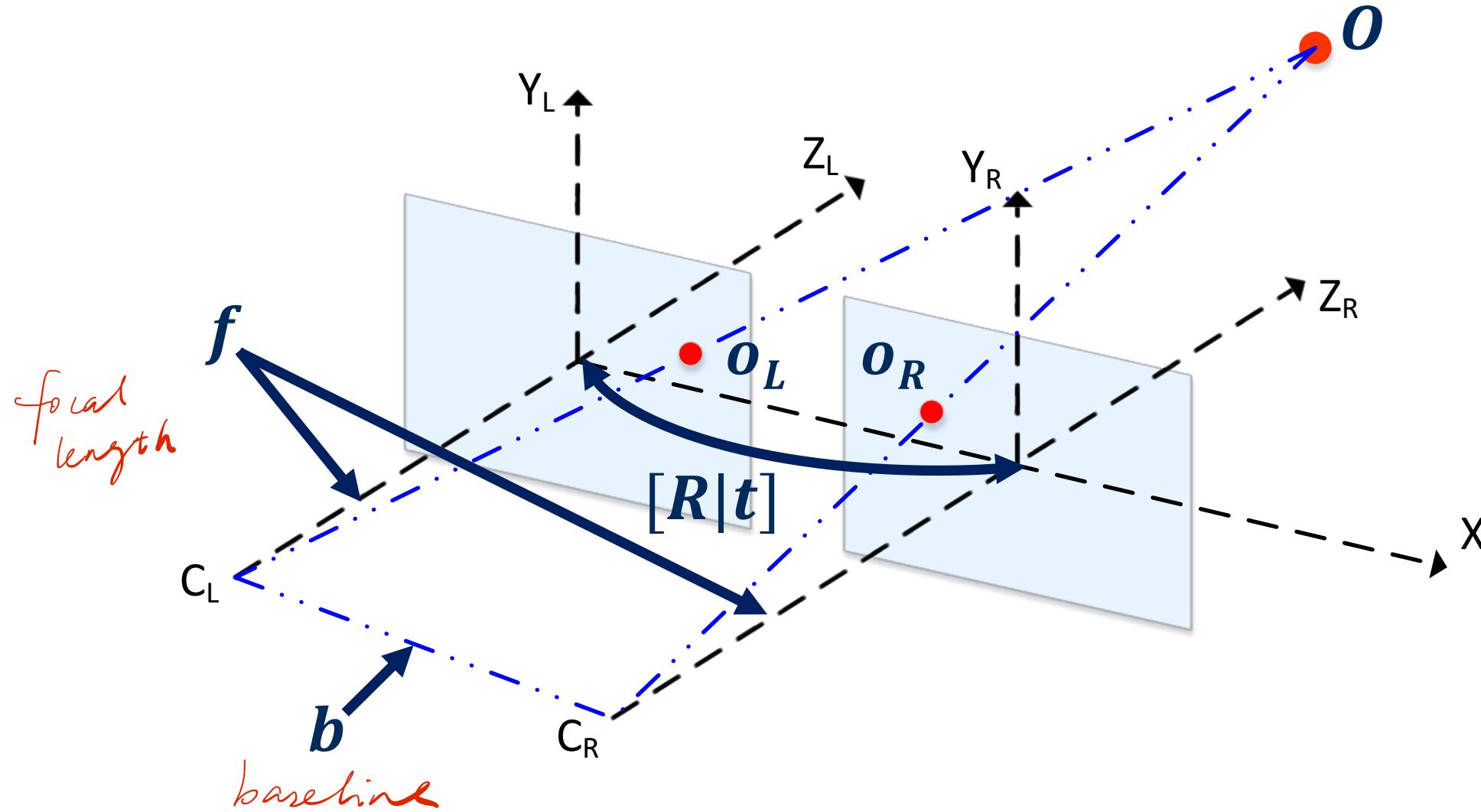
# Stereo Camera Model



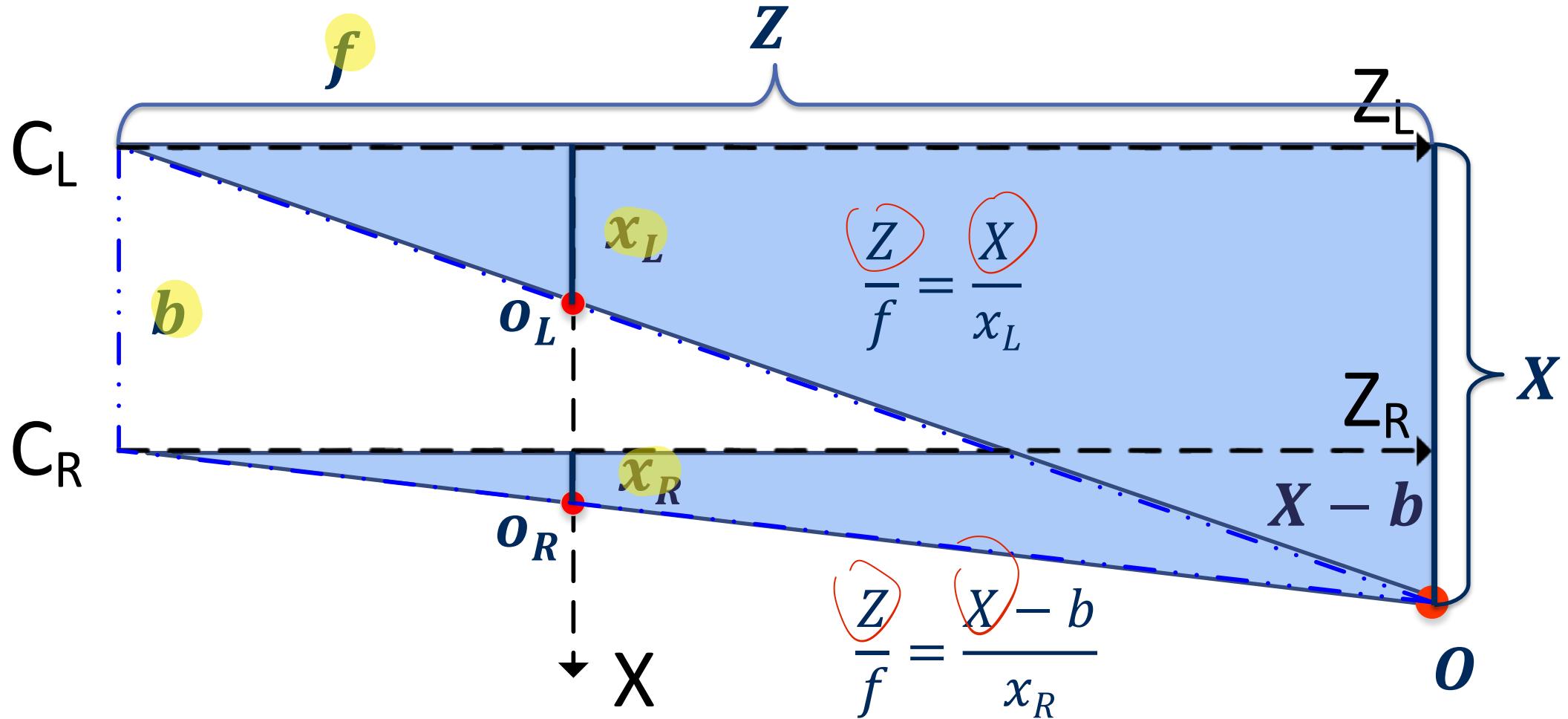
# Stereo Sensor: Assumptions

- Sensor is constructed from two identical cameras
- The two cameras have parallel optical axes
- Project to Bird's eye view for easier geometry

# Stereo Camera Model



# Stereo Camera Model



# Computing 3D Point Coordinates

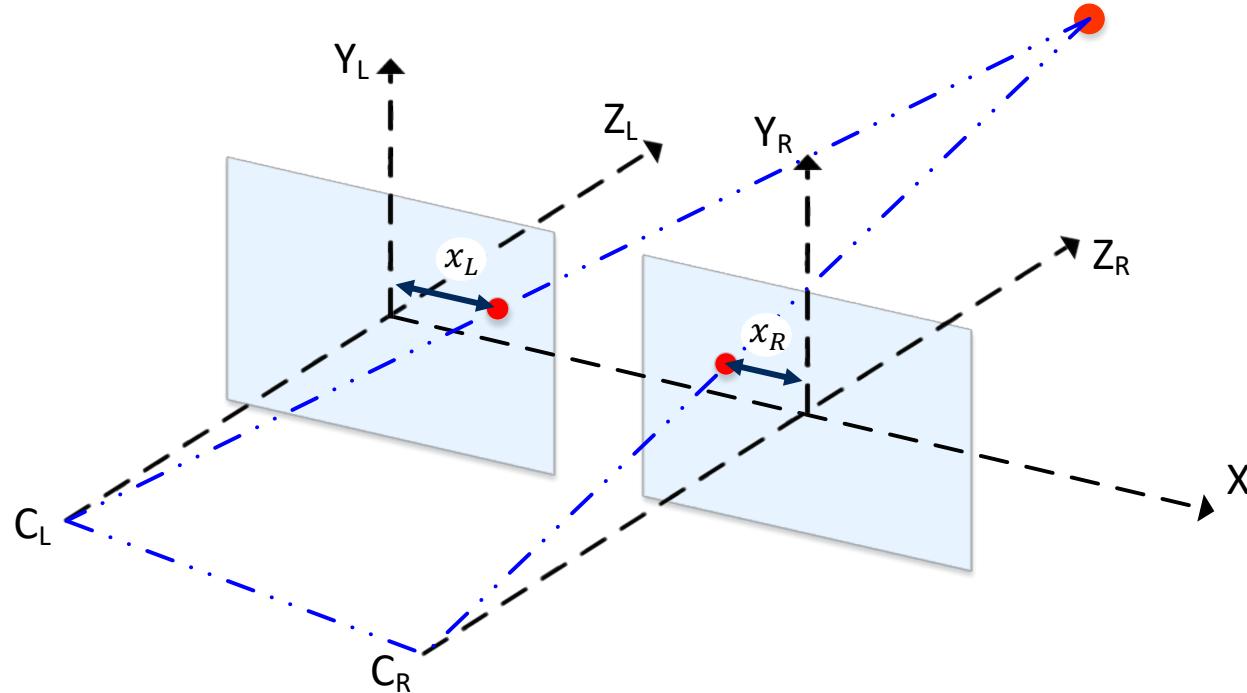
- Disparity:

$$d = x_L - x_R$$

$$\text{where } x_L = u_L - u_0$$

$$x_R = u_R - u_0$$

$$y_L = v_L - v_0$$



# Computing 3D Point Coordinates

Main stereo relations:

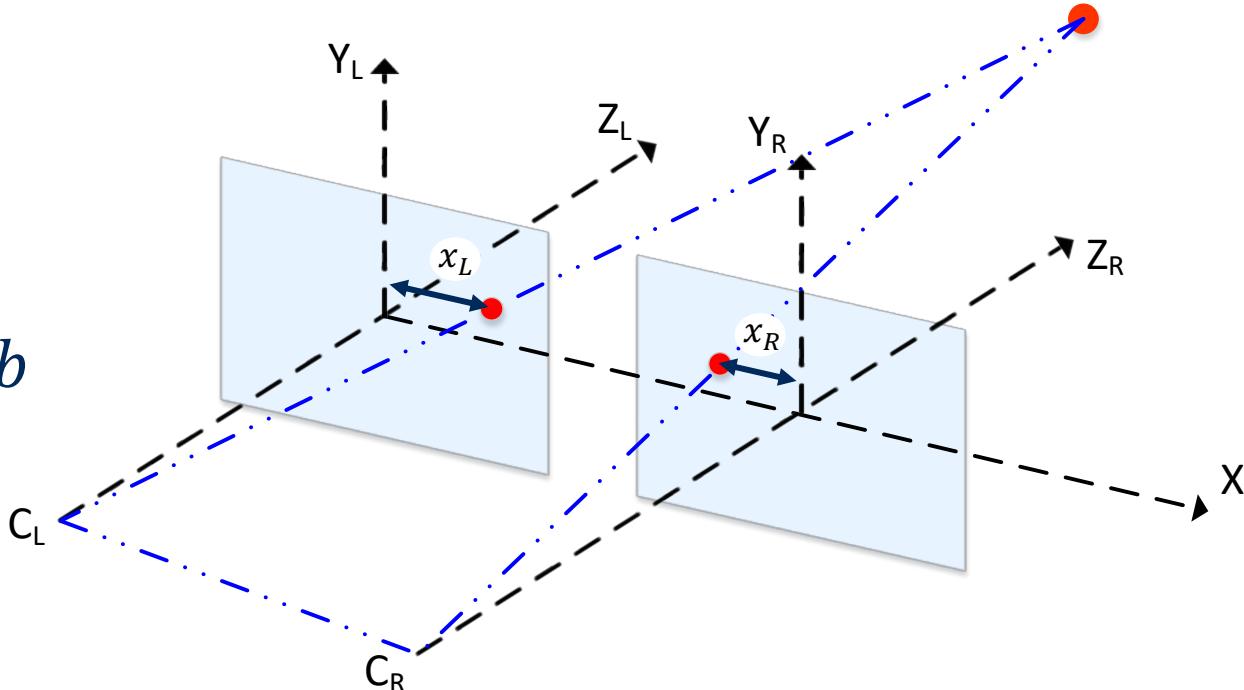
$$\frac{Z}{f} = \frac{X}{x_L} \rightarrow ZxL = fX$$

$$\frac{Z}{f} = \frac{X - b}{x_r} \rightarrow ZxR = fX - fb$$

$$Zx_R = Zx_L - fb$$

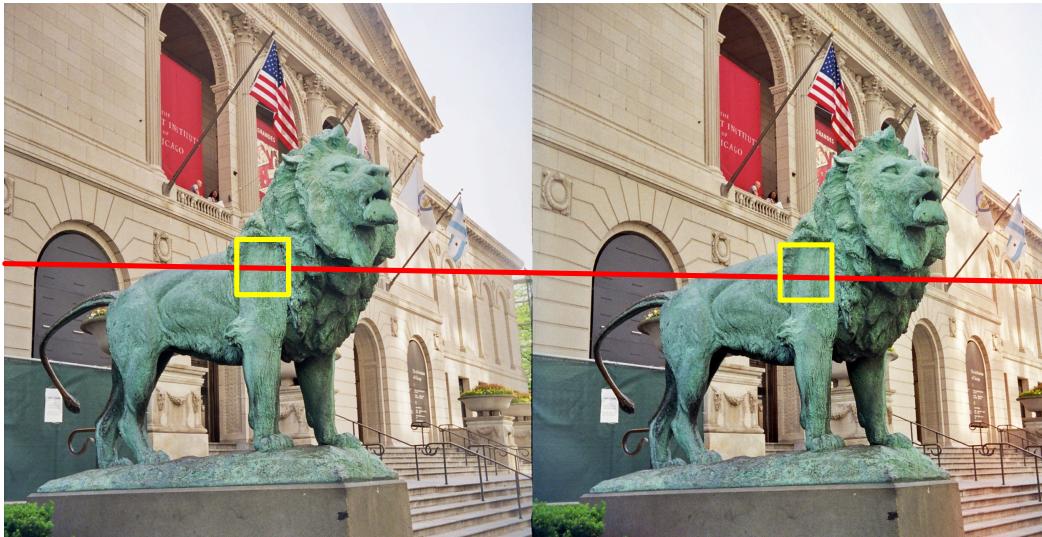
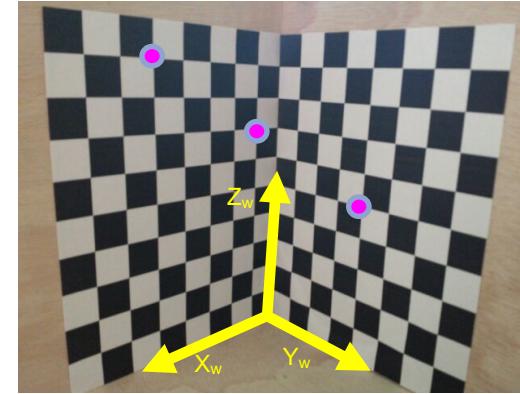
$$Z = \frac{fb}{d}$$

From above:  $X = \frac{Zx_L}{f}, \quad Y = \frac{Zy_L}{f}$



# Computing 3D Point Coordinates

- Two main problems:
  - We need to know  $f, b, u_0, v_0$ 
    - Use stereo camera calibration
  - We need to find corresponding  $x_R$  for each  $x_L$ 
    - Use disparity computation algorithms



# Summary

- Stereopsis as a phenomenon was well known as early as the 19th century.
- Given the geometric transformation between the two cameras of a stereo sensor, and the disparity, you can estimate the 3D location of pixel.
- Next: Disparity Computation

# Visual Depth Perception: Computing The Disparity

Course 3, Module 1, Lesson 3 – Part 2



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# Computing 3D Point Coordinates

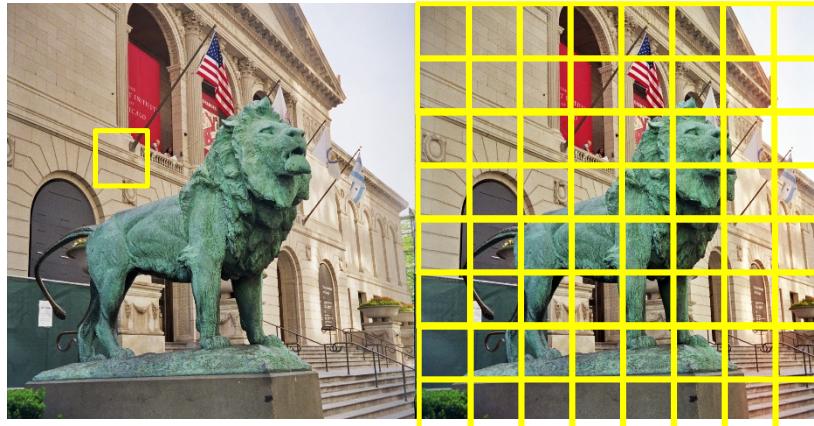
- Two main problems:
  - We need to know  $f, b, u_0, v_0$ 
    - Use stereo camera calibration
  - We need to find corresponding  $x_R$  for each  $x_L$ 
    - Use disparity computation algorithms

Stereo equations:

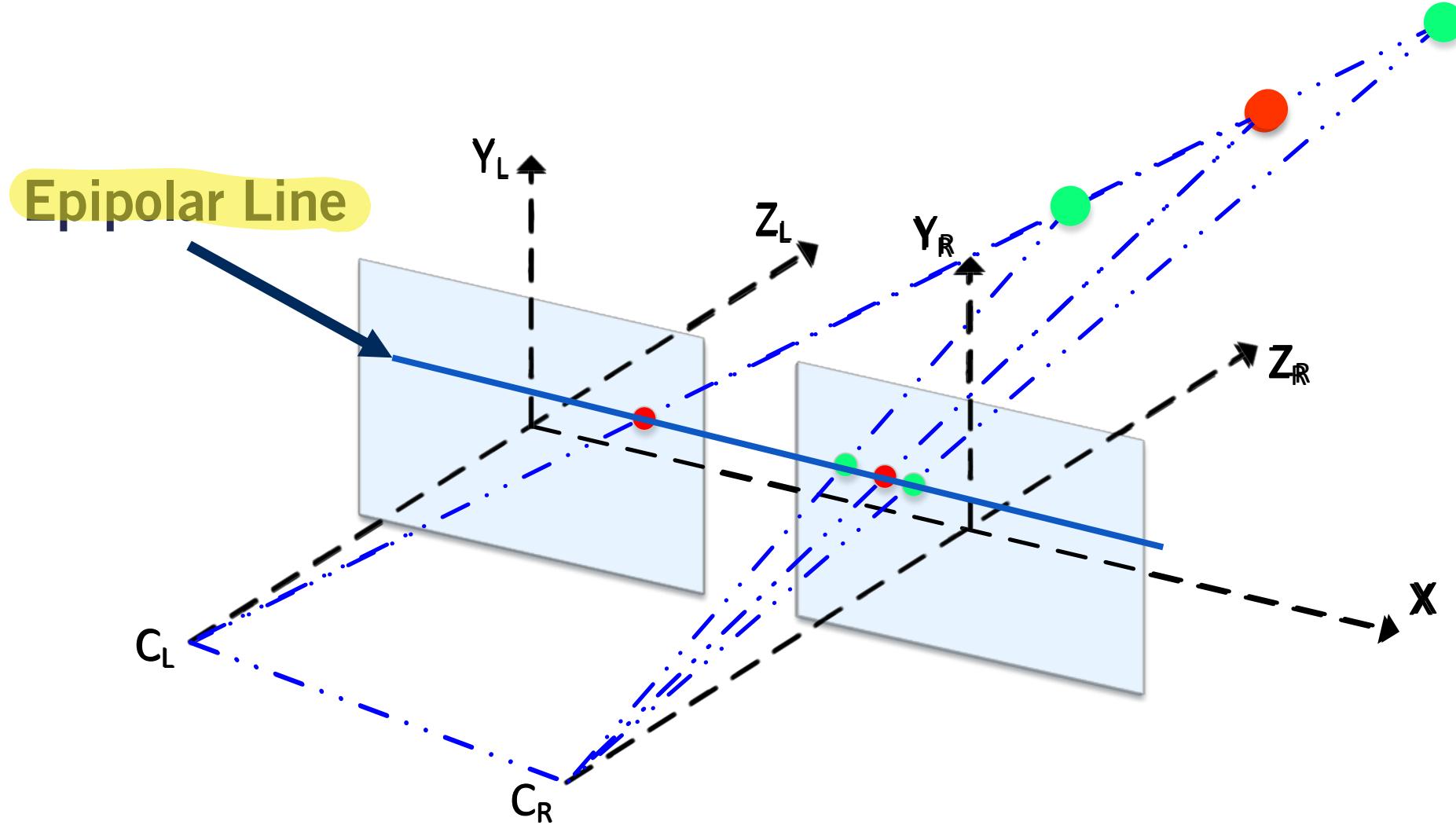
$$Z = \frac{fb}{x_L - x_R} = \frac{fb}{d},$$
$$X = \frac{Zx_L}{f}, \quad Y = \frac{Zy_L}{f}$$

# Computing 3D Point Coordinates (Review)

- **Disparity:** The difference in image location of the same 3D point under perspective to two different cameras
- **Correspond** pixels in the left image to those in the right image to find matches
- **Brute Force Solution:**
  - Search the whole image for each pixel?

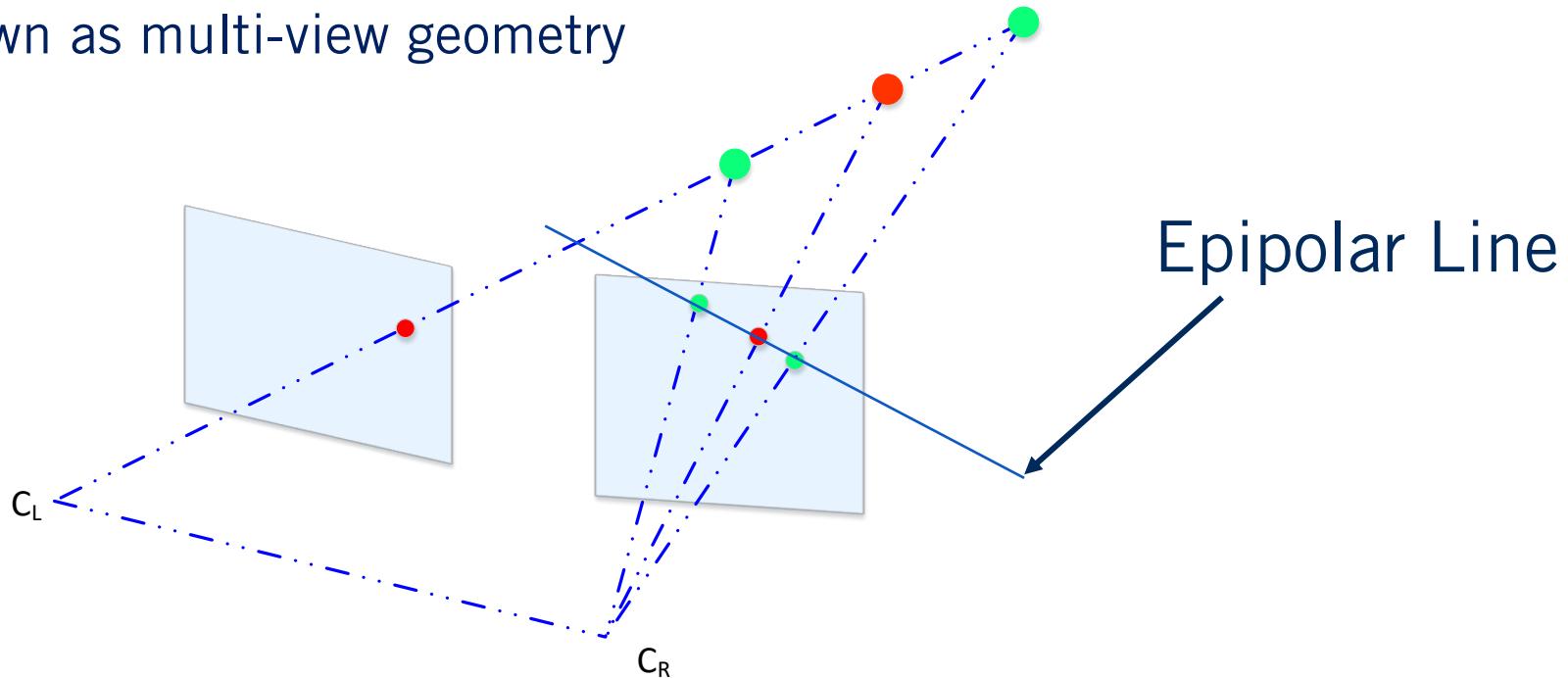


# Epipolar Constraint for Correspondence



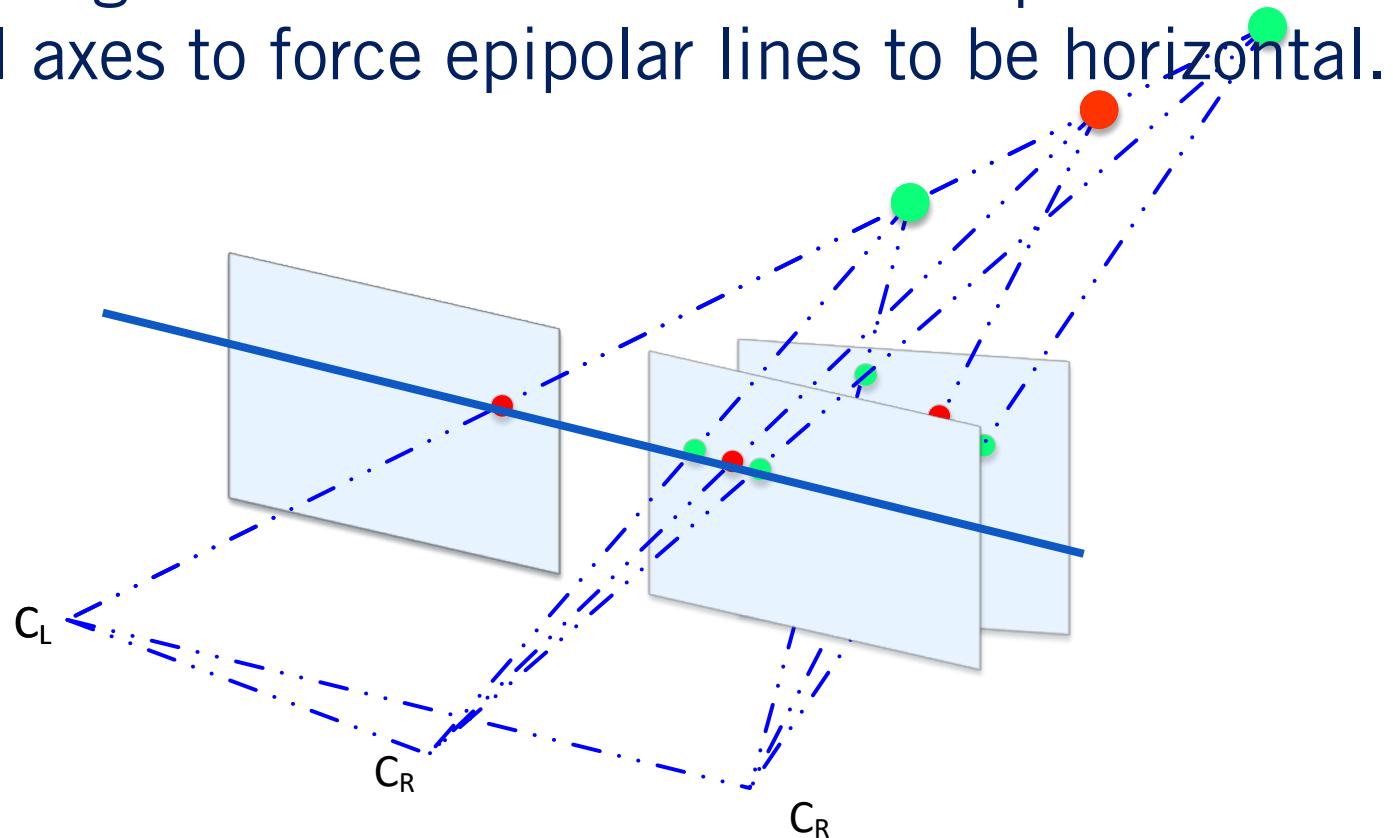
# Non-Parallel Optical Axes

- Horizontal epipolar lines only occur when the optical axes of the two cameras are parallel.
- If this condition is not met, epipolar lines will be skewed
  - Known as multi-view geometry



# Disparity Computation

- We can use **stereo rectification** to warp images originating from two cameras with non-parallel optical axes to force epipolar lines to be horizontal.

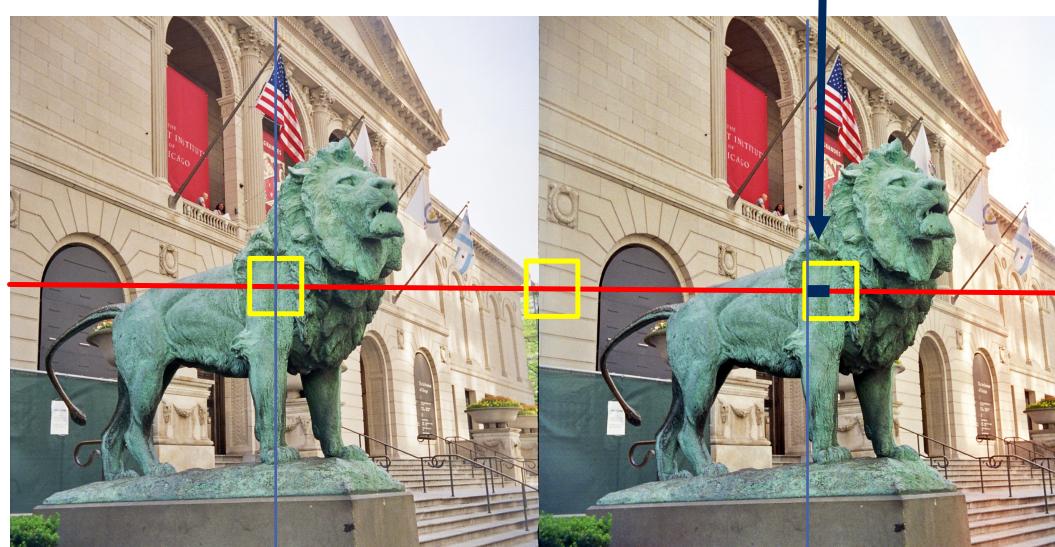


# A basic Stereo Algorithm

**Given:** Rectified Images and Stereo Calibration.

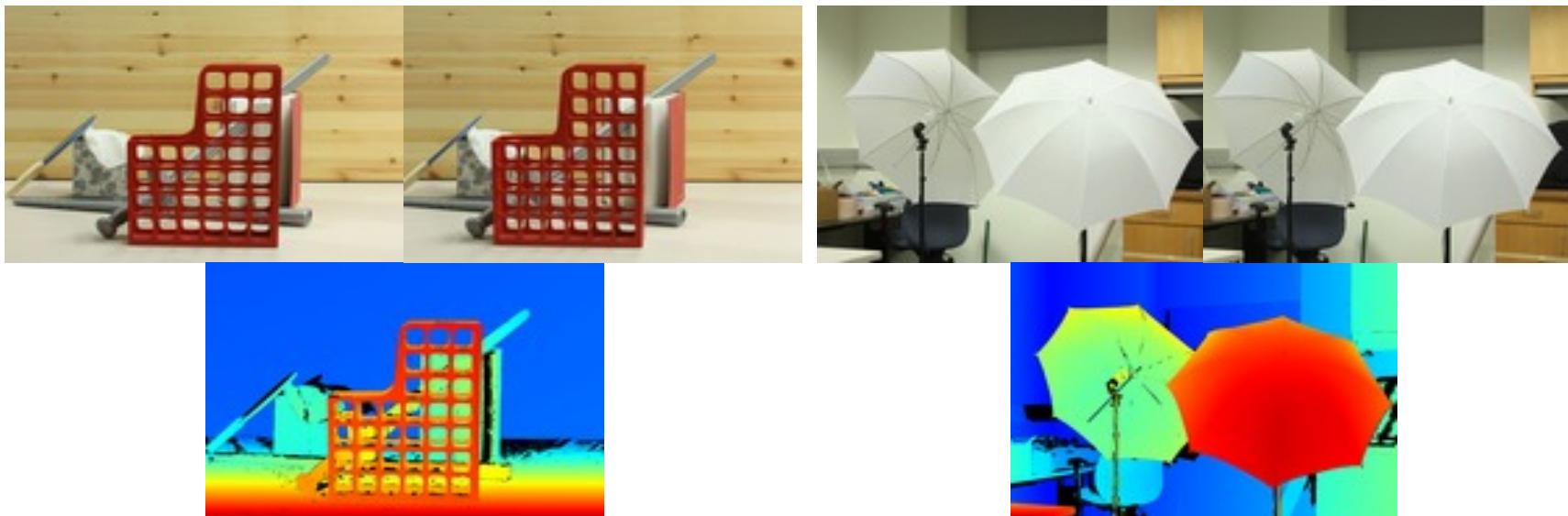
For each epipolar line,

1. Take each pixel on this line in the left image
2. Compare these left image pixels to every pixel in the right image on the same epipolar line
3. Pick the pixel that has minimum cost
4. Compute disparity,  $d$



# Stereo Matching

- Stereo matching is a very well-studied problem in computer vision
- Survey at: <http://vision.middlebury.edu/stereo/eval3/>



# Summary

- Disparity estimation can be performed through stereo matching algorithms
- Efficient solutions exist as the problem is constrained with epipolar constraints
- **Next: Image Filtering**