Trajectory Propagation

Course 4, Module 6, Lesson 1

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Learning Objectives

- Understand the difference between kinematic and dynamic motion models
- Recall the bicycle model from Course 1
- Generate trajectories given control inputs and a motion model

Kinematic vs. Dynamic Model

Particle Kinematic Model

$$\ddot{x} = a$$

- Disregards mass and inertia of the robot
- Uses linear and angular velocities (and/or derivatives) as input

Particle Dynamic Model

$$M\ddot{x} + B\dot{x} = F$$

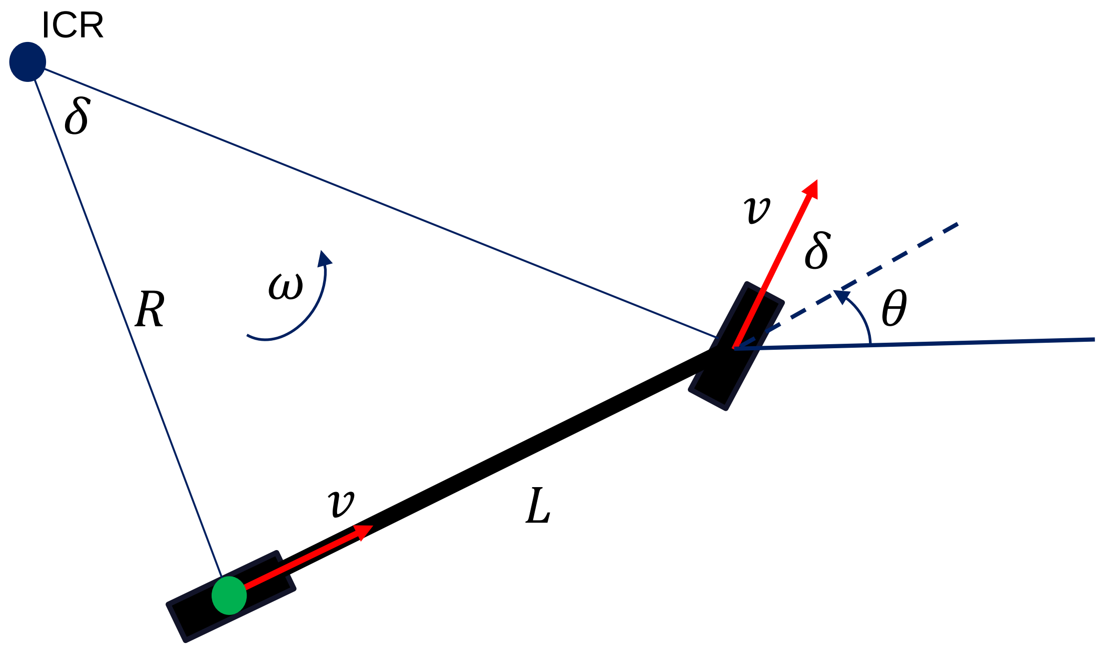
- Takes mass and inertia into consideration
- Uses forces and torques as inputs

Recall: Kinematic Bicycle Model

- x and y correspond to base link position of the robot
- θ corresponds to heading of the chassis with respect to x-axis
- δ is the steering angle input, v is the velocity input

Do not have access to x, y directly.

$$\dot{x} = v\cos(\theta)$$
$$\dot{y} = v\sin(\theta)$$
$$\dot{\theta} = \frac{v\tan(\delta)}{L}$$
$$\delta_{min} \leq \delta \leq \delta_{max}$$
$$v_{min} \leq v \leq v_{max}$$



Kinematic Model Discretization

- Discretization of differential equations allows for efficient computation of trajectories
- Recursive definition saves computation time

$$x_n = \sum_{i=0}^{n-1} v_i \cos(\theta_i)\Delta t = x_{n-1} + v_{n-1}\cos(\theta_{n-1})\,\Delta t$$

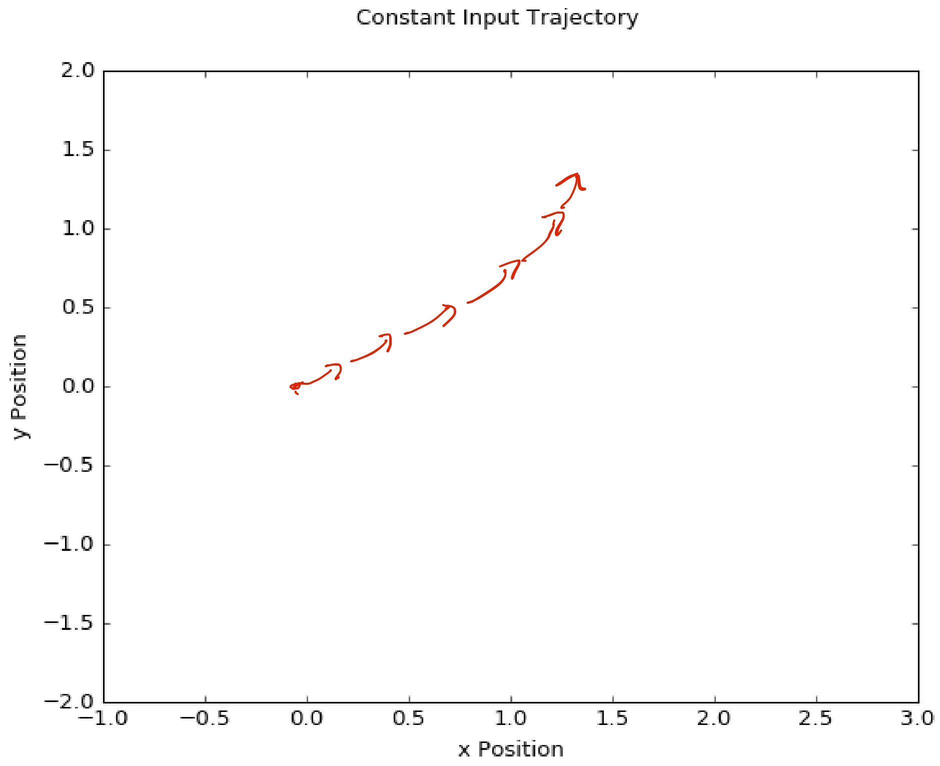
$$y_n = \sum_{i=0}^{n-1} v_i \sin(\theta_i)\Delta t = x_{n-1} + v_{n-1}\sin(\theta_{n-1})\,\Delta t$$

$$\theta_n = \sum_{i=0}^{n-1} \frac{v_i \tan(\delta_i)}{L}\Delta t = \theta_{n-1} + \frac{v_{n-1}\tan(\delta_{n-1})}{L}\Delta t$$

Constant Velocity and Steering Angle Example

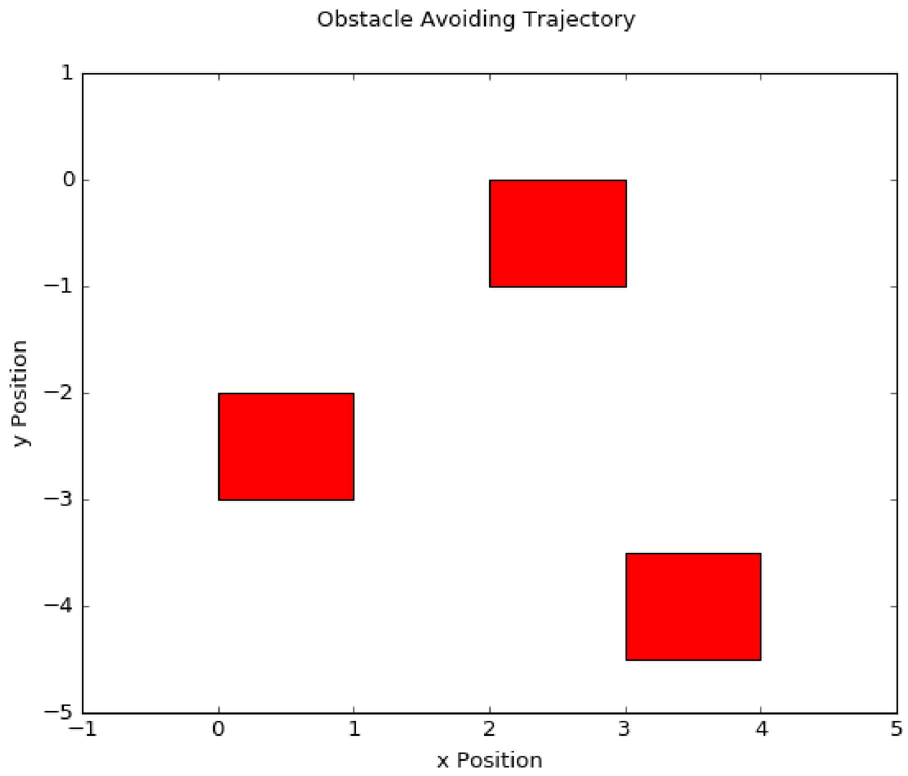
- For a given control sequence, we can compute the vehicle's trajectory
- Useful for prediction as well

Other agents: guess on the control inputs
→ estimate the future trajectory



Varying Input for Obstacle Avoidance

- To avoid obstacles, we require more complex maneuvers
- We can vary the steering input according to a steering function to navigate complex scenarios
- Main objective of local planning is to compute the control inputs (or trajectory) required to navigate to goal point without collision



Summary

- Reviewed the difference between kinematic and dynamic motion models
- Learned how to generate trajectories given our bicycle model

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