Geometric steering control II – Stanley Controller

Course 1, Module 6, Lesson 3

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Learning Objectives

- Derive the Stanley path tracking controller
- Analyze the evolution of heading and crosstrack errors
- Evaluate convergence from arbitrary starting points

Stanley Controller Approach

- Stanley method is the path tracking approach used by Stanford University's Darpa Grand Challenge team
    - Uses the center of the front axle as a reference point
    - Look at both the error in heading and the error in position relative to the closest point on the path
    - Define an intuitive steering law to
        - Correct heading error
        - Correct position error
        - Obey max steering angle bounds

Heading control law

- Combine three requirements:
  - Steer to align heading with desired heading (proportional to heading error)

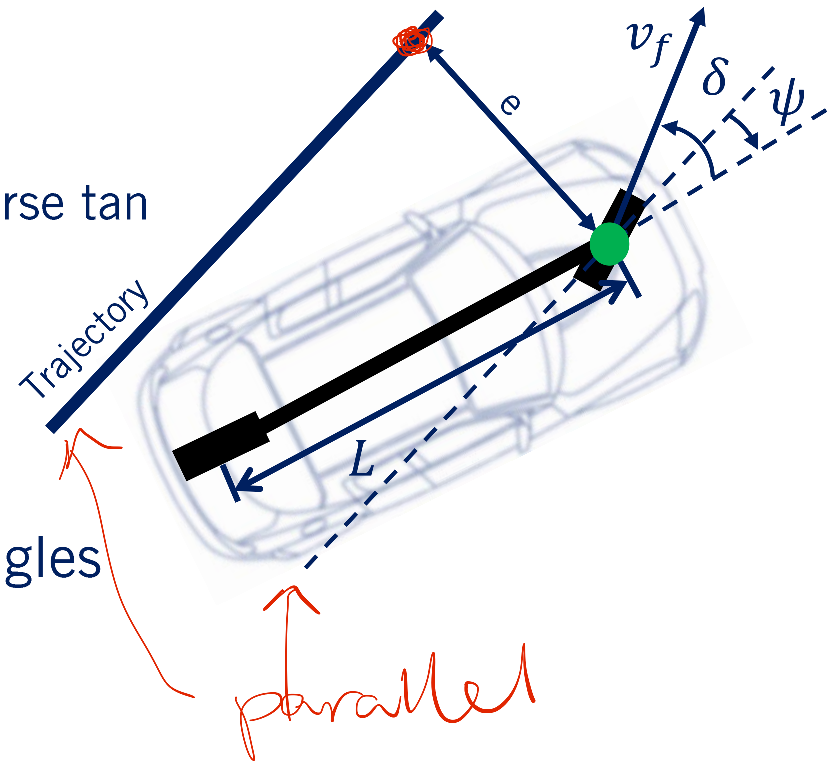
    $$\delta(t) = \psi(t)$$

  - Steer to eliminate crosstrack error
    - Essentially proportional to error
    - Inversely proportional to speed
    - Limit effect for large errors with inverse tan
    - Gain k determined experimentally

    $$\delta(t) = \tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)$$

  - Maximum and minimum steering angles

    $$\delta(t) \in [\delta_{min}, \delta_{max}]$$

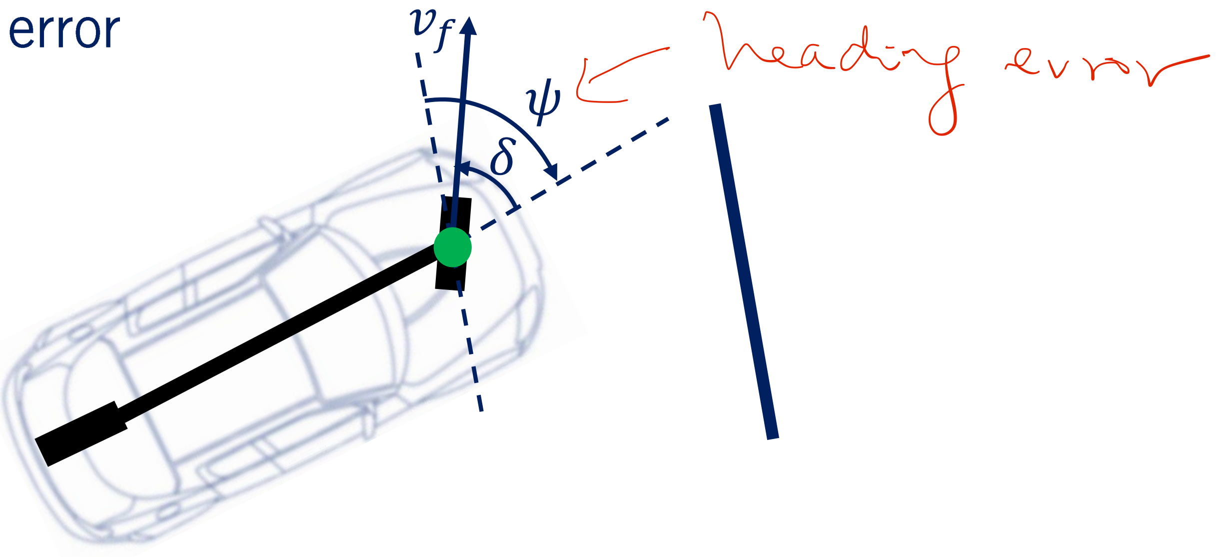


Combined steering law

- Stanley Control Law

$$\delta(t) = \psi(t) + \tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right), \quad \delta(t) \in [\delta_{min}, \delta_{max}]$$

- For large heading error, steer in opposite direction
  - The larger the heading error, the larger the steering correction
  - Fixed at limit beyond maximum steering angle, assuming no crosstrack error

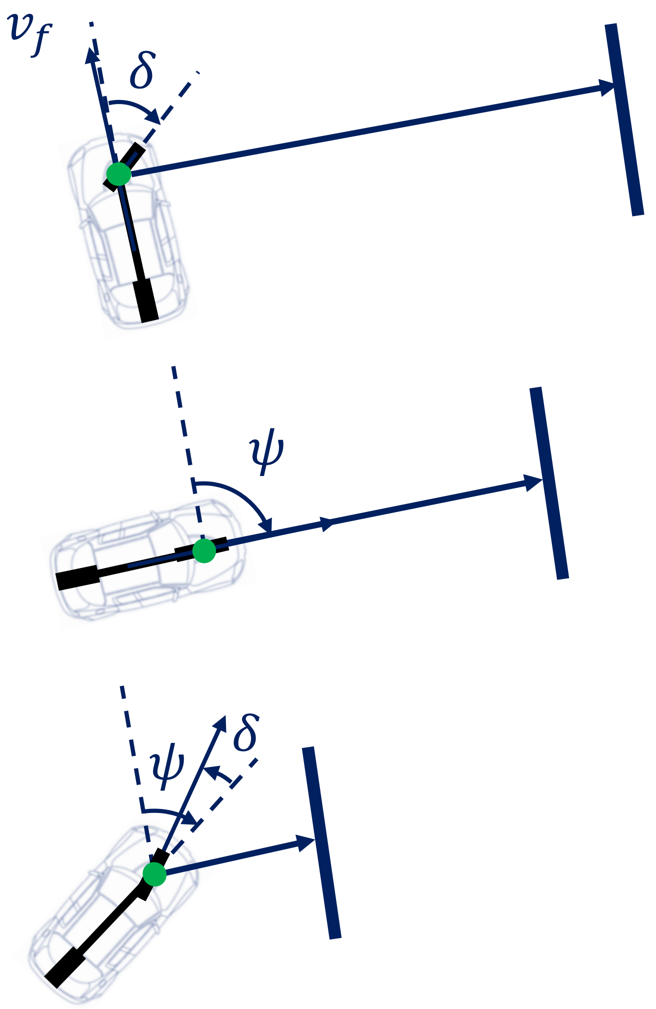


Combined steering law

- For larger positive crosstrack error

$$\tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right) \approx \frac{\pi}{2} \quad \rightarrow \delta(t) \approx \psi(t) + \frac{\pi}{2}$$

  - As heading changes due to steering angle, the heading correction counteracts the crosstrack correction, and drives the steering angle back to zero
  - The vehicle approaches the path, crosstrack error drops, and steering command starts to correct heading alignment.



Error Dynamics

- The error dynamics when not at maximum steering angle are:

$$\dot{e}(t) = -v_f(t)\sin(\psi(t) - \delta(t)) = -v_f(t)\sin\left(\tan^{-1}\left(\frac{ke(t)}{v_f(t)}\right)\right)$$

$$= \frac{-ke(t)}{\sqrt{1 + \left(\frac{ke(t)}{v_f}\right)^2}}$$

- For small crosstrack errors, leads to exponential decay characteristics

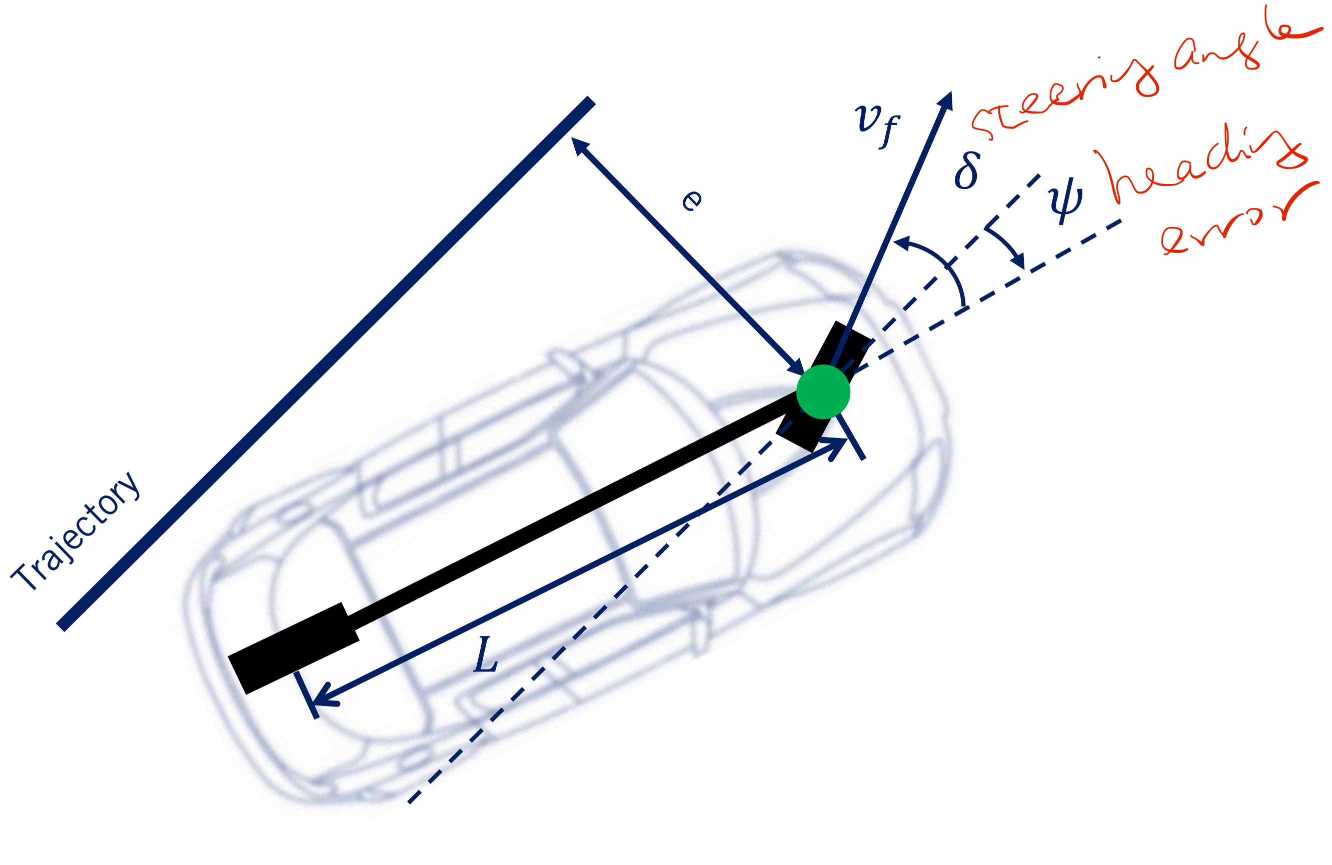
$$\dot{e}(t) \approx -ke(t)$$ → 1st order ODE

↓

exponential

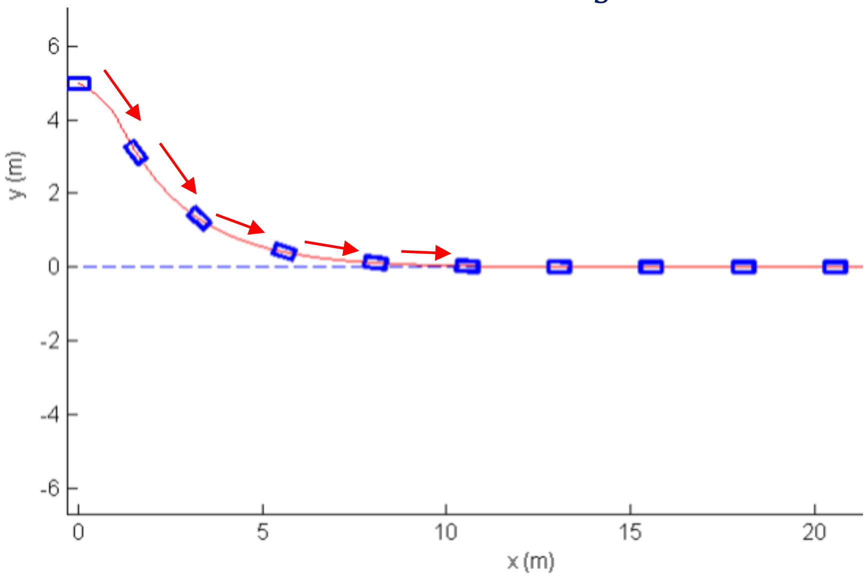
Case Study

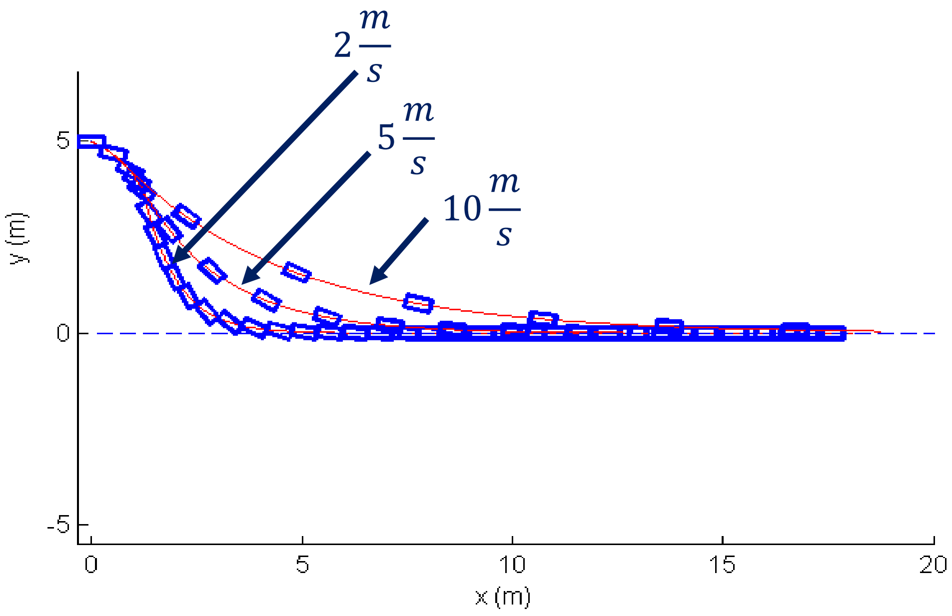
- Two scenarios:
    - Large initial crosstrack error
    - Large initial heading error



Case Study 1

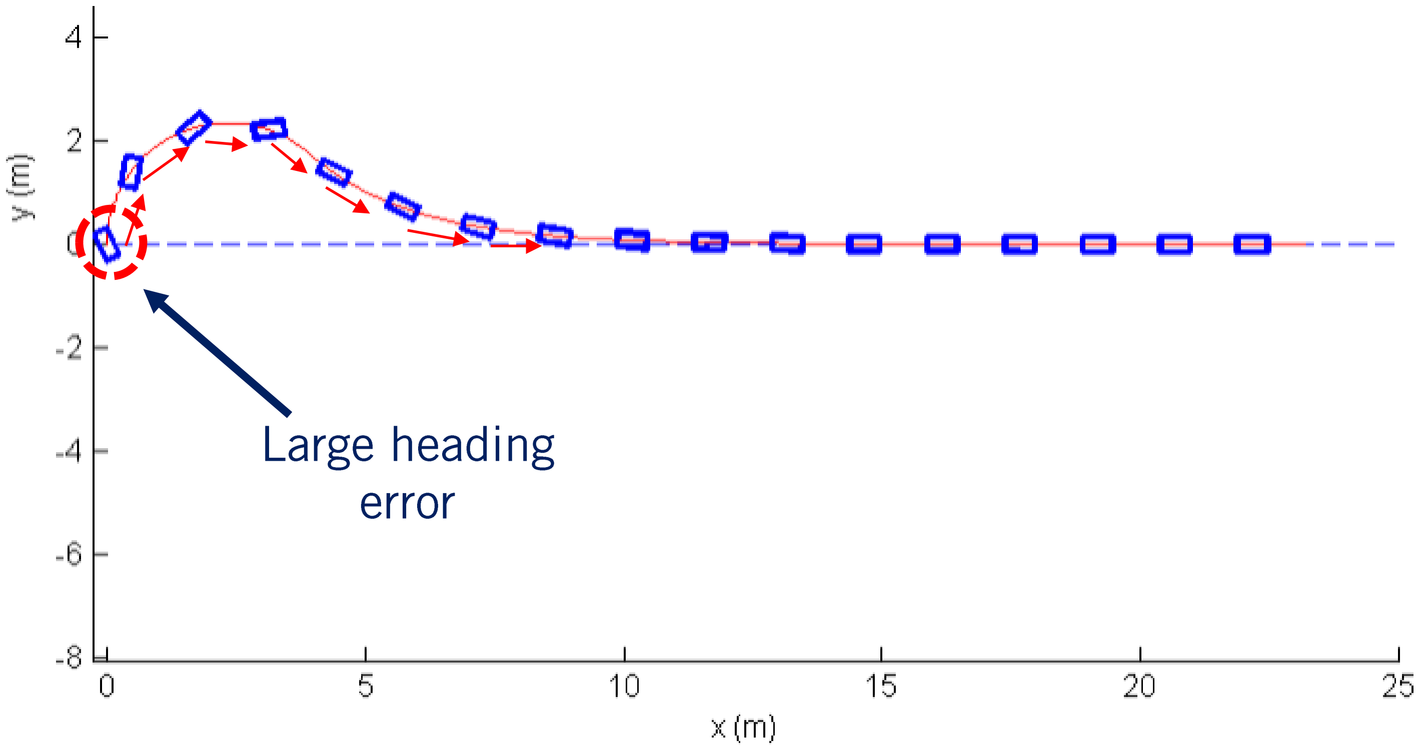
- Large initial crosstrack error
  - Crosstrack error of 5 meters
  - Max steer $\delta = 25^o$, forward speed of $v_f = 5\ \frac{m}{s}$
  - Gain $k = 2.5$, length $L = 1\ m$
  - Effect of speed variation
    - $v_f = 2, 5, 10\ \frac{m}{s}$





Case Study 2

- Large initial heading error
  - Max steer $\delta = 25^o$, forward speed of $v_f = 5\,\frac{m}{s}$
  - Gain $k = 2.5$, length $L = 1m$



Adjustment

- Low speed operation
  - Inverse speed can cause numerical instability
  - Add softening constant to controller

$$\delta(t) = \psi(t) + \tan^{-1}\left(\frac{ke(t)}{k_s + v_f(t)}\right)$$

- Extra damping on heading
  - Becomes an issue at higher speeds in real vehicle
- Steer into constant radius curves
  - Improves tracking on curves by adding a feedforward term on heading

curved path
with high curvature

Summary

In this lesson, you learned
- How to apply the Stanley controller to path tracking
- What the convergence properties of the Stanley controller are
- How to improve real-world performance of the Stanley controller

What is next?
- Advanced control strategies such as Model Predictive Control (MPC) for lateral control