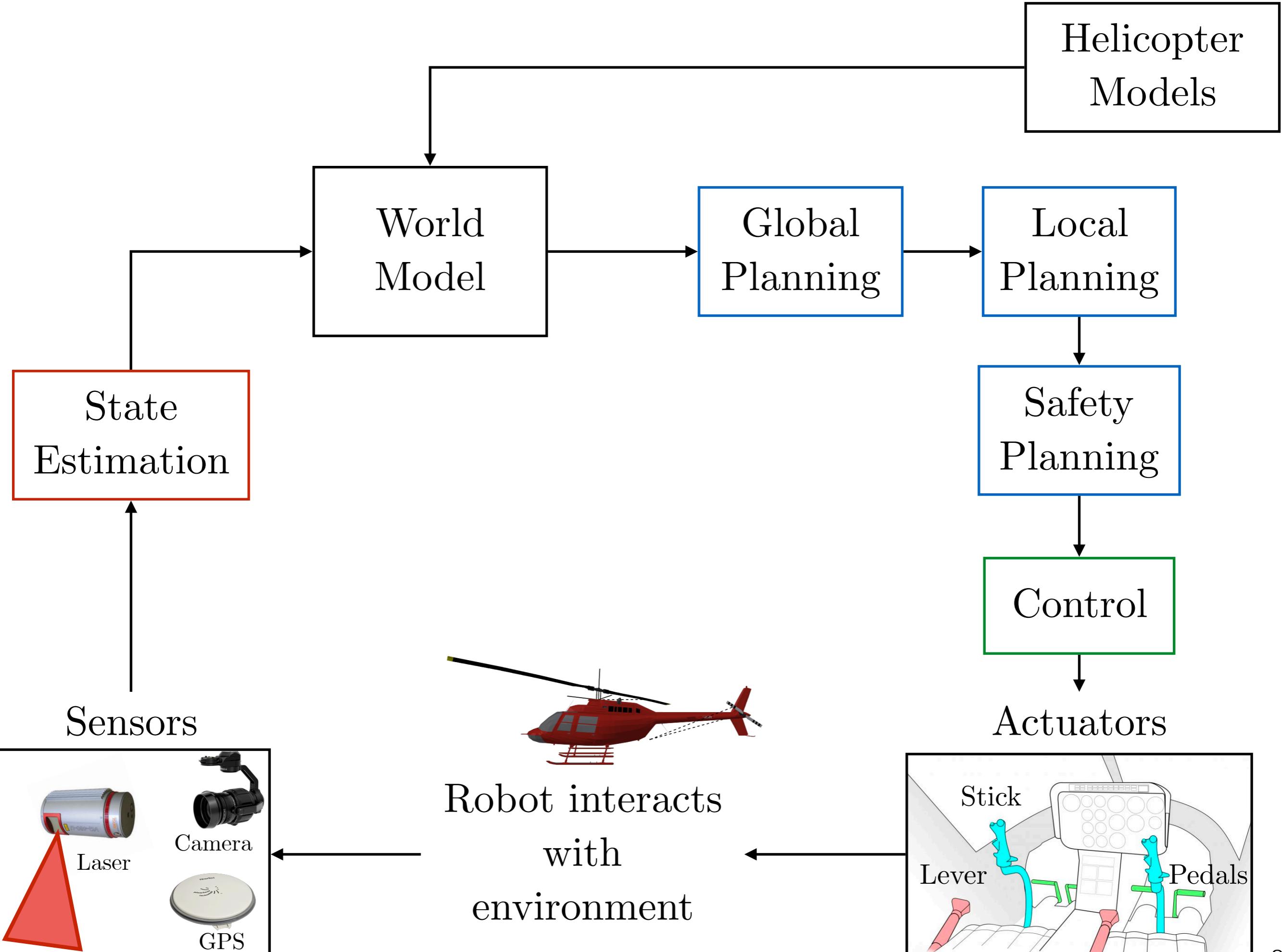


# Introduction to State Estimation

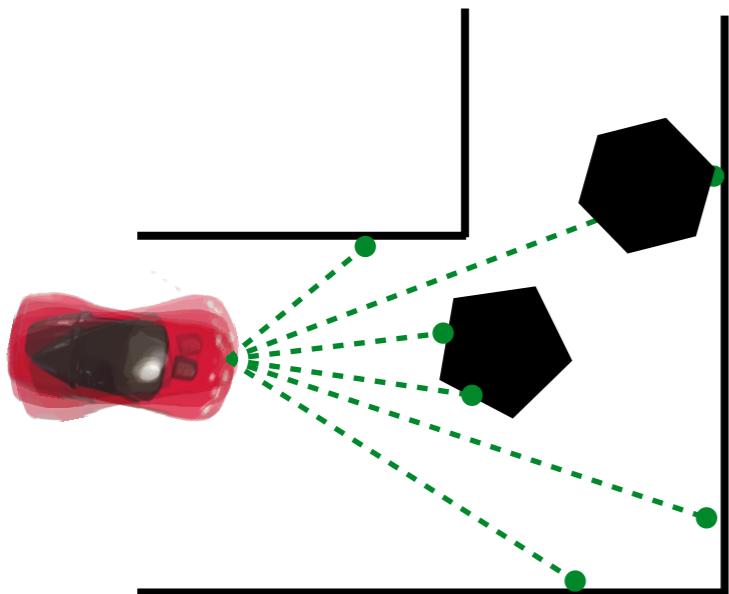
Instructor: Chris Mavrogiannis

TAs: Kay Ke, Gilwoo Lee, Matt Schmittle

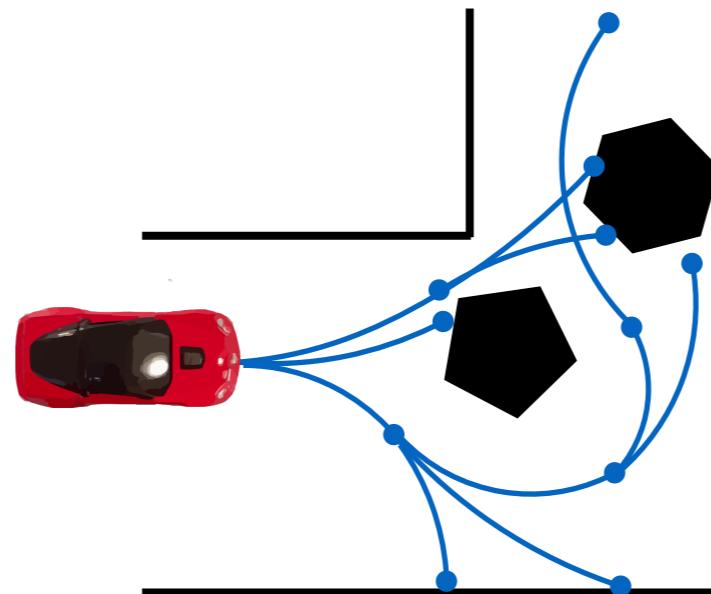
\*Slides based on or adapted from Sanjiban Choudhury



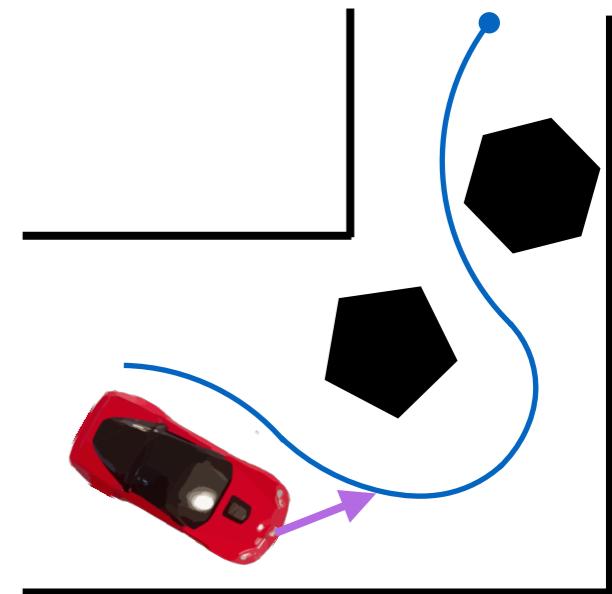
**Estimate  
state**



**Plan a  
sequence of  
motions**



**Control  
robot to  
follow plan**



# Today's objective

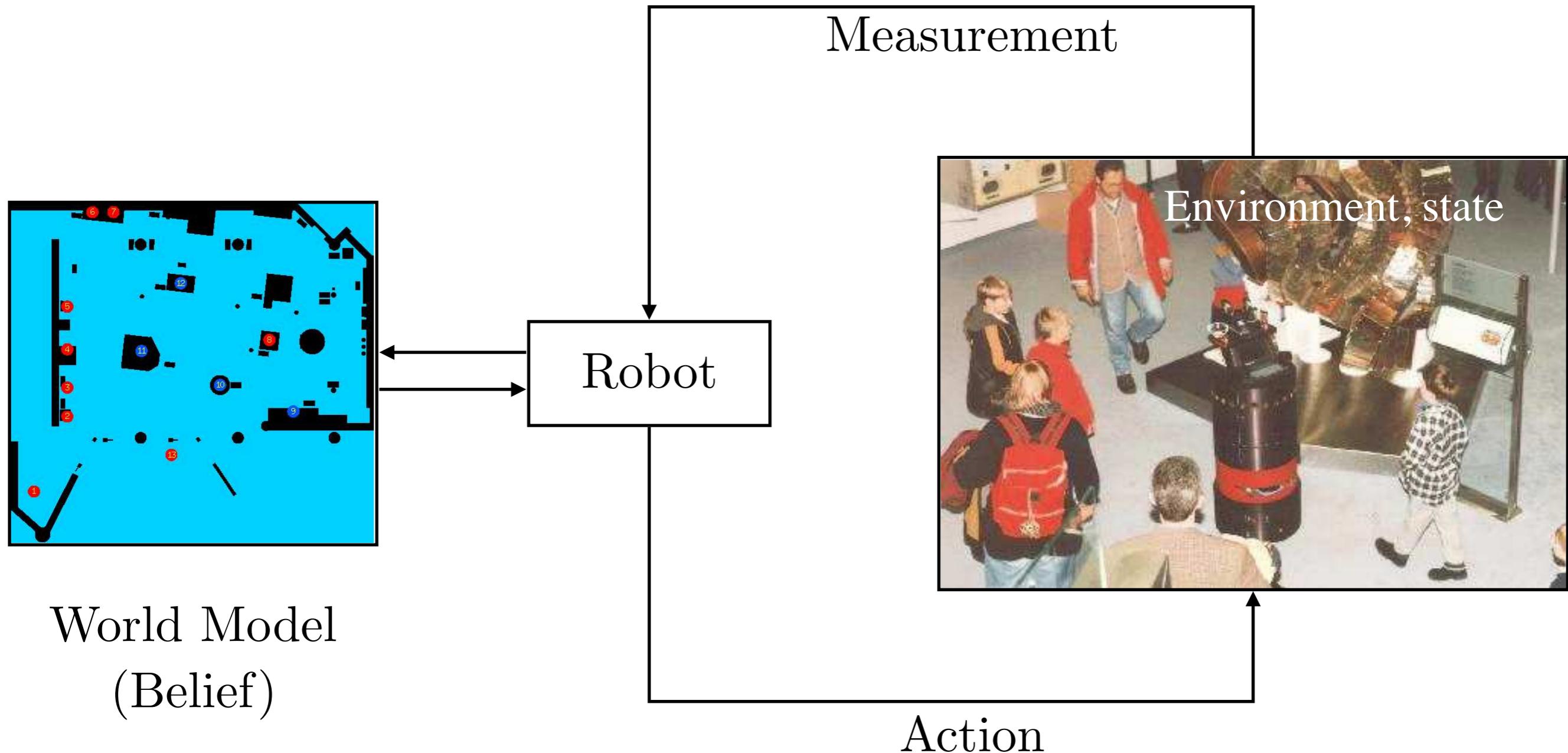
1. Formulate state estimation as a Bayes filtering problem
2. Discuss various components of a Bayes filter
3. Intuition behind Bayes filtering

# A robot interacting with the world



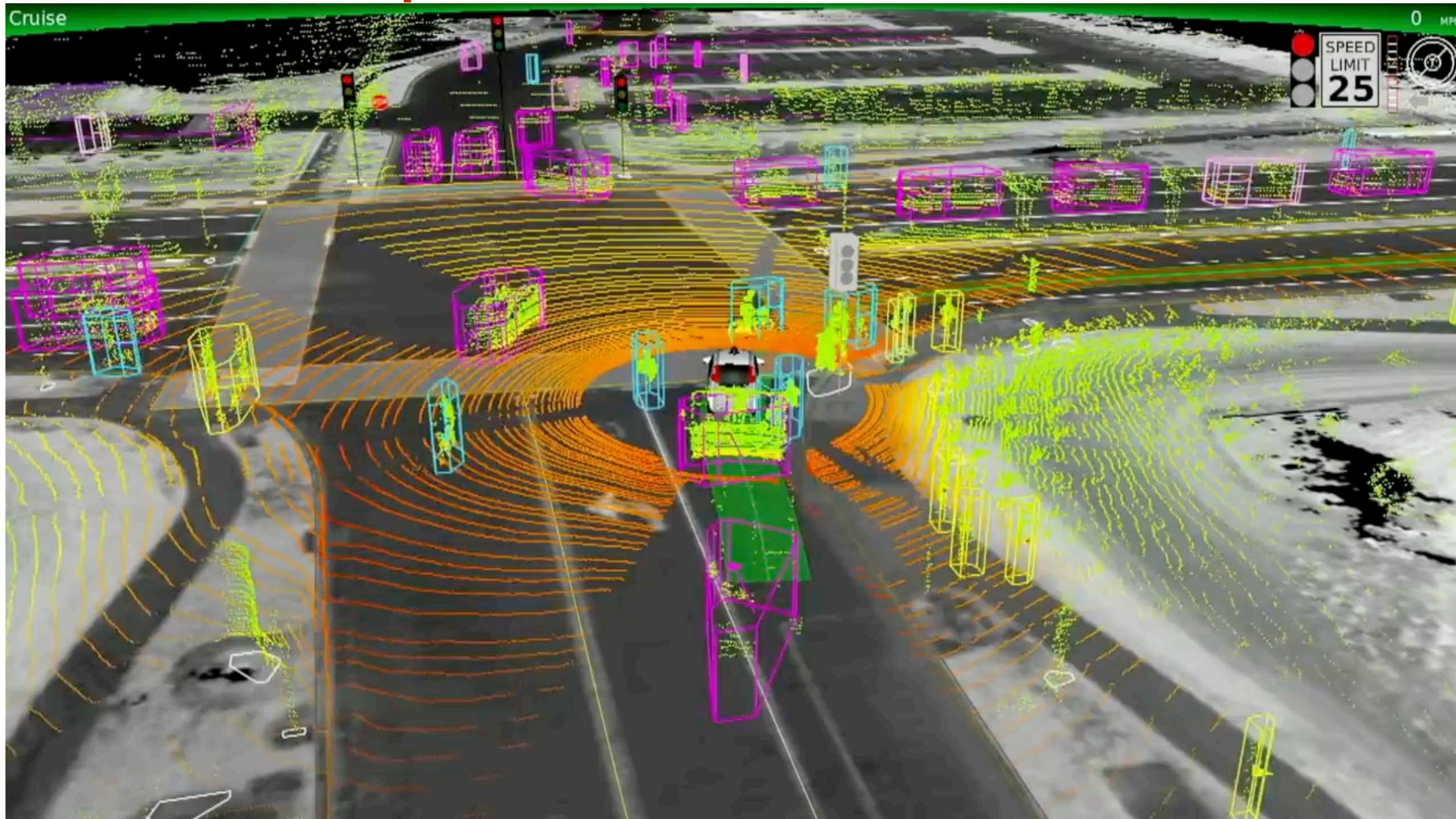
The **Minerva**  
**Experience**

# A robot interacting with the world



# Bayes filtering in action!

# Example 1: Rich information



Measurement:

LiDAR, camera, GPS

World Model (Belief):

Location, cars, people..

Action:

Steering, speed

## Example 2: Medium information

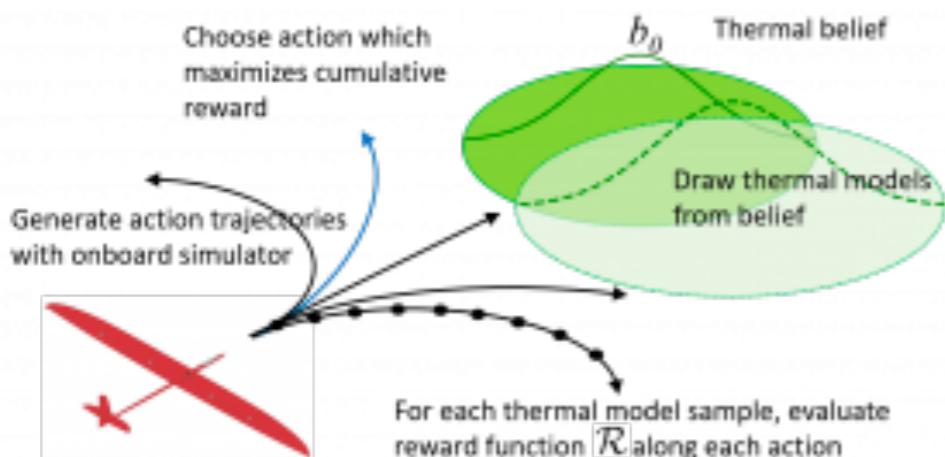


Measurement:  
Planar LiDAR

World Model (Belief):  
State of robot

Action:  
Pitch, Roll

# Example 3: Very low information

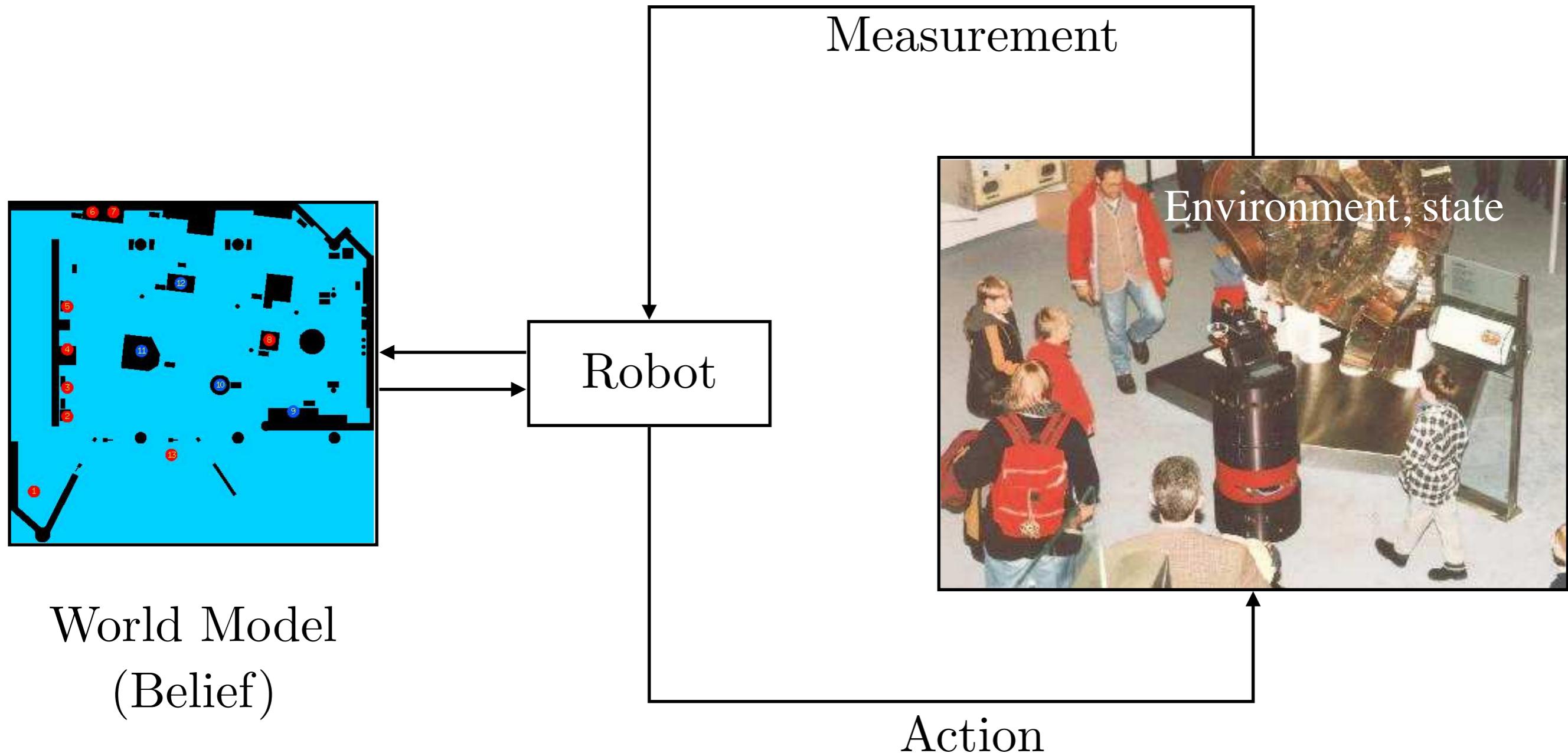


Measurement: Pitot tube (airspeed), GPS

World Model (Belief): Thermal belief

Action: Location of wind thermal sensor

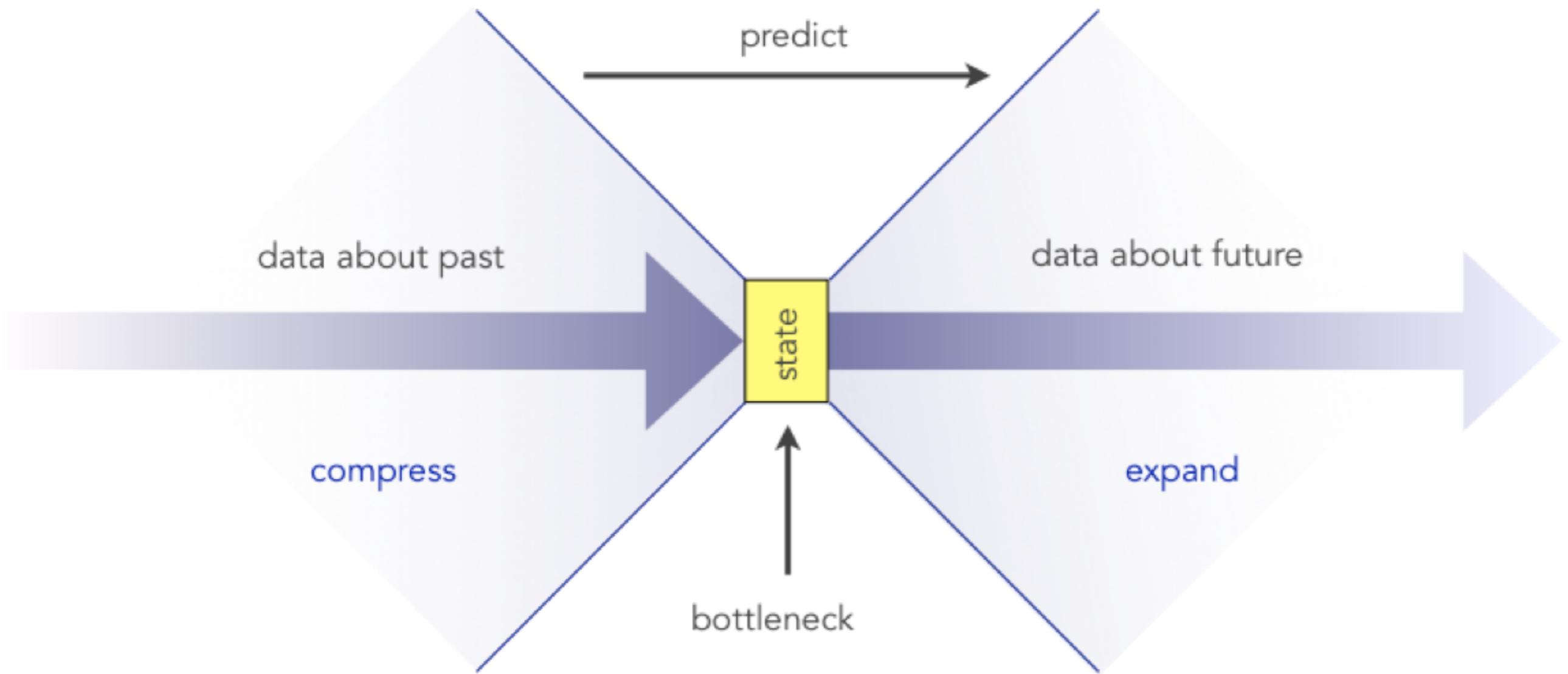
# A robot interacting with the world





How do we formally define this problem?

# State: A very abstract definition



State: statistic of history sufficient to predict the future

# State ( $x_t$ )

Collection of variables sufficient to predict the ~~future~~  
(future that we care about)

What are some examples of state?

1. Pose of a robot - Usually 6 dof (3 position, 3 for orientation)  
- 3 dof for planar mobile robot (x, y, heading)
2. Configuration of a manipulator - Collection of joint angles
3. Location of objects in environment

State can be static/dynamic, discrete/continuous/hybrid

# Measurement ( $z_t$ )

Measurements are sensor values that provide  
information about the current state.

(Measurement does not always tell you state directly!!)

What are some examples of measurements?

1. GPS - absolute information about robot pose
2. Laser scan - relative geometric information between pose and environment
3. Camera image - information about color / texture  
(harder to model)

# Action ( $u_t$ )

Actions affect how a **state changes** from one time to another

What are some examples of actions?

1. Active forces applied by the robot - (measure motor currents, force torque sensors, odometers)
2. Passive actions that change environment - weather (can detect with sensors)
3. NOP actions - doing nothing is also an action. State does not change.

# Fundamental problem: State is hidden

All the robot sees is a stream of actions and measurements

$$u_1, z_1, u_2, z_2, u_3, z_3, \dots$$

But robot never sees the state

$$x_1, x_2, x_3, \dots$$

# Fundamental problem: State is hidden

But all decision making depends on knowing state

Solution: Estimate belief over state

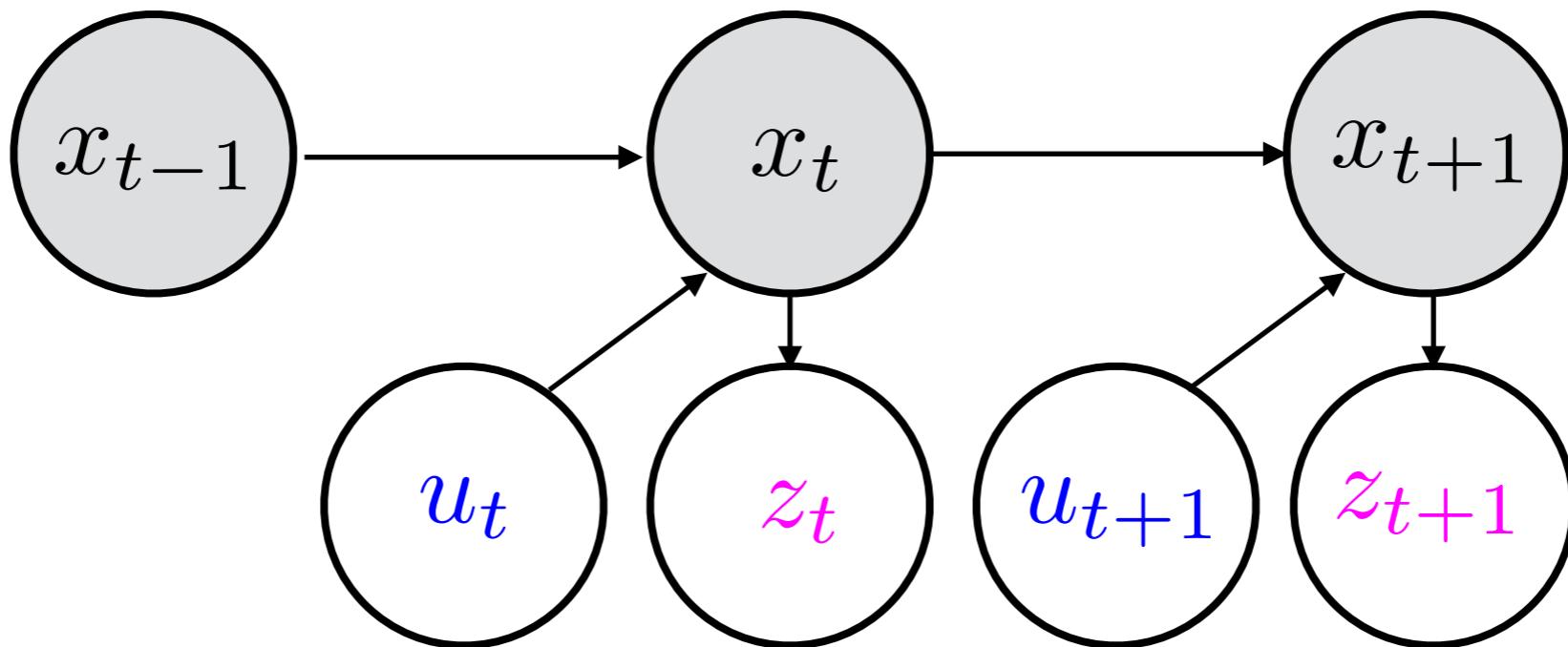
$$bel(x_t) = P(x_t | z_{1:t}, u_{1:t})$$

Belief is a probability of each possible state given history

Also called Posterior / Information state / State of knowledge

Represent belief? Parametric (Gaussian), Non-parametric (Histogram)

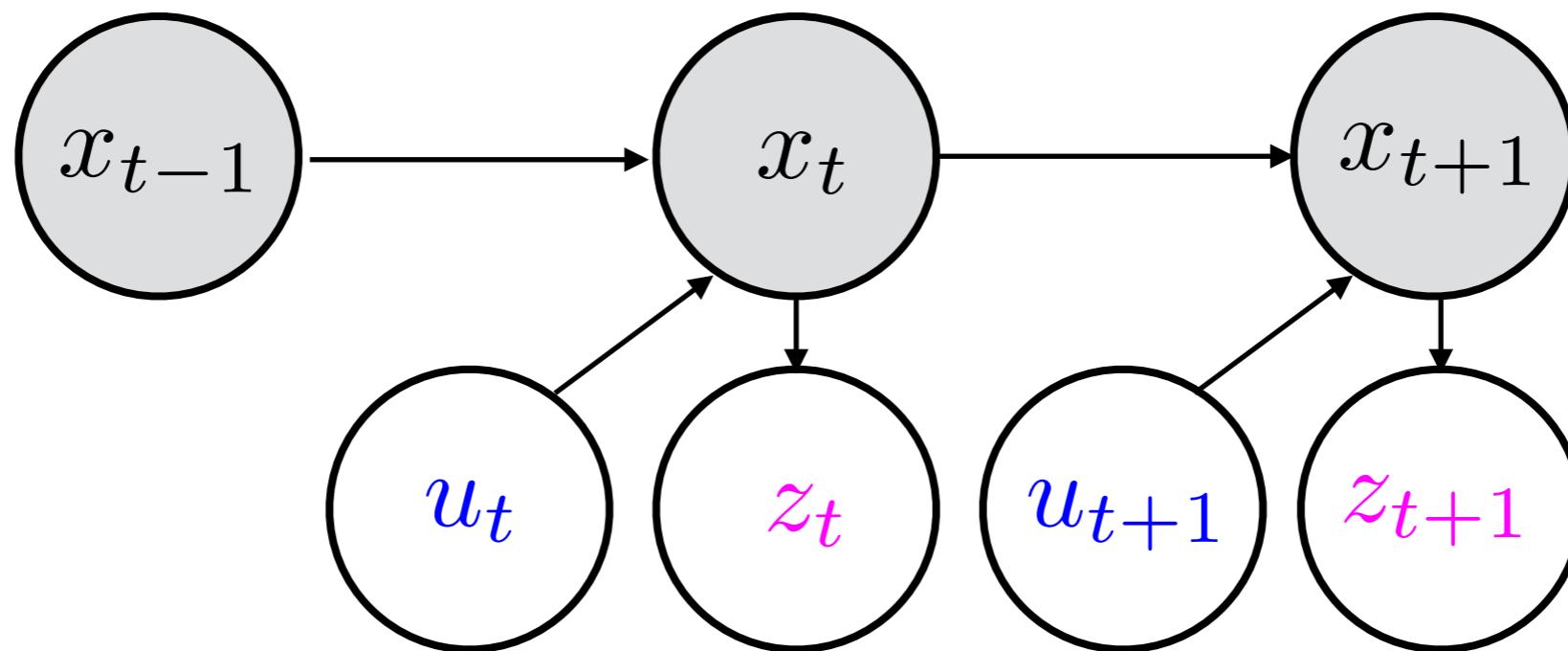
# Let's think about causality of events



Assumptions:

1. Robot receives a stream of measurements / actions.
2. One measurement / action per time-step.

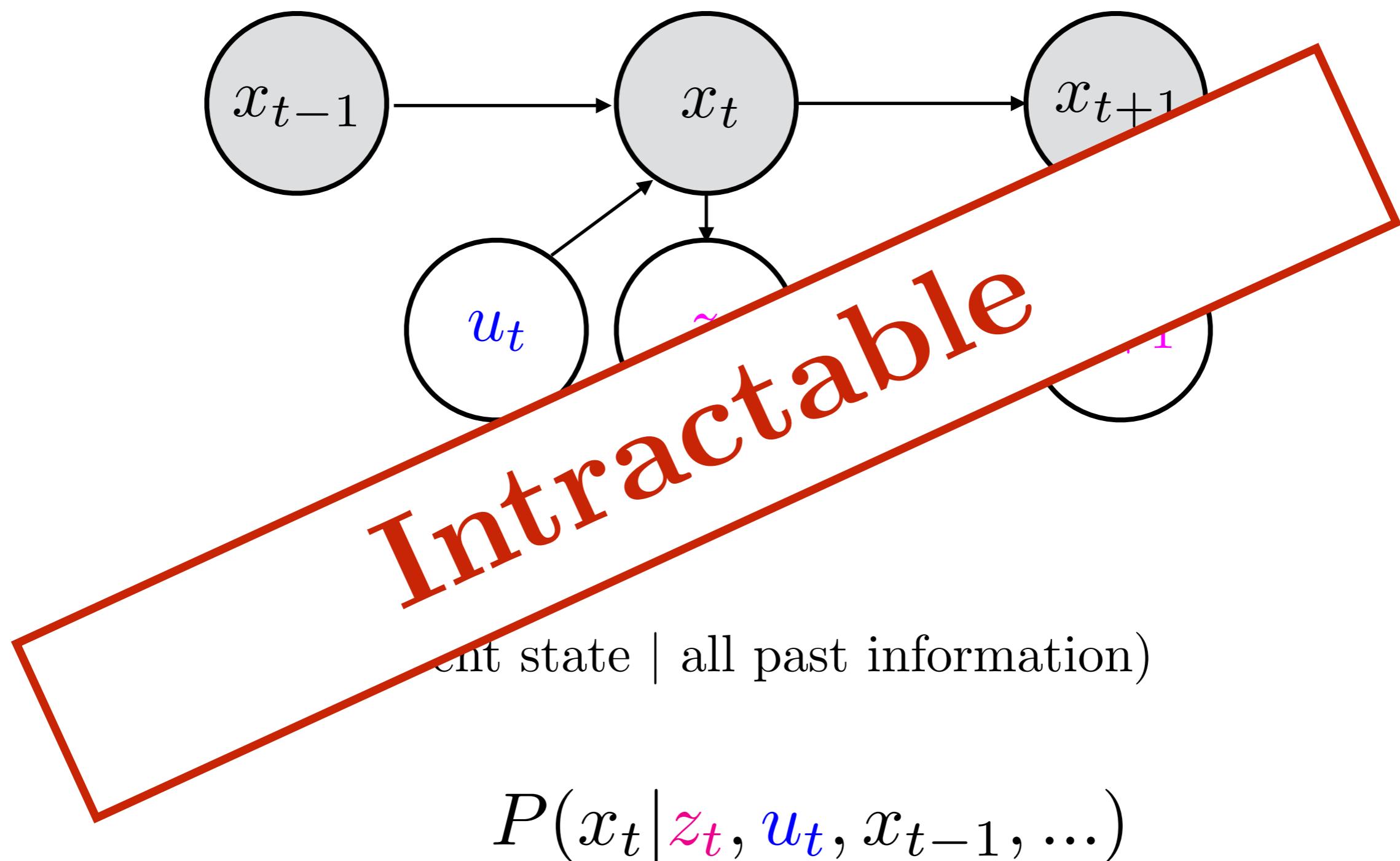
# Problem: How do we estimate belief?



$P(\text{ current state} \mid \text{all past information})$

$$P(x_t \mid z_t, u_t, x_{t-1}, \dots)$$

# Problem: How do we estimate belief?

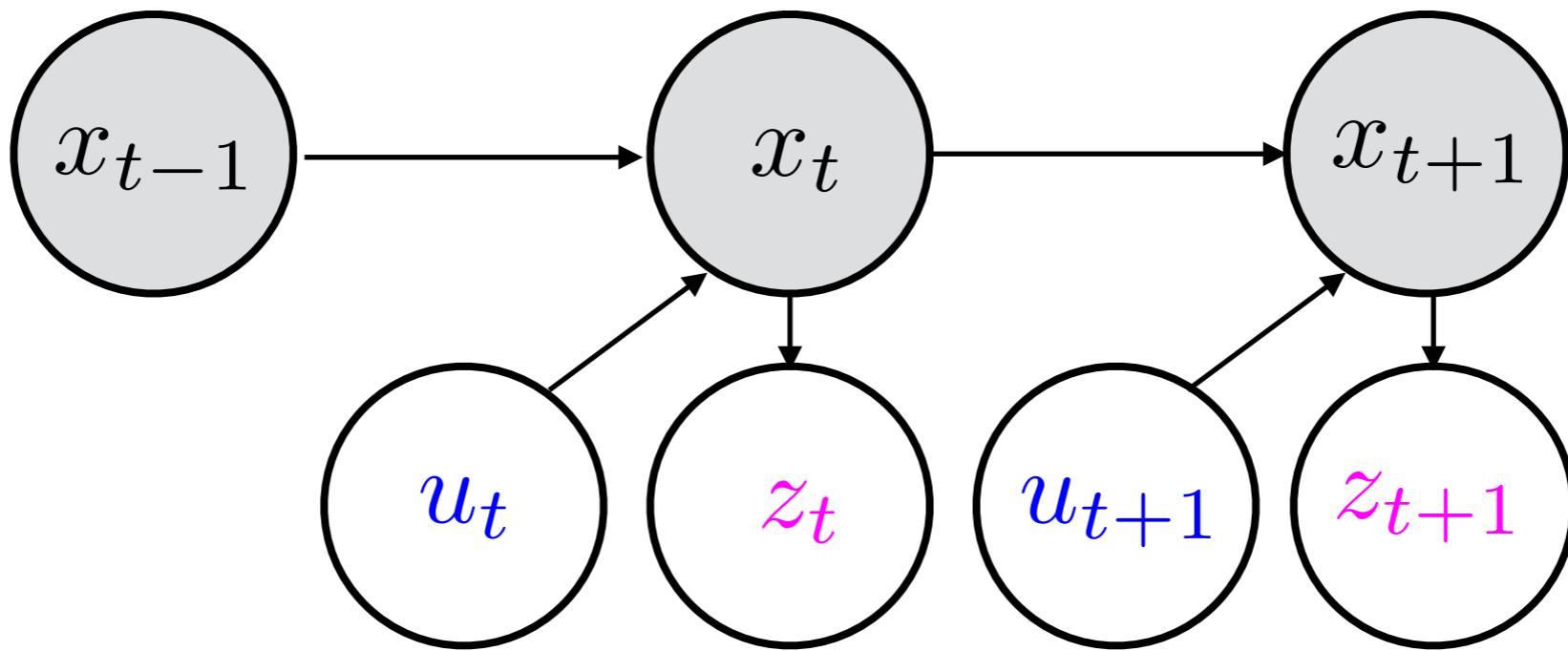


# Solution



Andrey Andreyevich Markov (1856 - 1922)

# Solution: Markov assumption



Markov assumption :

Future state **conditionally independent** of past actions,  
measurements **given** present state.

$$P(x_t | \textcolor{blue}{u_t}, x_{t-1}, z_{t-1}, u_{t-1}, \dots) = P(x_t | \textcolor{blue}{u_t}, x_{t-1})$$

$$P(\textcolor{magenta}{z_t} | x_t, u_t, x_{t-1}, z_{t-1}, u_{t-1}, \dots) = P(\textcolor{magenta}{z_t} | x_t)$$

# Reminder: Conditional Independence

$$P(A|B, C) = P(A|C)$$

iff  $A, B$  conditionally independent given  $C$

# Probabilistic models

State transition probability / dynamics / motion model

$$P(x_t | x_{t-1}, u_t)$$

Measurement probability / Observation model

$$P(z_t | x_t)$$

# When does Markov not hold?

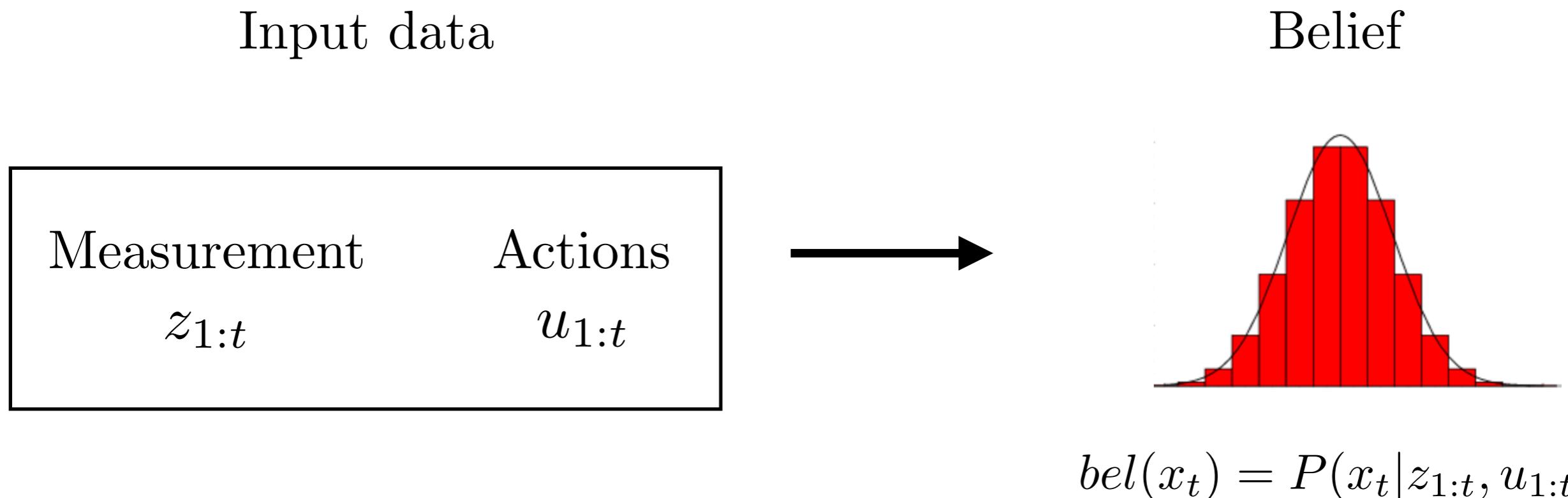
$$P(x_t | x_{t-1}, u_t) \quad P(\textcolor{magenta}{z}_t | x_t)$$

whenever state doesn't capture all requisite information

- Camera images at different times of the day
- Unmodelled pedestrians in front of laser
- Steady gusts of wind

# Central Question:

How do we tractably calculate belief?



Ans: Bayes filter!

# Bayes filter in a nutshell

Key Idea: Apply Markov to get a **recursive** update!

# Bayes filter in a nutshell

Step 0. Start with the belief at time step t-1

$$bel(x_{t-1})$$



$$bel(x_{t-1})$$

# Bayes filter in a nutshell

Step 1: Prediction - push belief through dynamics given action

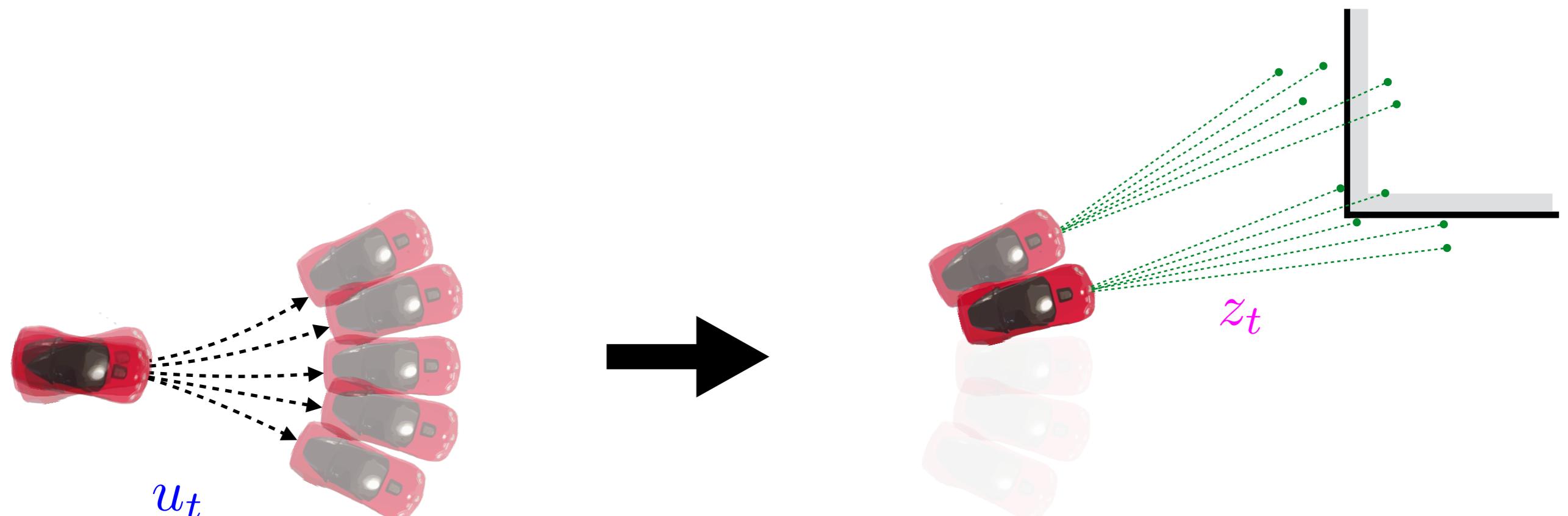
$$(\text{discrete}) \quad \overline{bel}(x_t) = \sum P(x_t | \mathbf{u}_t, x_{t-1}) bel(x_{t-1})$$



# Bayes filter in a nutshell

Step 2: Correction - apply Bayes rule given measurement

$$bel(x_t) = \eta P(z_t|x_t) \overline{bel}(x_t)$$



$$bel(x_{t-1}) \quad \overline{bel}(x_t)$$

$$bel(x_t)$$

$$\eta = \frac{1}{\sum P(z_t|x_t) \overline{bel}(x_t)}$$

# Bayes filter in a nutshell

**Key Idea:** Apply Markov to get a recursive update!

Step 0. Start with the belief at time step t-1

$$bel(x_{t-1})$$

Step 1: Prediction - push belief through dynamics given **action**

$$\overline{bel}(x_t) = \sum P(x_t | \textcolor{blue}{u}_t, x_{t-1}) bel(x_{t-1})$$

Step 2: Correction - apply Bayes rule given **measurement**

$$bel(x_t) = \eta P(\textcolor{violet}{z}_t | x_t) \overline{bel}(x_t)$$

# Bayes filter is a powerful tool



Localization

Mapping

SLAM

POMDP