Lateral Dynamic Bicycle Model

Course 1, Module 4, Lesson 5

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Learning Objectives

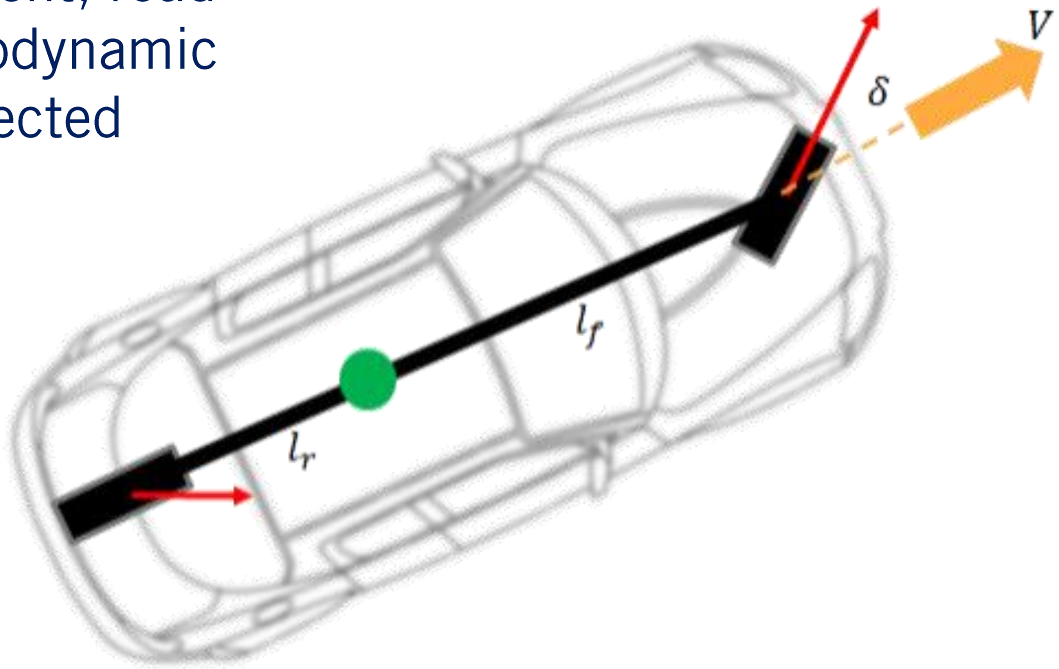
- Build a dynamic model of a car using the kinematic bicycle model as a starting point

- Convert to standard state space representation

Vehicle Model to Bicycle Model

- Assumptions
  - Longitudinal velocity is constant
  - Left and right axle are lumped into a single wheel (bicycle model)
  - Suspension movement, road inclination and aerodynamic influences are neglected

nonlinear dynamics



Lateral Dynamics

- Lateral dynamics can be written as

β: vehicle slip angle

$V = \omega R$

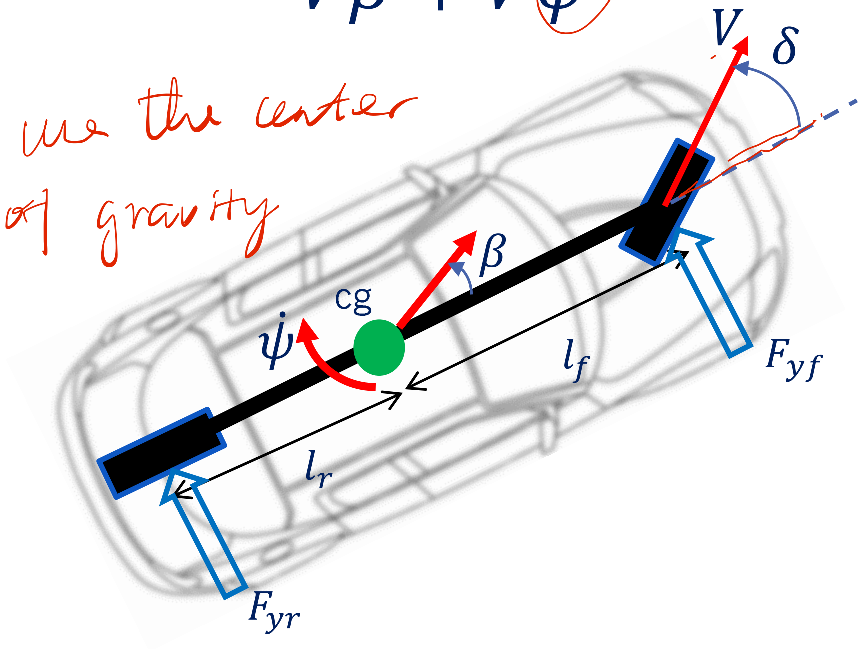
$\omega = \dot{\psi}$

Lateral acceleration

$$a_y = \ddot{y} + \omega^2 R$$
$$= V\dot{\beta} + V\dot{\psi}$$

heading rate of change

use the center of gravity



side slip rate

yaw rate

front and rear tire forces

vehicle mass

$$mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr}$$

vehicle velocity

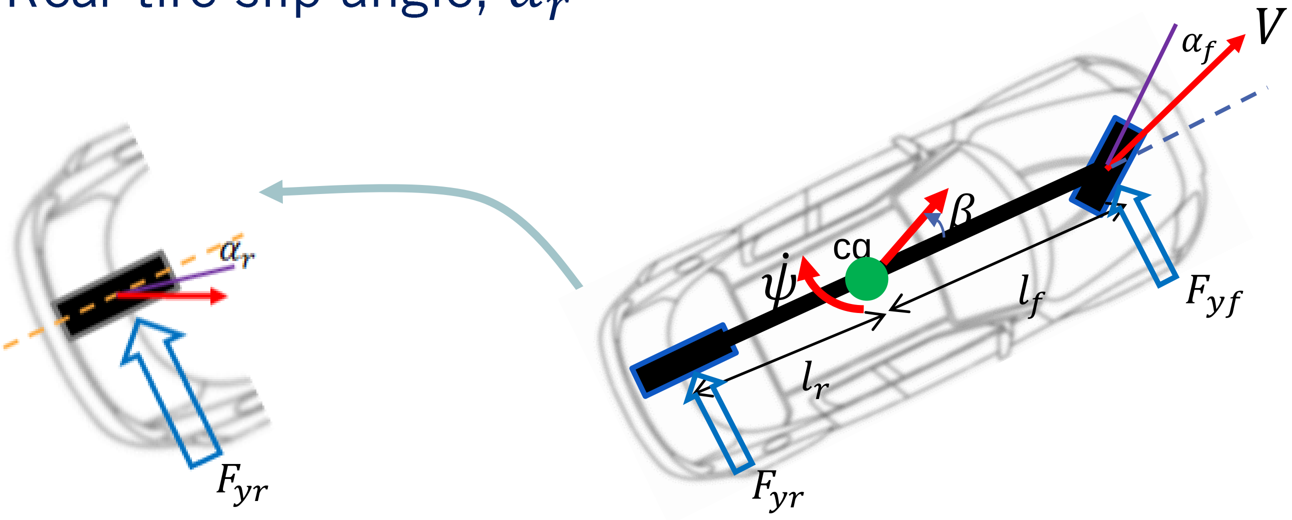
$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

vehicle inertia

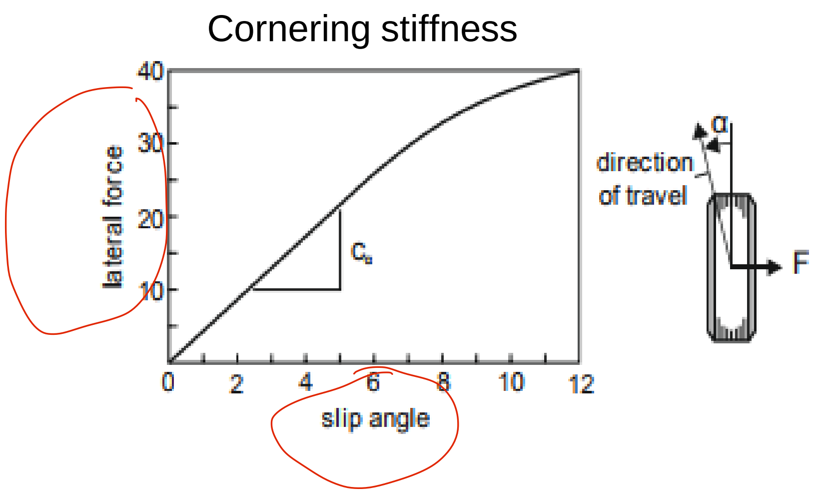
center of gravity distance from front and rear tires

Tire Slip Angles

- Many different tire slip models
- For small tire slip angles, the lateral tire forces are approximated as a linear function of tire slip angle
- Tire variables
  - Front tire slip angle, α_f
  - Rear tire slip angle, α_r



Front and Rear Tire Forces



- C_f :linearized cornering stiffness of the front wheel

$$F_{yf} = C_f \alpha_f = C_f \left(\delta - \beta - \frac{l_f \dot{\psi}}{V} \right)$$

steering angle

- C_r : linearized cornering stiffness of the rear wheel

$$F_{yr} = C_r \alpha_r = C_r \left(-\beta + \frac{l_r \dot{\psi}}{V} \right)$$

Lateral and Yaw Dynamics

- From the previous slide formulations:

$$F_{yf} = C_f \alpha_f = C_f \left(\delta - \beta - \frac{l_f \dot{\psi}}{V} \right)$$

$$F_{yr} = C_r \alpha_r = C_r \left(-\beta + \frac{l_r \dot{\psi}}{V} \right)$$

Substitute the lateral forces

$$mV(\dot{\beta} + \dot{\psi}) = F_{yf} + F_{yr}$$

$$I_z \ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

Rearranging the equations

$$\dot{\beta} = \frac{-(C_r + C_f)}{mV} \beta + \left(\frac{C_r l_r - C_f l_f}{mV^2} - 1 \right) \dot{\psi} + \frac{C_f}{mV} \delta$$

rate of change of yaw rate

$$\ddot{\psi} = \frac{C_r l_r - C_f l_f}{I_z} \beta - \frac{C_r l_r^2 + C_f l_f^2}{I_z V} \dot{\psi} + \frac{C_f l_f}{I_z} \delta$$

Standard State Space Representation

- State Vector: $X_{lat} = [y \quad \beta \quad \psi \quad \dot{\psi}]^T$

lateral position   side slip angle   yaw angle   yaw rate

$$\boxed{\dot{X}_{lat} = A_{lat} X_{lat} + B_{lat} \delta}$$

$$A_{lat} = \begin{bmatrix} 0 & V & V & 0 \\ 0 & -\dfrac{C_r + C_f}{mV} & 0 & \dfrac{C_r l_r - C_f l_f}{mV^2} - 1 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{C_r l_r - C_f l_f}{I_z} & 0 & -\dfrac{C_r l_r^2 + C_f l_f^2}{I_z V} \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 \\ \dfrac{C_f}{mV} \\ 0 \\ \dfrac{C_f l_f}{I_z} \end{bmatrix}$$

Summary

What we have learned from this lesson?

- Formulated the lateral dynamics of a bicycle model
- Defined a state space representation of lateral model

What is next?

- Vehicle actuation system models
    - Throttle, brake & steering