

# Time to Collision

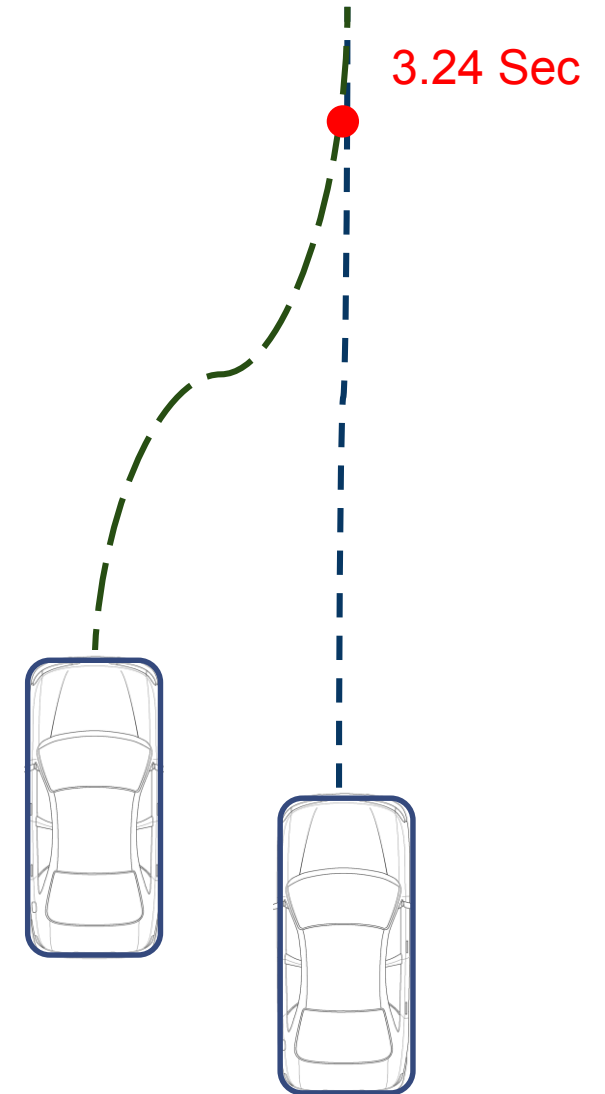
COURSE 4, MODULE 4, LESSON 3



UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE & ENGINEERING

# Definition of Time to Collision

- Assuming all dynamic object continue along their predicted path:
  - Will then be a collision between any of the objects?
  - If so how far into the future?
- Time to Collision is comprised of:
  - Collision point between the two dynamic objects
  - Prediction of the time to arrive to the collision point
- Requirements for Accuracy:
  - Accurate predicted trajectories for all dynamic objects (position, heading and velocity)
  - Accurate dynamic objects geometries

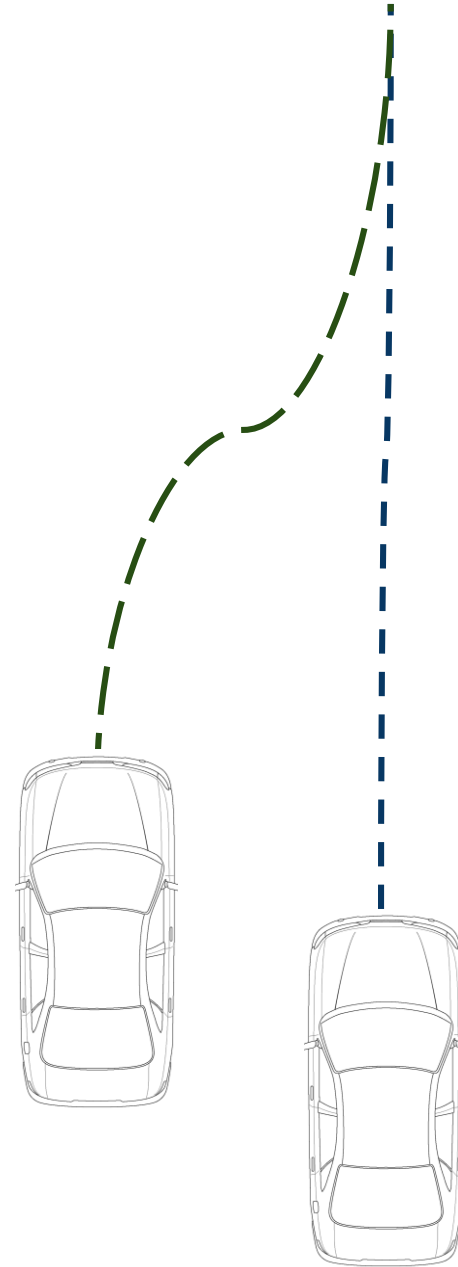


# Approaches to calculate time to collision

- Two basic approaches to calculating time to collision:
  - Simulation approach
  - Estimation approach

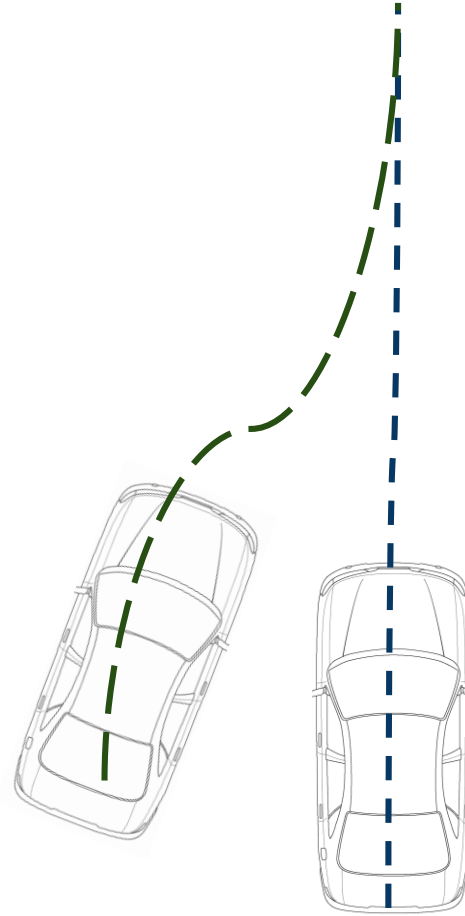
# Simulation approach

- Simulate the movement of each vehicle as time passes



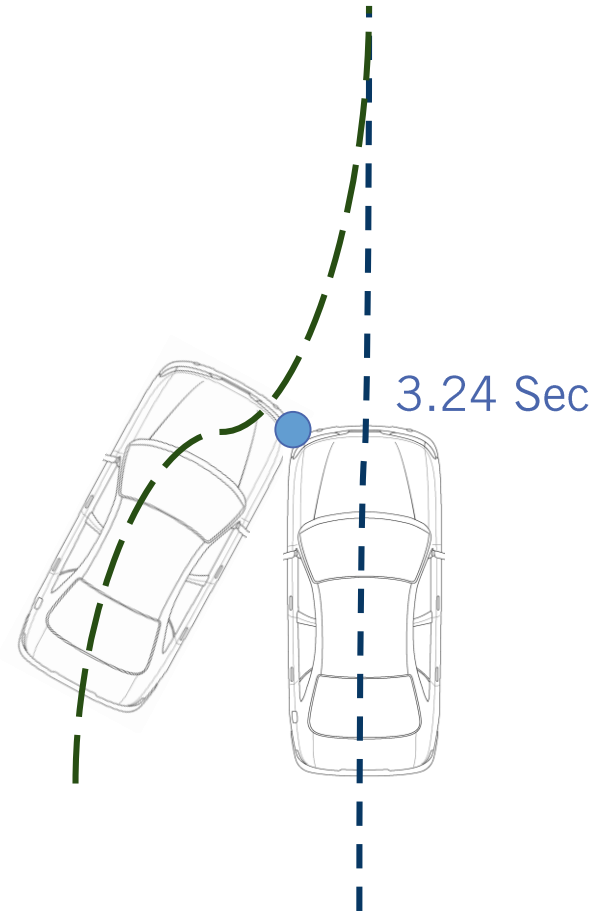
# Simulation approach

- Simulate the movement of each vehicle as time passes
- Taking account of the vehicle model over time



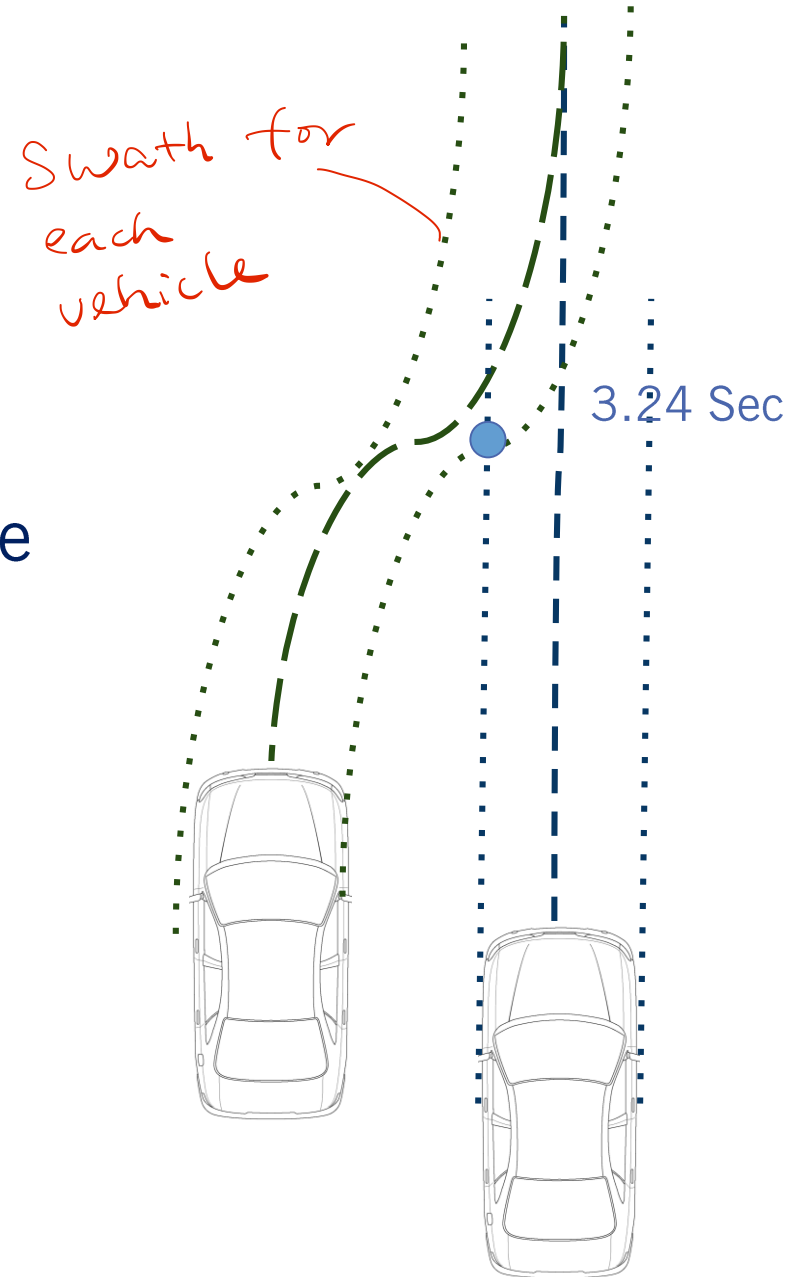
# Simulation approach

- Simulate the movement of each vehicle as time passes
- Taking account of the vehicle model over time
- Checking if any part of the two dynamic object has collided



# Estimation approach

- Geometries of the vehicles are approximated over duration of the predicted path
- Collision point is estimated based the cars predictions
- Many assumptions are usually made by this method usually to estimate time to collision



# Relative Strengths and Weaknesses

## Simulation Approach

- **Computationally expensive**  
*step by step evaluations*  
*↳ object geometry intersection computations*
- **Higher accuracy if simulated with high fidelity**
- **Offline Applications (Dataset evaluation or Simulations)**

## Estimation Approach

- **Computationally inexpensive**
  - Memory footprint
  - Computational time
- **Less accurate due to approximations and estimations**
- **Real Time Applications (In Car Prediction)**  
*online*



# Simulation approach Pseudocode

## Inputs:

- $D$  – list of all dynamic objects + planned path of ego vehicle
  - Predicted paths
- $dt$  – time between simulation steps  $\Rightarrow$  can be different from the predicted path time steps.
- $N_c$  – number of circles for collision approximation

## Outputs:

- $P_{coll}$  – list of all collision points
- $TTC$  – list of all times to collision points  
Time to collision

# Simulation approach Pseudocode

---

## Algorithm Constant Velocity TTC( $D, T, dt, N_c$ )

---

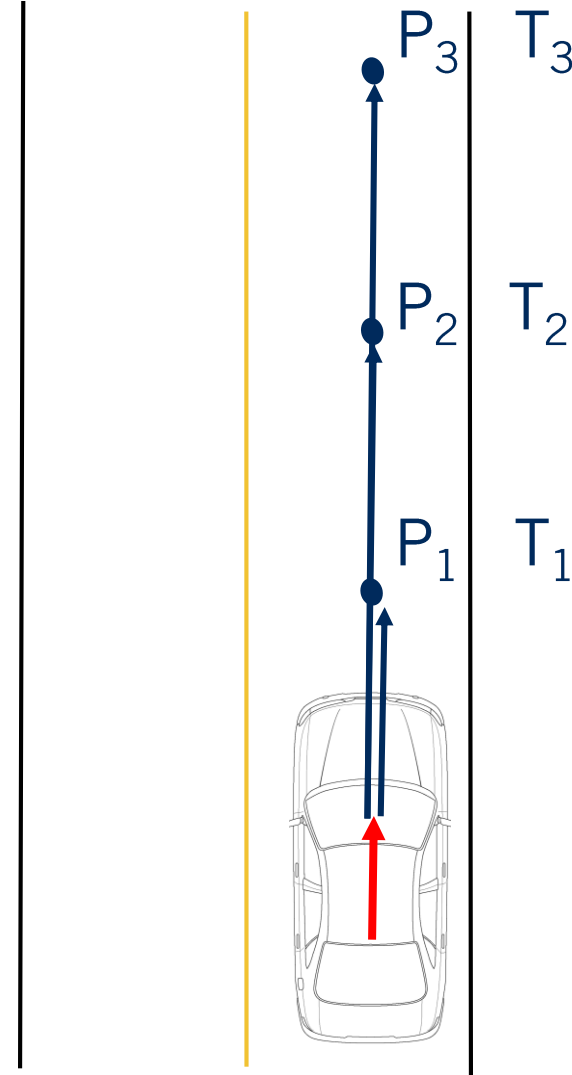
```
1.   $t \leftarrow 0$ 
2.   $x_0 = x_{obj}$ 
3.  while  $t < T$  do
4.     $t = t + dt$ 
5.    for  $i \in \{1, \dots |D|\}$  do
6.       $d_i.x_t \leftarrow \text{PositionEstimation}(d_i, t)$ 
7.      for  $j \in \{i, \dots |D|\}$  do
8.         $d_j.x_t \leftarrow \text{PositionEstimation}(d_j, t)$ 
9.         $P_{coll,ij} \leftarrow \text{CollisionEstimation}(d_i.x_t, d_j.x_t, N_c)$ 
10.       if  $P_{coll,ij}$  then
11.          $TTC_{ij} \leftarrow t$ 
12.       end
13.     end
14.   end
15. end while
16. return  $P_c, TTC$ 
```

---

# Estimation of Dynamic Object State

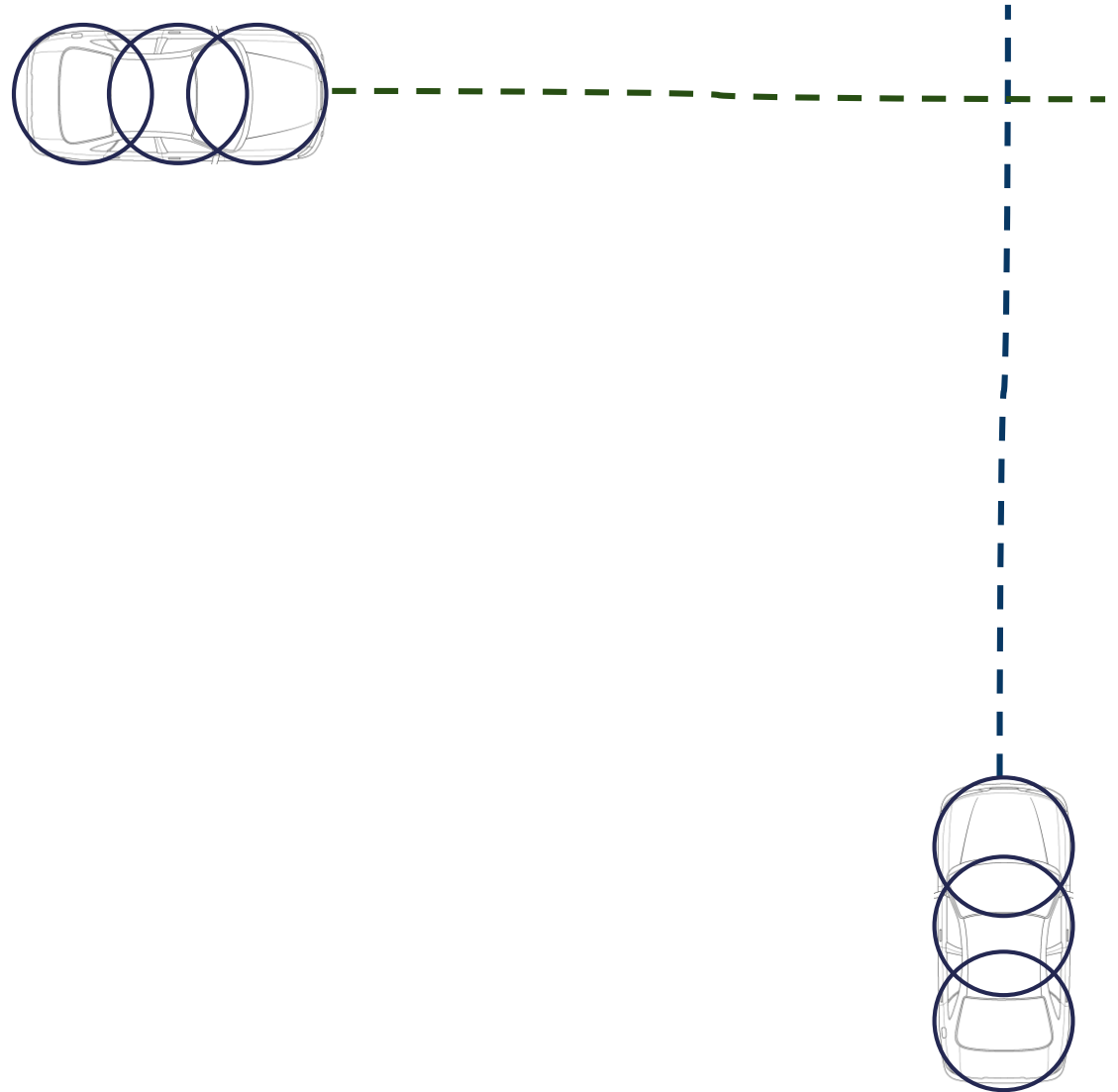
- Each predicted vehicle state has a predicted time at each location
- Find the closest vehicle state along the predicted path to the current simulation time

$T = 3$  seconds



# Efficient Collision Detection Method

1. Represent each car as a set of circles



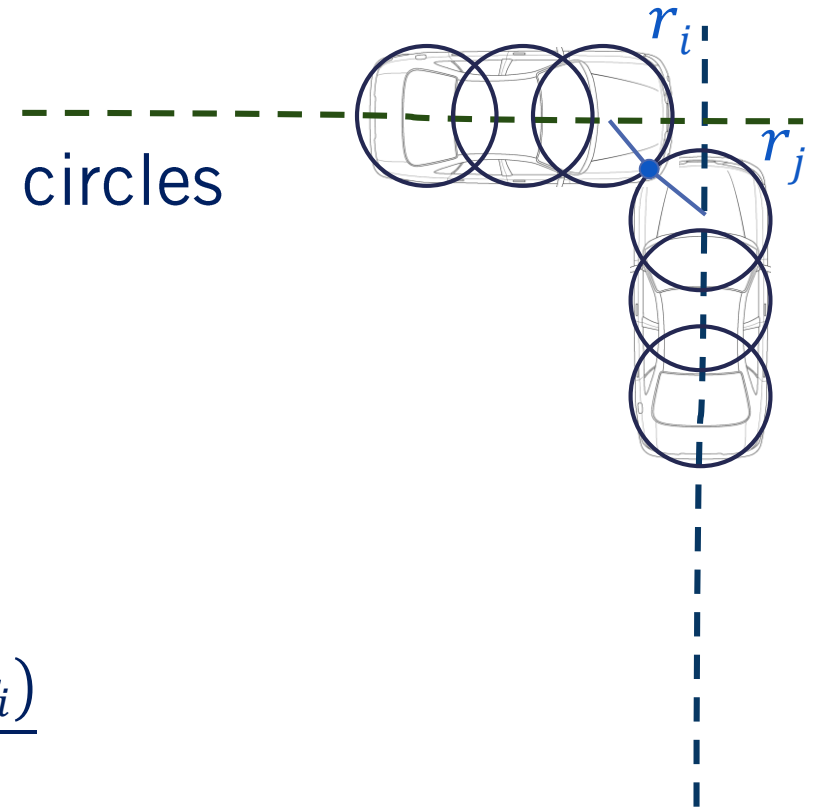
# Efficient Collision Detection Method

1. Represent each car as a set of circles
2. Check if a collision will occur between two circles

$$d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}, \quad r_i + r_j \geq d_{i,j}$$

3. Calculate collision point

$$C_X = \frac{(x_i * r_j) + (x_j * r_i)}{(r_i + r_j)}, \quad C_Y = \frac{(y_i * r_j) + (y_j * r_i)}{(r_i + r_j)}$$



# Efficient Collision Detection Method

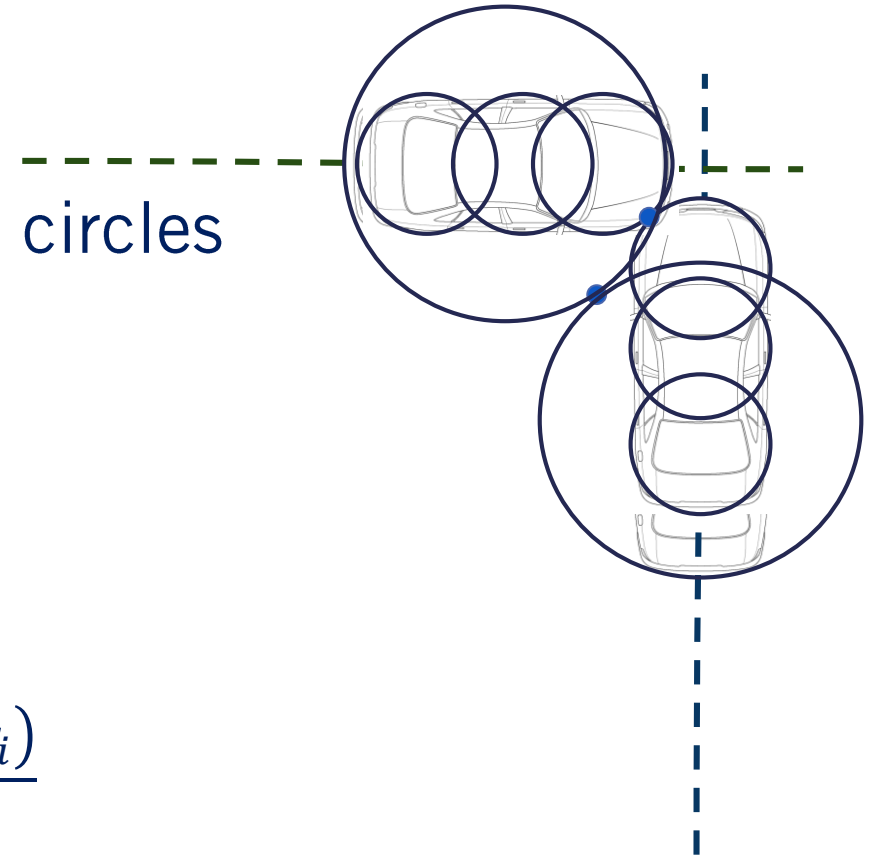
1. Represent each car as a set of circles
2. Check if a collision will occur between two circles

$$d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}, \quad r_i + r_j \geq d_{i,j}$$

3. Calculate collision point

$$C_X = \frac{(x_i * r_j) + (x_j * r_i)}{(r_i + r_j)}, \quad C_Y = \frac{(y_i * r_j) + (y_j * r_i)}{(r_i + r_j)}$$

- Tradeoff between:
  - Accuracy
  - Number of computations



# Summary

- Define time to collision and all its components
- Identify two approaches to calculate time to collisions and discuss their strengths and weaknesses
  - Simulation approach
  - Estimation approach
- Outline a simulation based algorithm to calculate time to collision
- **Next:** Behavior Planning and its Principles