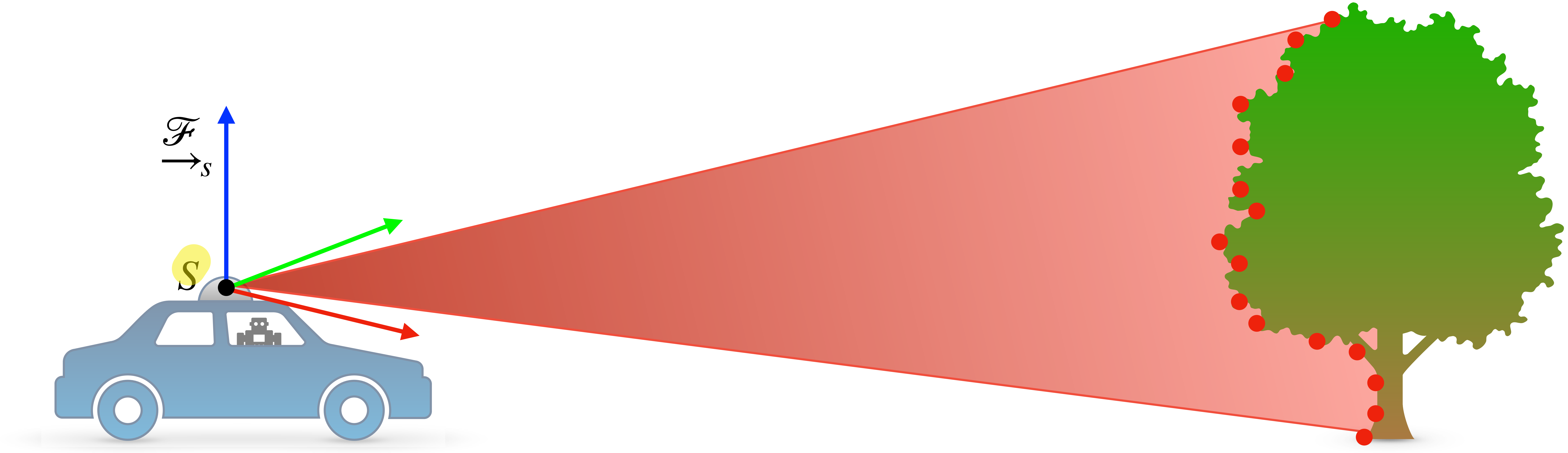
MODULE 4 LESSON 3

POSE ESTIMATION FROM LIDAR DATA

State Estimation via Point Set Registration

State Estimation via Point Set Registration



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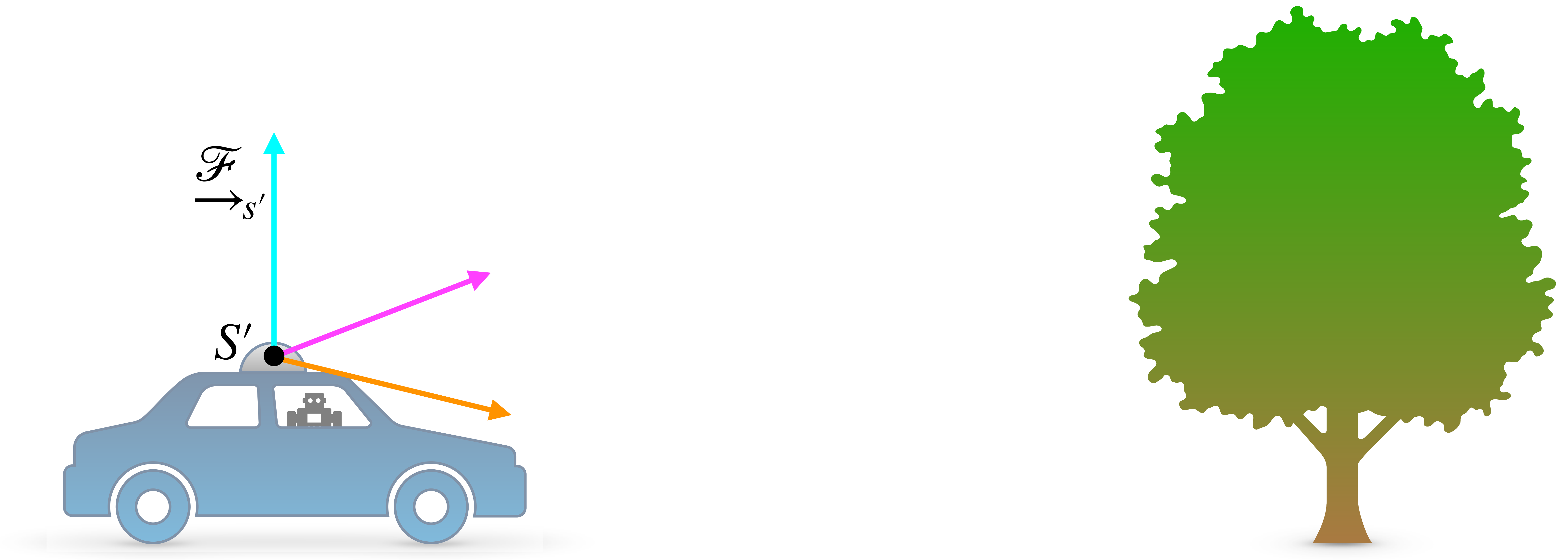

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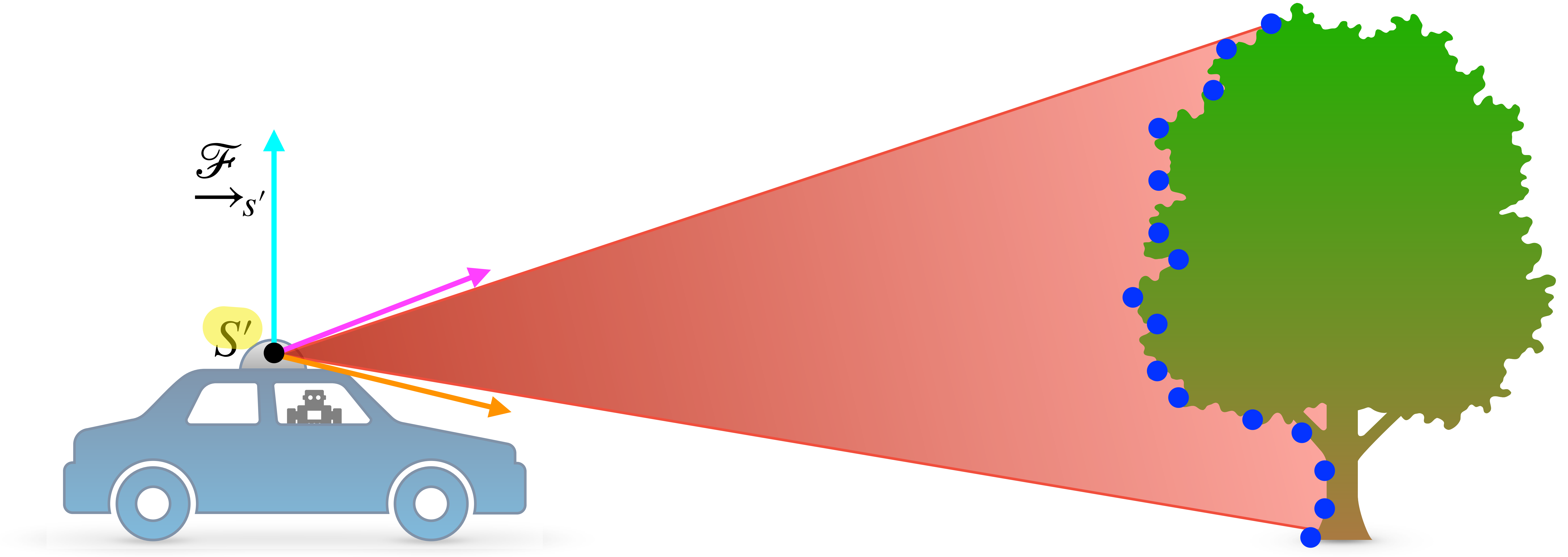
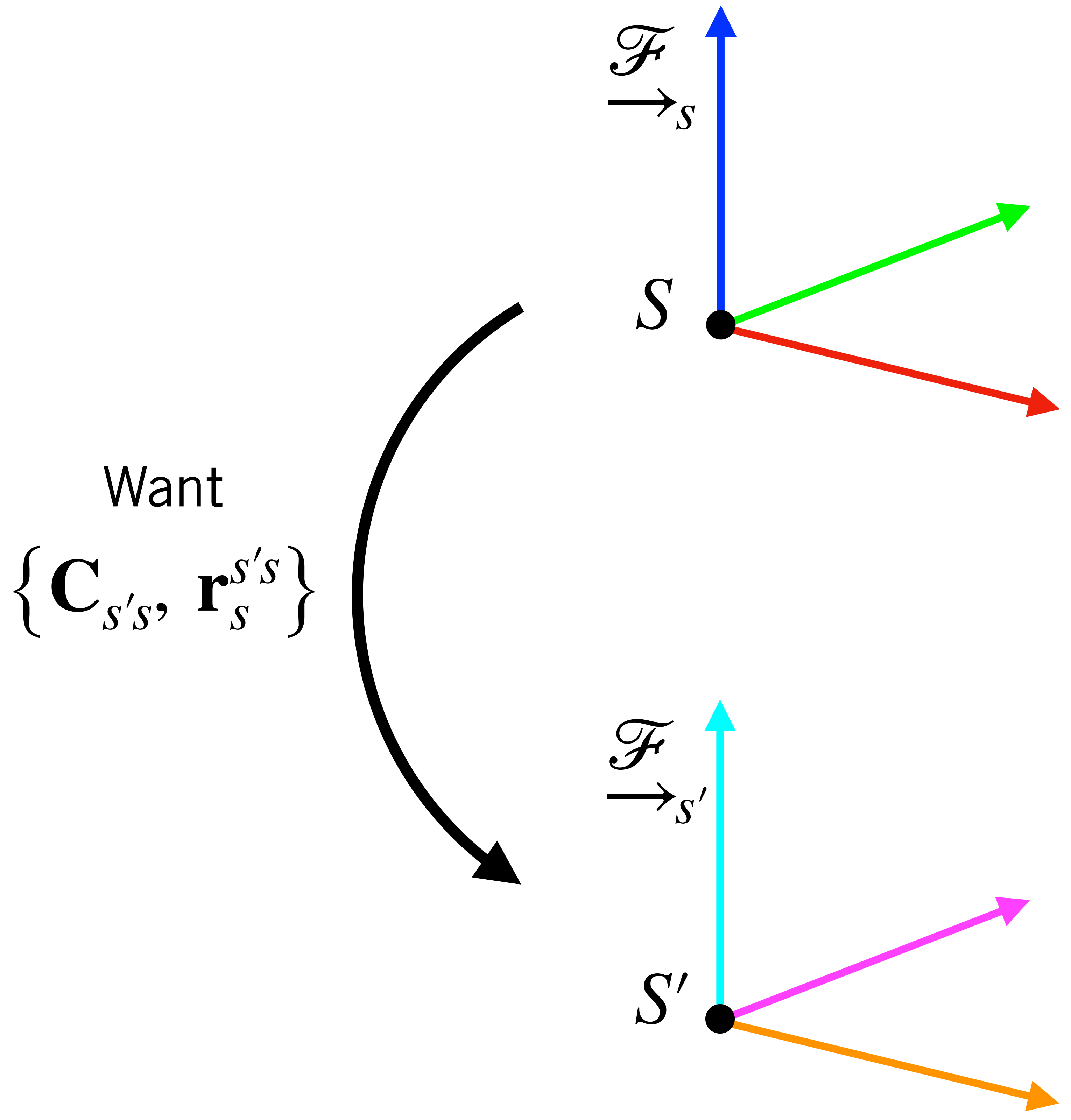
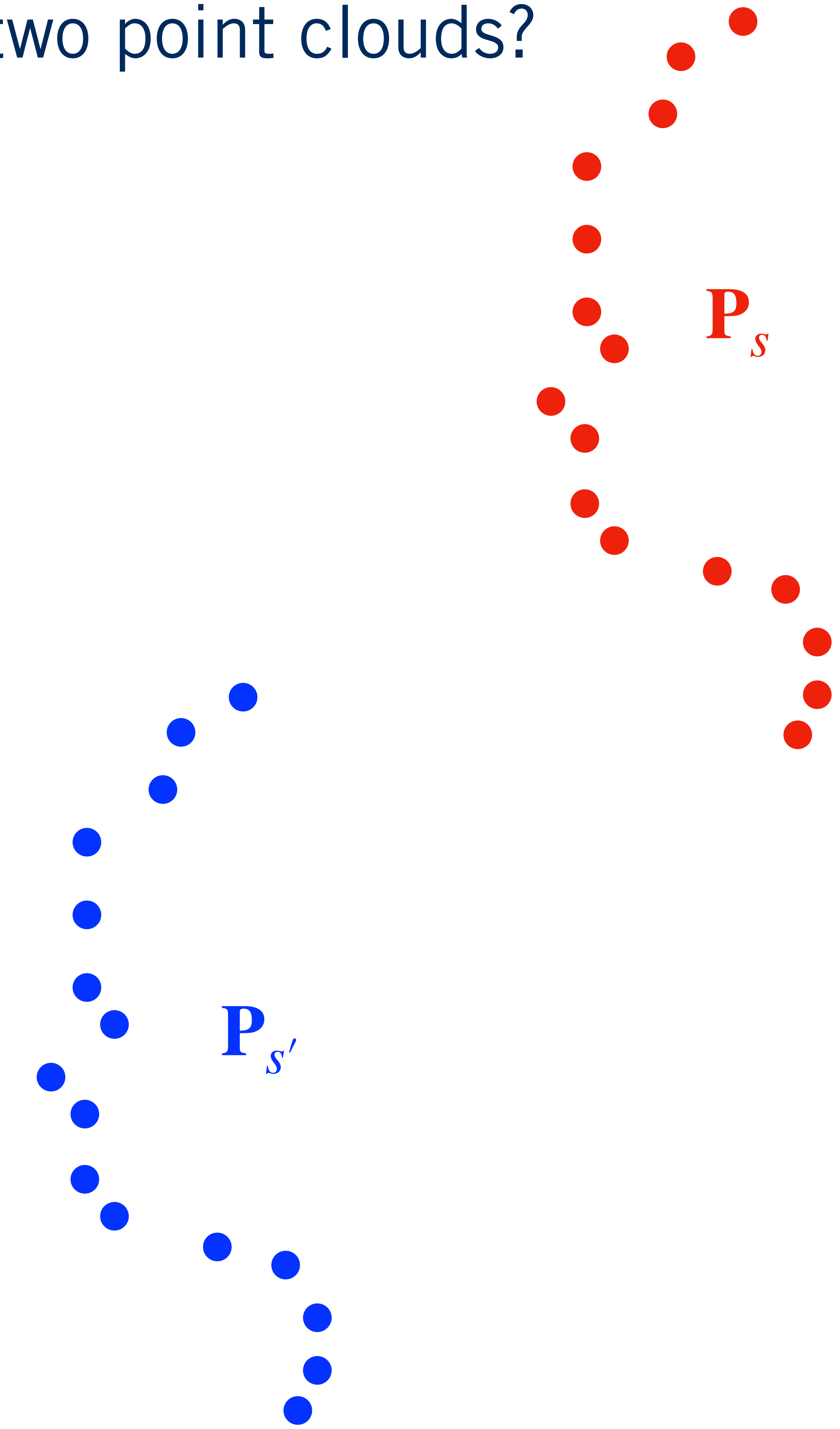


State Estimation via Point Set Registration

What motion of the car best aligns the two point clouds?





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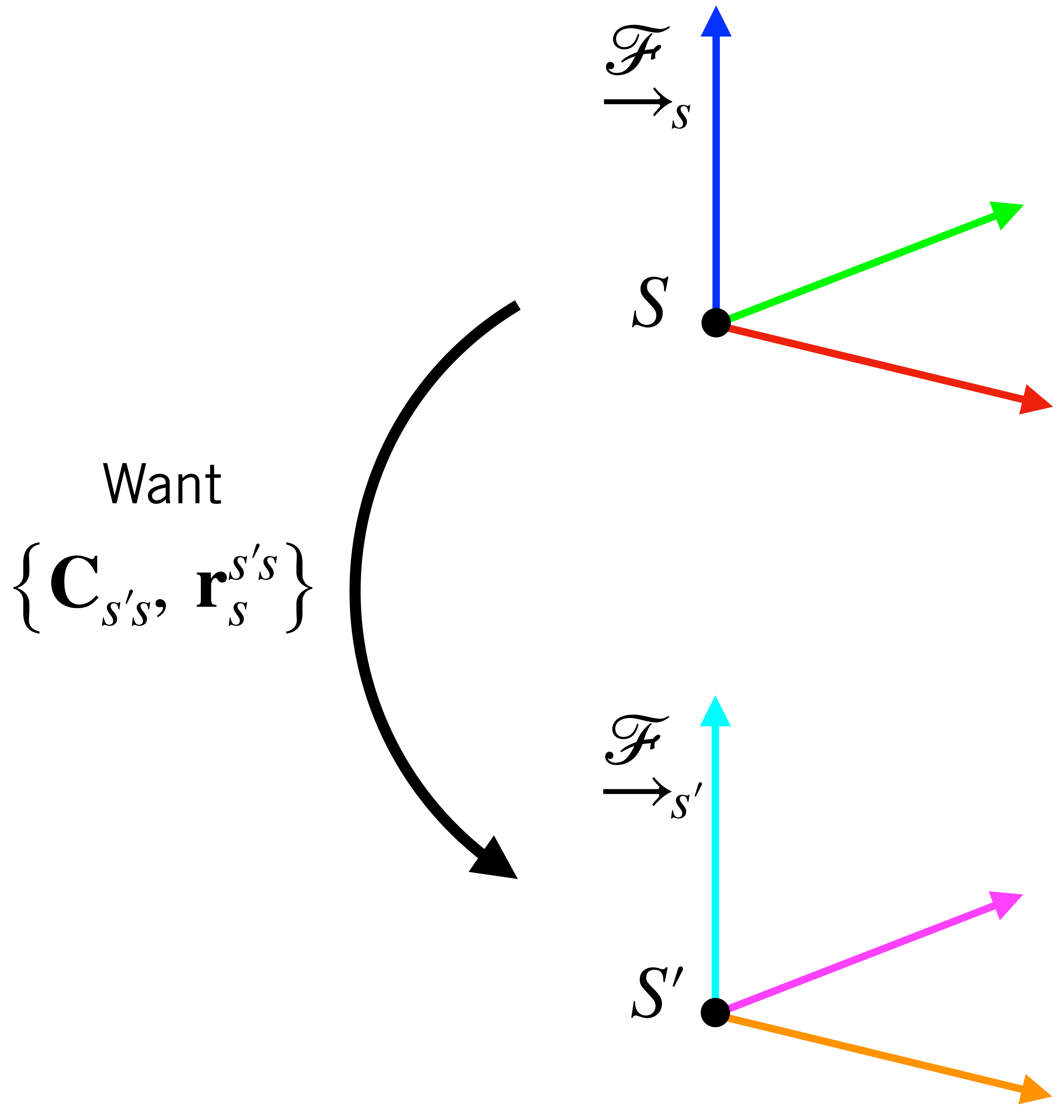

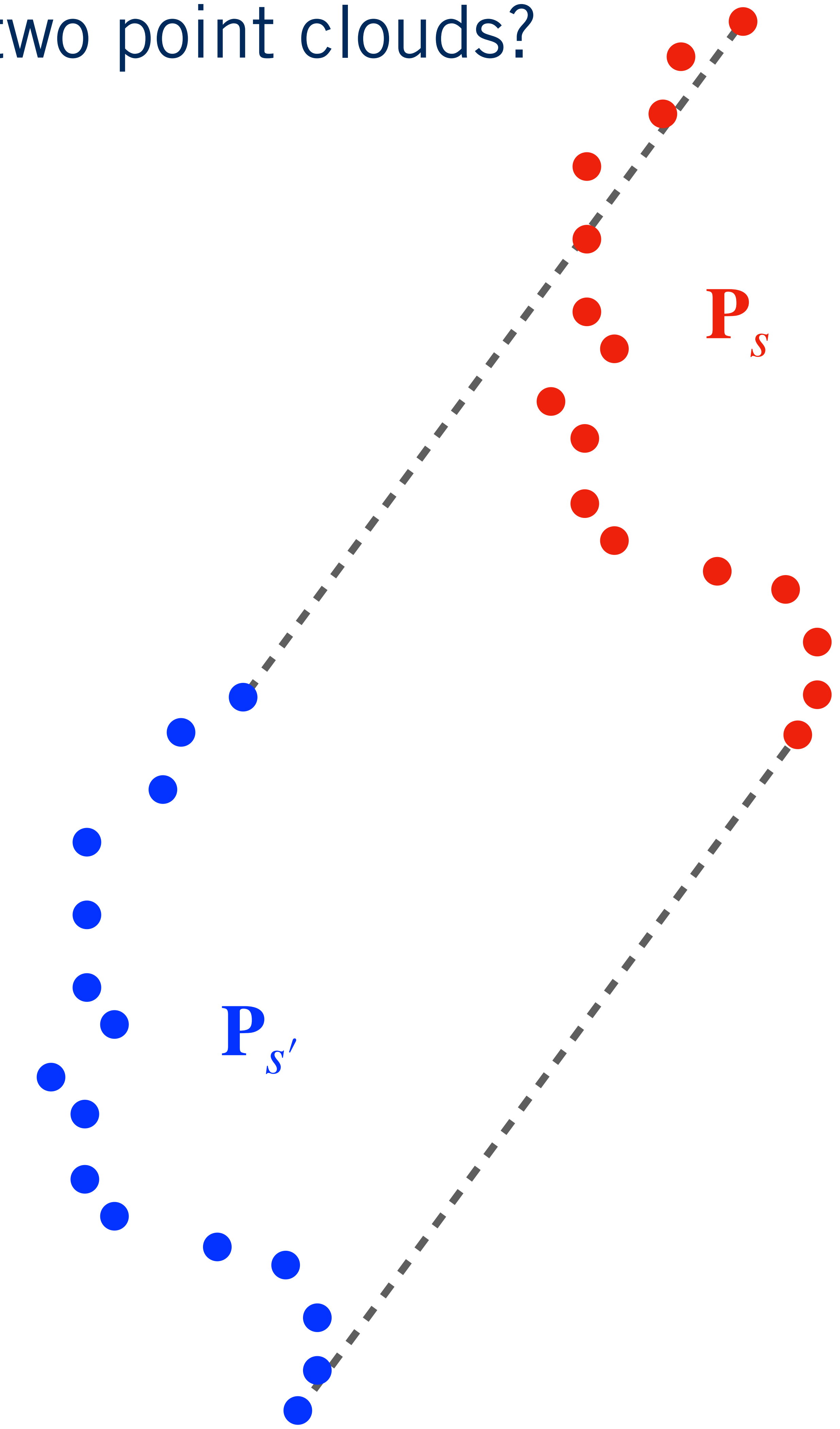

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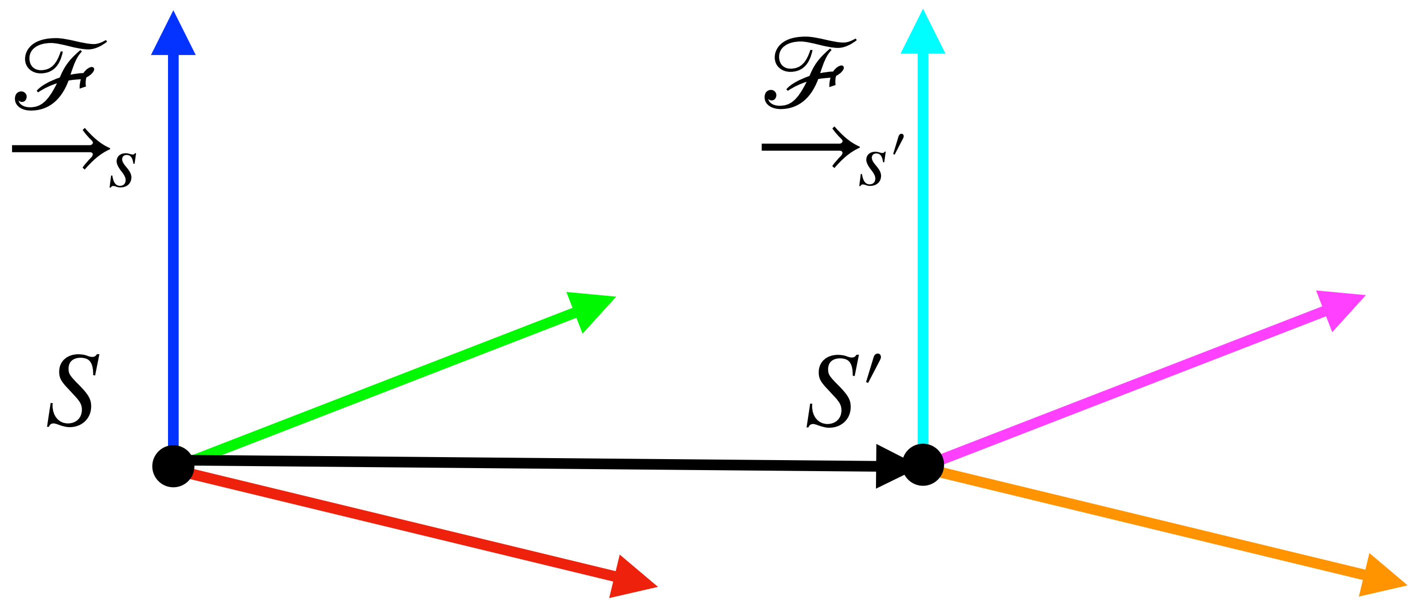
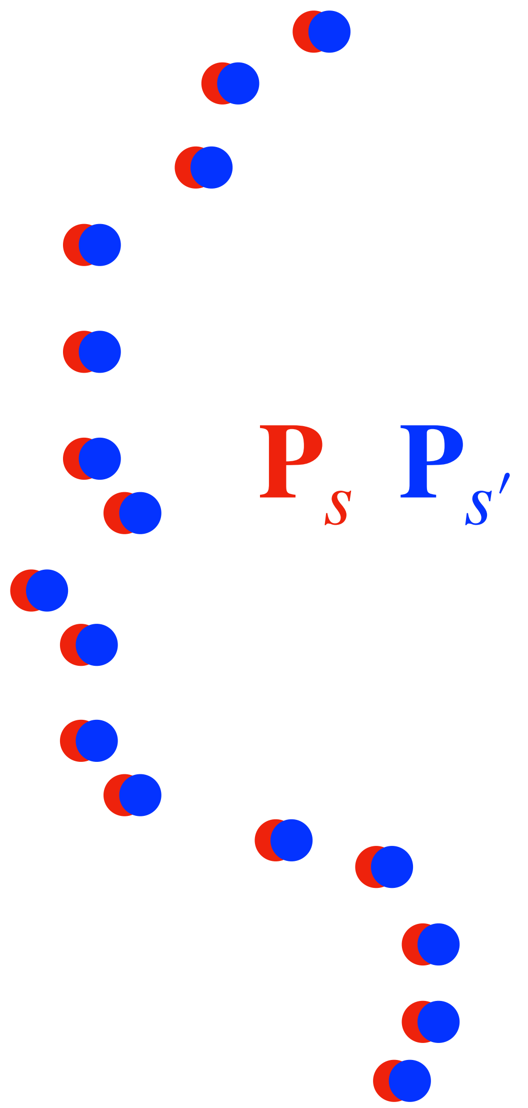


$$\left\{ \hat{\mathbf{C}}_{s's}, \hat{\mathbf{r}}_s^{s's} \right\}$$



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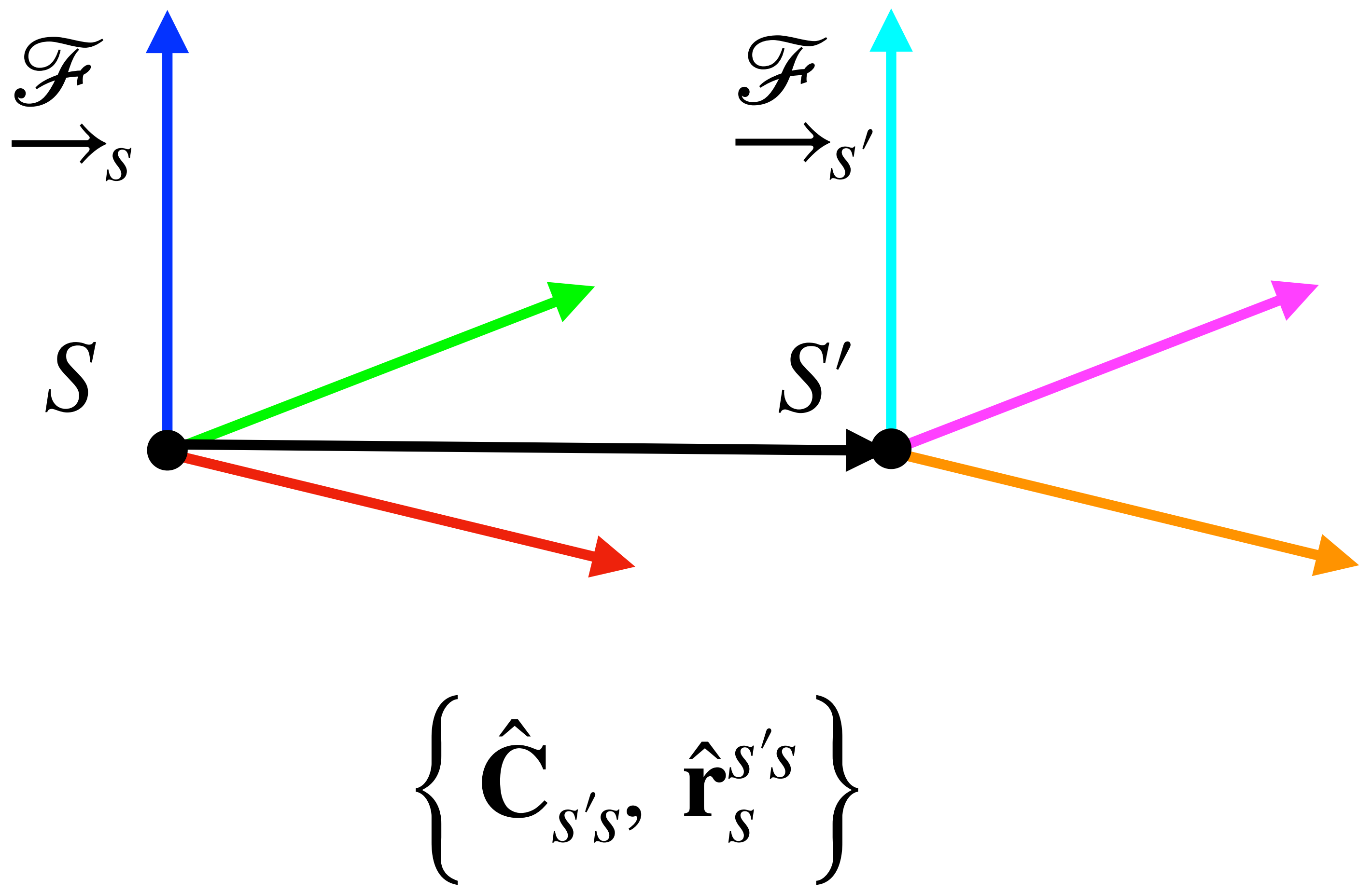
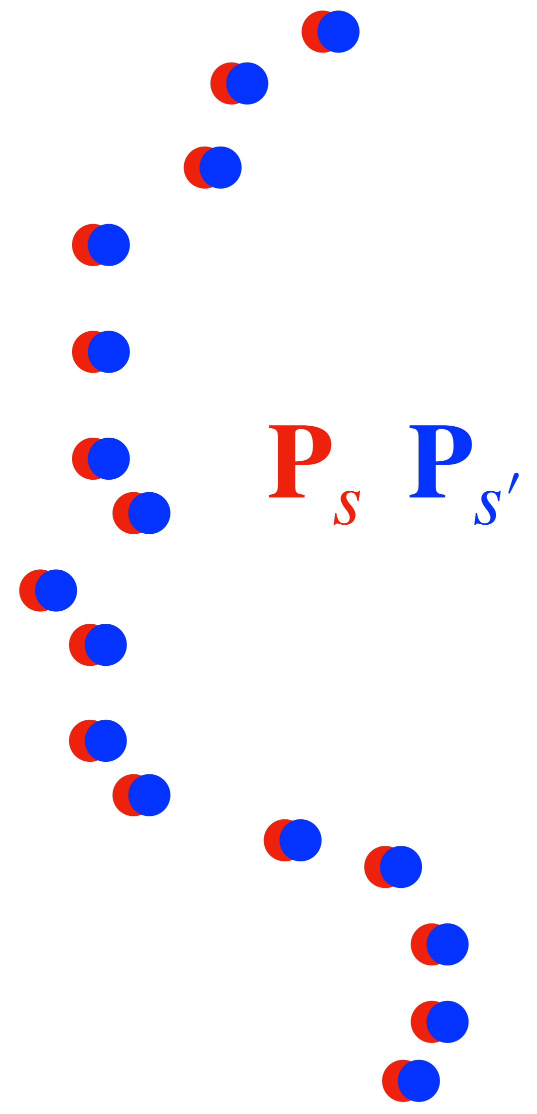
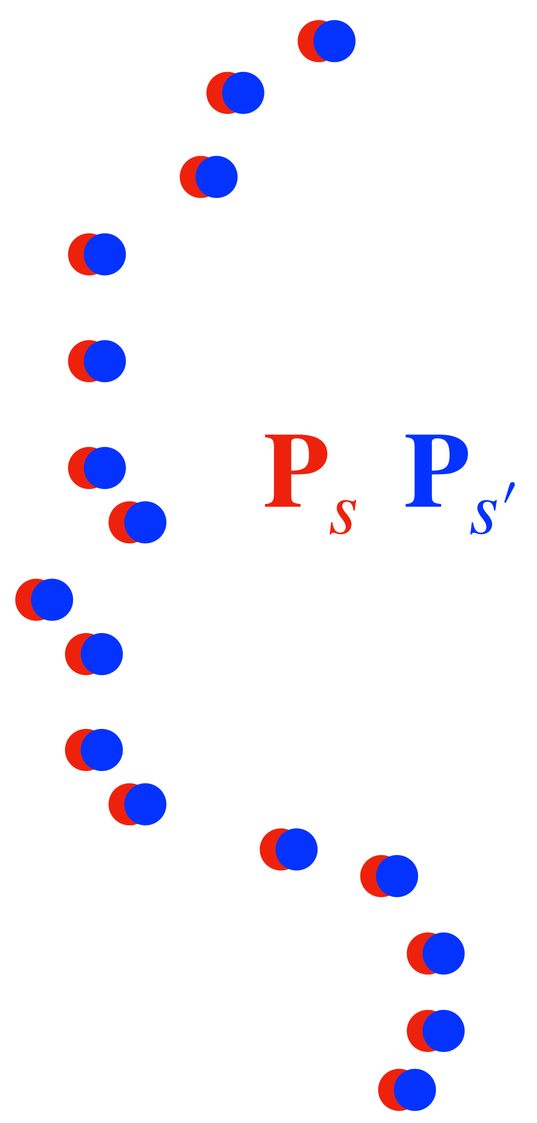




Problem: We don't know which points correspond to each other

The Iterative Closest Point (ICP) Algorithm

Intuition: When the optimal motion is found, corresponding points will be closer to each other than to other points



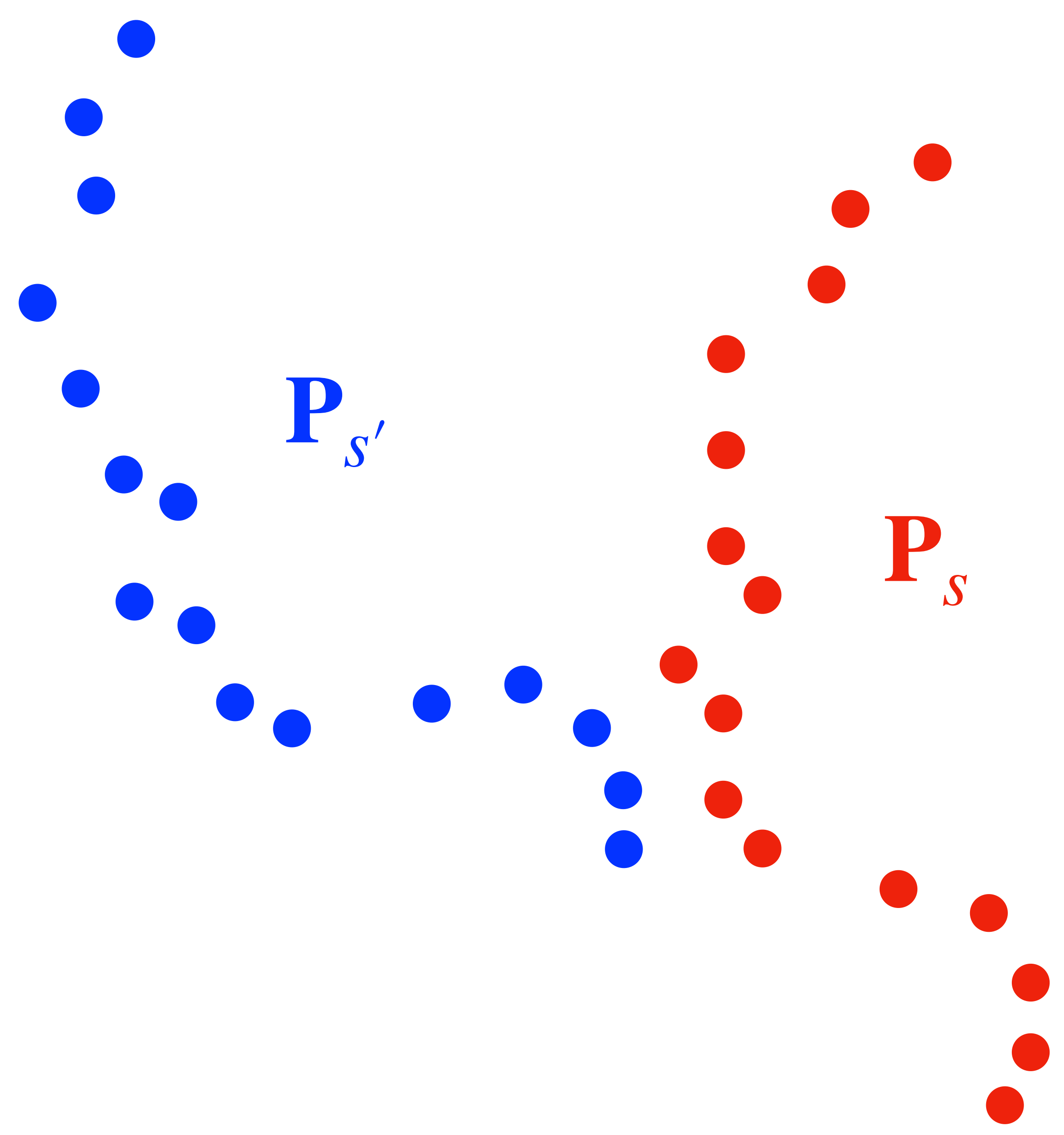
Heuristic: For each point, the best candidate for a corresponding point is the point that is closest to it right now

The Iterative Closest Point (ICP) Algorithm

Procedure:

1. Get an initial guess for the transformation $\left\{ \check{\mathbf{C}}_{s's}, \check{\mathbf{r}}_s^{s's} \right\}$

a motion model from IMU
or wheel odometry.

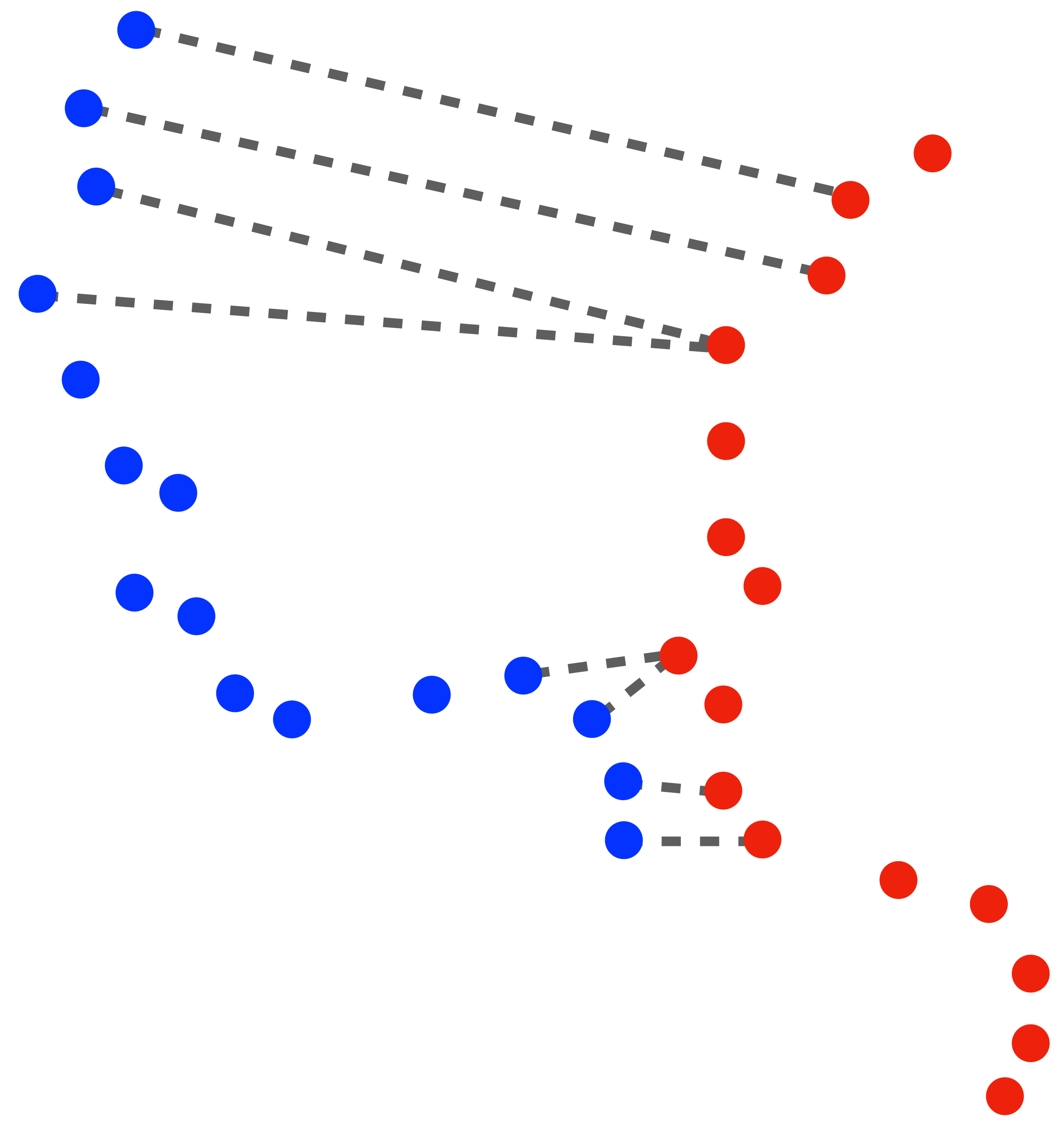


The Iterative Closest Point (ICP) Algorithm

Procedure:

1. Get an initial guess for the transformation $\left\{\check{\mathbf{C}}_{s's}, \check{\mathbf{r}}_s^{s's}\right\}$

2. Associate each point in $\mathbf{P}_{s'}$ with the nearest point in $\mathbf{P}_s$



The Iterative Closest Point (ICP) Algorithm

Procedure:

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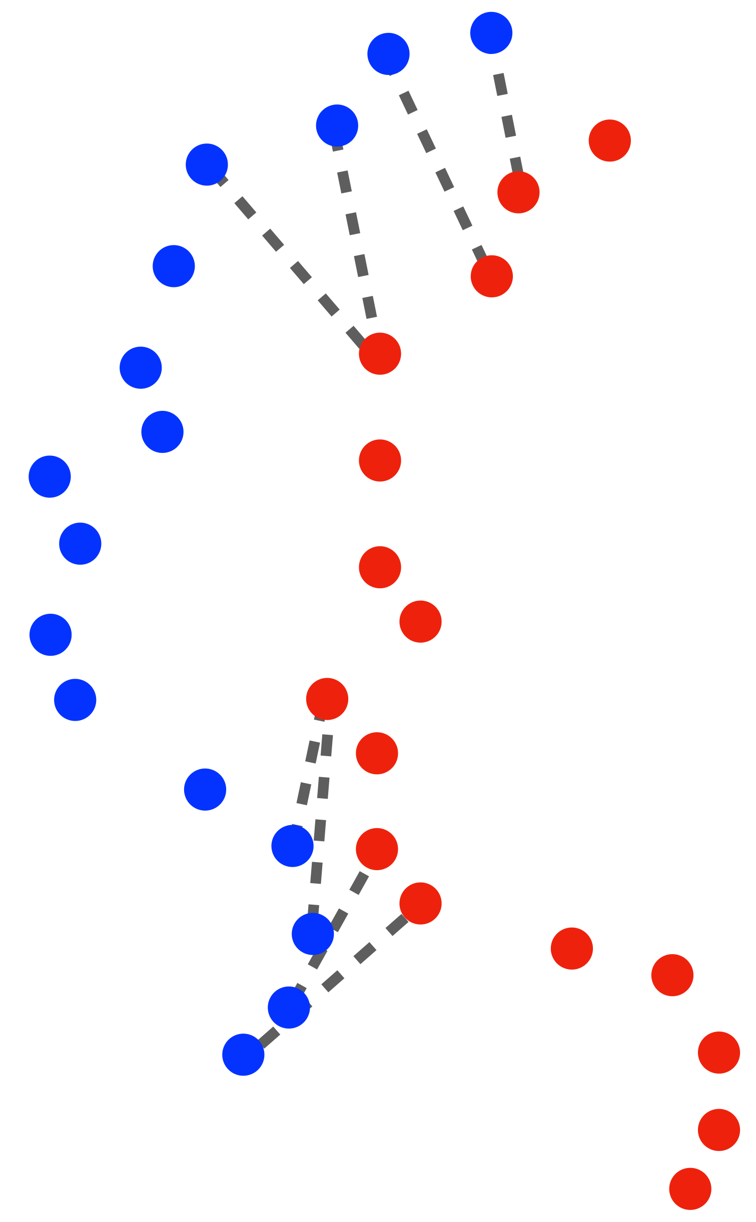
3. Solve for the optimal transformation $\left\{ \hat{\mathbf{C}}_{s's}, \hat{\mathbf{r}}_s^{s's} \right\}$



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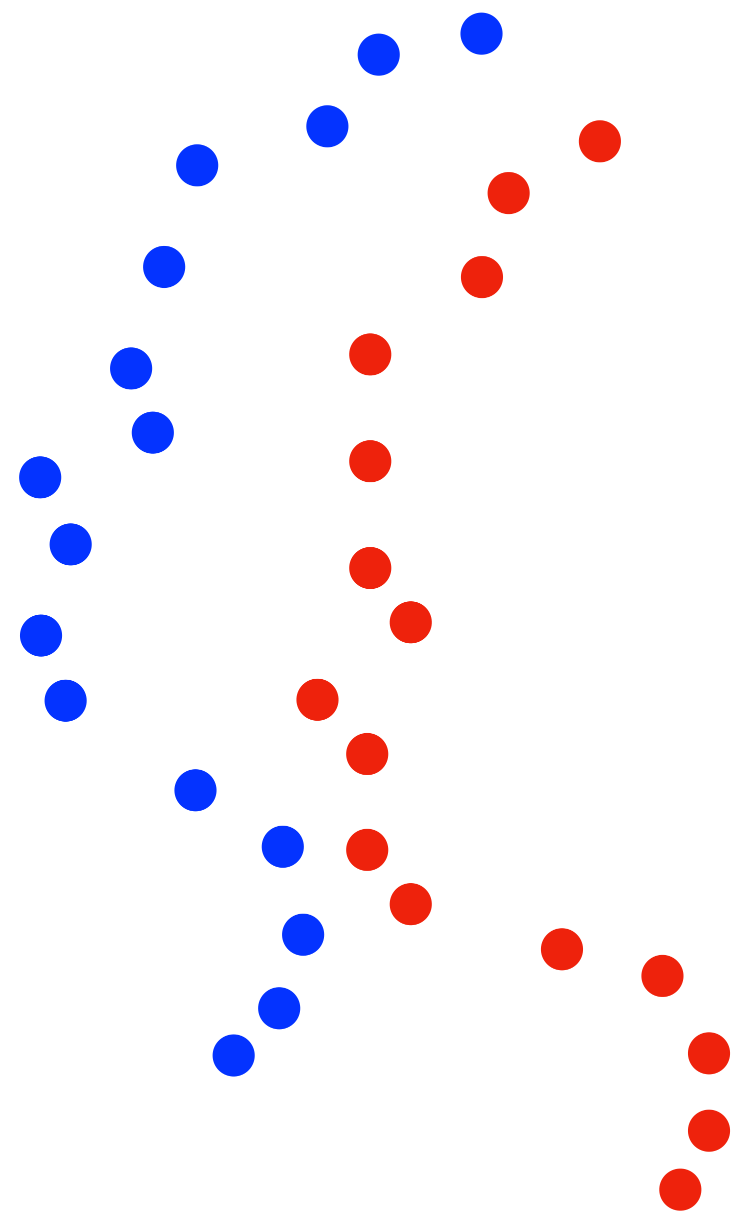
4. Repeat until convergence



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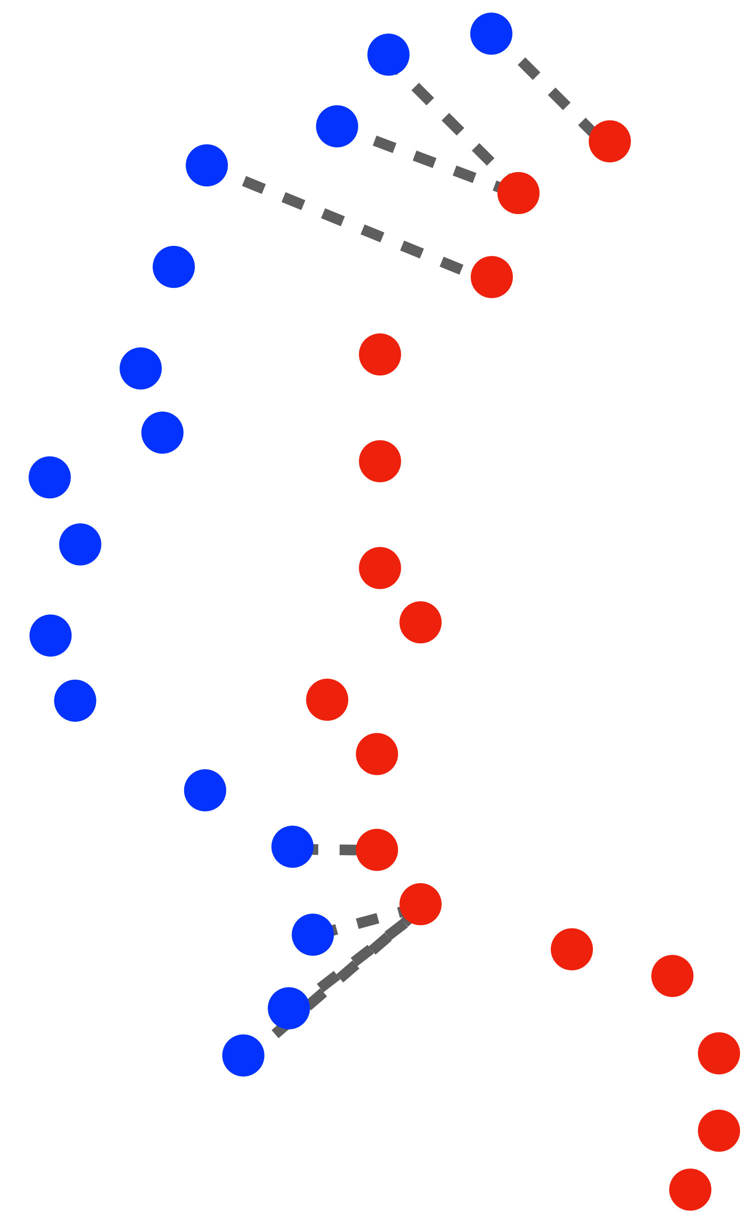

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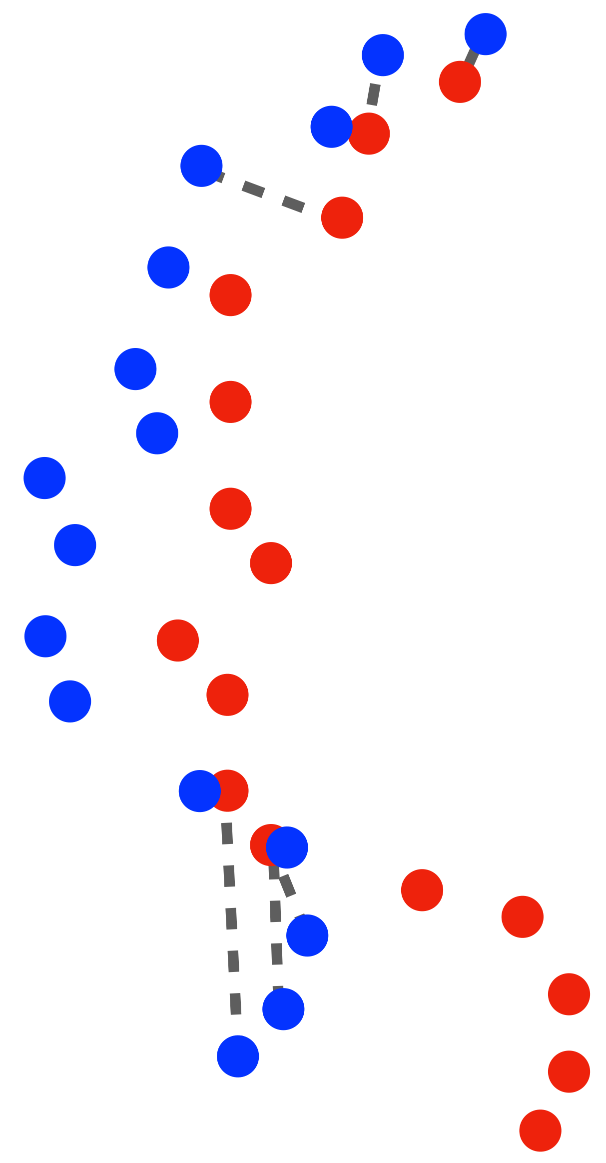

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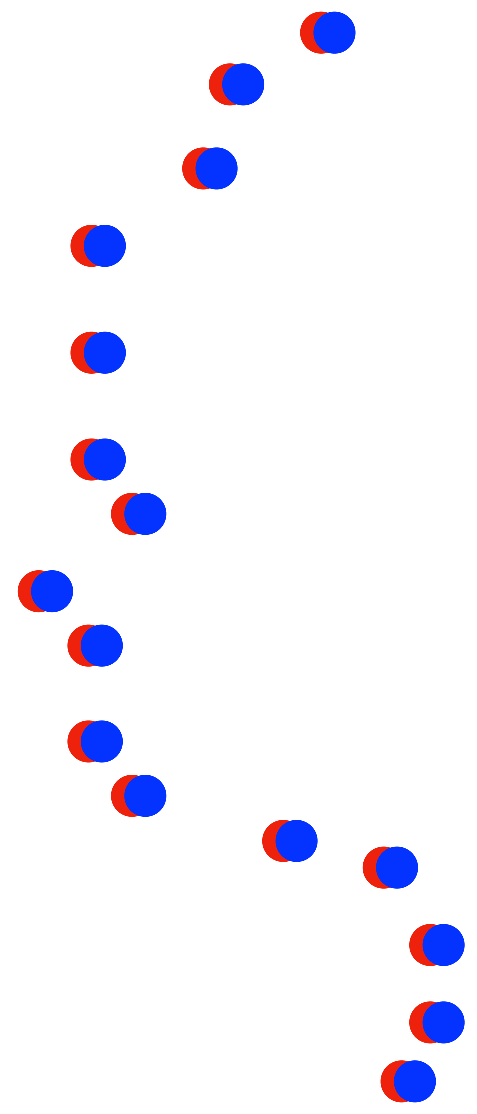

ICP | Solving for the Optimal Transformation

How do we solve for the optimal $\left\{ \hat{\mathbf{C}}_{s's}, \hat{\mathbf{r}}_s^{s's} \right\}$ at each step?

Use least-squares!

$$\left\{ \hat{\mathbf{C}}_{s's}, \hat{\mathbf{r}}_s^{s's} \right\} = \operatorname{argmin}_{\{\mathbf{C}_{s's}, \mathbf{r}_s^{s's}\}} \mathscr{L}_{\mathrm{LS}} \left(\mathbf{C}_{s's}, \mathbf{r}_s^{s's} \right)$$

$$\mathscr{L}_{\mathrm{LS}} \left(\mathbf{C}_{s's}, \mathbf{r}_s^{s's} \right) = \sum_{j=1}^{n} \left\| \mathbf{C}_{s's} \left(\mathbf{p}_s^{(j)} - \mathbf{r}_s^{s's} \right) - \mathbf{p}_{s'}^{(j)} \right\|_2^2$$

Careful: Rotations need special treatment because they don't behave like vectors!

ICP | Solving for the Optimal Transformation

1. Compute the *centroids* of each point cloud

$$\boldsymbol{\mu}_s = \frac{1}{n}\sum_{j=1}^{n} \mathbf{p}_s^{(j)} \qquad\qquad \boldsymbol{\mu}_{s'} = \frac{1}{n}\sum_{j=1}^{n} \mathbf{p}_{s'}^{(j)}$$

2. Compute a matrix capturing the *spread* of the two point clouds

$$\mathbf{W}_{s's} = \frac{1}{n}\sum_{j=1}^{n} \left(\mathbf{p}_s^{(j)} - \boldsymbol{\mu}_s\right)\left(\mathbf{p}_{s'}^{(j)} - \boldsymbol{\mu}_{s'}\right)^T$$

ICP | Solving for the Optimal Transformation

3. Use the *singular value decomposition* of the matrix to get the optimal rotation

singular values (scaling)

diagonal matrix

$$\mathbf{U}\mathbf{S}\mathbf{V}^T = \mathbf{W}_{s's}$$

$\mathbf{U}^T\mathbf{U} = \mathbf{1}$ (rotation)

$\mathbf{V}^T\mathbf{V} = \mathbf{1}$ (rotation)

Do not want scaling.

$$\hat{\mathbf{C}}_{s's} = \mathbf{U}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det \mathbf{U} \det \mathbf{V} \end{bmatrix}\mathbf{V}^T$$

This term ensures that we get a proper rotation without any reflection

ICP | Solving for the Optimal Transformation

3. Use the *singular value decomposition* of the matrix to get the optimal rotation

singular values
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This term ensures that we get a
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4. Use the optimal rotation to get the optimal translation by *aligning the centroids*

$$\hat{\mathbf{r}}_s^{s's} = \boldsymbol{\mu}_s - \hat{\mathbf{C}}_{s's}^T \boldsymbol{\mu}_{s'}$$

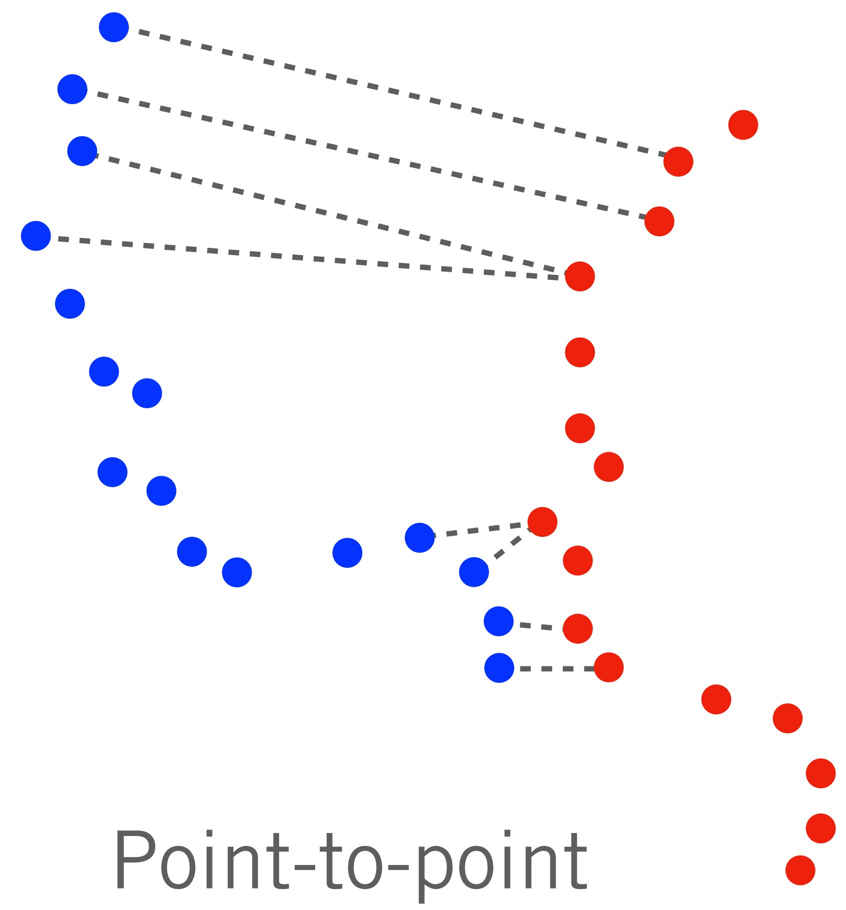
ICP | Estimating Uncertainty

We can obtain an estimate of the covariance matrix of the ICP solution using this formula:

$$\underbrace{\mathrm{cov}(\hat{\mathbf{x}})}_{\text{covariance of estimated motion parameters}} \simeq \left[\left(\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^2} \right)^{-1} \frac{\partial^2 \mathcal{L}}{\partial \mathbf{z} \partial \mathbf{x}} \underbrace{\mathrm{cov}(\mathbf{z})}_{\text{covariance of all measurements (both point clouds)}} \frac{\partial^2 \mathcal{L}}{\partial \mathbf{z} \partial \mathbf{x}}^T \left(\frac{\partial^2 \mathcal{L}}{\partial \mathbf{x}^2} \right)^{-1} \right]_{\mathbf{x}=\hat{\mathbf{x}}}$$

ICP | Variants

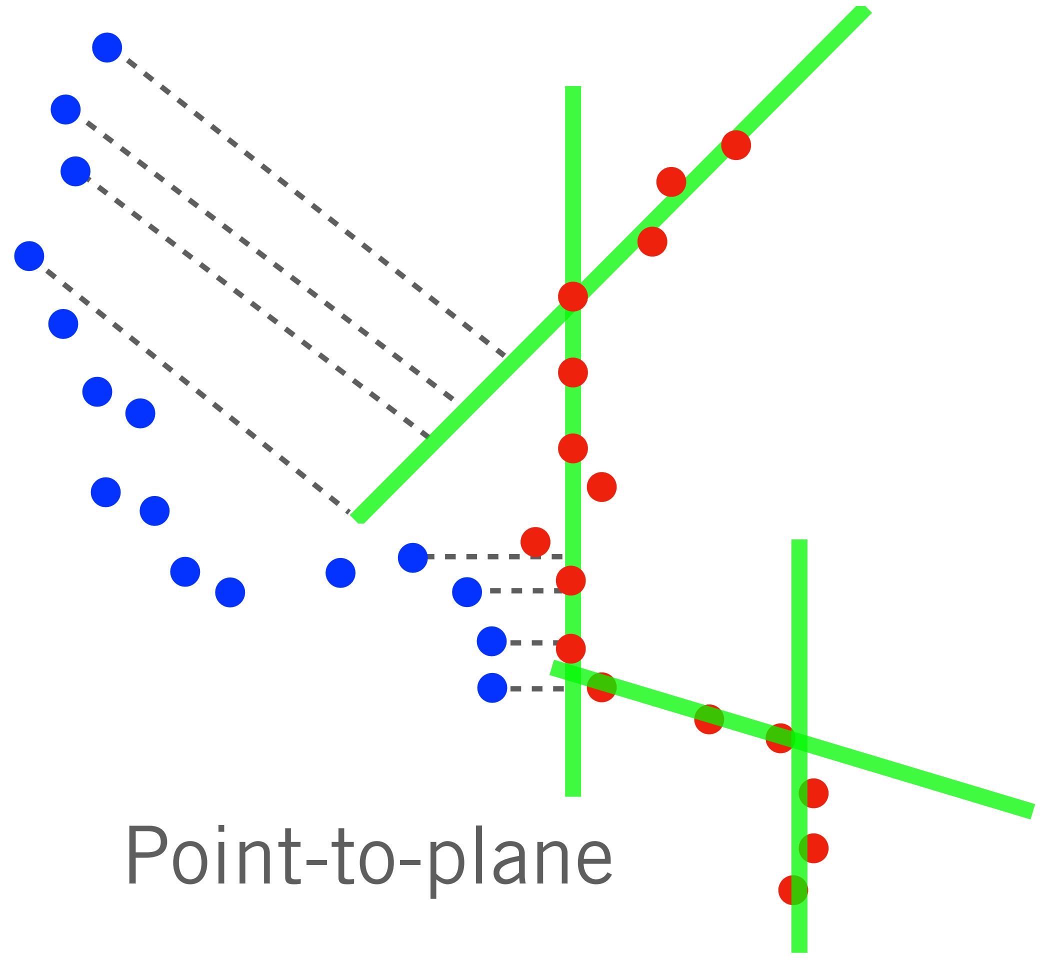
Point-to-point ICP minimizes the *Euclidean distance* between each point in $\mathbf{P}_{s'}$ and the *nearest point* in $\mathbf{P}_s$



Point-to-point

Point-to-plane ICP minimizes the *perpendicular distance* between each point in $\mathbf{P}_{s'}$ and the *nearest plane* in $\mathbf{P}_s$

- This tends to work well in structured environments like cities



Point-to-plane

Outliers | Objects in Motion

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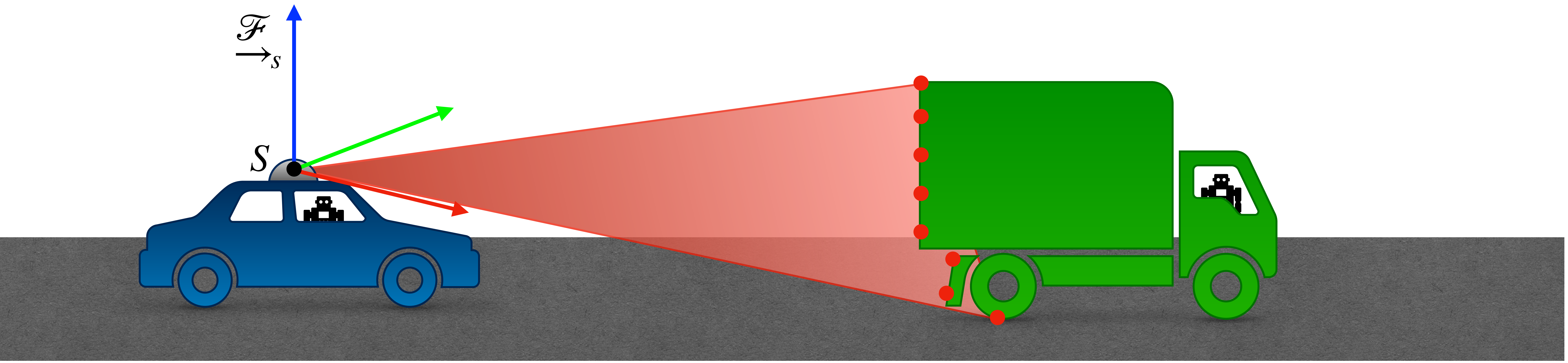
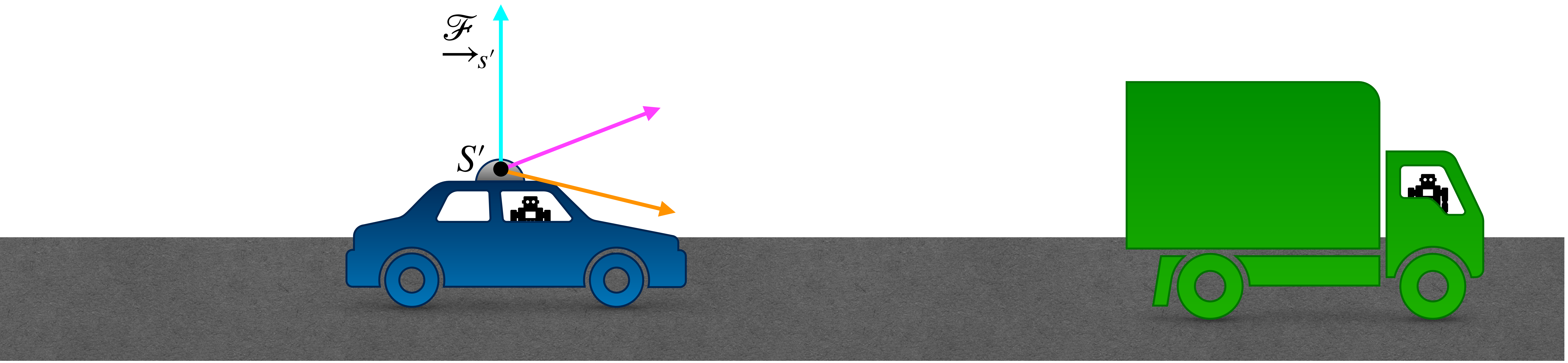


Outliers | Objects in Motion



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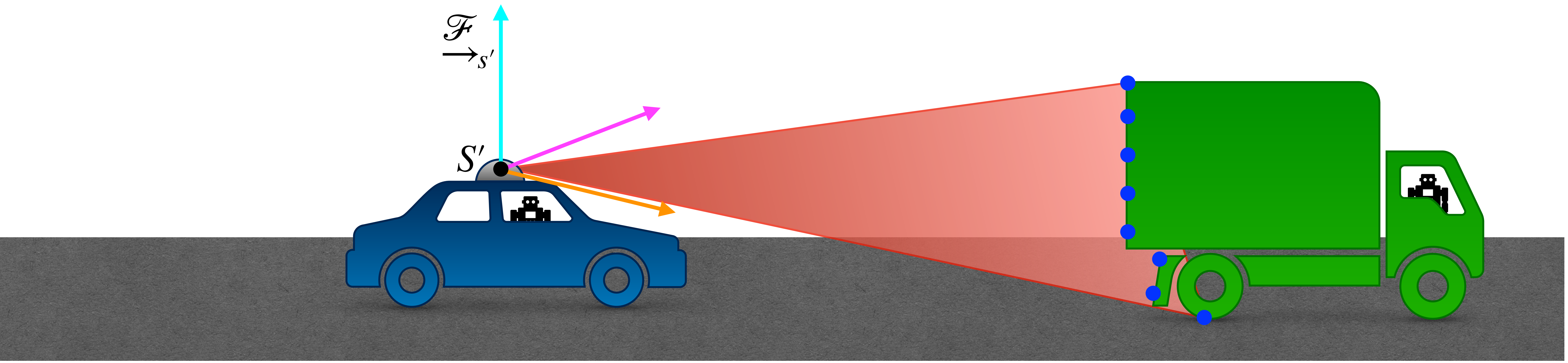

Outliers | Objects in Motion

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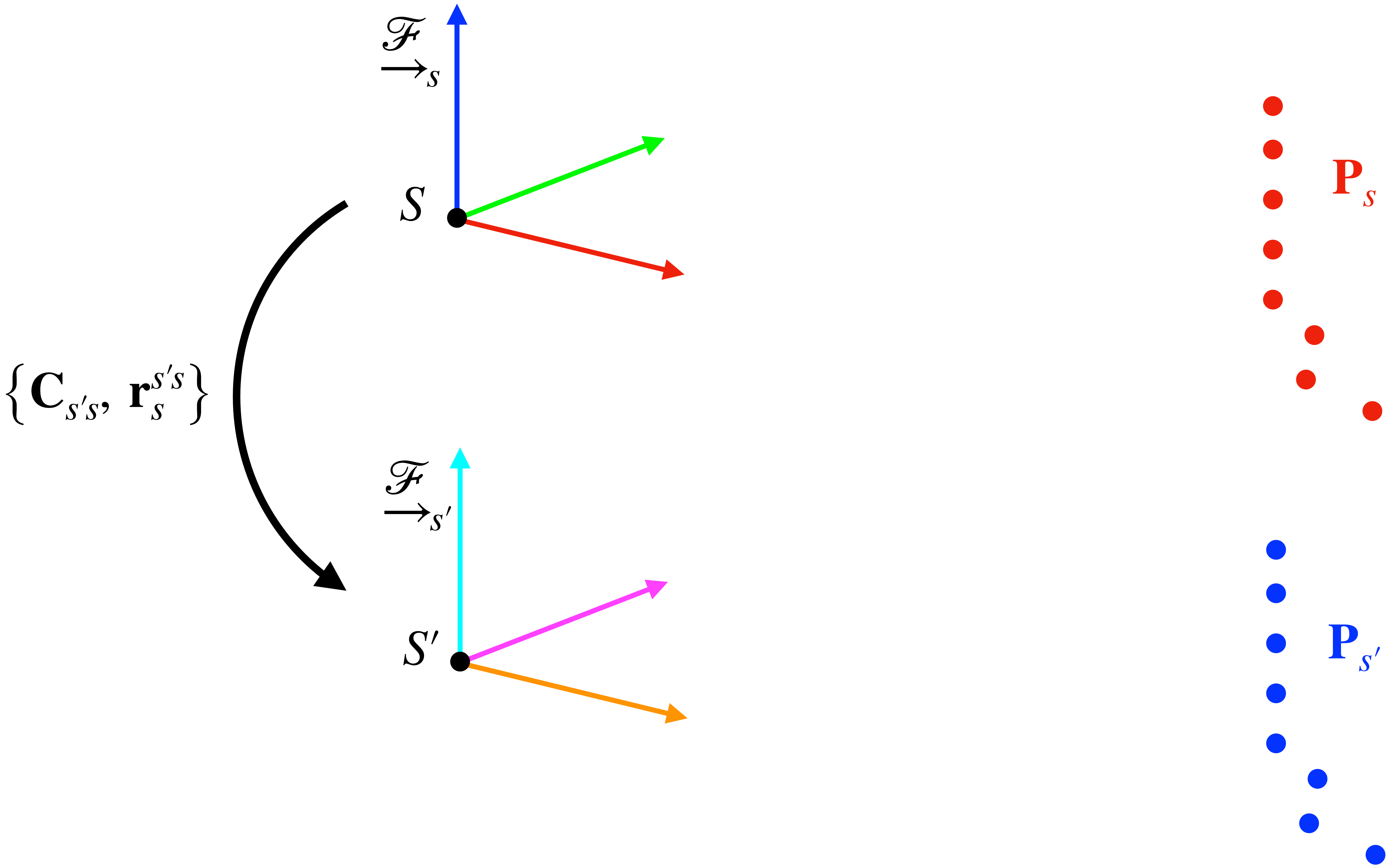

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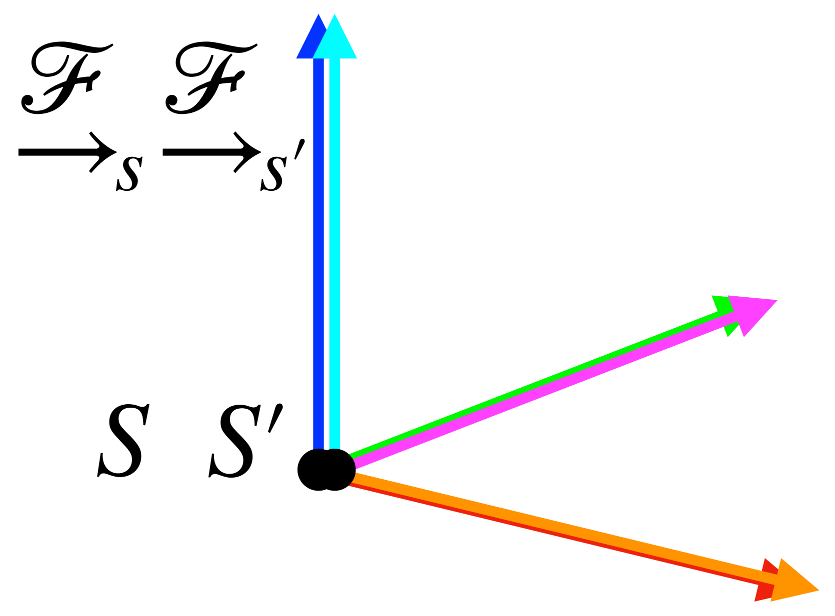
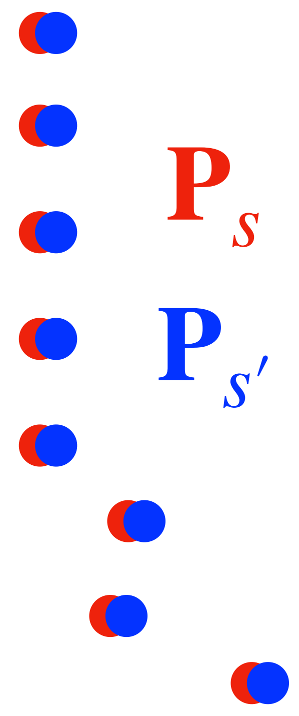




ICP alone is not always enough!

Outliers | Robust Loss Functions

Error term

$$\mathbf{e}^{(j)} = \mathbf{C}_{s's}\left(\mathbf{p}_s^{(j)} - \mathbf{r}_s^{s's}\right) - \mathbf{p}_{s'}^{(j)}$$

Least-squares

$$\mathscr{L} = \sum_{j=1}^{n} \mathbf{e}^{(j)\,T}\mathbf{e}^{(j)}$$

Outliers | Robust Loss Functions

Large errors induced by

Less sensitive to outliers

No closed-form solution

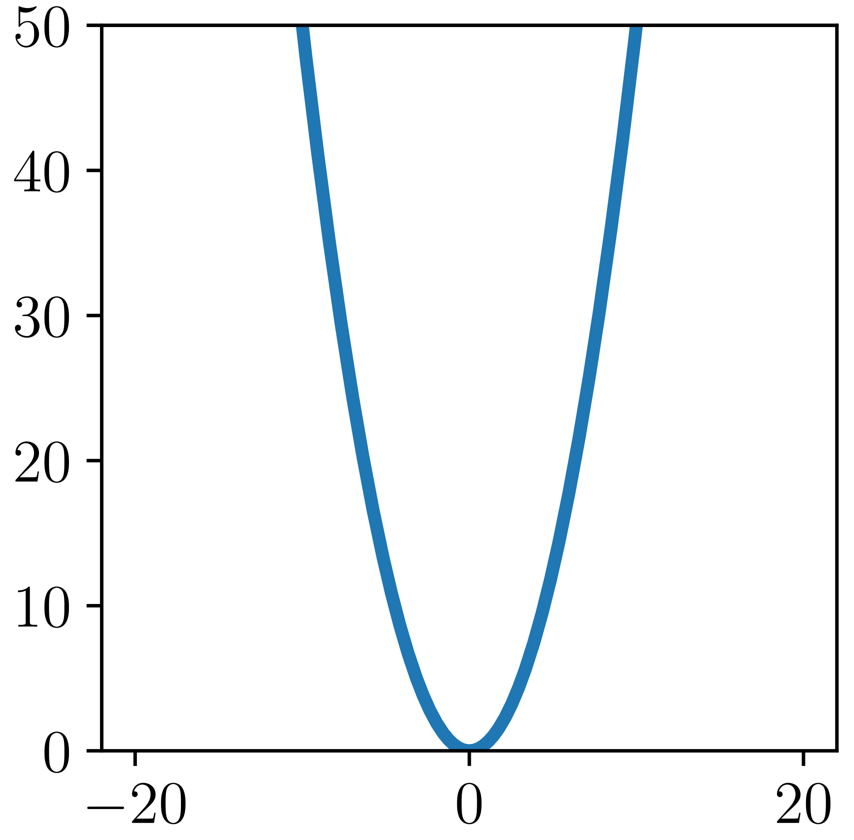
Error term

$$\mathbf{e}^{(j)} = \mathbf{C}_{s's}\left(\mathbf{p}_s^{(j)} - \mathbf{r}_s^{s's}\right) - \mathbf{p}_{s'}^{(j)}$$

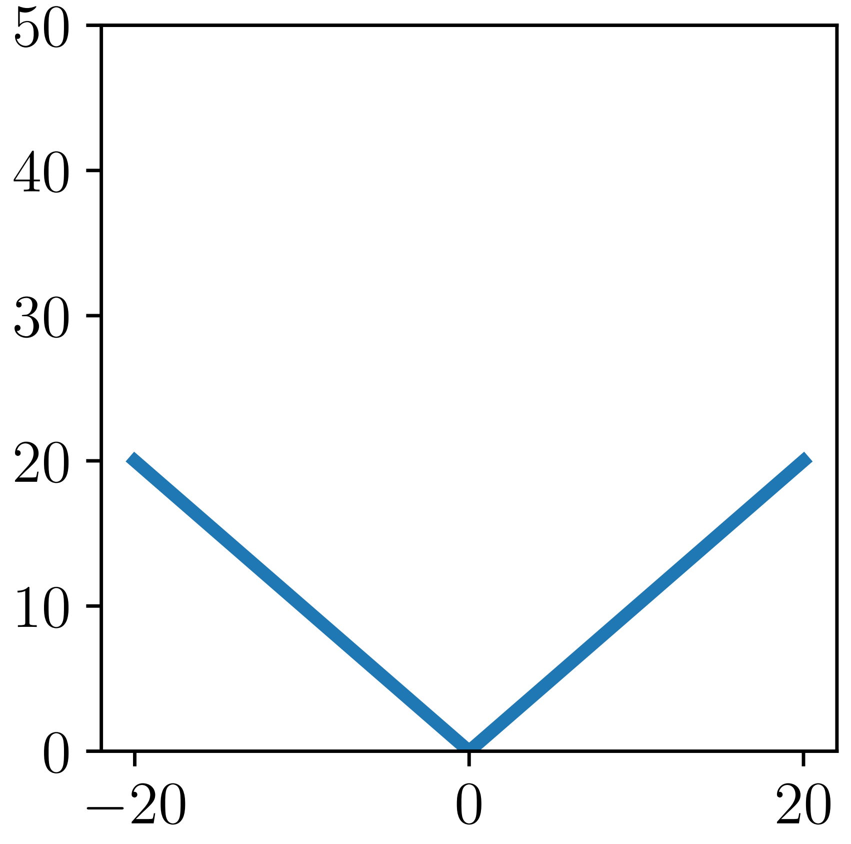
Robust loss

$$\mathscr{L} = \sum_{j=1}^{n} \rho\left(\mathbf{e}^{(j)}\right)$$

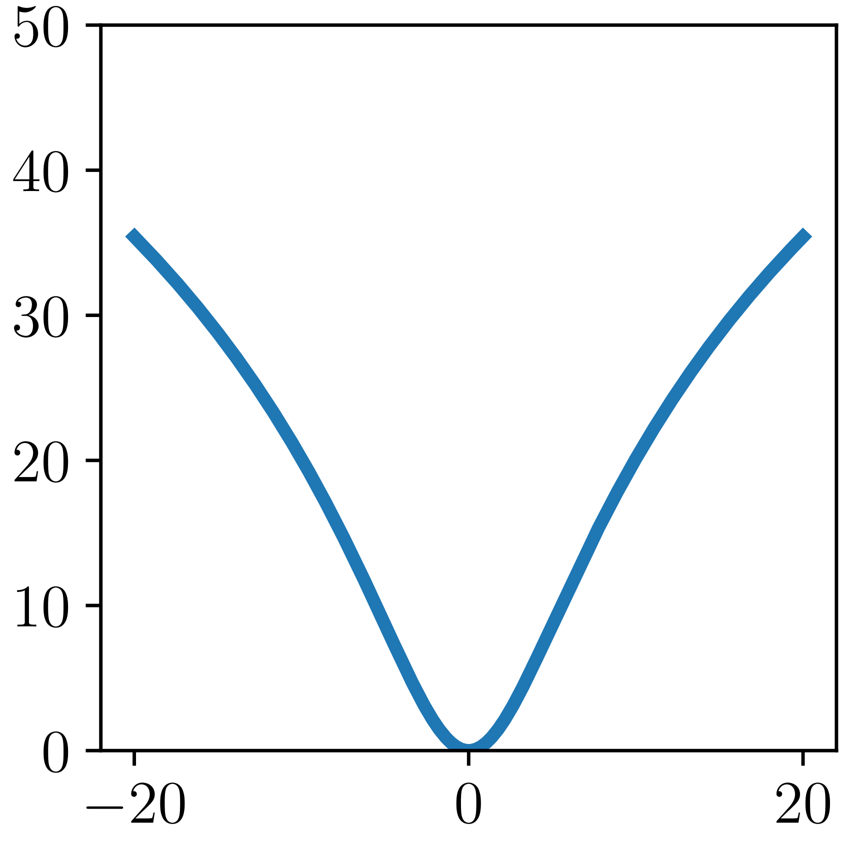
Squared error (L_2)



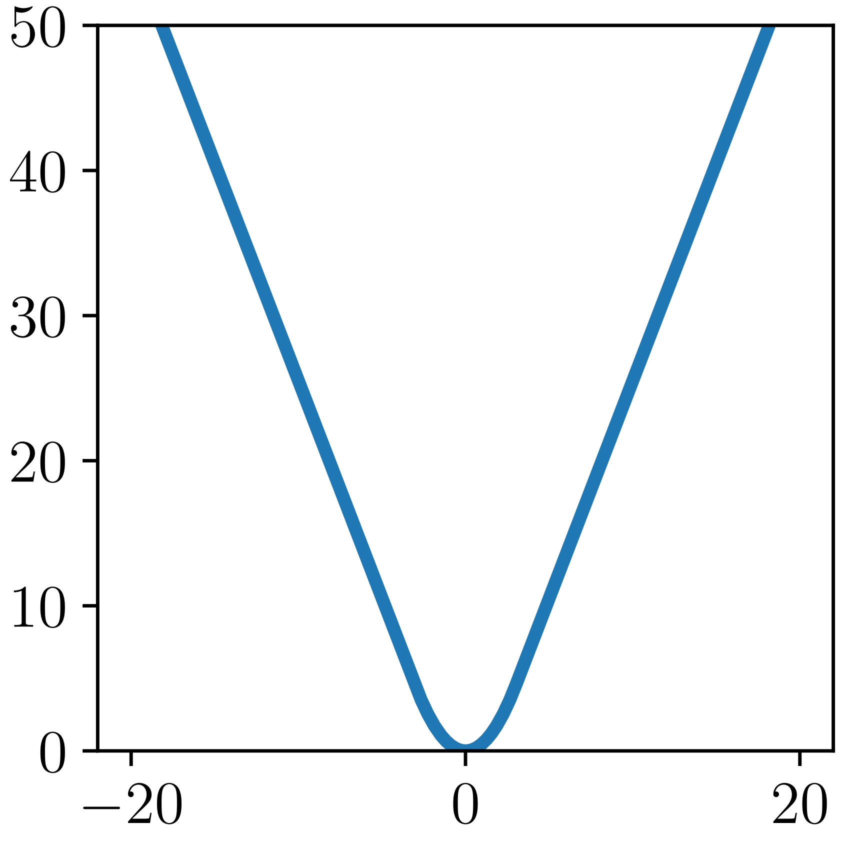
Absolute error (L_1)



Cauchy loss



Huber loss



$$\rho(\mathbf{e}) = \frac{1}{2}\|\mathbf{e}\|_2^2 = \frac{1}{2}\mathbf{e}^T\mathbf{e}$$

$$\rho(\mathbf{e}) = \|\mathbf{e}\|_1 = \sum_i |e_i|$$

$$\rho(\mathbf{e}) = \frac{k^2}{2}\log\left(1 + \frac{1}{k^2}\|\mathbf{e}\|_2^2\right)$$

$$\rho(e) = \begin{cases} \frac{1}{2}\|\mathbf{e}\|_2^2 \\ k\left(\|\mathbf{e}\|_1 - \frac{k}{2}\right) \end{cases}$$

Summary | Pose Estimation from LIDAR Data

- ICP is a way to determine the motion of a self-driving car by aligning point clouds from LIDAR or other sensors

- ICP *iteratively* minimizes the distance between points in each point cloud

- ICP is sensitive to outliers caused by moving objects, which can be partly mitigated using robust loss functions

Summary | LIDAR Sensing

- LIDAR measures distances using laser light and the *time-of-flight* equation

- LIDAR scans are stored as *points clouds* that can be manipulated using common spatial operations (e.g., translation, rotation, scaling)

- The Iterative Closest Point (ICP) algorithm is one way of using LIDAR to localize a self-driving car