

Creating a Road Network Graph

Course 4, Module 3, Lesson 1



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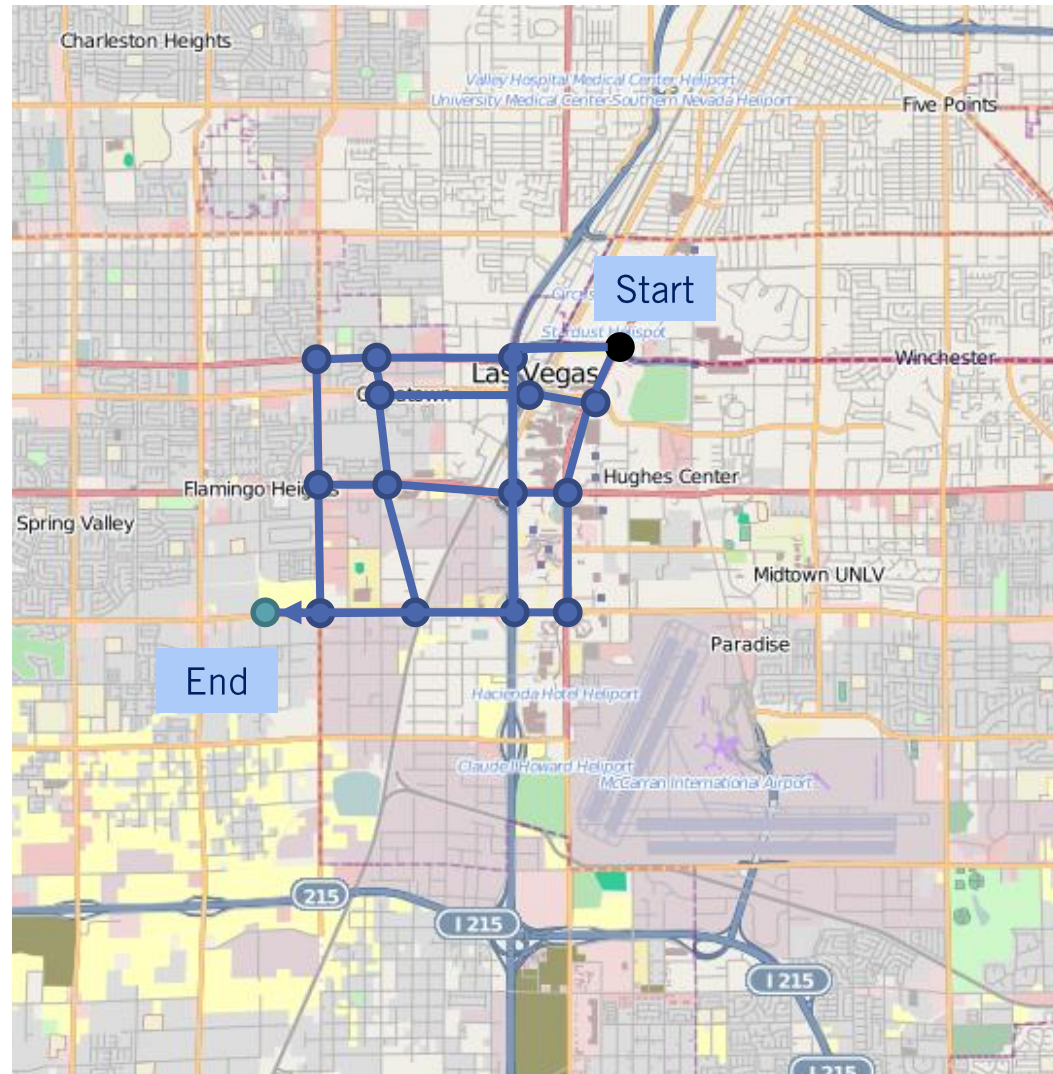
Mission Planning

Highest level planning
problem.

- Order of km.

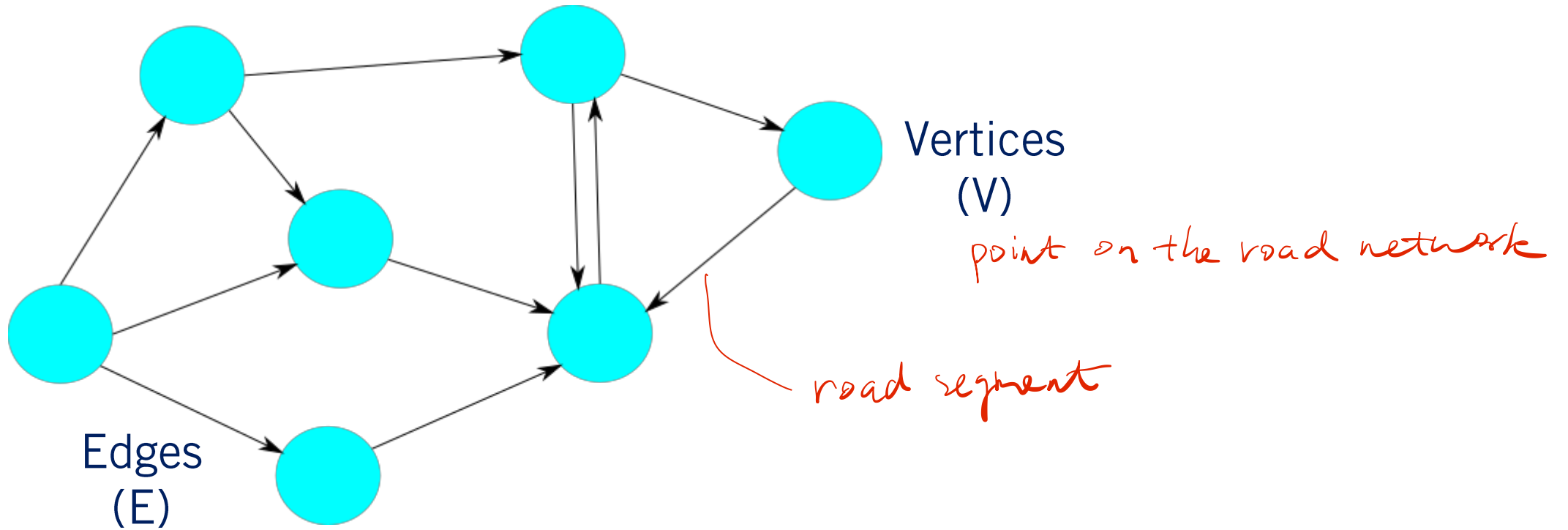
- Focus on
speed limit,
road length,
traffic flow rate,
& road closures

abstracting away lower-level details. like
rules of the road and
other agents present



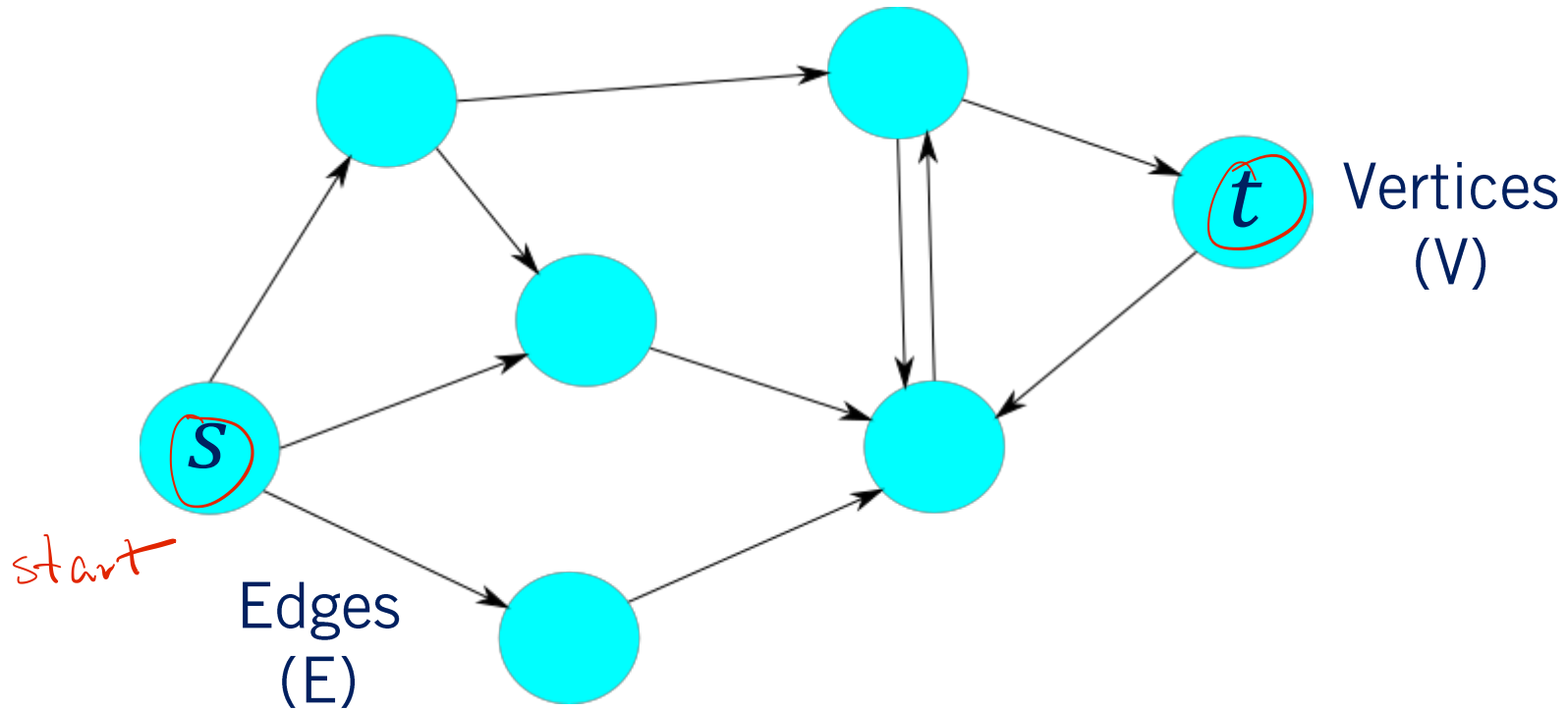
Graphs

Graph: $G = (V, E)$



Graphs

Graph: $G = (V, E)$



Breadth First Search (BFS)

Algorithm BFS(G, s, t)

```
1.  open  $\leftarrow$  Queue() First in First out
2.  closed  $\leftarrow$  Set()
3.  predecessors  $\leftarrow$  Dict()
4.  open.enqueue( $s$ )
5.  while !open.isEmpty() do
6.     $u \leftarrow$  open.dequeue()
7.    if isGoal( $u$ ) then
8.      return extractPath( $u$ , predecessors)
9.    for all  $v \in u$ .successors()
10.     if  $v \in$  closed or  $v \in$  open then
11.       continue
12.     open.enqueue( $v$ )
13.     predecessors[ $v$ ]  $\leftarrow u$ 
14.  closed.add( $u$ )
```

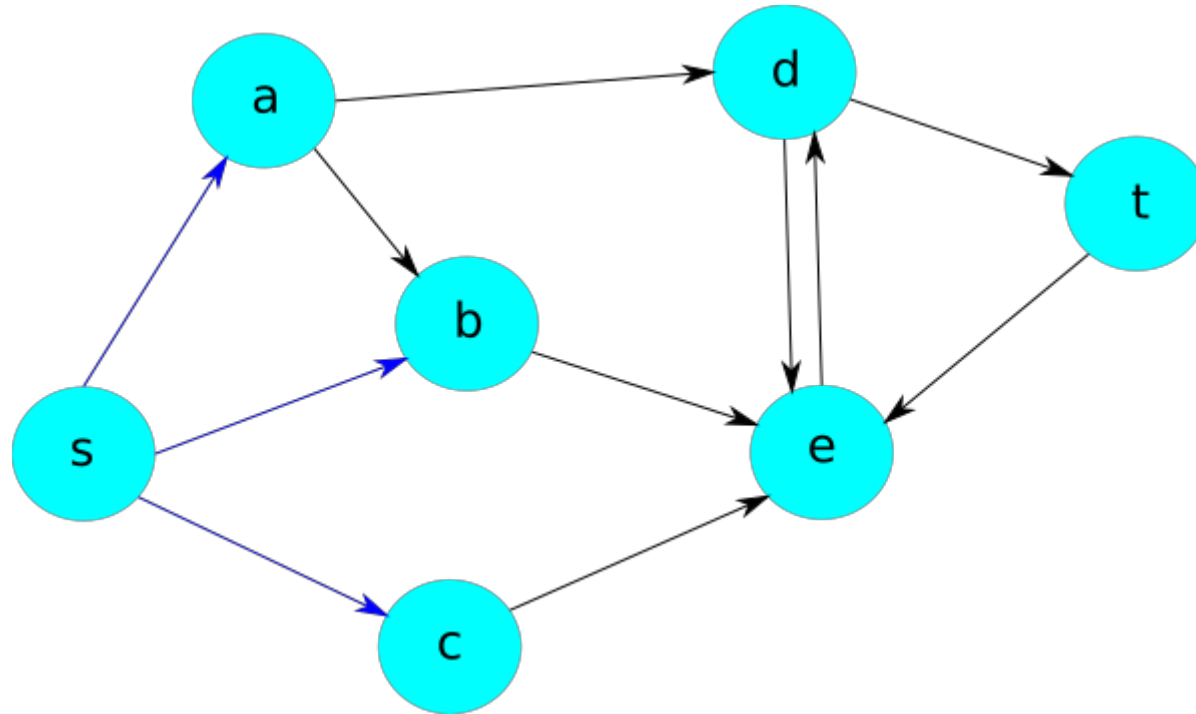
Example - First Wavefront

Open Queue:

a
b
c

Closed Set: s

Predecessors: s



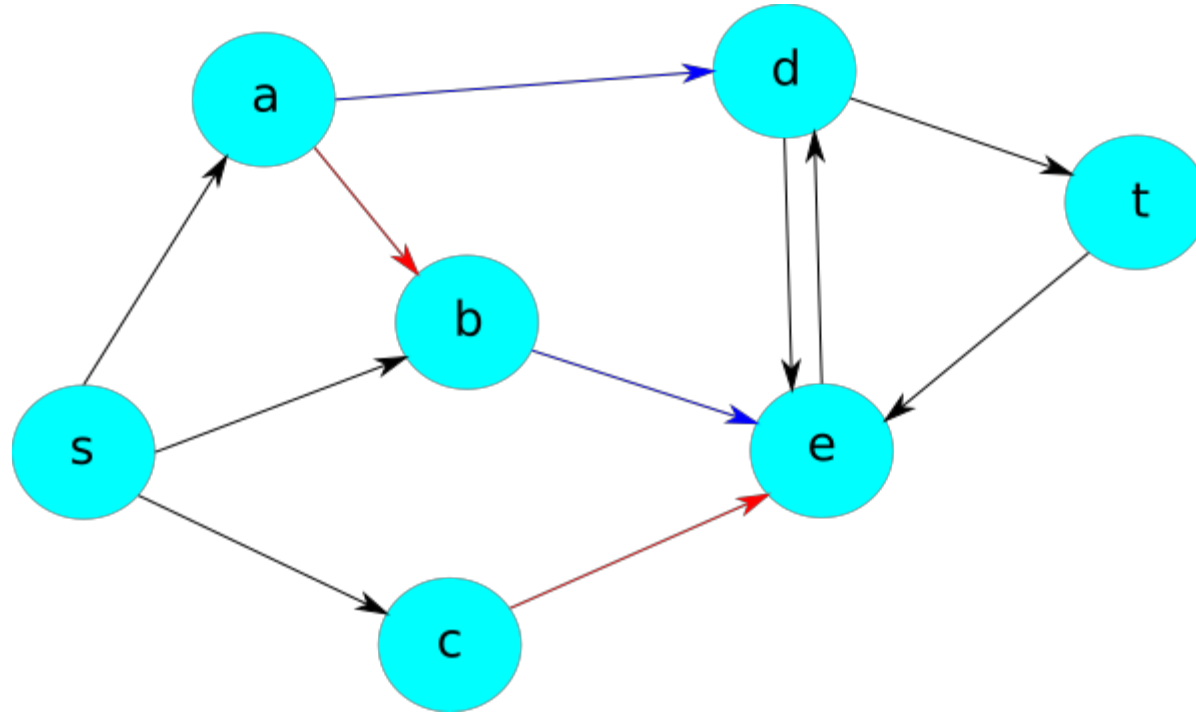
Example - Second Wavefront

Open Queue:

d
e

Closed Set: s

a
b
c



Example - Third Wavefront

Open Queue:

t

Closed Set: s

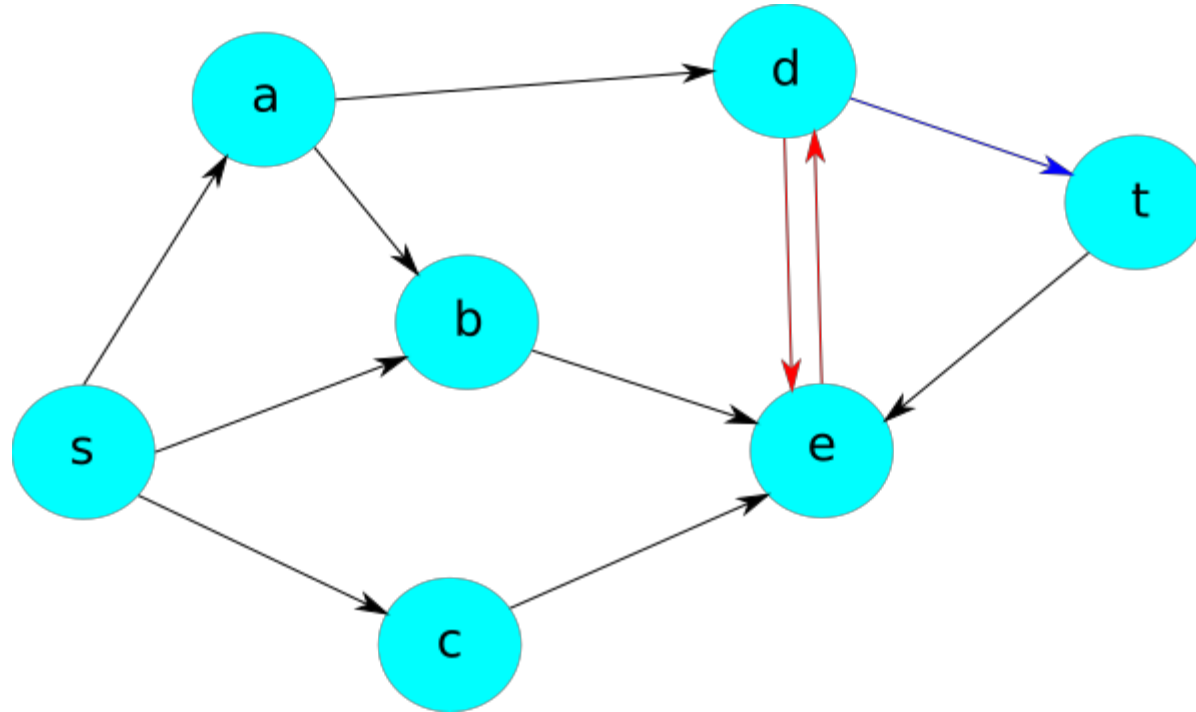
a

b

c

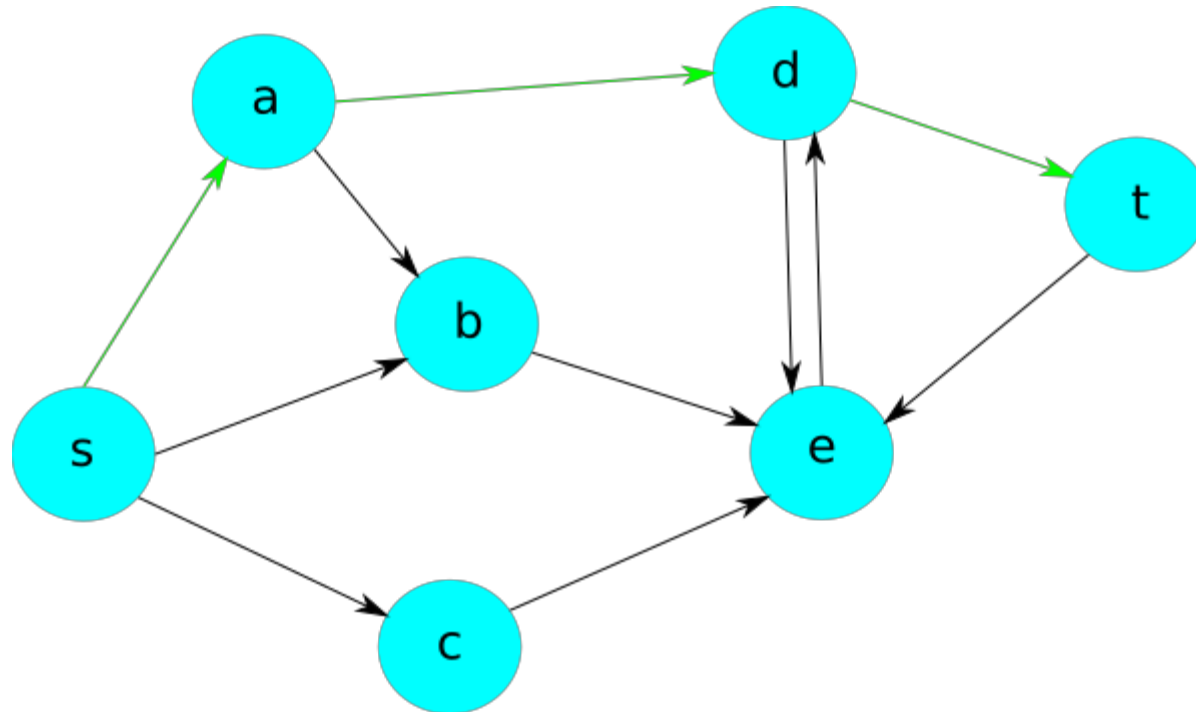
d

e



Example - Optimal Path

Final Path: s
a
d
t



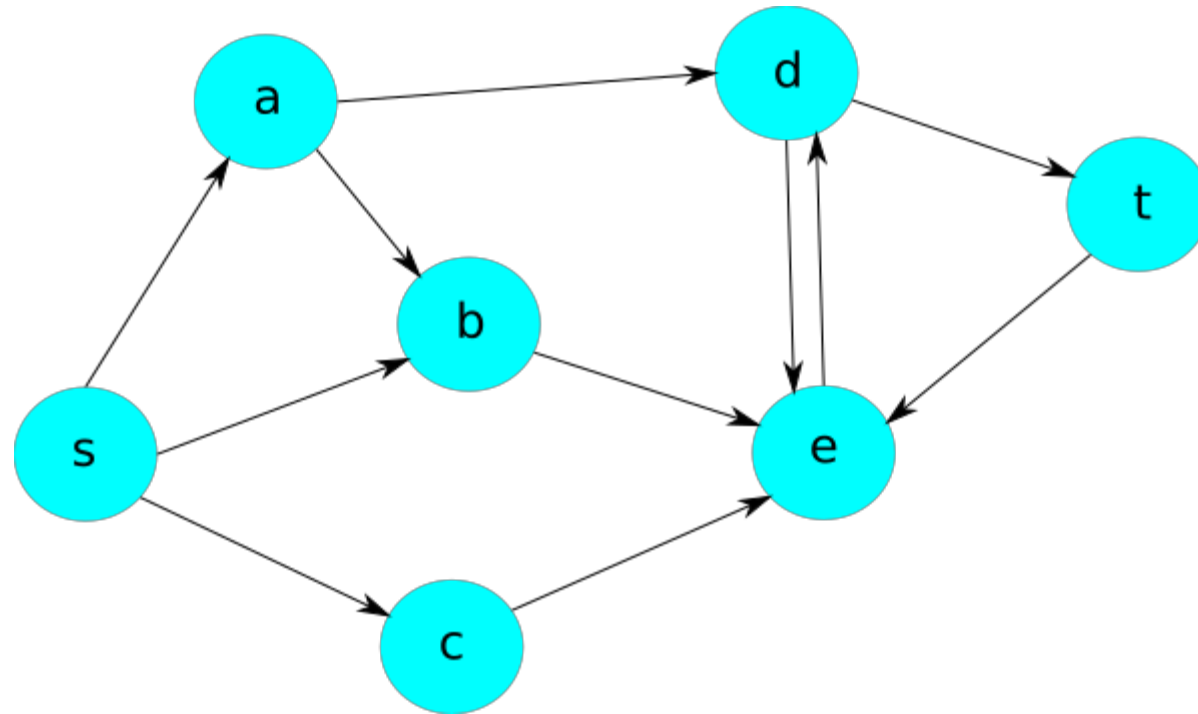
Summary

- Recognize the mission planning problem as a map-level navigation problem
- Learned how to embed a graph in the map
 - Vertices connected by road segments, which correspond to edges
- Learned how to use BFS to search an unweighted graph for the shortest path to the destination

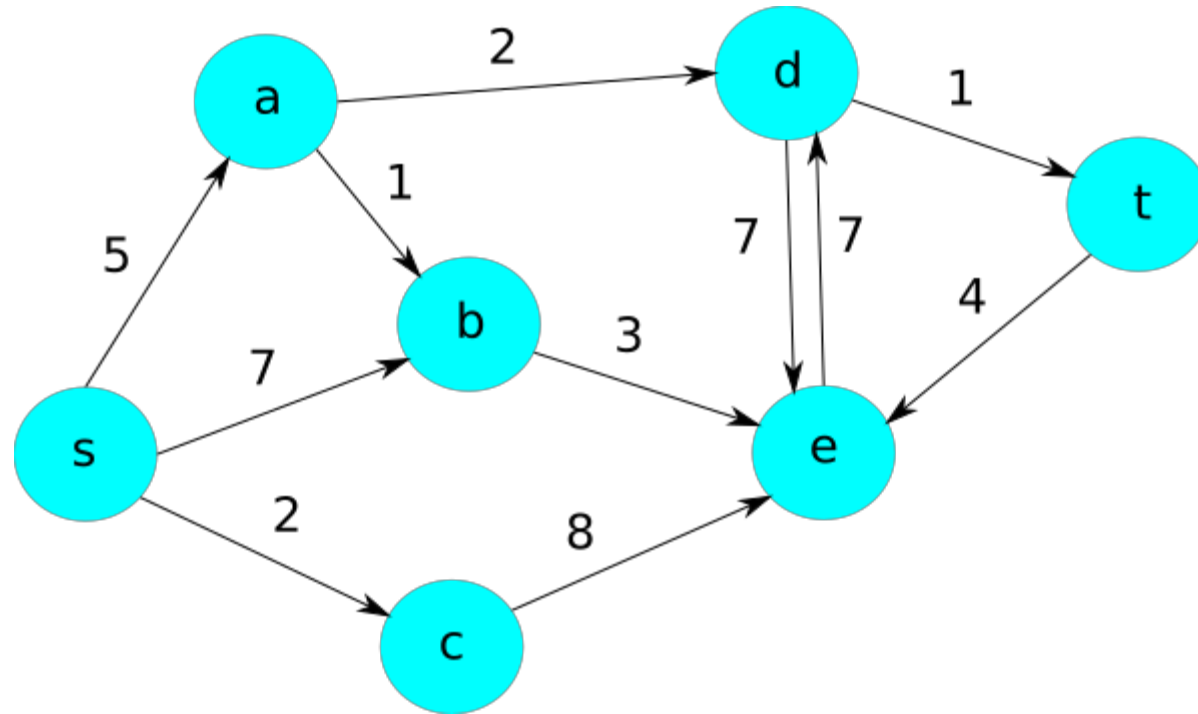
Dijkstra's Shortest Path Search

Course 4, Module 3, Lesson 2

Unweighted Graph



Weighted Graph



Dijkstra's Algorithm

Algorithm Dijkstra's(G, s, t)

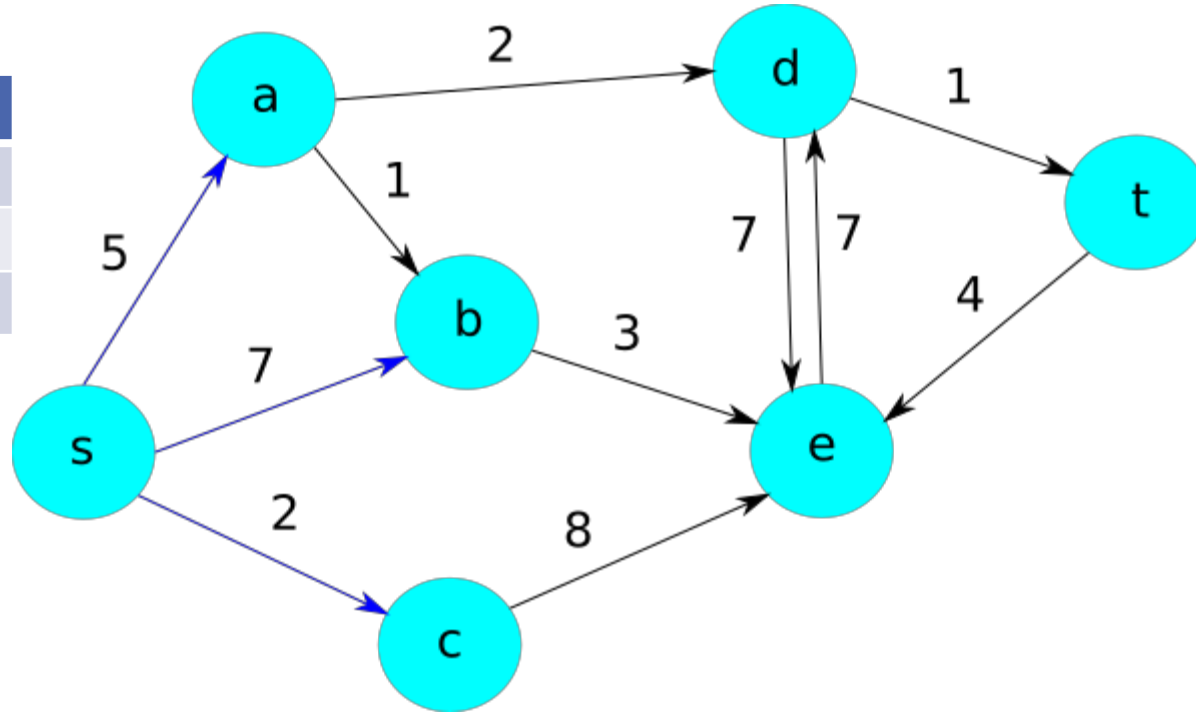
```
1.  open  $\leftarrow$  MinHeap()
2.  closed  $\leftarrow$  Set()
3.  predecessors  $\leftarrow$  Dict()
4.  open.push( $s, 0$ )
5.  while !open.isEmpty() do
6.     $u, uCost \leftarrow$  open.pop()
7.    if isGoal( $u$ ) then
8.      return extractPath( $u$ , predecessors)
9.    for all  $v \in u$ .successors()
10.     if  $v \in$  closed then
11.       continue
12.      $uvCost \leftarrow$  edgeCost( $G, u, v$ )
13.     if  $v \in$  open then
14.       if  $uCost + uvCost < open[v]$  then
15.          $open[v] \leftarrow uCost + uvCost$ 
16.          $predecessors[v] \leftarrow u$ 
17.     else
18.       open.push( $v, uCost + uvCost$ )
19.        $predecessors[v] \leftarrow u$ 
20.  closed.add( $u$ )
```

Example - Processing s

Open Min Heap:

Node	Cost to vertex
c	2
a	5
b	7

Closed Set: s

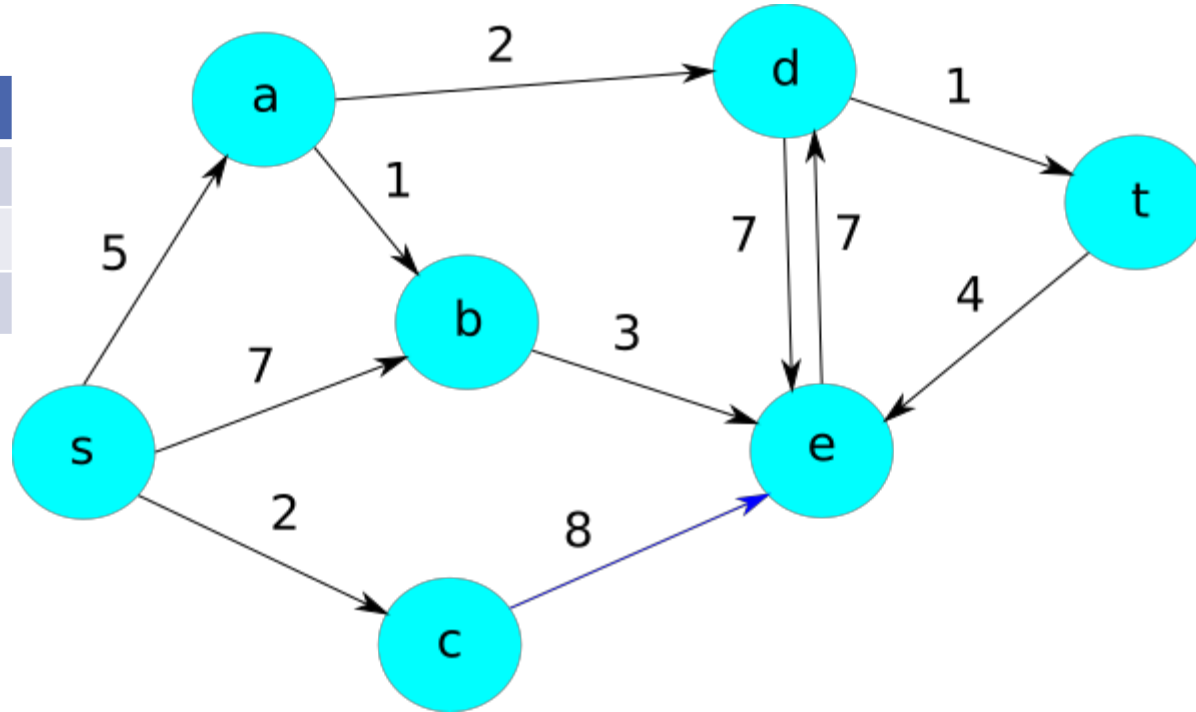


Example - Processing c

Open Min Heap:

Node	Cost to vertex
a	5
b	7
e	10

Closed Set: s
c

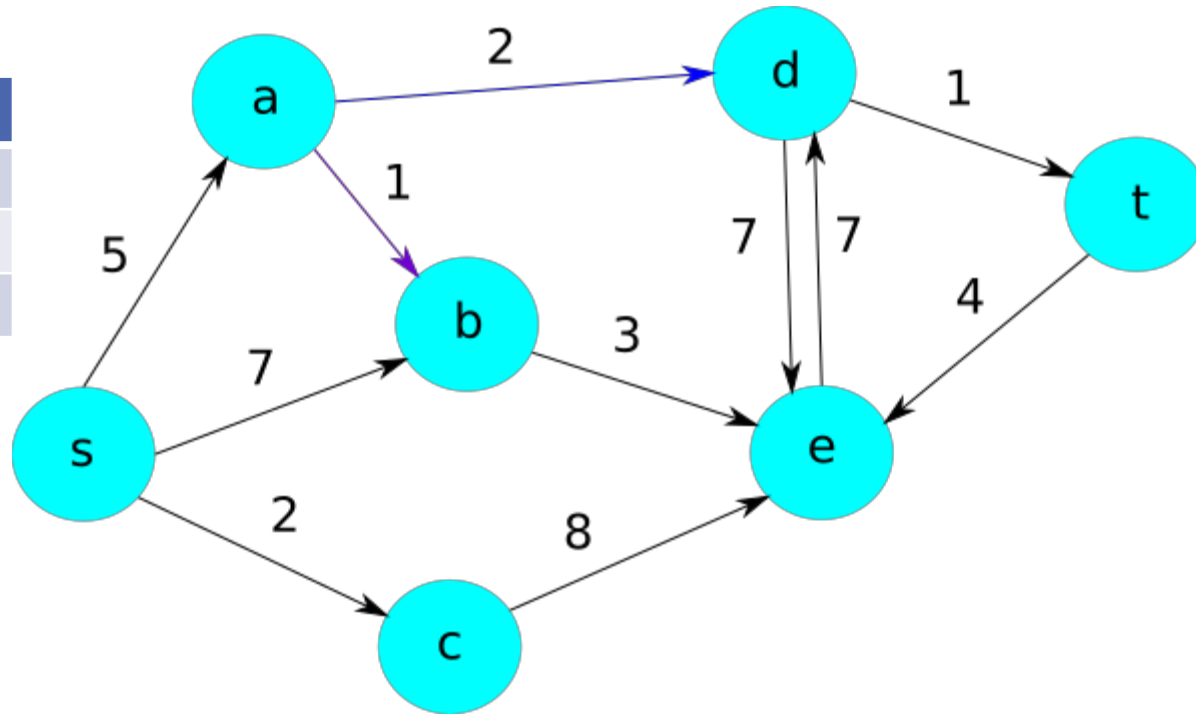


Example - Processing a

Open Min Heap:

Node	Cost to go
b	6
d	7
e	10

Closed Set: s
c
a

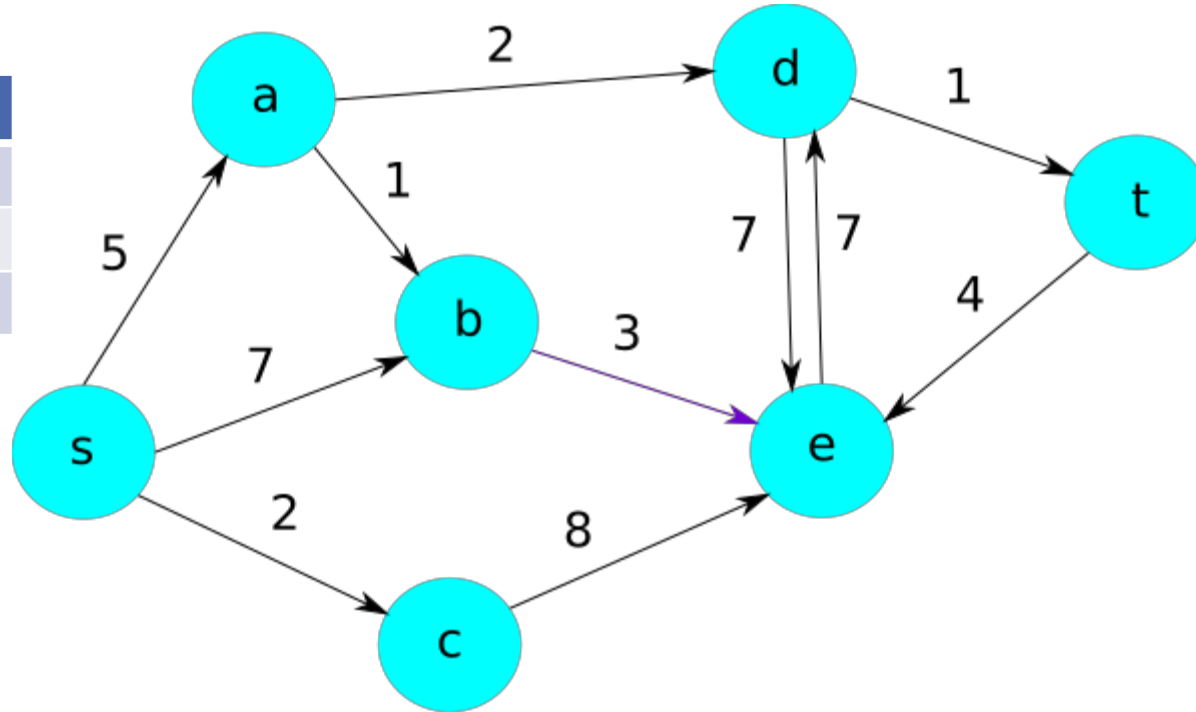


Example - Processing b

Open Min Heap:

Node	Cost to go
d	7
e	9

Closed Set: s
c
a
b

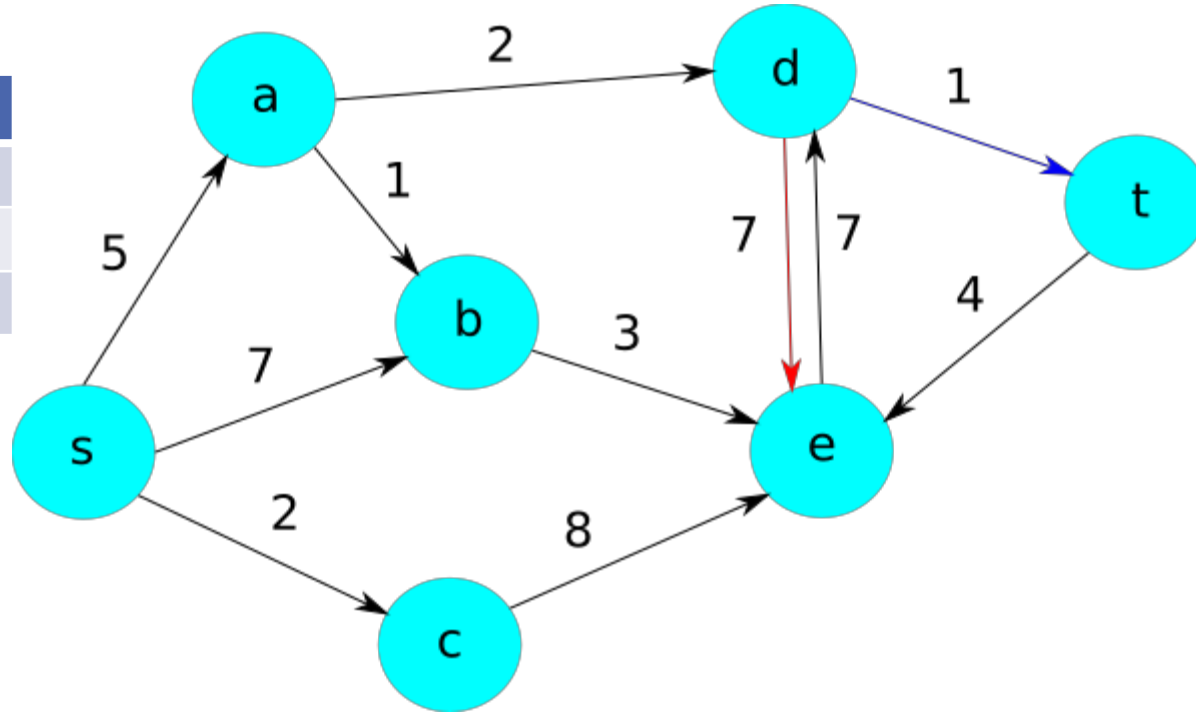


Example - Processing d

Open Min Heap:

Node	Cost to go
t	8
e	9

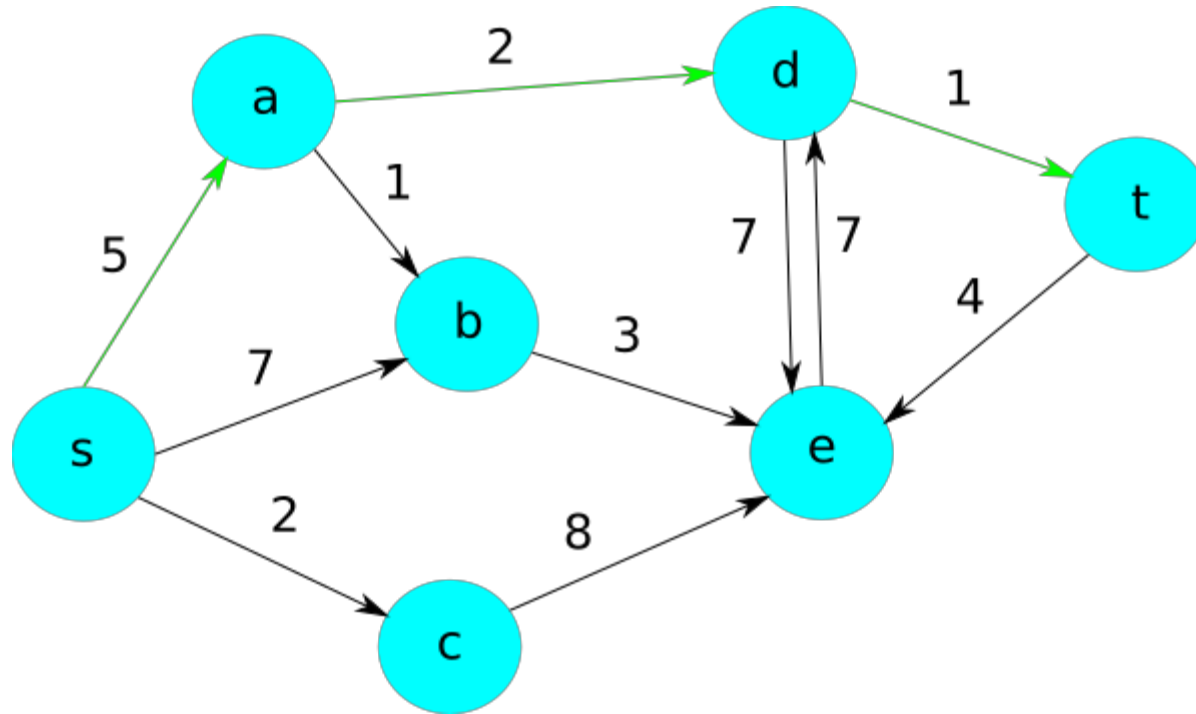
Closed Set: s
c
a
b
d



Example - Optimal Path

Final path: s
a
d
t

Closed Set: s
c
a
b
d



Search on a Map

- Example - map of Berkeley, California
 - 2,097 vertices
 - 5,740 edges
- Example - map of New York City, New York
 - 54,837 vertices
 - 140,497 edges

