

Motion Planning Constraints

Course 4, Module 1, Lesson 2



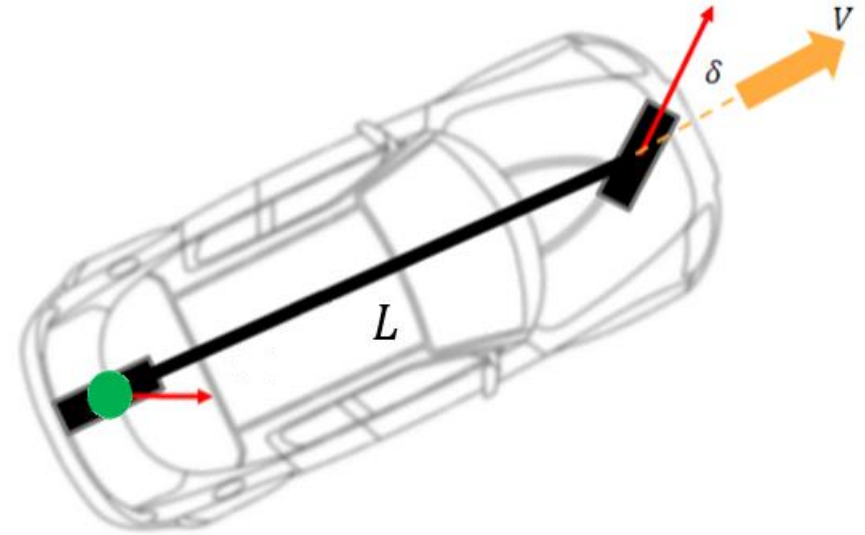
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Learning Objectives

- Understand how the vehicle's kinematics and dynamics constrain the motion planning problem
- Understand how static and dynamic obstacles affect motion planning
- Understand the impact of regulatory elements on motion planning

Bicycle Model

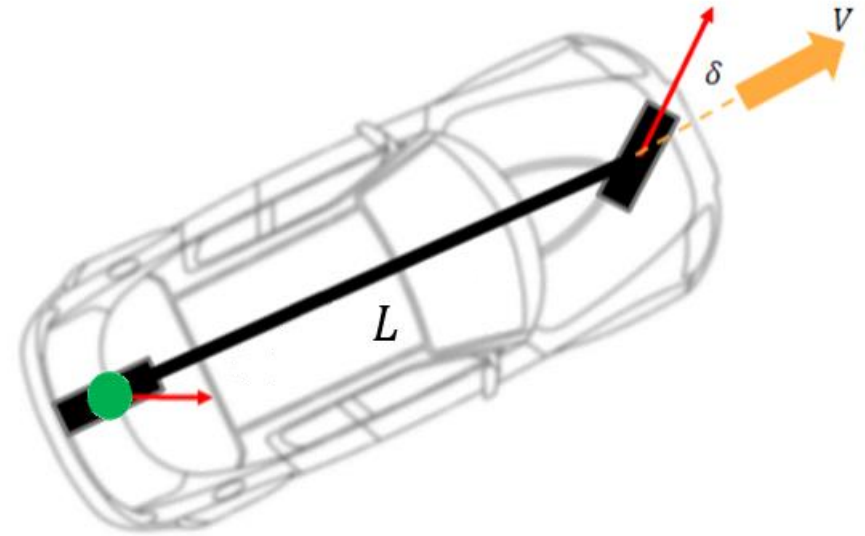
- Kinematics simplified to bicycle model (as discussed in Course 1)



Bicycle Model

- Kinematics simplified to bicycle model (as discussed in Course 1)
- Bicycle model imposes curvature constraint on our path planning process

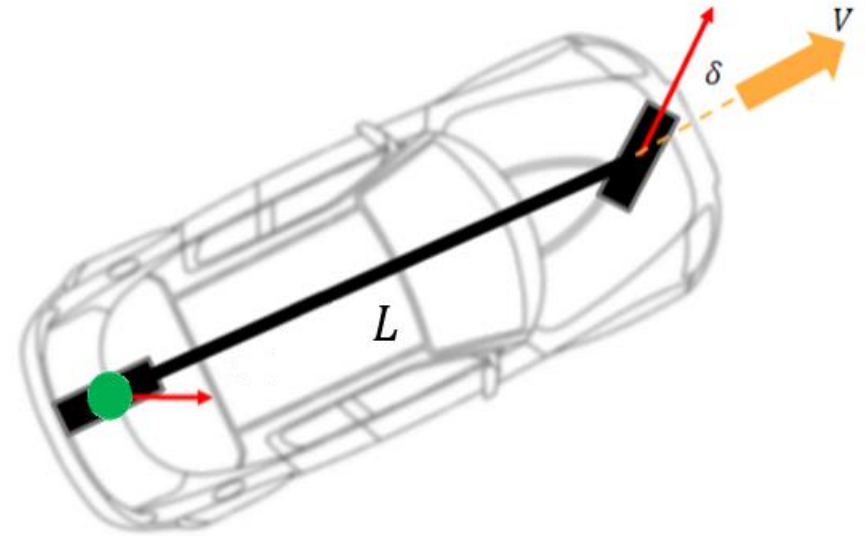
limited steering angles similar to a car



$$\dot{\theta} = \frac{V \tan(\delta)}{L}$$
$$|\kappa| \leq \kappa_{max}$$

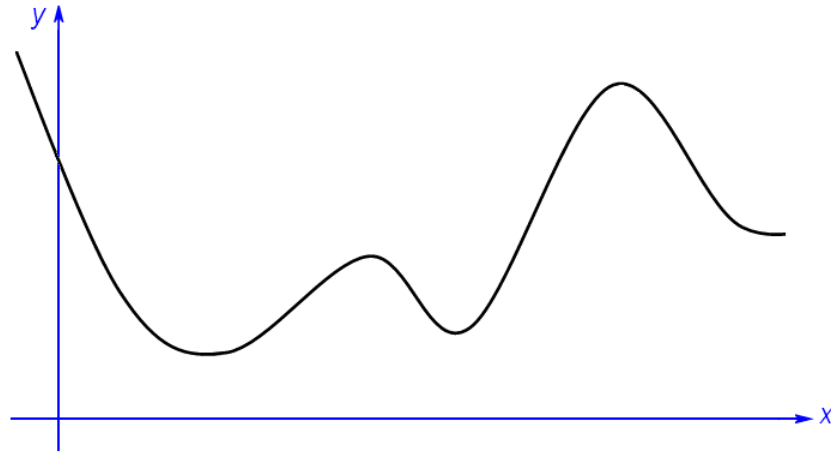
Bicycle Model

- Kinematics simplified to bicycle model (as discussed in Course 1)
- Bicycle model imposes curvature constraint on our path planning process
- Curvature constraint is non-holonomic
 - Non-holonomic constraints reduce the directions a mobile robot can travel at any point
 - Makes motion planning challenging

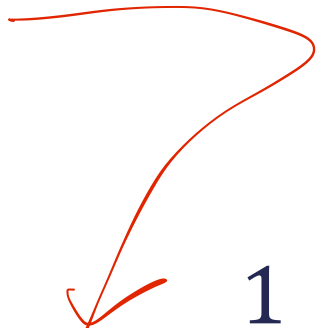


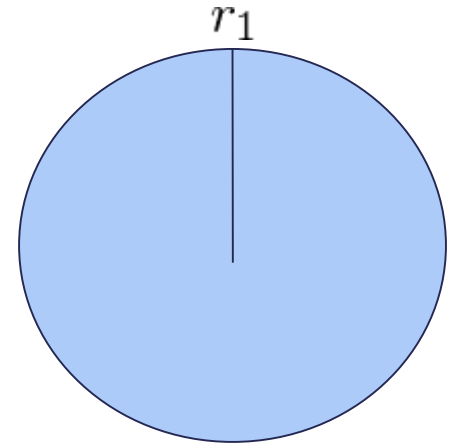
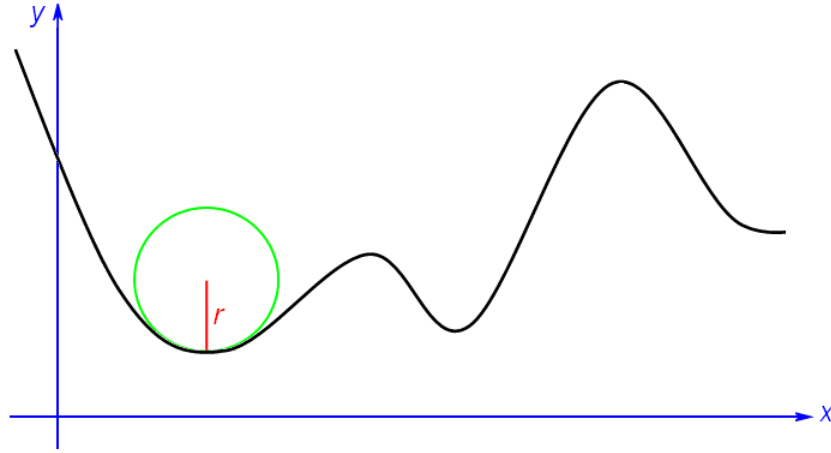
$$\dot{\theta} = \frac{V \tan(\delta)}{L}$$
$$|\kappa| \leq \kappa_{max}$$

Curvature



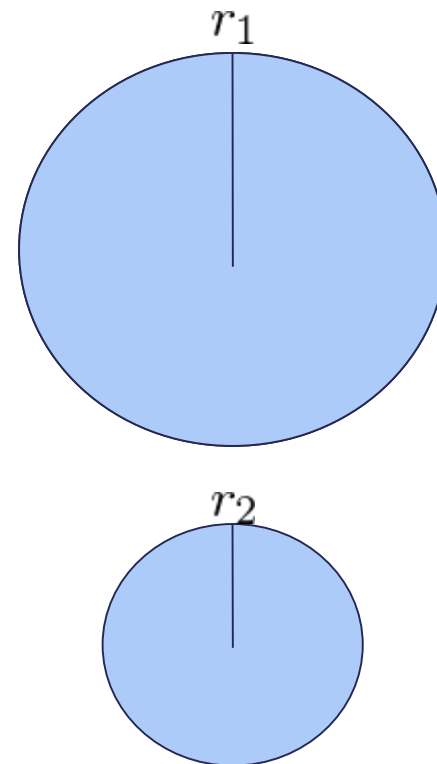
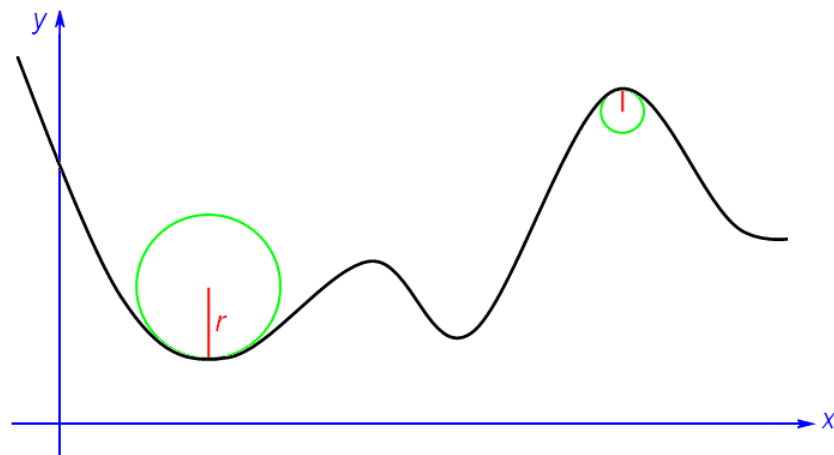
Curvature


$$\kappa = \frac{1}{r}$$



Curvature

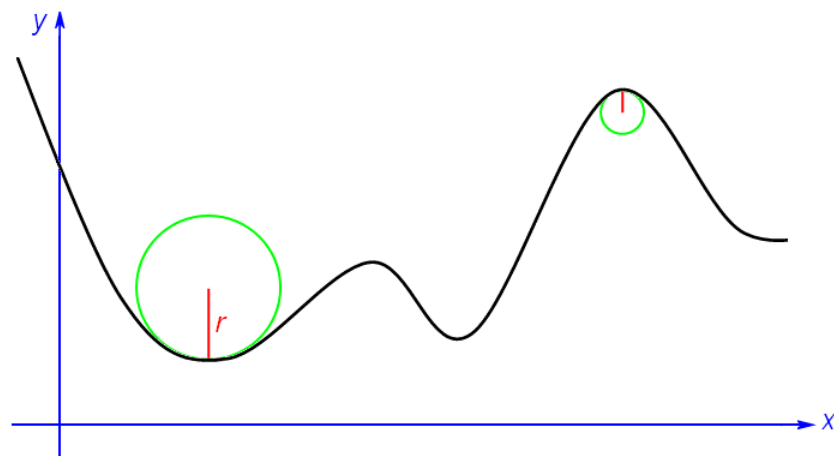
$$\kappa = \frac{1}{r}$$



$$r_2 < r_1$$
$$\kappa_2 > \kappa_1$$

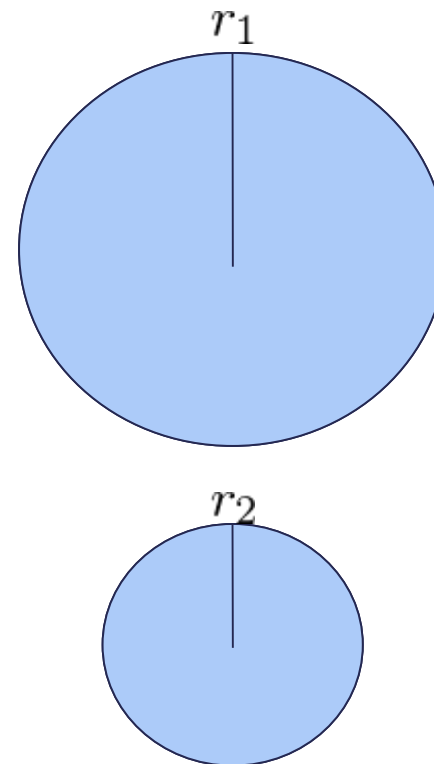
Curvature

$$\kappa = \frac{1}{r}$$



derivative

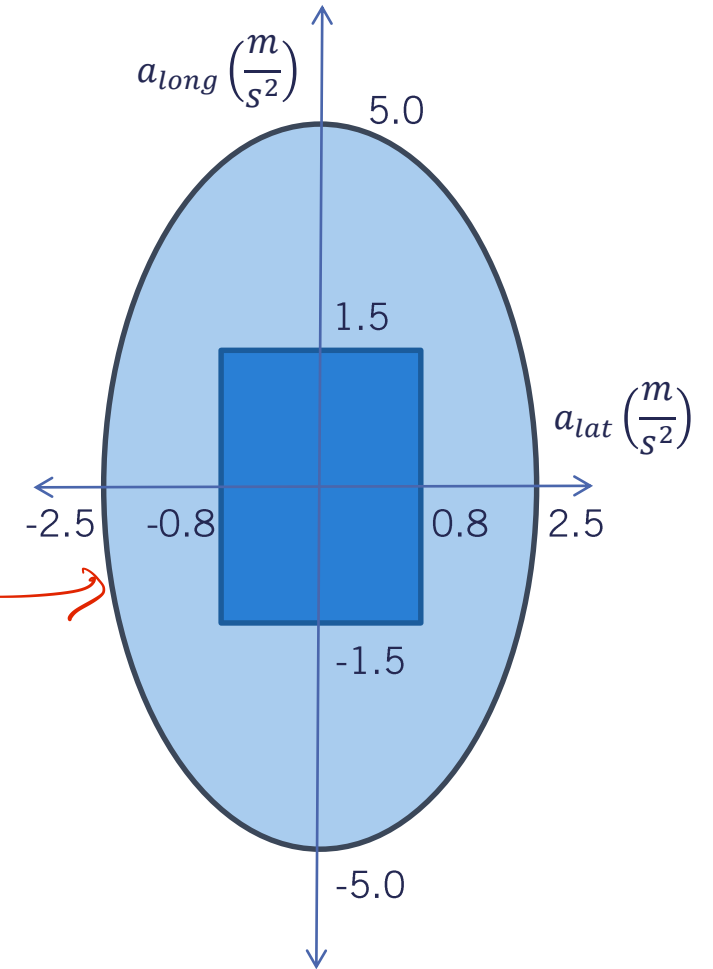
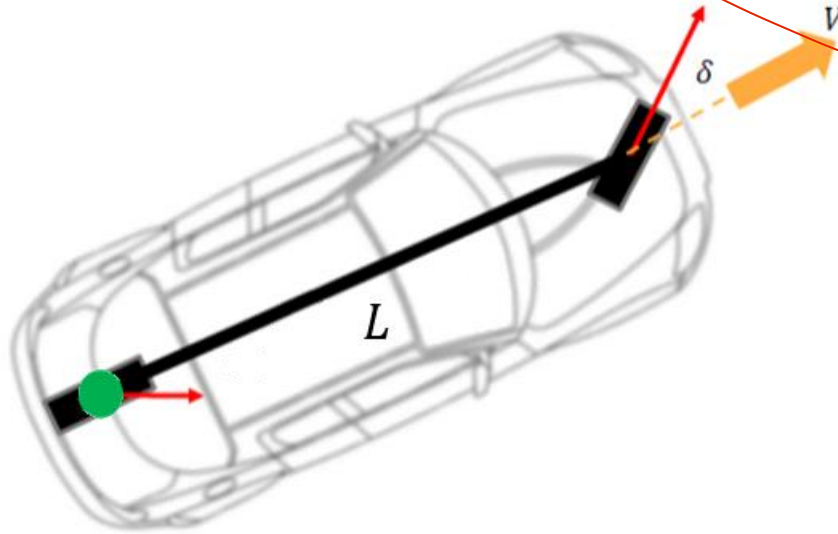
$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$



$$\begin{aligned} r_2 &< r_1 \\ \kappa_2 &> \kappa_1 \end{aligned}$$

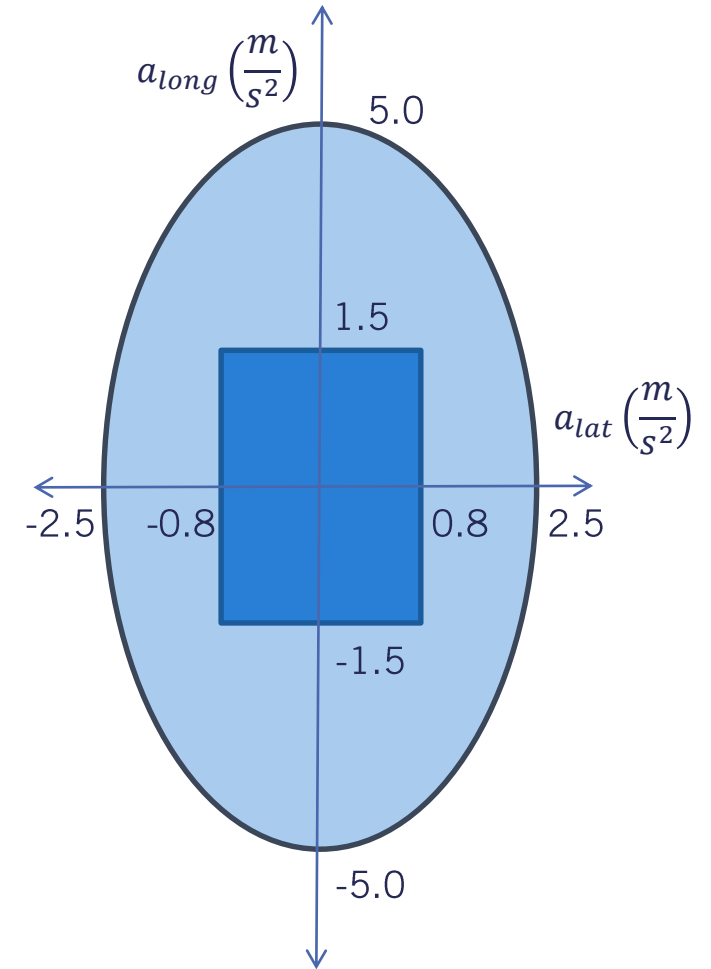
Vehicle Dynamics

- Recall: friction ellipse denotes maximum magnitude of tire forces before **stability** loss



Vehicle Dynamics

- Recall: friction ellipse denotes maximum magnitude of tire forces before stability loss
- Friction forces are extreme limit; more useful constraint is accelerations tolerable by passengers
 - Given by “comfort rectangle” range of lateral and longitudinal accelerations



Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
 - Lateral acceleration is a function of instantaneous turning radius of path and velocity

$$a_{lat} = \frac{v^2}{r}, \quad a_{lat} \leq a_{lat_{max}}$$

Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
 - Lateral acceleration is a function of instantaneous turning radius of path and velocity
- Recall: instantaneous curvature is inverse of instantaneous turning radius

$$a_{lat} = \frac{v^2}{r}, \quad a_{lat} \leq a_{lat_{max}}$$

$$\kappa = \frac{1}{r}$$

Dynamics and Curvature

- Friction limits and comfort restrict lateral acceleration
 - Lateral acceleration is a function of instantaneous turning radius of path and velocity
- Recall: instantaneous curvature is inverse of instantaneous turning radius
- Substituting, velocity is constrained by path curvature and lateral acceleration

$$a_{lat} = \frac{v^2}{r}, \quad a_{lat} \leq a_{lat_{max}}$$

+

$$\kappa = \frac{1}{r}$$

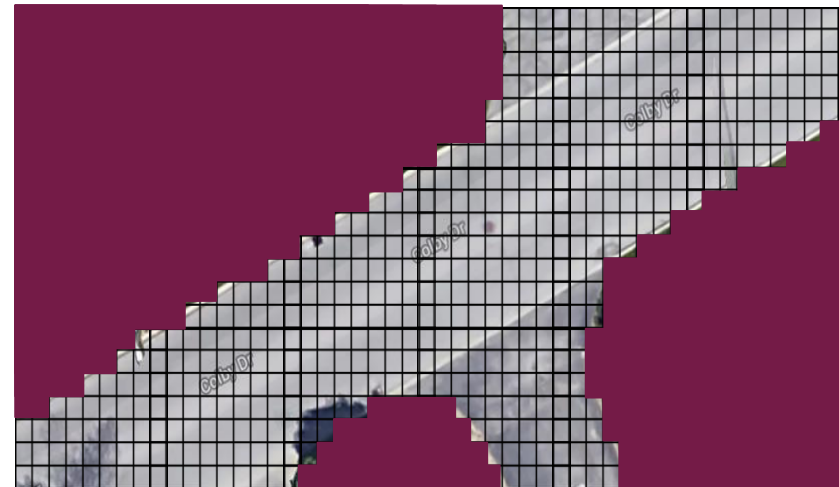
|| ✓

$$v^2 \leq \frac{a_{lat_{max}}}{\kappa}$$

← curvature of the path

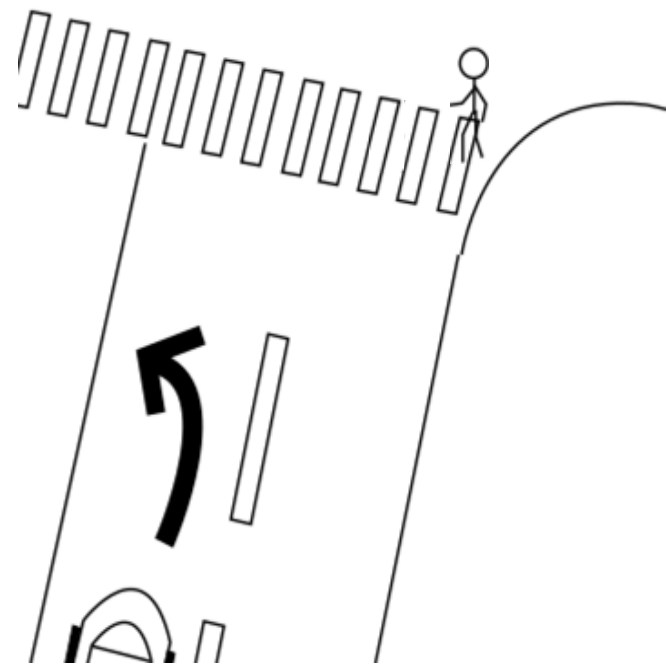
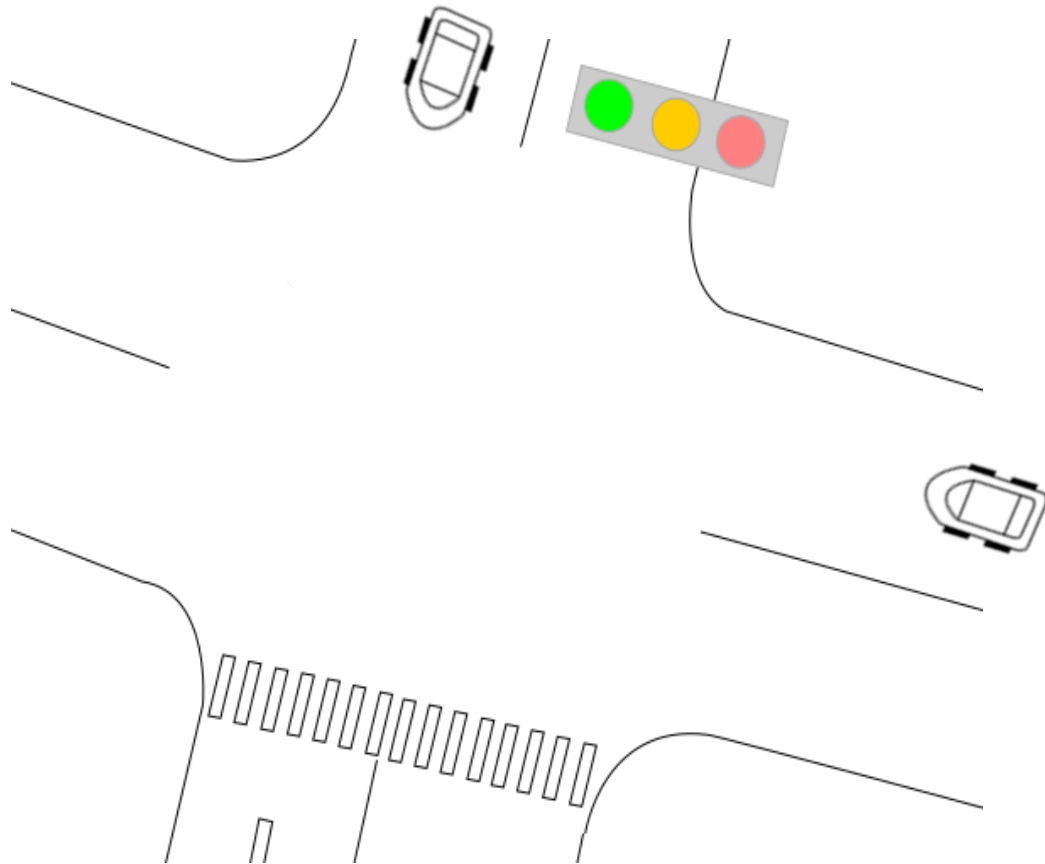
Static Obstacles

- Static obstacles block portions of workspace
 - Occupancy grid encoding stores obstacle locations
- Static obstacle constraints satisfied by performing collision checking
 - Can check for collisions using the swath of the vehicle's path *union of ego vehicle positions*
 - Can also check for closest obstacle along ego vehicle's path

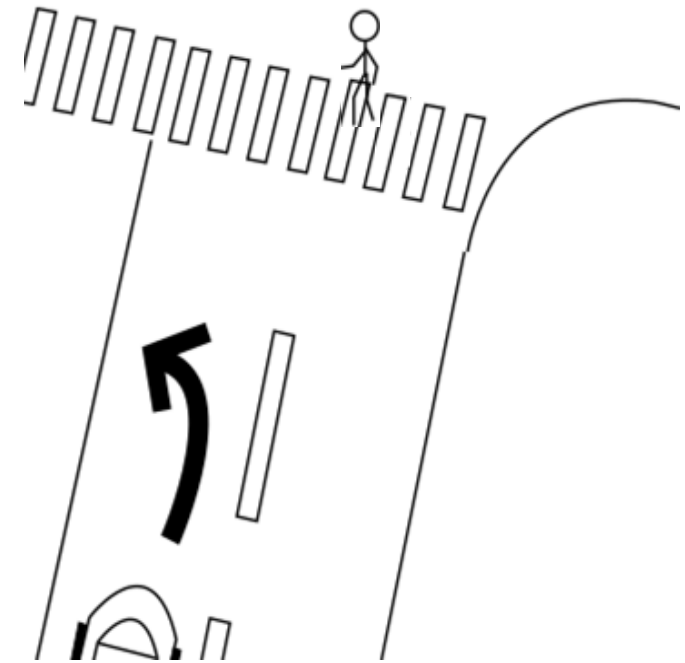
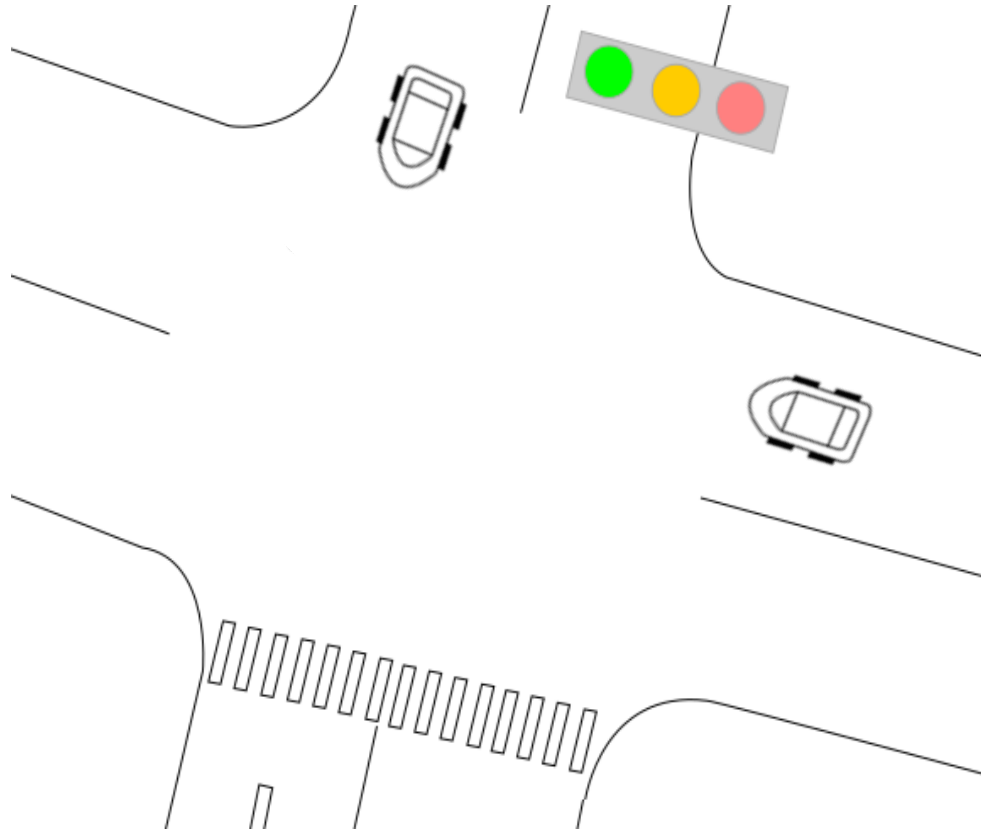


Dynamic Obstacles

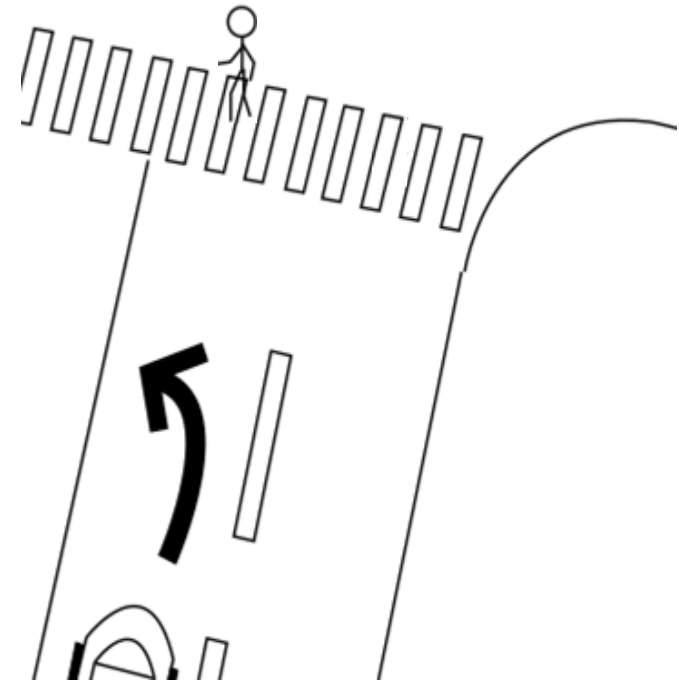
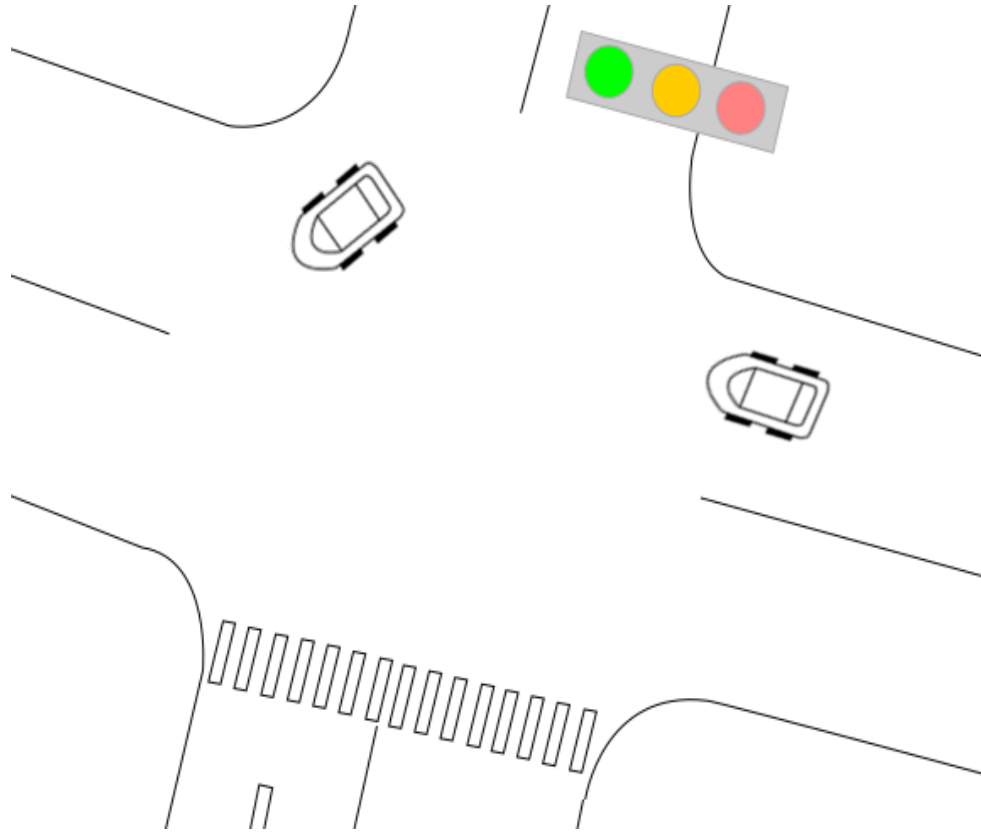
prediction



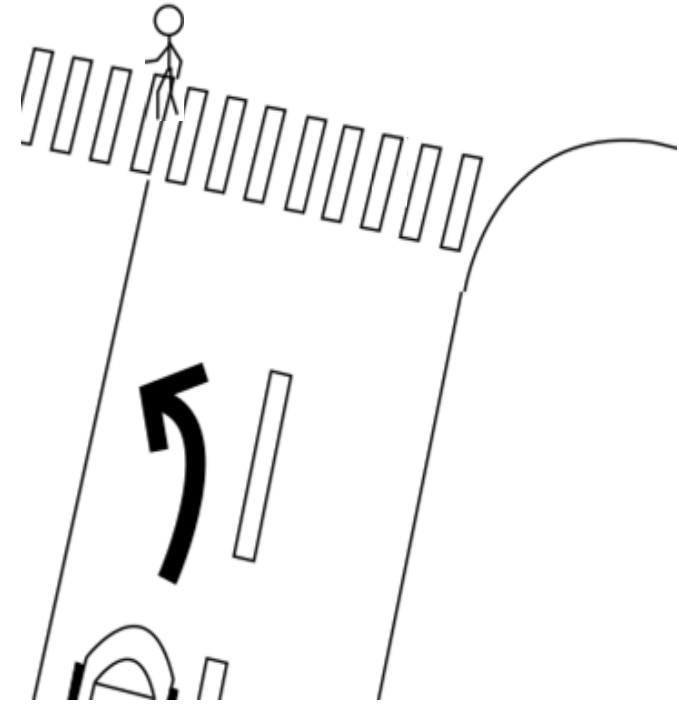
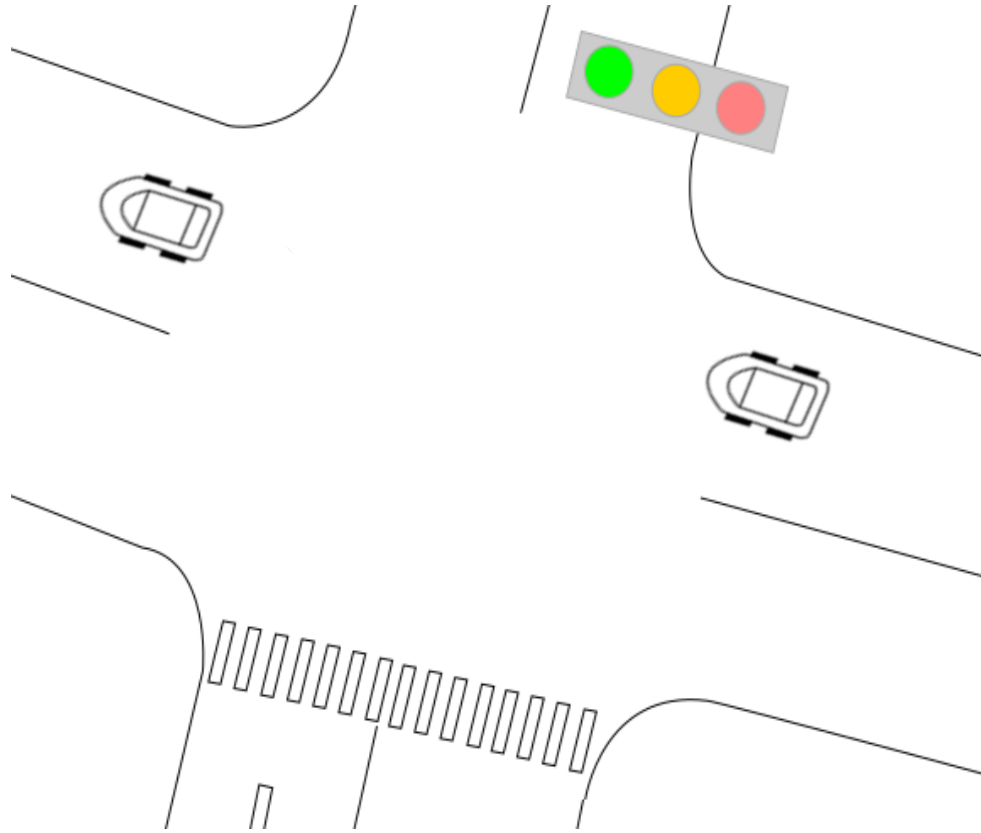
Dynamic Obstacles



Dynamic Obstacles

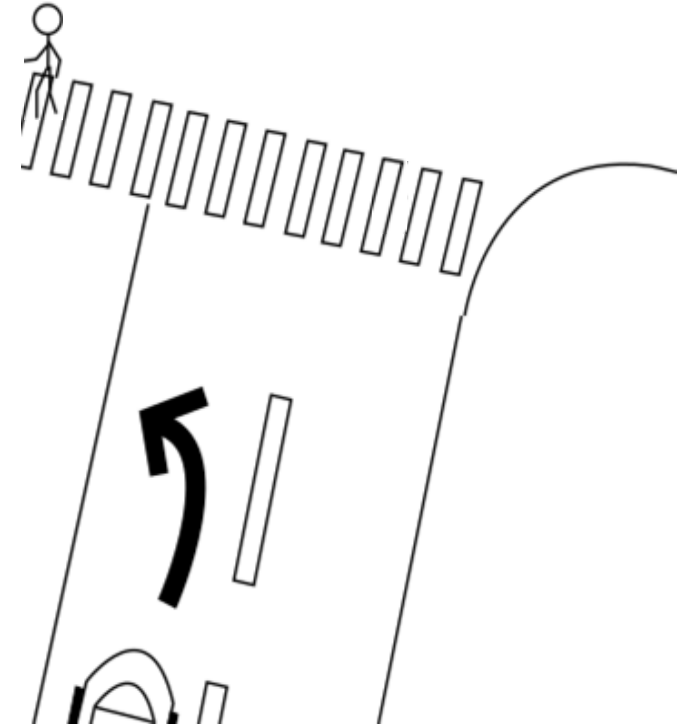
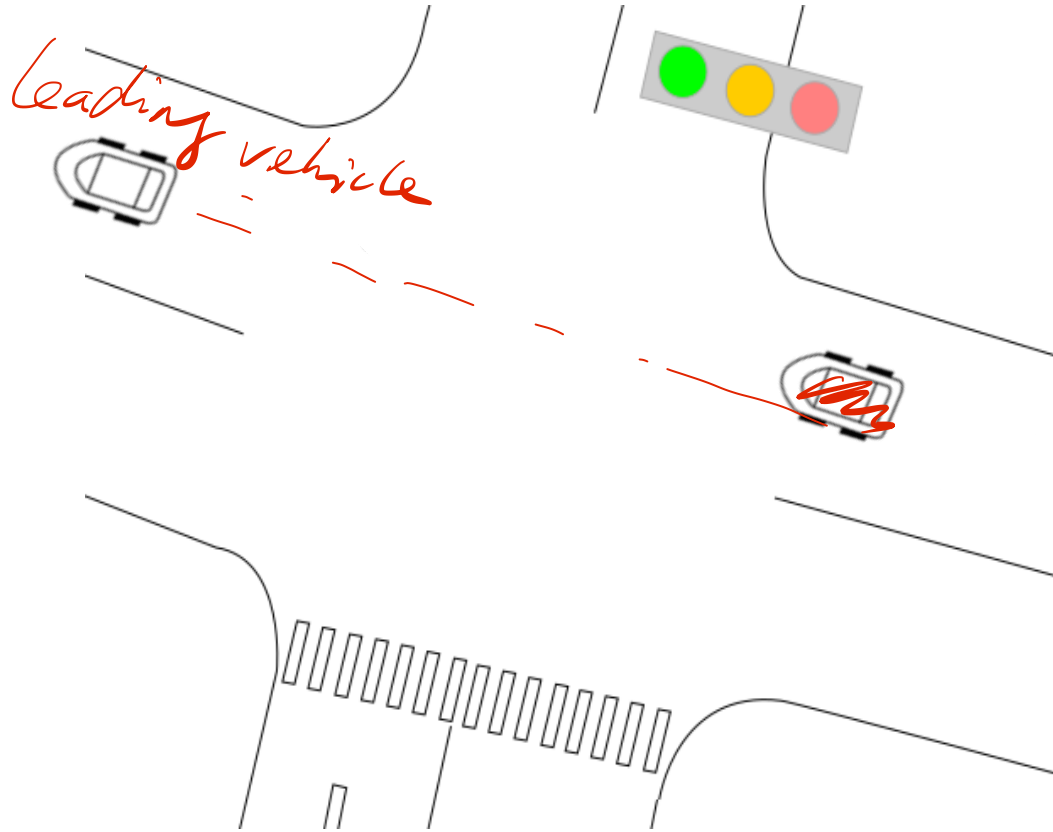


Dynamic Obstacles



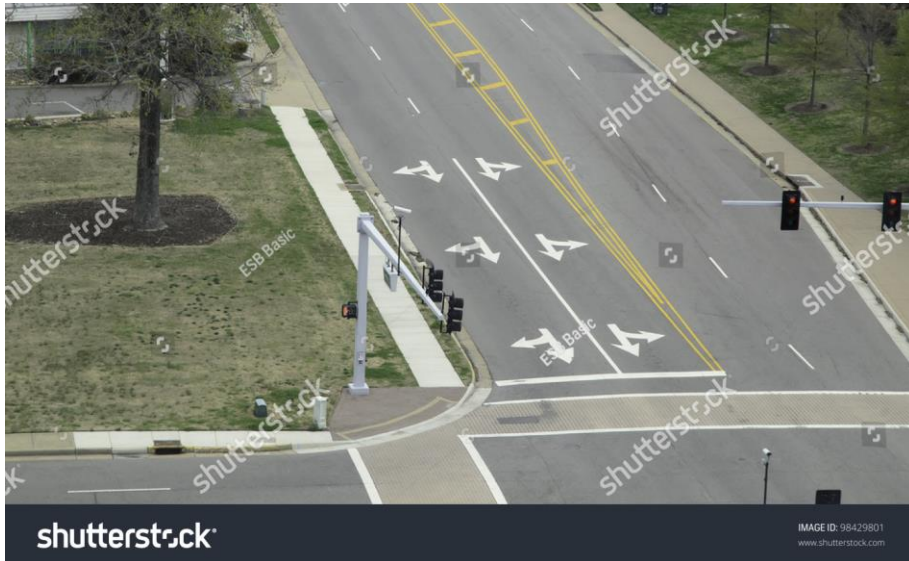
Dynamic Obstacles

{ behavior planning (maneuver decisions)
local planning



Rules of the Road and Regulatory Elements

- Lane constraints restrict path locations
- Signs, traffic lights influence vehicle behaviour



Summary

- Explained the kinematic and dynamic constraints present in the motion planning problem, and how path curvature affects them
- Discussed how static and dynamic obstacles impact our planning process
- Discussed how regulatory elements impact our driving behaviour



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