

Camera Calibration

Course 3, Module 1, Lesson 2

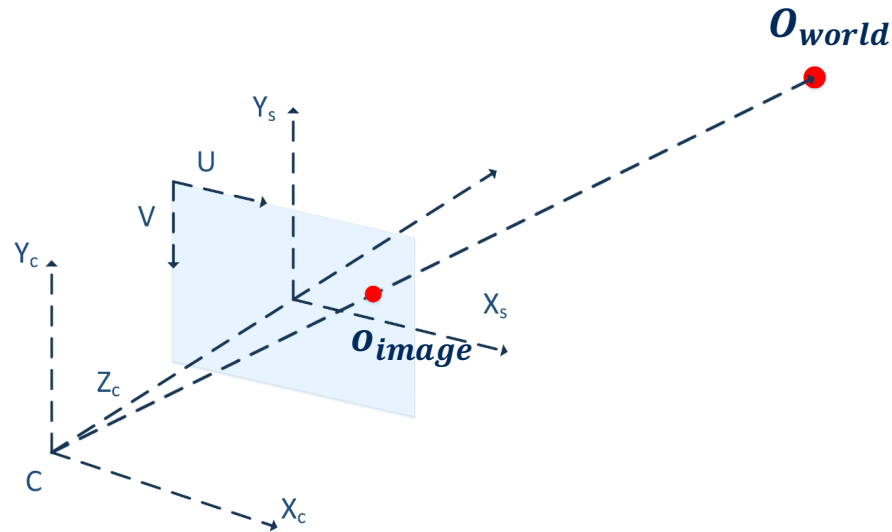


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Computing the Projection

- Projection from World \rightarrow Camera:

$$\mathbf{o}_{image} = PO = K[R|t]\mathbf{o}_{world}$$



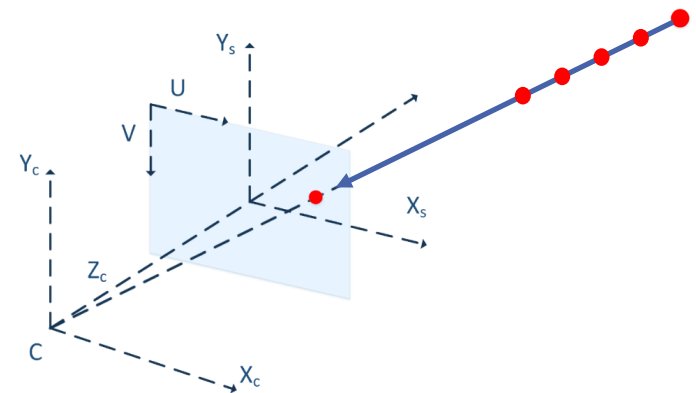
Computing the Projection

- **World** coordinates to **Image** coordinates:

$$o_{image} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K[R|t] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- **Image** coordinates to **Pixel** coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{z} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} su \\ sv \\ s \end{bmatrix}$$



Scale: s

lost when projecting from 3D to 2D

Camera Calibration: Problem Formulation

- World coordinates and Camera coordinates known:

$$O_{image} = P O_{world} = K [R | t] O_{world}$$

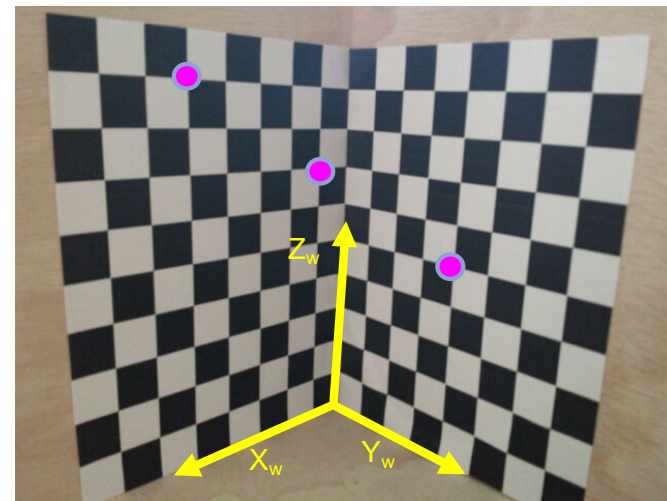
intrinsic parameters (pointing to K)
extrinsic rotation (pointing to R)
extrinsic translation (pointing to t)

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Calibration: Problem Formulation

- Use scenes with **known geometry** to:
 - Correspond 2D image coordinates to 3D world coordinates
 - Find the **Least Squares Solution** (or non-linear solution) of the parameters of P

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Camera Calibration: Problem Formulation

- Expanding the equation results in:

$$su = p_{11} X + p_{12} Y + p_{13} Z + p_{14} \quad (1)$$

$$sv = p_{21} X + p_{22} Y + p_{23} Z + p_{24} \quad (2)$$

$$s = p_{31} X + p_{32} Y + p_{33} Z + p_{34} \quad (3)$$

Camera Calibration: Problem Formulation

- Move to LHS:

$$su - p_{11} X - p_{12} Y - p_{13} Z - p_{14} = 0 \quad (1)$$

$$sv - p_{21} X - p_{22} Y - p_{23} Z - p_{24} = 0 \quad (2)$$

$$s - p_{31} X - p_{32} Y - p_{33} Z - p_{34} = 0 \quad (3)$$

Camera Calibration: Problem Formulation

- Replace Eq. (3) in Eq. (1) and Eq. (2) to get 2 equations per point:

$$p_{31}Xu + p_{32}Yu + p_{33}Zu + p_{34}u - p_{11}X - p_{12}Y - p_{13}Z - p_{14} = 0$$

$$p_{31}Xv + p_{32}Yv + p_{33}Zv + p_{34}v - p_{21}X - p_{22}Y - p_{23}Z - p_{24} = 0$$

Camera Calibration: Problem Formulation

- If we have N 3D points and their corresponding N 2D projections, set up **homogeneous linear system**
- Solved with Singular Value Decomposition (SVD)

$$\begin{bmatrix}
 X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1 X_1 & -u_1 Y_1 & -u_1 Z_1 & -u_1 \\
 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1 X_1 & -v_1 Y_1 & -v_1 Z_1 & -v_1 \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_N X_N & -u_N Y_N & -u_N Z_N & -u_N \\
 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & X_1 & -v_N X_N & -v_N Y_N & -v_N Z_N & -v_N
 \end{bmatrix}
 \begin{bmatrix}
 p_{11} \\
 p_{12} \\
 p_{13} \\
 p_{14} \\
 p_{21} \\
 p_{22} \\
 p_{23} \\
 p_{24} \\
 p_{31} \\
 p_{32} \\
 p_{33} \\
 p_{34}
 \end{bmatrix}
 = 0$$

Camera Calibration: Linear Methods

- Advantages of such a simple linear system are:
 - Easy to formulate
 - Closed form solution
- Disadvantages of such a simple linear system are:
 - Does not directly provide camera parameters
 - Does not model radial distortion and other complex phenomena
 - Does not allow for constraints such as known focal length to be imposed

solving P

Factoring the P matrix

Projection Matrix: $P = K[R|t]$

3D Camera Center projects to zero:

$$PC = 0$$

$$K[R|t]C = 0$$

$$K(RC + t) = 0$$

$$t = -RC$$

Substituting in to projection matrix:

$$P = K[R|-RC]$$

$$P = [KR|-KRC]$$

Let $M = KR$

RQ factorization

*3D camera center
 $PC = 0$*

Factorizing the P matrix

$$P = [M | -MC]$$

$$M = \underbrace{\mathcal{R}}_{3 \times 3} \underbrace{Q}_{3 \times 3}$$

\mathcal{R} is not our Rotation Matrix!

upper
triangular
matrix

an orthogonal
basis

Factorizing the P matrix

$$M = \mathcal{R}Q = KR$$

- Intrinsic Calibration Matrix (upper triangular):

$$K = \mathcal{R}$$

- Rotation Matrix (orthogonal):

$$R = Q$$

- Translation Vector:

$$t = -K^{-1}P[:, 4] = -K^{-1}MC$$

Last column

Summary

- The camera matrix P can be found through a process known as camera calibration
- The intrinsic and extrinsic camera parameters can be extracted from the P matrix using RQ factorization
- **Next: Stereo Vision**

Calibration Tools – MOVE TO SUPPLEMENTAL MATERIALS

- **OpenCV:**

- https://docs.opencv.org/master/d4/d94/tutorial_camera_calibration.html

- **Matlab:**

- <https://www.mathworks.com/help/vision/ug/single-camera-calibrator-app.html>

- **ROS:**

- http://wiki.ros.org/camera_calibration