Populating Occupancy Grids from LIDAR Scan Data

Course 4, Module 2, Lesson 2 – Part 1

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Learning Objectives

- Issue with the Bayesian Probability Update
- Present a solution utilizing log odds
- Bayesian log odds update derivation

Bayesian Update Of The Occupancy Grid - Summary

- Bayes' theorem is applied at each update step for each cell

Normalizer constant

$$bel_t(m^i) = np(y_t|m^i)bel_{t-1}(m^i)$$

Current measurement

Previous belief map

- There's a problem!

Issue With Standard Bayesian Update

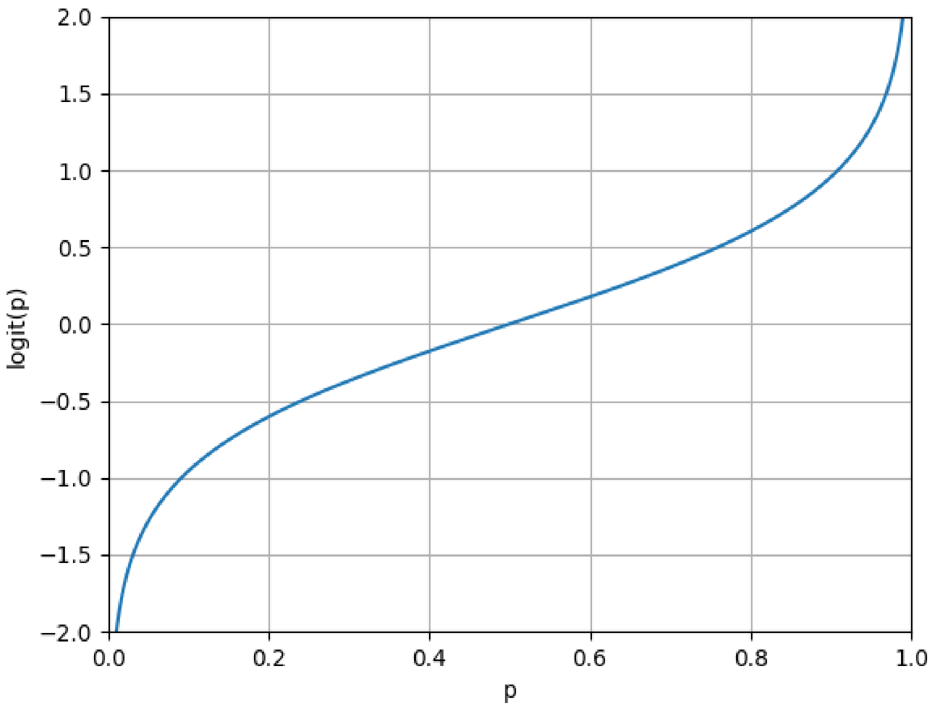
- Update a single unoccupied grid cell

$$bel_t(m) = np(y_t|m)bel_{t-1}(m)$$

0.000000008 0.000012 0.000638

- Multiplication of numbers close to zero is hard for computers
- Store the log odds ratio rather than probability

$$bel_t(m) \to (-\infty, \infty)$$



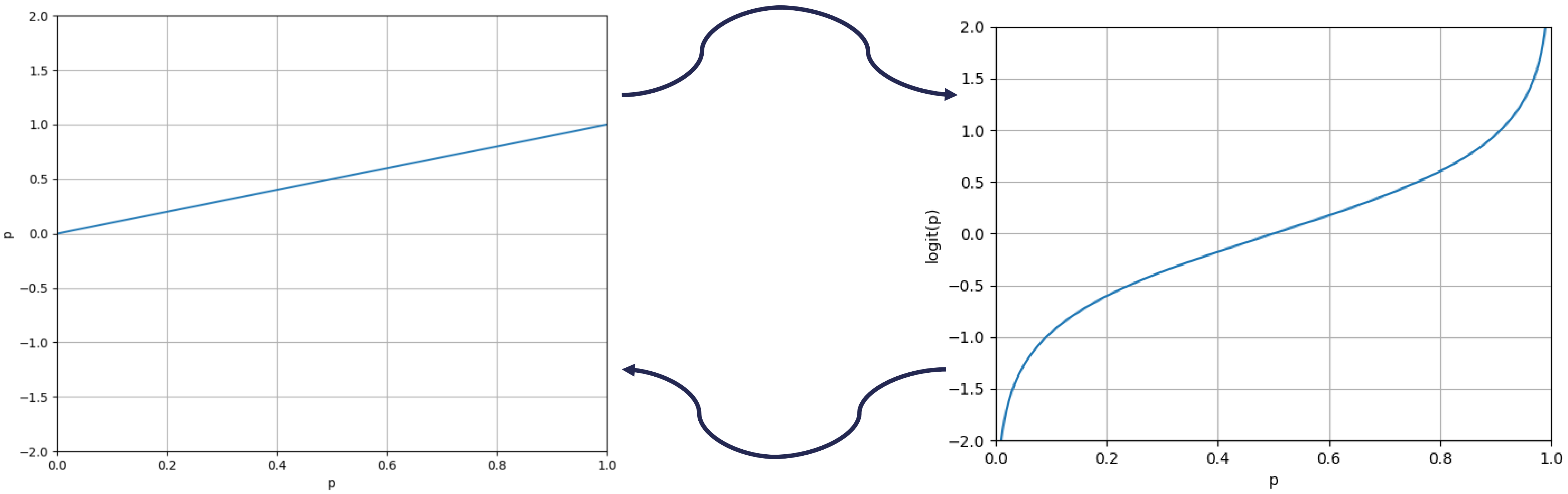
logit function

$$\log\left(\frac{p}{1-p}\right)$$

probability

Conversion

$$logit(p) = \log\left(\frac{p}{1-p}\right)$$



$$p = \frac{e^{logit(p)}}{1 + e^{logit(p)}}$$

Bayesian Log Odds Single Cell Update Derivation

- Applying Bayes' rule:

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

$$p(m^i|y_{1:t}) = \frac{p(y_t|y_{1:t-1}, m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

Current map cell

Sensor measurement for given cell

Pulling out current measurement y_t from past measurements $y_{1:t-1}$

- Applying the Markov assumption:

$$p(m^i|y_{1:t}) = \frac{p(y_t|m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

Bayesian Log Odds Single Cell Update Derivation

$$p(m^i|y_{1:t}) = \frac{p(y_t|m^i)p(m^i|y_{1:t-1})}{p(y_t|y_{1:t-1})}$$

- Applying Bayes' rule to measurement model:

$$\operatorname{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$p(y_t|m^i) = \frac{p(m^i|y_t)p(y_t)}{p(m^i)}$$

- Yields:

$$p(m^i|y_{1:t}) = \frac{p(m^i|y_t)p(y_t)p(m^i|y_{1:t-1})}{p(m^i)p(y_t|y_{1:t-1})}$$

Bayesian Log Odds Single Cell Update Derivation

- Denominator: $1 - p$

$$p(\neg m^i | y_{1:t}) = 1 - p(m^i | y_{1:t}) = \frac{p(\neg m^i | y_t) p(y_t) p(\neg m^i | y_{1:t-1})}{p(\neg m^i) p(y_t | y_{1:t-1})}$$

- Logit function

$$\operatorname{logit}(p) = \log\left(\frac{p}{1-p}\right) \qquad \frac{p(m^i | y_{1:t})}{p(\neg m^i | y_{1:t})} = \frac{\dfrac{p(m^i | y_t) \cancel{p(y_t)} p(m^i | y_{1:t-1})}{p(m^i) \cancel{p(y_t | y_{1:t-1})}}}{\dfrac{p(\neg m^i | y_t) \cancel{p(y_t)} p(\neg m^i | y_{1:t-1})}{p(\neg m^i) \cancel{p(y_t | y_{1:t-1})}}}$$

Bayesian Log Odds Single Cell Update Derivation

- Simplifying like terms results in:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)p(\neg m^i)p(m^i|y_{1:t-1})}{p(\neg m^i|y_t)p(m^i)p(\neg m^i|y_{1:t-1})}$$

- Can rewrite by taking $\neg p$ to $1 - p$:

$$\frac{p(m^i|y_{1:t})}{p(\neg m^i|y_{1:t})} = \frac{p(m^i|y_t)\left(1 - p(m^i)\right)p(m^i|y_{1:t-1})}{\left(1 - p(m^i|y_t)\right)p(m^i)\left(1 - p(m^i|y_{1:t-1})\right)}$$

- Finally, taking the log:

$$\mathrm{logit}\left(p(m^i|y_{1:t})\right) = \mathrm{logit}\left(p(m^i|y_t)\right) + \mathrm{logit}\left(p(m^i|y_{1:t-1})\right) - \mathrm{logit}\left(p(m^i)\right)$$

logit of the belief cell i is occupied at time t

$$\boxed{l_{t,i} = \mathrm{logit}\left(p(m^i|y_t)\right) + l_{t-1,i} - l_{0,i}}$$

Bayesian log odds Update

Inverse Measurement Model

Previous belief

Initial belief

set to 50% uniformally, no prior information

$$l_{t,i} = \text{logit}\left(p(m^i|y_t)\right) + l_{t-1,i} - l_{0,i}$$

- Numerically stable (due to logit mapping)
- Computationally efficient (addition)

Summary

- Identified issue with the Bayesian probability update
- Presented a solution utilizing log odds
- Bayesian log odds update derivation

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Learning Objectives

- Create a simple Inverse Measurement Model
- Discuss an improvement using Bresenham line algorithm

Inverse Measurement Module

$$l_{t,i} = \text{logit}\boxed{\left(p\left(m^i|y_t\right)\right)} + l_{t-1,i} - l_{0,i}$$

- State of the occupancy grid given a measurement
- So far we have only seen the following measurement model:

$$p\left(y_t\middle|m^i\right)$$

  - State of the occupancy grid given a measurement
- A inverse measurement model is needed!

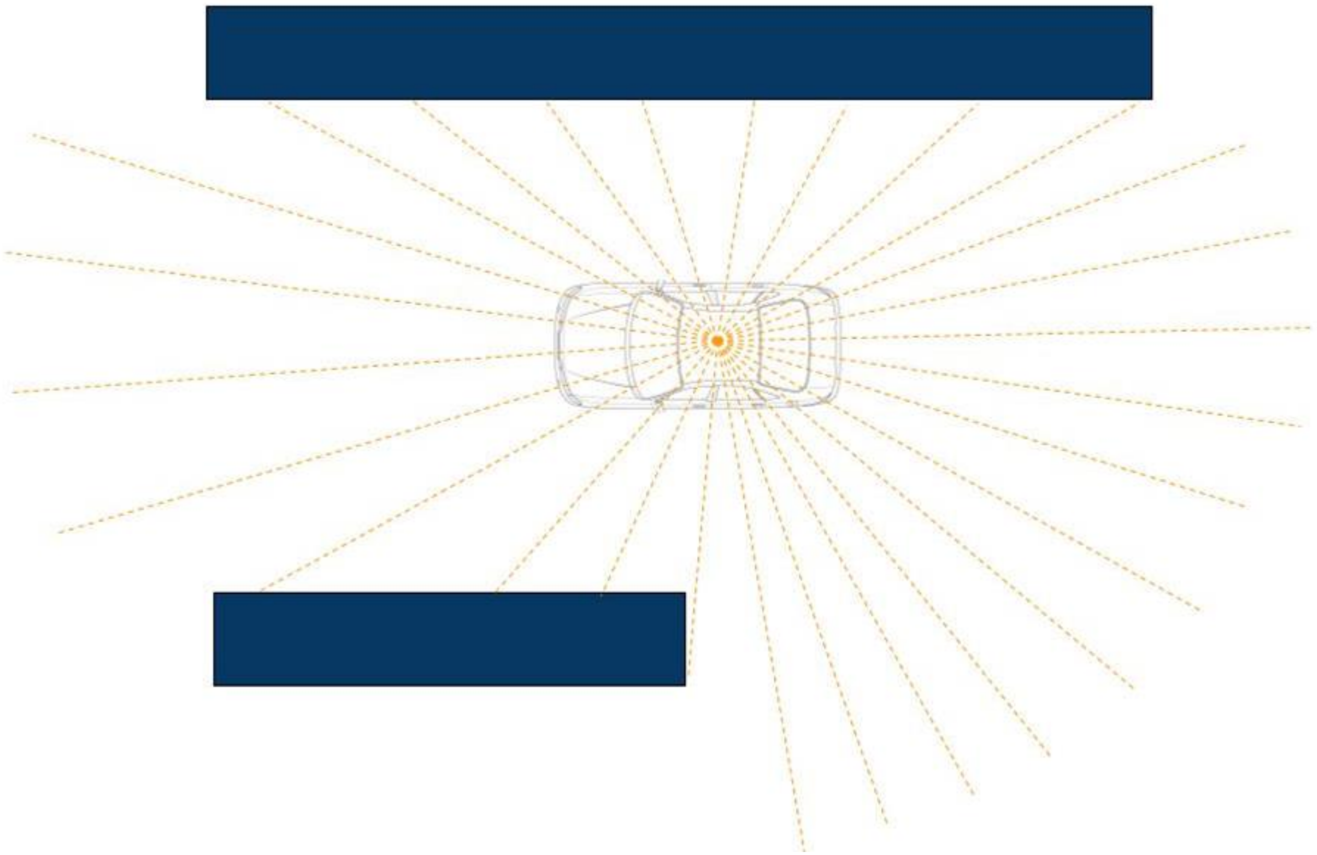
Inverse Measurement Module

- Scanner bearing:

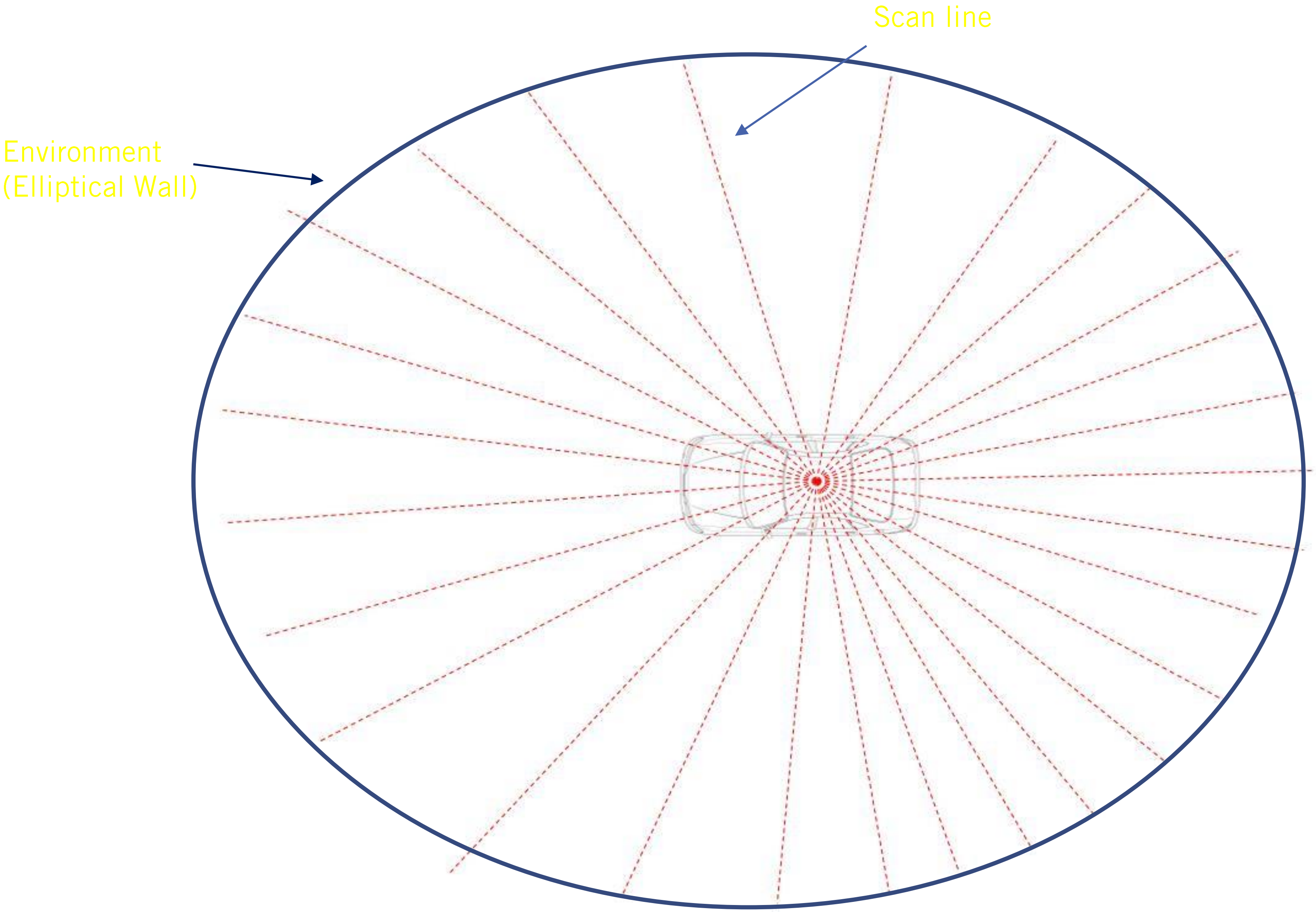
$$\phi^s = [-\phi^s_{max} \quad \cdots \quad \phi^s_{max}] \qquad \phi^s_j \in \phi^s$$

- Scanner ranges:

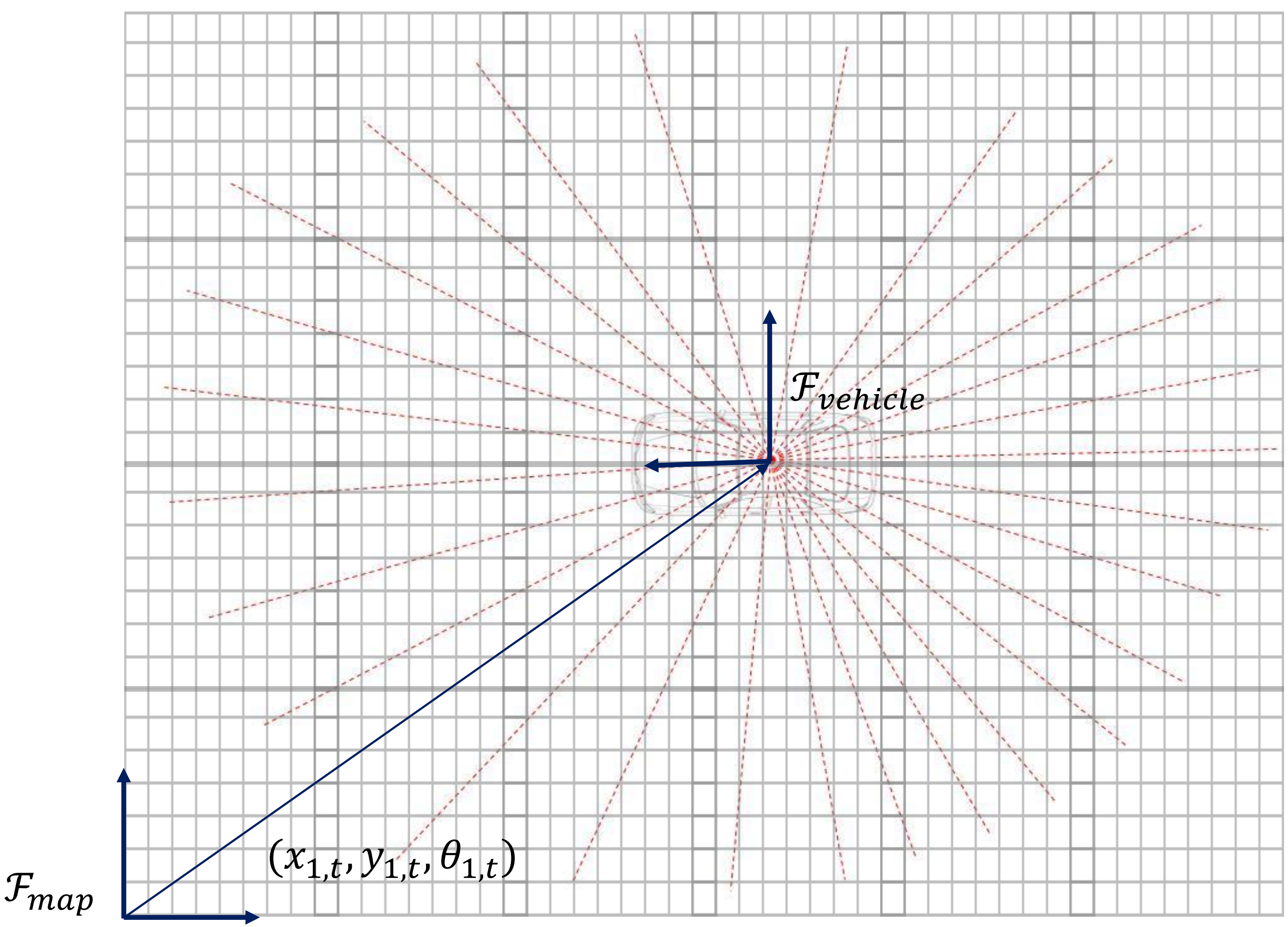
$$r^s = \left[r^s_1 \quad \cdots \quad r^s_j\right] \qquad r^s_j \in [0, r^s_{max}]$$



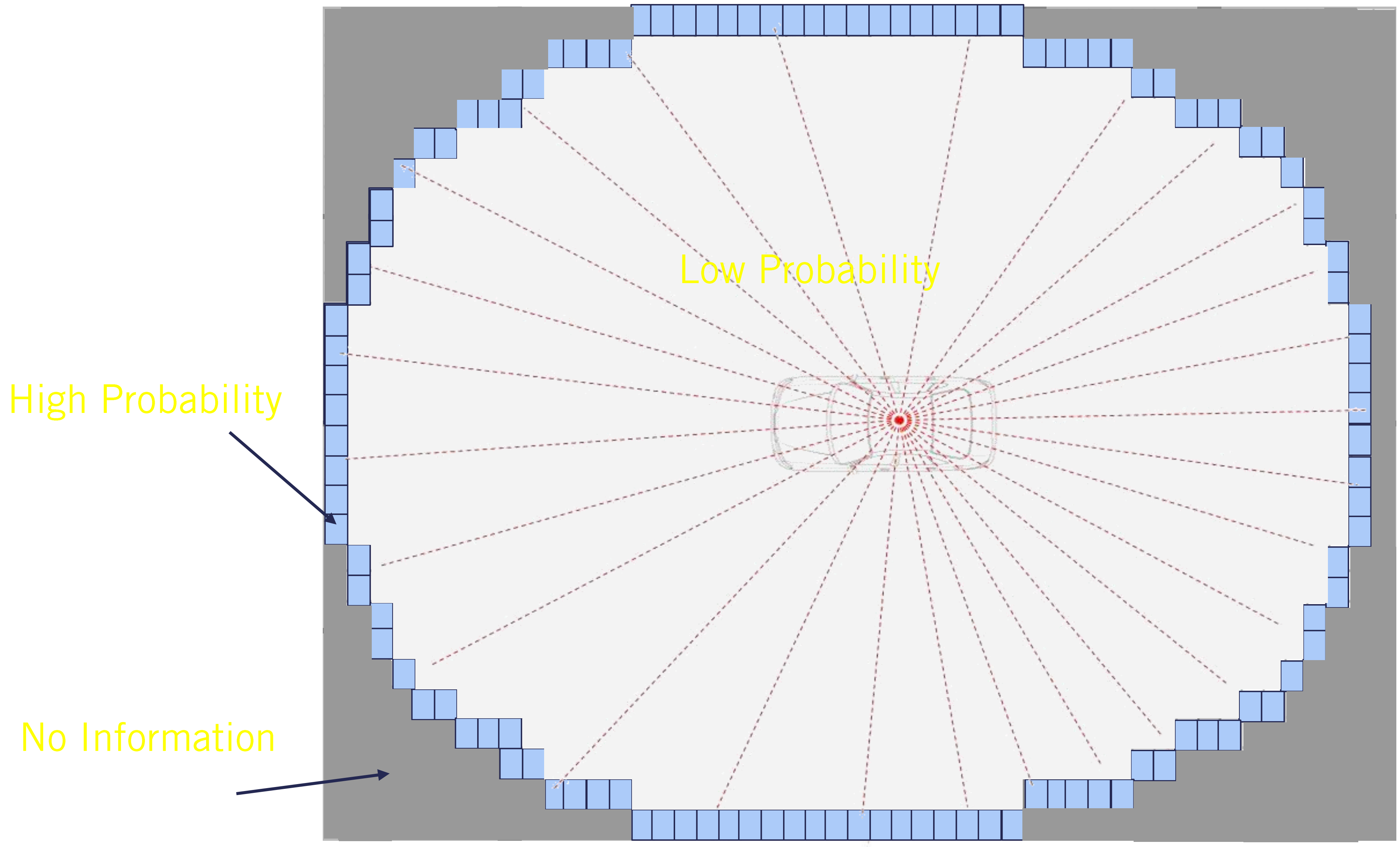
Inverse Measurement Module



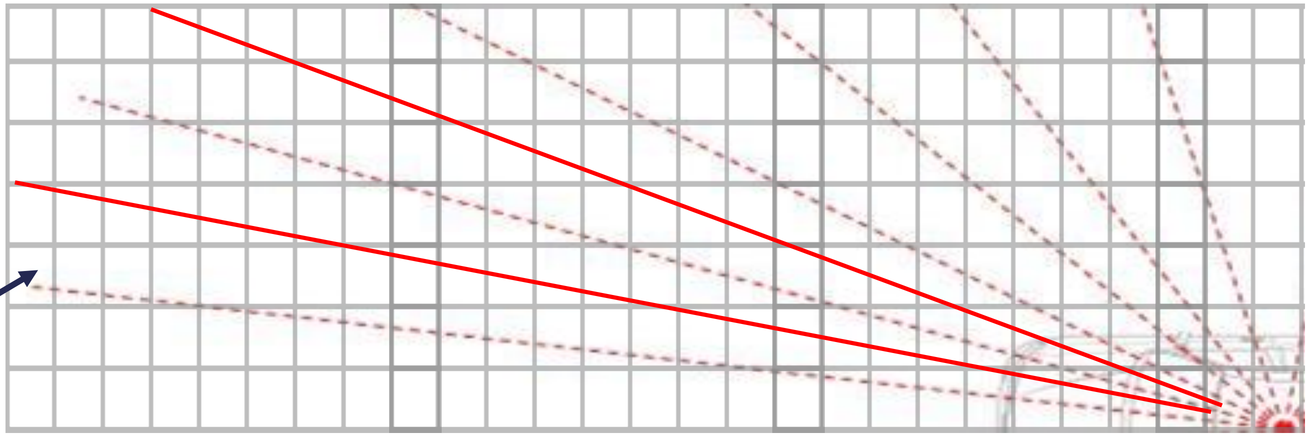
Inverse Measurement Module



Inverse Measurement Module



Inverse Measurement Module – To be fixed



(r^i, ϕ^i)

Closest relative bearing:

$$k = \operatorname{argmin}\left(\left|\phi^i - \phi_j^s\right|\right)$$

Relative range:

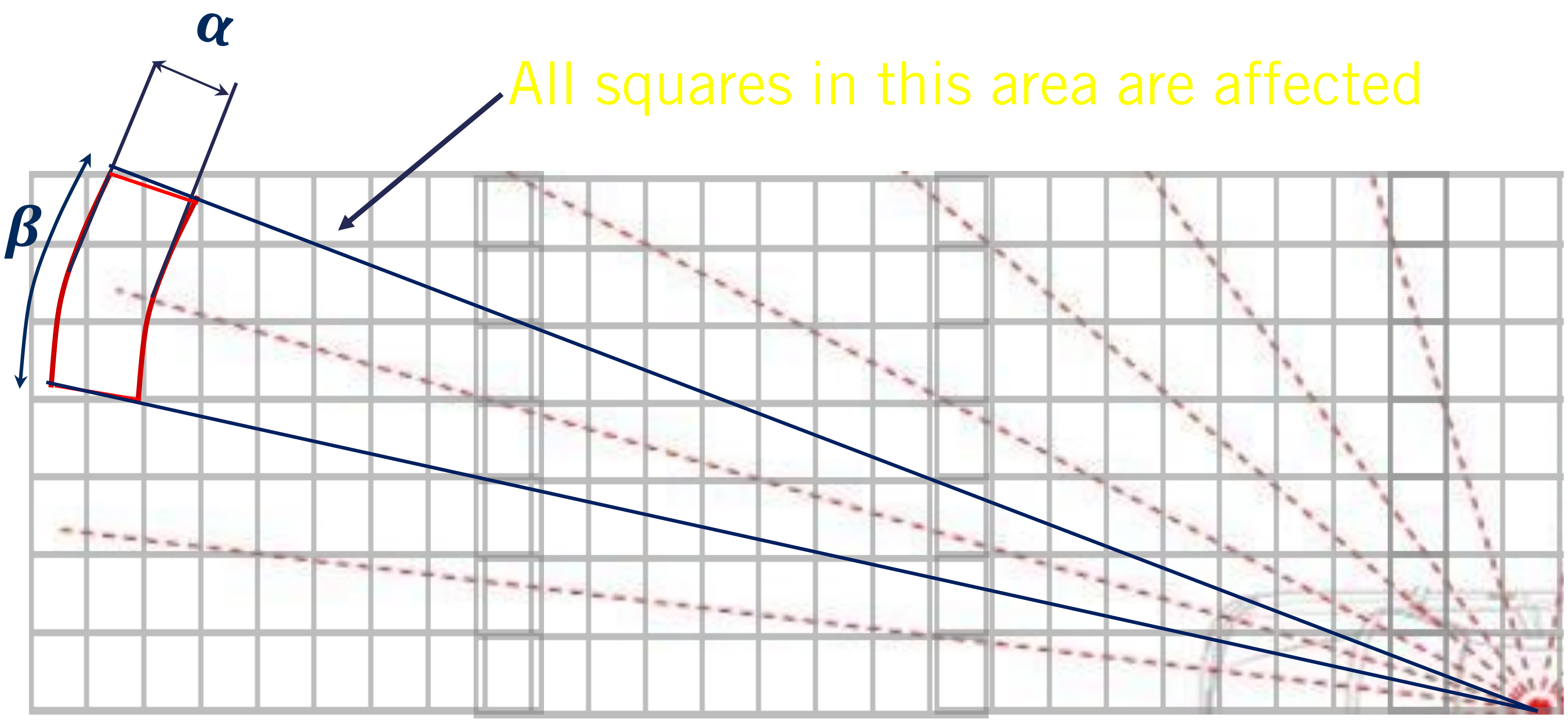
$$r^i = \sqrt{\left(m_x^i - x_{1,t}\right)^2 + \left(m_y^i - x_{2,t}\right)^2}$$

Relative bearing:

$$\phi^i = \tan^{-1}\left(\frac{m_y^i - x_{2,t}}{m_x^i - x_{1,t}}\right) - x_{3,t}$$

Inverse Measurement Module

- **α** - defines the affected range for high probability
- **β** - defines the affected angle for low and high probability



Inverse Measurement Module - Algorithm

- No Information

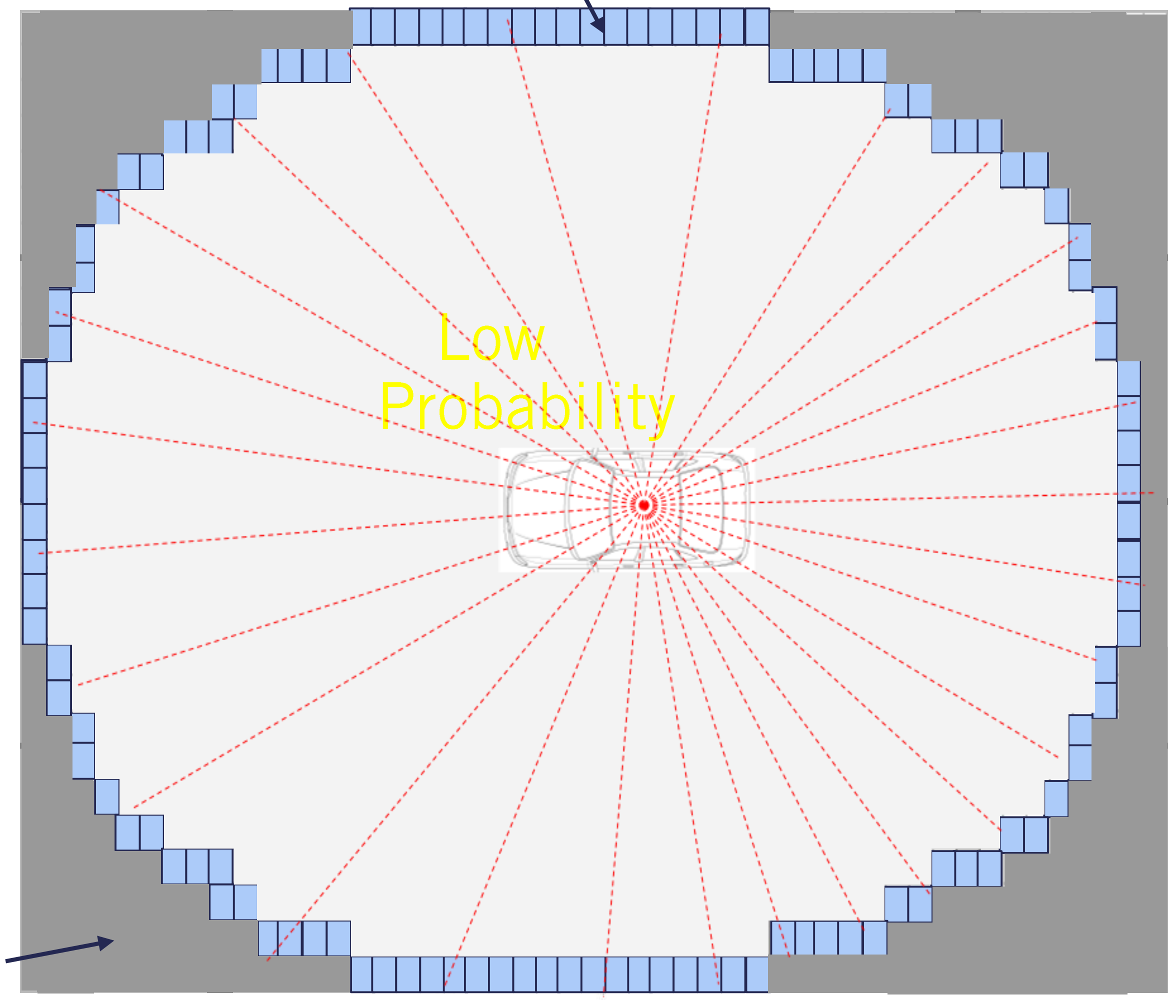
  if $r^i > \min(r^s_{max})$ or $|\phi^i - \phi^s_k| > \beta/2$

- High probability

  if $r^s_k < r^s_{max}$ and $|r^i - r^s_k| > \alpha/2$

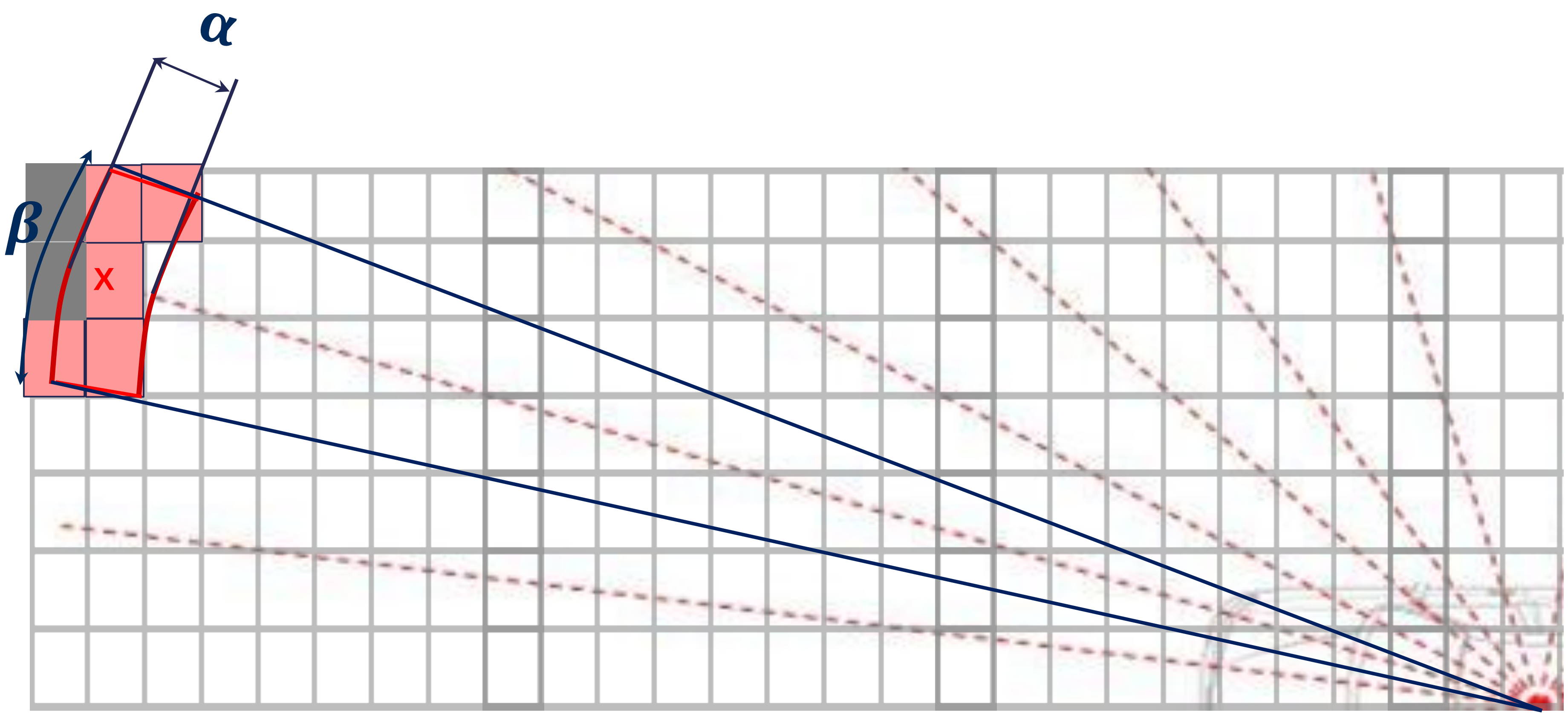
- Low probability

  if $r^i < r^s_k$



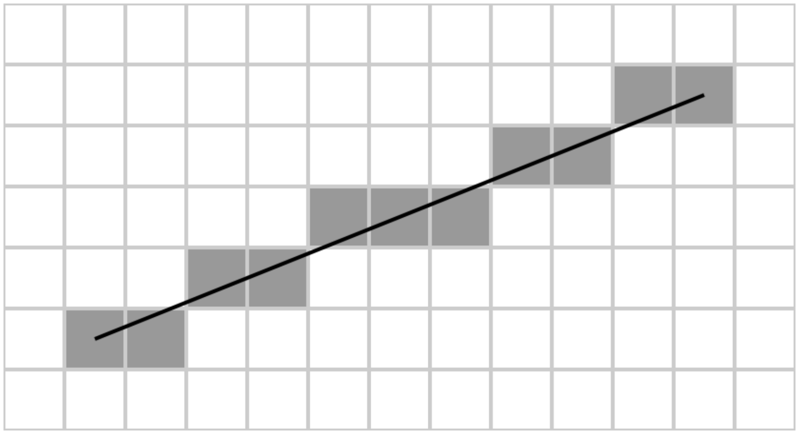
Inverse Measurement Module

- Example – red cells denote high probability of occupied, given measurement denoted by red x.



Inverse Measurement Module With Ray Tracing

- Ray tracing algorithm using Bresenham's line algorithm
  - Rasterized line algorithm
  - Uses very cheap fixed point operations for fast calculations
- Perform update on each beam from the LIDAR rather then each cell on the grid
  - Preforms far fewer updates (ignores no information zone)
  - Much cheaper per operation



Summary

- Create a simple Inverse Measurement Model
- Discuss an improvement using Bresenham line algorithm