

# Dynamic Windowing

Course 4, Module 6, Lesson 4



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# Recall: Kinematic Bicycle Model

- Inputs are linear velocity and steering angle
- No consideration of higher-order terms

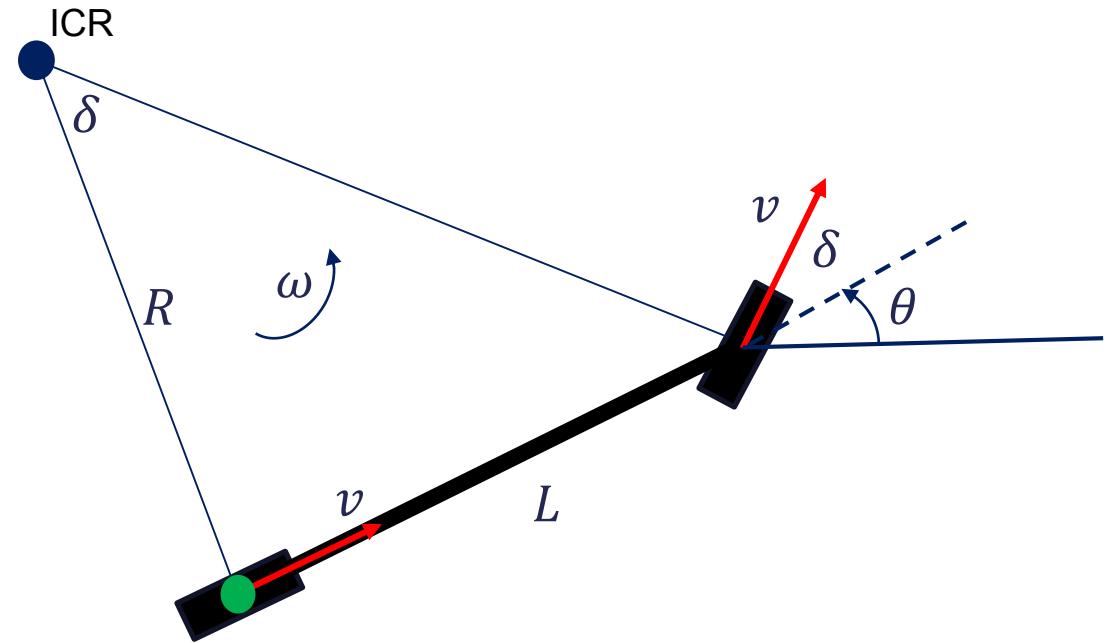
$$\dot{\theta} = \frac{v \tan(\delta)}{L}$$

$$\dot{x} = v \cos(\theta)$$

$$\dot{y} = v \sin(\theta)$$

$$\delta_{min} \leq \delta \leq \delta_{max}$$

$$v_{min} \leq v \leq v_{max}$$



# Bicycle Model + Acceleration Constraints

- Higher order terms handled by adding constraints
- More comfort for passengers, but less maneuverability

$$\dot{\theta} = \frac{v \tan(\delta)}{L}$$

$$\dot{x} = v \cos(\theta)$$

$$\dot{y} = v \sin(\theta)$$

$$\delta_{min} \leq \delta \leq \delta_{max}$$

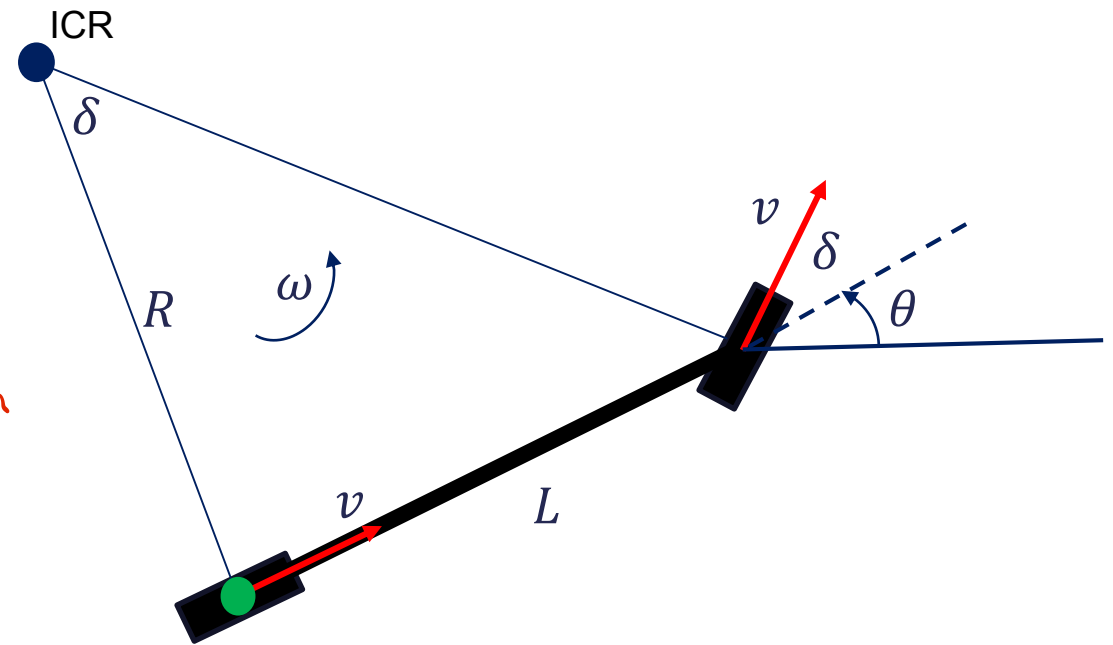
$$v_{min} \leq v \leq v_{max}$$

*angular acceleration*

$$\ddot{\theta}_{min} \leq \ddot{\theta} \leq \ddot{\theta}_{max}$$

$$\ddot{x}_{min} \leq \ddot{x} \leq \ddot{x}_{max}$$

*linear acceleration*



# Constraint in Terms of Steering Angle

- Angular acceleration constraint may prevent us from selecting certain maneuvers based on current angular velocity
- Change in steering angle between planning cycles is bounded
- Similar logic applies for changes in linear velocity inputs between planning cycles

$$\dot{\theta} = \frac{v \tan(\delta)}{L}$$

$$|\ddot{\theta}| = \left| \frac{\dot{\theta}_2 - \dot{\theta}_1}{\Delta t} \right|$$

$$|\tan(\delta_2) - \tan(\delta_1)| \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

# Example

- Given current steering angle and the angular acceleration bound, which candidate trajectories do not violate that bound?

$$v = 1 \text{ m/s}$$

current steering angle

$$\delta_1 = \frac{\pi}{8}$$

$$\Delta t = 1 \text{ s}$$

$$\delta_{min} = -\frac{\pi}{4}$$

$$L = 1 \text{ m}$$

$$\delta_{max} = \frac{\pi}{4}$$

$$|\ddot{\theta}| \leq 0.6 \text{ s}^{-2}$$

radian/s<sup>2</sup>

# Example

- Changing our steering angle to  $-\frac{\pi}{8}$  or  $-\frac{\pi}{4}$  violates our angular acceleration constraint

$$|\tan(\delta_2) - \tan(\delta_1)| \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$|\tan(\pi/4) - \tan(\pi/8)| = 0.586 \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$|\tan(\pi/8) - \tan(\pi/8)| = 0 \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

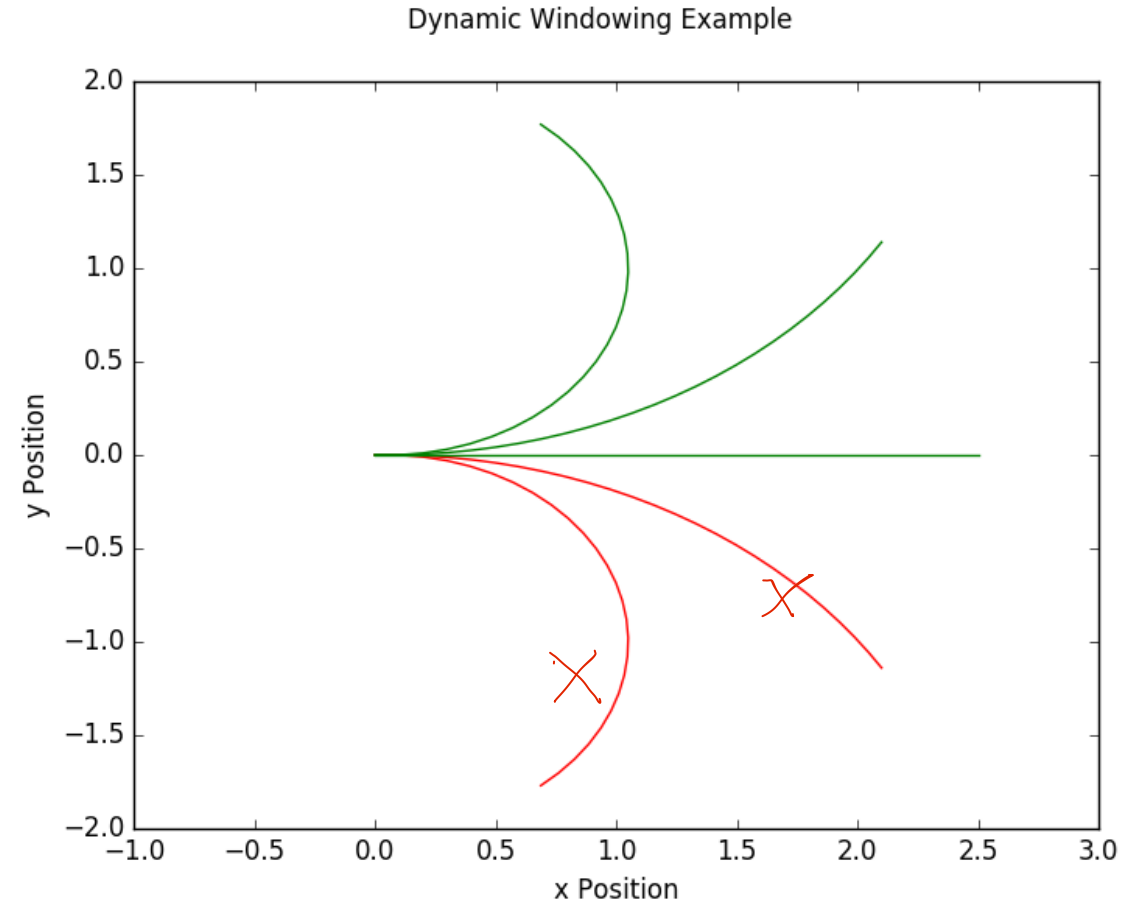
$$|\tan(0) - \tan(\pi/8)| = 0.414 \leq \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$\times |\tan(-\pi/8) - \tan(\pi/8)| = 0.828 > \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

$$\times |\tan(-\pi/4) - \tan(\pi/8)| = 1.414 > \frac{\ddot{\theta}_{max} L \Delta t}{v}$$

# Comparing Trajectories

- Trajectories that exceed the angular acceleration constraint are coloured red
- Added constraints reduce manoeuvrability of the robot



# Summary

- Introduced linear and angular acceleration constraints to our motion planning problem
- Discussed dynamic windowing and how it allows us to handle these new constraints in the trajectory rollout algorithm