Probabilistic Models

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*Slides based on or adapted from Sanjiban Choudhury

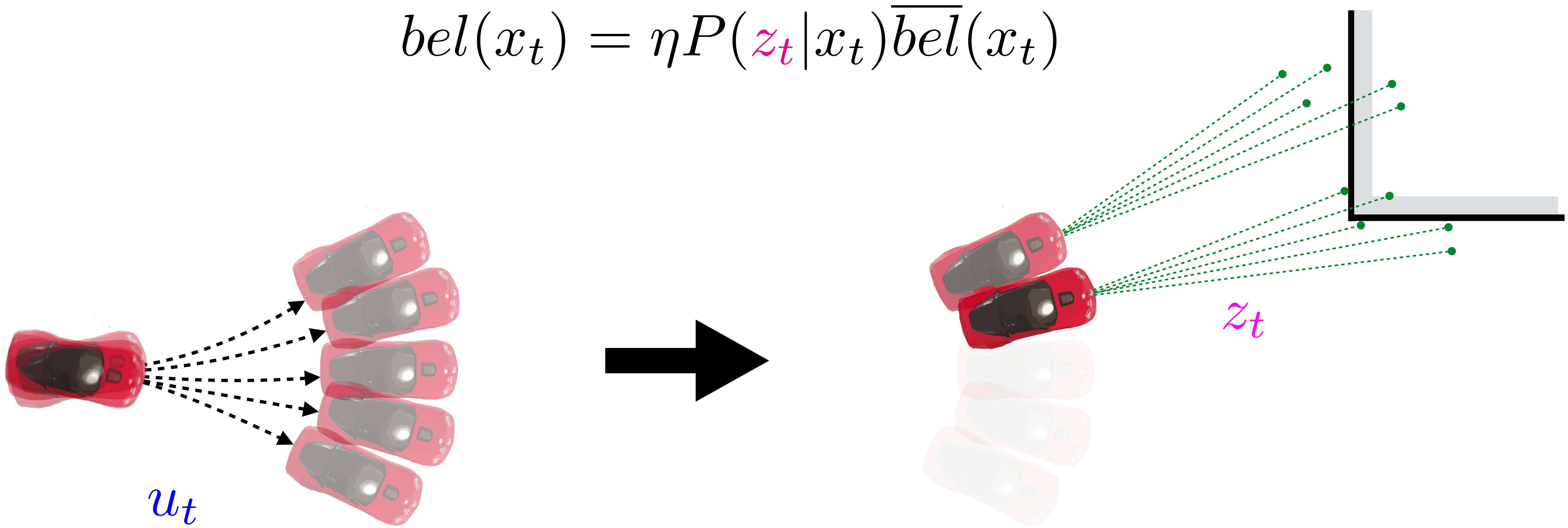
Bayes filter in a nutshell

Step 1: Prediction - push belief through dynamics given action

$$\overline{bel}(x_t) = \int P(x_t|u_t, x_{t-1})bel(x_{t-1})dx_{t-1}$$

Step 2: Correction - apply Bayes rule given measurement

$$bel(x_t) = \eta P(z_t|x_t)\overline{bel}(x_t)$$



$bel(x_{t-1})$ $\overline{bel}(x_t)$ $bel(x_t)$

Today's objective

1. Briefly discuss different paradigms of Bayes filtering

2. Probabilistic motion model

3. Look again at Bayes filter if there are questions and time

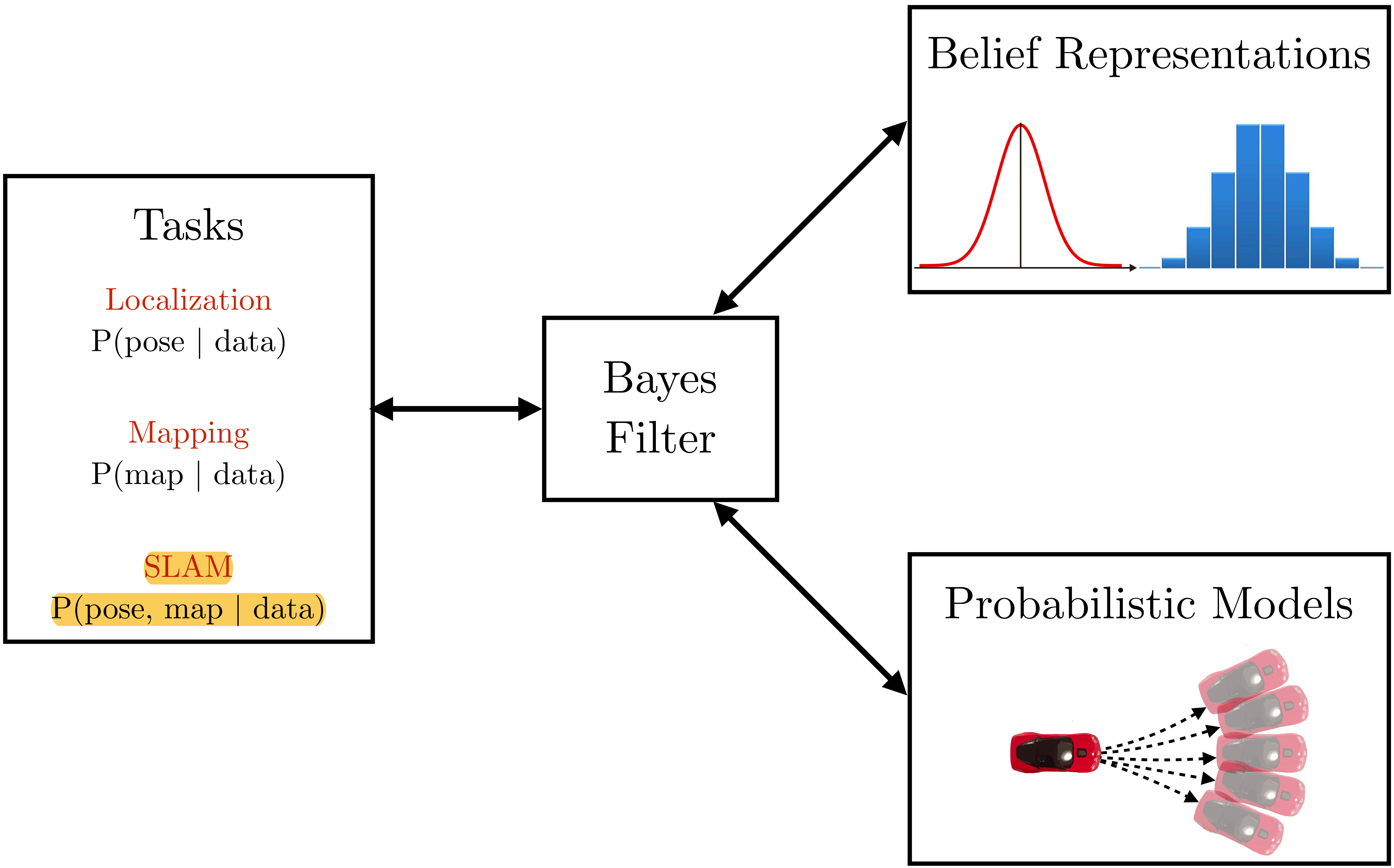
Bayes filter is a powerful tool

Localization Mapping SLAM POMDP

Casting different tasks as Bayes filtering

Tasks	State	Action	Measurement
Localization			
Mapping			
SLAM			
Pursuit-Evasion			

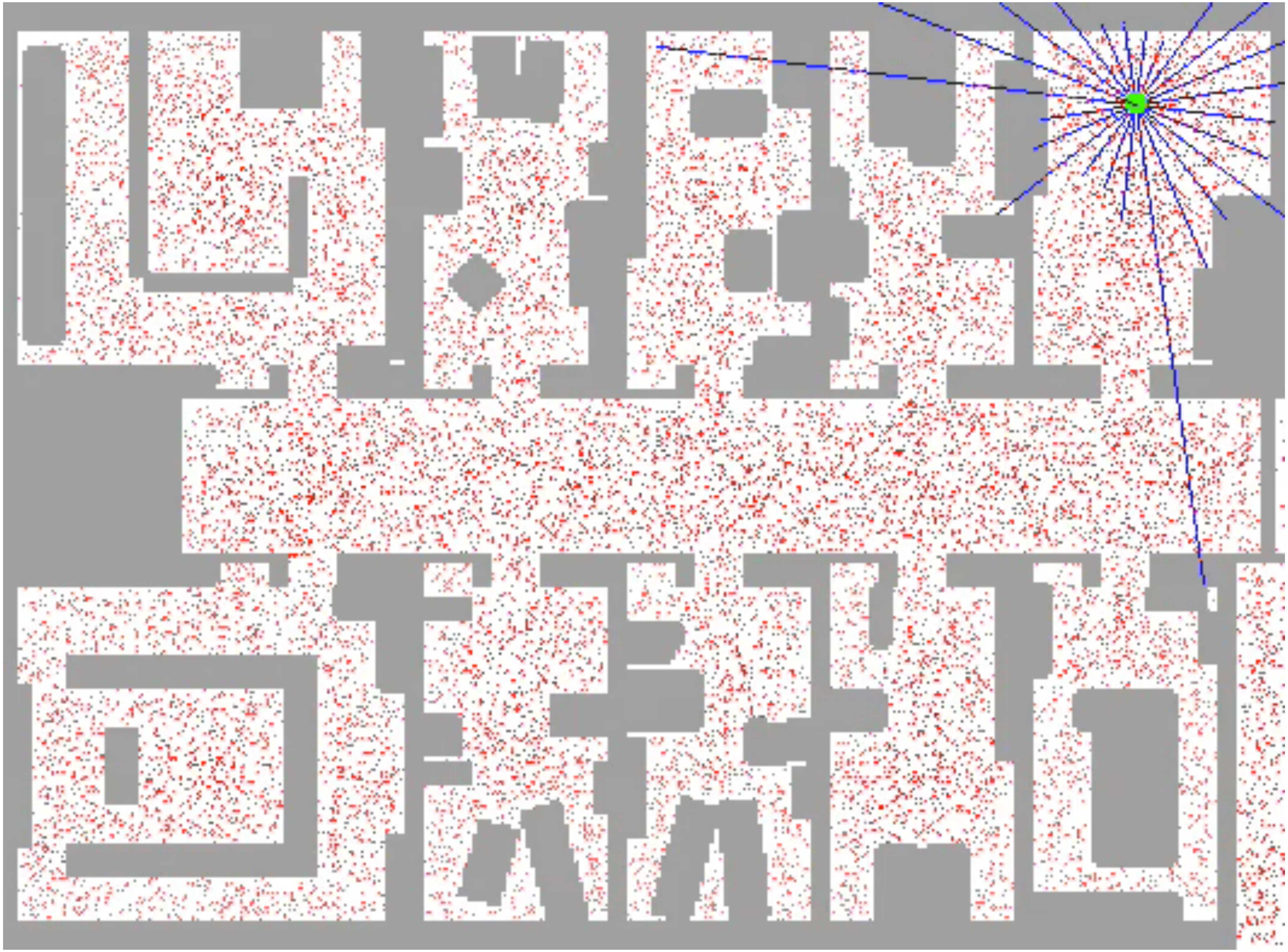
Assembling Bayes filter



Tasks that we will cover

Tasks	**Belief Representation**	**Probabilistic Models**
Localization P(pose | data) (Week 3)	Gaussian / Particles	Motion model Measurement model
Mapping P(map | data) (Week 4)	Discrete (binary)	Inverse measurement model
SLAM P(pose, map | data) (Week 4)	Particles+Gaussian (pose, landmarks)	Motion model, measurement model, correspondence model

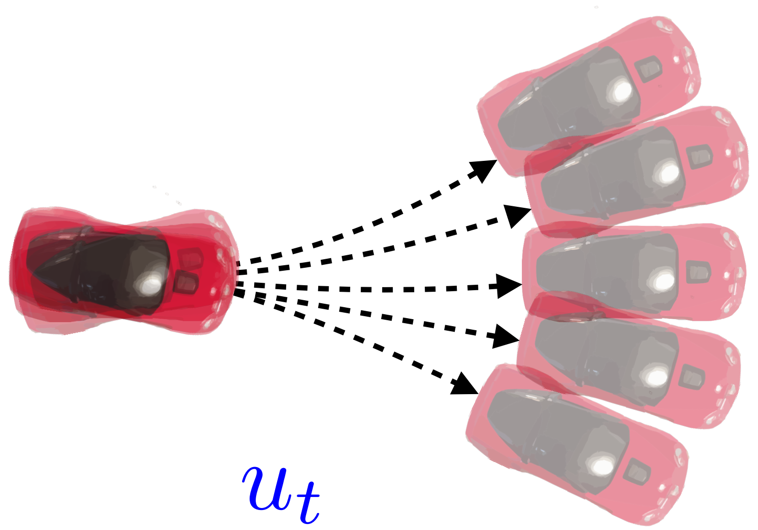
What is localization?



Probabilistic models in localization

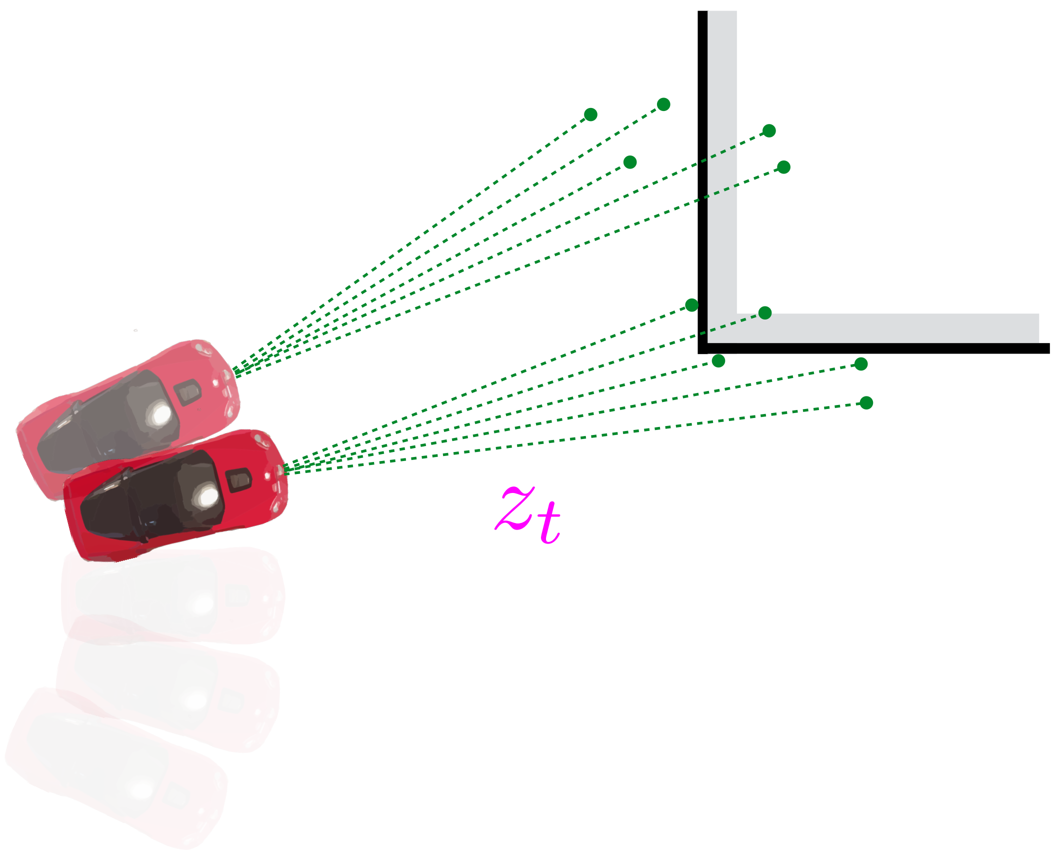
Motion model

$$P(x_t|u_t, x_{t-1})$$



Measurement model

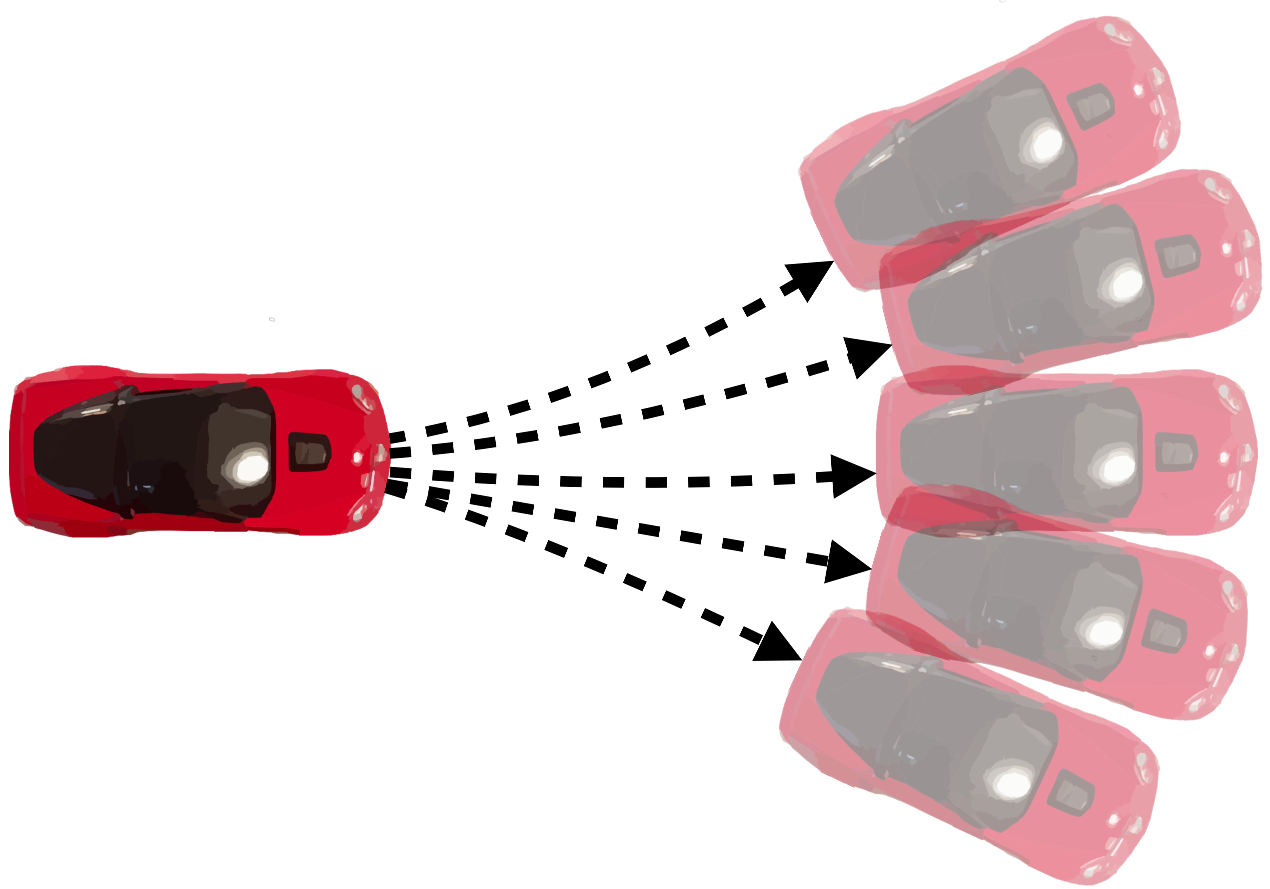
$$P(z_t|x_t)$$



How do we think about models?

Motion Model

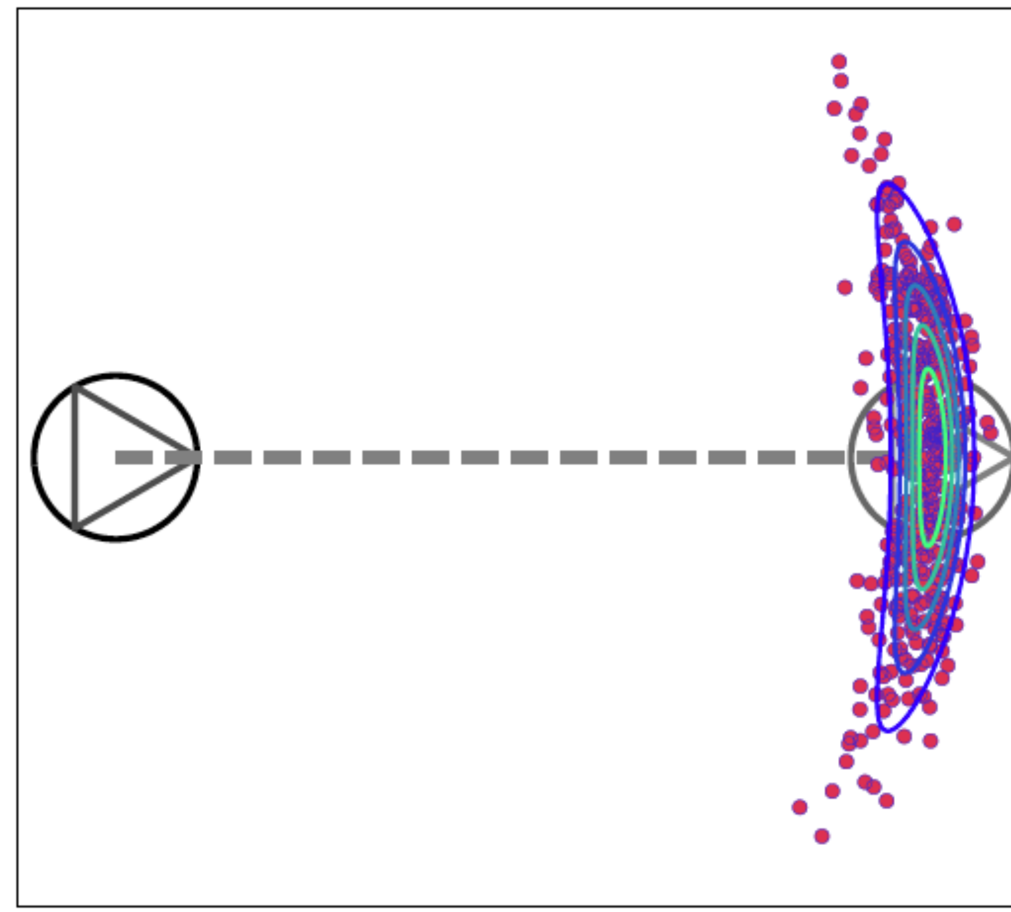
$$P(x_t | u_t, x_{t-1})$$



Spectrum of motion models

(Redbull simulator)

vs



Highest fidelity models of everything

Simple model with lots of noise

Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

Why is the motion model probabilistic?

If we know how to write out equations of motion,
we should be able to exactly predict where a body ends up, right?

What are the sources of ~~noise~~ stochasticity?

Three questions you should ask

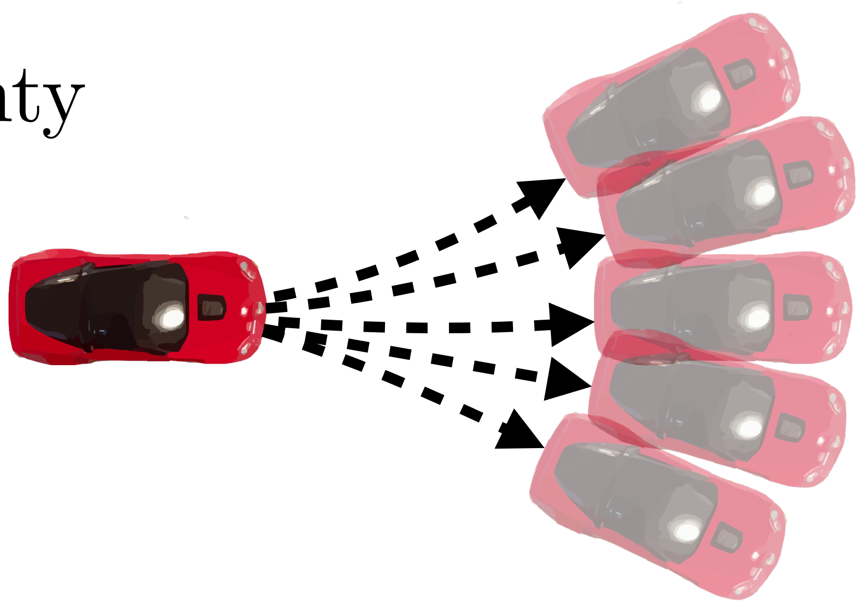
1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

What is the practical goal of modeling?

In theory - try to accurately model uncertainty



In practice - do we really need this?

1. We need something that is computationally cheap

(Bayes filter will sample repeatedly from this)

2. We need just enough stochasticity to explain any measurements we may see

(bayes filter will use measurements to hone in on the right state)

3. We need a model that can deal with unknown unknowns

(No matter what the model, we need to overestimate uncertainty)

Key Idea:

Simple model + Stochasticity

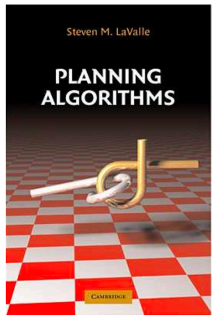
Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

Kinematics: A simple car



Chapter 13

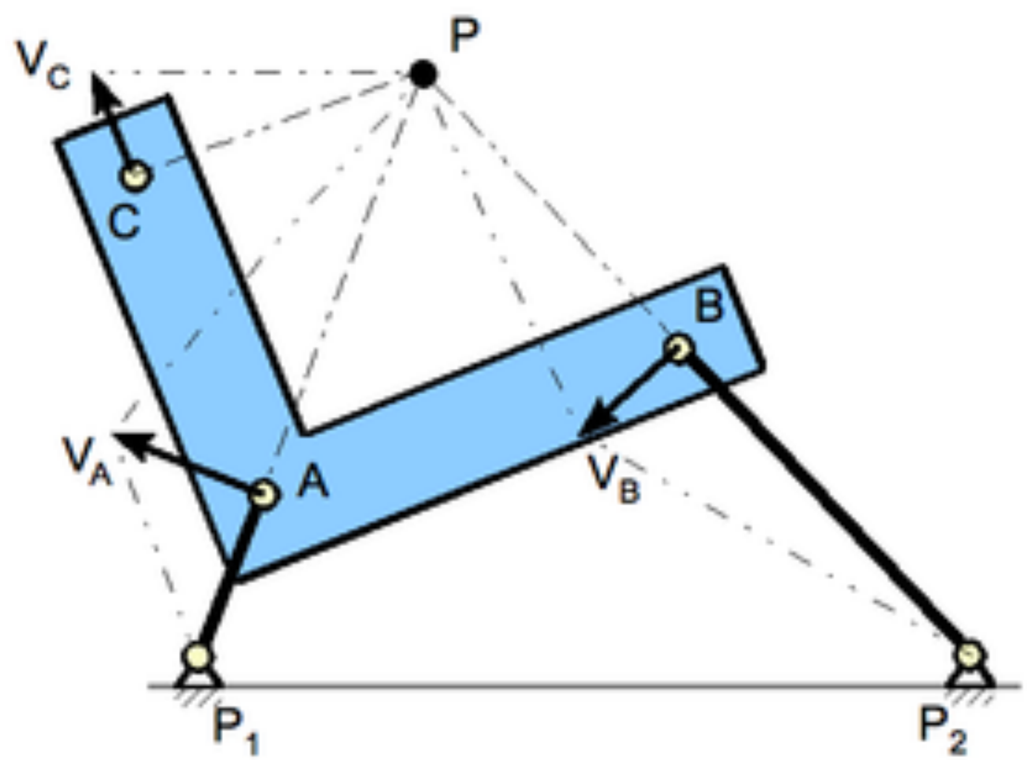
Kinematic model governs how wheel speeds map to robot velocities

Dynamic model governs how wheel torques map to robot accelerations

We will ignore the dynamics and focus on the kinematics
(assume we can set the speed directly)

Assume wheels rolls on hard, flat, horizontal ground without slipping

Aside I: Instant centre of rotation

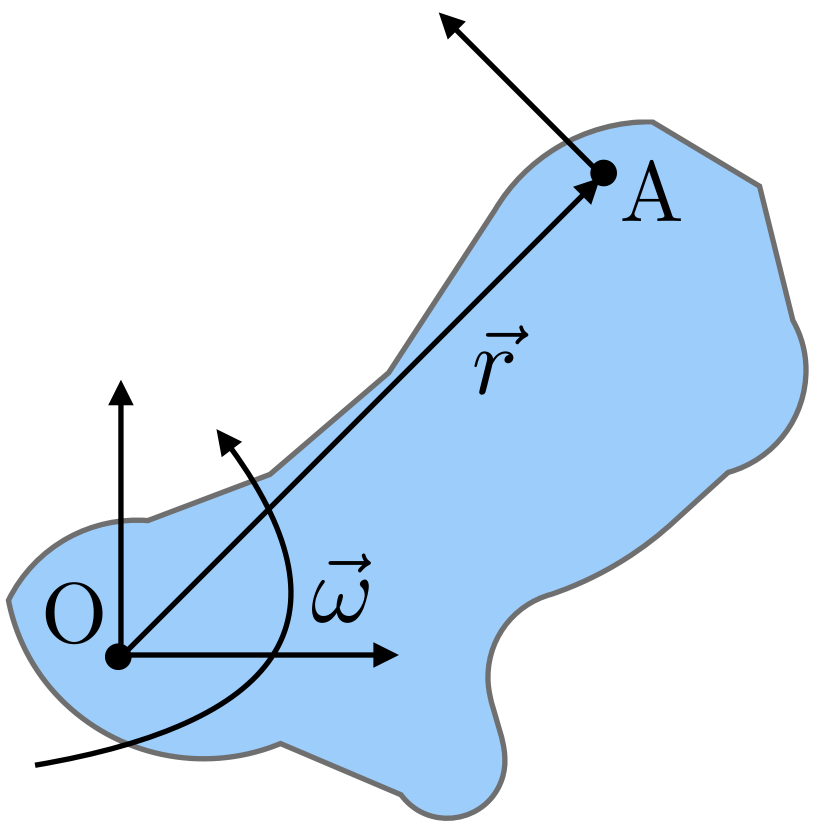


Any displacement (translation + rotation) of a rigid body can be modeled as pure rotation about an instant centre of rotation (CoR).

rigid body = a solid body whose deformation is negligible

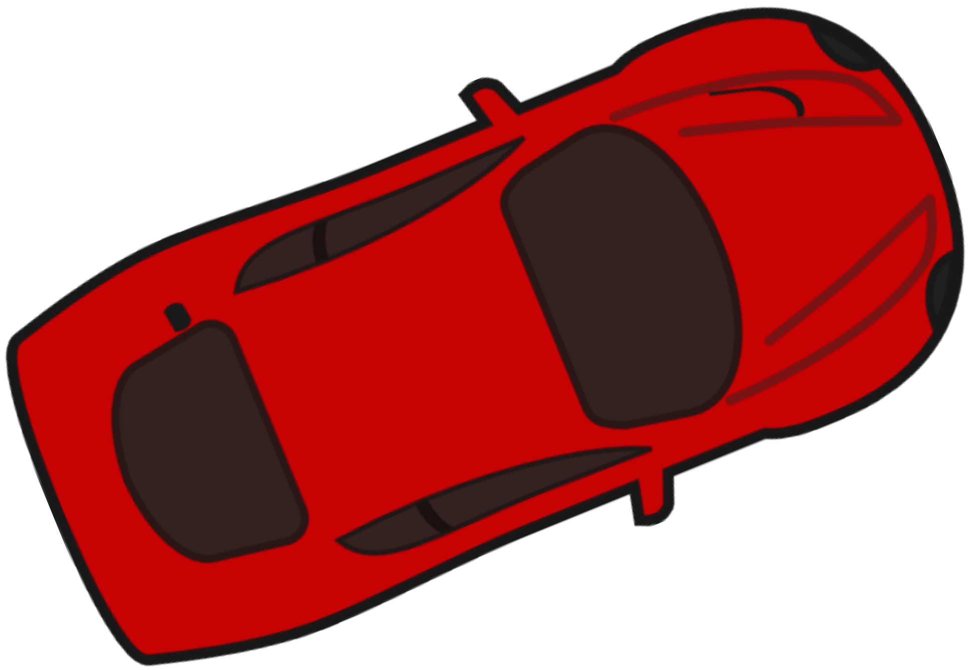
Aside II: Velocity on Rotating Body

$$\vec{V} = \vec{\omega} \times \vec{r} = |\vec{\omega}||\vec{r}| \sin(\theta)$$



Motion model

$$P(x_t | u_t, x_{t-1})$$

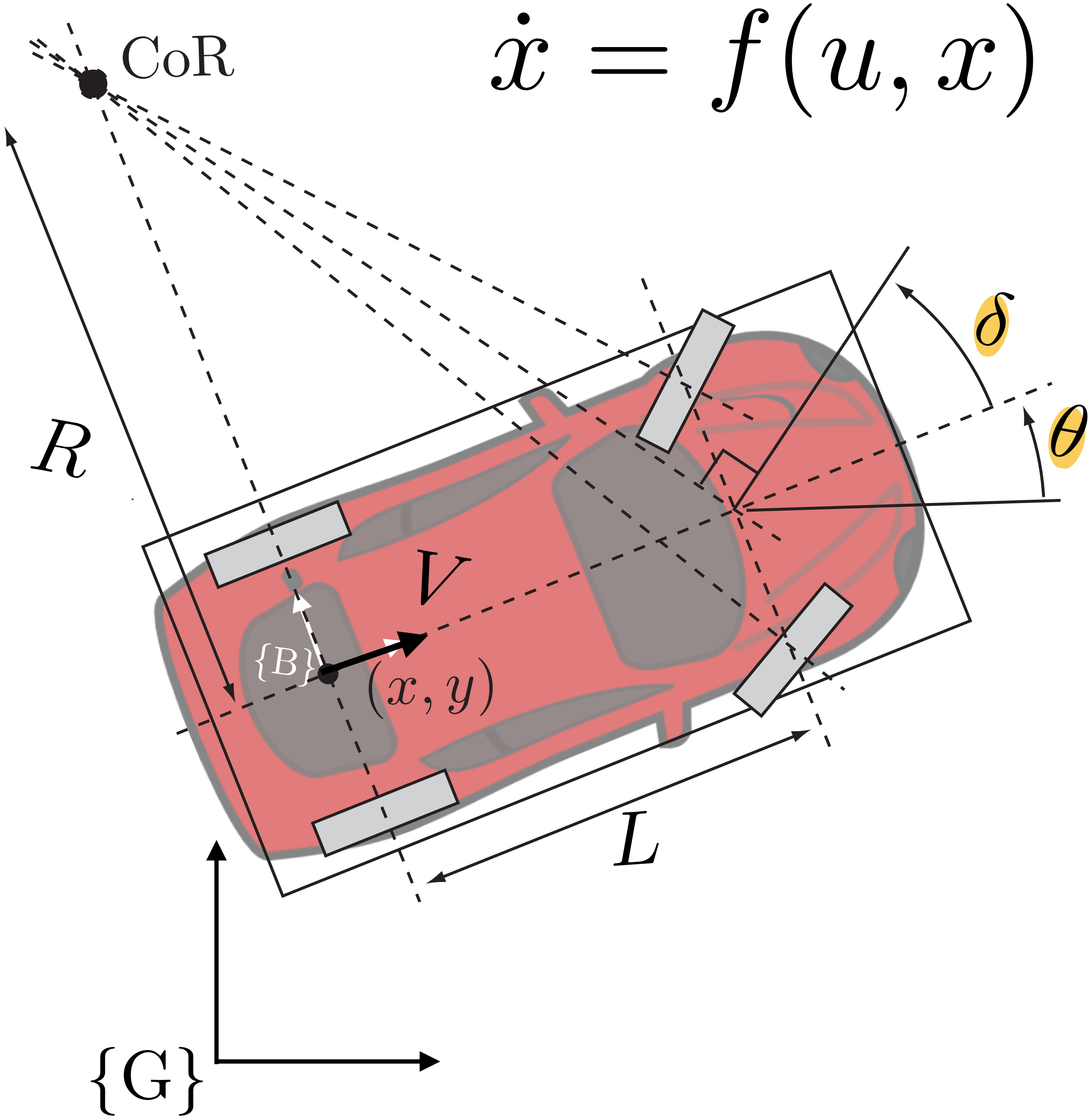


$$\begin{bmatrix} \text{x} \\ \text{y} \\ \theta \end{bmatrix} \begin{array}{l} \text{(x-coord)} \\ \text{(y-coord)} \\ \text{(heading)} \end{array}$$

$$\begin{bmatrix} V \\ \delta \end{bmatrix} \begin{array}{l} \text{(speed)} \\ \text{(steering angle)} \end{array}$$

Motion model: Equations of Motion

$$\dot{x} = f(u, x)$$



$$\tan\delta = \frac{L}{R}$$

$$R = \frac{L}{\tan\delta}$$

$$\omega = \frac{V}{R} = \frac{V\tan\delta}{L}$$

$$\dot{x} = V\cos(\theta)$$

$$\dot{y} = V\sin(\theta)$$

$$\dot{\theta} = \omega = \frac{V\tan\delta}{L}$$

Discretize: Numerical integration

$$\dot{\theta} = \frac{V}{L} \tan \delta$$

Assume that steering angle is piece-wise constant from t to t+1

$$\theta_{t+1} - \theta_t = \int_t^{t+\Delta t} \dot{\theta} dt \quad \Rightarrow \quad \theta_{t+1} = \theta_t + \frac{V}{L} \tan \delta \Delta t$$

$$x_{t+1} - x_t = \int_t^{t+\Delta t} \dot{x} dt = \int_t^{t+\Delta t} V \cos \theta(t) dt = \int_t^{t+\Delta t} \frac{V \cos \theta}{\dot{\theta}} \frac{d\theta}{dt} dt$$

$$= \frac{L}{\tan \delta} \int_t^{t+\Delta t} \cos \theta d\theta = \frac{L}{\tan \delta} (\sin \theta_{t+1} - \sin \theta_t)$$

$$y_{t+1} - y_t = \frac{L}{\tan \delta} (-\cos \theta_{t+1} + \cos \theta_t)$$

Why is the motion model probabilistic?

If we know how to write out equations of motion,
we should be able to exactly predict where a body ends up, right?

What are the sources of ~~noise~~ stochasticity?

Category	**Example**
Control signal error	Voltage discretization, communication lag
Unknown physics parameters	Friction of carpet, tire pressure
Incorrect physics	Ignoring tire deformation, ignoring wheel slippage

Stochasticity

1. Control signal error

$$\hat{V} \sim \mathcal{N}(V, \sigma_v^2) \qquad \hat{\delta} \sim \mathcal{N}(\delta, \sigma_\delta^2)$$

2. Unknown physics parameters

$$\hat{L} \sim \mathcal{N}(L, \sigma_L^2)$$

3. Incorrect physics

$$\hat{x} \sim \mathcal{N}(x, \sigma_x^2) \qquad \hat{y} \sim \mathcal{N}(y, \sigma_y^2) \qquad \hat{\theta} \sim \mathcal{N}(\theta, \sigma_\theta^2)$$

Good Exercises

1. Equations of motion for front axle

2. Equations of motion for centre of mass