Advanced Vehicle Control Methods

Course 1, Module 6, Lesson 4

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

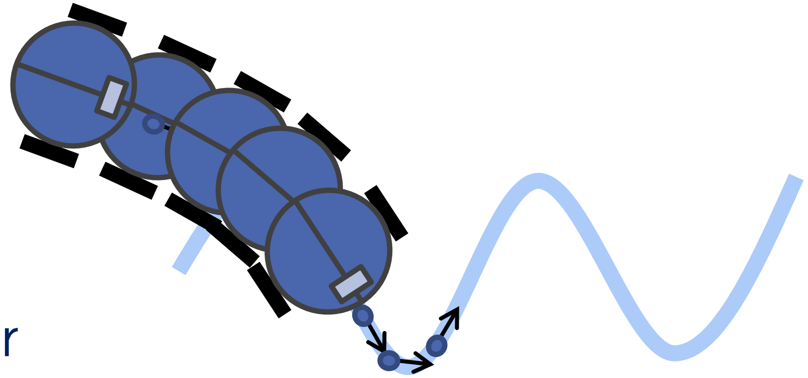
Learning Objectives

In this video, we will…

- Describe the MPC architecture and the concept of receding horizon control
- Formulate an MPC optimization problem for both linear and nonlinear models
- Apply MPC to joint longitudinal and lateral vehicle control

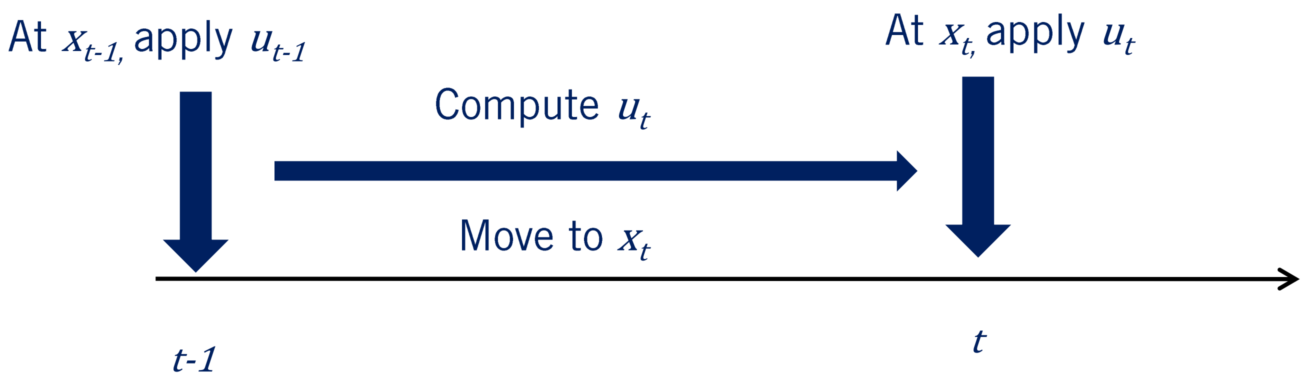
Model predictive control

- Model predictive control (MPC)
  - Numerically solving an optimization problem at each time step
  - Receding horizon approach
- Advantages of MPC
  - Straightforward formulation
  - Explicitly handles constraints
  - Applicable to linear or nonlinear models
- Disadvantages of MPC
  - Computationally expensive

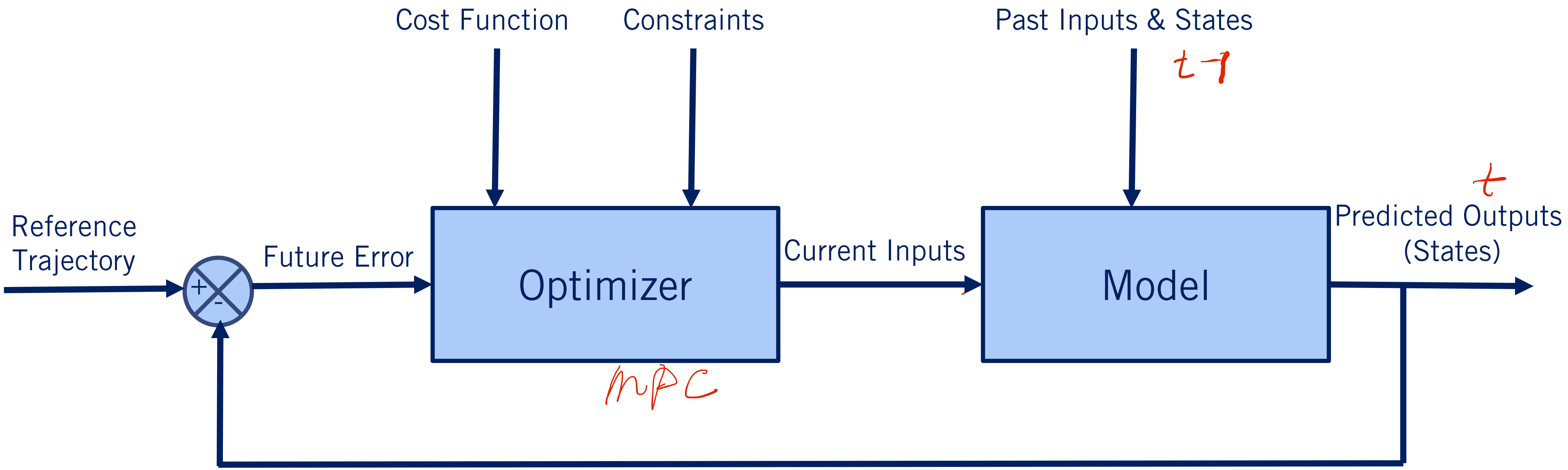


Receding horizon Control

- Receding Horizon Control Algorithm
  - Pick receding horizon length (T)
  - For each time step, t
  - Set initial state to predicted state, x_t
    - Perform optimization over finite horizon t to T while traveling from x_{t-1} to x_t
    - Apply first control command, u_t, from optimization at time t



MPC structure



Linear MPC formulation

- Linear time-invariant discrete time model:

$$x_{t+1} = Ax_t + Bu_t$$

- MPC seeks to find control policy U

$$U = \{u_{t|t}, u_{t+1|t}, u_{t+2|t}, \dots\}$$

- Objective function - regulation:

$$J(x(t), U) = \sum_{j=t}^{t+T-1} x_{j|t}^T Q x_{j|t} + u_{j|t}^T R u_{j|t}$$

- Objective function - tracking:

Weight matrix

$$\delta x_{j|t} = x_{j|t,des} - x_{j|t} \qquad J(x(t), U) = \sum_{j=t}^{t+T-1} \delta x_{j|t}^T Q \delta x_{j|t} + u_{j|t}^T R u_{j|t}$$

Linear MPC SOLUTION

- Unconstrained, finite horizon, discrete time problem formulation:

$$\min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) = x_{t+T|t}^{\mathrm{T}} \, Q_f \, x_{t+T|t} + \sum_{j=t}^{t+T-1} x_{j|t}^{\mathrm{T}} Q x_{j|t} + u_{j|t}^{\mathrm{T}} R u_{j|t}$$

$$s.t. \qquad x_{j+1|t} = A x_{t|t} + B u_{t|t} \, , \qquad t \le j \le t + T - 1$$

LQR

- Linear quadratic regulator, provides a closed form solution
  - Full state feedback: $u_t = -K x_t$
  - Control gain K is a matrix
  - Refer to supplemental materials

(Non)Linear MPC formulation

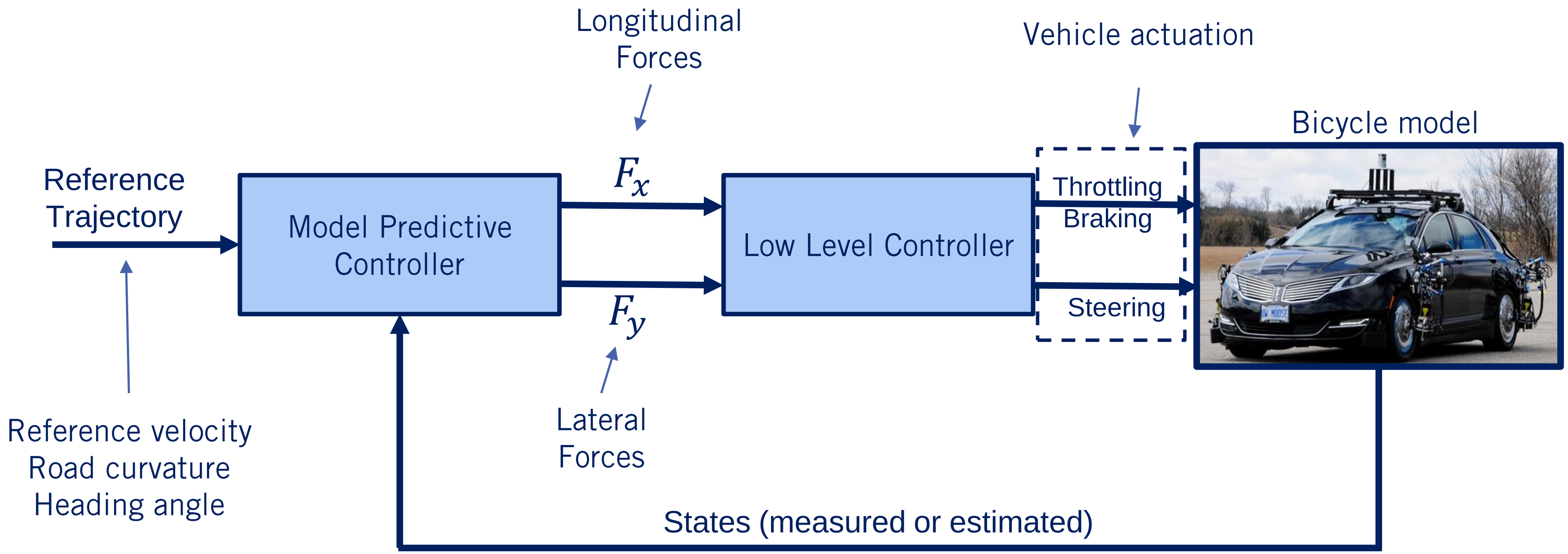
- Constrained (non)linear finite horizon discrete time case

$$\min_{U \triangleq \{u_{t|t}, u_{t+1|t}, \dots\}} J(x(t), U) = \sum_{j=t}^{t+T} C(x_{j|t}, u_{j|t})$$

$$\begin{aligned}
s.t. \quad & x_{j+1|t} = f\left(x_{j|t}, u_{j|t}\right), && t \le j \le t+T-1 \\
& x_{min} \le x_{j+1|t} \le x_{max}, && t \le j \le t+T-1 \\
& u_{min} \le u_{j|t} \le u_{max}, && t \le j \le t+T-1 \\
& g\left(x_{j|t}, u_{j|t}\right) \le 0, && t \le j \le t+T-1 \\
& h\left(x_{j|t}, u_{j|t}\right) = 0, && t \le j \le t+T-1
\end{aligned}$$

- No closed form solution, must be solved numerically

Vehicle Lateral Control



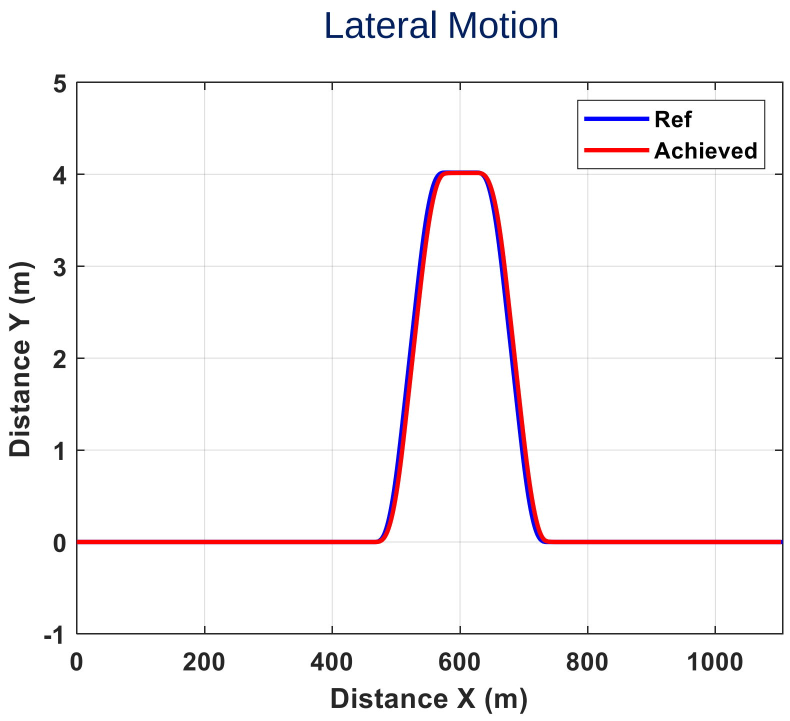
Model Predictive Controller

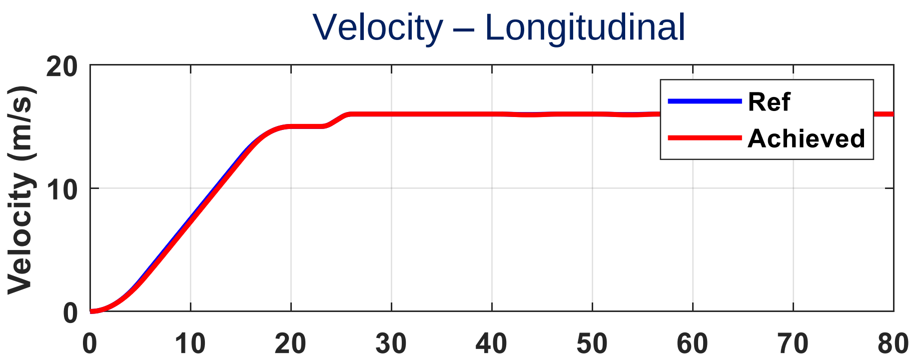
- Cost Function - Minimize
  - Deviation from desired trajectory
  - Minimization of control command magnitude
- Constraints - Subject to
  - Longitudinal and lateral dynamic models
  - Tire force limits

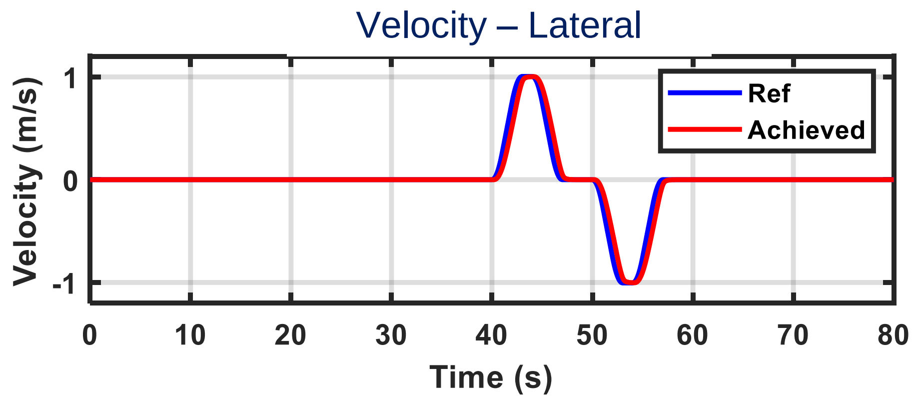
- Can incorporate low level controller, adding constraints for:
  - Engine map
  - Full dynamic vehicle model
  - Actuator models
  - Tire force models

Vehicle Lateral Control

- Vehicle trajectory (double lane change)

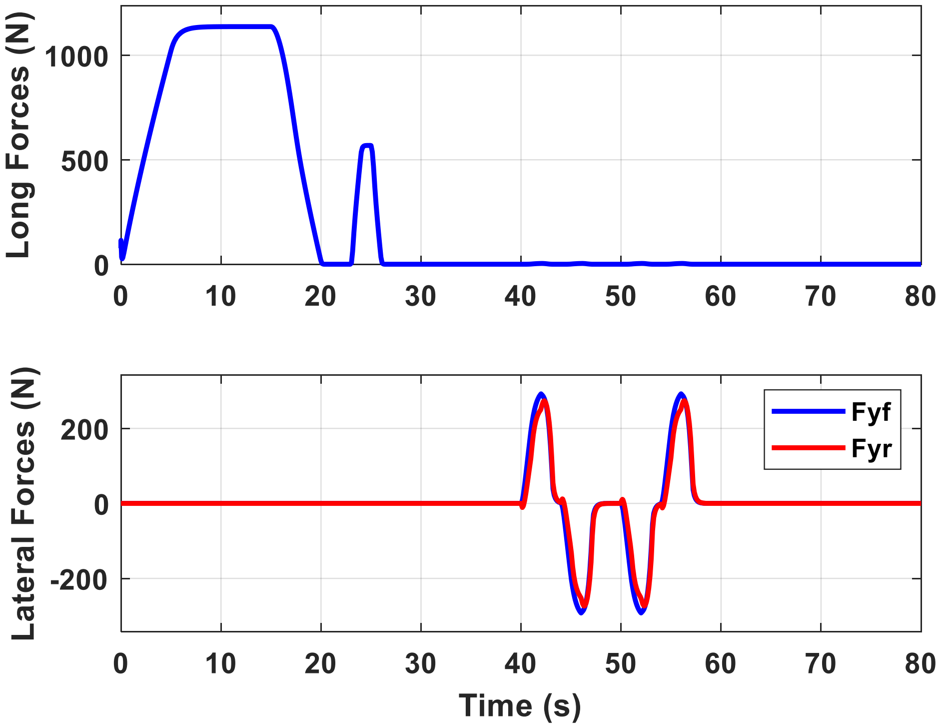




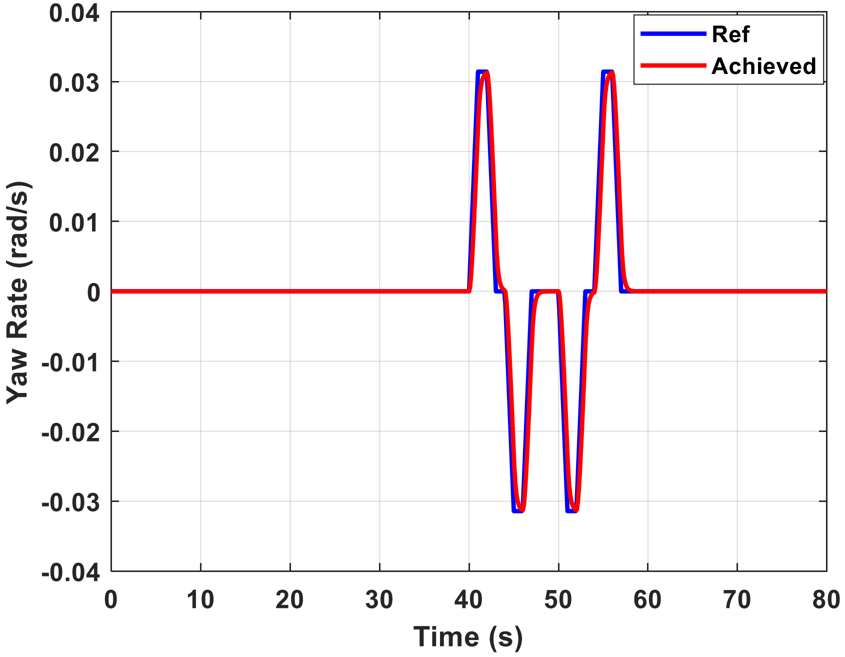


Vehicle Lateral Control

Tire Forces



Yaw Rate



Summary

What we have learned from this lesson:

- The concepts of Model Predictive Control (MPC)
- MPC costs and constraints
- Applied MPC to a lane change maneuver

Summary

In this module, you …

- Defined the lateral path tracking problem
- Applied two geometric path tracking controllers, the pure pursuit and Stanley controllers, to the path tracking problem.
- And define a model predictive controller for joint lateral and longitudinal control.

What is next?

- Apply lateral and longitudinal control in the Carla simulator