

Probabilistic Models II

Instructor: Chris Mavrogiannis

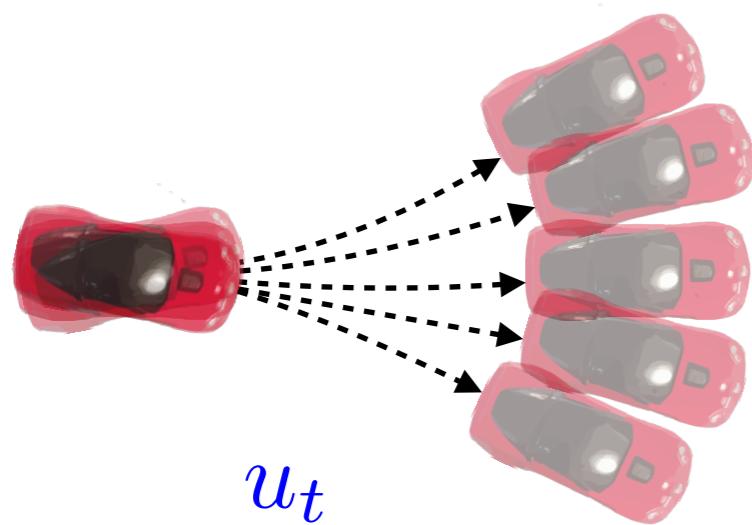
TAs: Kay Ke, Gilwoo Lee, Matt Schmittle

*Slides based on or adapted from Sanjiban Choudhury

Probabilistic Models in Localization

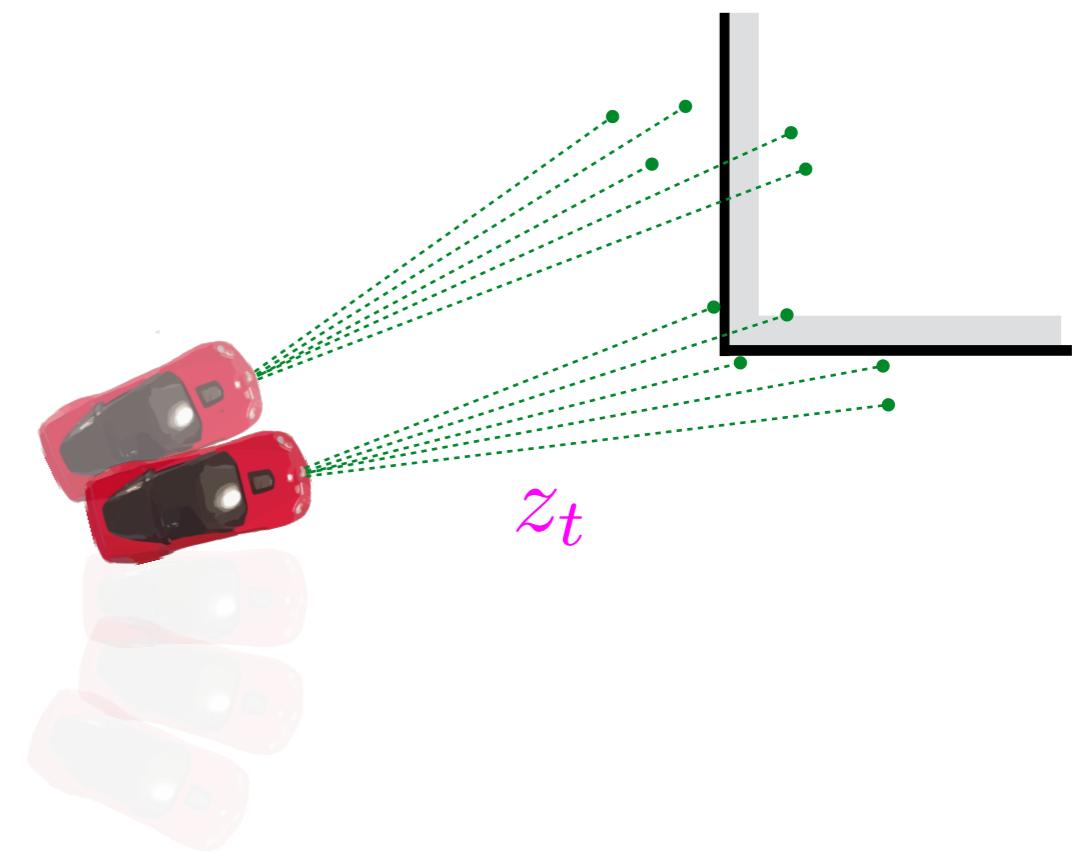
Motion model

$$P(x_t | \mathbf{u}_t, x_{t-1})$$



Measurement model

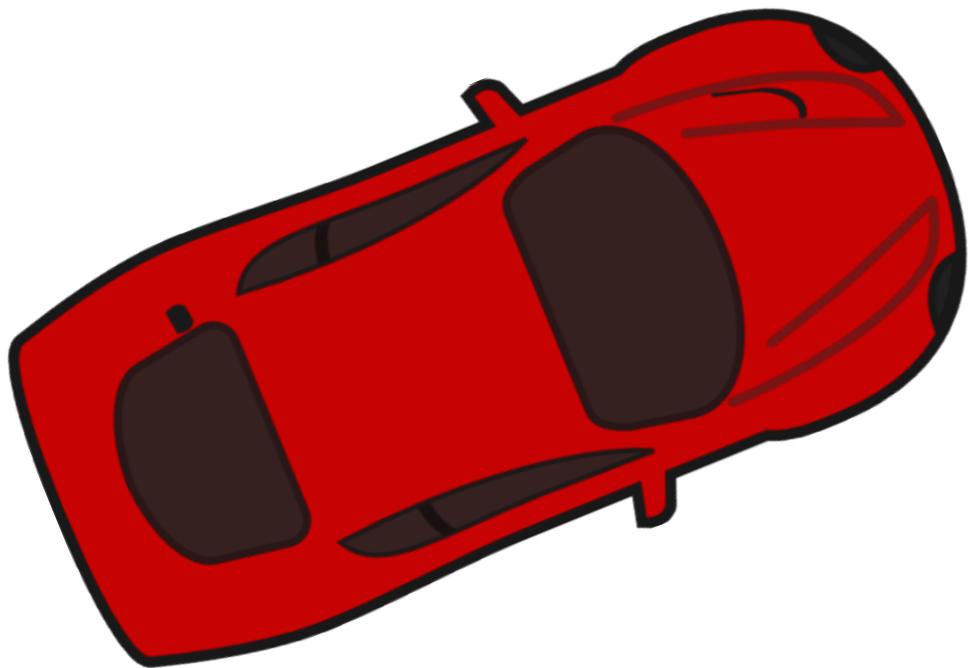
$$P(\mathbf{z}_t | x_t)$$



Key Idea:

Simple model + Stochasticity

Simple Model



$$\dot{x} = V \cos(\theta)$$

$$\dot{y} = V \sin(\theta)$$

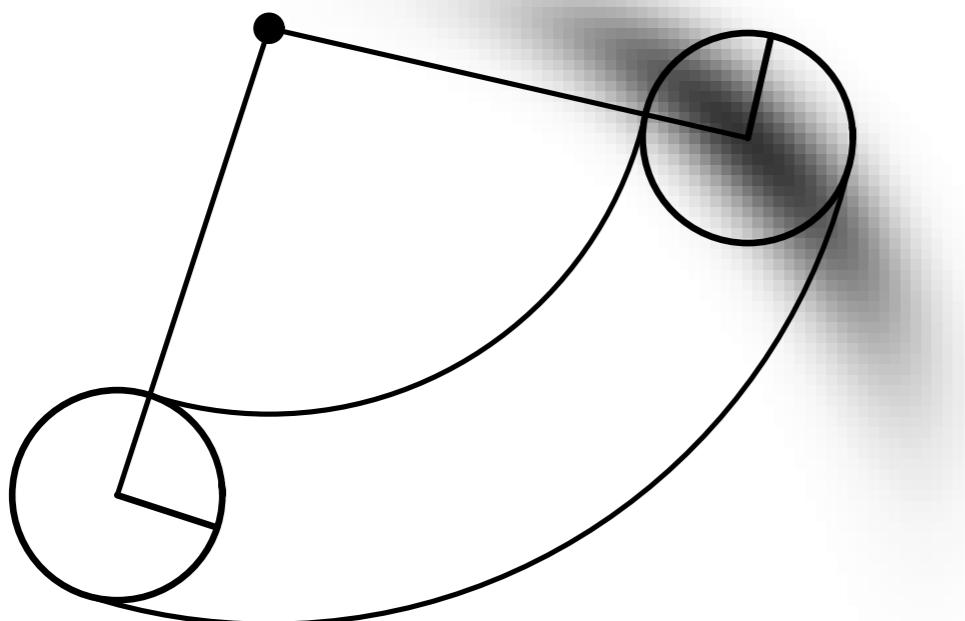
$$\dot{\theta} = \omega = \frac{V \tan \delta}{L}$$

Why is it simple?

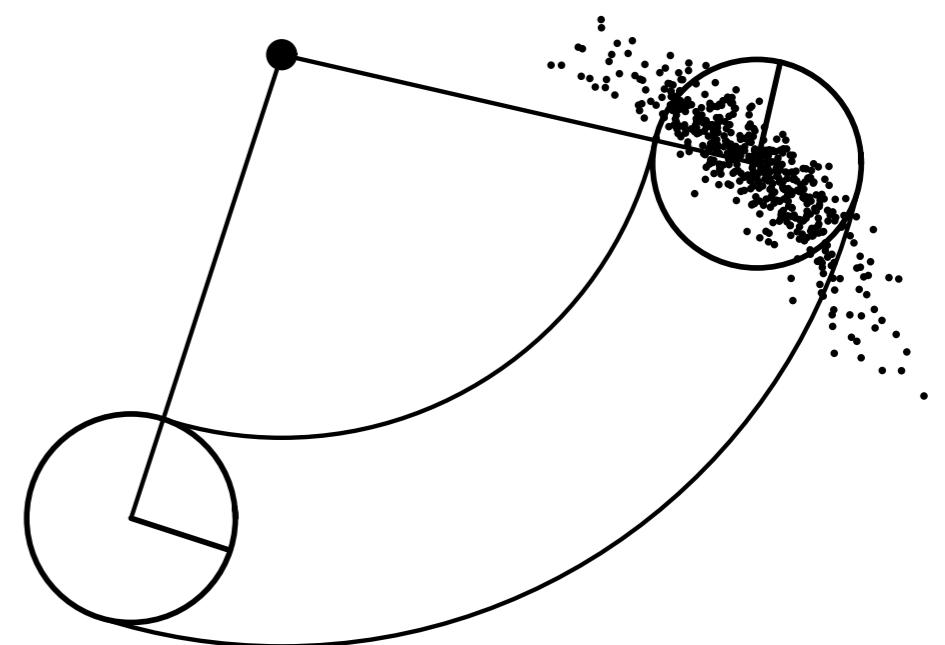
- Simplified physics
- Assumes perfect actuation
- Assumes perfect knowledge of design parameters

Accounting for Stochasticity

$$\dot{x} = f(u, x) + \text{Noise}$$



Probability density
function

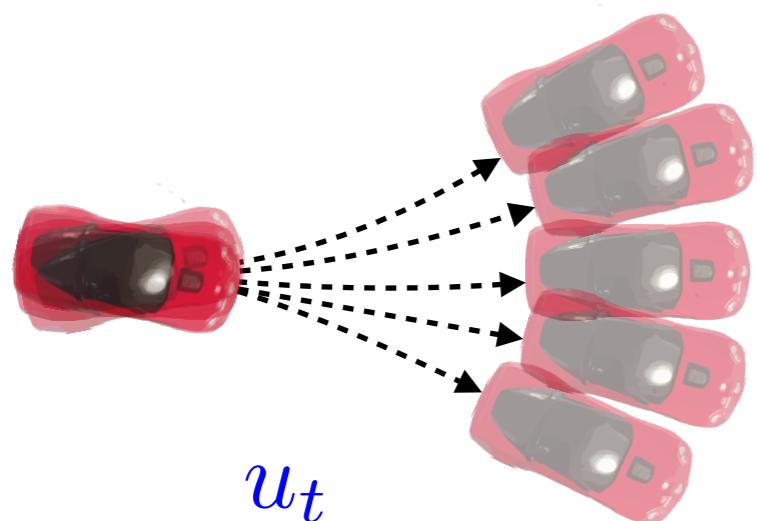


Samples from the
pdf

Probabilistic models in localization

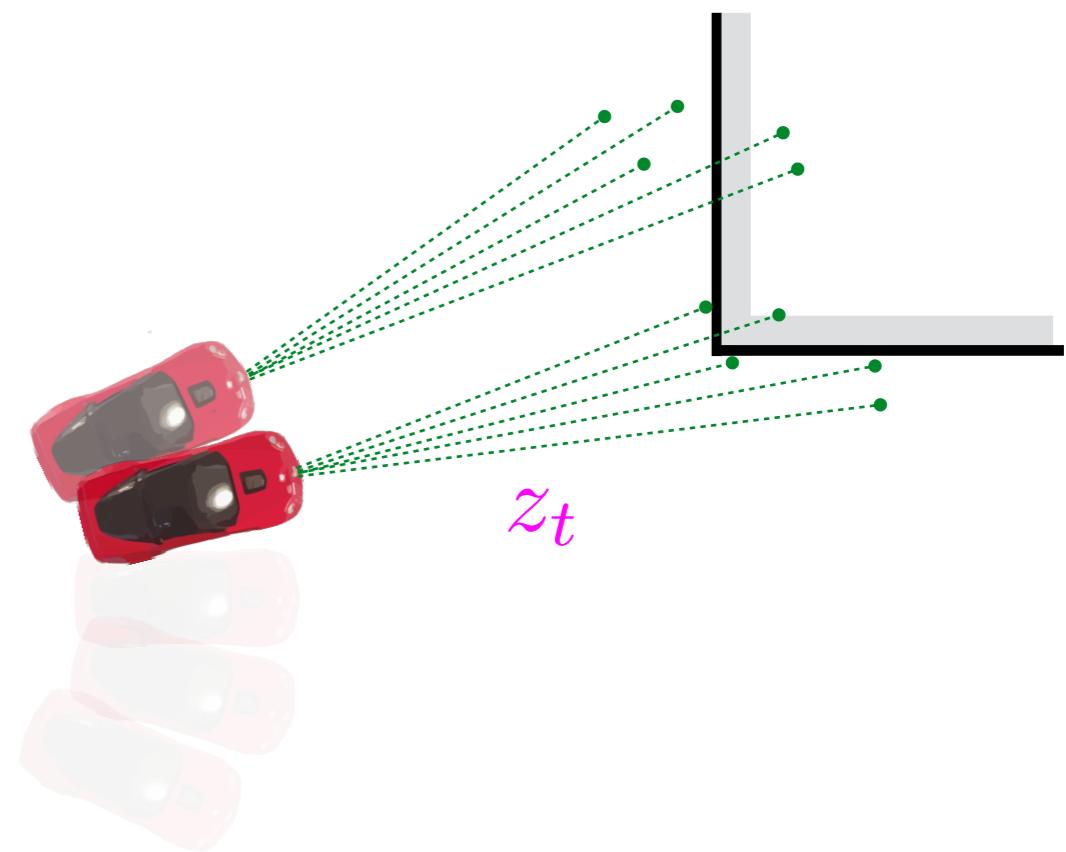
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Measurement model

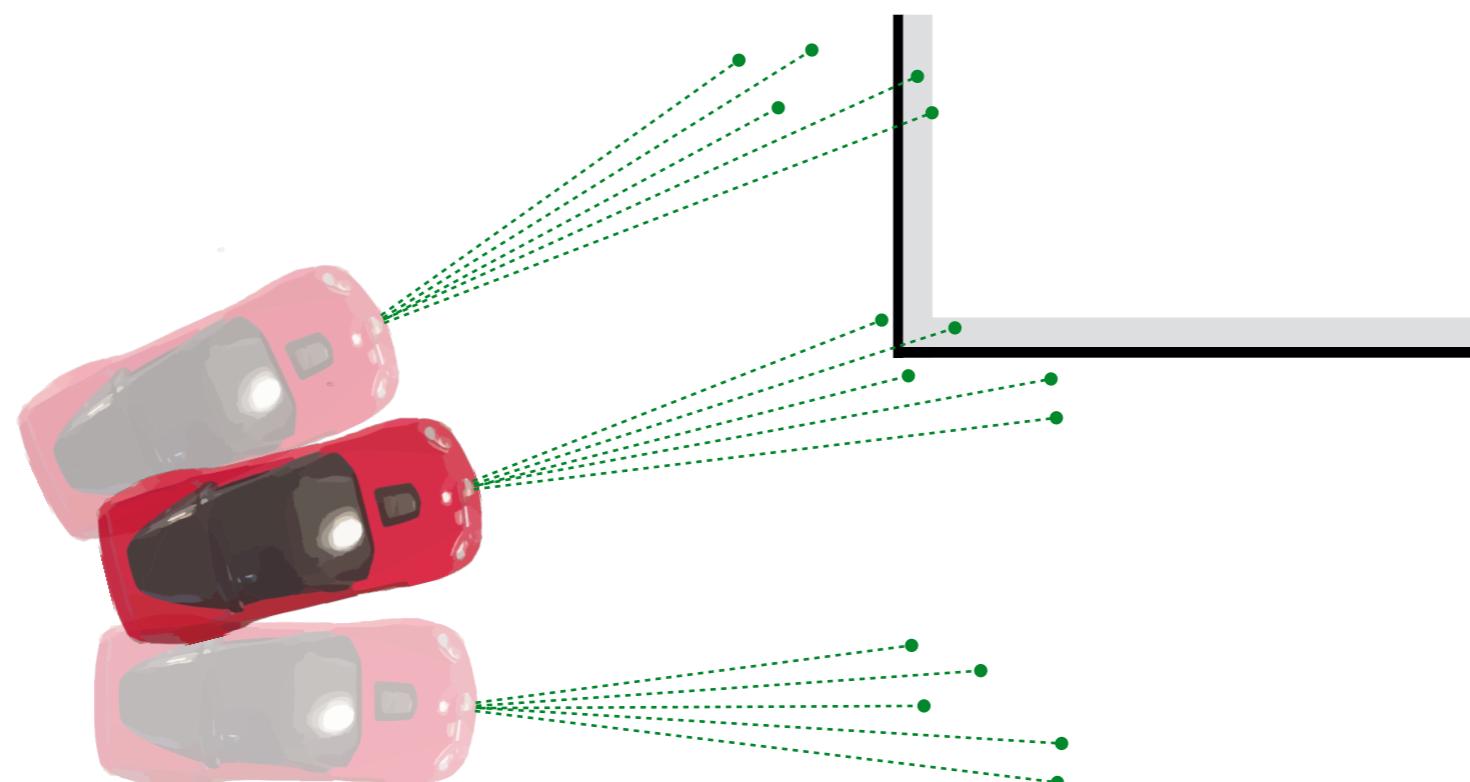
$$P(\mathbf{z}_t | x_t)$$



Measurement Model

$$P(z_t | x_t, m)$$

sensor
reading state map

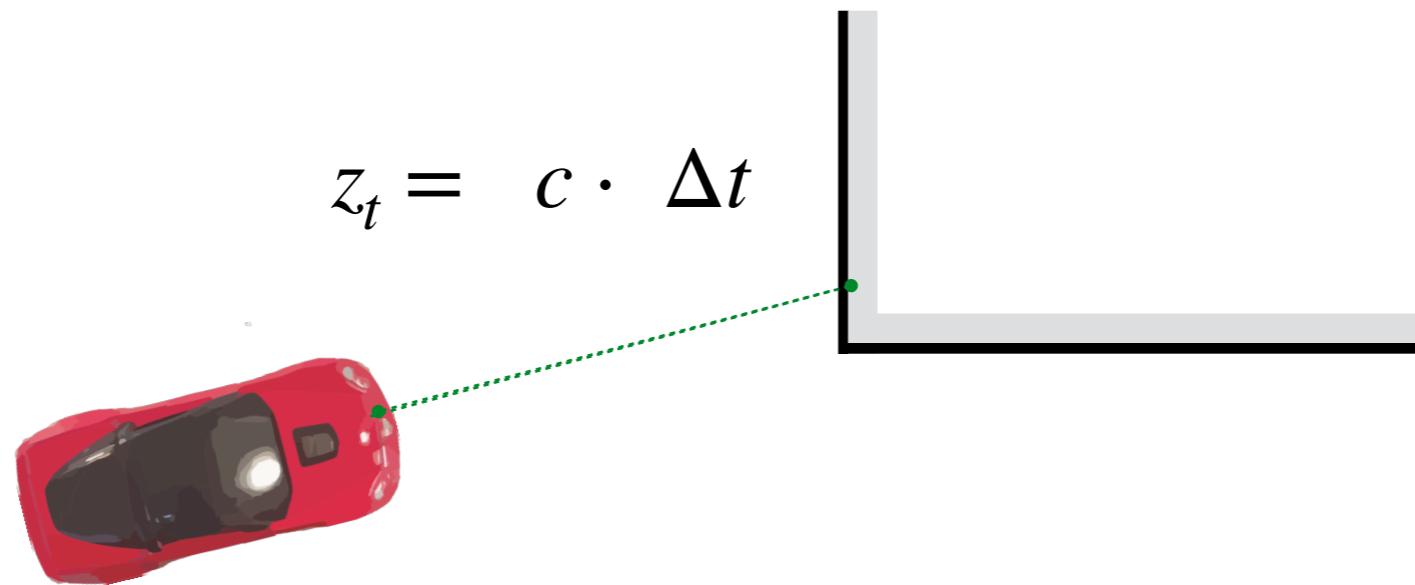


LiDAR

- Light + RaDAR
- Light Detection And Ranging
- A distance sensor
- Everywhere, self-driving cars ... your racecar
- Edward Hutchinson Synge (1930)

How does a LiDAR work?

- Source emits photons
- Photons bounce on objects
- Return to receiver
- Measure time between emission-reception
- Multiply by speed of light
- Get distance measurement

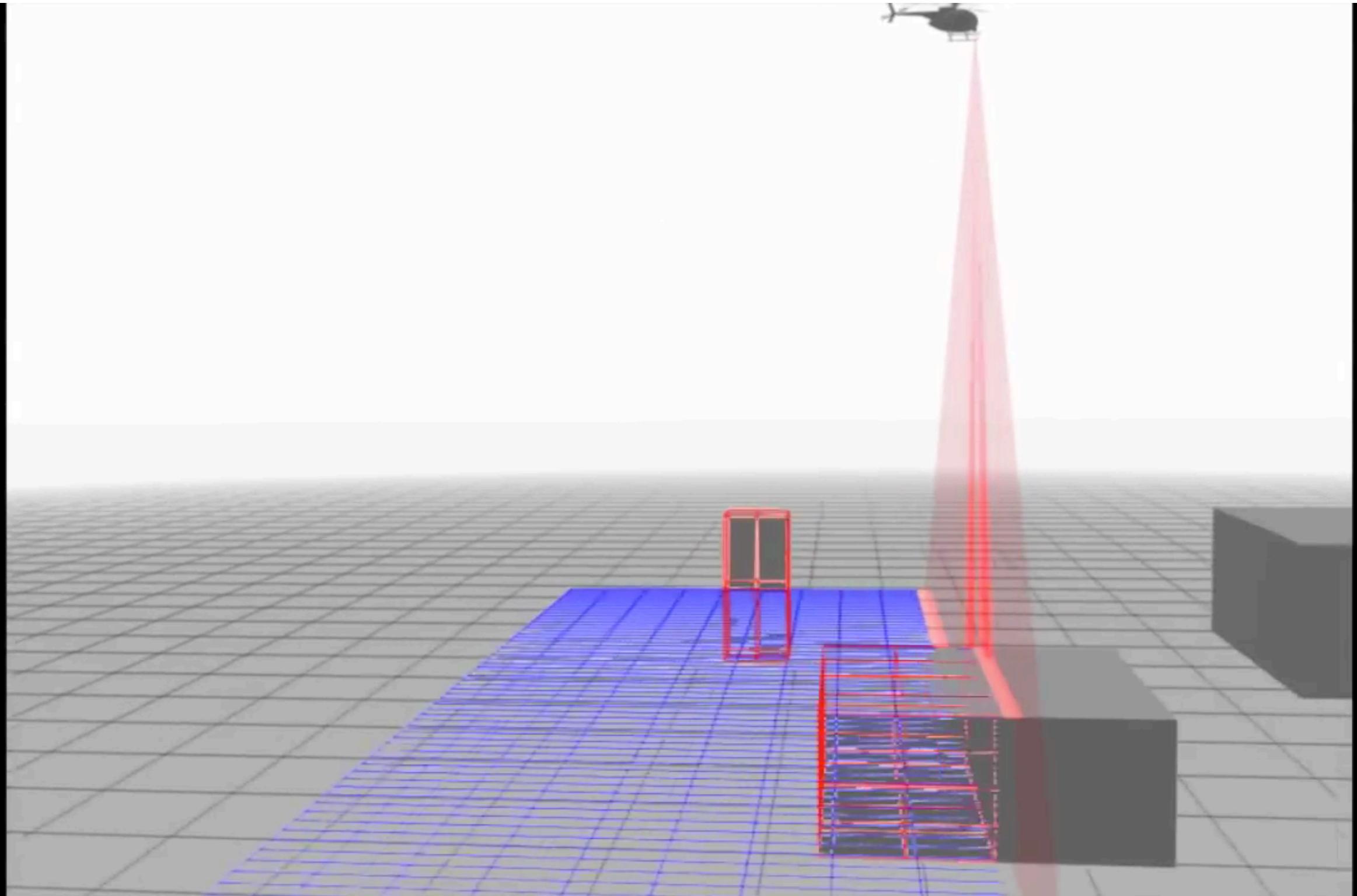


How does a LiDAR work?



<https://www.youtube.com/watch?v=NZKvf1cXe8s&t=67s>

Working with Lasers in the Real World



(courtesy Lyle Chamberlain)

Some Pros & Cons

- Fast ($\sim 1M$ photons per second)
- Accurate
- Does not require external light
- Works remarkably well in the real world
- High resolution images
- Photons pass through glass
- Sensitive to weather conditions (rain, snow, ...)
- High-res LiDAR might be expensive

Three Questions You Should Ask

1. Why is the model probabilistic?
2. What defines a good model?
3. What model should I use for my robot?

Why is the Measurement Model Probabilistic?

Several sources of stochasticity

Three Questions You Should Ask

1. Why is the model probabilistic?
2. What defines a good model?
3. What model should I use for my robot?

What Defines a Good Model?

Good news: LiDAR is very precise!

A handful of measurements is enough to localize robot

However, has distinct modes of failures

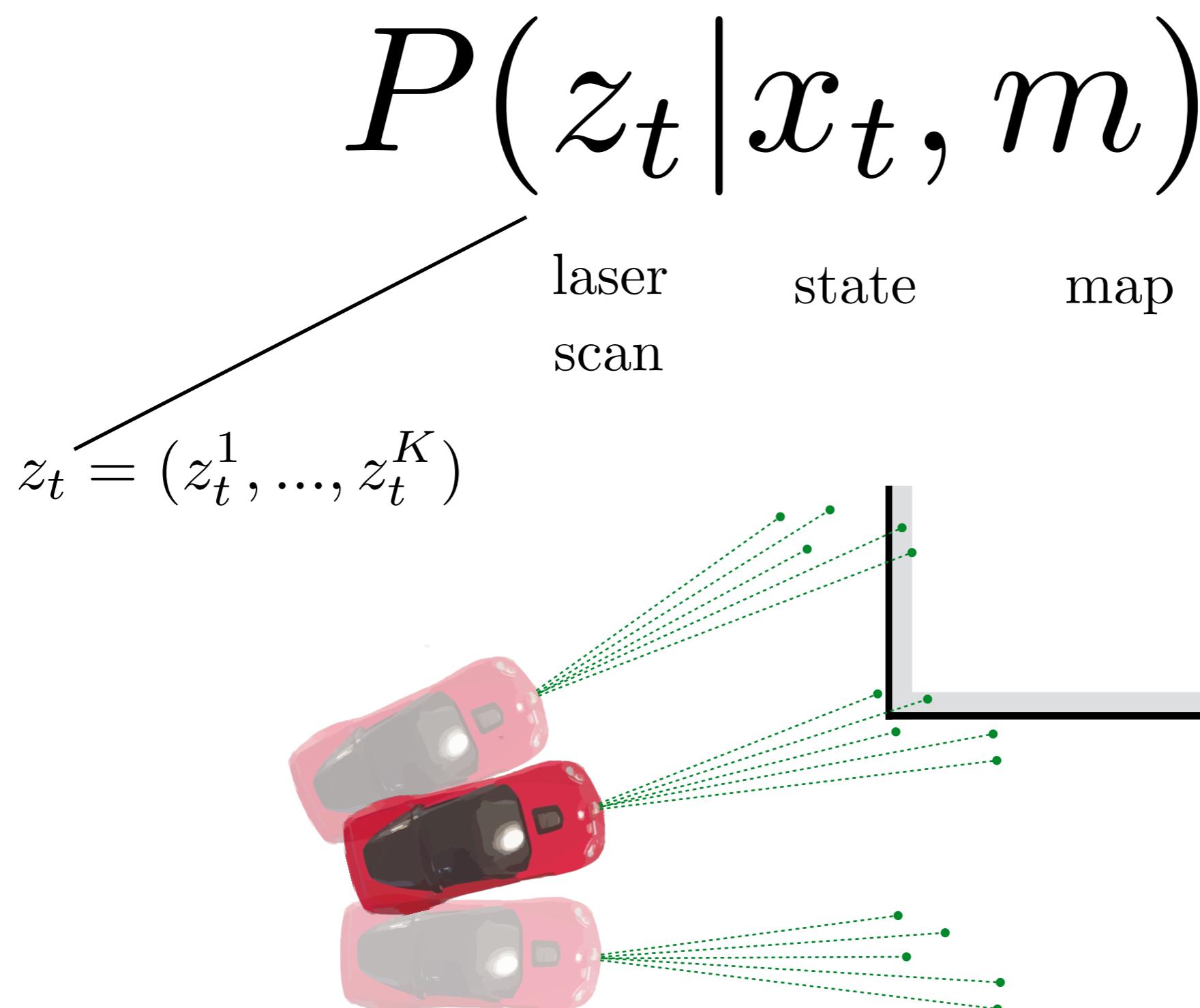
Problem: Overconfidence in measurement can be catastrophic

Solution: Anticipate specific types of failures and add stochasticity accordingly.

Three questions you should ask

1. Why is the model probabilistic?
2. What defines a good model?
3. What model should I use for my robot?

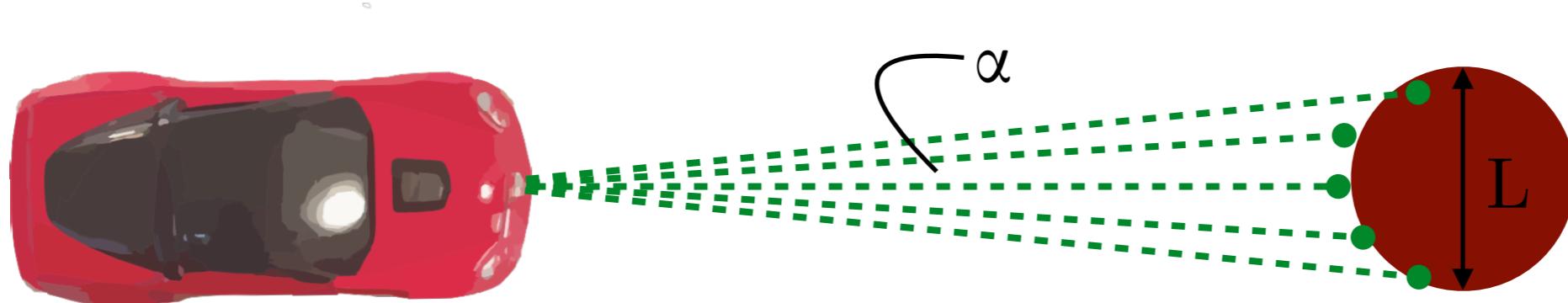
Measurement Model for LiDAR



Beam-based Model for LiDAR

Assume individual beams are conditionally independent given map

When is this assumption invalid?



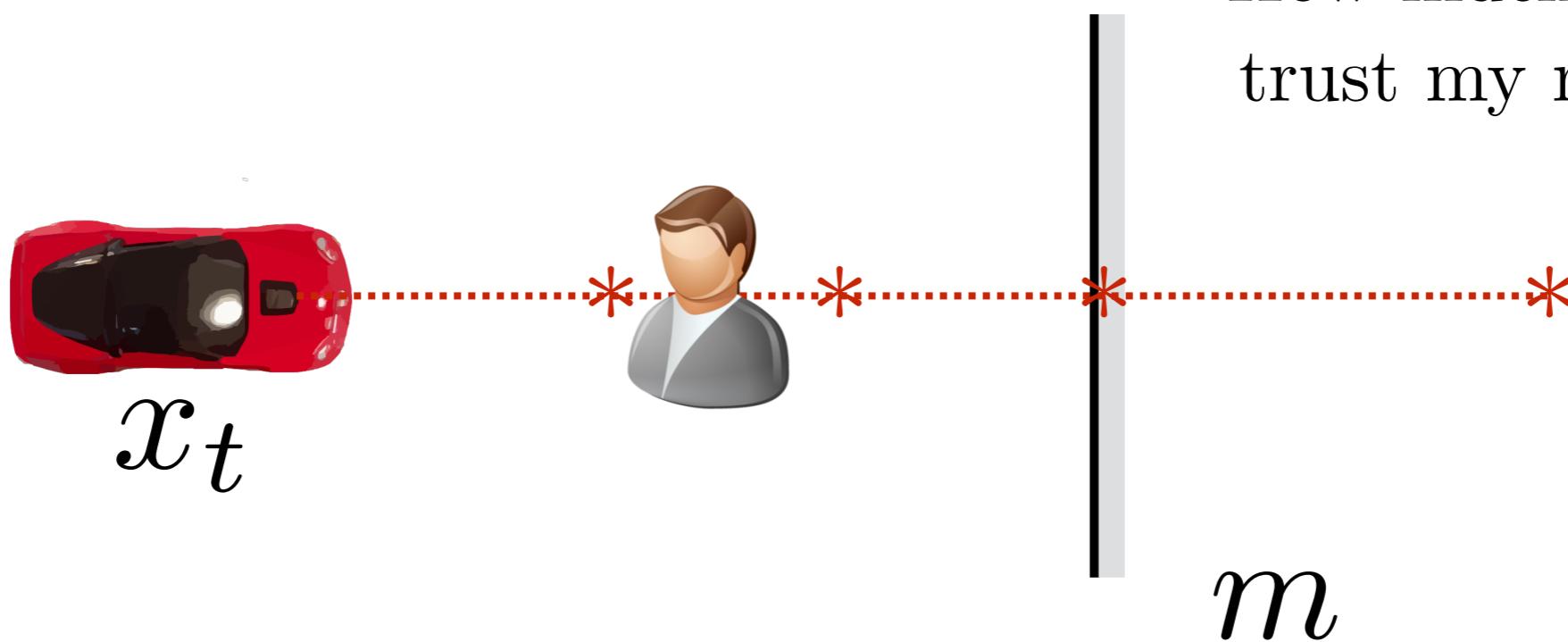
Example: Small α + Large L = Beams correlated

Measurement Model for Single Beam

$$P(z_t^k | x_t, m)$$

distance state map
value

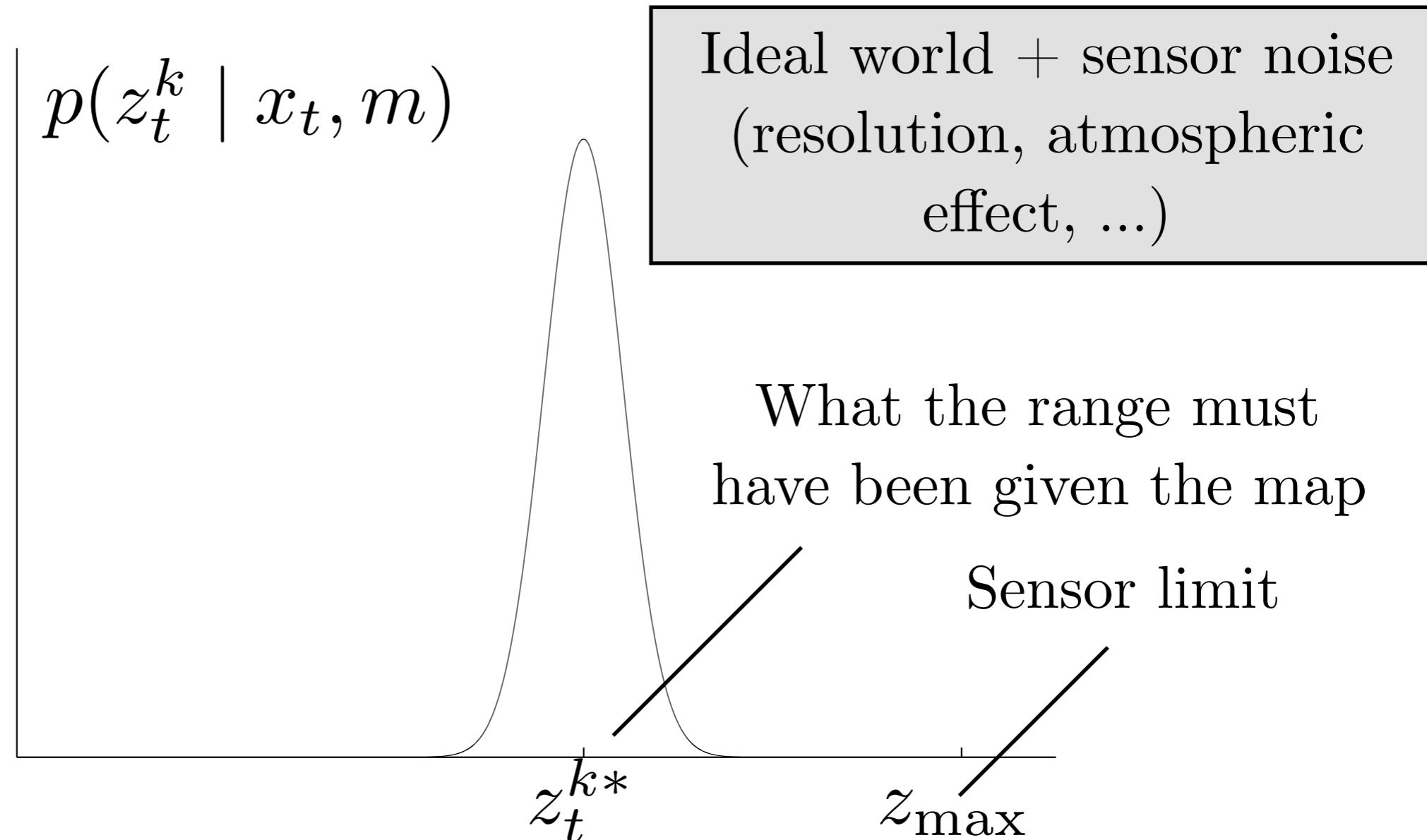
How much should I
trust my readings?



Typical Sources of Stochasticity

1. Simple measurement noise in distance value
2. Presence of unexpected objects
3. Sensor failures
4. Randomness

Factor 1: Simple Measurement Noise



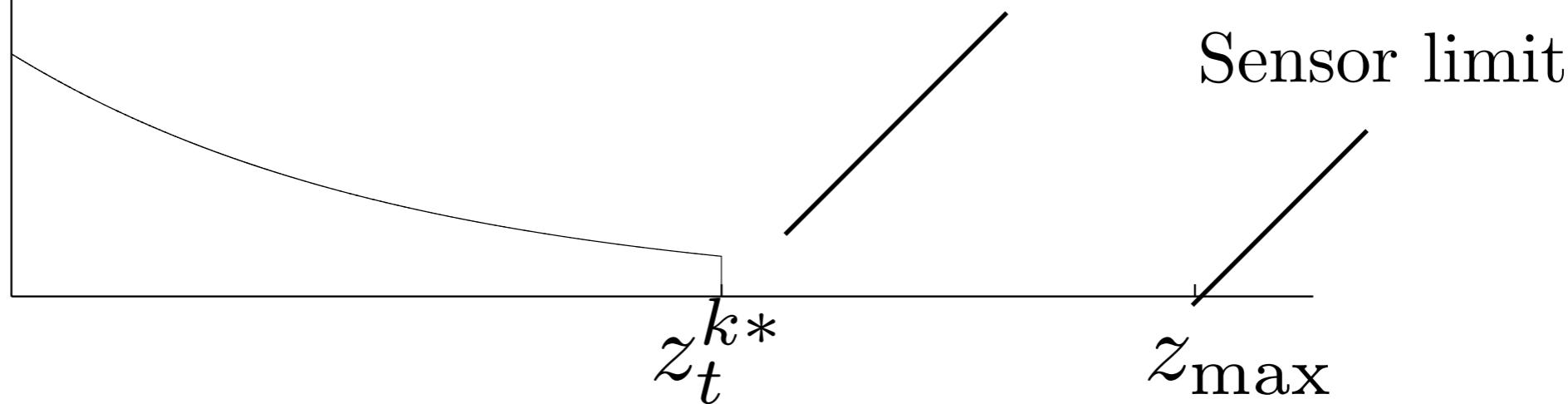
$$p_{\text{hit}}(z_t^k | x_t, m) = \begin{cases} \eta \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \leq z_t^k \leq z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 2: Unexpected Objects

$$p(z_t^k \mid x_t, m)$$

The shorter the reading, the higher the likelihood of an unexpected object (e.g., a human)

What the range must have been given the map



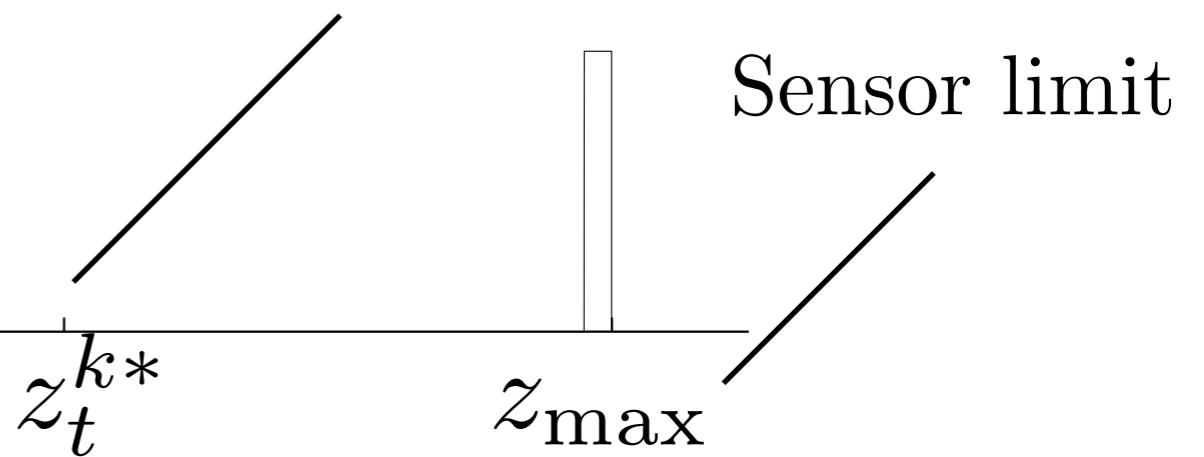
$$p_{\text{short}}(z_t^k \mid x_t, m) = \begin{cases} \eta \lambda_{\text{short}} e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \leq z_t^k \leq z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

Factor 3: Sensor Failures

$$p(z_t^k \mid x_t, m)$$

Physics (e.g., black, light-absorbing objects, ...);
happens frequently

What the range must
have been given the map



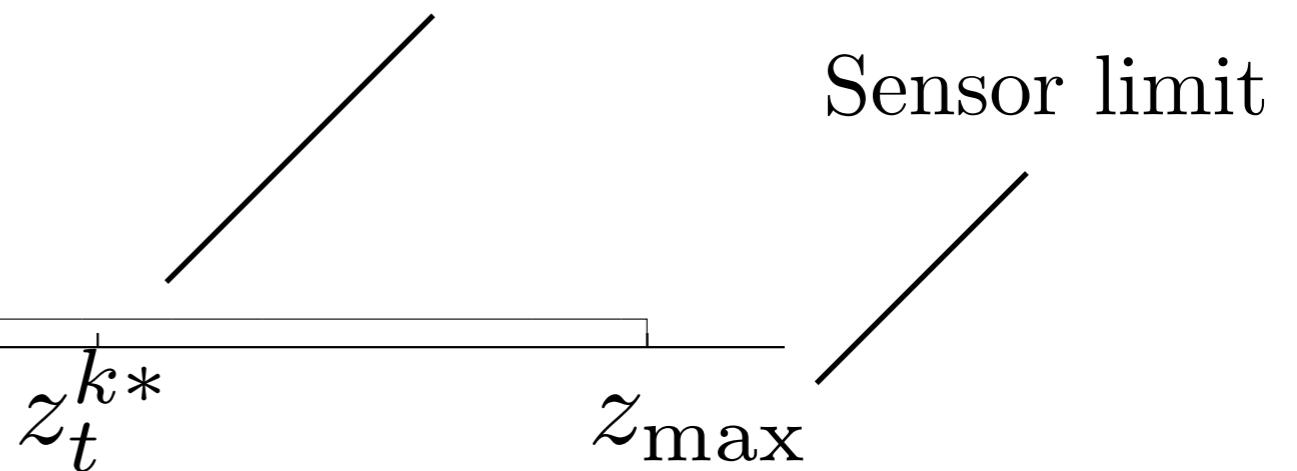
$$p_{\max}(z_t^k \mid x_t, m) = I(z = z_{\max}) = \begin{cases} 1 & \text{if } z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 4: Random Measurements

$$p(z_t^k \mid x_t, m)$$

Unexplained measurements
(e.g., bouncing off walls,
cross-talk between sensors)

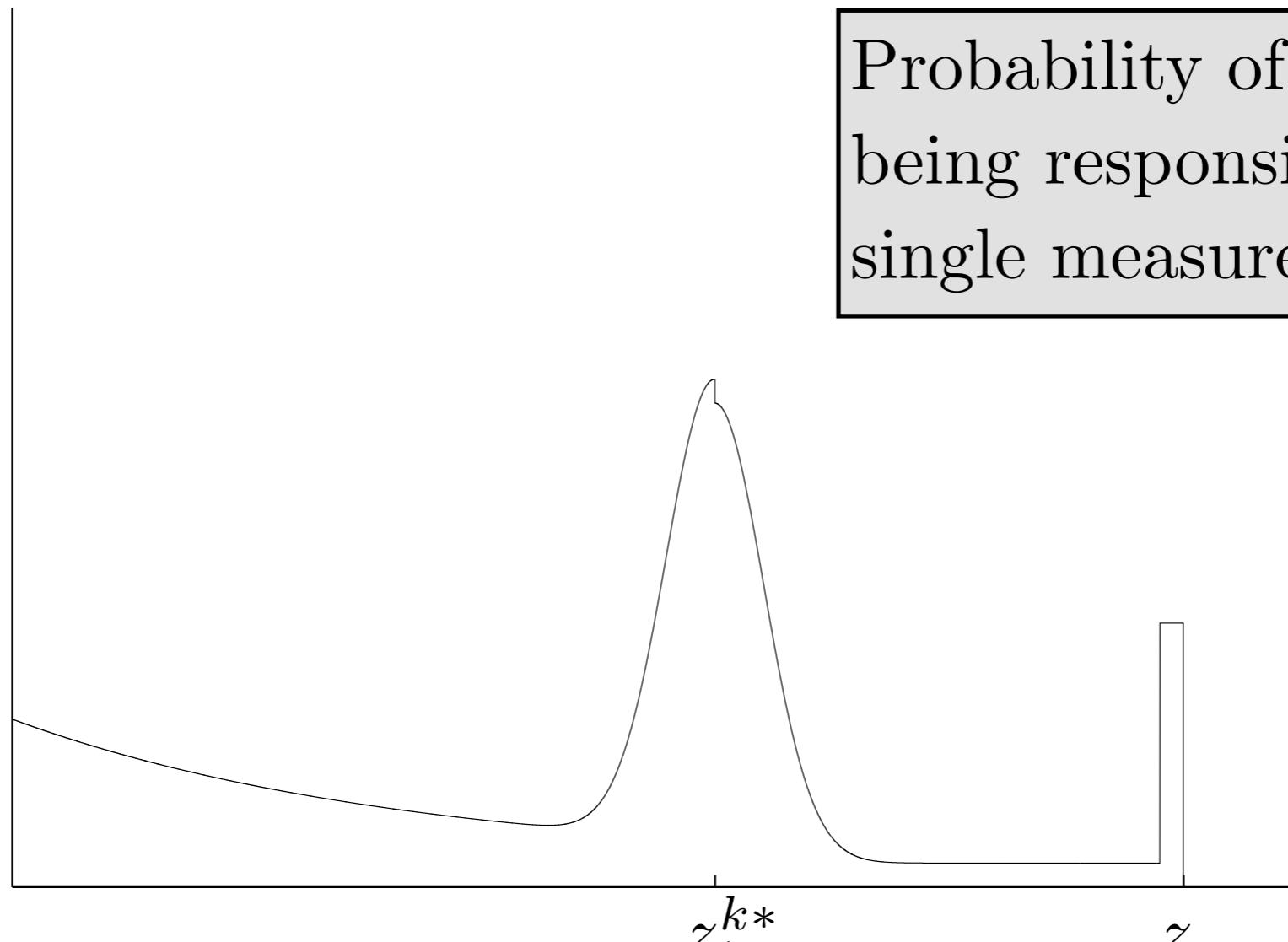
What the range must
have been given the map



$$p_{\text{rand}}(z_t^k \mid x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \leq z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Combined Measurement Model

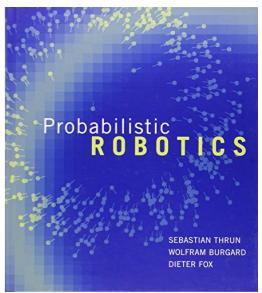
Probability of each mode being responsible for a single measurement.



Weights summing up to 1

$$p(z_t^k | x_t, m) = \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\max} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k | x_t, m) \\ p_{\text{short}}(z_t^k | x_t, m) \\ p_{\max}(z_t^k | x_t, m) \\ p_{\text{rand}}(z_t^k | x_t, m) \end{pmatrix}$$

Measurement Algorithm



Chapter 6

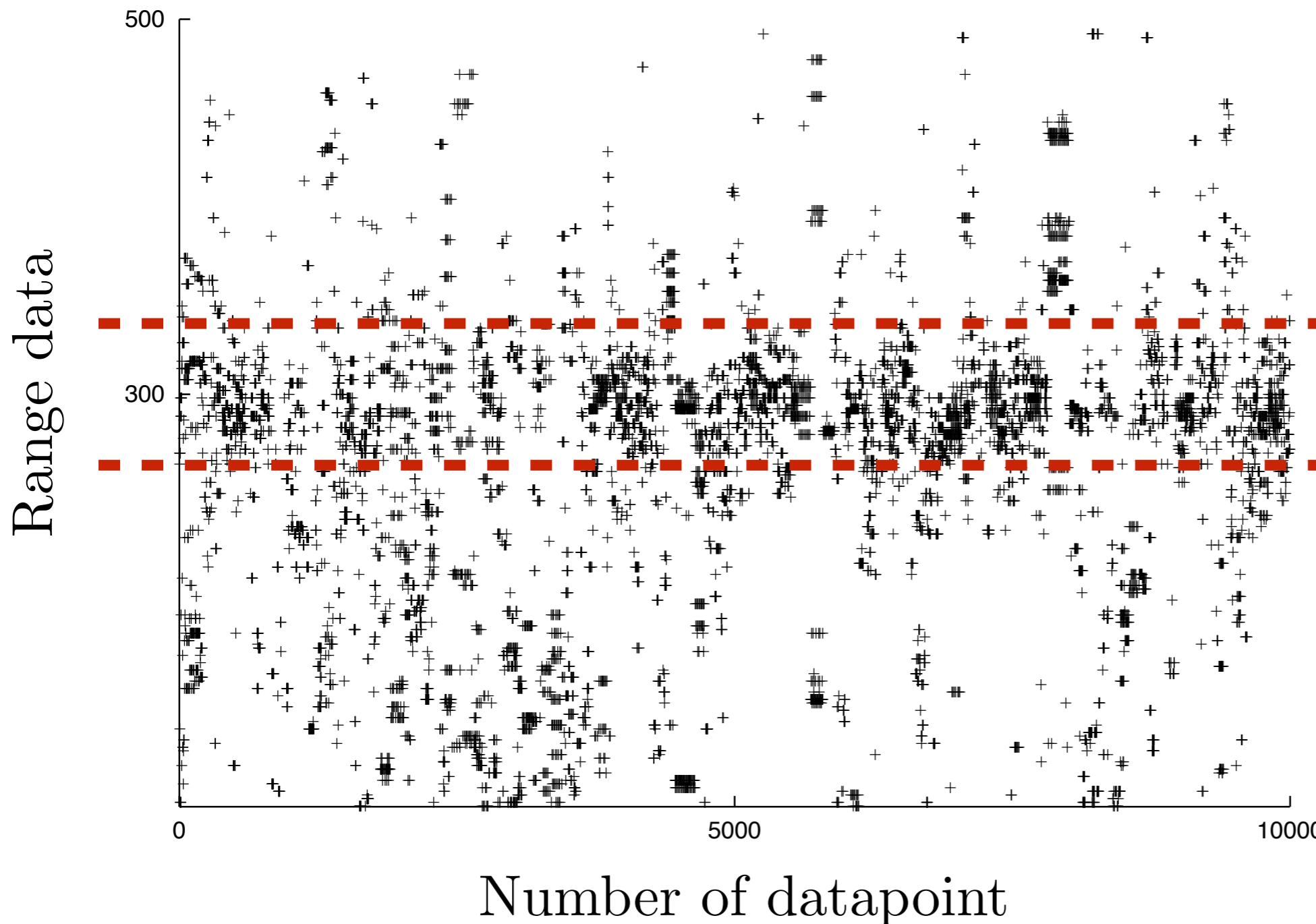
Input: State of the robot x , Map m , True laser scan z

Output: Probability p

1. Use x to figure out the pose of the sensor
2. Ray-cast (shoot out rays) from sensor to the map
3. Get back a simulated laser-scan z^*
4. Go over every ray in z^* and compare with z . Compute combined likelihood based on how much they match / mismatch.
5. Multiply all probabilities to get p

Question: How do we Tune Parameters?

In theory: Collect lots of data and optimize parameters to maximize data likelihood



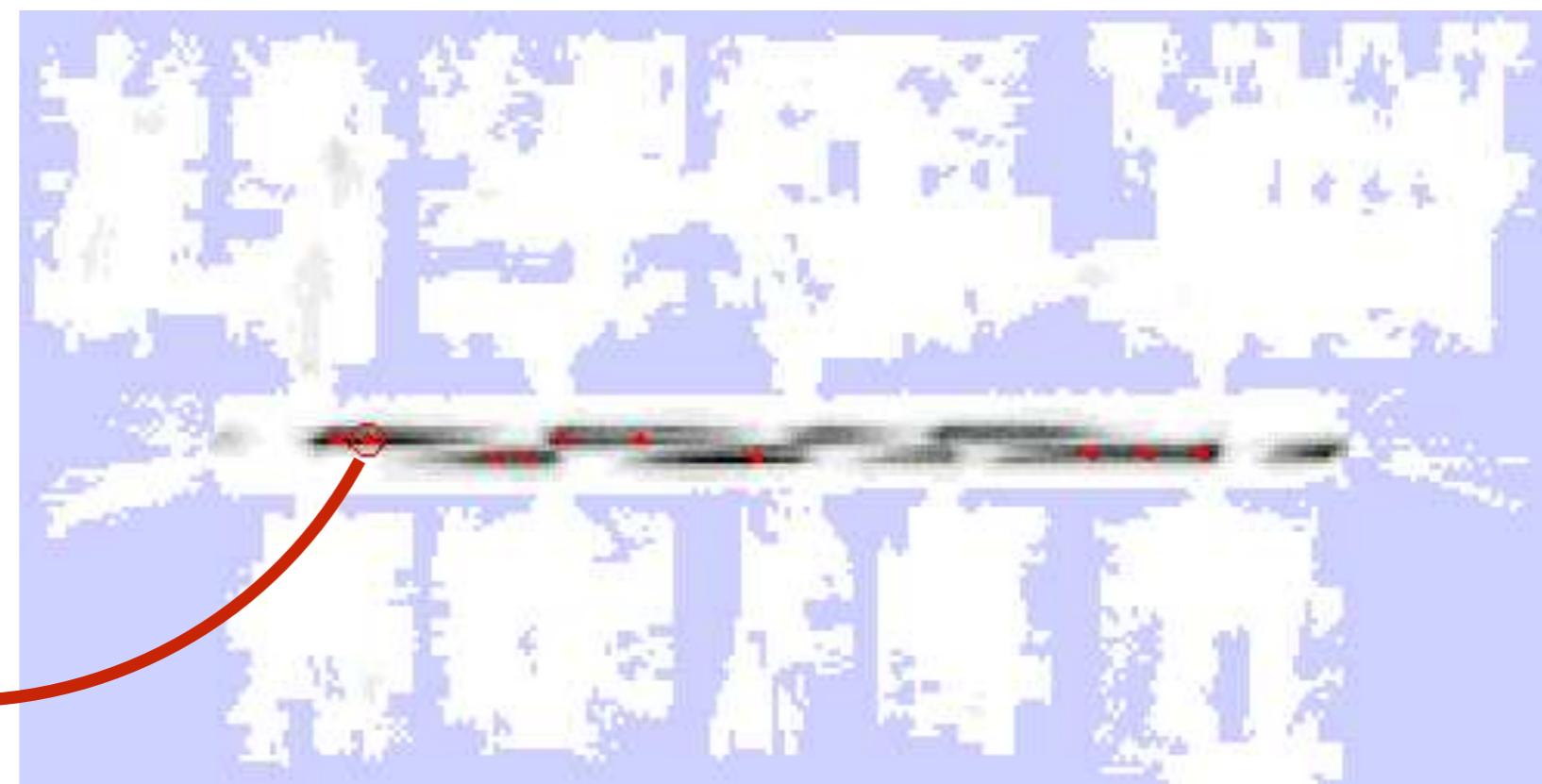
Example:
Place a robot
300 cm from
a wall and
collect lots of
data

Question: How do we Tune Parameters?

In practice: Simulate a scan and plot the likelihood from different positions



Actual scan



Likelihood at various locations

Problem: Overconfidence

$$P(z_t | x_t, m) = \prod_{i=1}^K P(z_t^k | x_t, m)$$

Independence assumption may result in repetition of mistakes