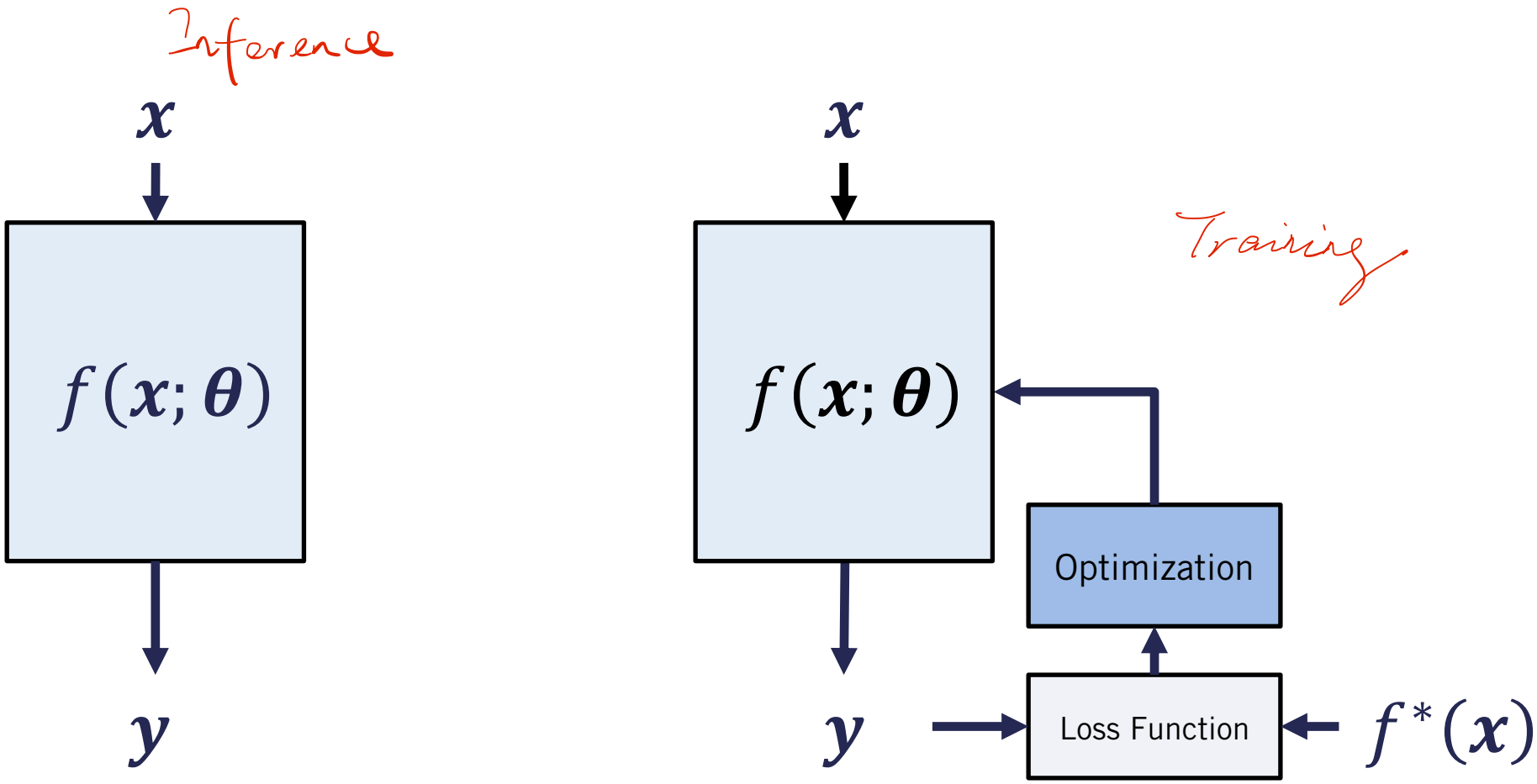
Output Layers and Loss Functions

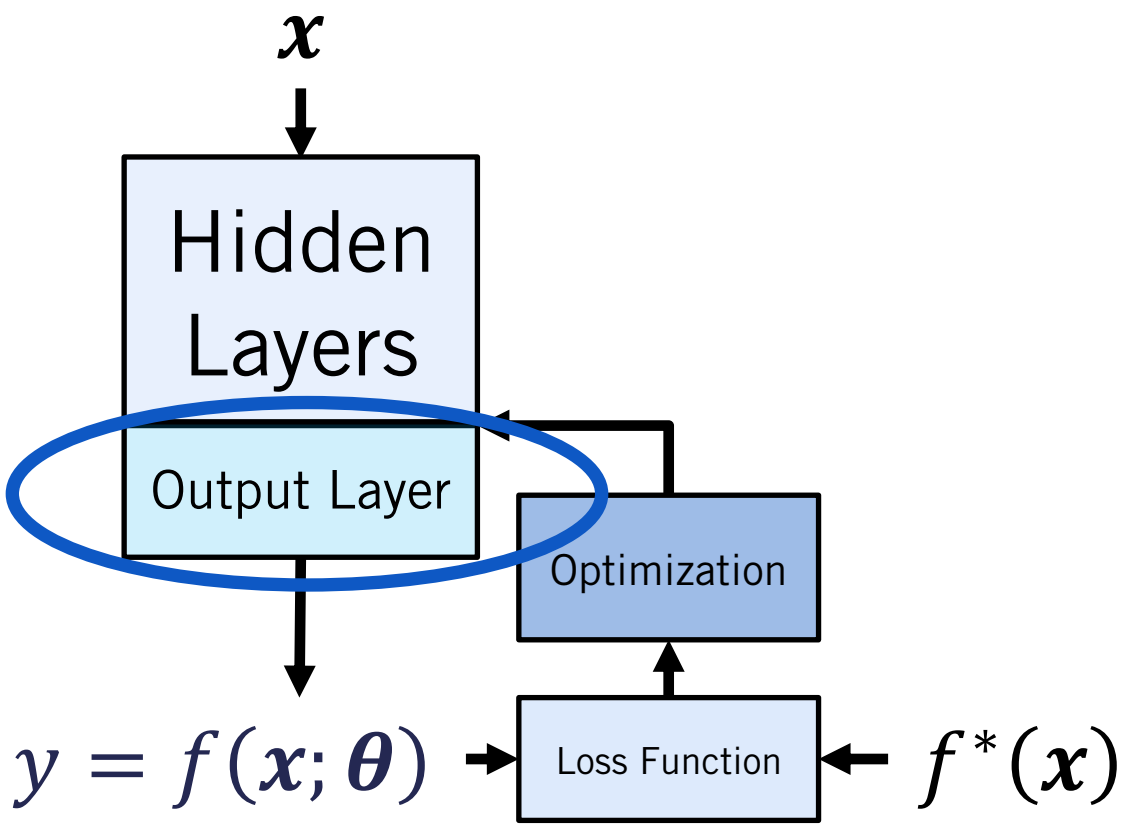
Course 3, Module 3, Lesson 2

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Machine Learning Algorithm Design

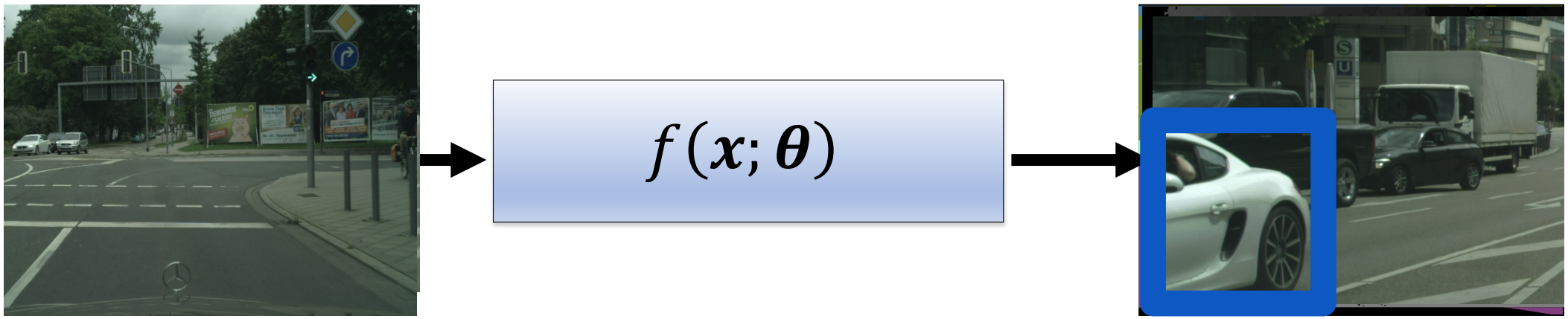


Artificial Neural Networks



Tasks: Classification and Regression

- **Classification:** Given input $\boldsymbol{x}$ map it to one of $\boldsymbol{k}$ classes or categories.
  - Image classification, semantic segmentation
- **Regression:** Given input $\boldsymbol{x}$ map it to a real number
  - Depth prediction, bounding box estimation



Classification: Softmax Output Layers

- **Softmax output layers** are most often used as the output of a classifier, to represent the **probability distribution** over K different classes

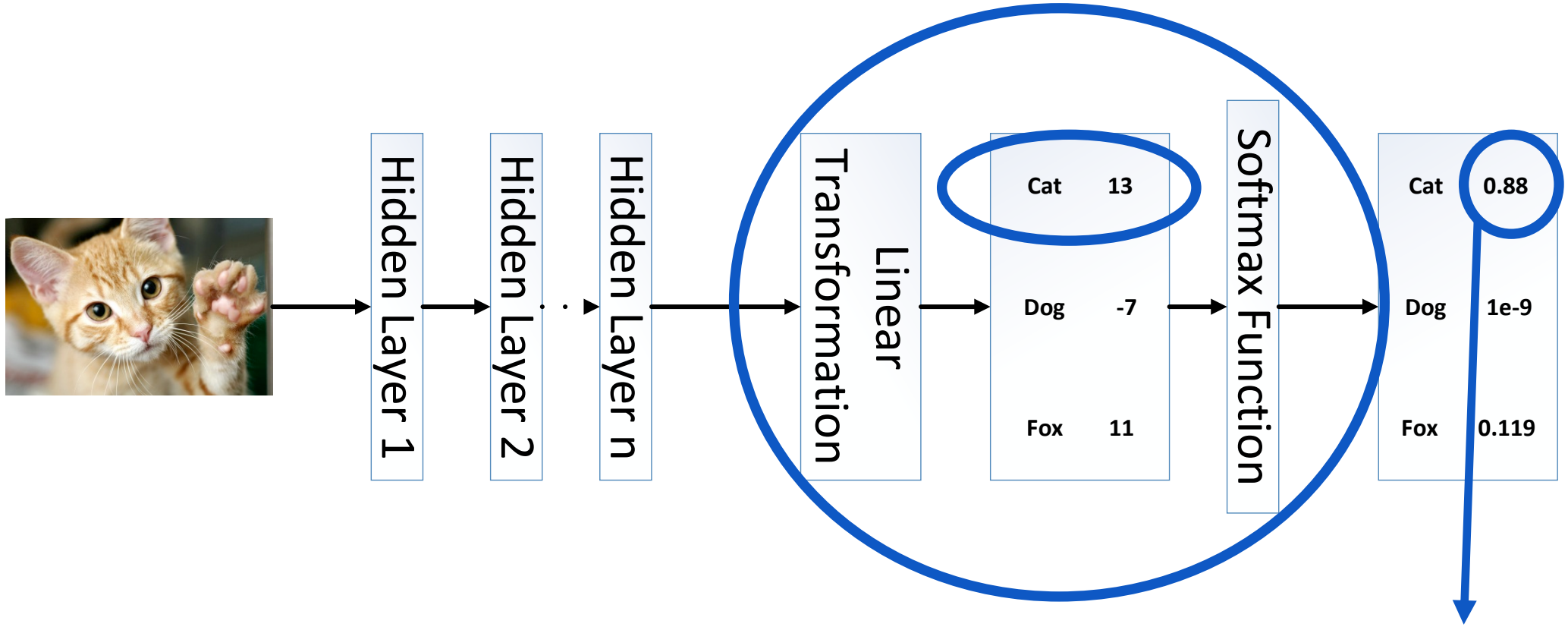
- The Softmax output layer is comprised of:
  - A linear transformation:

$$z = W^T h + b$$

  - Followed by the **Softmax** function:

$$Softmax(z_i) = \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

Classification: Softmax Output Layers



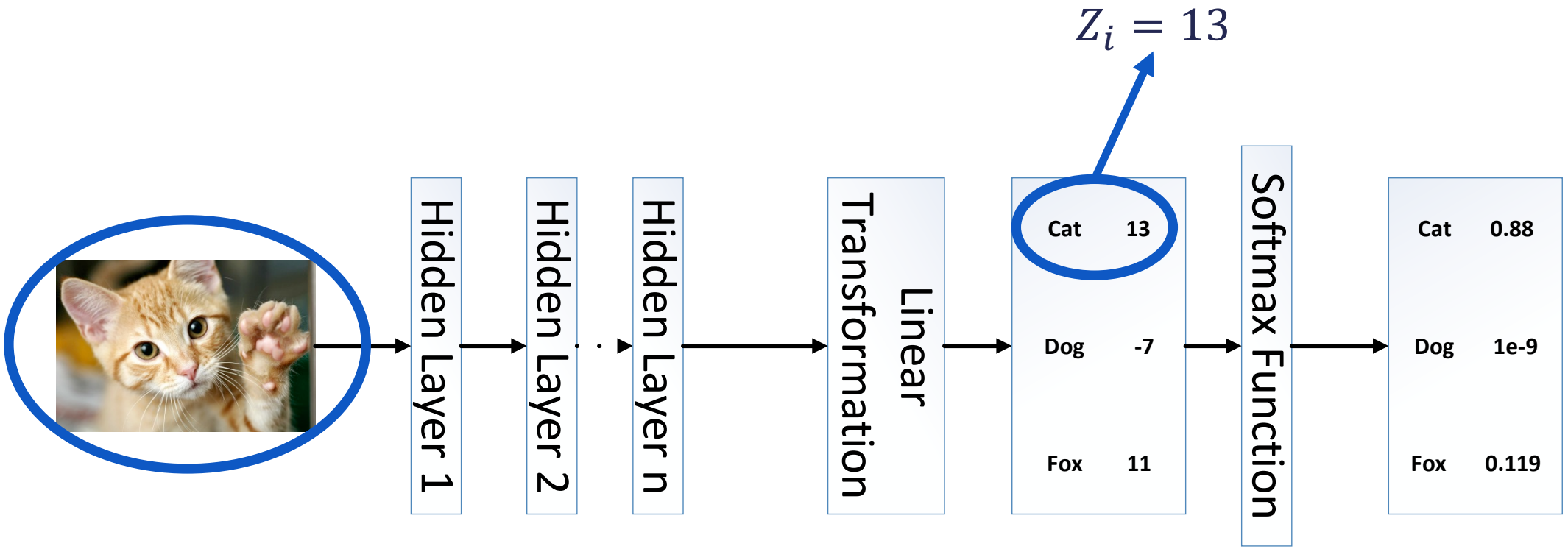
$$Softmax(h_{cat}) = \frac{\exp(13)}{\exp(13) + \exp(-7) + \exp(11)} = 0.88$$

Classification: Cross-Entropy Loss Function

- By considering the output of the softmax output layer as a probability distribution, the **Cross Entropy Loss** function is derived using **maximum likelihood** as:

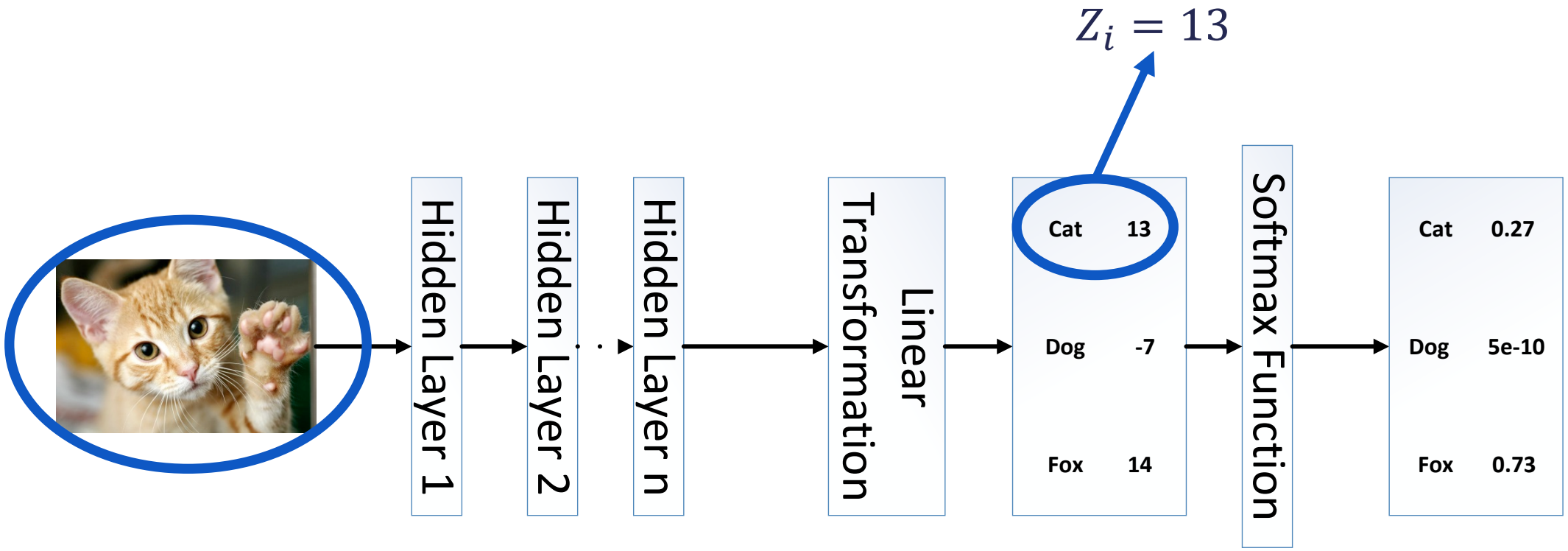
$$L(\theta) = -\log\big(Softmax(z_i)\big) = -z_i + \log \sum_j \exp(z_j)$$

Classification: Softmax Output Layers



$$L(\theta) = -z_i + \log \sum_j \exp(z_j) = -13 + \log(\exp(13) + \exp(-7) + \exp(11)) = 0.12$$

Classification: Softmax Output Layers



$$L(\theta) = -z_i + \log \sum_j \exp(z_j) = -13 + \log(\exp(13) + \exp(-7) + \exp(14)) = 1.31$$

Regression: Linear Output Layers

- **Linear Output Units** are based only on an affine transformation with no non-linearity

$$z = W^T h + b$$

- **Linear Output Units** are usually used with the **Mean Squared Error** loss function to model the **mean** of a probability distribution:

$$L(\theta) = \sum_i \left(z_i - f^*(x_i)\right)^2$$

Summary

- To build a machine learning model you need:
  - A model
  - A loss function
  - An optimization procedure
- Loss functions are chosen based on the task at hand

- **Next: Optimization**