Longitudinal Vehicle Model

Course 1, Module 4, Lesson 4

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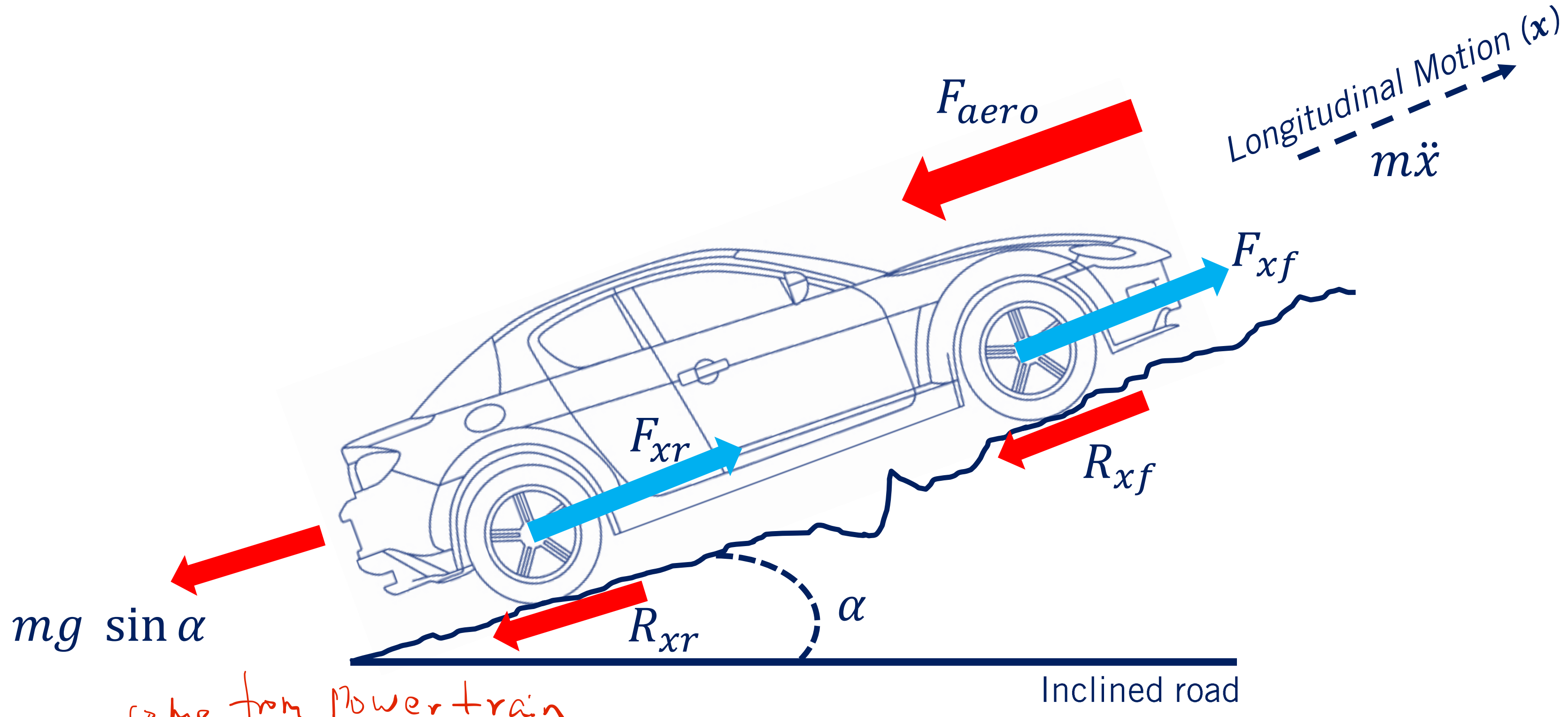
Learning Objectives

- Define dynamic force balance on a vehicle
- Describe powertrain component models
- Connect models to create a full longitudinal motion model

Longitudinal Vehicle Model



come from powertrain

Vehicle acceleration	Front & rear tire forces	Aerodynamic forces	Front & rear road rolling resistance	Gravitational force due to the road inclination

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin\alpha$$

Simplified Longitudinal Dynamics

- The full longitudinal dynamics

$$m\ddot{x} = F_{xf} + F_{xr} - F_{aero} - R_{xf} - R_{xr} - mg\sin\alpha$$

- Let F_x - total longitudinal force: $F_x = F_{xf} + F_{xr}$
- Let R_x - total rolling resistance: $R_x = R_{xf} + R_{xr}$
- Assume α is a small angle: $\sin\alpha = \alpha$

- Then the simplified longitudinal dynamics become

$$\underbrace{m\ddot{x}}_{\text{Inertial Term}} = \underbrace{F_x}_{\text{Traction Force}} - \underbrace{\boxed{F_{aero} - R_x - mg\alpha}}_{\text{Total Resistant Forces } (F_{Load})}$$

Simple Resistance Force Models

- Total resistance load:

$$F_{load} = F_{aero} + R_x + mg\alpha$$

- The aerodynamic force can depend on air density, frontal area, on the speed of the vehicle

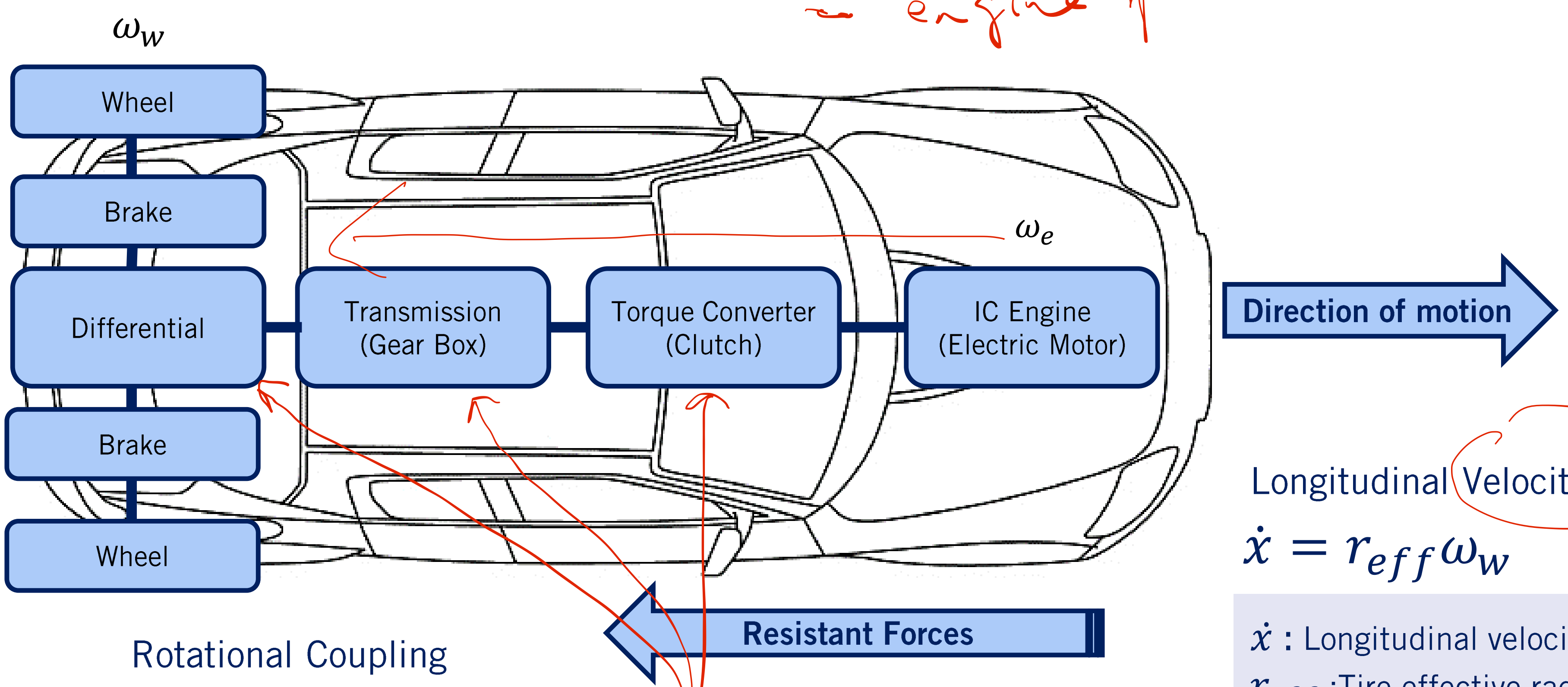
$$F_{aero} = {}^{1}\!/_{2}\, C_a \rho A \dot{x}^2 = c_a \dot{x}^2$$

- The rolling resistance can depend on the tire normal force, tire pressures and vehicle speed

$$R_x = N(\hat{c}_{r,0} + \hat{c}_{r,1}|\dot{x}| + \cancelto{0}{\hat{c}_{r,2}\dot{x}^2}) \approx c_{r,1}|\dot{x}|$$

Powertrain Modeling

wheel speed
→ engine speed



Rotational Coupling

$$\omega_w = GR\omega_t = GR\omega_e$$

ω_w :wheel angular speed
ω_t :turbine angular speed
ω_e :engine angular speed
GR: Combined gear ratios

Longitudinal Velocity

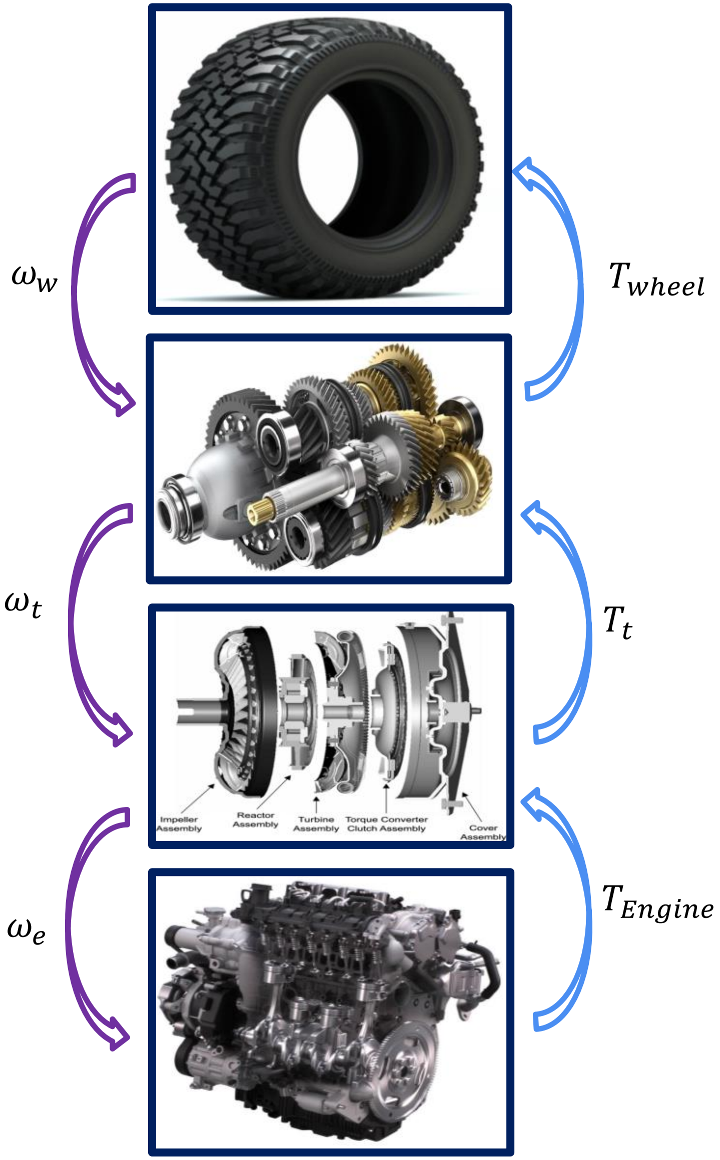
$$\dot{x} = r_{eff}\omega_w$$

$\dot{x}$: Longitudinal velocity
r_{eff} :Tire effective radius

Longitudinal acceleration

$$\ddot{x} = r_{eff}GR\dot{\omega}_e$$

Power Flow in Powertrain



Wheel

$$I_w \dot{\omega}_w = T_{wheel} - r_{eff} F_x$$

Tire force calculated earlier

$$T_{wheel} = I_w \dot{\omega}_w + r_{eff} F_x$$

Transmission

$$I_t \dot{\omega}_t = T_t - (GR) T_{wheel}$$

$$I_t \dot{\omega}_t = T_t - GR(I_w \dot{\omega}_w + r_{eff} F_x)$$

Torque Converter

$$\omega_t = \omega_e$$

$$T_t = (I_t + I_w GR^2)\dot{\omega}_e + GR r_{eff} F_x$$

Engine

$$I_e \dot{\omega}_e = T_{Engine} - T_t$$

$$I_e \dot{\omega}_e = T_{Engine} - (I_t + I_w GR^2)\dot{\omega}_e - GR r_{eff} F_x$$

Engine Dynamics

- Tire force in terms of inertia and load force:

$$F_x = m\ddot{x} + F_{load} = mr_{eff}GR\dot{\omega}_e + F_{load}$$

- Combining with our engine dynamics model yields:

$$\underbrace{(I_e + I_t + I_w GR^2 + m(GR^2)r_{eff}^2)}_{J_e}\dot{\omega}_e = T_{Engine} - (GR)(r_{eff}F_{Load})$$

- Finally, the engine dynamic model simplifies to

$$J_e\dot{\omega}_e = T_{Engine} - \underbrace{(GR)(r_{eff}F_{Load})}_{\text{Total Load Torque } (T_{Load})}$$

engine torque → accelerator pedal position

brake torque → brake pedal position

Summary

What we have learned from this lesson?

- Vehicle longitudinal dynamics, resistance forces
- Powertrain components and component models
- Unified longitudinal dynamic model for speed control

What is next?

- The lateral dynamics of a vehicle