Camera Calibration

Course 3, Module 1, Lesson 2

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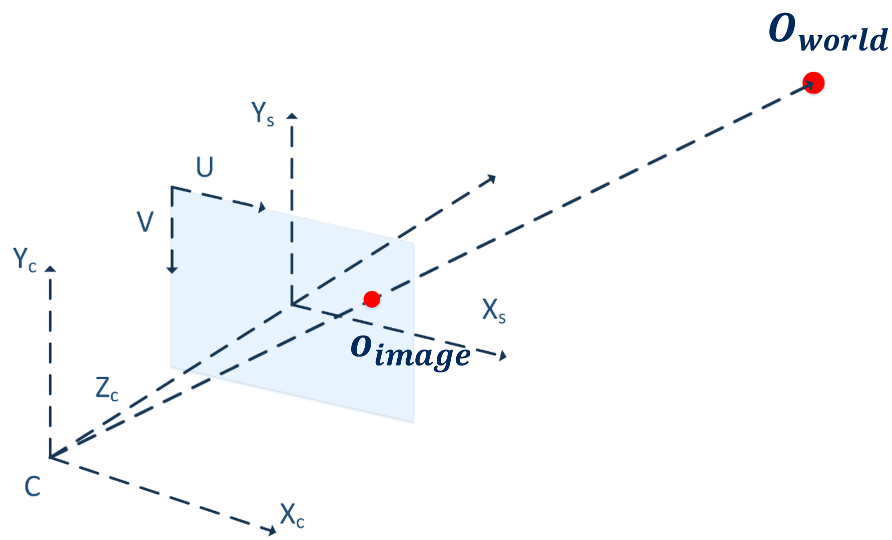
Learning Objectives

- Learn how to find the camera matrix P through a process called camera calibration
- Learn how to extract the intrinsic and extrinsic parameters from the camera P matrix

Computing the Projection

- Projection from World → Camera:

$$\boldsymbol{o_{image}} = PO = K[R|t]\boldsymbol{O_{world}}$$



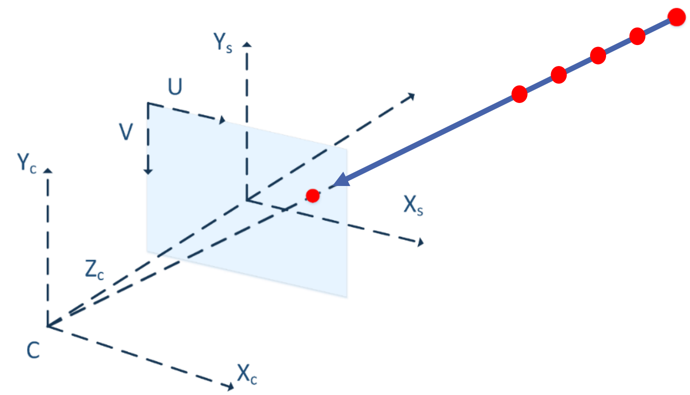
Computing the Projection

- **World** coordinates to **Image** coordinates:

$$o_{image} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K[R|t] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- **Image** coordinates to **Pixel** coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{z} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} su \\ sv \\ s \end{bmatrix}$$



Scale: s

lost when projecting from 3D to 2D

Camera Calibration: Problem Formulation

- World coordinates and Camera coordinates known:

intrinsic parameters

$$o_{image} = P\boldsymbol{O}_{\boldsymbol{world}} = K[R|t]\boldsymbol{O}_{\boldsymbol{world}}$$

extrinsic rotation

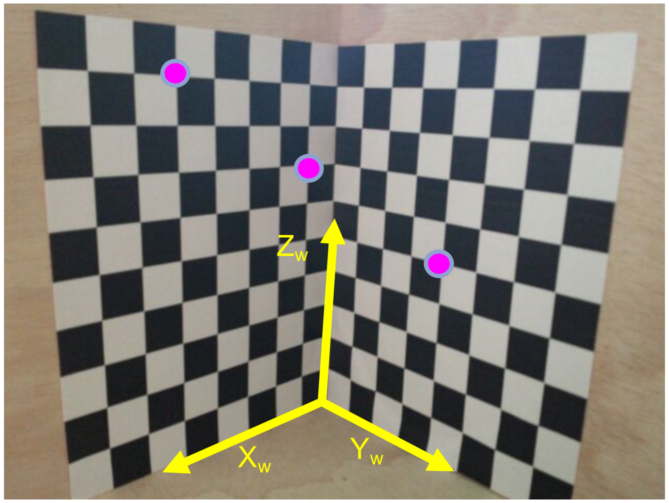
extrinsic translation

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Calibration: Problem Formulation

- Use scenes with **known geometry** to:
  - Correspond 2D image coordinates to 3D world coordinates
  - Find the **Least Squares Solution** (or non-linear solution) of the parameters of $\boldsymbol{P}$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Camera Calibration: Problem Formulation

- Expanding the equation results in:

$$su = p_{11}\,X + p_{12}\,Y + p_{13}\,Z + p_{14} \qquad (1)$$
$$sv = p_{21}\,X + p_{22}\,Y + p_{23}\,Z + p_{24} \qquad (2)$$
$$s = p_{31}\,X + p_{32}\,Y + p_{33}\,Z + p_{34} \qquad (3)$$

Camera Calibration: Problem Formulation

- Move to LHS:

$$
\begin{aligned}
su - p_{11}X - p_{12}Y - p_{13}Z - p_{14} &= 0 \qquad (1)\\
sv - p_{21}X - p_{22}Y - p_{23}Z - p_{24} &= 0 \qquad (2)\\
s - p_{31}X - p_{32}Y - p_{33}Z - p_{34} &= 0 \qquad (3)
\end{aligned}
$$

Camera Calibration: Problem Formulation

- Replace Eq. (3) in Eq. (1) and Eq. (2) to get 2 equations per point:

$$p_{31}Xu + p_{32}Yu + p_{33}Zu + p_{34}u - p_{11}\,X - p_{12}\,Y - p_{13}\,Z - p_{14} = 0$$
$$p_{31}Xv + p_{32}Yv + p_{33}Zv + p_{34}v - p_{21}\,X - p_{22}\,Y - p_{23}\,Z - p_{24} = 0$$

Camera Calibration: Problem Formulation

- If we have N 3D points and their corresponding N 2D projections, set up **homogeneous linear system**
- Solved with Singular Value Decomposition (SVD)

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NX_N & -u_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & X_1 & -v_NX_N & -v_NY_N & -v_NY_N & -u_N \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = 0$$

Camera Calibration: Linear Methods

- Advantages of such a simple linear system are:
  - Easy to formulate
  - Closed form solution

- Disadvantages of such a simple linear system are:

Can you think of some disadvantages?

solving P

  - Does not directly provide camera parameters
  - Does not model radial distortion and other complex phenomena
  - Does not allow for constraints such as known focal length to be imposed

Factoring the P matrix

Projection Matrix: $P = K[R|t]$

RQ factorization

3D Camera Center projects to zero:

$$PC = 0$$

$$K[R|t]C = 0$$

$$K(RC + t) = 0$$

$$t = -RC$$

Substituting in to projection matrix:

$$P = K[R|-RC]$$

3D camera center

$PC = 0$

$$P = [KR|-KRC]$$

Let $M = KR$

Factorizing the P matrix

$$P = [M|-MC]$$

$$M = \underbrace{\mathcal{R}}_{3\times3}\ \underbrace{Q}_{3\times3}$$

an orthogonal basis

upper triangular matrix

$\mathcal{R}$ is not our Rotation Matrix!

Factorizing the P matrix

$$M = \mathcal{R}Q = KR$$

- **Intrinsic Calibration Matrix (upper triangular):**

$$K = \mathcal{R}$$

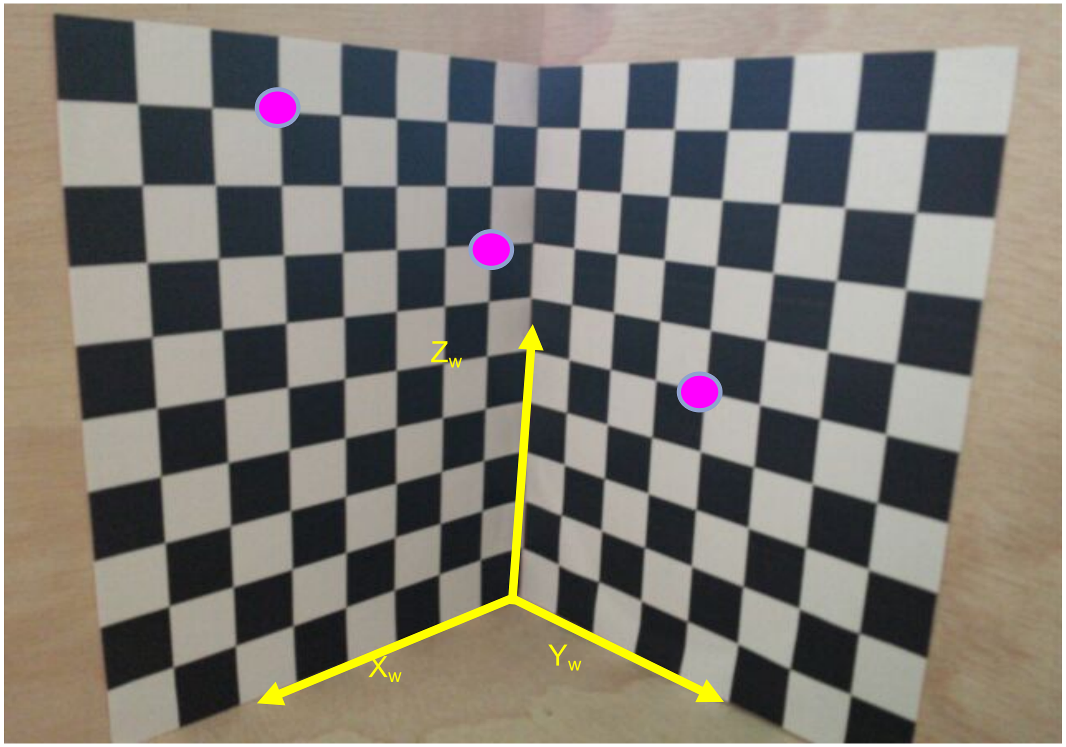
- **Rotation Matrix (orthogonal):**

$$R = Q$$

- **Translation Vector:**

$$t = -K^{-1}\underbrace{P[:,4]}_{\text{Last column}} = -K^{-1}MC$$

Camera Calibration



Summary

- The camera matrix P can be found through a process known as camera calibration
- The intrinsic and extrinsic camera parameters can be extracted from the P matrix using RQ factorization

- **Next: Stereo Vision**

Calibration Tools – MOVE TO SUPPLEMENTAL MATERIALS

- **OpenCV**:
  - https://docs.opencv.org/master/d4/d94/tutorial_camera_calibration.html
- **Matlab:**
  - https://www.mathworks.com/help/vision/ug/single-camera-calibrator-app.html
- **ROS:**
  - http://wiki.ros.org/camera_calibration