Proportional-Integral-Derivative (PID) Control

Course 1, Module 5, Lesson 1

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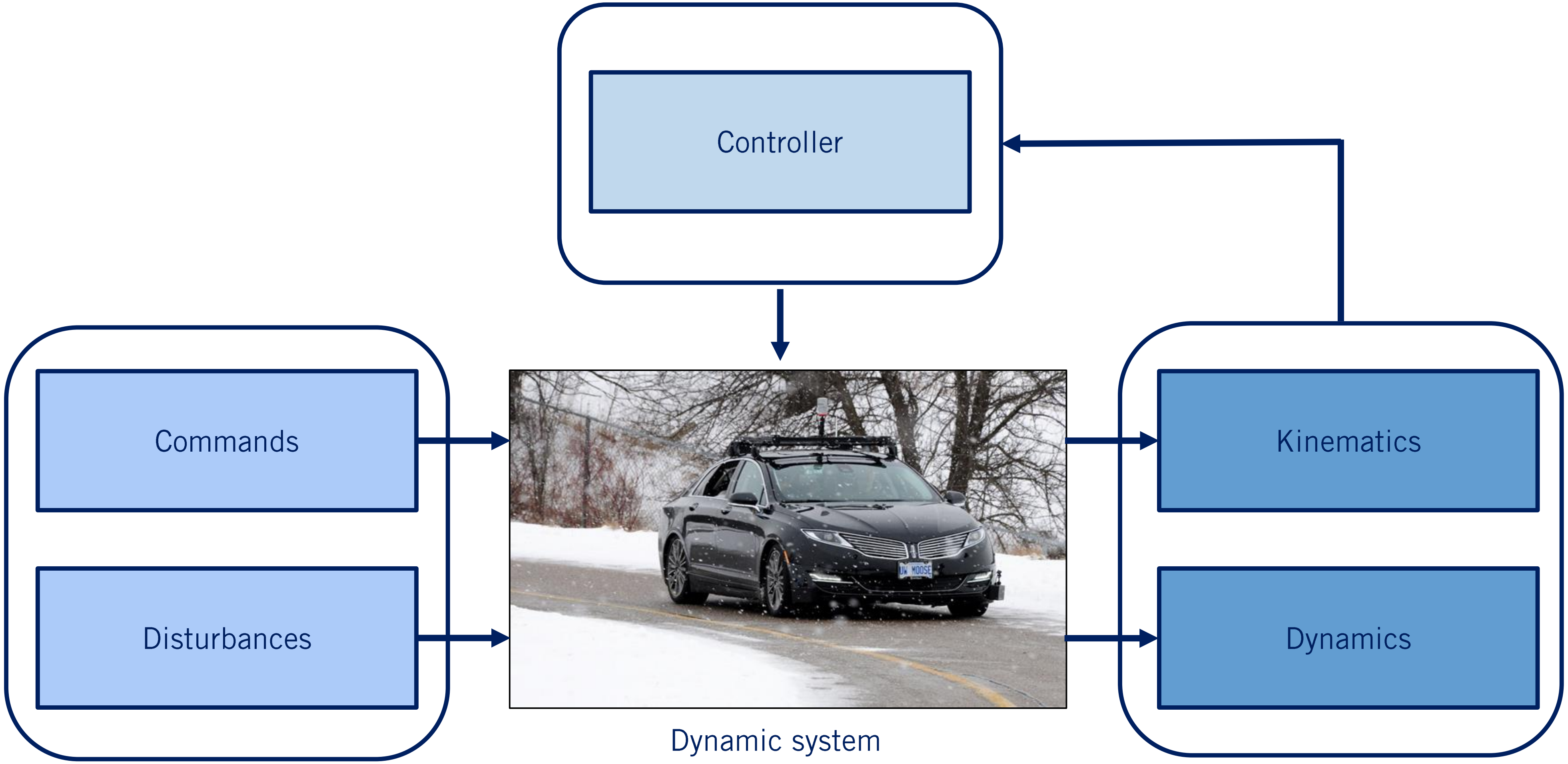
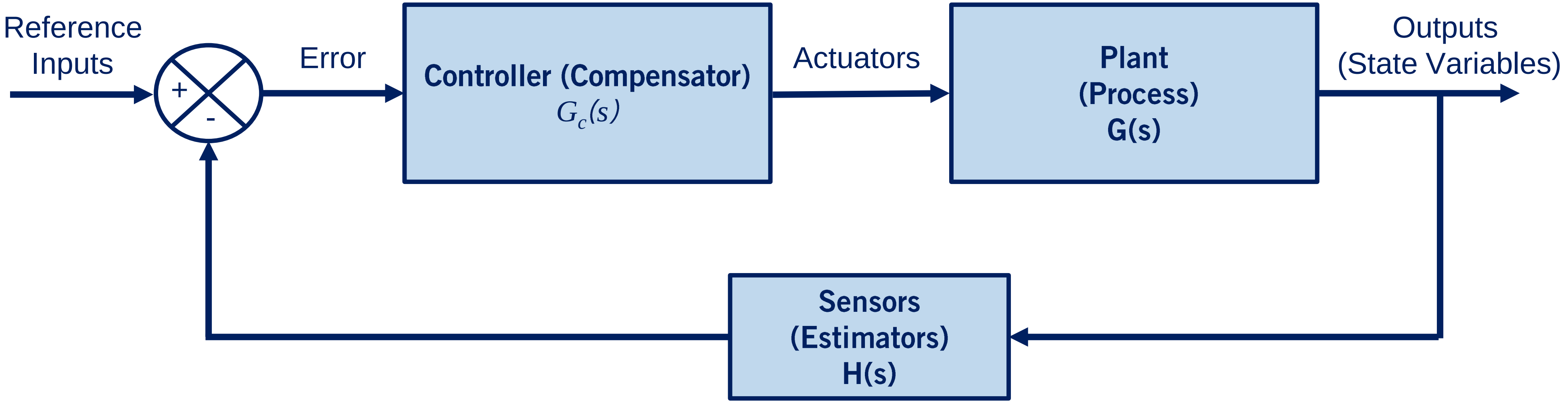
Learning Objectives

- In this module, you’ll
  - Review basics of linear time-invariant (LTI) control
  - Develop proportional-integral-derivative (PID) controllers for longitudinal vehicle control
  - Combine feedforward and feedback control to improve speed tracking performance
- In this video, you’ll
  - Review the basics of LTI control
  - Explore the design of PID controllers

Control Development



Typical Feedback Control Loop



Plant System or Process

- System Representation:
  - The plant system could be linear or nonlinear
  - Plant representation: state-space form and transfer functions
  - Linear time-invariant systems can be expressed using transfer functions

- Transfer Function:
  - A transfer function G is a relation between input U and output Y

$$Y(s) = G(s)U(s) \qquad s = \sigma + j\omega$$

  - Expressed in the Laplace domain, as a function of s, a complex variable

$$Y(s) = G(s)U(s) = \frac{N(s)}{D(s)}U(s)$$

  - Zeros – roots of numerator , Poles – roots of denominator

Controller or Compensator

- Control algorithms can vary from simple to complex

- Some simple algorithms, widely used in industry:
  - Lead-lag controllers
  - PID controllers

- More complex algorithms
  - Nonlinear methods: Feedback linearization, Backstepping, Sliding mode
  - Optimization methods: Model predictive control

PID Controller

- In the time domain:

$$u(t) = K_P e(t) + K_I \int_0^t e(t)dt + K_D \dot{e}(t)$$

where K_P, K_I, K_D are the proportional, integral and derivative gains

- In the Laplace domain:

$$U(s) = G_c(s)E(s) = \left(K_P + \frac{K_I}{s} + K_D s\right)E(s)$$

$$= \left(\frac{K_D s^2 + K_P s + K_I}{s}\right)E(s)$$

Proportional-Integral Derivative Controller

two zeros added

$$G_c(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

one pole added

- The pole is at the origin
- The zeros can be arbitrary places on the complex plane
- PID controller design selects zero locations, by selecting P, I, and D gains
- There are several algorithms to select the PID gains (e.g. Zeigler-Nichols)

Characteristics of P, I, and D Gains

A step input (reference signal)

90% reference value

settle within 5% of reference

Closed Loop Response	Rise Time	Overshoot	Settling Time	Steady State Error
Increase K_P	Decrease	Increase	Small change	Decrease
Increase K_I	Decrease	Increase	Increase	Eliminate
Increase K_D	Small change	Decrease	Decrease	Small change

respond to rate of change of error

By properly tuning the PID gains

Example: Second Order System

Mass-Spring-Damper System

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Dynamic Equation of the System

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Laplace Transform (s-domain)

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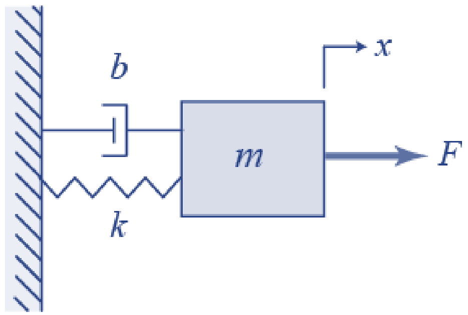
Transfer Function of Output to Input



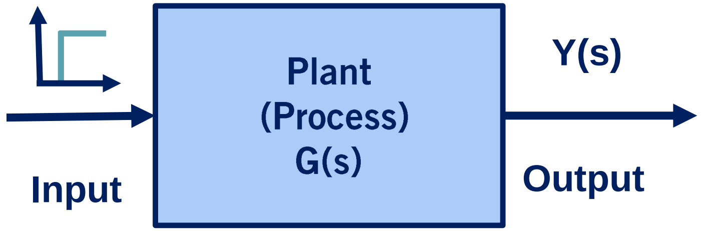
$$m\ddot{x} + b\dot{x} + kx = F, \qquad x(0) = 0$$

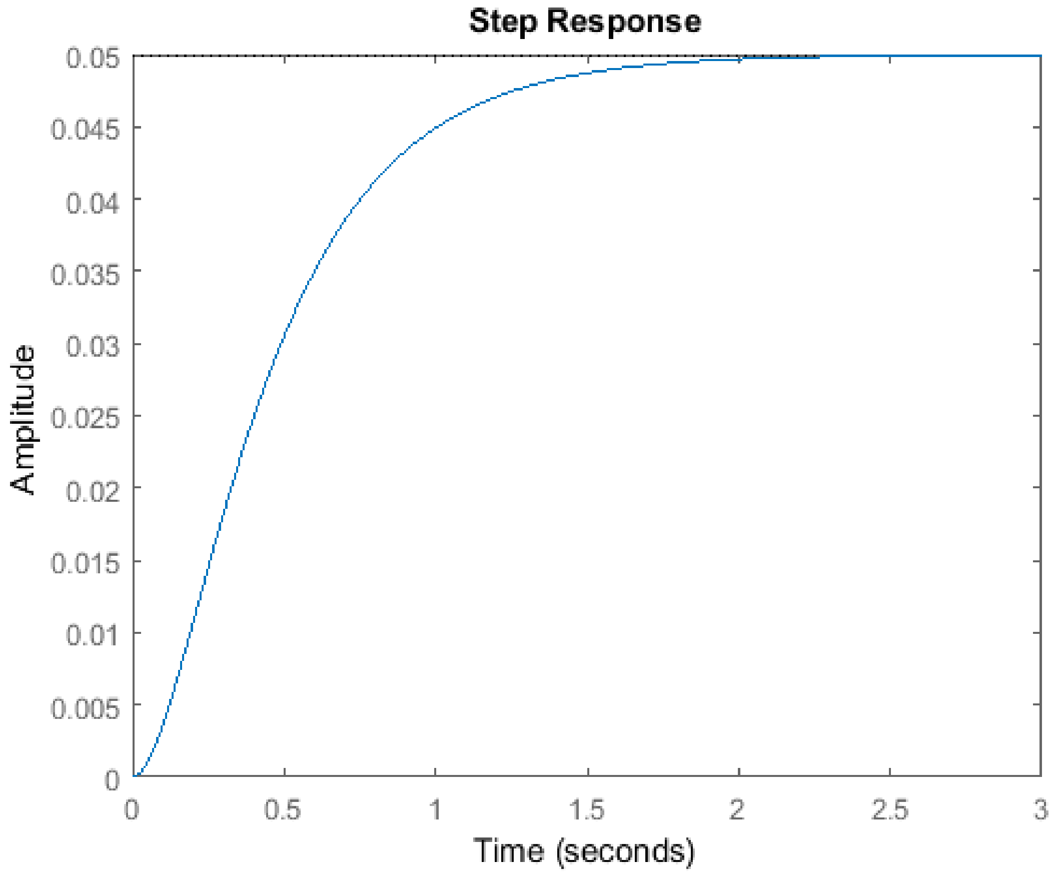
$$ms^2X(s) + bsX(s) + kX(s) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Open-Loop Step Response

- Let $m = 1$, $b = 10$, $k = 20$, $F = 1$.

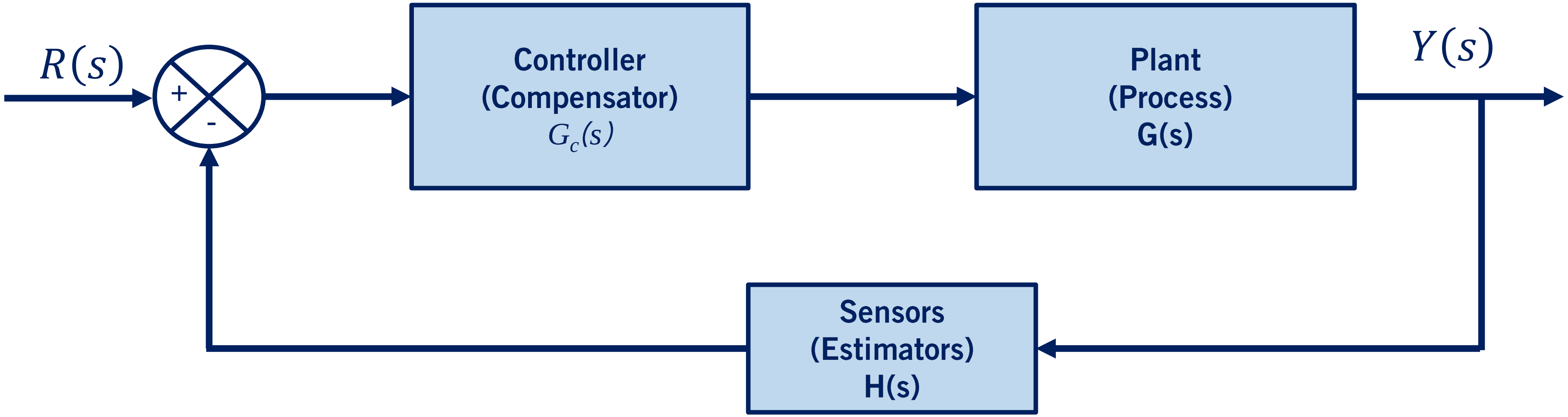




Closed-loop Response

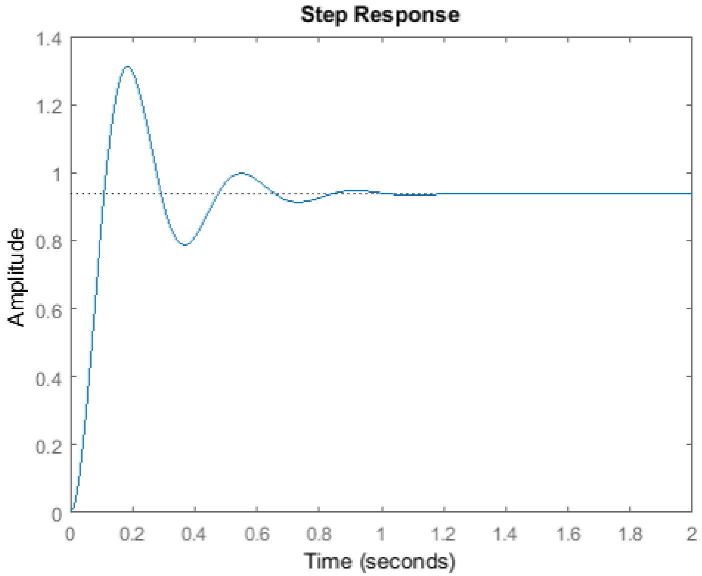
- For the unity feedback, i.e., $H(s) = 1$, the closed loop system is given by,

$$\frac{Y(s)}{R(s)} = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)}$$



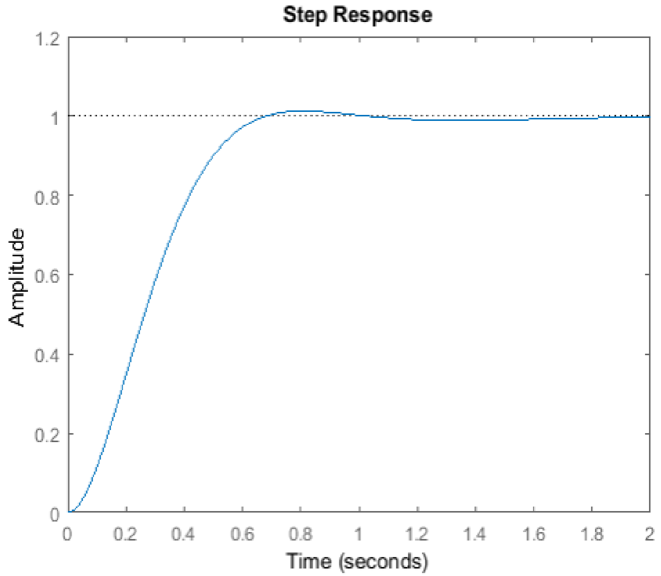
Step Response

P Controller



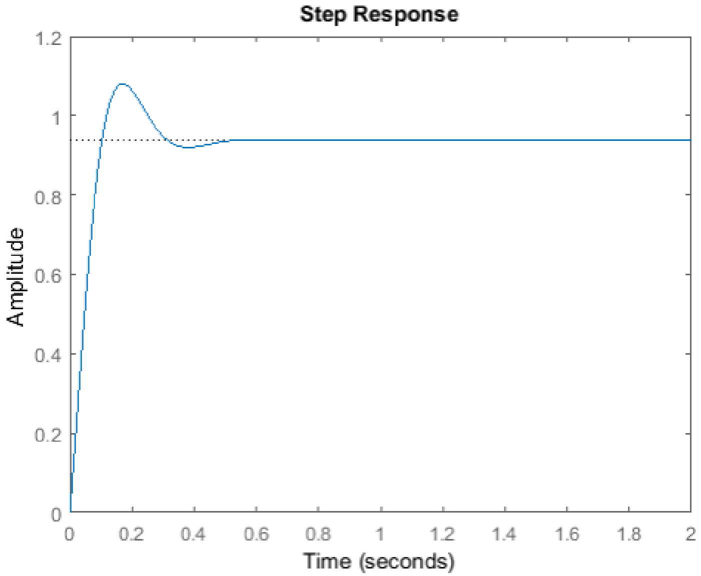
$$G_P(s) = K_P$$

PD Controller



$$G_{PD}(s) = K_P + sK_D$$

PI Controller

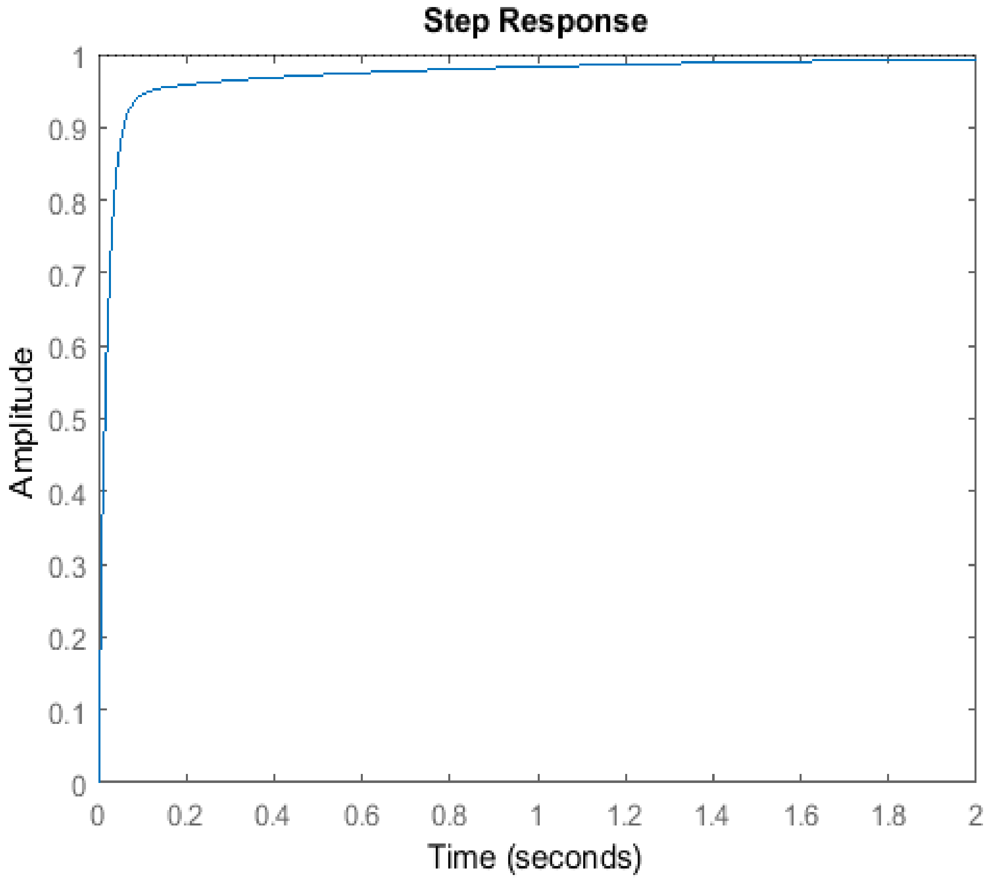


$$G_{PI}(s) = K_P + \frac{K_I}{s}$$

Step Response for PID Control

- PID controller is given by,

$$G_{PID}(s) = \left(K_P + K_D s + \frac{K_I}{s}\right)$$



- The closed loop system becomes,

$$G_{CL}(s) = \frac{K_D s^2 + K_P s + K_I}{s^3 + (10 + K_D)s^2 + (20 + K_P)s + K_I}$$

Summary

What we have learned from this lesson:

- Why we employ controller and its importance
- PID controller (simple but useful and applied) along with the tuning method

What is next?

- Longitudinal speed control of a vehicle using PID controller