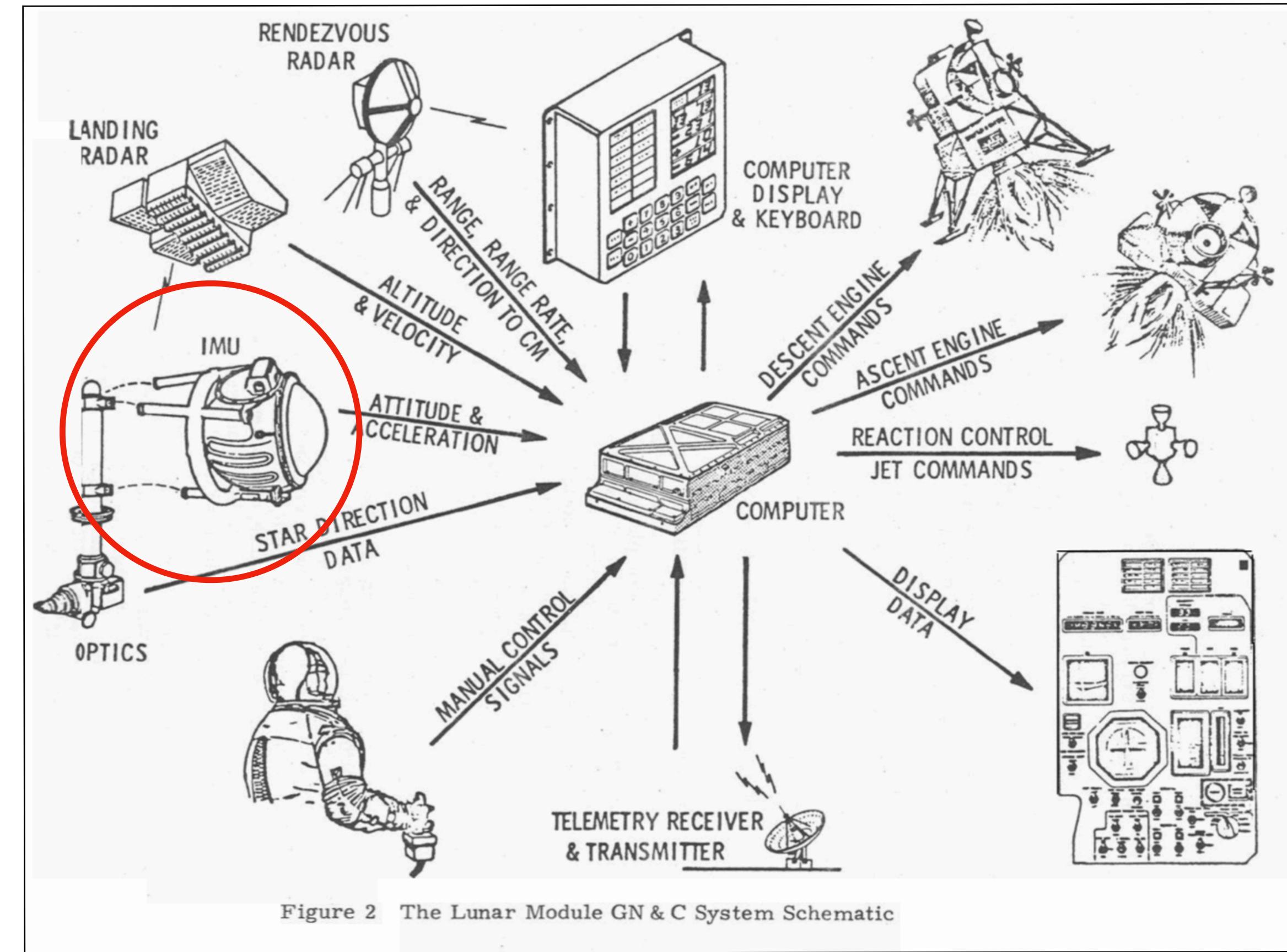
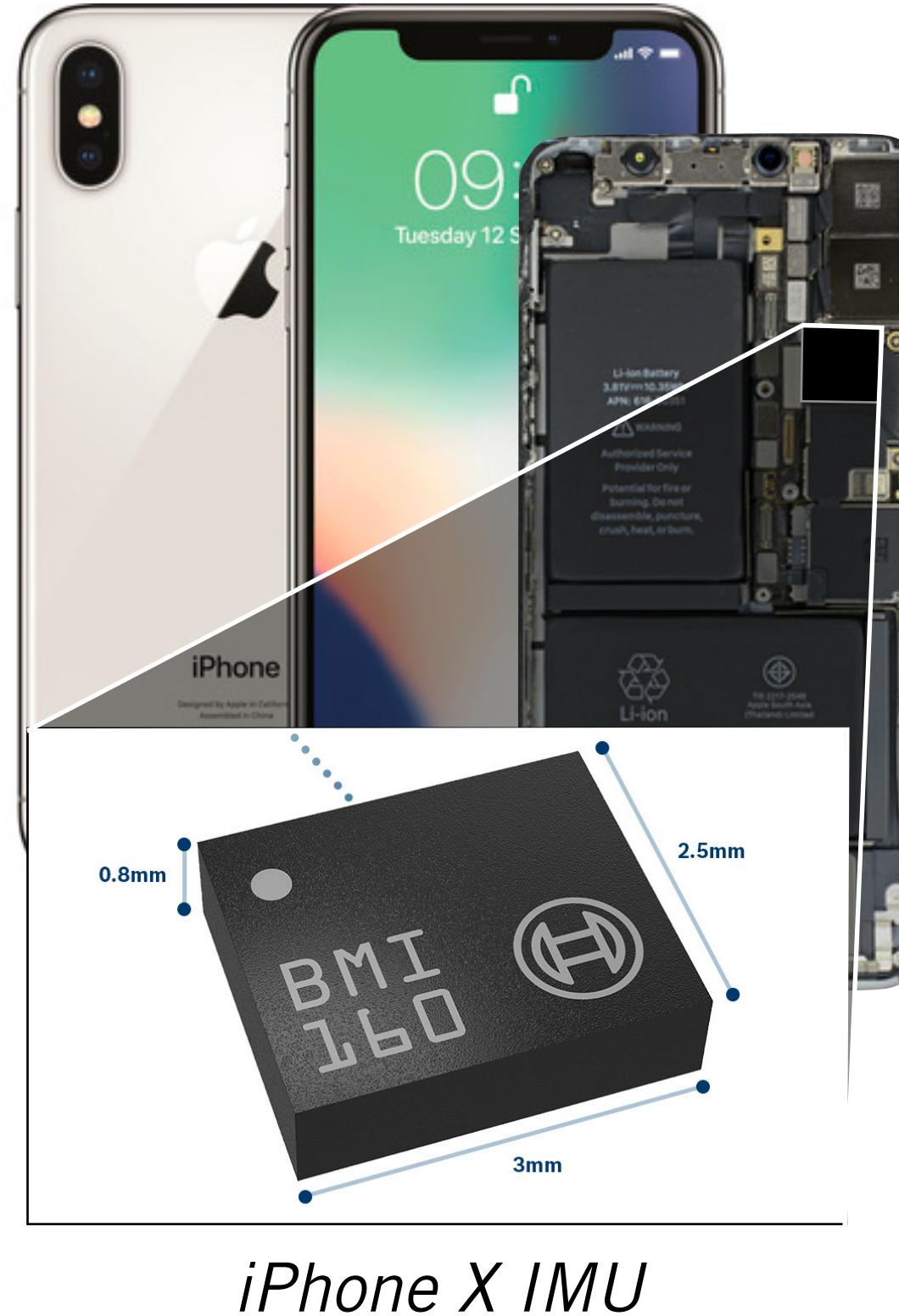


Module 3 | Lesson 2

THE INERTIAL MEASUREMENT UNIT

The Inertial Measurement Unit



Apollo Lunar Module Guidance Navigation & Control

The Inertial Measurement Unit

- An IMU is typically composed of
 - 3 **gyroscopes** (measure angular rotation rates about three separate axes)
 - 3 **accelerometers** (measure accelerations along three orthogonal axes)
- IMUs come in many form factors; cost varies from ~\$10 to ~\$100K+

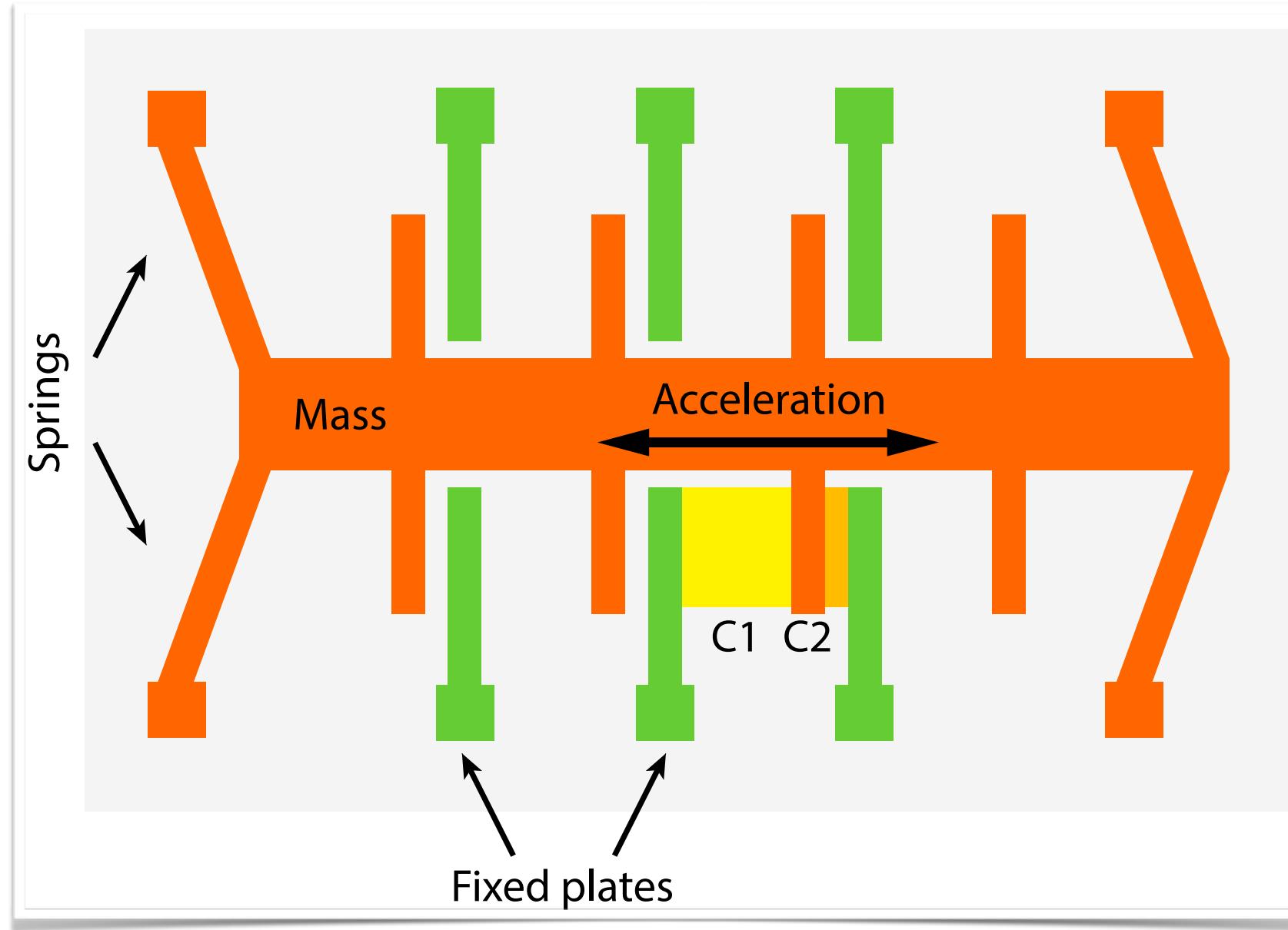


The Gyroscope

- Gyroscope: γῦρος gûros, “circle” + σκοπέω skopéō, “to look”
 - Historically: a spinning disc that maintains a specific orientation relative to *inertial space*, providing an orientation reference
 - Modern spinning-disk gyroscopes spin at up to 24,000 RPM!
- Microelectromechanical systems (MEMS) are much smaller and cheaper
 - Measure rotational *rates* instead of orientation directly
 - Measurements are **noisy** and **drift** over time



The Accelerometer



- Accelerometers measure acceleration relative to *free-fall* - this is also called the *proper acceleration* or *specific force*:

$$\mathbf{a}_{\text{meas}} = \mathbf{f} = \frac{\mathbf{F}_{\text{non-gravity}}}{m}$$

Sitting still at your desk, your *proper* acceleration is g upwards! (think of the 'normal' force holding you up)

In localization, we typically require the acceleration relative to a fixed reference frame

- 'coordinate' acceleration
- computed using fundamental equation for accelerometers in a gravity field:

$$\mathbf{f} + \mathbf{g} = \ddot{\mathbf{r}}_i$$

The Accelerometer | Examples

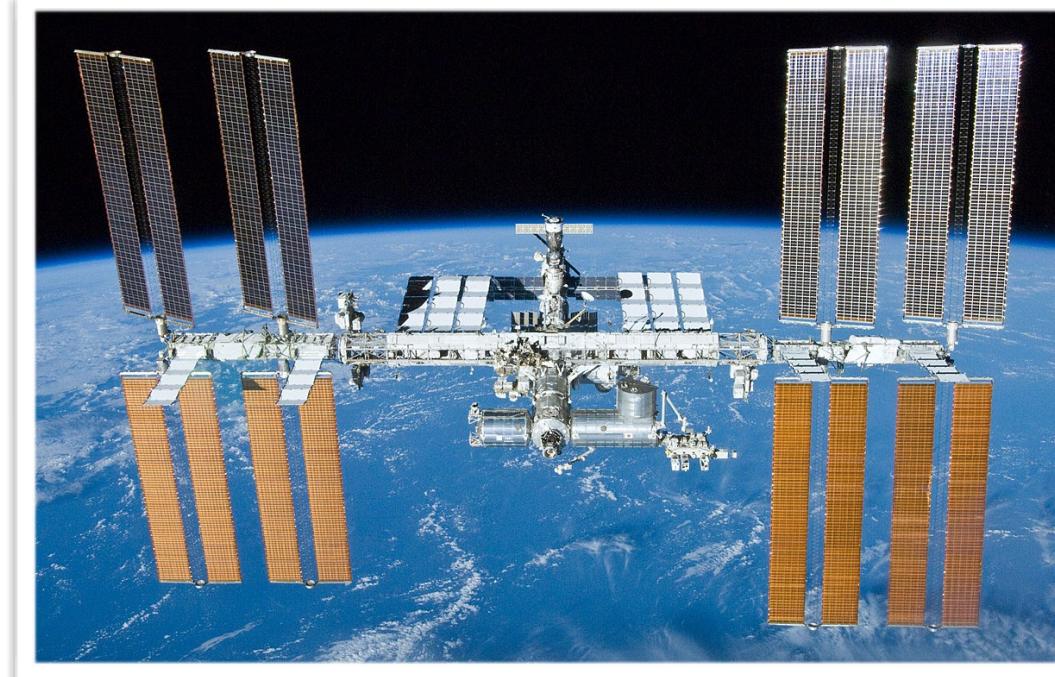
- An accelerometer in a stationary car measures:

$$\mathbf{f} = \ddot{\mathbf{r}}_i - \mathbf{g} \approx \mathbf{0} - \mathbf{g} \approx -\mathbf{g} \quad g \text{ 'up'}$$



- An accelerometer on the International Space Station measures:

$$\mathbf{f} = \ddot{\mathbf{r}}_i - \mathbf{g} \approx \mathbf{g} - \mathbf{g} \approx \mathbf{0} \quad (\text{zero-g!})$$



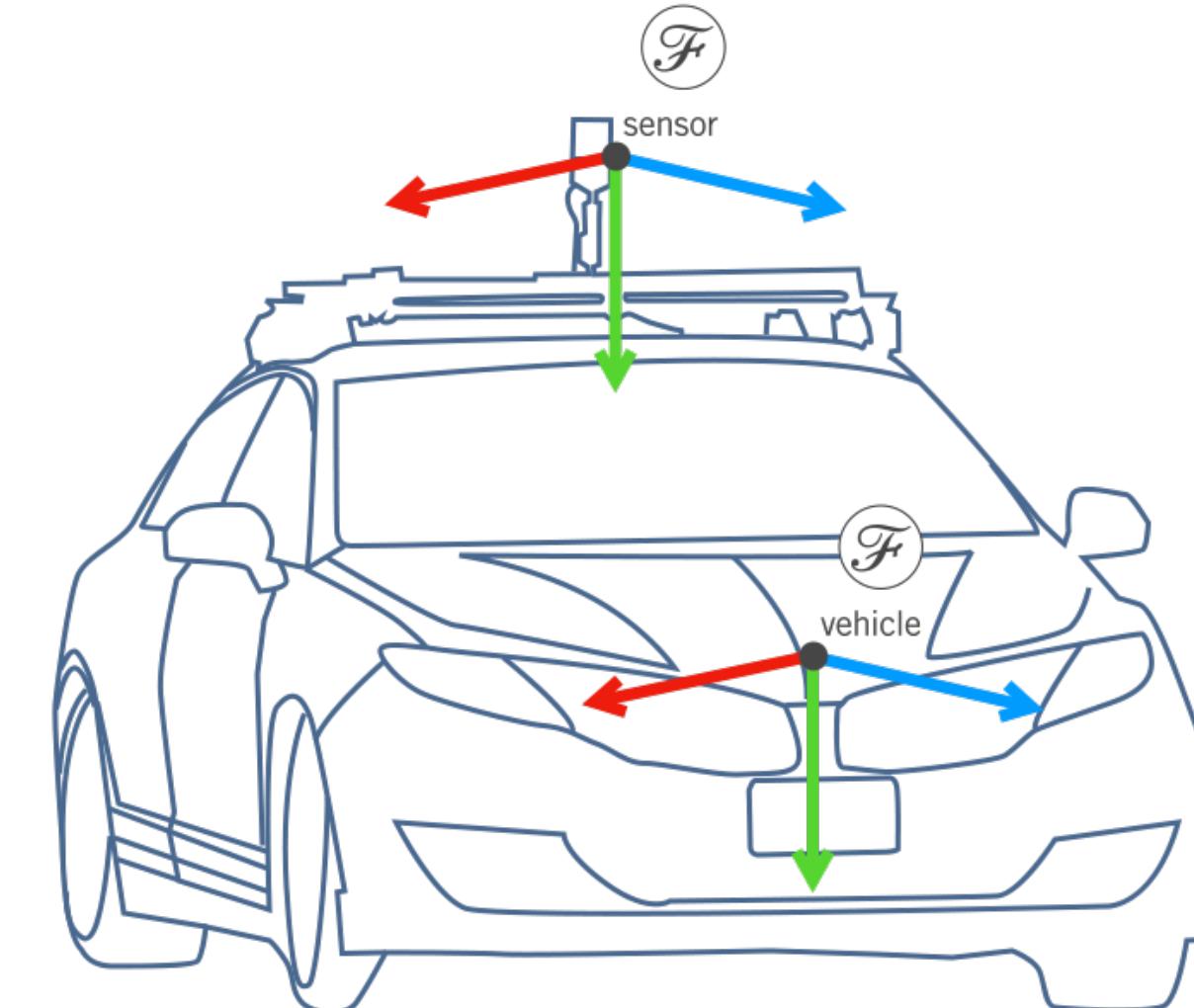
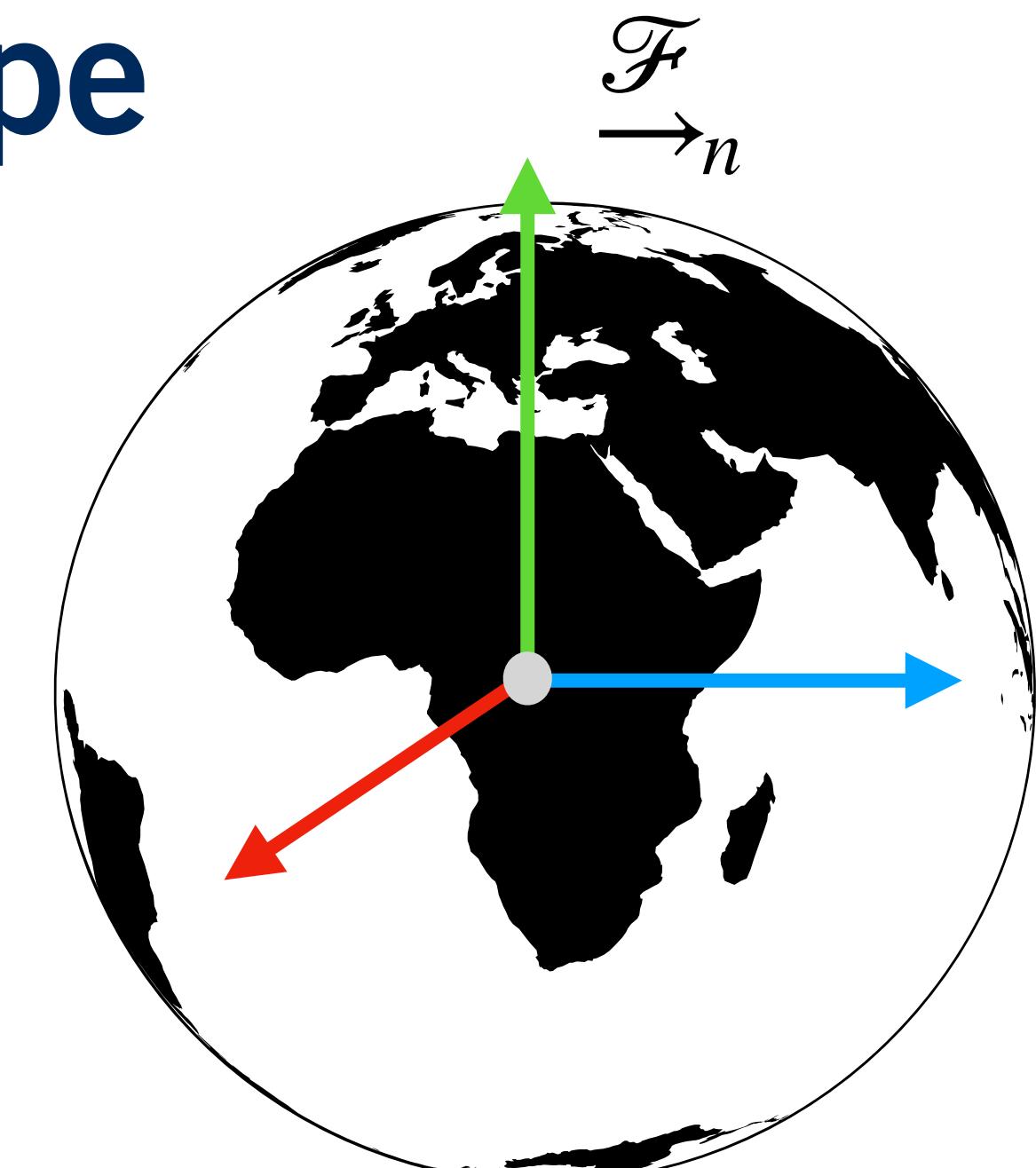
Measurement Model: Gyroscope

$$\boldsymbol{\omega}(t) = \boldsymbol{\omega}_s(t) + \mathbf{b}_{\text{gyro}}(t) + \mathbf{n}_{\text{gyro}}(t)$$

$\boldsymbol{\omega}_s(t)$ angular velocity of the sensor expressed in the sensor frame.

$\mathbf{b}_{\text{gyro}}(t)$ slowly evolving bias

$\mathbf{n}_{\text{gyro}}(t)$ noise term



Measurement Model: Accelerometer

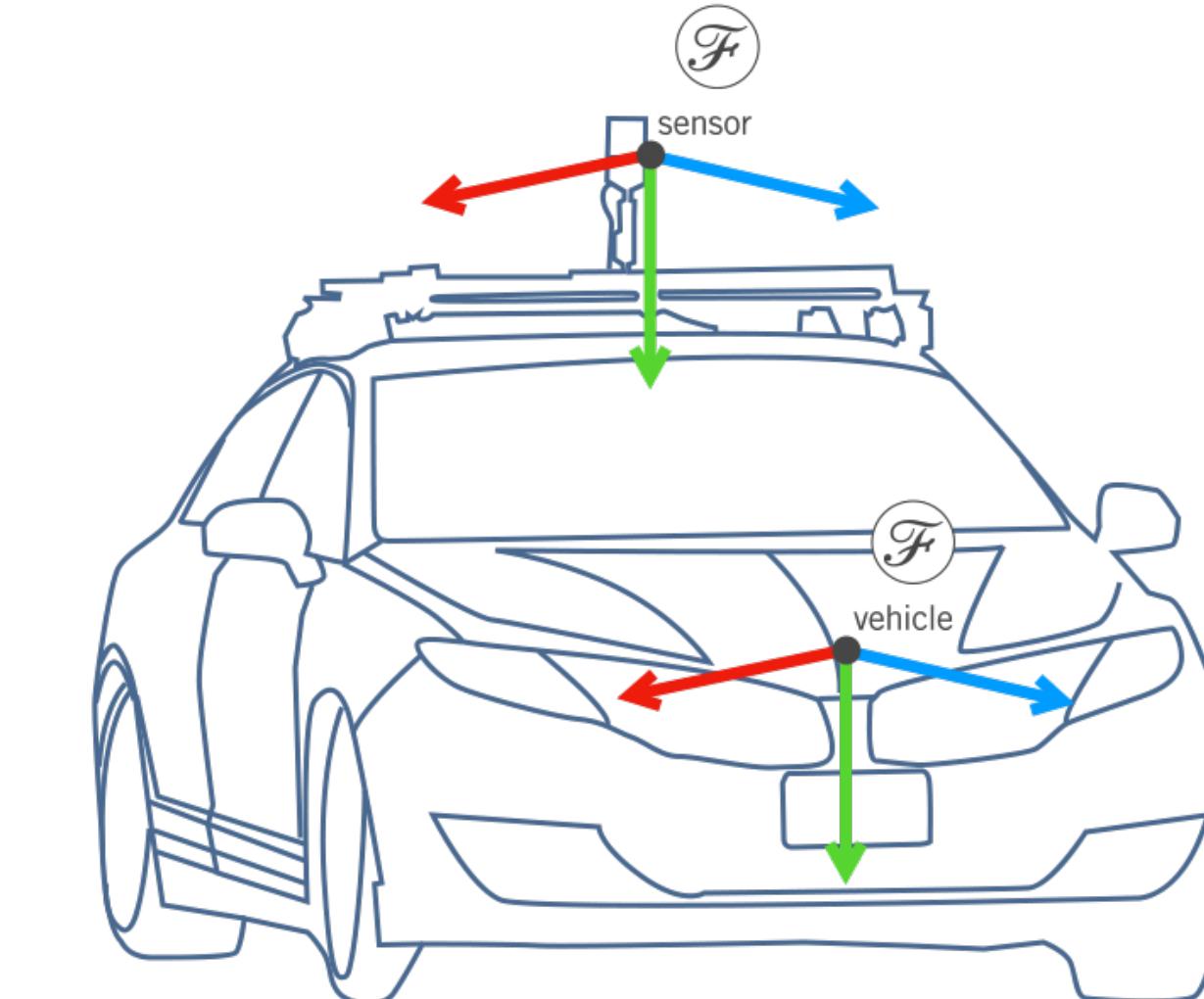
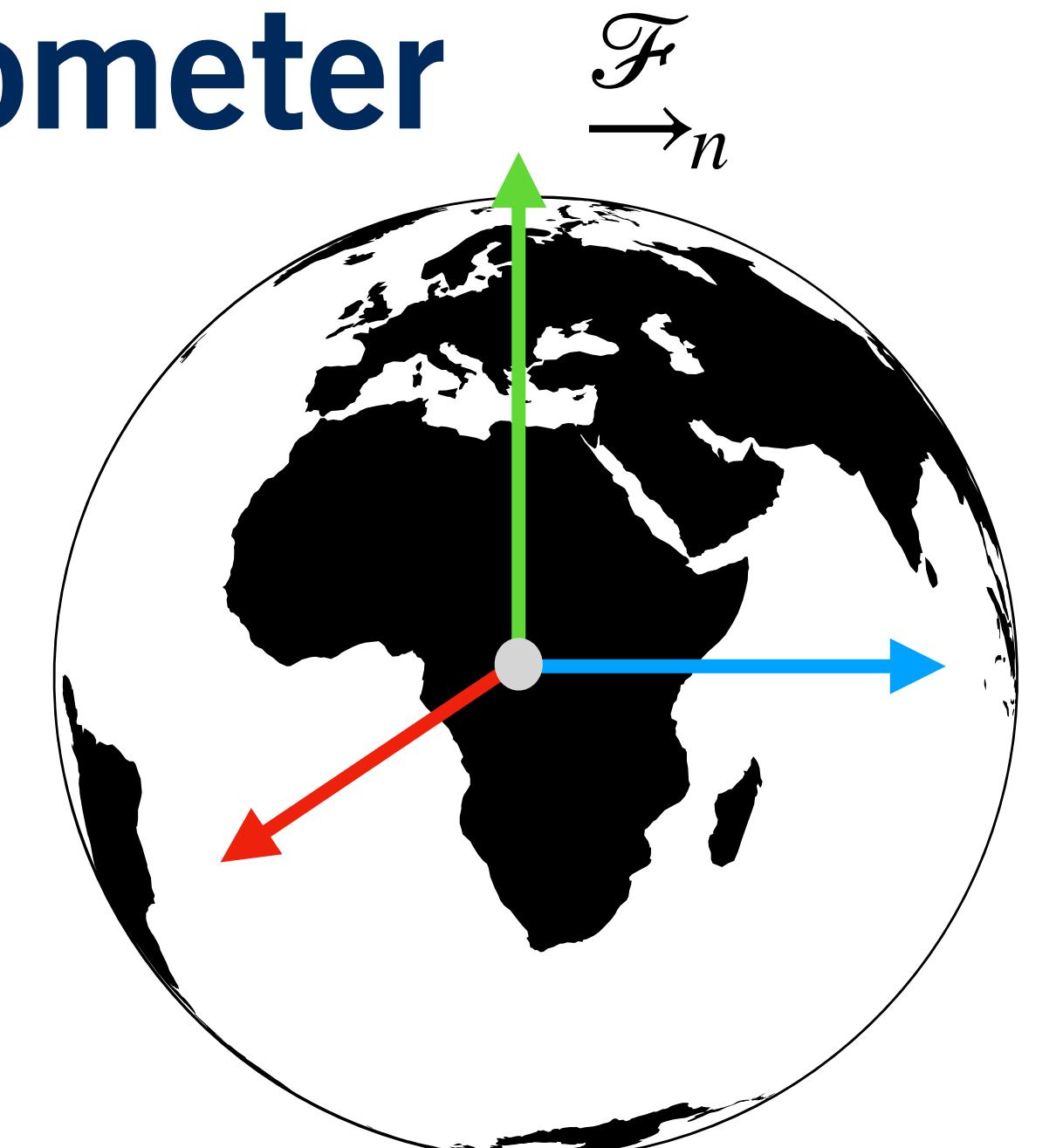
$$\mathbf{a}(t) = \mathbf{C}_{sn}(t)(\dot{\mathbf{r}}_n^{sn}(t) - \mathbf{g}_n) + \mathbf{b}_{accel}(t) + \mathbf{n}_{accel}(t)$$

$\mathbf{C}_{sn}(t)$ orientation of the sensor (computed by integrating the rotational rates from the gyroscope)

$\mathbf{b}_{accel}(t)$ bias term

$\mathbf{n}_{accel}(t)$ noise term

\mathbf{g}_n gravity in the navigation frame



Inertial Navigation: Important Notes

- When using an IMU for localization, keep in mind:
 1. If we inaccurately keep track of $\mathbf{C}_{sn}(t)$, we incorporate components of \mathbf{g}_n into $\dot{\mathbf{r}}_n^{sn}(t)$. This will ultimately lead to terrible estimates of position ($\mathbf{r}_n^{sn}(t)$).
 2. Both measurement models **ignore** the effect of **Earth's rotation**.
 3. We only consider **strapdown IMUs** - where the individual sensors are rigidly attached to the vehicle and are not gimballed.

Summary | The Inertial Measurement Unit

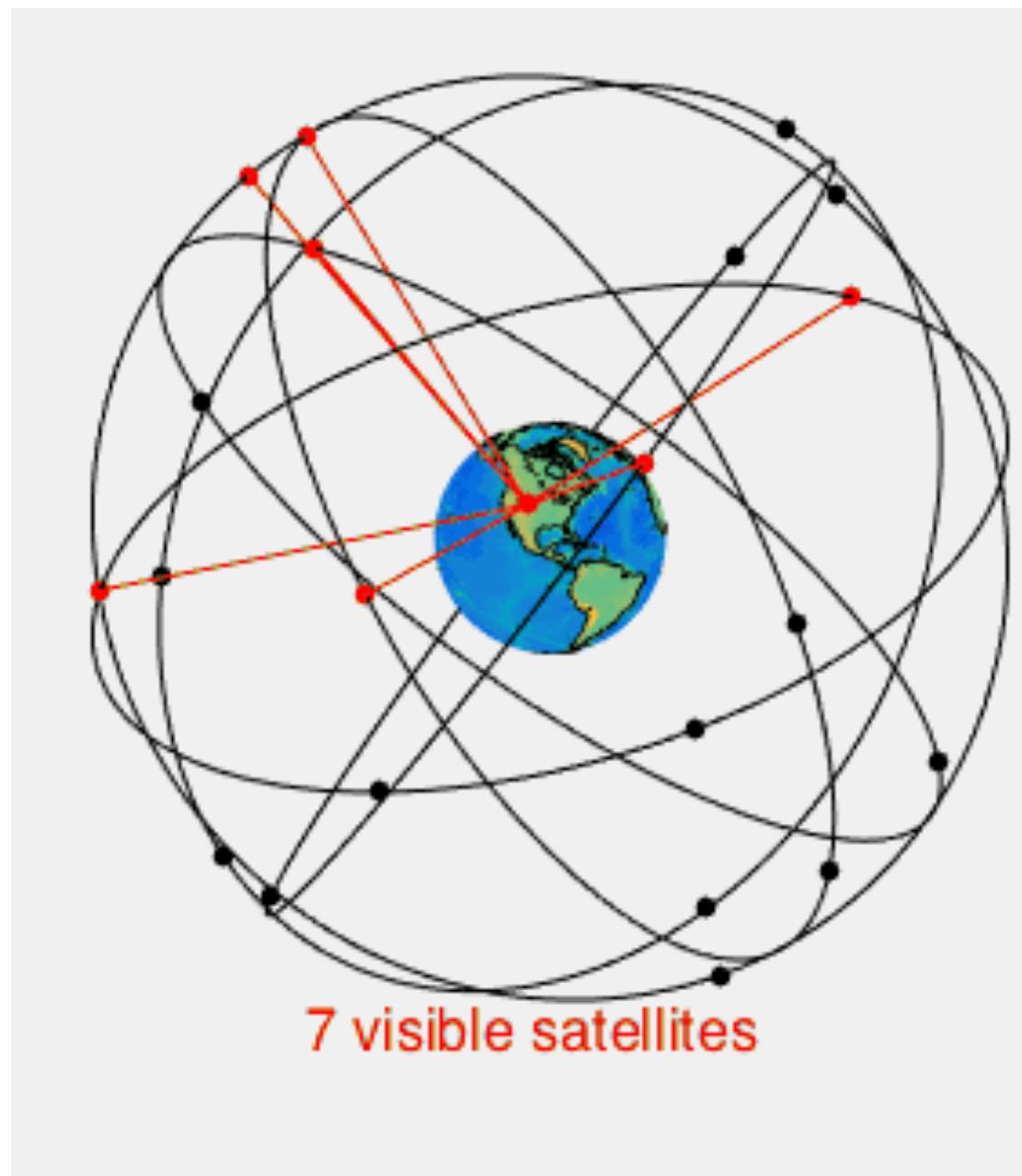
- A 6-DOF IMU is composed of three gyroscopes and three accelerometers, mounted orthogonally.
- A strapdown gyroscope measures a rotational rate in the sensor frame.
- A strapdown accelerometer measures a specific force (or acceleration relative to free-fall) in the sensor frame.

Module 3 | Lesson 3

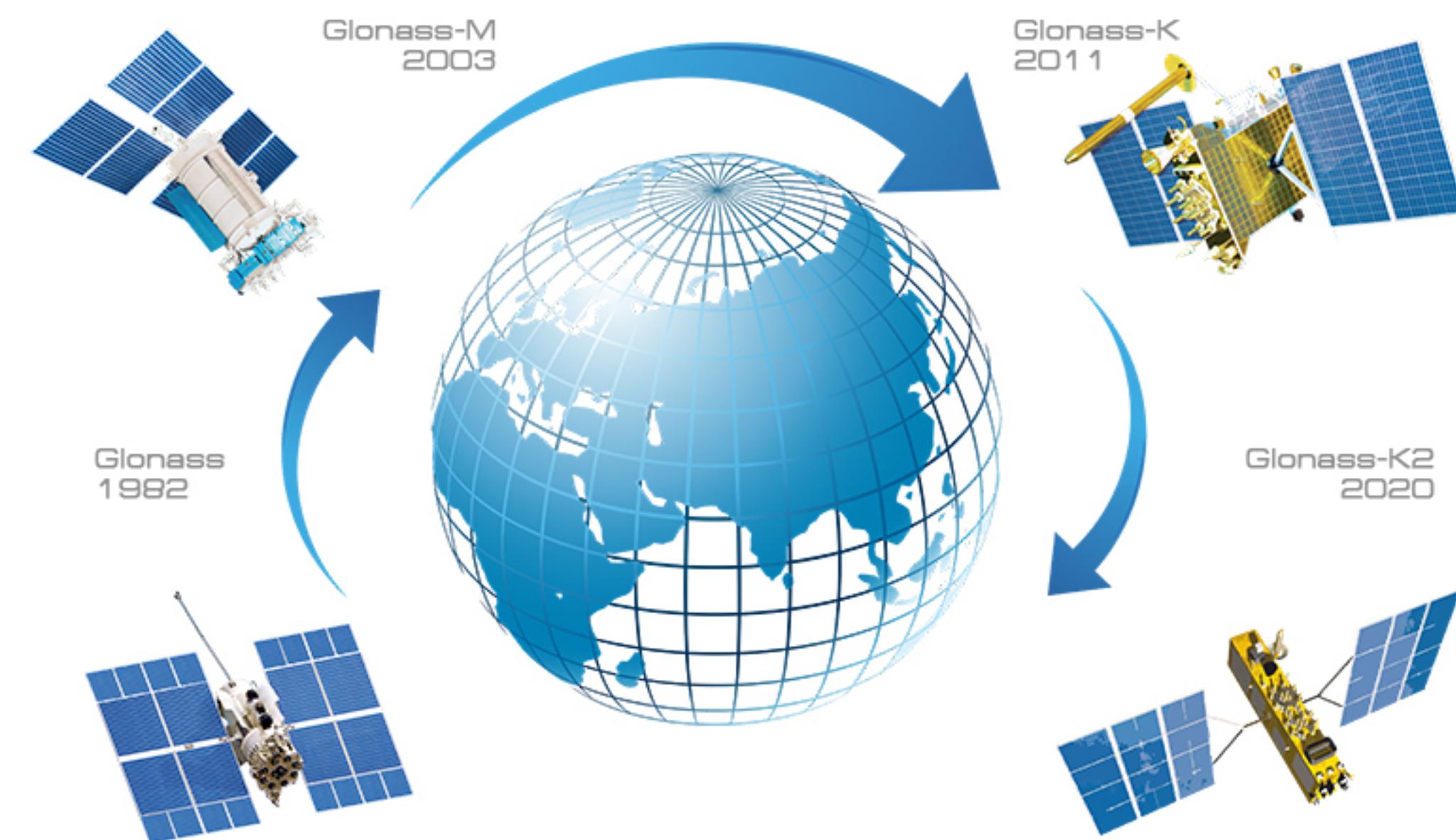
GLOBAL NAVIGATION SATELLITE SYSTEMS

GNSS | Accurate Global Positioning

- **Global Navigation Satellite System (GNSS)** is a catch-all term for a satellite system(s) that can be used to pinpoint a receiver's position anywhere in the world



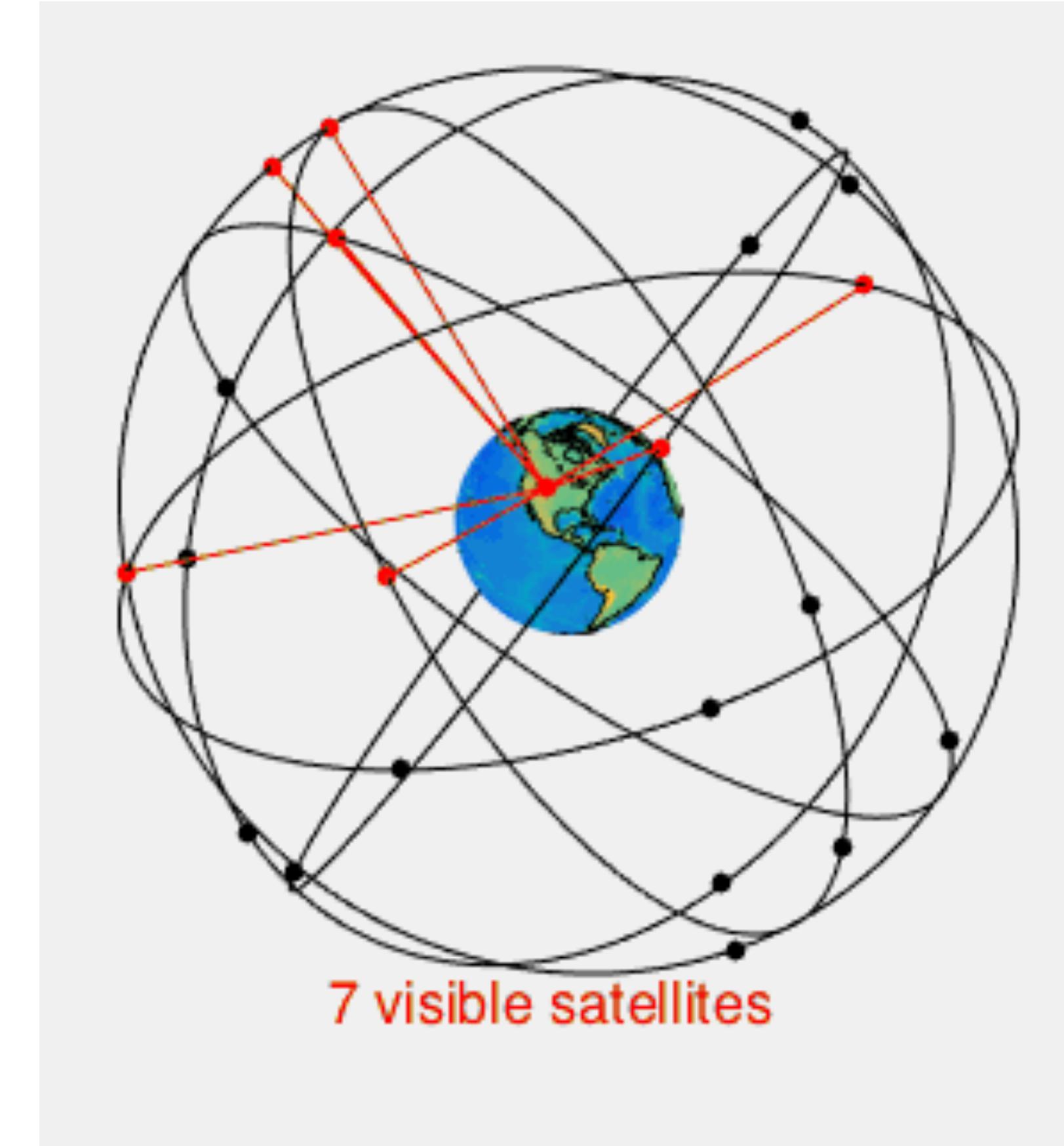
Global Positioning System (GPS)



Globalnaya navigatsionnaya
sputnikovaya sistema (GLONASS)

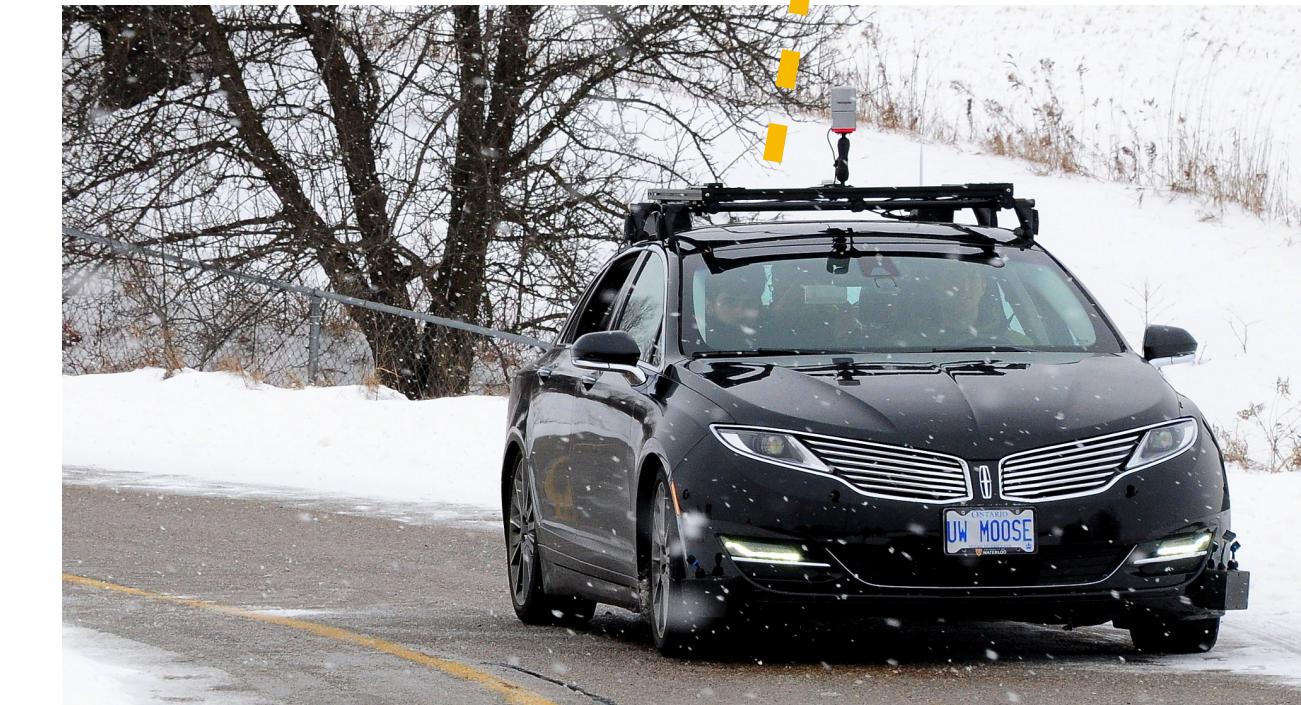
GPS | Global Positioning System

- Composed of 24 to 32 satellites in 6 orbital planes
 - *Altitude of ~20,200 km (12,550 miles)*
 - *Orbital period of ~12 hours*
- Each satellite broadcasts on two frequencies:
 - *L1 (1575.42 MHz, civilian + military)*
 - *L2 (1227.6 MHz, military)*



GPS I Computing Position

- Each GPS satellite transmits a signal that encodes
 1. its *position*
(via accurate ephemeris information)
 2. time of signal transmission
(via onboard atomic clock)
- To compute a GPS position fix in the Earth-centred frame, the receiver uses the **speed of light** to compute distances to each satellite based on time of signal arrival



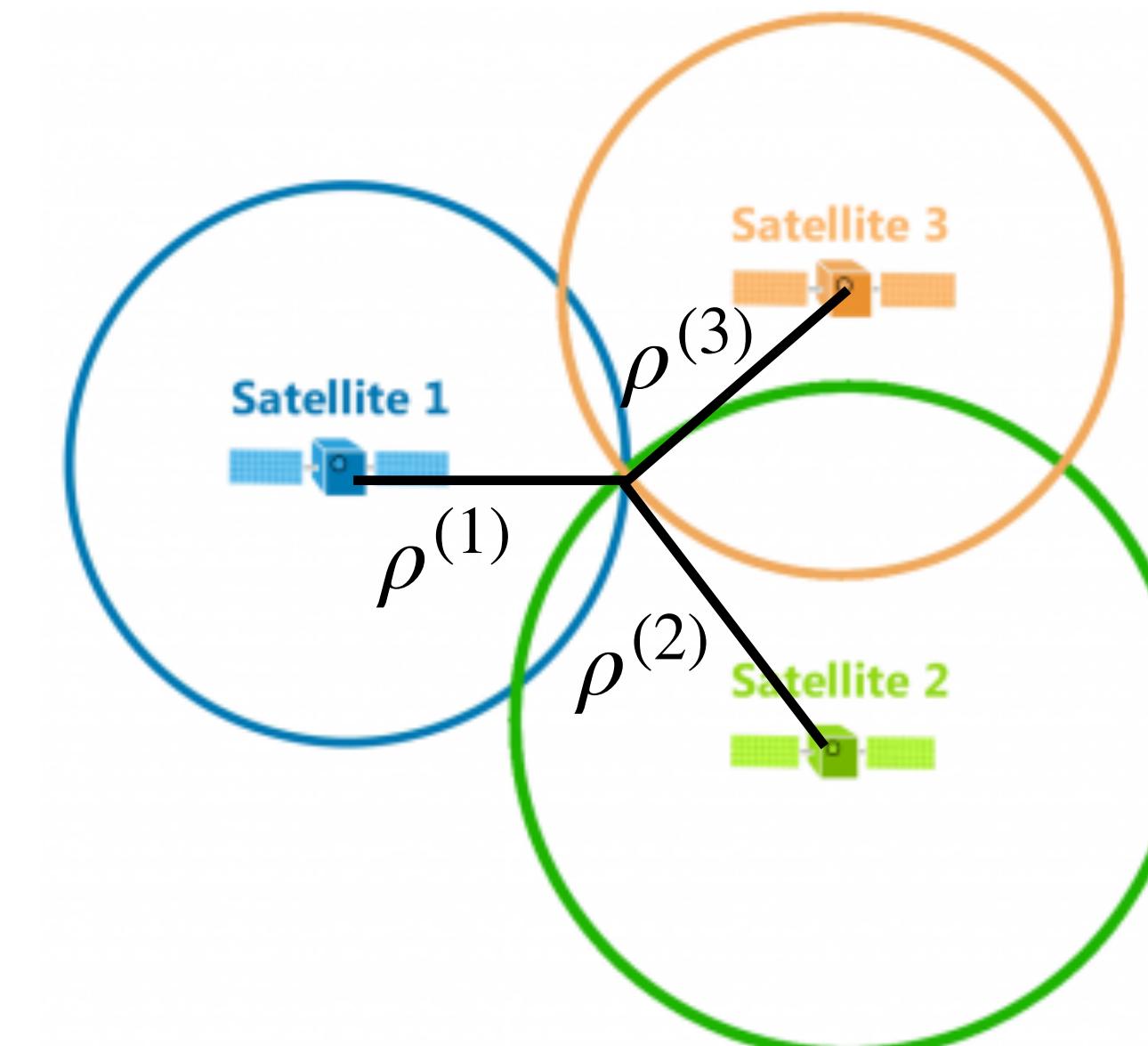
At least **four** satellites are required to solve for 3D position, three if only 2D is required (e.g., if altitude is known)

Trilateration

- For each satellite, we measure the **pseudorange** as follows:

$$\rho^{(i)} = c(t_r - t_s) = \sqrt{(\mathbf{p}^{(i)} - \mathbf{r})^T (\mathbf{p}^{(i)} - \mathbf{r})} + c\Delta t_r + c\Delta t_a^{(i)} + \eta^{(i)}$$

\mathbf{r} receiver (3D) position
 $\mathbf{p}^{(i)}$ position of satellite i
 Δt_r receiver clock error
 $\Delta t_a^{(i)}$ atmospheric propagation delay
 η measurement noise
 c speed of light
 t_s, t_r time sent, time received



Trilateration in 2D

Trilateration

- By using at least 4 satellites, we can solve for:

\mathbf{r} receiver (3D) position

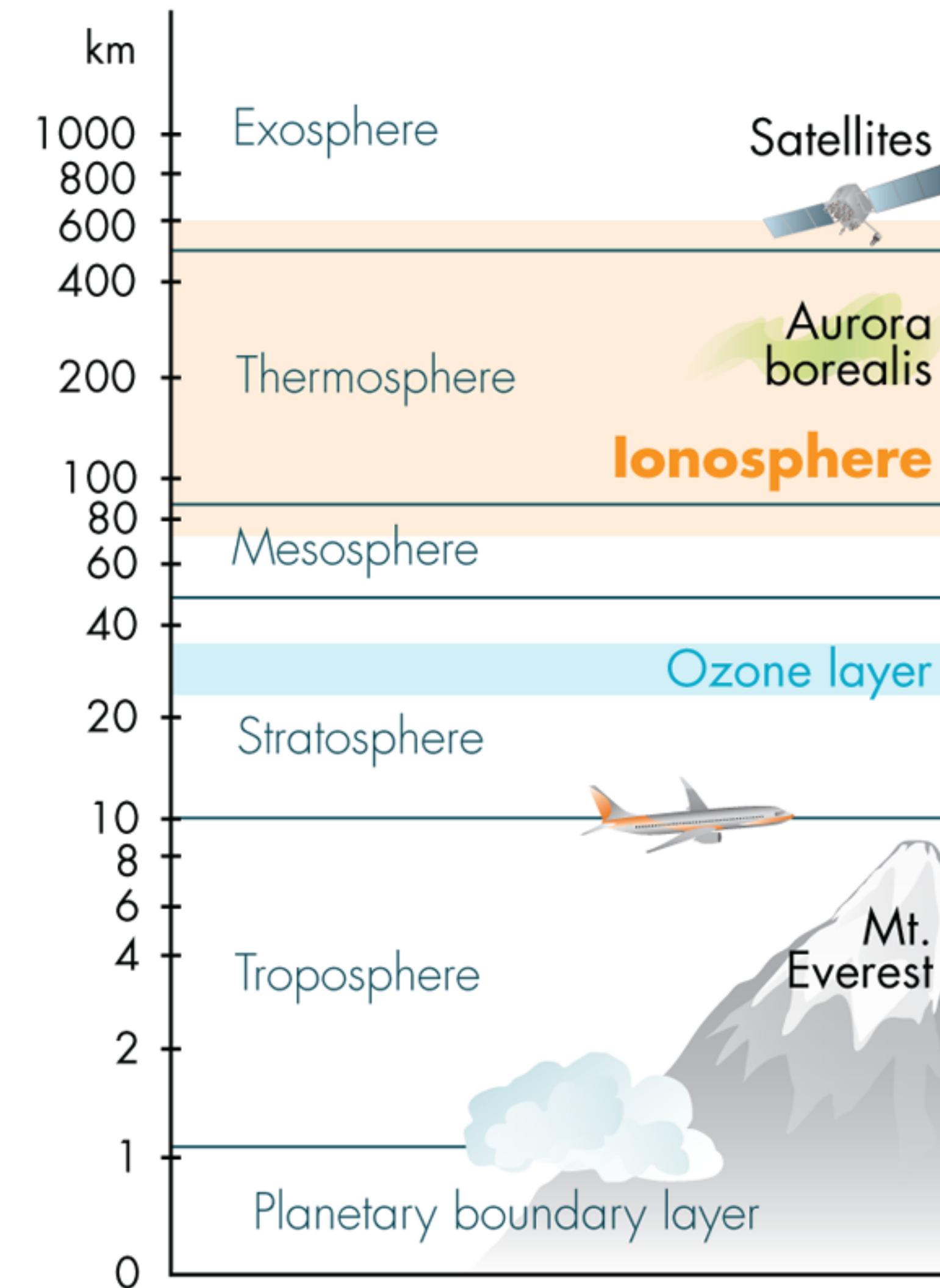
Δt_r receiver clock error



Multilateration in 3D

GPS | Error Sources (I)

- **Ionospheric delay**
 - Charged ions in the atmosphere affect signal propagation
- **Multipath effects**
 - Surrounding terrain, buildings can cause unwanted reflections



GPS | Error Sources (II)

- Ephemeris & clock errors
 - A clock error of 1×10^{-6} s gives a 300 m position error!
- Geometric Dilution of Precision (GDOP)
 - The configuration of the visible satellites affects position precision.



Poor config - high GDOP



Good config - low GDOP

GPS | Improvements

Basic GPS	Differential GPS (DGPS)	Real-Time Kinematic (RTK) GPS
mobile receiver	mobile receiver + fixed base station(s)	mobile receiver + fixed base station
no error correction	estimate error caused by atmospheric effects	estimate relative position using phase of carrier signal
~10 m accuracy	~10 cm accuracy	~2 cm accuracy