:# Assignment 1: Approximation of mathematical functions

```
import math
import numpy
import matplotlib.pyplot as plt
%matplotlib inline
```

Question 1: Approximating $\log$

Function $\log$ is defined on positive real numbers: $\log(x) : \mathcal{R}^+ \to \mathcal{R}$. The Taylor series for $\log$ has the following form:

$$\begin{aligned}\log(x) &= \frac{(x-1)^1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots \\ &= \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}\end{aligned}$$

and converges to $\log$ for $0 < x < 2$.

1. Implement `log1` that approximates $\log(x),\ 0 < x < 2$.

2.
    - Plot `math.log` and `log1` for $0.25 \leq x \leq 4$ for $n = 1, 2, 4, 8$.
    - Plot the **absolute** error of `log1` compared to `math.log` on $0.01 \leq x \leq 1$ for $n = 1, 8$. Use plt.plot:

3. Implement `log2` that approximates $\log(x),\ 0 < x < \infty$. *Hint:* $\log(e^N \cdot x) = N + \log x$.

4.
    - Plot `math.log` and `log2` for $0.1 \leq x \leq 100$ for $n = 1, 2, 4, 8$.
    - Plot the **relative** error of `log2` compared to `math.log` on $0.1 \leq x \leq 100$ for $n = 1, 8$. Use plt.plot:

5. What are the maximum absolute and relative errors of `log1` compared to `log2` on interval $0.25 \leq x \leq 1.75$?

Question 2: Using approximate versions of $\log$ and $\exp$

1. Use `logfact` from lecture 1 notebook with `math.log` and `math.exp`, to implement the multinomial coefficients for $m = 3$ on the log scale:

$$\binom{n}{k_1, k_2, \ldots, k_m} = \frac{n!}{k_1!k_2!\cdots k_m!}$$

2. Use `log2` from question 1 and `exp2` from the lecture 2 notebook to implement the approximate computation of the multinomial coefficients for $m = 3$:

3. Compute $\binom{10}{2,2,6}$, and $\binom{150}{100,25,25}$. Compare the running times of `multinomial coefficients` using `log2` and `exp2` vs. `math.log` and `math.exp`.

4. What are the absolute and relative error of `multinomial coefficients` for each combination of arguments — $\binom{10}{2,2,6}$, and $\binom{150}{100,25,25}$?

5. For which n(number of Taylor series terms) the absolute difference between the *relative errors* of $\binom{1000}{900,50,50}$ computed for `n` and `n+1` is smaller than 0.1%? Use the approximation from Q2.2