We consider a system with C = 40 circuits that serves a mix of services, requiring different levels of throughput, resulting in different consumption of circuits.

Part 1 : Voice calls only

We start by a unique service, consisting of voice calls consuming c_v = 1 circuit each and staying in the system for an exponential service time of parameter $\mu_v = 1/T_v$, with $T_v=180$ seconds.

1. Compute the blocking probability using the Erlang-B formula for a Poisson arrival rate of $\lambda_v \in [0.01; 0.5]$ calls per second. Deduce the maximal arrival rate for a blocking of 1%.
2. Construct the transition matrix Q of the corresponding continuous time Markov process and compute the steady-state probabilities by solving numerically the balance equations (matrix inversion). Compute the blocking rates and compare with 1.

Part 2 : Voice and video calls

We consider now a mix of two services, voice calls defined above and video streaming calls consuming c_s = 5 circuits each and staying in the system for an exponential service time of parameter $\mu_s = 1/T_s$, with $T_s=2$ minutes. We suppose that the aggregated arrival process of calls for each class is Poisson and that the arrival rate of video calls is $\lambda_s = 0.2\lambda_v$. Define, for class *i*, $E_i = {}^{\lambda_i}/_{\mu_i}$, the traffic in Erlang.

This is a multi-Erlang loss system, and the capacity constraint is given by:

$$n_s c_s + n_v c_v \leq C \qquad (1)$$

where n_i is the number of active users of class *i*. Blocking states for class *i* calls are acceptable states (verifying equation 1, i.e. belonging to the acceptable state space A) and for which:

$$n_s c_s + n_v c_v + c_i > C \qquad (2)$$

3. Construct the transition matrix of the corresponding continuous time Markov process and solve the balance equations. Deduce the blocking probability for each of the classes. Define the maximum voice arrival rate so that the blocking for both classes is below 1%.
4. The steady state probabilities can be computed using the multi-Erlang formula:

$$p(n_s, n_v) = \frac{1}{G} {}^{E_s^{n_s}}/_{n_s!} \, {}^{E_v^{n_v}}/_{n_v!}$$

   with the normalizing constant:

$$G = \sum_{(n_s,n_v)\in \mathrm{A}} {}^{E_s^{n_s}}/_{n_s!} \, {}^{E_v^{n_v}}/_{n_v!}$$

   Compute analytically the blocking rates for the different classes and compare with the result of 3.