Homework 1

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Task 1

Prove:

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \mathbb{E}\,\xi)^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 + (\mathbb{E}\,\xi - \overline{x})^2$$

Solution:

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \mathbb{E}\,\xi)^2 = \frac{1}{n}(\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n}(2\,\mathbb{E}\,\xi * x_i) + \sum_{i=1}^{n}(\mathbb{E}\,\xi)^2)$$

$$= \frac{1}{n}(\sum_{i=1}^{n} x_i^2 - 2\,\mathbb{E}\,\xi \sum_{i=1}^{n} x_i + n\,\mathbb{E}^2\,\xi)$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{2n}{n}\,\mathbb{E}\,\xi\overline{x} + \mathbb{E}^2\,\xi$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{x}\,\mathbb{E}\,\xi + \mathbb{E}^2\,\xi + \overline{x}^2 + \overline{x}^2 - 2\overline{x}^2$$

$$= (\mathbb{E}^2\,\xi - 2\overline{x}\,\mathbb{E}\,\xi + \overline{x}^2) + (\frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{x}^2 + \overline{x}^2)$$

$$= (\mathbb{E}\,\xi - \overline{x})^2 + \frac{1}{n}\sum_{i=1}^{n}(x_i^2 - 2\overline{x}x_i + \overline{x}^2)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 + (\mathbb{E}\,\xi - \overline{x})^2$$