Sequential Phase Linking: Progressive integration of SAR images for operational phase estimation

Dana El Hajjar[1,2], Yajing Yan[1], Guillaume Ginolhac[1], Mohammed Nabil El Korso[2]

[1] Université Savoie Mont Blanc, Laboratoire d'Informatique, Systèmes, Traitement de l'Information et de la Connaissance (LISTIC), Annecy, France
[2] Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des Signaux et Systèmes (L2S), Gif-sur-Yvette, France

This work is funded by the ANR REPED-SARIX project(ANR-21-CE23-0012-01) of the French national Agency of research, and Neptune 3 project.

Context

Motivations

- Huge SAR data volumes thanks to ongoing and upcoming SAR missions
- Traditional MT-InSAR methods struggle to efficiently integrate newly acquired data, often requiring reprocessing of the entire datasets.

Contributions

We aim to develop a sequential approach based on joint Maximum Likelihood Estimation (MLE) [3] that:

- efficiently handles newly acquired data
- yields similar performances as offline approaches with a much lower computational cost
- can be extended to a more robust version to handle non-Gaussian data distributions

Background

Covariance matrix structure

The covariance matrix for the SAR image time series is

$$\widetilde{\boldsymbol{\Sigma}} = \widetilde{\boldsymbol{\Psi}} \odot \widetilde{\mathbf{w}}_\theta \widetilde{\mathbf{w}}_\theta^H$$

where $\widetilde{\boldsymbol{\Psi}}$ is the real core of the covariance matrix and $\widetilde{\mathbf{w}}_\theta$ denotes the vector of the exponential of the complex phases ($\widetilde{\mathbf{w}}_\theta = [e^{j\theta_0}, \dots, e^{j\theta_l}]$).

Phase Linking approach : MLE-PL

A joint estimation of $\widetilde{\boldsymbol{\Psi}}$ and $\widetilde{\mathbf{w}}_\theta$ is proposed [3]

$$\begin{array}{ll} \underset{\widetilde{\mathbf{w}}_\theta, \widetilde{\boldsymbol{\Psi}}}{\min} & \mathcal{L}(\widetilde{\boldsymbol{\Psi}} \odot \widetilde{\mathbf{w}}_\theta \widetilde{\mathbf{w}}_\theta^H) \\ \text{subject to} & \widetilde{\boldsymbol{\Psi}} \text{ real}, \widetilde{\mathbf{w}}_\theta \in \mathbb{T}_l,\ \theta_1 = 0 \end{array}$$

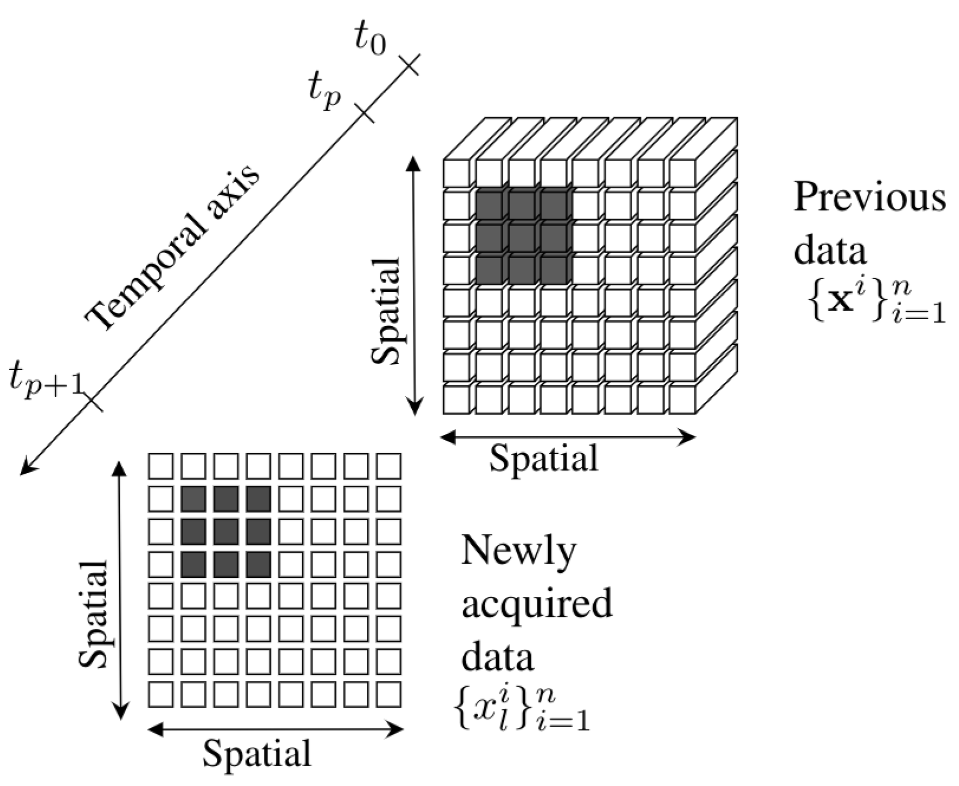
where $\mathbb{T}_l = \left\{ \widetilde{\mathbf{w}} \in \mathbb{C}^l \,|\, |[\widetilde{w}]_i| = 1, \forall i \in [1, l] \right\}$ is the l-torus of phase only complex vectors.

Data modeling

We consider a stack of $l = p + 1$ SAR images, for a given pixel, we denote $\{\widetilde{\mathbf{x}}^i\}_{i=1}^n$ the local homogeneous spatial neighborhood of size n, where $\widetilde{\mathbf{x}}^i \in \mathbb{C}^l$ $\forall i \in [\![1, n]\!]$, i.e.,

$$\widetilde{\mathbf{x}}^i = [\underbrace{x_1^i, \dots, x_p^i}_{\mathbf{x}^i}, x_l^i]^T \in \mathbb{C}^l$$

where $\mathbf{x}^i \in \mathbb{C}^p$ denotes the multivariate pixel of the previous data. Each pixel of the local patch is assumed to be distributed as a zero mean Complex Circular Gaussian (CCG) , i.e. $\widetilde{\mathbf{x}} \sim \mathcal{CN}(0, \widetilde{\boldsymbol{\Sigma}})$.



The hermitian structured covariance matrix, can be written as

$$\widetilde{\boldsymbol{\Sigma}} = \begin{pmatrix} \boldsymbol{\Sigma} & w_{\theta_l}^* \text{diag}(\mathbf{w}_\theta)\boldsymbol{\gamma}^T \\ \boldsymbol{\gamma}\text{diag}(\mathbf{w}_\theta)^H w_{\theta_l} & \gamma_l \end{pmatrix}$$

Sequential Phase Linking (S-MLE-PL)

The negative log-likelihood for the entire dataset, can be expressed as

$$\mathcal{L}_G(\boldsymbol{\gamma}, \gamma_l, w_{\theta_l}) = -\sum_{i=1}^n \mathcal{L}_G^i(x_{p+1}^i | \mathbf{x}^i; \boldsymbol{\gamma}, \gamma_l, w_{\theta_l}) + \mathcal{L}_G^i(\mathbf{x}^i)$$

According to [1], $x_l^i | \mathbf{x}^i \sim \mathcal{CN}(\mu_x^i, \sigma_x^2)$ where
$\mu_x^i = w_{\theta_l} \boldsymbol{\gamma} \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{x}^i$
and $\sigma_x^2 = \gamma_l - \boldsymbol{\gamma} \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \text{diag}(\hat{\mathbf{w}}_\theta^H) \boldsymbol{\gamma}^T$.

$$\mathcal{L}_G(\boldsymbol{\gamma}, \gamma_l, w_{\theta_l}) \propto n \log(v) + \sum_{i=1}^n \frac{y^{i*} y^i}{v}$$

where $y^i = x_l^i - w_{\theta_l} \boldsymbol{\gamma}\, \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \mathbf{x}^i$
and $v = \gamma_l - \boldsymbol{\gamma}\, \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \text{diag}(\hat{\mathbf{w}}_\theta) \boldsymbol{\gamma}^T$

In this work, we propose to estimate simultaneously the coherence and the new phase difference [2]

$$\begin{array}{ll} \underset{\boldsymbol{\gamma}, \gamma_l, \theta_l}{\min} & \mathcal{L}_G(\boldsymbol{\gamma}, \gamma_l, w_{\theta_l}) \\ \text{subject to} & \boldsymbol{\gamma}, \gamma_l \text{ real},\ |w_{\theta_l}| = 1,\ \theta_1 = 0 \end{array}$$

The above problem is addressed using a **Block Coordinate Descent (BCD)** algorithm.

Algorithm

1: **Input**: Samples $\{\tilde{\mathbf{x}}^i\}_{i=1}^n$, $\hat{\boldsymbol{\Sigma}}$, diag$(\hat{\mathbf{w}}_\theta)$
2: **repeat** 3,4,5
3: Update $\boldsymbol{\gamma}$

$$\boldsymbol{\gamma} = \left(\sum_{i=1}^n w_{\theta_l}^* x_l^i \mathbf{L}^i - w_{\theta_l} x_l^{i*} \mathbf{L}^{i*}\right) \left(2 \sum_{i=1}^n \text{Re}(\mathbf{M}^i)\right)^{-1}$$

4: Update γ_l

$$\gamma_l = \frac{1}{n} \sum_{i=1}^n (x_l^i - w_{\theta_l} \boldsymbol{\gamma} \mathbf{L}^{iH})^* (x_l^i - w_{\theta_l} \boldsymbol{\gamma} \mathbf{L}^{iH}) + k$$

5: Update w_{θ_l}

$$w_{\theta_l} = \mathcal{P}_{\mathbb{T}_1}\left(\left(\sum_{i=1}^n x_l^i \mathbf{L}^i \boldsymbol{\gamma}^T\right) \cdot \left(\sum_{i=1}^n \boldsymbol{\gamma} \mathbf{M}^i \boldsymbol{\gamma}^T\right)^{-1} \right)$$

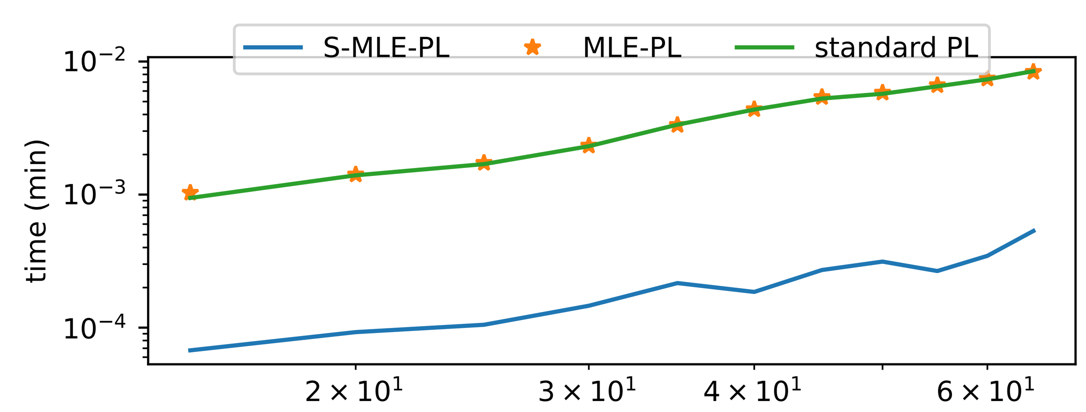
6: **until** convergence
7: **Output**: $\boldsymbol{\gamma}$, γ_l, and w_{θ_l}

where $\mathbf{N} = \text{diag}(\hat{\mathbf{w}}_\theta)^H \hat{\boldsymbol{\Sigma}}^{-1} \text{diag}(\hat{\mathbf{w}}_\theta)$, $k = \boldsymbol{\gamma} \mathbf{N} \boldsymbol{\gamma}^T$, $\mathbf{L}^i = \mathbf{x}^{iH} \hat{\boldsymbol{\Sigma}}^{-1} \text{diag}(\hat{\mathbf{w}}_\theta)$, and $\mathbf{M}^i = \mathbf{L}^{iH} \mathbf{L}^i$.

Computational efficiency

Simulation Setup:

- **Time Series Size**: $l = p + 1 = 20$ images.
- **Sample size**: $n = 2 \times l$
- **Coherence Matrix**: $[\widetilde{\boldsymbol{\Psi}}]_{ij} = \rho^{|i-j|}$ with $\rho = 0.7$.
- **Phase Differences**: $\Delta_{i,i-1} = \theta_i - \theta_{i-1} = \frac{2}{l}$ rad.
- **Sample Generation**: $\widetilde{\mathbf{x}}^i \sim \mathcal{N}(0, \widetilde{\boldsymbol{\Sigma}})$

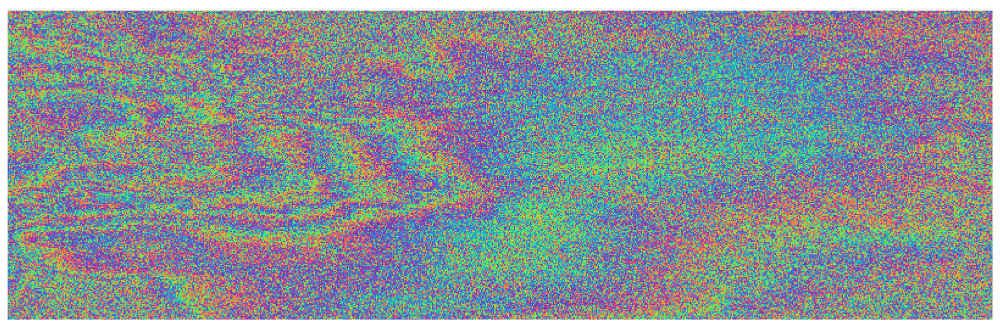


Comparison of computation time versus l for standard Phase Linking (PL), Phase Linking based on Maximum Likelihood Estimation (MLE-PL) and Sequential Phase Linking based on Maximum Likelihood Estimation (S-MLE-PL).

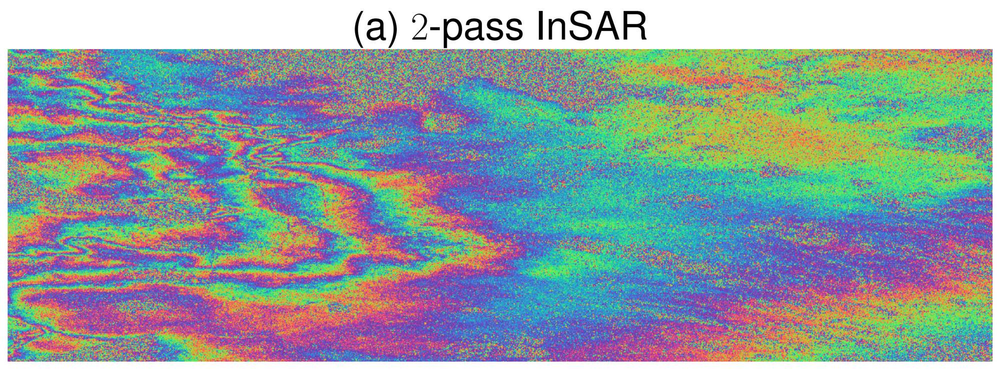
Real world application

Mexico City dataset:

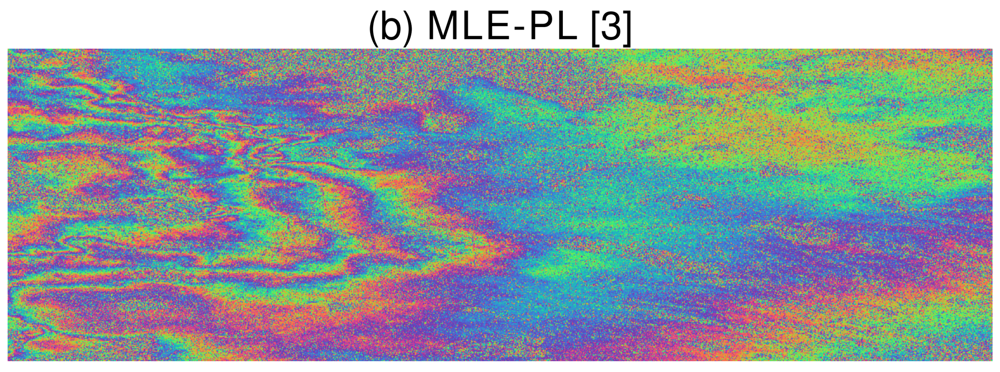
- **Time series size:** $l = 20$ SAR images Sentinel-1 acquired every 12 days
- **Acquisition time span:** 14 August 2019 - 10 April 2020 (8 months)
- **Multi-looking window size:** $n = 64$



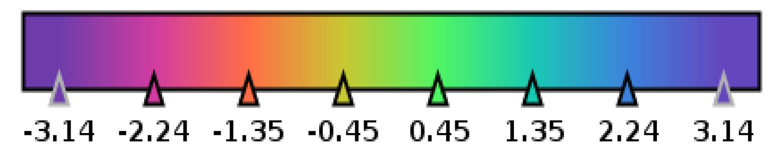
(a) 2-pass InSAR



(b) MLE-PL [3]



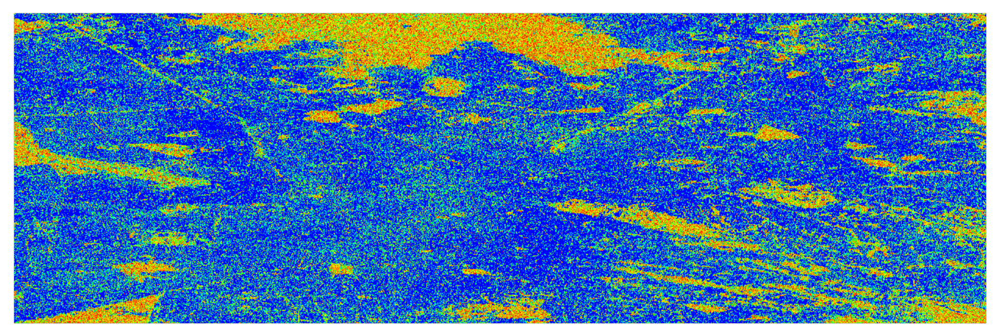
(c) S-MLE-PL



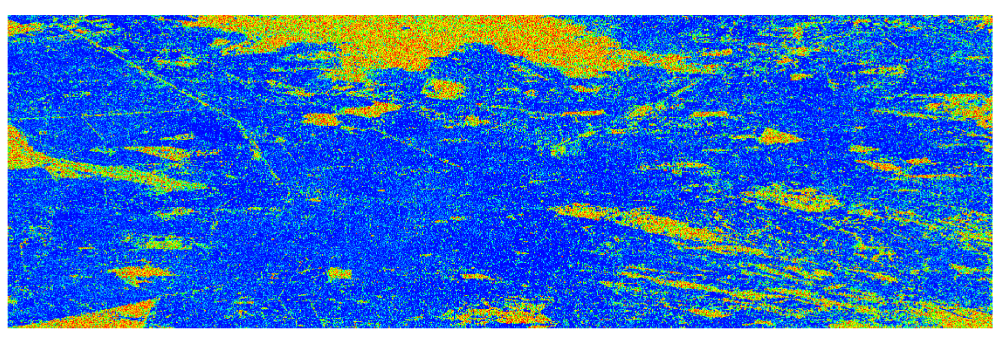
Close-up view of the longest temporal baseline interferogram (14 August 2019 - 10 April 2020): (a) 2-pass InSAR (b) MLE-PL [3] and (c) S-MLE-PL in case $l = 20$ and $n = 64$.

The quality of the estimated interferometric phases may be assessed by the goodness of the fit between the observed phases and the estimated

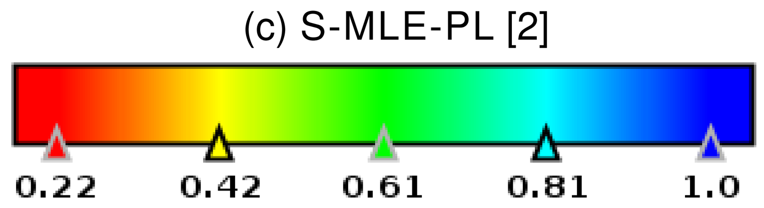
$$\gamma_{\text{post}} = \frac{\text{Re}\left(\sum_{q=1}^l \sum_{i=q+1}^l \mathrm{e}^{(\Delta\theta_{iq} - (\hat{\theta}_i - \hat{\theta}_q))}\right)}{l(l-1)/2}$$



(a) MLE-PL [3]



(c) S-MLE-PL [2]



Conclusions and perspectives

We present a novel sequential approach that allows incorporating efficiently new images in InSAR phase estimation in a PL framework.

Perspectives

- generalization of S-MLE-PL to a block of new SAR images
- estimate the displacement time series and compare the results with GPS data

References

[1] T. W. Anderson. An introduction to multivariate statistical analysis, volume 2. Wiley New York, 1958.
[2] D. El Hajjar, Y. Yan, G. Ginolhac, and M.N. El Korso. Sequential phase linking: progressive integration of SAR images for operational phase estimation. In IGARSS International Geoscience and Remote Sensing Symposium. IEEE, 2024.
[3] P. Vu, A. Breloy, F. Brigui, Y. Yan, and G. Ginolhac. Robust phase linking in insar. IEEE Transactions on Geoscience and Remote Sensing, 61:1–11, 2023.