A TRANSLATING FORT INTO $\mathcal{SROIQ}$

In this section we illustrate the translation of FORT. For the application of the first, second, third, fourth, and fifth steps, we only show it on a single axiom; the specific existential dependence axiom in section A.1. Then we illustrate the output of the fifth step in section A.2, and that of sixth step in section A.3.

A.1 Applying steps (1-5) on a single axiom

Consider the definition of the specific existential dependence relation from FORT: An entity x is specifically existentially dependent entity y, denoted SED(x, y), iff; at any time t, x cannot exist at t unless y exists at t; & x and y are different entities; & x exists at some t (D).

$$\forall(x,y)SED(x,y) \rightarrow \forall t(E(x,t) \rightarrow E(y,t)) \wedge \neg(x=y) \wedge \exists tE(x,t) \tag{D}$$

In the following, we express a binary predicate $R(x, y)$ as R_{xy}, and the predicate $x = y$ as Eq_{xy}.

A.1.1 Transforming to Clausal Form:

$$\begin{aligned}&\forall(x,y)[\neg SED_{xy} \vee (\forall t(\neg E_{xt} \vee E_{yt}) \wedge \neg Eq_{xy} \wedge \exists tE_{xt})]\wedge\\&[(\exists t(E_{xt} \wedge \neg E_{yt}) \vee Eq_{xy} \vee \forall t\neg E_{xt}) \vee SED_{xy}]\end{aligned} \tag{NNF}$$

$$\begin{aligned}&\forall x,y,m\exists n,a\forall b[\neg SED_{xy} \vee ((\neg E_{xm} \vee E_{ym}) \wedge \neg Eq_{xy} \wedge E_{xn})]\wedge\\&[((E_{xa} \wedge \neg E_{ya}) \vee Eq_{xy} \vee \neg E_{xb}) \vee SED_{xy}]\end{aligned} \tag{PNF}$$

Upon skolemization, substitute n and a by the skolem functions f and g respectively as $\{n \leftarrow f(x,y,m)\}$ and $\{a \leftarrow g(x,y,m)\}$. For simplicity we will write $f(x,y,m)$ as f and $g(x,y,m)$ as g.

$$\begin{aligned}&[\neg SED_{xy} \vee ((\neg E_{xm} \vee E_{ym}) \wedge \neg Eq_{xy} \wedge E_{xf})]\wedge\\&[((E_{xg} \wedge \neg E_{yg}) \vee Eq_{xy} \vee \neg E_{xb}) \vee SED_{xy}]\end{aligned} \tag{SNF}$$

$$\begin{aligned}&(\neg SED_{xy} \vee \neg E_{xm} \vee E_{ym}) \wedge (\neg SED_{xy} \vee \neg Eq_{xy}) \wedge (\neg SED_{xy} \vee E_{xf})\\&\wedge(E_{xg} \vee Eq_{xy} \vee \neg E_{xb} \vee SED_{xy}) \wedge (\neg E_{yg} \vee Eq_{xy} \vee \neg E_{xb} \vee SED_{xy})\end{aligned} \tag{CNF}$$

A.1.2 Rewriting as Horn rules:

$$SED_{xy} \wedge E_{xm} \rightarrow E_{ym} \tag{R1}$$
$$SED_{xy} \rightarrow \neg Eq_{xy} \tag{R2}$$
$$SED_{xy} \rightarrow E_{xf} \tag{R3}$$

A.1.3 Qualifying Expressible Horn rules:

R3 does not qualify since the variables are not enclosed.

$$SED_{xy} \wedge E_{xm} \rightarrow E_{ym} \tag{R1}$$
$$SED_{xy} \rightarrow \neg Eq_{xy} \tag{R2}$$

A.1.4 Constructing the rule graphs:

$$G = \langle\{x,y,m\}, \{SED_{xy}, E_{ym}\}, \emptyset, y, m : E\rangle \tag{G1}$$
$$G = \langle\{x,y\}, \{SED_{xy}\}, \emptyset, x, y : \neg Eq\rangle \tag{G2}$$

A.1.5 Converting into axioms:

$$SED^{-} \circ E \sqsubseteq E \tag{A1}$$
$$SED \sqsubseteq nEq, where nEq = \neg Eq \tag{A2}$$

A.2 the non-structured set of SROIQ axioms

We illustrate below the output of step 5; $FORT_{S5}$ as the set of 120 non-structured axioms, and the structures R_{NS} and I resembling the set of non-simple roles and the set of proposition builders, respectively.

$$\mathrm{SED}^{-} \circ \mathrm{E} \sqsubseteq \mathrm{E} \quad \text{(a1)}$$
$$\mathrm{SED} \sqsubseteq \neg\mathrm{equal} \quad \text{(a2)}$$
$$\mathrm{SED} \circ \mathrm{negE} \sqsubseteq \varnothing \quad \text{(a3)}$$
$$\mathrm{SED} \circ \mathrm{SED} \sqsubseteq \mathrm{SED} \quad \text{(a4)}$$
$$\mathrm{componentOf} \sqsubseteq \mathrm{partOf} \quad \text{(a5)}$$
$$\mathrm{Tra}(\mathrm{componentOf}) \quad \text{(a6)}$$
$$\mathrm{Irr}(\mathrm{componentOf}) \quad \text{(a7)}$$
$$\mathrm{Asy}(\mathrm{componentOf}) \quad \text{(a8)}$$
$$\mathrm{componentOf} \sqsubseteq \mathrm{properPartOf} \quad \text{(a9)}$$
$$\mathrm{componentOf} \circ \neg\mathrm{PartOf}^{-} \sqsubseteq \varnothing \quad \text{(a10)}$$
$$\mathrm{overlaps} \circ \mathrm{componentOf} \sqsubseteq \varnothing \quad \text{(a11)}$$
$$\mathrm{elementOf} \sqsubseteq \mathrm{partOf} \quad \text{(a12)}$$
$$\mathrm{elementOf} \sqsubseteq \mathrm{SED} \quad \text{(a13)}$$
$$\mathrm{Tra}(\mathrm{elementOf}) \quad \text{(a14)}$$
$$\mathrm{Irr}(\mathrm{elementOf}) \quad \text{(a15)}$$
$$\mathrm{Asy}(\mathrm{elementOf}) \quad \text{(a16)}$$
$$\mathrm{elementOf} \sqsubseteq \mathrm{properPartOf} \quad \text{(a17)}$$
$$\mathrm{elementOf} \circ \neg\mathrm{PartOf}^{-} \sqsubseteq \varnothing \quad \text{(a18)}$$
$$\mathrm{overlaps} \circ \mathrm{elementOf} \sqsubseteq \varnothing \quad \text{(a19)}$$
$$\mathrm{Tra}(\mathrm{partOf}) \quad \text{(a20)}$$
$$\mathrm{Ref}(\mathrm{partOf}) \quad \text{(a21)}$$
$$\mathrm{equal} \sqsubseteq \mathrm{partOf} \quad \text{(a22)}$$
$$\mathrm{equal} \sqsubseteq \mathrm{partOf}- \quad \text{(a23)}$$
$$\mathrm{Tra}(\mathrm{equal}) \quad \text{(a24)}$$
$$\mathrm{Ref}(\mathrm{equal}) \quad \text{(a25)}$$
$$\mathrm{Sym}(\mathrm{equal}) \quad \text{(a26)}$$
$$\mathrm{properPartOf} \sqsubseteq \mathrm{partOf} \quad \text{(a27)}$$
$$\mathrm{properPartOf} \sqsubseteq \neg\mathrm{partOf}- \quad ||\mathrm{Dis}(\mathrm{properPartOf}, \mathrm{partOf}-) \quad \text{(a28)}$$
$$\mathrm{Tra}(\mathrm{properPartOf}) \quad \text{(a29)}$$
$$\mathrm{Irr}(\mathrm{properPartOf}) \quad \text{(a30)}$$
$$\mathrm{Asy}(\mathrm{properPartOf}) \quad \text{(a31)}$$
$$\mathrm{partOf}- \circ \mathrm{partOf} \sqsubseteq \mathrm{overlaps} \quad \text{(a32)}$$
$$\mathrm{Ref}(\mathrm{overlaps}) \quad \text{(a33)}$$
$$\mathrm{Sym}(\mathrm{overlaps}) \quad \text{(a34)}$$
$$\mathrm{partOf} \circ \mathrm{partOf}- \sqsubseteq \mathrm{underlaps} \quad \text{(a35)}$$
$$\mathrm{Ref}(\mathrm{underlaps}) \quad \text{(a36)}$$
$$\mathrm{Sym}(\mathrm{underlaps}) \quad \text{(a37)}$$
$$\mathrm{overcross} \sqsubseteq \mathrm{overlaps} \quad \text{(a38)}$$
$$\mathrm{overcross} \sqsubseteq \neg\mathrm{partOf} \quad ||\mathrm{Dis}(\mathrm{overcross}, \mathrm{partOf}) \quad \text{(a39)}$$
$$\mathrm{Ref}(\mathrm{overcross}) \quad \text{(a40)}$$
$$\mathrm{Sym}(\mathrm{overcross}) \quad \text{(a41)}$$
$$\mathrm{undercross} \sqsubseteq \mathrm{underlaps} \quad \text{(a42)}$$
$$\mathrm{undercross} \sqsubseteq \neg\mathrm{partOf}- \quad ||\mathrm{Dis}(\mathrm{undercross}, \mathrm{partOf}-) \quad \text{(a43)}$$

$$\text{properOverlap} \sqsubseteq \text{overcross} \tag{a44}$$
$$\text{properOverlap} \sqsubseteq \text{overcross}^- \tag{a45}$$
$$\text{negPartOf} \equiv \neg\text{partOf} \tag{a46}$$
$$\text{negOverlaps} \equiv \neg\text{overlaps} \tag{a47}$$
$$\text{properUnderlap} \sqsubseteq \text{undercross} \tag{a48}$$
$$\text{properUnderlap} \sqsubseteq \text{undercross}^- \tag{a49}$$
$$\text{properExtension} \sqsubseteq \neg\text{partOf}$$
$$||\text{Dis}(\text{properExtension}, \text{partOf}) \tag{a50}$$
$$\text{properExtension} \sqsubseteq \text{partOf}^- \tag{a51}$$
$$\text{C}_{\text{overlaps}} \equiv \exists\text{overlaps}.\top \tag{a52}$$
$$\text{C}_{\text{overlaps}} \sqsubseteq \exists\text{R}_{\text{overlaps}}.\text{SELF} \tag{a53}$$
$$\text{R}_{\text{overlaps}} \circ \neg\text{overlaps} \circ \text{underlaps} \sqsubseteq \text{overlaps} \tag{a54}$$
$$\text{C}_{\text{partOf}} \equiv \exists\text{partOf}.\top \tag{a55}$$
$$\text{C}_{\text{partOf}} \sqsubseteq \exists\text{R}_{\text{partOf}}.\text{SELF} \tag{a56}$$
$$\text{R}_{\text{partOf}} \circ \neg\text{partOf} \circ \text{overlaps} \sqsubseteq \text{partOf} \tag{a57}$$
$$\text{Ref}(\text{connected}) \tag{a58}$$
$$\text{Sym}(\text{connected}) \tag{a59}$$
$$\text{externallyConnected} \sqsubseteq \text{connected} \tag{a60}$$
$$\text{externallyConnected} \sqsubseteq \neg\text{overlaps}$$
$$||\text{Dis}(\text{externallyConnected}, \text{overlaps}) \tag{a61}$$
$$\text{tangentialPartOf} \sqsubseteq \text{partOf} \tag{a62}$$
$$\text{internalPartOf} \sqsubseteq \text{partOf} \tag{a63}$$
$$\text{internalPartOf} \sqsubseteq \neg\text{tangentialPartOf}$$
$$||\text{Dis}(\text{internalPartOf}, \text{tangentialPartOf}) \tag{a64}$$
$$\text{Ref}(\text{EL}) \tag{a65}$$
$$\text{Tra}(\text{EL}) \tag{a66}$$
$$\text{partOf} \sqsubseteq \text{EL} \tag{a67}$$
$$\text{partOf} \circ \text{R}_{\text{EL}} \sqsubseteq \text{EL} \tag{a68}$$
$$\text{C}_{\text{EL}} \equiv \exists\text{EL}.\top \tag{a69}$$
$$\text{C}_{\text{EL}} \sqsubseteq \exists\text{R}_{\text{EL}}.\text{SELF} \tag{a70}$$
$$\text{EL} \circ \text{partOf} \sqsubseteq \text{EL} \tag{a71}$$
$$\text{EL} \circ \text{L} \sqsubseteq \text{WL} \tag{a72}$$
$$\text{L}^- \circ \text{EL} \circ \text{L} \sqsubseteq \text{partOf} \tag{a73}$$
$$\text{L}^- \circ \text{L} \sqsubseteq \text{equal} \tag{a74}$$
$$\text{partOf}^- \circ \text{L} \sqsubseteq \text{PL} \tag{a75}$$
$$\neg\text{partOf} \circ \text{PL} \sqsubseteq \varnothing \tag{a76}$$
$$\text{PL} \circ \text{L}^- \sqsubseteq \varnothing \tag{a77}$$
$$\text{tangentialPartOf}^- \circ \text{L} \sqsubseteq \text{TPL} \tag{a78}$$
$$\neg\text{tangentialPartOf} \circ \text{TPL} \sqsubseteq \varnothing \tag{a79}$$
$$\text{TPL} \circ \text{L}^- \sqsubseteq \varnothing \tag{a80}$$
$$\text{internalPartOf}^- \circ \text{L} \sqsubseteq \text{IPL} \tag{a81}$$
$$\neg\text{internalPartOf} \circ \text{IPL} \sqsubseteq \varnothing \tag{a82}$$
$$\text{IPL} \circ \text{L}^- \sqsubseteq \varnothing \tag{a83}$$
$$\text{L} \circ \text{partOf} \sqsubseteq \text{WL} \tag{a84}$$
$$\text{WL} \circ \neg\text{partOf}^- \sqsubseteq \varnothing \tag{a85}$$
$$\text{WL}^- \circ \neg\text{L} \sqsubseteq \varnothing \tag{a86}$$

$$\mathrm{L} \circ \mathrm{tangentialPartOf} \sqsubseteq \mathrm{TWL} \tag{a87}$$
$$\mathrm{TWL} \circ \neg\mathrm{tangentialPartOf}^- \sqsubseteq \emptyset \tag{a88}$$
$$\mathrm{TWL}^- \circ \neg\mathrm{L} \sqsubseteq \emptyset \tag{a89}$$
$$\mathrm{L} \circ \mathrm{internalPartOf} \sqsubseteq \mathrm{IWL} \tag{a90}$$
$$\mathrm{IWL} \circ \neg\mathrm{internalPartOf}^- \sqsubseteq \emptyset \tag{a91}$$
$$\mathrm{IWL}^- \circ \neg\mathrm{L} \sqsubseteq \emptyset \tag{a92}$$
$$\mathrm{L} \sqsubseteq \mathrm{PL} \tag{a93}$$
$$\mathrm{L} \sqsubseteq \mathrm{WL} \tag{a94}$$
$$\mathrm{L} \circ \mathrm{partOf}^- \sqsubseteq \mathrm{PL} \tag{a95}$$
$$\mathrm{L} \circ \mathrm{tangentialPartOf}^- \sqsubseteq \mathrm{TPL} \tag{a96}$$
$$\mathrm{L} \circ \mathrm{internalPartOf}^- \sqsubseteq \mathrm{IPL} \tag{a97}$$
$$\mathrm{PL} \circ \mathrm{partOf}^- \sqsubseteq \mathrm{PL} \tag{a98}$$
$$\mathrm{TPL} \circ \mathrm{partOf}^- \sqsubseteq \mathrm{TPL} \tag{a99}$$
$$\mathrm{IPL} \circ \mathrm{partOf}^- \sqsubseteq \mathrm{IPL} \tag{a100}$$
$$\mathrm{partOf} \circ \mathrm{WL} \sqsubseteq \mathrm{WL} \tag{a101}$$
$$\mathrm{internalPartOf} \circ \mathrm{IWL} \sqsubseteq \mathrm{IWL} \tag{a102}$$
$$\mathrm{partOf} \circ \mathrm{PL} \sqsubseteq \mathrm{PL} \tag{a103}$$
$$\mathrm{IPL} \sqsubseteq \mathrm{PL} \tag{a104}$$
$$\mathrm{TPL} \sqsubseteq \mathrm{PL} \tag{a105}$$
$$\mathrm{IWL} \sqsubseteq \mathrm{WL} \tag{a106}$$
$$\mathrm{TWL} \sqsubseteq \mathrm{WL} \tag{a107}$$
$$\mathrm{Irr}(\mathrm{memberOf}) \tag{a108}$$
$$\mathrm{Asy}(\mathrm{memberOf}) \tag{a109}$$
$$\mathrm{memberOf} \sqsubseteq \mathrm{properPartOf} \tag{a110}$$
$$\mathrm{negMemberOf} \equiv \neg\mathrm{memberOf} \tag{a111}$$
$$\mathrm{properOverlap}^- \circ \mathrm{memberOf} \sqsubseteq \neg\mathrm{memberOf} \tag{a112}$$
$$\mathrm{properPartOf}^- \circ \mathrm{memberOf} \sqsubseteq \neg\mathrm{memberOf} \tag{a113}$$
$$\mathrm{properPartOf} \circ \mathrm{memberOf} \sqsubseteq \neg\mathrm{memberOf} \tag{a114}$$
$$\mathrm{overlaps} \circ \mathrm{memberOf} \circ \mathrm{R}_{\mathrm{memberOf}} \sqsubseteq \mathrm{overlaps} \tag{a115}$$
$$\mathrm{C}_{\mathrm{memberOf}} \equiv \exists\mathrm{memberOf}^-.\top \tag{a116}$$
$$\mathrm{C}_{\mathrm{memberOf}} \sqsubseteq \exists\mathrm{R}_{\mathrm{memberOf}}.\mathrm{SELF} \tag{a117}$$
$$\mathrm{Irr}(\mathrm{constitutes}) \tag{a118}$$
$$\mathrm{Asy}(\mathrm{constitutes}) \tag{a119}$$
$$\mathrm{Tra}(\mathrm{constitutes}) \tag{a120}$$
$$\mathrm{partOf} \circ \mathrm{R}_{\mathrm{constitutes}} \sqsubseteq \mathrm{elementOf} \tag{a121}$$
$$\mathrm{C}_{\mathrm{constitutes}} \equiv \exists\mathrm{constitutes}.\top \tag{a122}$$
$$\mathrm{C}_{\mathrm{constitutes}} \sqsubseteq \exists\mathrm{R}_{\mathrm{constitutes}}.\mathrm{SELF} \tag{a123}$$
$$\mathrm{negEqual} \equiv \neg\mathrm{equal} \tag{a124}$$

Based on the preceding 124 axioms, we specify the set of non-simple roles R_{NS}, and the set of proposition builders I (all proposition builder are stated afterwards) as follows. In R_{NS}, the denotation $rolename^n$ refers to a rolename that is added to R_{NS} because of axiom number n in S_5 i.e. axiom n lead to the non-simplicity of the role *rolename*.

$R_{NS} = \{E^{1}, negEqual^{2}, SED^{4}, partOf^{5}, componentOf^{6}, elementOf^{14}, ppartOf^{9,17}, equal^{24}, overlaps^{32}, underlaps^{35}, EL^{66}, WL^{72}, PL^{75,95}, TPL^{78}, IPL^{81}, TWL^{87}, IWL^{90}, negMemberOf^{108}, constitutes^{114}\}$.

$I = \{\mathbb{I}_2, \mathbb{I}_5, \mathbb{I}_9, \mathbb{I}_{12}, \mathbb{I}_{13}, \mathbb{I}_{17}, \mathbb{I}_{22}, \mathbb{I}_{23}, \mathbb{I}_{27}, \mathbb{I}_{32}, \mathbb{I}_{35}, \mathbb{I}_{38}, \mathbb{I}_{42}, \mathbb{I}_{44}, \mathbb{I}_{45}, \mathbb{I}_{48}, \mathbb{I}_{49}, \mathbb{I}_{51}, \mathbb{I}_{54}, \mathbb{I}_{57}, \mathbb{I}_{60}, \mathbb{I}_{62}, \mathbb{I}_{63}, \mathbb{I}_{67}, \mathbb{I}_{68}, \mathbb{I}_{72}, \mathbb{I}_{73}, \mathbb{I}_{74}, \mathbb{I}_{75}, \mathbb{I}_{78}, \mathbb{I}_{81}, \mathbb{I}_{84}, \mathbb{I}_{87}, \mathbb{I}_{90}, \mathbb{I}_{93}, \mathbb{I}_{94}, \mathbb{I}_{95}, \mathbb{I}_{96}, \mathbb{I}_{97}, \mathbb{I}_{98}, \mathbb{I}_{99}, \mathbb{I}_{100}, \mathbb{I}_{101}, \mathbb{I}_{102}, \mathbb{I}_{103}, \mathbb{I}_{104}, \mathbb{I}_{105}, \mathbb{I}_{106}, \mathbb{I}_{107}, \mathbb{I}_{110}, \mathbb{I}_{112}, \mathbb{I}_{113}, \mathbb{I}_{114}, \mathbb{I}_{115}, \mathbb{I}_{121}\}$.

$$
\begin{aligned}
&\mathbb{I}_2 \rightarrow (SED \prec negEqual) \land (SED^- \prec negEqual)\\
&\mathbb{I}_5 \rightarrow (componentOf \prec partOf) \land (componentOf^- \prec partOf)\\
&\mathbb{I}_9 \rightarrow (componentOf \prec ppartOf) \land (componentOf^- \prec ppartOf)\\
&\mathbb{I}_{12} \rightarrow (elementOf \prec partOf) \land (elementOf^- \prec partOf)\\
&\mathbb{I}_{13} \rightarrow (elementOf \prec SED) \land (elementOf^- \prec SED)\\
&\mathbb{I}_{17} \rightarrow (elementOf \prec ppartOf) \land (elementOf^- \prec ppartOf)\\
&\mathbb{I}_{22} \rightarrow (equal \prec partOf) \land (equal^- \prec partOf)\\
&\mathbb{I}_{23} \rightarrow (equal \prec partOf^-) \land (equal^- \prec partOf^-)\\
&\mathbb{I}_{27} \rightarrow (ppartOf \prec partOf) \land (ppartOf^- \prec partOf)\\
&\mathbb{I}_{32} \rightarrow (partOf \prec overlaps) \land (partOf^- \prec overlaps)\\
&\mathbb{I}_{35} \rightarrow (partOf \prec underlaps) \land (partOf^- \prec underlaps)\\
&\mathbb{I}_{38} \rightarrow (overcross \prec overlaps) \land (overcross^- \prec overlaps)\\
&\mathbb{I}_{42} \rightarrow (undercross \prec underlaps) \land (undercross^- \prec underlaps)\\
&\mathbb{I}_{44} \rightarrow (poverlaps \prec overcross) \land (poverlaps^- \prec overcross)\\
&\mathbb{I}_{45} \rightarrow (poverlaps \prec overcross^-) \land (poverlaps^- \prec overcross^-)\\
&\mathbb{I}_{48} \rightarrow (punderlaps \prec undercross) \land (punderlaps^- \prec undercross)\\
&\mathbb{I}_{49} \rightarrow (punderlaps \prec undercross^-) \land (punderlaps^- \prec undercross^-)\\
&\mathbb{I}_{51} \rightarrow (pExtension \prec partOf^-) \land (pExtension^- \prec partOf^-)\\
&\mathbb{I}_{54} \rightarrow (R_{overlaps} \prec overlaps) \land (R^-_{overlaps} \prec overlaps)\\
&\land (negOverlaps \prec overlaps) \land (negOverlaps^- \prec overlaps)\\
&\land (underlaps \prec overlaps) \land (underlaps^- \prec overlaps)\\
&\mathbb{I}_{57} \rightarrow (R_{partOf} \prec partOf) \land (R^-_{partOf} \prec partOf)\\
&\land (negPartOf \prec partOf) \land (negPartOf^- \prec partOf)\\
&\land (overlaps \prec partOf) \land (overlaps^- \prec partOf)\\
&\mathbb{I}_{60} \rightarrow (externallyConnected \prec connected)\\
&\land (externallyConnected^- \prec connected)\\
&\mathbb{I}_{62} \rightarrow (tangentialPartOf \prec partOf) \land\\
&(tangentialPartOf^- \prec partOf)\\
&\mathbb{I}_{63} \rightarrow (internalPartOf \prec partOf) \land (internalPartOf^- \prec partOf)\\
&\mathbb{I}_{67} \rightarrow (partOf \prec EL) \land (partOf^- \prec EL)\\
&\mathbb{I}_{68} \rightarrow (R_{EL} \prec EL) \land (R^-_{EL} \prec EL)\\
&\land (partOf \prec EL) \land (partOf^- \prec EL)\\
&\mathbb{I}_{72} \rightarrow (EL \prec WL) \land (EL^- \prec WL) \land (L \prec WL) \land (L^- \prec WL)\\
&\mathbb{I}_{73} \rightarrow (L \prec partOf) \land (L^- \prec partOf)\\
&\land (EL \prec partOf) \land (EL^- \prec partOf)\\
&\mathbb{I}_{74} \rightarrow (L \prec equal) \land (L^- \prec equal)
\end{aligned}
$$

$\mathbb{I}_{75} \rightarrow (partOf \prec PL) \wedge (partOf^{-} \prec PL) \wedge (L \prec PL) \wedge (L^{-} \prec PL)$
$\mathbb{I}_{78} \rightarrow (TP \prec TPL) \wedge (TP^{-} \prec TPL) \wedge (L \prec TPL) \wedge (L^{-} \prec TPL)$
$\mathbb{I}_{81} \rightarrow (IP \prec IPL) \wedge (IP^{-} \prec IPL) \wedge (L \prec IPL) \wedge (L^{-} \prec IPL)$
$\mathbb{I}_{84} \rightarrow (L \prec WL) \wedge (L^{-} \prec WL) \wedge (partOf \prec WL) \wedge (partOf^{-} \prec WL)$
$\mathbb{I}_{87} \rightarrow (L \prec TWL) \wedge (L^{-} \prec TWL) \wedge (TP \prec TWL) \wedge (TP^{-} \prec TWL)$
$\mathbb{I}_{90} \rightarrow (L \prec IWL) \wedge (L^{-} \prec IWL) \wedge (IP \prec IWL) \wedge (IP^{-} \prec IWL)$
$\mathbb{I}_{93} \rightarrow (L \prec PL) \wedge (L^{-} \prec PL)$
$\mathbb{I}_{94} \rightarrow (L \prec WL) \wedge (L^{-} \prec WL)$
$\mathbb{I}_{95} \rightarrow (L \prec PL) \wedge (L^{-} \prec PL) \wedge (partOf \prec PL) \wedge (partOf^{-} \prec PL)$
$\mathbb{I}_{96} \rightarrow (L \prec TPL) \wedge (L^{-} \prec TPL) \wedge (TP \prec TPL) \wedge (TP^{-} \prec TPL)$
$\mathbb{I}_{97} \rightarrow (L \prec IPL) \wedge (L^{-} \prec IPL) \wedge (IP \prec IPL) \wedge (IP^{-} \prec IPL)$
$\mathbb{I}_{98} \rightarrow (partOf \prec PL) \wedge (partOf^{-} \prec PL)$
$\mathbb{I}_{99} \rightarrow (partOf \prec TPL) \wedge (partOf^{-} \prec TPL)$
$\mathbb{I}_{100} \rightarrow (partOf \prec IPL) \wedge (partOf^{-} \prec IPL)$
$\mathbb{I}_{101} \rightarrow (partOf \prec WL) \wedge (partOf^{-} \prec WL)$
$\mathbb{I}_{102} \rightarrow (partOf \prec IWL) \wedge (partOf^{-} \prec IWL)$
$\mathbb{I}_{103} \rightarrow (partOf \prec PL) \wedge (partOf^{-} \prec PL)$
$\mathbb{I}_{104} \rightarrow (IPL \prec PL) \wedge (IPL^{-} \prec PL)$
$\mathbb{I}_{105} \rightarrow (TPL \prec PL) \wedge (TPL^{-} \prec PL)$
$\mathbb{I}_{106} \rightarrow (IWL \prec WL) \wedge (IWL^{-} \prec WL)$
$\mathbb{I}_{107} \rightarrow (TWL \prec WL) \wedge (TWL^{-} \prec WL)$
$\mathbb{I}_{110} \rightarrow (memberOf \prec ppartOf) \wedge (memberOf^{-} \prec ppartOf)$
$\mathbb{I}_{112} \rightarrow (properOverlap \prec negMemberOf) \wedge$
$(properOverlap^{-} \prec negMemberOf) \wedge (memberOf \prec negMemberOf)$
$\wedge (memberOf^{-} \prec negMemberOf)$
$\mathbb{I}_{113} \rightarrow (ppartOf \prec negMemberOf) \wedge (ppartOf^{-} \prec negMemberOf)$
$\wedge (memberOf \prec negMemberOf) \wedge (memberOf^{-} \prec negMemberOf)$
$\mathbb{I}_{114} \rightarrow (ppartOf \prec negMemberOf) \wedge (ppartOf^{-} \prec negMemberOf)$
$\wedge (memberOf \prec negMemberOf) \wedge (memberOf^{-} \prec negMemberOf)$
$\mathbb{I}_{115} \rightarrow (memberOf \prec overlaps) \wedge (memberOf^{-} \prec overlaps)$
$\wedge (R_{memberOf} \prec overlaps) \wedge (R^{-}_{memberOf} \prec overlaps)$
$\mathbb{I}_{121} \rightarrow (partOf \prec elementOf) \wedge (partOf^{-} \prec elementOf)$
$\wedge (R_{constitutes} \prec elementOf) \wedge (R^{-}_{constitutes} \prec elementOf)$

A.3 generalization and establishing decidability

We show below how step 6 is applied on $FORT_{S5}$ by applying both simplicity and regularity rules, and we provide the final output of the procedure; the structured SROIQ set $FORT_{S6}$.

A.3.1 Applying the simplicity rule. Upon checking the non-simple roles in R_{NS} and the corresponding axioms in S_5 in which they are found, the simplicity rule obliges the suppression of some axioms:
$S'_5 = S_5 - \{a_7, a_8, a_{15}, a_{16}, a_{28}, a_{30}, a_{31}, a_{39}, a_{43}, a_{50}, a_{61}, a_{118}, a_{119}\}$

A.3.2 Applying the regularity rule. Based on S'_5, we construct the (irregular) role hierarchy by translating each *RIA* into a regular order between its roles and add it into the role hierarchy. The resulting hierarchy is shown in figure 1. It shows three incompatibilities due to the orders shown *blue* which violate the totality of the regularity of the role hierarchy. We identify the following incompatibilities in function of the axioms that induced them as follows:

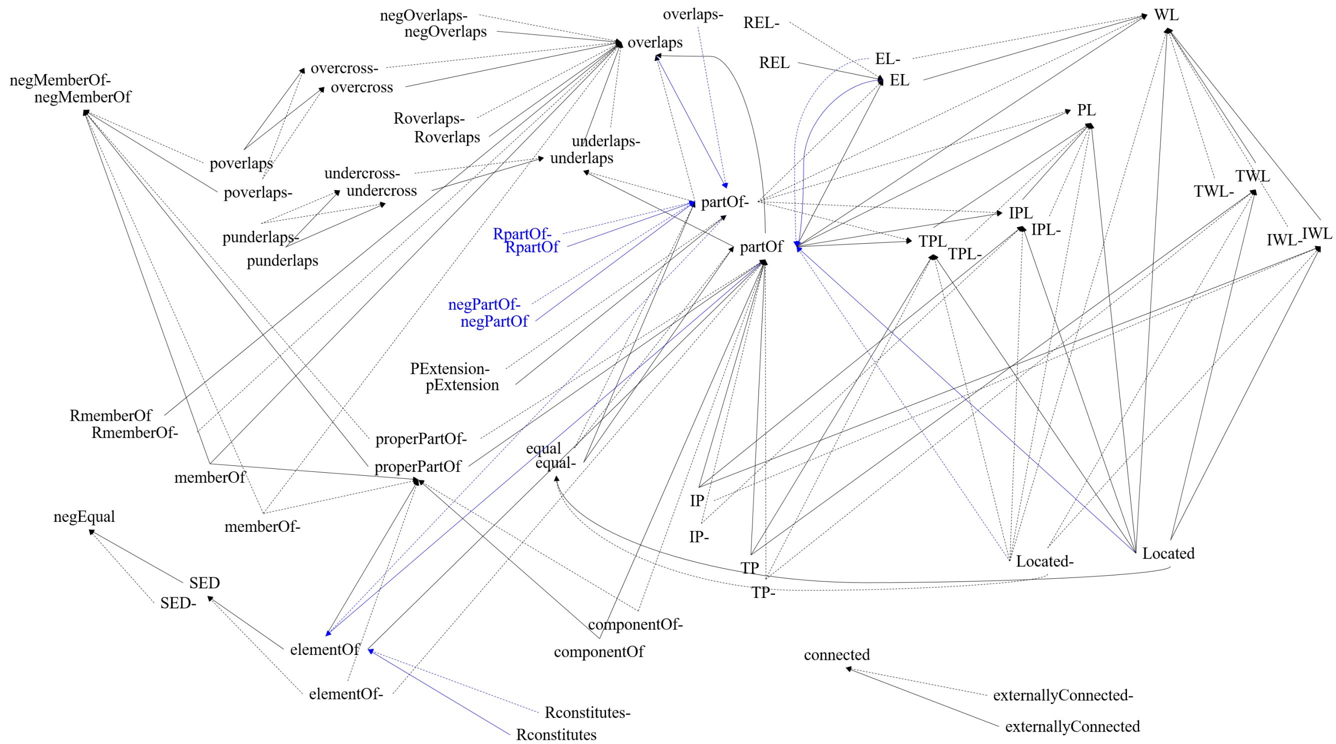

Figure 1: The role hierarchy of FORT presenting the regular orders in black, and the irregular ones in blue.

$m_1 : \mathbb{I}_{57} \longrightarrow \neg\mathbb{I}_{32}$

$m_2 : \mathbb{I}_{73} \longrightarrow \neg\mathbb{I}_{67} \wedge \neg\mathbb{I}_{68}$

$m_3 : \mathbb{I}_{121} \longrightarrow \neg\mathbb{I}_{12}$

Thus, besides S_5' we have the following structures:

$\mathcal{M} = \{m_1, m_2, m_3\}$

$\mathcal{U} = \{a_{57}, a_{32}, a_{73}, a_{67}, a_{68}, a_{121}, a_{12}\}$

Now, we are able to compute the structured subsets of $\mathcal{U}$ by computing the sub-theories as follows:

$\mathbb{M}_1 = \{m_1\}, where \mathbb{U}_1 = \langle\{a_{57}\}, \{a_{32}\}\rangle$

$\mathbb{M}_2 = \{m_2\}, where \mathbb{U}_2 = \langle\{a_{73}\}, \{a_{67}, a_{68}\}\rangle$

$\mathbb{M}_3 = \{m_3\}, where \mathbb{U}_3 = \langle\{a_{121}\}, \{a_{12}\}\rangle$

As a last step, from each tuple $\mathbb{U}_n$, we make the choice of suppressing one set yielding in S_6 as a structured subset of S_5', such that $S_6 = S_5' - \{a_{57}, a_{73}, a_{121}\}$.

As final considerations, the final structured subset S_6 consists of 108 axioms where the inputted set S_5 consisted of 124 axioms. Thus 16 axioms (a_7, a_8, a_{15}, a_{16}, a_{28}, a_{30}, a_{31}, a_{39}, a_{43}, a_{50}, a_{61}, a_{118}, a_{119}, a_{57}, a_{73}, a_{121}) were suppressed in total upon applying the simplicity and regularity rules.