

A TRANSLATING FORT INTO SROIQ

In this section we illustrate the translation of FORT. For the application of the first, second, third, fourth, and fifth steps, we only show it on a single axiom; the specific existential dependence axiom in section A.1. Then we illustrate the output of the fifth step in section A.2, and that of sixth step in section A.3.

A.1 Applying steps (1-5) on a single axiom

Consider the definition of the specific existential dependence relation from FORT: An entity x is specifically existentially dependent entity y , denoted $SED(x, y)$, iff; at any time t , x cannot exist at t unless y exists at t ; & x and y are different entities; & x exists at some t (D).

$$\forall(x, y) SED(x, y) \rightarrow \forall t (E(x, t) \rightarrow E(y, t)) \wedge \neg(x = y) \wedge \exists t E(x, t) \quad (D)$$

In the following, we express a binary predicate $R(x, y)$ as R_{xy} , and the predicate $x = y$ as Eq_{xy} .

A.1.1 Transforming to Clausal Form: Upon skolemization SNF, we substitute n and a by the skolem functions f as $\{n \leftarrow f(x, y, t)\}$. For simplicity we will write $f(x, y, m)$ as f .

$$\forall(x, y) [\neg SED_{xy} \vee (\forall t (\neg E_{xt} \vee E_{yt}) \wedge \neg Eq_{xy} \wedge \exists t E_{xt})] \quad (NNF)$$

$$\forall x, y, t \exists n \forall b [\neg SED_{xy} \vee ((\neg E_{xt} \vee E_{yt}) \wedge \neg Eq_{xy} \wedge E_{xn})] \quad (PNF)$$

$$\neg SED_{xy} \vee ((\neg E_{xt} \vee E_{yt}) \wedge \neg Eq_{xy} \wedge E_{xf}) \wedge \quad (SNF)$$

$$(\neg SED_{xy} \vee \neg E_{xt} \vee E_{yt}) \wedge (\neg SED_{xy} \vee \neg Eq_{xy}) \wedge (\neg SED_{xy} \vee E_{xf}) \quad (CNF)$$

A.1.2 Rewriting as Horn rules:

$$SED_{xy} \wedge E_{xt} \rightarrow E_{yt} \quad (R1)$$

$$SED_{xy} \rightarrow \neg Eq_{xy} \quad (R2)$$

$$SED_{xy} \rightarrow E_{xf} \quad (R3)$$

A.1.3 Qualifying Expressible Horn rules: R3 does not qualify since the variables are not enclosed.

$$SED_{xy} \wedge E_{xt} \rightarrow E_{yt} \quad (R1)$$

$$SED_{xy} \rightarrow \neg Eq_{xy} \quad (R2)$$

A.1.4 Constructing the rule graphs:

$$G = \langle \{x, y, m\}, \{SED_{xy}, E_{yt}\}, \emptyset, y, t : E \rangle \quad (G1)$$

$$G = \langle \{x, y\}, \{SED_{xy}\}, \emptyset, x, y : \neg Eq \rangle \quad (G2)$$

A.1.5 Converting into axioms:

$$SED^- \circ E \sqsubseteq E \quad (A1)$$

$$SED \sqsubseteq nEq, \text{ where } nEq = \neg Eq \quad (A2)$$

A.2 the non-structured set of SROIQ axioms

We illustrate below the output of step 5; $FORT_{S5}$ as the set of 120 non-structured axioms, and the structures R_{NS} and I resembling the set of non-simple roles and the set of proposition builders, respectively.

$SED^- \circ E \sqsubseteq E$	(a1)
$SED \sqsubseteq \neg equal$	(a2)
$SED \circ negE \sqsubseteq \emptyset$	(a3)
$SED \circ SED \sqsubseteq SED$	(a4)
$componentOf \sqsubseteq partOf$	(a5)
$Tra(componentOf)$	(a6)
$Irr(componentOf)$	(a7)
$Asy(componentOf)$	(a8)
$componentOf \sqsubseteq properPartOf$	(a9)
$componentOf \circ \neg PartOf^- \sqsubseteq \emptyset$	(a10)
$overlaps \circ componentOf \sqsubseteq \emptyset$	(a11)
$elementOf \sqsubseteq partOf$	(a12)
$elementOf \sqsubseteq SED$	(a13)
$Tra(elementOf)$	(a14)
$Irr(elementOf)$	(a15)
$Asy(elementOf)$	(a16)
$elementOf \sqsubseteq properPartOf$	(a17)
$elementOf \circ \neg PartOf^- \sqsubseteq \emptyset$	(a18)
$overlaps \circ elementOf \sqsubseteq \emptyset$	(a19)
$Tra(partOf)$	(a20)
$Ref(partOf)$	(a21)
$equal \sqsubseteq partOf$	(a22)
$equal \sqsubseteq partOf^-$	(a23)
$Tra(equal)$	(a24)
$Ref(equal)$	(a25)
$Sym(equal)$	(a26)
$properPartOf \sqsubseteq partOf$	(a27)
$properPartOf \sqsubseteq \neg partOf^-$	
$ Dis(properPartOf, partOf^-)$	(a28)
$Tra(properPartOf)$	(a29)
$Irr(properPartOf)$	(a30)
$Asy(properPartOf)$	(a31)
$partOf^- \circ partOf \sqsubseteq overlaps$	(a32)
$Ref(overlaps)$	(a33)
$Sym(overlaps)$	(a34)
$partOf \circ partOf^- \sqsubseteq underlaps$	(a35)
$Ref(underlaps)$	(a36)
$Sym(underlaps)$	(a37)
$overcross \sqsubseteq overlaps$	(a38)
$overcross \sqsubseteq \neg partOf$	
$ Dis(overcross, partOf)$	(a39)
$Ref(overcross)$	(a40)
$Sym(overcross)$	(a41)
$undercross \sqsubseteq underlaps$	(a42)
$undercross \sqsubseteq \neg partOf^-$	
$ Dis(undercross, partOf^-)$	(a43)

$\text{properOverlap} \sqsubseteq \text{overcross}$	(a44)
$\text{properOverlap} \sqsubseteq \text{overcross-}$	(a45)
$\text{negPartOf} \equiv \neg \text{partOf}$	(a46)
$\text{negOverlaps} \equiv \neg \text{overlaps}$	(a47)
$\text{properUnderlap} \sqsubseteq \text{undercross}$	(a48)
$\text{properUnderlap} \sqsubseteq \text{undercross-}$	(a49)
$\text{properExtension} \sqsubseteq \neg \text{partOf}$	
$\quad \text{Dis}(\text{properExtension}, \text{partOf})$	(a50)
$\text{properExtension} \sqsubseteq \text{partOf-}$	(a51)
$C_{\text{overlaps}} \equiv \exists \text{overlaps}. \top$	(a52)
$C_{\text{overlaps}} \sqsubseteq \exists R_{\text{overlaps}}. \text{SELF}$	(a53)
$R_{\text{overlaps}} \circ \neg \text{overlaps} \circ \text{underlaps} \sqsubseteq \text{overlaps}$	(a54)
$C_{\text{partOf}} \equiv \exists \text{partOf}. \top$	(a55)
$C_{\text{partOf}} \sqsubseteq \exists R_{\text{partOf}}. \text{SELF}$	(a56)
$R_{\text{partOf}} \circ \neg \text{partOf} \circ \text{overlaps} \sqsubseteq \text{partOf}$	(a57)
$\text{Ref}(\text{connected})$	(a58)
$\text{Sym}(\text{connected})$	(a59)
$\text{externallyConnected} \sqsubseteq \text{connected}$	(a60)
$\text{externallyConnected} \sqsubseteq \neg \text{overlaps}$	
$\quad \text{Dis}(\text{externallyConnected}, \text{overlaps})$	(a61)
$\text{tangentialPartOf} \sqsubseteq \text{partOf}$	(a62)
$\text{internalPartOf} \sqsubseteq \text{partOf}$	(a63)
$\text{internalPartOf} \sqsubseteq \neg \text{tangentialPartOf}$	
$\quad \text{Dis}(\text{internalPartOf}, \text{tangentialPartOf})$	(a64)
$\text{Ref}(\text{EL})$	(a65)
$\text{Tra}(\text{EL})$	(a66)
$\text{partOf} \sqsubseteq \text{EL}$	(a67)
$\text{partOf} \circ R_{\text{EL}} \sqsubseteq \text{EL}$	(a68)
$C_{\text{EL}} \equiv \exists \text{EL}. \top$	(a69)
$C_{\text{EL}} \sqsubseteq \exists R_{\text{EL}}. \text{SELF}$	(a70)
$\text{EL} \circ \text{partOf} \sqsubseteq \text{EL}$	(a71)
$\text{EL} \circ L \sqsubseteq \text{WL}$	(a72)
$L- \circ \text{EL} \circ L \sqsubseteq \text{partOf}$	(a73)
$L- \circ L \sqsubseteq \text{equal}$	(a74)
$\text{partOf-} \circ L \sqsubseteq \text{PL}$	(a75)
$\neg \text{partOf} \circ \text{PL} \sqsubseteq \emptyset$	(a76)
$\text{PL} \circ L- \sqsubseteq \emptyset$	(a77)
$\text{tangentialPartOf-} \circ L \sqsubseteq \text{TPL}$	(a78)
$\neg \text{tangentialPartOf} \circ \text{TPL} \sqsubseteq \emptyset$	(a79)
$\text{TPL} \circ L- \sqsubseteq \emptyset$	(a80)
$\text{internalPartOf-} \circ L \sqsubseteq \text{IPL}$	(a81)
$\neg \text{internalPartOf} \circ \text{IPL} \sqsubseteq \emptyset$	(a82)
$\text{IPL} \circ L- \sqsubseteq \emptyset$	(a83)
$L \circ \text{partOf} \sqsubseteq \text{WL}$	(a84)
$\text{WL} \circ \neg \text{partOf-} \sqsubseteq \emptyset$	(a85)
$\text{WL-} \circ \neg L \sqsubseteq \emptyset$	(a86)

$L \circ \text{tangentialPartOf} \sqsubseteq \text{TWL}$	(a87)
$\text{TWL} \circ \neg \text{tangentialPartOf} \sqsubseteq \emptyset$	(a88)
$\text{TWL} \circ \neg L \sqsubseteq \emptyset$	(a89)
$L \circ \text{internalPartOf} \sqsubseteq \text{IWL}$	(a90)
$\text{IWL} \circ \neg \text{internalPartOf} \sqsubseteq \emptyset$	(a91)
$\text{IWL} \circ \neg L \sqsubseteq \emptyset$	(a92)
$L \sqsubseteq \text{PL}$	(a93)
$L \sqsubseteq \text{WL}$	(a94)
$L \circ \text{partOf} \sqsubseteq \text{PL}$	(a95)
$L \circ \text{tangentialPartOf} \sqsubseteq \text{TPL}$	(a96)
$L \circ \text{internalPartOf} \sqsubseteq \text{IPL}$	(a97)
$\text{PL} \circ \text{partOf} \sqsubseteq \text{PL}$	(a98)
$\text{TPL} \circ \text{partOf} \sqsubseteq \text{TPL}$	(a99)
$\text{IPL} \circ \text{partOf} \sqsubseteq \text{IPL}$	(a100)
$\text{partOf} \circ \text{WL} \sqsubseteq \text{WL}$	(a101)
$\text{internalPartOf} \circ \text{IWL} \sqsubseteq \text{IWL}$	(a102)
$\text{partOf} \circ \text{PL} \sqsubseteq \text{PL}$	(a103)
$\text{IPL} \sqsubseteq \text{PL}$	(a104)
$\text{TPL} \sqsubseteq \text{PL}$	(a105)
$\text{IWL} \sqsubseteq \text{WL}$	(a106)
$\text{TWL} \sqsubseteq \text{WL}$	(a107)
$\text{Irr}(\text{memberOf})$	(a108)
$\text{Asy}(\text{memberOf})$	(a109)
$\text{memberOf} \sqsubseteq \text{properPartOf}$	(a110)
$\text{negMemberOf} \equiv \neg \text{memberOf}$	(a111)
$\text{properOverlap} \circ \text{memberOf} \sqsubseteq \neg \text{memberOf}$	(a112)
$\text{properPartOf} \circ \text{memberOf} \sqsubseteq \neg \text{memberOf}$	(a113)
$\text{properPartOf} \circ \text{memberOf} \sqsubseteq \neg \text{memberOf}$	(a114)
$\text{overlaps} \circ \text{memberOf} \circ R_{\text{memberOf}} \sqsubseteq \text{overlaps}$	(a115)
$C_{\text{memberOf}} \equiv \exists \text{memberOf} . \top$	(a116)
$C_{\text{memberOf}} \sqsubseteq \exists R_{\text{memberOf}} . \text{SELF}$	(a117)
$\text{Irr}(\text{constitutes})$	(a118)
$\text{Asy}(\text{constitutes})$	(a119)
$\text{Tra}(\text{constitutes})$	(a120)
$\text{partOf} \circ R_{\text{constitutes}} \sqsubseteq \text{elementOf}$	(a121)
$C_{\text{constitutes}} \equiv \exists \text{constitutes} . \top$	(a122)
$C_{\text{constitutes}} \sqsubseteq \exists R_{\text{constitutes}} . \text{SELF}$	(a123)
$\text{negEqual} \equiv \neg \text{equal}$	(a124)

Based on the preceding 124 axioms, we specify the set of non-simple roles R_{NS} , and the set of proposition builders I (all proposition builder are stated afterwards) as follows. In R_{NS} , the denotation rolename^n refers to a rolename that is added to R_{NS} because of axiom number n in S_5 i.e. axiom n lead to the non-simplicity of the role rolename .

$R_{NS} = \{E^1, negEqual^2, SED^4, partOf^5, componentOf^6, elementOf^{14}, ppartOf^{9,17}, equal^{24}, overlaps^{32}, underlaps^{35}, EL^{66}, WL^{72}, PL^{75,95}, TPL^{78}, IPL^{81}, TWL^{87}, IWL^{90}, negMemberOf^{108}, constitutes^{114}\}.$

$I = \{\mathbb{I}_2, \mathbb{I}_5, \mathbb{I}_9, \mathbb{I}_{12}, \mathbb{I}_{13}, \mathbb{I}_{17}, \mathbb{I}_{22}, \mathbb{I}_{23}, \mathbb{I}_{27}, \mathbb{I}_{32}, \mathbb{I}_{35}, \mathbb{I}_{38}, \mathbb{I}_{42}, \mathbb{I}_{44}, \mathbb{I}_{45}, \mathbb{I}_{48}, \mathbb{I}_{49}, \mathbb{I}_{51}, \mathbb{I}_{54}, \mathbb{I}_{57}, \mathbb{I}_{60}, \mathbb{I}_{62}, \mathbb{I}_{63}, \mathbb{I}_{67}, \mathbb{I}_{68}, \mathbb{I}_{72}, \mathbb{I}_{73}, \mathbb{I}_{74}, \mathbb{I}_{75}, \mathbb{I}_{78}, \mathbb{I}_{81}, \mathbb{I}_{84}, \mathbb{I}_{87}, \mathbb{I}_{90}, \mathbb{I}_{93}, \mathbb{I}_{94}, \mathbb{I}_{95}, \mathbb{I}_{96}, \mathbb{I}_{97}, \mathbb{I}_{98}, \mathbb{I}_{99}, \mathbb{I}_{100}, \mathbb{I}_{101}, \mathbb{I}_{102}, \mathbb{I}_{103}, \mathbb{I}_{104}, \mathbb{I}_{105}, \mathbb{I}_{106}, \mathbb{I}_{107}, \mathbb{I}_{110}, \mathbb{I}_{112}, \mathbb{I}_{113}, \mathbb{I}_{114}, \mathbb{I}_{115}, \mathbb{I}_{121}\}.$

$$\begin{aligned}
\mathbb{I}_2 &\rightarrow (SED < negEqual) \wedge (SED^- < negEqual) \\
\mathbb{I}_5 &\rightarrow (componentOf < partOf) \wedge (componentOf^- < partOf) \\
\mathbb{I}_9 &\rightarrow (componentOf < ppartOf) \wedge (componentOf^- < ppartOf) \\
\mathbb{I}_{12} &\rightarrow (elementOf < partOf) \wedge (elementOf^- < partOf) \\
\mathbb{I}_{13} &\rightarrow (elementOf < SED) \wedge (elementOf^- < SED) \\
\mathbb{I}_{17} &\rightarrow (elementOf < ppartOf) \wedge (elementOf^- < ppartOf) \\
\mathbb{I}_{22} &\rightarrow (equal < partOf) \wedge (equal^- < partOf) \\
\mathbb{I}_{23} &\rightarrow (equal < partOf^-) \wedge (equal^- < partOf^-) \\
\mathbb{I}_{27} &\rightarrow (ppartOf < partOf) \wedge (ppartOf^- < partOf) \\
\mathbb{I}_{32} &\rightarrow (partOf < overlaps) \wedge (partOf^- < overlaps) \\
\mathbb{I}_{35} &\rightarrow (partOf < underlaps) \wedge (partOf^- < underlaps) \\
\mathbb{I}_{38} &\rightarrow (overcross < overlaps) \wedge (overcross^- < overlaps) \\
\mathbb{I}_{42} &\rightarrow (undercross < underlaps) \wedge (undercross^- < underlaps) \\
\mathbb{I}_{44} &\rightarrow (poverlaps < overcross) \wedge (poverlaps^- < overcross) \\
\mathbb{I}_{45} &\rightarrow (poverlaps < overcross^-) \wedge (poverlaps^- < overcross^-) \\
\mathbb{I}_{48} &\rightarrow (punderlaps < undercross) \wedge (punderlaps^- < undercross) \\
\mathbb{I}_{49} &\rightarrow (punderlaps < undercross^-) \wedge (punderlaps^- < undercross^-) \\
\mathbb{I}_{51} &\rightarrow (pExtension < partOf^-) \wedge (pExtension^- < partOf^-) \\
\mathbb{I}_{54} &\rightarrow (R_{overlaps} < overlaps) \wedge (R_{overlaps}^- < overlaps) \\
&\wedge (negOverlaps < overlaps) \wedge (negOverlaps^- < overlaps) \\
&\wedge (underlaps < overlaps) \wedge (underlaps^- < overlaps) \\
\mathbb{I}_{57} &\rightarrow (R_{partOf} < partOf) \wedge (R_{partOf}^- < partOf) \\
&\wedge (negPartOf < partOf) \wedge (negPartOf^- < partOf) \\
&\wedge (overlaps < partOf) \wedge (overlaps^- < partOf) \\
\mathbb{I}_{60} &\rightarrow (externallyConnected < connected) \\
&\wedge (externallyConnected^- < connected) \\
\mathbb{I}_{62} &\rightarrow (tangentialPartOf < partOf) \wedge \\
&(tangentialPartOf^- < partOf) \\
\mathbb{I}_{63} &\rightarrow (internalPartOf < partOf) \wedge (internalPartOf^- < partOf) \\
\mathbb{I}_{67} &\rightarrow (partOf < EL) \wedge (partOf^- < EL) \\
\mathbb{I}_{68} &\rightarrow (R_{EL} < EL) \wedge (R_{EL}^- < EL) \\
&\wedge (partOf < EL) \wedge (partOf^- < EL) \\
\mathbb{I}_{72} &\rightarrow (EL < WL) \wedge (EL^- < WL) \wedge (L < WL) \wedge (L^- < WL) \\
\mathbb{I}_{73} &\rightarrow (L < partOf) \wedge (L^- < partOf) \\
&\wedge (EL < partOf) \wedge (EL^- < partOf) \\
\mathbb{I}_{74} &\rightarrow (L < equal) \wedge (L^- < equal)
\end{aligned}$$

$$\begin{aligned}
\mathbb{I}_{75} &\rightarrow (\text{partOf} < PL) \wedge (\text{partOf}^- < PL) \wedge (L < PL) \wedge (L^- < PL) \\
\mathbb{I}_{78} &\rightarrow (TP < TPL) \wedge (TP^- < TPL) \wedge (L < TPL) \wedge (L^- < TPL) \\
\mathbb{I}_{81} &\rightarrow (IP < IPL) \wedge (IP^- < IPL) \wedge (L < IPL) \wedge (L^- < IPL) \\
\mathbb{I}_{84} &\rightarrow (L < WL) \wedge (L^- < WL) \wedge (\text{partOf} < WL) \wedge (\text{partOf}^- < WL) \\
\mathbb{I}_{87} &\rightarrow (L < TWL) \wedge (L^- < TWL) \wedge (TP < TWL) \wedge (TP^- < TWL) \\
\mathbb{I}_{90} &\rightarrow (L < IWL) \wedge (L^- < IWL) \wedge (IP < IWL) \wedge (IP^- < IWL) \\
\mathbb{I}_{93} &\rightarrow (L < PL) \wedge (L^- < PL) \\
\mathbb{I}_{94} &\rightarrow (L < WL) \wedge (L^- < WL) \\
\mathbb{I}_{95} &\rightarrow (L < PL) \wedge (L^- < PL) \wedge (\text{partOf} < PL) \wedge (\text{partOf}^- < PL) \\
\mathbb{I}_{96} &\rightarrow (L < TPL) \wedge (L^- < TPL) \wedge (TP < TPL) \wedge (TP^- < TPL) \\
\mathbb{I}_{97} &\rightarrow (L < IPL) \wedge (L^- < IPL) \wedge (IP < IPL) \wedge (IP^- < IPL) \\
\mathbb{I}_{98} &\rightarrow (\text{partOf} < PL) \wedge (\text{partOf}^- < PL) \\
\mathbb{I}_{99} &\rightarrow (\text{partOf} < TPL) \wedge (\text{partOf}^- < TPL) \\
\mathbb{I}_{100} &\rightarrow (\text{partOf} < IPL) \wedge (\text{partOf}^- < IPL) \\
\mathbb{I}_{101} &\rightarrow (\text{partOf} < WL) \wedge (\text{partOf}^- < WL) \\
\mathbb{I}_{102} &\rightarrow (\text{partOf} < IWL) \wedge (\text{partOf}^- < IWL) \\
\mathbb{I}_{103} &\rightarrow (\text{partOf} < PL) \wedge (\text{partOf}^- < PL) \\
\mathbb{I}_{104} &\rightarrow (IPL < PL) \wedge (IPL^- < PL) \\
\mathbb{I}_{105} &\rightarrow (TPL < PL) \wedge (TPL^- < PL) \\
\mathbb{I}_{106} &\rightarrow (IWL < WL) \wedge (IWL^- < WL) \\
\mathbb{I}_{107} &\rightarrow (TWL < WL) \wedge (TWL^- < WL) \\
\mathbb{I}_{110} &\rightarrow (\text{memberOf} < \text{ppartOf}) \wedge (\text{memberOf}^- < \text{ppartOf}) \\
\mathbb{I}_{112} &\rightarrow (\text{properOverlap} < \text{negMemberOf}) \wedge \\
&(\text{properOverlap}^- < \text{negMemberOf}) \wedge (\text{memberOf} < \text{negMemberOf}) \\
&\wedge (\text{memberOf}^- < \text{negMemberOf}) \\
\mathbb{I}_{113} &\rightarrow (\text{ppartOf} < \text{negMemberOf}) \wedge (\text{ppartOf}^- < \text{negMemberOf}) \\
&\wedge (\text{memberOf} < \text{negMemberOf}) \wedge (\text{memberOf}^- < \text{negMemberOf}) \\
\mathbb{I}_{114} &\rightarrow (\text{ppartOf} < \text{negMemberOf}) \wedge (\text{ppartOf}^- < \text{negMemberOf}) \\
&\wedge (\text{memberOf} < \text{negMemberOf}) \wedge (\text{memberOf}^- < \text{negMemberOf}) \\
\mathbb{I}_{115} &\rightarrow (\text{memberOf} < \text{overlaps}) \wedge (\text{memberOf}^- < \text{overlaps}) \\
&\wedge (R_{\text{memberOf}} < \text{overlaps}) \wedge (R_{\text{memberOf}}^- < \text{overlaps}) \\
\mathbb{I}_{121} &\rightarrow (\text{partOf} < \text{elementOf}) \wedge (\text{partOf}^- < \text{elementOf}) \\
&\wedge (R_{\text{constitutes}} < \text{elementOf}) \wedge (R_{\text{constitutes}}^- < \text{elementOf})
\end{aligned}$$

A.3 generalization and establishing decidability

We show below how step 6 is applied on $FORT_{S_5}$ by applying both simplicity and regularity rules, and we provide the final output of the procedure; the structured SROIQ set $FORT_{S_6}$.

A.3.1 Applying the simplicity rule. Upon checking the non-simple roles in R_{NS} and the corresponding axioms in S_5 in which they are found, the simplicity rule obliges the suppression of some axioms:

$$S'_5 = S_5 - \{a_7, a_8, a_{15}, a_{16}, a_{28}, a_{30}, a_{31}, a_{39}, a_{43}, a_{50}, a_{61}, a_{118}, a_{119}\}$$

A.3.2 Applying the regularity rule. Based on S'_5 , we construct the (irregular) role hierarchy by translating each *RIA* into a regular order between its roles and add it into the role hierarchy. The resulting hierarchy is shown in figure 1. It shows three incompatibilities due to the orders shown *blue* which violate the totality of the regularity of the role hierarchy. We identify the following incompatibilities in function of the axioms that induced them as follows:

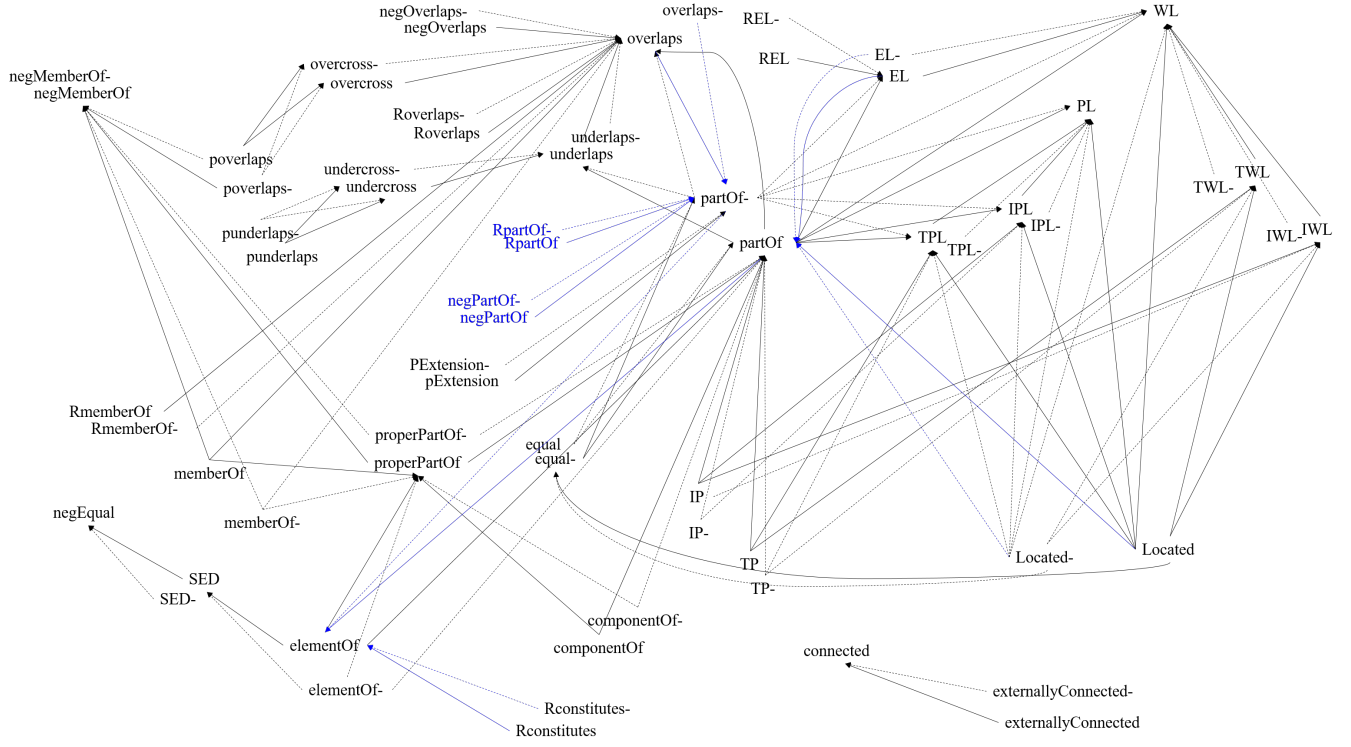


Figure 1: The role hierarchy of FORT presenting the regular orders in black, and the irregular ones in blue.

$$m_1 : \mathbb{I}_{57} \rightarrow \neg \mathbb{I}_{32}$$

$$m_2 : \mathbb{I}_{73} \rightarrow \neg \mathbb{I}_{67} \wedge \neg \mathbb{I}_{68}$$

$$m_3 : \mathbb{I}_{121} \rightarrow \neg \mathbb{I}_{12}$$

Thus, besides S'_5 we have the following structures:

$$\mathcal{M} = \{m_1, m_2, m_3\}$$

$$\mathcal{U} = \{a_{57}, a_{32}, a_{73}, a_{67}, a_{68}, a_{121}, a_{12}\}$$

Now, we are able to compute the structured subsets of \mathcal{U} by computing the sub-theories as follows:

$$\mathbb{M}_1 = \{m_1\}, \text{ where } \mathbb{U}_1 = \langle \{a_{57}\}, \{a_{32}\} \rangle$$

$$\mathbb{M}_2 = \{m_2\}, \text{ where } \mathbb{U}_2 = \langle \{a_{73}\}, \{a_{67}, a_{68}\} \rangle$$

$$\mathbb{M}_3 = \{m_3\}, \text{ where } \mathbb{U}_3 = \langle \{a_{121}\}, \{a_{12}\} \rangle$$

As a last step, from each tuple \mathbb{U}_n , we make the choice of suppressing one set yielding in S_6 as a structured subset of S'_5 , such that $S_6 = S'_5 - \{a_{57}, a_{73}, a_{121}\}$.

As final considerations, the final structured subset S_6 consists of 108 axioms where the inputted set S_5 consisted of 124 axioms. Thus 16 axioms ($a_7, a_8, a_{15}, a_{16}, a_{28}, a_{30}, a_{31}, a_{39}, a_{43}, a_{50}, a_{61}, a_{118}, a_{119}, a_{57}, a_{73}, a_{121}$) were suppressed in total upon applying the simplicity and regularity rules.