Causual Inference Note

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1 PSM Model Setup

For an individual i, the outcome depends on whether they receive a certain treatment:

$$y_i = \begin{cases} y_{1i}, & \text{if } D_i = 1 \\ y_{0i}, & \text{if } D_i = 0 \end{cases} \tag{1}$$

- D_i indicates whether individual i receives the treatment, where 1 represents treated, and 0 represents untreated.
- y_{1i} represents the outcome for individual i if treated.
- y_{0i} represents the outcome for individual i if untreated.

Given the observable covariates x_i, the probability of an individual i receiving the treatment is defined as:

$$p(x_i) = \Pr(D_i = 1 \mid x = x_i) = E(D_i \mid x_i) \tag{2}$$

Based on Equations (1) and (2), the **Average Treatment Effect on the Treated (ATT)** is given by:

$$ATT = E[y_{1i} - y_{0i} \mid D_i = 1] \tag{3}$$

$$= E[E[y_{1i} - y_{0i} \mid D_i = 1, p(x_i)]] \tag{4}$$

$$= E[E[y_{1i} \mid D_i = 1, p(x_i)] - E[y_{0i} \mid D_i = 0, p(x_i)] \mid D_i = 1] \tag{5}$$

1.1 Definition of ATE

The treatment effect is defined as a random variable:

$$y_{1i} - y_{0i} \tag{6}$$

The expected value of this effect is called the **Average Treatment Effect (ATE)**:

$$ATE = E(y_{1i} - y_{0i}) \tag{7}$$

ATE represents the expected treatment effect for a randomly selected individual from the population, regardless of whether they received the treatment.

1.2 Definition of ATT

If we consider only those who actually participated in the treatment, we define the **Average Treatment Effect on the Treated (ATT)**:

$$ATT = E(y_{1i} - y_{0i} \mid D_i = 1) \tag{8}$$

ATT measures the **average effect of the treatment for those who received it**.

1.3 Importance of ATT vs. ATE

For policymakers, **ATT is often more important** because it directly measures the effect on those who received the intervention. However, ATE and ATT are generally **not equal**.

1.4 Estimation Challenges and Selection Bias

Since we cannot observe both y_{0i} and y_{1i} for the same individual, estimating ATE or ATT is challenging. A naive comparison between the treated and untreated groups results in selection bias:

$$E(y_{1i} \mid D_i = 1) - E(y_{0i} \mid D_i = 0) \tag{9}$$

This difference consists of two terms:

$$E(y_{1i} \mid D_i = 1) - E(y_{0i} \mid D_i = 1) + E(y_{0i} \mid D_i = 1) - E(y_{0i} \mid D_i = 0) \tag{10}$$

- The first term represents the **true ATT**:

$$ATT = E(y_{1i} \mid D_i = 1) - E(y_{0i} \mid D_i = 1) \tag{11}$$

- The second term represents **selection bias**:

$$E(y_{0i} \mid D_i = 1) - E(y_{0i} \mid D_i = 0) \tag{12}$$

If selection bias is present, a direct comparison between treated and untreated individuals **does not accurately estimate ATT or ATE**.

1.5 PSM Assumptions

1.6 Common Support Assumption

For any possible value of x_i, the propensity score must satisfy:

$$0 < p(x_i) < 1 \tag{13}$$

This assumption ensures that there is **overlap between the treated and control groups**, making it possible to find comparable units.

1.7 Balancing Assumption

$$D_i \perp (y_{1i}, y_{0i}) \mid p(x_i) \tag{14}$$

This assumption states that **conditional on the propensity score $p(x_i)$, treatment assignment is as good as random**. That is, for a given $p(x_i)$, there are no systematic differences between the treatment and control groups, meaning the treatment effect is entirely due to the treatment itself.