Chapter 3

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Introduction

This chapter introduces a taxonomy of assignment mechanisms.

The assignment mechanism describes, as a function of all covariates and of all potential outcomes, the probability of any vector of assignments.

We consider three basic restrictions on assignment mechanisms:

1. individualistic assignment: This limits the dependence of a particular unit's assignment probability on the values of covariates and potential outcomes for other units.

2. probabilistic assignment: This requires the assignment mechanism to imply a non-zero probability for each treatment value, for every unit.

3. unconfounded assignment: This disallows dependence of the assignment mechanism

Introduction

Following Cochran (1965), we also make a distinction between experiments, where the assignment mechanism is both known and controlled by the researcher, and observational studies, where the assignment mechanism is not known to, or not under the control of, the researcher.

We consider three classes of assignment mechanisms, covered in parts II, III and IV, and V of the book, respectively.

The first class, studied in Part II, corresponds to what we call sic*classical randomized experiments*. Here the assignment mechanism satisfies all three restrictions on the assignment process, and, moreover, the researcher knows and controls the functional form of the assignment mechanism.

Introduction

The second class of assignment mechanisms (covered in Parts III and IV) are denoted *regular assignment mechanisms*. Here, the assignment mechanism need not be under the control of, or known by, the researcher.

When the assignment mechanism is not under the control of the researcher, the restrictions on the assignment mechanism that make it regular are now usually assumptions, and they are typically not satisfied by design, as they are in classical randomized experiments.

In later chapters methods for assessing the plausibility of the assumptions, as well as investigating the sensitivity to violations of them, will be discussed .

Introduction

The regular observational study has been studied extensively from a theoretical perspective and is widely used in empirical work.

Many, but not all, of the methods applicable to randomized experiments can be used, but often modifications to the specific methods are critical to enhance the credibility of the results.

If the covariate distributions under the various treatment regimes are substantially different, i.e., *unbalanced*, it can be very important to have an initial *design* stage of the study.

In this design stage, the data on covariate values and treatment assignment (but, importantly, not the final outcome data) are analyzed in order to assemble samples with improved balance in covariate distributions, somewhat in parallel with the design stage of randomized experiments.

Introduction

Part V analyzes assignment mechanisms where the assignment itself is regular, but the treatment received is not equal to the treatment assigned for all units. Thus, although the treatment assigned *is* unconfounded, the treatment received *is not* unconfounded, because the probability of receiving the active versus control treatment depends on potential outcomes.

Such settings have arisen in the econometric literature to account for settings where individuals choose the treatment regime, at least partly based on expected relative benefits associated with the two treatment regimes.

Although, as a general matter, such optimizing behavior is not inconsistent with regular assignment mechanisms, in some cases it suggests assignment mechanisms associated with so-called *instrumental variable* methods.

Road Map

- Notation
- Defining the assignment mechanism, unit-level assignment probabilities, and the propensity score.
- Defining classical randomized experiments
- Defining regular assignment mechanisms as a special class of observational studies
- Discussing some non-regular assignment mechanisms.

Notation

Consider a population of N units, indexed by $i = 1, \ldots, N$.

The i^{th} unit in this population is characterized by a $1 \times K$ vector of covariates (or pre-treatment variables or attributes), X_i, with $\mathbf{X}$ the $N \times K$ matrix of covariates in the population with i^{th} row equal to X_i.

For each unit there are two potential outcomes, $Y_i(0)$ and $Y_i(1)$. $Y_i(0)$ denotes the outcome under the control treatment, and $Y_i(1)$ denotes the outcome under the active treatment.

Note: tacitly accept the SUTVA across the N units indexed by i.

Notation

Let $\mathbf{Y}(0)$ and $\mathbf{Y}(1)$ denote the N–component vectors (or the N–vectors for short) of the potential outcomes.

More generally, the potential outcomes could themselves be multi-component row vectors, in which case $\mathbf{Y}(0)$ and $\mathbf{Y}(1)$ would be matrices.

With L outcomes the ith rows is equal to $(Y_{i1}(0), Y_{i2}(0), \ldots, Y_{iL}(0))$, and $(Y_{i1}(1), Y_{i2}(1), \ldots, Y_{iL}(1))$, respectively

Notation

The N-component vector of treatment assignments is denoted by $\mathbf{W}$, with i-th component $W_i \in \{0, 1\}$, with $W_i = 0$ if the unit is a 'control', and $W_i = 1$ if the unit is 'treated'.

Let $N_c = \sum_{i=1}^{N}(1 - W_i)$ and $N_t = \sum_{i=1}^{N} W_i$, with $N_c + N_t = N$.

The realized and possibly observed potential outcomes

$$Y_i^{\text{obs}} = Y_i(W_i) = \begin{cases} Y_i(0) & \text{if } W_i = 0, \\ Y_i(1) & \text{if } W_i = 1, \end{cases} \tag{1}$$

and the missing potential outcomes:

$$Y_i^{\text{mis}} = Y_i(1 - W_i) = \begin{cases} Y_i(1) & \text{if } W_i = 0, \\ Y_i(0) & \text{if } W_i = 1. \end{cases} \tag{2}$$

$\mathbf{Y}^{\text{obs}}$ and $\mathbf{Y}^{\text{mis}}$ are the corresponding N-vectors.

Notation

Note that

$$Y_i(0) = \begin{cases} Y_i^{\mathrm{mis}} & \text{if } W_i = 1, \\ Y_i^{\mathrm{obs}} & \text{if } W_i = 0, \end{cases} \quad \text{and} \quad Y_i(1) = \begin{cases} Y_i^{\mathrm{mis}} & \text{if } W_i = 0, \\ Y_i^{\mathrm{obs}} & \text{if } W_i = 1. \end{cases} \tag{3}$$

This characterization illustrates that the causal inference problem is fundamentally a missing data problem: if we impute the missing outcomes, we "know" all the potential outcomes and thus the value of any causal estimand.

Assignment Probabilities

Definition

(ASSIGNMENT MECHANISM)
Given a population of N units, the assignment mechanism is a row-exchangeable function $\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$, taking on values in $[0, 1]$, satisfying

$$\sum_{\mathbf{W}\in\{0,1\}^N} \Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = 1,$$

for all $\mathbf{X}$, $\mathbf{Y}(0)$, and $\mathbf{Y}(1)$.

The set $\mathbb{W} = \{0, 1\}^N$ is the set of all N-vectors with all elements equal to 0 or 1.

By the assumption that the function $\Pr(\cdot)$ is row exchangeable, we mean that the order in which we list the N units within the vectors or matrices is irrelevant to the value of the function.

Assignment Probabilities

Note that $\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$ is the probability that a particular value for the full assignment—first two units treated, third a control, fourth treated, *etc.*—will occur (i.e. *not* the probability of being 'treated').

The definition requires that the probabilities across the full set of 2^N possible assignment vectors $\mathbf{W}$ sum to one.

Note that some assignment vectors $\mathbf{W}$ may have zero probability for the units to be 'treated' or 'control' .

To rule out this possibility by assigning zero probability to the vector of assignments $\mathbf{W}$ with $W_i = 0$ and $W_i = 1$ for all i, or perhaps even assign zero probability to all vectors of assignments other than those with $\sum_{i=1}^{N} W_i = N/2$, for even values of the population size N.

Assignment Probabilities

Definition

(UNIT ASSIGNMENT PROBABILITY)
The unit-level assignment probability for unit i is

$$p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = \sum_{\mathbf{W}:W_i=1} \Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)).$$

Here we sum the probabilities across all possible assignment vectors $\mathbf{W}$ for which $W_i = 1$.

Out of the set of 2^N different assignment vectors, half (that is 2^{N-1}) have the property that $W_i = 1$.

The probability that unit i is assigned to the control treatment is $1 - p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$.

Assignment Probabilities

Note that according to this definition, the probability that unit i receives the treatment can be a function of, not only its own covariates X_i and potential outcomes $Y_i(1)$ and $Y_i(0)$, but also of the covariate values and potential outcomes of other units in the population.

Definition

(FINITE POPULATION PROPENSITY SCORE)
The propensity score at x is the average unit assignment probability for units with $X_i = x$,

$$e(x) = \frac{1}{N(x)} \sum_{i:X_i=x} p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$$

where $N(x) = \#\{i = 1, \ldots, N | X_i = x\}$ is the number of units with $X_i = x$. For values x with $N(x) = 0$, the propensity score is defined to be zero.

Assignment Probabilities

Example 1

Suppose we have two units. Then there are four (2^2) possible values for $\mathbf{W}$,

$$\mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}.$$

We conduct a randomized experiment where all treatment assignments have equal probability. Then the assignment mechanism is equal to

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = 1/4, \quad \text{for } \mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}. \tag{4}$$

In this case the unit assignment probability $p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$ is equal to $1/2$ for both units $i = 1, 2$. In a randomized experiment with no covariates, the propensity score is equal to the unit assignment probabilities, here all equal to $1/2$. $\square$

Assignment Probabilities

Example 2

We conduct a randomized experiment with two units where only those assignments with exactly one treated and one control unit are allowed. Then the assignment mechanism is

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = \begin{cases} 1/2 & \text{if } \mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}, \\ \\ 0 & \text{if } \mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}. \end{cases} \tag{5}$$

This does not change the unit level assignment probability, which remains equal to 1/2 for both units, and so is the propensity score. □

Assignment Probabilities

EXAMPLE 3

The unit with more to gain from the active treatment (using a coin toss in the case of a tie) is assigned to the treatment group, and the other to the control group.

Assignment Probabilities: Example 3

$$\Pr(\mathbf{W}|\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1)) = \begin{cases} 1 & \text{if } Y_2(1)-Y_2(0) > Y_1(1)-Y_1(0) \text{ and } \mathbf{W} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ 1 & \text{if } Y_2(1)-Y_2(0) < Y_1(1)-Y_1(0) \text{ and } \mathbf{W} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ 1/2 & \text{if } Y_2(1)-Y_2(0) = Y_1(1)-Y_1(0) \text{ and } \mathbf{W} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ 0 & \text{if } \mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}, \\ 0 & \text{if } Y_2(1)-Y_2(0) < Y_1(1)-Y_1(0) \text{ and } \mathbf{W} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ 0 & \text{if } Y_2(1)-Y_2(0) > Y_1(1)-Y_1(0) \text{ and } \mathbf{W} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \end{cases} \quad (6)$$

In this example the unit-level treatment probabilities $p_i(\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1))$ are equal to zero, one or a half, depending whether the gain for unit i is smaller or larger than for the other unit, or equal. Given that there are no covariates, the propensity score remains a constant, equal to $1/2$ in this case. This is a type of assignment mechanism that we often rule out when attempting to infer causal effects. □

Assignment Probabilities

EXAMPLE 4: sequential randomized experiment

There are three units, and thus eight possible values for **W**,

$$\mathbf{W} \in \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

Suppose there is a covariate X_i measuring the order in which the units entered the experiment, $X_i \in \{1, 2, 3\}$. Without loss of generality, assume $X_i = i$ and assume $W_1 = 1$ from coin toss, then $W_2 = 0$, and the reverse if $W_1 = 0$ from the coin toss.

The assignment of $i = 3$ depends on the outcomes of individuals 1 and 2. He/she is assigned 'treatment' if $Y^{\text{obs}} \leq Y_2^{\text{obs}}$ and to 'control' if the 'unequality' is the reversed.

Assignment Probabilities: Example 4, formally:

$$\Pr(\mathbf{W}|\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1),\mathbf{X}) = \left\{ \begin{array}{ll} 1/2 & \text{if } Y_1(0) > Y_2(1), \text{ and } \mathbf{W} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \\ 1/2 & \text{if } Y_1(1) \geq Y_2(0), \text{ and } \mathbf{W} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \\ 1/2 & \text{if } Y_1(0) \leq Y_2(1), \text{ and } \mathbf{W} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \\ 1/2 & \text{if } Y_1(1) < Y_2(0), \text{ and } \mathbf{W} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \end{array} \right\} \tag{7}$$

In this case the unit assignment probability is equal to $1/2$ for the first two units,

$$p_2(\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1)) = p_2(\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1)) = 1/2,$$

and, for unit 3, equal to

$$p_3(\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1)) = \begin{cases} 0 & \text{if } Y_1(0) > Y_2(1), \text{ and } Y_1(1) < Y_2(0), \\ 1 & \text{if } Y_1(1) \geq Y_2(0), \text{ and } Y_1(0) \leq Y_2(1), \\ 1/2 & \text{otherwise.} \end{cases}$$

Because the covariates identify the unit, the propensity score is equal to the unit assignment probabilities. Thus, for $x = 1$ and $x = 2$ the propensity score is equal to $1/2$. If $x = 3$, the propensity score is equal to $p_3(\mathbf{X},\mathbf{Y}(0),\mathbf{Y}(1))$. $\square$

Restrictions on the Assignment Mechanism

Before classifying the various types of assignment mechanisms, we present three general properties that assignment mechanisms may satisfy.

Definition

(Individualistic Assignment)
An assignment mechanism $\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$ is individualistic if, for some function $q(\cdot) \in [0, 1]$,

$$p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = q(X_i, Y_i(0), Y_i(1)), \quad \text{for all } i = 1, \ldots, N,$$

and

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = c \cdot \prod_{i=1}^{N} q(X_i, Y_i(0), Y_i(1))^{W_i} \left(1 - q(X_i, Y_i(0), Y_i(1))\right)^{1-W_i},$$

for $(\mathbf{W}, \mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) \in \mathbb{A}$, for some set $\mathbb{A}$, and zero elsewhere. (c is the constant that ensures that the probabilities sum up to unity.)

Restrictions on the Assignment Mechanism

Individualistic assignment is violated in sequential experiments such as Example 4.

Given individualistic assignment, the propensity score simplifies to:

$$e(x) = \frac{1}{N_x} \sum_{i:X_i=x} q(X_i, Y_i(0), Y_i(1)).$$

Definition

(Probabilistic Assignment)
An assignment mechanism $\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1))$ is probabilistic if the probability of assignment to treatment for unit i is strictly between zero and one:

$$0 < p_i(\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) < 1, \quad \text{for each possible } \mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1),$$

for all $i = 1, \ldots, N$.

Restrictions on the Assignment Mechanism

Definition

(UNCONFOUNDED ASSIGNMENT)
An assignment mechanism is unconfounded if it does not depend on the potential outcomes:

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = \Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}'(0), \mathbf{Y}'(1)),$$

for all $\mathbf{W}$, $\mathbf{X}$, $\mathbf{Y}(0)$, $\mathbf{Y}(1)$, $\mathbf{Y}'(0)$, and $\mathbf{Y}'(1)$.

Thus, if the assignment is unconfounded, we can drop the two potential outcomes as arguments and write the assignment mechanism as $\Pr(\mathbf{W}|\mathbf{X})$.

The assignment mechanisms in Examples 1 and 2 are, but those in Examples 3 and 4 are not, unconfounded.

Restrictions on the Assignment Mechanism

Under individualistic assignment and unconfoundedness:

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = c \cdot \prod_{i=1}^{N} e(X_i)^{W_i} \cdot (1 - e(X_i))^{1-W_i}. \tag{8}$$

Note that, under unconfoundedness, the propensity score is no longer just the average assignment probability for units with covariate value $X_i = x$; it can also be interpreted as the unit-level assignment probability for all units with $X_i = x$.

Given individualistic assignment, the combination of probabilistic and unconfounded assignment is referred to as *strongly ignorable treatment assignment* (Rosenbaum and Rubin, 1983a).

More generally, *ignorable treatment assignment* refers to the weaker restriction where the assignment mechanism can be written in terms of $\mathbf{W}$, $\mathbf{X}$, and $\mathbf{Y}^{\text{obs}}$ only, without dependence on $\mathbf{Y}^{\text{mis}}$ (Rubin, 1978).

Assignment Mechanisms and Super-populations

Often the sample of size N is viewed as a random sample from a super-population.

Sampling from the super-population generates a joint sampling distribution on the quadruple of unit-level variables $(Y_i(0), Y_i(1), W_i, X_i)$, $i = 1, \ldots, N$.

More explicitly, we assume the $(Y_i(0), Y_i(1), W_i, X_i)$ are independently and identically distributed draws from a distribution indexed by a parameter (θ, ψ). We write this in factored form as

$$f_{W|Y(0),Y(1),X}(W_i|Y_i(0), Y_i(1), X_i, \phi) \cdot f_{Y(0),Y(1)|X}(Y_i(0), Y_i(1)|X_i, \theta) \cdot f_X(X_i|\psi), \quad (9)$$

where the parameters are in their respective parameter spaces, and the full parameter vector is (θ, ψ, ϕ).

Assignment Mechanisms and Super-populations

In this setting we define the propensity score as

Definition

(SUPER-POPULATION PROPENSITY SCORE)
The propensity score at x is the population average unit assignment probability for units with $X_i = x$,

$$e(x) = \mathbb{E}_{\mathrm{SP}}\left[f_{W|Y(0),Y(1),X}(1|Y_i(0), Y_i(1), X_i, \phi) f_{Y(0),Y(1)|X}(Y_i(0), Y_i(1)|X_i, \theta) \middle| X_i = x \right],$$

for all x in the support of X_i; $e(x)$ is here a function of ϕ, a dependence that we usually suppress notationally.

The "SP" subscript on the expectations operator indicates that the expectation is taken over the distribution generated by random sampling, from the superpopulation.

Assignment Mechanisms and Super-populations

Definition

(Super-population Probabilistic Assignment)
An assignment mechanism is super-population probabilistic if the probability of assignment to treatment for unit i is strictly between zero and one:

$$0 < f_{W|Y(0),Y(1),X}(1|Y_i(0), Y_i(1), X_i, \phi) < 1, \quad \text{for each possible } X_i,\, Y_i(0),\, Y_i(1).$$

Definition

(Super-population Unconfounded Assignment)
An assignment mechanism is superpopulation unconfounded if it does not depend on the potential outcomes:

$$f_{W|Y(0),Y(1),X}(w|y_0, y_1, x, \phi) = f_{W|Y(0),Y(1),X}(w|y_0', y_1', x, \phi),$$

for all y_0, y_1, x, y_0', y_1', ϕ, and for $w = 0, 1$.

Randomized Experiments

Definition

(Randomized Experiments)
A randomized experiment is an assignment mechanism that (i) is probabilistic, and (ii) has a known functional form that is controlled by the researcher.

Definition

(Classical Randomized Experiments)
A classical randomized experiment is a randomized experiment with an assignment mechanism that is (*i*) individualistic, and (*ii*) unconfounded.

Randomized Experiments

The definition of a classical randomized experiment rules out sequential experiments as in Example 4.

In sequential experiments, the assignment for units assigned in a later stage of the experiment generally depends on observed outcomes for units assigned earlier in the experiment.

A leading case of a classical randomized experiment is a *completely randomized experiment*, where, *a priori*, the number of randomized treated units, N_t, is fixed.

Randomized Experiments

Because the unit assignment probability is $q = N_t/N$ the assignment mechanism equals

$$\Pr(\mathbf{W}|\mathbf{X}, \mathbf{Y}(0), \mathbf{Y}(1)) = \begin{cases} 1 \Big/ \begin{pmatrix} N \\ N_t \end{pmatrix} & \text{if } \sum_{i=1}^{N} W_i = N_t, \\ 0 & \text{otherwise,} \end{cases}$$

where the number of distinct values of the assignment vector with N_t units out of N assigned to the active treatment is

$$\begin{pmatrix} N \\ N_t \end{pmatrix} = \frac{N!}{N_t! \cdot (N - N_t)!}, \qquad \text{with} \quad J! = J(J-1)\ldots 1.$$

Other examples of classical randomized experiments include stratified randomized experiments and paired randomized experiments, discussed in Chapters 9 and 10.

Observational Studies: Regular Assignment Mechanisms

In Parts III and IV, we discuss cases where the exact assignment probabilities may be unknown to the researcher, but the researcher still has substantial information concerning the assignment mechanism.

In general we refer to designs with unknown assignment mechanisms as *observational studies*:

Definition

(Observational Studies)
An assignment mechanism corresponds to an observational study if the functional form of the assignment mechanism is unknown.

Observational Studies: Regular Assignment Mechanisms

The special case of an observational study (Part **III**) is a *regular assignmentmechanism*:

Definition

(REGULAR ASSIGNMENT MECHANISMS)
An assignment mechanism is regular if
(i), the assignment mechanism is individualistic,
(ii), the assignment mechanism is probabilistic, and
(iii), the assignment mechanism is unconfounded.

If the assignment mechanism is known (i.e. known functional form) the assignment mechanism corresponds to a classical randomized experiment but now, as the functional form is not known, the assignment mechanism corresponds to an observational study with a regular assignment mechanism.

Observational Studies: Regular Assignment Mechanisms

Part III focuses on the design stage of studies where the assumption of a regular assignment mechanism is viewed as plausible.

In this design stage, we focus on the data on treatment assignment and pretreatment variables only, without seeing the outcome data.

The concern in this stage is balance in the covariate distributions between treated and control groups. In completely and stratified randomized experiments, expected balance is 'guaranteed' by design, but in observational studies this balance needs to be created by special analyses.

In Part IV we discusses methods of analysis for causal inference with regular assignment mechanisms in some detail. Even if in many cases the assumption may appear too strong for an assignment, it is a very important starting point for many studies.

Observational Studies: Irregular Assignment Mechanisms

Part VI discusses another class of assignment mechanisms. The focus is on settings where *assignment to treatment* may differ for some units from the *receipt of treatment*, but where assignment to treatment itself is unconfounded.

This class of assignment mechanisms includes noncompliance in randomized experiments.

Often in these designs, the receipt of treatment can be viewed as "latently regular", that is, it would be regular given some additional covariates that are not fully observed.

To conduct inference in such settings, it is often useful to invoke additional conditions, in particular *exclusion restrictions*, which rule out the presence of particular causal effects.