SP25 8732: Homework 2

Danbo CHEN

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1 Question 1

(a) Solution:

- . /// 1.A
- > reg price sqrft bdrms

sqrft							
	Coefficient	Std. err	. t	P> t	[95% co	nf.	interval
Total	28984060.7	87	333150.12	3	t MSE	=	491.76
Residual	20797100.3	86	241826.74		quared R-squared	=	0.2825 0.2741
Model	8186960.43	1	8186960.4		o > F	=	0.0000
				•	, 86)	=	33.85
Source	SS	df	MS	Numl	per of obs	=	88
reg sqrft b	drms						
_cons	-19.315	31.04662	-0.62	0.536	-81.04399		42.414
bdrms	15.19819	9.483517	1.60	0.113	-3.657582		34.05396
sqrft	.1284362	.0138245	9.29	0.000	.1009495		.1559229
price	Coefficient	Std. err.	t	P> t	[95% con	f. i	nterval]
Total	917854.506	87	10550.0518	3 Root	MSE	=	63.045
					R-squared	=	0.6233
Residual	580009.152 337845.354	2 85	290004.576 3974.65122		-	=	0.0000 0.6319
Model	500000 450	2	200004 576	F(2, Prob	•	=	72.96
	SS	df	MS		er of obs	=	88

. predict e, resid

. reg price e

Source	SS	df	MS	Number of ob	os =	88
				F(1, 86)	=	51.33
Model	343066.064	1	343066.064	Prob > F	=	0.0000
Residual	574788.441	86	6683.58653	R-squared	=	0.3738
				Adj R-square	ed =	0.3665
Total	917854.506	87	10550.0518	Root MSE	=	81.753
price	Coefficient	Std. err.	t	P> t [95%	conf.	interval]
е	.1284362	.0179268	7.16	0.000 .0927	7989	.1640736
_cons	293.546	8.714921	33.68	0.000 276.2	2213	310.8707
-						

The coefficients of the residual's component e_i are equal to the $\hat{\beta}_1$ estimated in the first regression. By definition, the coefficient $\hat{\beta}_1$ is the partial effect of $SQRFT_i$ on $PRICE_i$ holding everything else fixed. With this procedure, estimating the residuals of $SQRFT_i$ on $BDRMS_i$, we account for the variations in $SQRFT_i$ which does not depend on the variable $SQRFT_i$. In other words, we keep fixed the variation of $SQRFT_i$ due to change in $BDRMS_i$, and using the residuals we just see the partial effect of $SQRFT_i$ on prices.

(b) Solution:

- . /// 1.B
- . reg bdrms sqrft

Source	SS	df	MS	Numbei	of obs	=	88
Model Residual	17.3972288 44.1936803	1 86	17.3972288 .513880004		, F	=	33.85 0.0000 0.2825
Total	61.5909091	87	.707941484	_	Adj R-squared Root MSE		0.2741 .71685
bdrms	Coefficient	Std. err.	t	P> t	[95% cor	nf.	interval]
sqrft _cons	.0007747 2.008077	.0001332 .2788063	5.82 7.20	0.000 0.000	.00051 1.453829		.0010394 2.562326

. gen delta_1 = _b[sqrft]

. reg price sqrft

Source	SS	df	MS		per of obs	s =	88
Model Residual	569801.074 348053.432	1 86	569801.074 4047.13293	Prob R-so	F(1, 86) Prob > F R-squared Adj R-squared Root MSE		140.79 0.0000 0.6208 0.6164
Total	917854.506	87	10550.0518	_			63.617
price	Coefficient	Std. err.	t	P> t	[95% (conf.	interval]
sqrft _cons	.140211 11.20415	.0118166 24.74261	11.87 0.45	0.000 0.652	.11672 -37.982		.1637017 60.39082

. reg price sqrft bdrms

Source	SS	df	MS	Numbe	er of obs	=	88
				F(2,	85)	=	72.96
Model	580009.152	2	290004.576	Prob	> F	=	0.0000
Residual	337845.354	85	3974.65122	R-squ	uared	=	0.6319
				Adj I	R-squared	=	0.6233
Total	917854.506	87	10550.0518	Root	MSE	=	63.045
	·						
price	Coefficient	Std. err.	t	P> t	[95% co	nf.	interval]
sqrft	.1284362	.0138245	9.29	0.000	.100949	5	.1559229
bdrms	15.19819	9.483517	1.60	0.113	-3.65758	2	34.05396
cons	-19.315	31.04662	-0.62	0.536	-81.0439	9	42.414

- . gen beta_1_hat = _b[sqrft]
- . gen beta_2_hat = _b[bdrms]

. gen error = beta_1_tilde - beta_1_hat - beta_2_hat*delta_1

Results are presented in Table 1.

Table 1: Estimation results from above steps.

$ ilde{\delta}_1$	$ ilde{eta}_1$	\hat{eta}_1	\hat{eta}_2	$\tilde{\beta}_1 - \hat{\beta}_1 - \hat{\beta}_2 \tilde{\delta}_1$
0.0007747	0.140211	0.1284362	15.19819	-3.23e-09

We can verify that:

$$\tilde{\beta}_1 = \hat{\beta}_1 + \hat{\beta}_2 \tilde{\delta}_1$$

To derive the expectation result, we take the conditional expectation on both sides and use the fact

that the **OLS estimator** is conditionally unbiased:

$$\mathbb{E}[\hat{\beta}_1] = \beta_1, \quad \mathbb{E}[\hat{\beta}_2] = \beta_2.$$

Furthermore, $**\tilde{\delta}_1$ is a function of the data matrix X^{**} :

$$\tilde{\delta}_1 = (X_1' X_1)^{-1} X_1' X_2.$$

Thus:

$$\mathbb{E}[\hat{\beta}_1 \mid X] = \mathbb{E}[\hat{\beta}_1 + \hat{\beta}_2 \tilde{\delta}_1 \mid X] = \beta_1 + \beta_2 \tilde{\delta}_1.$$

We consider a regression model:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i,$$

where $**X_{i2}$ is omitted** from the regression. Define:

$$X_i = \begin{bmatrix} 1 \\ SQRFT_i \\ BDRMS_i \end{bmatrix}, \quad Z_{i1} = \begin{bmatrix} 1 \\ SQRFT_i \end{bmatrix}, \quad Z_{i2} = \begin{bmatrix} BDRMS_i \end{bmatrix}.$$

Let:

$$\gamma_1 = (\beta_0, \beta_1)', \quad \gamma_2 = (\beta_2).$$

$$\hat{\beta} = (Z_1' Z_1)^{-1} Z_1' Y.$$

Substituting $Y = Z\beta + \varepsilon$:

$$\hat{\beta} = (Z_1' Z_1)^{-1} Z_1' (Z\beta + \varepsilon).$$

Expanding:

$$\hat{\beta} = \gamma_1 + P_{1,2}\gamma_2 + (Z_1'Z_1)^{-1}Z_1'\varepsilon.$$

Taking Expectation:

$$\mathbb{E}[\hat{\beta} \mid X] = \gamma_1 + P_{1.2}\gamma_2.$$

Since Z_2 is a scalar, $\gamma_2 = \beta_2$, and:

$$P_{1.2} = (\tilde{\delta}_0, \tilde{\delta}_1)'.$$

Thus, we obtain:

$$\mathbb{E}\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} \middle| X = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \tilde{\delta}_0 \\ \tilde{\delta}_1 \end{bmatrix} \beta_2.$$

The second row confirms the required result.

2 Question 2

P2: (a) When $Z \times R$, $\beta_{1V} = (Z'X)^{-1}ZY = (Z'X)^{-1}Z'(X'\beta + E) = (Z'X)^{-1}Z'X'\beta + (Z'X)^{-1}Z'E$ $=\beta+(z'x)^{\dagger}z'\epsilon.=\beta+(z'z)^{\dagger}(z'z'\epsilon)$ $\beta + (\sum_{j=1}^{n} (Z_{i}X_{i}^{j}))^{-1} (\sum_{j=1}^{n} Z_{i}^{j} E)$ $\Rightarrow \rho E Z_{i}X_{i} = E Q Z_{i}E_{i}$ $\vec{\beta}_{\text{IV}} - \beta = \left(\sum_{i=1}^{n} Z_i X_i' \right)^{-1} \left(\sum_{i=1}^{n} Z_i' \mathcal{E} \right), \quad \vec{\beta}_{\text{Sols}} = \left(Z_i' X \right)^{-1} Z_i' y, \quad \text{Let } P_{\text{ZZ}} Z_{\text{ZZ}}$ X= BX = Z(Z'Z) Z'X β25LS = (X'β2X) - X'β2 y = (X'Z(Z'Z) - Z'X) - X'Z(Z'Z) - Z'X y βzsls - β = [X'Z(Z'Z) + Z'X) + X'Z(Z'Z) + Z'E (b) Moment of X: and Zi, E[XiXi]= Qxx < io, rank Qxx = k E[Zizi] = Qzz < wo. rank Qzz = L E[zixi]=Qzx<\u00f6 rankQzx=k. By CLT and WLLN, Jn(βωω - β)=Jn((方言XiZi)(方言ZiZi))(方言ZiXi)) (方は対域行るZiZi), (万方言ZiEi) -> (Qxz · Qxz · Qxxx) · Qxz · Qxz Jn. n = in. t = E: E: -> (N(0, SZ) By In(Busis -B) = 2 (Qxz · Q+z Q+x) Qxz Q+z · N(0. S2) = N(0.V) V= (Qxz Qzz Qzx) - Qxz (Qzz S Qzz Qzx) (Qxz Qzz Qxx)

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C. If under Homoskedasticity, E(Ei) = 62
         Then (Jn n I MZiEi) -> N(0, 62 (22)
           V can red > V= 6 (Qxx Qx Qx Qxx)
                A sample analog \hat{V} = \hat{S}^2 (\hat{Q_{XS}} \hat{Q_{SS}} \hat{Q_{SX}})^{-1} where
                     2x2= + x'Z; Q22= + xZ'Z; Q2x = +Z'X
                     82 = 1 = = + = ( yi-xiBus)2
              in I under heteroskedasticity, \vec{\Sigma} = \frac{1}{n} \sum_{i=1}^{n} z_i z_i' \hat{z}_i' = \frac{1}{n} \sum_{i=1}^{n} z_i z_i' (y_i - x_i' \beta_{asis})^2
                                                                                                                                                                                                                       1, 128,2
  d. (i) Under Homoskedatily, \hat{V} = \hat{\delta}^{2} (\hat{Q}_{xx} \hat{Q}_{xx} \hat{Q}_{xx})^{-1}
                                                   \widehat{\xi_{i}} = y_{i} - x'\beta = x'\beta + \xi_{i} - x'\beta = x'(\beta - \beta) + \xi_{i}
\widehat{\xi_{i}}^{*} = (y_{i} - x'\beta)^{2} = (x'_{i}(\beta - \beta))^{2} + \xi_{i})^{2} = (\beta - \beta)'x'_{i}x'_{i}(\beta - \beta) + 2(\beta - \beta)'x_{i}\xi_{i} + \xi_{i}^{*}
                                                       G_{12} = \frac{1}{2} \frac{1}{2} \hat{\xi}_{1}^{2} = \frac{1}{2} \frac{1}{2} \hat{\xi}_{1}^{2} + 2(\beta \hat{\beta})' \frac{1}{2} \hat{\chi}_{1} \hat{\xi}_{1} + (\beta \hat{\beta})' \frac{1}{2} \hat{\chi}_{1} \hat{\chi}_{1}
                                                  By WLLN, 1/2, E; > 62;
                                                 Since Exik < 10 and E Ei < 10, E xix E. < 10, we can get
                                                                                                                        The XiE: -> 0 (By WILN)
                                               By WLLN, h = XiXi -> E XiXi = as n > 00
                                           Thus: \hat{6} \rightarrow 6, \hat{V} \rightarrow V.
                  (ii) If under hetereskedasticity,
                                                           1 = 1 = ZiZi (yi - 8xi Bises)
                                                                          = \frac{1}{n} \sum z_i z_i' z_i^2 + 2 \cdot \frac{1}{n} \sum_{i=1}^{n} z_i z_i' [\beta - \beta]' x_i z_i [\beta - \beta]' x_i x_i' (\beta - \beta)
                                         「これ」という。E[ziziをう] < D
                                                   IZEL = TIZIK ZILLIB-B) XIEI] = (B-B) TIZIK ZIK ZIL TIEI
                                                                                       - E[Zik Zil XiEi] < DO.
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3 Question 3

. eststo: ivregress 2sls lwage exper tenure (educ = sibs feduc meduc), first

First-stage regressions

Number of obs = 722 F(5, 716) = 73.49Prob > F = 0.0000 R-squared = 0.3391 Adj R-squared = **0.3345** Root MSE = 1.8247

Coefficient	Std. err.	t	P> t	[95% conf.	interval]
2009287	.0173033	-11.61	0.000	2348999	1669575
.037542	.0141963	2.64	0.008	.0096706	.0654135
1085301	.0317095	-3.42	0.001	1707849	0462754
.1568147	.0256903	6.10	0.000	.1063774	.2072521
.110667	.030085	3.68	0.000	.0516017	.1697323
13.17707	.4193578	31.42	0.000	12.35375	14.00039
	2009287 .037542 1085301 .1568147 .110667	.037542 .0141963 1085301 .0317095 .1568147 .0256903 .110667 .030085	2009287 .0173033 -11.61 .037542 .0141963 2.64 1085301 .0317095 -3.42 .1568147 .0256903 6.10 .110667 .030085 3.68	2009287 .0173033 -11.61 0.000 .037542 .0141963 2.64 0.008 1085301 .0317095 -3.42 0.001 .1568147 .0256903 6.10 0.000 .110667 .030085 3.68 0.000	2009287 .0173033 -11.61 0.0002348999 .037542 .0141963 2.64 0.008 .0096706 1085301 .0317095 -3.42 0.0011707849 .1568147 .0256903 6.10 0.000 .1063774 .110667 .030085 3.68 0.000 .0516017

Instrumental variables 2SLS regression

Number of obs = 722 Wald chi2(3) = 76.58 Prob > chi2 0.0000 R-squared 0.0641 Root MSE .40543

lwage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
educ	.1376307	.0183256	7.51	0.000	.1017133	.1735482
exper	.033928	.0058711	5.78	0.000	.0224208	.0454351
tenure	.0073686	.0032532	2.27	0.024	.0009925	.0137447
_cons	4.481904	.2992201	14.98	0.000	3.895443	5.068365

- . // B
- . eststo: reg educ exper tenure sibs feduc meduc

Source	SS	df	MS		er of obs	=	722
				` '	716)	=	73.49
Model	1223.3278	5	244.665559	9 Prob	> F	=	0.0000
Residual	2383.88689	716	3.32945096	5 R-sq	uared	=	0.3391
				———— Adj R-squared		=	0.3345
Total	3607.21468	721	5.00307168	3 Root	MSE	=	1.8247
educ	Coefficient	Std. err.	t	P> t	[95% cor	nf.	interval]
exper	2009287	.0173033	-11.61	0.000	2348999	9	1669575
tenure	.037542	.0141963	2.64	0.008	.0096706	6	.0654135
sibs	1085301	.0317095	-3.42	0.001	1707849	9	0462754
feduc	.1568147	.0256903	6.10	0.000	.1063774	4	.2072521
meduc	.110667	.030085	3.68	0.000	.0516017	7	.1697323
_cons	13.17707	.4193578	31.42	0.000	12.3537	5	14.00039

(est2 stored)

. predict educ_hat

(option xb assumed; fitted values)
(213 missing values generated)

. eststo: reg lwage educ_hat exper tenure

722	=	er of obs	Numbe	MS	df	SS	Source
26.37	=	718)	- F(3,				
0.0000	=	> F	7 Prob	4.19587887	3	12.5876366	Model
0.0993	=	uared	1 R-squ	.159086761	718	114.224295	Residual
0.0955	=	R-squared	– Adj R				
.39886	=	MSE	4 Root	.1758834	721	126.811931	Total
interval]	nf.	[95% cor	P> t	t F	Std. err.	Coefficient	lwage
.1730252	2	.1022362	0.000	7.63	.0180283	.1376307	educ hat
	4	.0225884	0.000	5.87	.0057759	.033928	exper
.0452675							
.0452675 .0136518	-	.0010853	0.022	2.30	.0032004	.0073686	tenure

(est3 stored)

- . // For the Table
- . reg educ exper tenure sibs feduc meduc

Source	SS	df	MS		er of obs	=	722
Model Residual	1223.3278 2383.88689	5 716	244.665559 3.32945096	Prob R-sq	uared	= =	73.49 0.0000 0.3391 0.3345
Total	3607.21468	721	5.00307168	——— Adj R-squar 307168 Root MSE		=	1.8247
educ	Coefficient	Std. err.	t	P> t	[95% c	onf.	interval]
exper tenure sibs feduc meduc _cons	2009287 .037542 1085301 .1568147 .110667 13.17707	.0173033 .0141963 .0317095 .0256903 .030085 .4193578	-11.61 2.64 -3.42 6.10 3.68 31.42	0.000 0.008 0.001 0.000 0.000	23489 .00967 17078 .10637 .05160 12.353	06 49 74 17	1669575 .0654135 0462754 .2072521 .1697323 14.00039

. reg lwage educ_hat exper tenure

Source	SS	df	MS	Numbe	er of obs	=	722
				- F(3,	718)	=	26.37
Model	12.5876366	3	4.1958788	7 Prob	> F	=	0.0000
Residual	114.224295	718	.15908676	1 R-squ	ıared	=	0.0993
			————— Adj R-squared		=	0.0955	
Total	126.811931	721	.1758834 Root MSE		=	.39886	
lwage	Coefficient	Std. err.	t	P> t	[95% co	onf.	interval]
educ_hat exper tenure _cons	.1376307 .033928 .0073686 4.481904	.0180283 .0057759 .0032004 .2943666	7.63 5.87 2.30 15.23	0.000 0.000 0.022 0.000	.102236 .022588 .001089	34 53	.1730252 .0452675 .0136518 5.059826

We can notice that all estimates β_j are identical to those obtained before, but that the standard errors in the second equation are different. In the second case, the estimate of σ^2 is wrong. Its

computation includes the variance of the sum of the error term in the second equation and the error term from the first stage regression. Since \hat{X} is a projection of X, 2SLS will have higher variance (even if OLS is inconsistent). This confirms the fact that, when OLS is consistent, it is better than 2SLS. But when X is endogenous, we must account for endogeneity using the 2SLS procedure. In summary, we can conclude that standard errors from the second approach are invalid.

•	// C				
	eststo:	reg educ	sibs	feduc	meduc

722	=	ber of obs	N	MS	df	SS	Source
65.20	=	, 718)	— F				
0.0000	=	b > F	16 P	257.42714	3	772.281437	Model
0.2143	=	quared	99 R	3.9483749	718	2834.93324	Residual
0.2108	=	R-squared	— Д				
1.987	=	t MSE	5 8 R	5.003071	721	3607.21468	Total
interval	nf.	[95% cor	P> t	t	Std. err.	Coefficient	educ
0259594	4	1613124	0.00	-2.72	.0344713	0936359	sibs
.2639447	5	.156063	0.00	7.64	.0274748	.2100041	feduc
.1949646	8	.0666098	0.00	4.00	.032689	.1307872	meduc
11.06809	2	9.660422	0.00	28.91	.3585001	10.36426	cons

(est4 stored)

. predict educ_tilde

(option xb assumed; fitted values)
(213 missing values generated)

. eststo: reg lwage educ_tilde exper tenure

722	=	per of obs		MS	df	SS	Source
26.45 0.0000	=	, 718) o > F	` .	4.2064079	3	12.619224	Model
0.0995	=	quared	8 R-sc	.15904276	718	114.192707	Residual
0.0957 .3988	=	R-squared t MSE	_	.175883	721	126.811931	Total
interval]	onf.	[95% co	P> t	t	Std. err.	Coefficient	lwage
.1419396 .0139544 .018704 5.514315	19 74	.083953 000833 .006533	0.000 0.082 0.000 0.000	7.65 1.74 4.07 23.65	.0147677 .0037657 .0030986 .2153205	.1129465 .0065613 .0126207 5.091582	educ_tilde exper tenure _cons

(est5 stored)

- . // For the Table
- . reg educ sibs feduc meduc

Source	SS	df	MS	Numl	ber of obs	=	722
				F(3	, 718)	=	65.20
Model	772.281437	3	257.427146	6 Prol	b > F	=	0.0000
Residual	2834.93324	718	3.94837499		quared	=	0.2141
				- Adj	R-squared	=	0.2108
Total	3607.21468	721	5.00307168	Roo ⁻	t MSE	=	1.9871
educ	Coefficient	Std. err.	t	P> t	[95% co	nf.	interval]
sibs	0936359	.0344713	-2.72	0.007	161312	4	0259594
feduc	.2100041	.0274748	7.64	0.000	.156063	5	.2639447
meduc	.1307872	.032689	4.00	0.000	.066609	8	.1949646
_cons	10.36426	.3585001	28.91	0.000	9.66042	2	11.06809
reg lwage e	duc_tilde expe	r tenure					
r eg lwage e d Source	duc_tilde expe	r tenure df	MS		ber of obs	=	722
Source	SS	df		F(3	, 718)	=	26.45
Source Model	SS 12.619224	df 3	4.20640799	F(3)	, 718) b > F	=	26.45 0.0000
Source	SS	df		F(3) Prol R-se	, 718) b > F quared	=	26.45 0.0000 0.0995
Source Model	SS 12.619224	df 3	4.20640799	F(3) Prol R-so Adj	, 718) b > F	= = =	26.45 0.0000 0.0995
Source Model Residual	SS 12.619224 114.192707	df 3 718	4.20640799 .159042768	F(3) Prol R-so Adj	, 718) b > F quared R-squared t MSE	= = =	26.45 0.0000 0.0995 0.0957
Source Model Residual	SS 12.619224 114.192707 126.811931	df 3 718 721	4.20640799 .159042768 .1758834	F(3) Prol R-so Adj Roo	, 718) b > F quared R-squared t MSE	= = = = enf.	26.45 0.0000 0.0995 0.0957 .3988
Source Model Residual Total lwage	12.619224 114.192707 126.811931 Coefficient	df	4.20640799 .159042768 .1758834	F(3) Prol R-so Adj Roo	, 718) b > F quared R-squared t MSE	= = = = onf.	26.45 0.0000 0.0995 0.0957 .3988 interval]
Source Model Residual Total lwage	SS 12.619224 114.192707 126.811931 Coefficient .1129465	df 3 718 721 Std. err0147677	4.20640799 .159042768 .1758834 t	F(3) Prol R-si Adj Roo P> t	, 718) b > F quared R-squared t MSE [95% co	= = = = onf.	26.45 0.0000 0.0995 0.0957 .3988 interval]

The first stage is used to estimate the endogenous variable, and if the first stage is not correctly specified, the second stage will not be consistent. The error term in the second stage will not be orthogonal to the exogenous regressors and the estimator will be inconsistent.

. // ט . eststo: reg lwage educ exper tenure

Source	SS	df	MS		r of obs	=	935
Model Residual	25.6953278 139.960966	3 931	8.5651092 .15033401	3 R-squ	> F lared	=	56.97 0.0000 0.1551
Total	165.656294	934	.17736219	3	-squared MSE	=	0.1524 .38773
lwage	Coefficient	Std. err.	t	P> t	[95% co	nf.	interval]
educ exper tenure _cons	.0748638 .0153285 .0133748 5.496696	.0065124 .0033696 .0025872 .1105282	11.50 4.55 5.17 49.73	0.000 0.000 0.000 0.000	.06208 .008715 .008297 5.27978	6 4	.0876446 .0219413 .0184522 5.713609

(est6 stored)

. esttab, se ar2 replace

		/->	4-3	
(5)	(1) (6)	(2)	(3)	(4)
lwage	lwage lwage	educ	lwage	educ
uc	0.138***			
	0.0749*** (0.0183)			
	(0.00651)			
xper	0.0339***	-0.201***	0.0339***	
0.00656	0.0153*** (0.00587)	(0.0173)	(0.00578)	
(0.00377)	(0.00337)	,	,	
enure 0.0126** *	0.00737* 0.0134***	0.0375**	0.00737*	
	(0.00325)	(0.0142)	(0.00320)	
(0.00310)	(0.00259)			
ibs		-0.109***		-0.0936**
		(0.0317)		(0.0345)
				•
educ		0.157***		0.210***
		(0.0257)		(0.0275)
		(0.0257)		(0.02/3)
educ		0.111***		0.131***
		(0.0301)		(0.0327)
		, ,		, ,
duc_hat			0.138***	
			(0.0180)	
duc_tilde				
0.113***				
(0.0148)				
cons	4.482***	13.18***	4.482***	10.36***
5.092***	5.497*** (0.299)	(0.419)	(0.294)	(0.359)
(0.215)	(0.111)	/	/	,,
	722	722	722	722
722 dj. R-sq	935 0.060	0.335	0.095	0.211
0.096	0.152			

4 Question 4

```
(4) From the conclusion in Problem, we have
          \beta_{>566} = (\chi' \geq (Z' \geq) Z' \chi)^{-1} \chi' \geq (Z' \geq)^{-1} Z' \gamma = 0
= (\begin{bmatrix} >19.4 & 115.4 & 49.6 \\ 5>.4 & 34.6 & 61.4 \end{bmatrix} \begin{bmatrix} 536 & 118.7 & 59.2 \\ 188.7 & >11.9 & 42.9 \\ 59.2 & 42.9 & 194.7 \end{bmatrix} \begin{bmatrix} 219 & 52.4 \\ 115.4 & 34.6 \\ 49.6 & 61.4 \end{bmatrix})^{-1} 
                   = (1.728) (By MAPLUS)
   (b) \hat{x}^2 = \frac{e'e}{n} = \frac{Y'Y + \hat{\beta}'_{2516} X'X \hat{\beta}_{2516} - 2(X \hat{\beta}'_{2516})'Y}{n} = 7.788
        -> we can reject to at 5% level
  C) > CI = (1.6535 $ 1.96 : √0.6168 $ , $ 1.6535 + 1.96 · √0.6168)
  id) Ho: β,=1 and β=2. under Ho
         \begin{pmatrix} \beta^{1} - \beta_{1} \\ \beta_{2} - \beta_{3} \end{pmatrix} = \begin{pmatrix} 1.728 - 1 \\ 1.654 - 2 \end{pmatrix} = \begin{pmatrix} 0.728 \\ -0.347 \end{pmatrix}, var \begin{pmatrix} \beta^{1} - \beta_{1} \\ \beta^{2} - \beta_{3} \end{pmatrix} = \begin{pmatrix} 0.1333 & -0.1884 \\ -0.1884 & 0.6168 \end{pmatrix} 
            (\beta-\beta)' var(\beta)'(\beta-\beta) = 5.305b, p-value = 0.07045 > 0.0/
    By using Wald test,
         reject Ho. at 1%
```

5 Question 5

Ps. Y: = Z: B+1: (a) First, we regress Zi on \$ Wi (IV) to get Zi $\Xi_i = X_i = \delta_0 + \delta_1 W_i + V_i$ Since Xi= Wi = Xi + Vi, Zi = Xi = So+ 5, (Xi+Vi)+Mi $\delta_{i}^{2} = \frac{\text{CoV}(Z_{i}, W_{i})}{\text{to Var}(W_{i})} = \frac{\text{CoV}(Z_{i}, X_{i} + V_{i})}{\text{Var}(X_{i} + V_{i})} = \frac{\text{CoV}(Z_{i}, W_{i})}{\text{Var}(X_{i} + V_{i})} = \frac{\text{CoV}(Z_{i}, W_{i$ to, In second stage. We regress Y; on \vec{Z} ; which $\vec{Z}_i = \vec{\delta}_0 + \vec{\delta}_i w_i$, and unde homoskedacity assumption, In($\beta_i \vec{v} - \beta_i$) $\rightarrow d \vec{N}(0, V_{LV})$ $V_{LV} = \vec{\delta}_E^2 E[W_i \vec{Z}_i]^T E[W_i W_i'] E[Z_i W_i']^{-1}$ E[Wizi] = E[(Xi+Vi) Xi] = EXi+ EviXi = EXi E[W:W:] = E[(X:+V:)2] = E[X:] +E[(:v:] + Ev:2 = E[X:] + 5; :. VIV = 62 . E[X;] (E[X]+62) . - 1 = 62. E[X;]+62 (G). From (b). When $6\sqrt{1}=0$, V_{IV} have same distribution as DLS estimation.