(a) $Y^{(i)} = X\beta + u^{(i)}$, $i=1.2$

OLS estimitor $\hat{\beta} = (X'X)^{-1}X'y^{(i)}$ $i=1.2$.

According to Guass-markov conditions.

$E(\beta) = E((X'X)^{-1}X'y^{(i)}) = (X'X)^{-1}X'E(y^{(i)})$

$= (X'X)^{-1}X'X\beta = \beta$

Least square estimator of β is unbiased.

$Var(\beta) = Var((X'X)^{-1}X'y^{(i)}) =$

$= (X'X)^{-1}X' \, Var\, y^{(i)} \, X \cdot (X'X)^{-1}$

$= (X'X)^{-1}X' \cdot (\sigma^2 I_n) \cdot X \cdot (X'X)^{-1}$

$= \sigma^2 \cancel{(X'X)^{-1}X'X}(X'X)^{-1} = \sigma^2 (X'X)^{-1}$

(b) Two model have common β, and with above analysis, we can derive that.

$E(\hat{\beta}^{(1)}) = E(\hat{\beta}^{(2)}) = \beta$

(2) $Y^{(1)} = X_1\beta + u^{(1)}$ (1)

$Y^{(2)} = X_2\beta + u^{(2)}$ (2)

OLS estimator of β in model (1) is

$\hat{\beta}^{(1)} = (X^{(1)\prime}X^{(1)})^{-1}X^{(1)\prime}y^{(1)}$

ols estimator of β in model (8) is

$$\hat{\beta}^{(2)} = (X^{(2)\prime} X^{(2)})^{-1} X^{(2)\prime} y^{(2)}$$

Now. $E(\hat{\beta}^{(1)}) = E[(X^{(1)\prime}X^{(1)})^{-1}X^{(1)\prime}y^{(1)}]$

$$= (X^{(1)\prime}X^{(1)})^{-1}X^{(1)} E\, y^{(1)}$$

$$= (\cancel{X^{(1)\prime}X^{(1)}})^{-1} \cancel{X^{(1)}} \cdot X^{(1)}\beta = \beta$$

and $E(\hat{\beta}^{(2)}) = E[(X^{(2)\prime}X^{(2)})^{-1}X^{(2)\prime}(X^{(2)}\beta)] = \beta$.

Therefore, $\hat{\beta}^{(1)}$ and $\hat{\beta}^{(2)}$ are both unbiased estimator for β.

$Var(\hat{\beta}^{(1)}) = var((X^{(1)\prime}X^{(1)})^{-1}X^{(1)}y^{(1)}) = (X^{(1)\prime}X^{(1)})X^{\prime(1)}\cdot(\sigma^2 I_{n_1})X^{(1)}(X^{(1)\prime}X^{(1)})^{-1}$

$$= \sigma^2 (X^{(1)\prime}X^{(1)})^{-1}$$

Similarly, we get

$$Var(\hat{\beta}^{(2)}) = \sigma^2 (X^{(2)\prime}X^{(2)})^{-1}$$

(2b) Here we have seen that both estimators are equal. Both estimator are unbiased estimator of β.