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F7024T 16/17

Assignment # 5.

Magnetohydrodynamic pump (motor) and brake

Magnetohydrodynamics is a theory to analyze conducting liquid metals and plasmas. It is described by the Navier-Stokes equations

$$\rho \frac{D\mathbf{u}}{Dt} = \rho(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}) = -\nabla p + \mathbf{j}\times\mathbf{B} + \mu\nabla^2\mathbf{u}$$
$$\nabla\cdot\mathbf{u} = 0$$

together with Maxwells equations

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \textit{Faradays lag}$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nabla\times\mathbf{B} = \mu_0\,\mathbf{j} \quad \textit{Since the motion is slow compared to velocity of light}$$
$$\textit{the displacement current can be neglected}$$
$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{u}\times\mathbf{B}) \quad \textit{Ohms law}$$

In this assignment you will analyze the behavior of MHD pumps and MHD brake in a channel flow. The geometry is given by the figure below

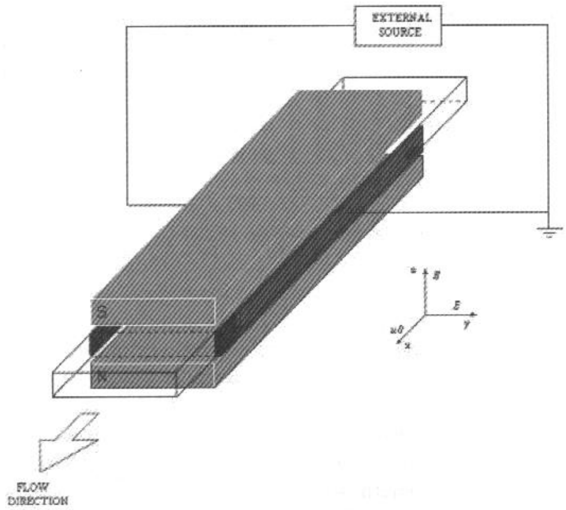


Figure 1. Microchannel geometry.

The magnetic field $\mathbf{B}$ is applied over a short region $|x| \le a$ a in the z-direction, and an external electric field $\mathbf{E} = -\nabla V$ is applied in the y-direction. The fluid flows in the x-direction. Since we have steady conditions the continuity equation for charge is

$$\nabla\cdot\mathbf{j} = 0 \quad \textit{together with Ohms law this gives}$$
$$\nabla\cdot(\sigma\mathbf{E}) = -\nabla\cdot(\sigma\mathbf{u}\times\mathbf{B})$$

A current flows across the fluid between the electrodes, so that the term $\mathbf{j}\times\mathbf{B}$ gives a force which is either accelerating (pump) or decelerating (brake). To analyze the flow follow the following instructions.

Note: In Comsols Electric current model Ohms law is not written in the form above used in MHD. In Comsol Ohms law is written in the usual way

$$\mathbf{J} = \sigma\mathbf{E}$$

$$\nabla\cdot\mathbf{J} = \nabla\cdot(\sigma\mathbf{E}) = Q$$

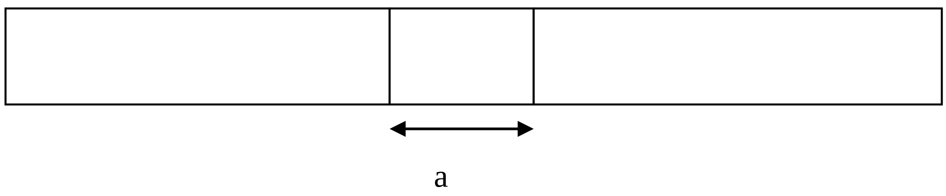
where Q is a source term. The extra term $\mathbf{u}\times\mathbf{B}$ in MHD should then be taken into account as a source term.

Instructions

1. Open Comsol and choose 2D Fluid dynamics, laminar flow and add AC/DC Electric currents 2D. Choose stationary study.

2. Add constants. The density as rho (1000[kg/m^3], (seawater), the viscosity as mu=0.001[kg/m/s]. B0=0.01[T], a=0.1[m] sigma=4 [S/m](sea water), Velocity U0=0.02[m/s], Potential V0=-1.5[V], k0=9.0[1/m]

3. Choose a rectangular region R1 with width 0.9[m] and height 0.06[m] with corner at (-1,-.03). Choose a rectangular region R2 with width 0.2[m] and height 0.06[m] with corner at (-0.1,-.03). Choose a rectangular region R3 with width 0.9[m] and height 0.06[m] with corner at (0.1,-.03).



4. Under Global definitions enter the variables

$$B_L = B_0 \exp(k0(x+a))$$

$$B_R = B_0 \exp(-k0(x-a))$$

7. Choose region 1 and put in mu and rho and expressions for the forces

$$\begin{aligned} F_x &= \text{sigma}*(-\text{BL}\cdot\text{Vy}-\text{BL}^\wedge 2\cdot \text{u}) \\ F_y &= \text{sigma}*(\text{BL}\cdot\text{Vx}-\text{BL}^\wedge 2\cdot \text{v}) \end{aligned} \qquad (1)$$

For region 2 choose the expressions

$$\begin{aligned} F_x &= \text{sigma*(-B0*Vy-B0^2*u)} \\ F_y &= \text{sigma*(B0*Vx-B0^2*v)} \end{aligned} \qquad (2)$$

For region 3 choose

$$\begin{aligned} F_x &= \text{sigma*(-BR*Vy-BR^2*u)} \\ F_y &= \text{sigma*(BR*Vx-BR^2*v)} \end{aligned} \qquad (3)$$

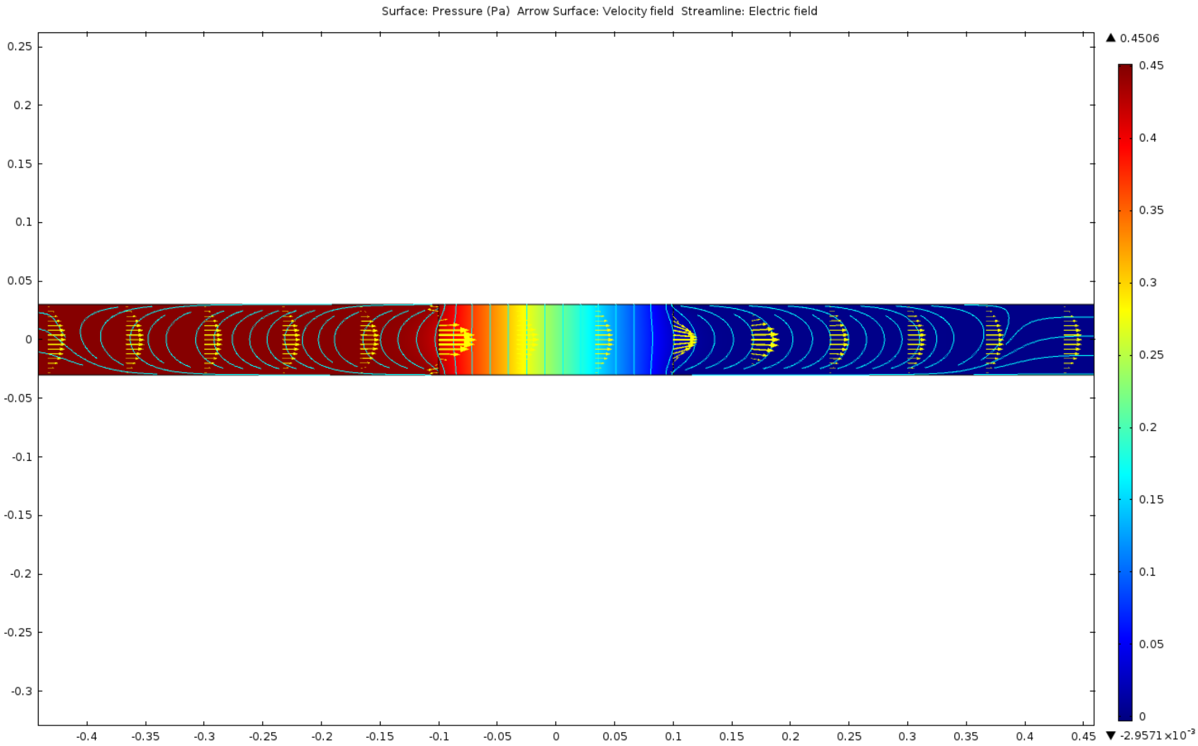
Note that Vy denotes the derivative of V with respect to y.

8. Choose boundary conditions for the flow
   Surface 1 Inflow with parabolic Poiseuille velocity profile(or Laminar flow)
   Surface 2-3 and 5-6 and 8-9 wall no slip.
   Surface 10 outlet.
9. Enter conditions for the electric current source
   For region 1 Qj=-BL*(vx-uy)*sigma-sigma*v*BLx (4)
   For region 2 Qj=-B0*(vx-uy)*sigma (5)
   For region 3 Qj=-BR*(vx-uy)*sigma-sigma*v*BRx (6)
   Choose boundary conditions
   Surface 1 and 10 ground
   Surface 2-3 and 8-9 electric insulation
   Surface 6 electric potential V0

   Surface 5 electric potential –V0
10. Choose mesh and compute.

Questions

a) Consider how the velocity profile changes along the channel. Vary the magnetic field and see how the profile changes. Also look at the pressure variation along the centerline of the channel. Increase the velocity and see how the pressure changes in the regions where the magnetic field is small and the region where it is large. Compare also the pressure distribution with the case of small magnetic field and large magnetic field.
b) Change the polarity of the electric field and consider the same questions as in a).
   Can you draw some conclusions which case is a pump and which case is a brake.
c) Consider the internet and look for a discussion about using MHD as a propulsion of motor boats
d) Write a report of the model with a derivation of the equations (1-6) used in the model.


Surface: Pressure (Pa) Arrow Surface: Velocity field Streamline: Electric field
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
▲ 0.4506
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
▼ -2.9571×10⁻³