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F7024T 16/17

Assignment # 6.

Convection and diffusion in a branched pipe

In this assignment we look at convection and diffusion of particles in a branched pipe, for instance the pipes of the human lung. We apply the Navier-Stokes equations together with the convection-diffusion equation for the concentration c, so

$$\frac{D\mathbf{u}}{Dt} = (\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}) = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u}$$
$$\nabla\cdot\mathbf{u} = 0$$

where ν is the kinematic viscosity of the air.

The convective-diffusion equation is

$$\frac{\partial c}{\partial t} + (\mathbf{u}\cdot\nabla)c = D\nabla^2 c$$

where D is the diffusion constant

*T*he diffusion coefficient for Brownian diffusion is given by

$$D = \frac{\kappa T}{k_b} = \frac{\kappa T\, Cu}{3\pi\rho\nu d_f},$$

where κ is Boltzmanns constant and T the absolute temperature. ρ is the density of air and ν is the viscosity of air and *a* is the pipe radius. Cu is a correction factor called Cunningham factor given by

$$Cu = 1 + \frac{\lambda}{d_f}(2.34 + 1.05\exp(-0.39\frac{d_f}{\lambda})),$$

where λ is the mean-free path of the air molecules, which is equal to 70 nm. Do the calculations for an inlet velocity corresponding to light, moderate and heavy physical activity in the interval of the average velocity ranging from 1-10 cm/s and a pipe radius of *a*=0.3 mm, corresponding to the 16th generation of the human lung. Choose the particle diameter $d_f = 20nm$.

Instructions

1. Open Comsol and choose 3D. Choose Laminar flow and Transport of diluted species. Consider steady flow.

2. Enter the physical parameters of the problem.

3. Import the 3D geometry from the file lungmodell.mphbin. in Canvas.

4. Scale the geometry imp1 under Transforms choose scale and select the geometry imp1. Choose scaling 0.0003. Form union and build selected. Under geometry choose Virtual operations and ignore edges. Select the curves 5,7,20,23. Click Build all.

5. Enter the boundary conditions, inlet and oulets for the flow.

6. Consider Transport of Diluted Species and select the model geometry. Under velocity field choose Velocity field(spf/fp1). Under Diffusion coefficiemt enter user defined diffusion D.

7. Enter the boundary condition for the concentration on the tube walls. Enter the inflow choosing c0=1[mol/m^3]. Define the outlets.

8. Choose a mesh and compute. Look at the slices of the velocity and concentration field.

9. First find the fraction of particles deposited to the wall by finding the ratio of particles flowing out of the system divided by the amount of particles flowing into the system. Plot the deposition as a function of the average velocity ranging from 1-10 cm/s. The deposition of particles to the wall at a specific surface is the surface integral of the wall-normal diffusive flux. Find the deposited fraction of particles to the different surfaces of the model for the physical activity corresponding to mean velocity 5 cm/s.

10. Write a report of the results.