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# Luleå Tekniska Universitet

## F7024T Multifysik, simulering och beräkning

*Assignment 3: Fluid flow past cylinder. Drag and lift forces*

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With supervisor  
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### Abstract

This work contains the result and analysis of the third COMSOL-laboratory exercise where the drag and lift force of a fluid, in this case water, on a cylinder has been simulated. It is found that the drag coefficient decreases logarithmically compared to the Reynold number, going from  $C_D \approx 200$  to  $C_D \approx 3$  between  $Re = 0$  and  $Re = 50$ , see fig. 1. Compared to wind tunnel data the drag coefficient seems to start at a higher value for low Reynold numbers, but decreased faster.

System did not converge for Reynold numbers much higher than 300, which is around where the system should change from laminar to turbulent flow<sup>a</sup>.

Secondly, introducing a rotation speed  $\Omega$ , a negative linear dependency between the lift coefficient  $C_L$  and the dimensionless coefficient  $Q = \frac{\Omega a}{V_\infty}$  could be observed. The linear plot started at  $C_L = 0$  for  $Q = 0$ , and decreases<sup>b</sup> with  $\approx 7$  for each increase in the rotational coefficient  $Q$ .

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<sup>a</sup>Derived from the only table at [1]

<sup>b</sup>Note that the negative sign only determines the rotation direction and is of little importance.

## 1 Introduction

COMSOL Multiphysics® is a general-purpose software platform, based on advanced numerical methods. It is a powerful tool useful to simulate flows; fields; forces and such in models provided either by files or built directly in COMSOL.

This report is a part of a written documentation of the COMSOL-laboratory exercises made in the course Multiphysics, Simulation and Computation at Luleå University of Technology. These exercises serve as practice in formulating mathematical models to describe physical and technical problems in a way that is suitable for implementation of the finite element method.

This work contains the result and analysis of the third COMSOL-laboratory exercise where the drag and lift force of a fluid, in this case water, on a cylinder has been simulated. This is done in two distinct studies.

In the first study the drag force will be simulated in which there will be no rotational speed on the affected cylinder. In this study the inflow speed,  $V_\infty$ , will be a function of the Reynold number, see eq. (1), to easily perform parametric sweeps. The drag coefficient  $C_D$  can be calculated using eq. (2), where  $F_D$  is the drag force extracted by COMSOL,  $A_{proj}$  the area projected in the direction of the stream. The assignment is to look how the Reynolds number affects the drag coefficient.

$$Re = \frac{V_\infty 2a\rho}{\mu} \quad (1)$$

$$C_D = \frac{2F_D}{\rho V_\infty^2 A_{proj}} \quad (2)$$

The second study will introduce a rotational velocity,  $\Omega$ , to the affected cylinder. This will generate a lift force on the water surrounding the cylinder. A lift coefficient can be calculated using eq. (5) where  $F_L$  is the lift force

and is calculated using eq. (4). The assignment is to compare how the rotational speed coefficient ( $Q$ ) affects the lift coefficient  $C_L$ .

$$Q = \frac{\Omega a}{V_\infty} \quad (3)$$

$$F_L = \rho V_\infty \Omega 2\pi a^2 \quad (4)$$

$$C_L = \frac{2F_L}{\rho V_\infty^2 A_{proj}} \quad (5)$$

$$\Rightarrow C_L = \frac{4\Omega\pi a^2}{V_\infty A_{proj}} \quad (6)$$

Drag and lift coefficients are dimensionless coefficients commonly used in fluid dynamics. The drag coefficient quantifies the drag or resistance of an object in a fluid environment such as air or water, while the lift coefficient relates the lift generated by an object to the properties of the fluid surrounding the object. Both drag and lift coefficients are relevant when building or simulating objects such as the wings of an airplane or looking at the fluid movements when an object is placed in the middle of a stream.[2][3]

## 2 Method

The exact method to calculate the coefficients and simulate the system is detailed and well explained in the instructions[4], but the general way to go about it is the same as most COMSOL projects.

1. Choose system type (fluids, laminar, 2D).
2. Introduce global parameters.
3. Build geometry.
4. Set study specifications for your system type:
  - Set fluid parameters.
  - Set Boundary conditions.

- Set initial conditions.
5. Build a mesh grid of your geometry.
  6. Compute system.

There are only two values not given or found by any formula to complete eqs. (2) and (5); these are the drag and lift forces,  $F_D$  and  $F_L$  respectively. These can be found simply by integrating your system in the  $x$ , resp.  $y$ -directions.

When all values to calculate  $C_D$  and  $C_L$  are known (where simulations are run to find  $F_D$  and  $F_L$ ) a global evaluation can be made on the simulation results.

## 2.1 Drag coefficient, $C_D$

To compare the drag coefficient to Reynolds number we need some way to increase the Reynolds number without messing up the system or any rules. Conveniently the only variable depending on Reynolds (and vice versa) is the inflow speed,  $V_\infty$ , see eq. (1). It is therefore very easy to make this speed a function of Reynolds number so that a simple parametric sweep can be done to the Reynolds number and the system will change accordingly.

The parametric sweep was run between 0.1 and 300 with tighter steps in the lower values, due to the more aggressive decline in the plotted results. COMSOL then finds the drag force for all nodes in the sweep and the global evaluation then evaluates the drag coefficient for all values of Reynolds number, saving the values to a 2D-plot as seen in fig. 1.

## 2.2 Lift coefficient, $C_L$

Comparing the lift coefficient to the rotational constant  $Q$  is rather straight forward. Since the rotational constant  $Q$  is negatively linearly dependent on  $V_\infty$  as is also the case

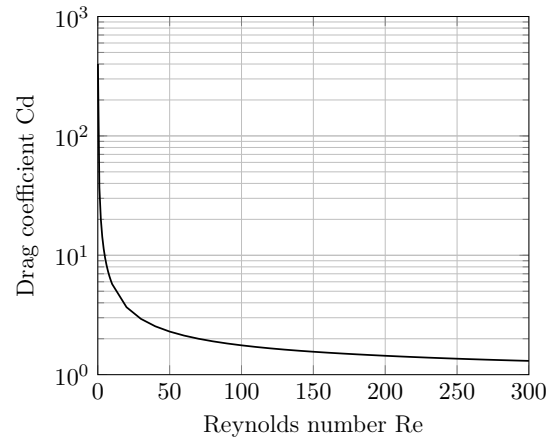


Figure 1: Drag coefficient compared to Reynold number

with the lift coefficient according to eq. (3) and eq. (6), they can simply be combined as eq. (7). It can be noted that  $C_L$  and  $Q$  are obviously linearly dependent meaning the larger the rotation the larger the amplitude on  $C_L$ . The sign will, since neither  $a$  nor  $A_{proj}$  are able to be negative, represent the rotational direction and as a result whether the lift coefficient will act to rise or lower the flow.

$$C_L = Q \frac{4\pi a}{A_{proj}} \quad (7)$$

A parametric sweep was performed on  $Q$  with values ranging from 1 through 40. The resulting 2D-plot can be viewed as fig. 2.

## 3 Results and interpretation

### 3.1 Drag coefficient, $C_D$

The evaluated correlation between  $C_D$  and  $Re$  seems to be logarithmically decreasing, but with Reynolds numbers at around 100 and higher it could be assumed to be rather linear with drag coefficient values decreasing slowly from 2.

It can be assumed that the linear region is

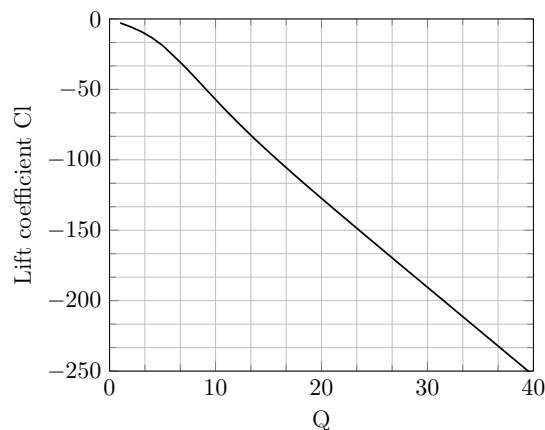


Figure 2: Lift coefficient compared to  $Q = \frac{\Omega a}{V_\infty}$

likely around where the system turns turbulent. Until the water flows with such speed there will be no real drag effect from the cylinder.

### 3.2 Lift coefficient, $C_L$

Comparing the lift coefficient to the rotational constant  $Q$  seems to be rather linear, as it should since the equation describing their relationship depends only on geometrical features, see eq. (7). This geometrical constant will act as a k-value and is equal to  $-7$  since  $C_L$  decreases by 7 for each step in  $Q$ .

Note that for the cylinder to create an upwards draft the lift coefficient would need to be an increase from 0 to 250 and not the opposite. This is due to the rotation of the system being calculated in reverse and doesn't really matter, but it sure is rather annoying.

## References

- [1] The Engineering Toolbox. Water flow in tubes - reynolds number. <http://www.engineeringtoolbox.com/>

reynold-number-water-flow-pipes-d\_574.html, 2018.

- [2] Wikipedia. Drag coefficient. [https://en.wikipedia.org/wiki/Drag\\_coefficient](https://en.wikipedia.org/wiki/Drag_coefficient), 2018.
- [3] Wikipedia. Lift coefficient. [https://en.wikipedia.org/wiki/Lift\\_coefficient](https://en.wikipedia.org/wiki/Lift_coefficient), 2018.
- [4] Multiphysics F7024T Hans Åkerstedt. Assignment #3, fluid flow past cylinder. drag and lift forces. Technical report, Department of Engineering Science and Mathematics, Mars 2017.

## 4 APPENDIX

### 4.1 Images

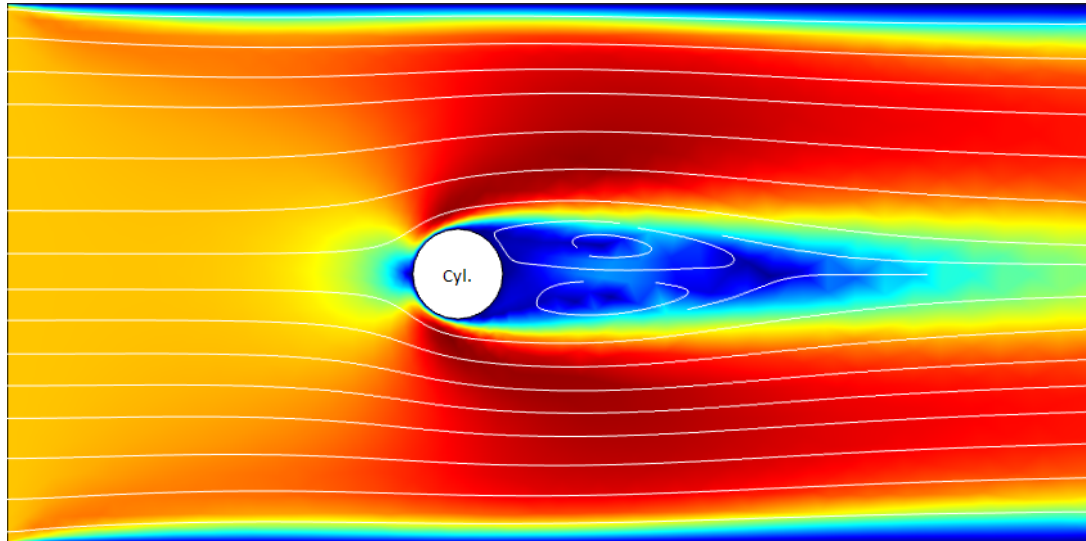


Figure 3: Integrated speed with no rotation of the cylinder and an Reynold number of 150.

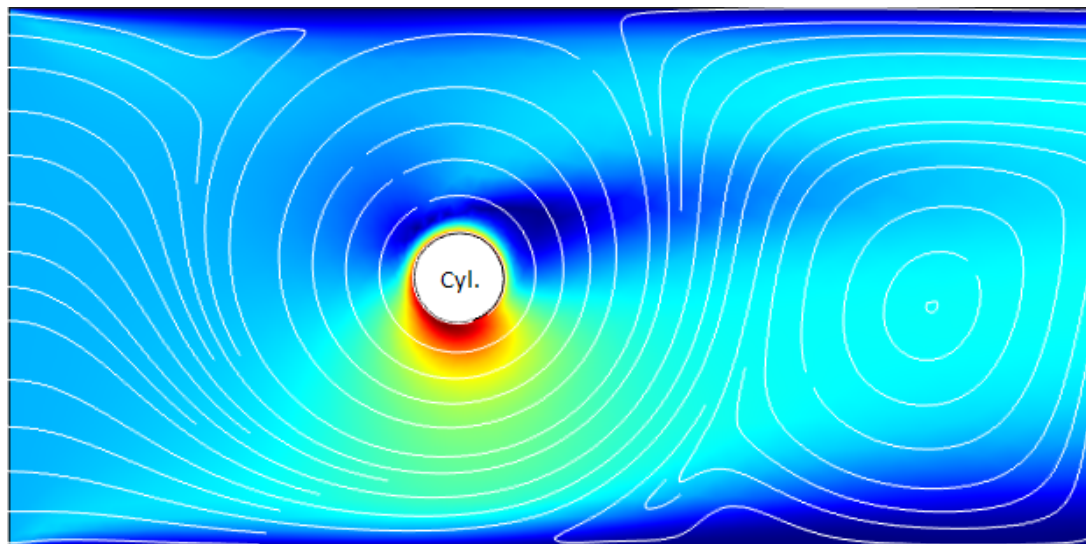


Figure 4: Integrated speed with low rotational constant of 3. Although speed is low the lift can be seen as the turquoise area rising below the cylinder. The “flowlines” are from where the system has a much higher rotational speed and the rise is much clearer.