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Luleå Tekniska Universitet
F7024T Multifysik, simulering och beräkning

Assignment 5: Magnetohydrodynamic pump and break

With supervisor
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Abstract

This work contains the result and analysis of the fifth COMSOL-laboratory exercise where the behavior of magnetohydrodynamic pumps and brakes are analyzed. The set of equations describing magnetohydrodynamics are the Navier-Stokes equations and Maxwells equations of electromagnetism. This laboratory exercise consists of four studies.

First the system is studied for negative vector potential, V0, configuration. The magnetic field, B0, is then studied through a parametric sweep, while the inflow velocity, U0, is kept static. Further on, the inflow velocity, U0, is then studied through a parametric sweep, while the magnetic field, B0, is kept static. The same study is then conducted again for the positive vector potential configuration.

For the negative vector potential configuration the results implies that the system works a break. Also, by variation of the magnetiv field the effect on the flow is increased. Furthermore, the variation of inflow velocity has an impact on the velocity field, but also on the pressure field. As for the study on the positive vector potential configuration the results implies that the system works as a pump. Morevoer, for the inflow velocity-sweep effects probaly caused by laminar-to-turbulent flow transistion is visible.

1

2 Introduction

COMSOL Multiphysics® is a general-purpose software platform, based on advanced numerical methods. It is a powerful tool useful to simulate flows; fields; forces and such in models provided either by files or built directly in COMSOL.

This report is a part of a written documentation of the COMSOL-laboratory exercises made in the course Multiphysics, Simulation and Computation at Luleå University of Technology. These exercises serve as practice in formulating mathematical models to describe physical and technical problems in a way that is suitable for implementation of the finite element method.

This work contains the result and analysis of the fifth COMSOL-laboratory exercise where behavior of magnetohydrodynamics (MHD) pumps and brakes are analyzed in a channel flow.

The principle of the MHD propulsion motor involves the electrification of gas or water. This can then be directed by an electric field, driving the vehicle in the opposite direction. Today, there are some existing functional protptypes with magnetohydrodynamic drive. However, due to the amount of energy needed to operate magnetohydrodynamic drive and the slow speed obtained this means of drive is considered impractical. As for the use of MHD drive for a boat motor the greatest limitation would be the condictivity of the fluid in which the boat will be traveling in.

Seawater consists of about 97% of water. The other 3% are, however, dissolved salts sodium chloride, magnesium chloride, sodium sulphate and other less common salts. In a MHD driven engine the magnesium and sodium ions are the charged particles used to push the vessel forward. This means that the speed of the vessel will be dependent on the salinity of the seawater. A consequence of this is that this system is not functional in fresh water, because fresh water almost consist of little to no dissolved salts[1].

As magnetohydrodynamics is a theory to analyze conducting liquid metals and plamsas, or the study of magnetic properties of electrically conducting fluids; the set of equations that descibe magnetohydrodynamics are the Navier-Stokes equations in combination with Maxwells equations of electromagnetism. Moreover, to solve this problem the differential equations is required to be solved simultaneously.

From this laboratory exercise there are a few questions to be answered with the help of the results obtained from the simulations

(i) Velocity changes along the channel
(ii) The variations of the magnetic field
(iii) The pressure variation along the centerline of the channel.
(iv) The pressure distribution with the case of small magnetic field and large magnetic field
(v) The effects of change in polarity
(vi) For wich case the channel works as a pump or as a break
(vii) Consider MHD as a propulsion for a motor boat

3 Method

3.1 General

The exact method to calculate the coefficients and simulate the system is detailed and well explained in the instructions[2], but the general way to go about it is the same as most COMSOL projects.

- Choose system type (fluids, laminar, $2D$).
- Introduce global parameters.

- Build geometry.
- Set study specifications for your system type:
  - Set fluid parameters.
  - Set Boundary conditions.
  - Set initial conditions.
- Build a mesh grid of your geometry.
- Compute system.

Following the instructions provided in the instructions there is not much that needs to be added. The parameters are listed in Table 1 The system is illustrated in Figure 1. Some parameters $B0$ and $U0$, magnetic field strength respective inflow velocity will be sweeped to look at their influence on the system. They will be sweeped in the intervals $B0 = [.01, .01, .17]$ [T] and $U0 = [.01, .01, .09]$ [m/s]. Another relevant parameter is $V0$, which represents the potential difference between the top center region (CR) and the bottom. The potential is set to $V0 = -1.5$ V for the first simulation and $V0 = 1.5$ V

Name	Symbol	Value	Unit
rho	ρ	$1E3$	$^{kg}/_{m^3}$
mu	μ	$1E-3$	$^{kg}/_{ms}$
a	a	.1	m
k0	k0	9	$^{1}/_{m}$
sigma	σ	4	$^{S}/_{m}$
B0	B	.05	T
U0	U	.02	$^{m}/_{s}$
V0	V	∓ 1.5	V

Table 1: Simulation Parameters

There are effectively four different studies performed in this laboration. $V0$ negative or positive, $B0$ static or sweep, $U0$ static or sweep. These will be talked about in section 3.3.

3.2 Evaluation formulas

Lorentz Force Law:

$$\bar{F} = Q(\bar{u} \times \bar{B}) \tag{1}$$

where Q is particle charge, $\bar{u}$ is PArticle velovity and $\bar{B}$ the magnetic field.

Navier Stokes Equation:

$$\rho\left(\frac{\delta \bar{u}}{\delta t} + (\bar{u}\cdot\nabla)\right) = -\nabla p + \mu\nabla^2\bar{u} + \bar{F} \tag{2}$$

Maxwells Equations:

$$\begin{cases} \nabla \times \bar{E} & = -\frac{\delta \bar{B}}{\delta t} \\ \nabla \cdot \bar{B} & = 0 \\ \nabla \times \bar{B} & = \mu_0 \bar{j} \\ \bar{j} & = \sigma(\bar{E} + \bar{u} \times \bar{B}) \end{cases} \tag{3}$$

The current 'j' is equal to the charge times velocity, $\bar{j} = Q\bar{u}$, which together with Equation 1 gives:

$$\bar{F} = \bar{j} \times \bar{B} \tag{4}$$

Looking at the figure describing the system, Figure 1, the following can be determined:

- E-field is in the y-direction
- B-field is in the z-direction
- Constant magnetic field *inside* the channel with strength B_0. This is the B-field of the channel-region.

Magnetic field outside channel

$$\begin{cases} B_L = B_0 * e^{k(x+a)} B_R = B_0 * e^{k(a-x)} \end{cases} \tag{5}$$

Expanding Equation 4 gives force composants:

$$\begin{cases} F_x = Bj_y \\ Fy_= - Bj_x \end{cases} \tag{6}$$

as

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ j_x & j_y & j_z \\ 0 & 0 & B \end{bmatrix} = \hat{i}j_xB - \hat{j}j_yB + 0$$

where $\bar{j}$-composants can be found by expanding Equation 3_4:

$$\bar{j} = \sigma(\bar{E} + \bar{u} \times \bar{B}) = \sigma(-\nabla V + \bar{u} \times \bar{B}) =$$

$$= \left\{ \begin{aligned} \bar{u} \times \bar{B} &= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_x & u_y & u_z \\ 0 & 0 & B \end{bmatrix} = \\ &= \hat{i} u_y B - \hat{j} u_x B \\ \nabla V &= \tfrac{\delta V}{\delta x}\hat{i} + \tfrac{\delta V}{\delta y}\hat{j} + \tfrac{\delta V}{\delta z}\hat{k} \end{aligned} \right\} =$$

$$= \sigma\left[\hat{i}\left(u_y B - \frac{\delta V}{\delta x}\right) - \hat{j}\left(u_x B + \frac{\delta V}{\delta y}\right)\right] \Rightarrow$$

$$\Rightarrow \begin{cases} j_x = \sigma\left(u_y B - \frac{\delta V}{\delta x}\right) \\ j_y = -\sigma\left(u_x B - \frac{\delta V}{\delta y}\right) \end{cases} \tag{7}$$

equation 7 will then be inserted in to equation 6.

$$\begin{cases} F_x &= -\sigma\left(B\frac{\delta V}{\delta y} + B^2 u_x\right) = \\ &= \sigma\left(-B\frac{\delta V}{\delta y} - B^2 u_x\right) \\ F_Y &= \sigma\left(B\frac{\delta V}{\delta y} - B^2 u_x\right) \end{cases} \tag{8}$$

To then get force components for each region, insert B-field of that region into equation 8, like the following equation 9.

$$\begin{cases} F_x &= \sigma(-B_i * V_y - B_i^2 * u) \\ F_y &= \sigma(B_i * V_y - B_i^2 * v) \end{cases} \tag{9}$$

where $i = [L, 0, R]$, indicating the respective regions Left, Center and Right.

Current sources for steady state, $\nabla \cdot \bar{j} = 0$ and ohms law gives the following:

$$\nabla \cdot (\sigma\bar{E}) = -\nabla \cdot (\sigma\bar{u} \times \bar{B}) = Q_j \tag{10}$$

where Q_j is the current source.

Expanding this current source gets us the following equation11.

$$\begin{aligned} Q_j &= -\nabla \cdot (\sigma\hat{i}u_y B - \sigma\hat{j}u_x B) = \\ &= -\sigma\left(\frac{\delta}{\delta x}(u_y B) - \frac{\delta}{\delta y}(u_x B)\right) = \\ &= -\sigma\left[\frac{\delta u_y}{\delta x}B + \frac{\delta B}{\delta x}u_y - \frac{\delta u_x}{\delta y}B - \frac{\delta B}{\delta y}\right] = \\ &= -\sigma\left[B\left(\frac{\delta u_y}{\delta x} - \frac{\delta u_x}{\delta y}\right) + \frac{\delta B}{\delta x}u_y - \frac{\delta B}{\delta y}u_x\right] \end{aligned} \tag{11}$$

A magnetic field constant in y-direction therefore gives the final equation 12.

$$Q_j = -\sigma B\left(\frac{\delta u_y}{\delta x} - \frac{\delta u_x}{\delta y}\right) - \sigma u_y\frac{\delta B}{\delta x} \tag{12}$$

Inserting the three different regions to this equation results in the electric current source equations 13, 14, 15.

$$Q_{j,L} = -B_L(v_x - u_y)\sigma - \sigma v B_{x,L} \tag{13}$$

$$Q_{j,0} = -B_0(v_x - u_y)\sigma \tag{14}$$

$$Q_{j,R} = -B_R(v_x - u_y)\sigma - \sigma v B_{x,R} \tag{15}$$

where $L, 0, R$ indicates the respective regions Left, Center and Right.

3.3 Studies

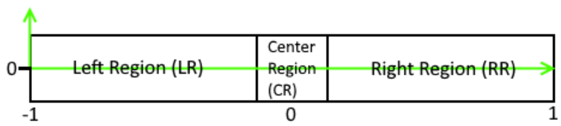


Figure 1: The simulated system. Green line is the line intersection.

As speed changes in the system can be unclear from $2D$ field plots a better suited type of data presentation had to be found. To accurately read the velocity profile a $1D$ two point intersection data set of the area was created between the points $[-1, 0]$to$[1, 0]$, see Figure 1. This intersection is used for all pressure and velocity comparisons.

Result groups used were the following:

- Surface $2D$ Pressure: for the main representation of the simulations.
  - Surface: Pressure
  - Arrow Surface: Velocity field
  - Streamline: Electric field
- Line $1D$: for the pressure and velocity profile comparisons along the channel. (**Note:** these will be toggled to have only one active at any given time.)
  - Line Graph: Pressure
  - Line Graph: Velocity
- Surface $2D$ Velocity: for fast Velocity field plots.
  - Surface: Velocity field
  - Arrow Surface: Velocity field

3.3.1 Magnetic field B0 sweep

To find the dependencies on the magnetic field the inflow velocity was kept static and the magnetic field, B0 swept. As mentioned in the introduction, section 2, it is swept between $B0 = .01$ and .17 T with a step size of .01 T.

To illustrate the interaction and changes it has on both the electric field and velocity the sweep is plotted as both a $2D$ field plot and a line graph $1D$ plot. The illustrated fields effect will serve the reader with an easy to understand connection to how the system reacts physically to the change.

For clearer comparison to the pressure and velocity along the channel it is plotted as line graphs. As the change in magnetic field always keeps at least either the start or end point of the plot static between sweep values, all $\bar{u}(B0)$ and $\bar{p}(B0)$ can be plotted together in their respective groups, allowing for clear and easy comparisons.

3.3.2 Inflow velocity U0 sweep

To find the dependencies on the inflow velocity the magnetic field was kept static and the inflow velocity, U0 swept. As mentioned in the introduction, section 2, it is swept between $U0 = .01$ and .09 $^m/_s$ with a step size of .01 T.

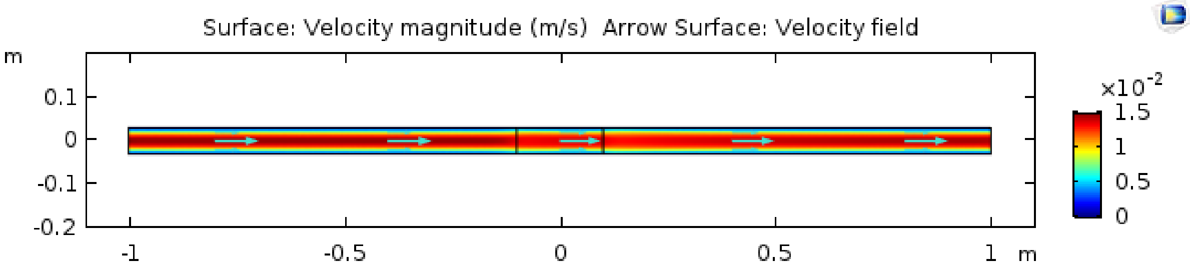
Similarly to the magnetic field sweep the system is plotted both as a $2D$ surface and a $1D$ line graph. The $2D$ field plots are again more important for illustrative purposes rather than purely analytical ones.

Again, similarly to the magnetic field plots, the inflow velocity sweep is plotted as line graphs for easier comparisons. However, when sweeping the inflow velocity effect on the flow speed along the channel, the values will look parallel with different magnitudes. This will obviously skew the results as the important observation is the interaction between the magnetic field and the velocity field. The sweeps, $\bar{u}(U0)$, must therefore be plotted separately to observe the differences in shape rather than magnitude.

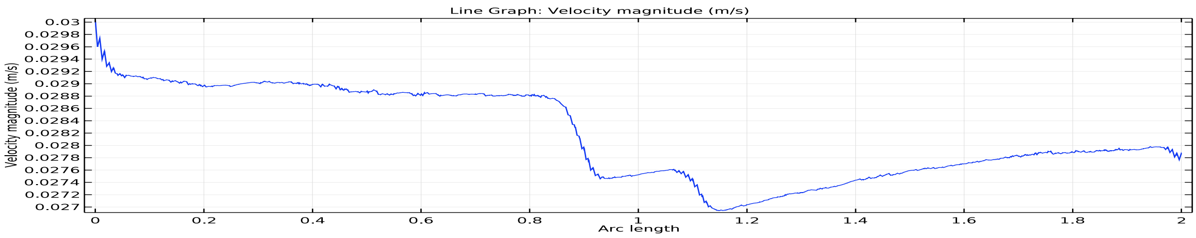
4 Results and Interpretation

4.1 Negative potential, $V0$

All simulations were first run with negative potential, as this was according to the instructions. The static simulation result can be seen in Figure 2. From these nothing interesting can be observed from the surface figure 2a, but looking at the channel in line figure 2b it is obvious that when the water hits the magnetic center region it aggressively slows down, and even if the water logarithmically increases towards the initial velocity it is never reached. This implies the system



(a) $2D$ surface velocity field plot of static system.



(b) $1D$ line graph of velocity along the channel.

Figure 2: Standard settings for static simulation of negative $V0$ V

will act as a break for any vessel using this configuration of potential voltage.

4.1.1 Magnetic field sweep, $B0$

Varying the magnetic field increases the effect on the flow. The collected plots for this subsection can be seen as Figure 4.

As we know from section 4.1 a negative potential will counteract the flow and act as a break; this is visible in sub-figures 4a,4b and 4c where for $B0 = .17$ T the velocity arrows are observably flattened by the breaking effect.

This increasing breaking effect can also be seen in sub-figure4dwhere the velocity dips lower the larger the magnitude set on the magnetic field. It can be observed that the velocity maximum breaking effect, $\zeta(B)\ {}^{m}/_{s}$, looks like the following:

$$\zeta(B, U = .02\ {}^{m}/_{s}) =$$

$$\begin{cases} .01475 - .01425 = .5m, & \text{for } B0 = .01 \text{ T} \\ .01475 - .0105 = 4.25m, & \text{for } B0 = .17 \text{ T} \end{cases}$$

assuming then ζ is approximately linearly dependent on the field strength, $B0$, it would fit expression 16.

$$\zeta(B) = .25m + .25m * B0 \qquad (16)$$

where m implies $*10^{-3}$.

Looking at the sub-figure4ethe pressure is negative until the CR where it increases rapidly to 0. This would imply the increased magnitude of the magnetic field, $B0$, somehow spreads the flow apart until it is bottlenecked while trying to pass the center region. possibly this is offset due to the simulation setup defining the output as $\rho_0 = 0$ Pa therefore pushing the input pressure lower, but this is a rather unlikely consequence of setting the output flow to zero-pressure. The pressure starts at $p_0 \approx -.4$ Pa at $B0 = .01$T and ends up at $p_0 \approx -7.5$ Pa at $B0 = .17$T.

4.1.2 Inflow velocity sweep, $U0$

Varying the inflow velocity has an obvious impact on the velocity field, a somewhat diminishing effects on the breaking field but also some effect on the pressure field. The collected plots for this sweep can be observed as Figure 5.

Observing figures 5a, 5b and 5c not much difference can be seen, except for a slight variation in the magnetic field to the right. Figures 5d and 5e are zoomed in on the right region and some curious effects on the electric field can be seen. This might be due to vortices appearing in the water as the center region acts as an obstacle in the flow; and the vortices then somehow affecting the electric field enough for it to register. This Effect can not be observed in any of the line graphs, so vortices seems like a stretch.

Since the velocity graphs can not be plotted in the same window, as their difference in bias magnitude will fade away any shapes in their respective graphs, they are plotted side to side for comparison, see figures 5f, 5g and 5h. It can be seen that the breaking effect, ζ, definitely as expected is not a proportional break, i.e. scales by a factor of the inflow, but rather most likely does not at all depend on the velocity field. A larger magnitude inflow straightens out and becomes almost linearly decreasing.

Figures 5i, 5j and 5k illustrates the inflow velocity sweep effect on the pressure. The pressure seems almost independent on the inflow velocity, except for the various sized overshoot. The line for $U0 = .07$, .08 and .09 are crooked and more unstable; black, purple and green respectively. This is most likely due to the system entering the laminar-turbulent transition region, which would also explain why the system does not converge for $U0 > .09\ ^m/_s$.

4.2 Positive potential, $V0$

After all simulations were run with negative potential in section 4.1, they are then run with positive potential. The static simulation result can be seen in Figure 3. Looking at the channel in line figure 3b the previous statement about the negative potential acting as a break is further solidified as when the water hits the magnetic center region in this simulation the flow aggressively speeds up, acting as an electric pump for any vessel using this configuration of potential voltage.

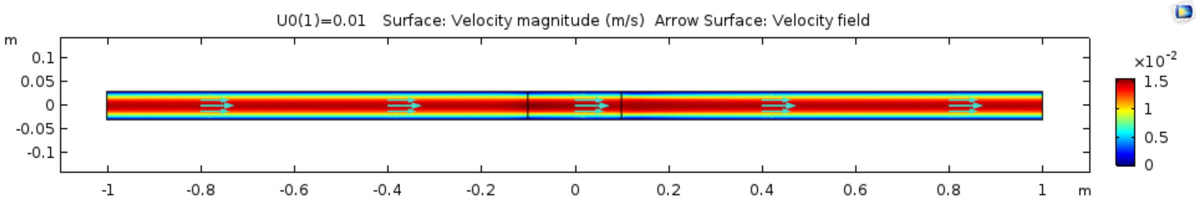
4.2.1 Magnetic field sweep, $B0$

As with section 4.1.1, varying the magnetic field increases the effect on the flow. The collected plots for this subsection can be seen as Figure 6.

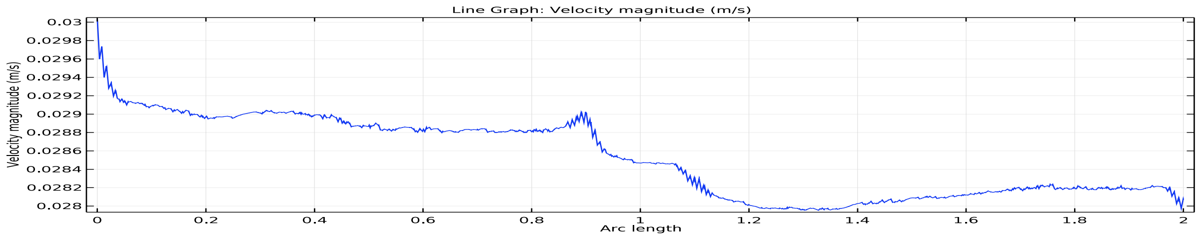
As the positive potential should make the system visibly act like a pump. This time however this is not as visible in the sub-figures, figures 6a,6b and 6c.

The pump effect can however be clearly observed in sub-figure 6d, where the velocity increases more the larger the magnitude set on the magnetic field. The differences of the peaks are lower in the pump than in the break (figure 4d) though, and in the cases of lowest magnetic field, $B0 = .01$ and .02 T, it even acts as a break. Most likely this is due to the center region acting as a reversed bottleneck, as observed in the pressure plots.

Looking at the sub-figure 6e the pressure looks like the pressure plot with negative $V0 = -1.5$ V (figure 4e) but perfectly mirrored around the x-axis. It is positive until the center region where it decreases rapidly to 0. Interpreting this is a little difficult to do; the larger the magnetic field the higher the pressure relative to the free flow after the center region. The magnitude of the magnetic field, $B0$, somehow concentrates the flow until it is bottlenecked while trying to pass the center region. The pressure starts at $p_0 \approx .5$ Pa at $B0 = .01$T and ends up at $p_0 \approx 7.6$ Pa at $B0 = .17$T.



(a) $2D$ surface velocity field plot of static system.



(b) $1D$ line graph of velocity along the channel.

Figure 3: Standard settings for static simulation of positive $V0 = 1.5\ V$

4.2.2 Inflow velocity sweep, $U0$

The collected plots for this sweep can be observed as Figure 5.

Figure 7d shows the entire swiped region for $U0 = .06\ ^m/_s$; with the inflow velocity this time acting in accordance to the magnetic field direction. It can be seen that the electric field in the left region is pushed together while the electric field in the right region gets pulled out. This pushing together effect can be seen in both the left region figures, figures 7a and 7e. Some curious effect, similar to the effect observed in the negative potential case, can again be observed in the right region plots, figures 7c and 7g.

The exact same effect can be observed for the velocity line plots as the negative potential case. Again the effect of the magnetic field can be seen to diminish with larger velocities, see figures 7h, 7i and 7j.

Figures 7k, 7l and 7m illustrates the inflow velocity sweep effect on the pressure. Again the pressure seems almost independent on the inflow velocity, except for the various sized overshoot. The unstable line graph for $U0 = .09$ is very clear in figures 7k and 7m, further supporting the hypothesis of the system system entering the laminar-turbulent transition region at $U0 \approx .09\ ^m/_s$.

References

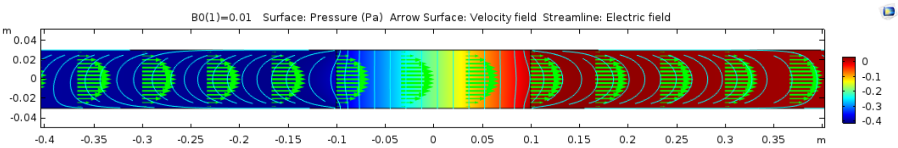
[1] Luimes, Maat, Smits, Vervoort. Report magneto hydrodynamics. Technical report, Maritime University of Applied Sciences, January 2012. `http://www.maritimesymposium-rotterdam.nl/uploads/Route/Magneto%20Hydrodynamics.pdf`.

[2] Multiphysics F7024T Hans Åkerstedt. Assignment #5, magnetohydrodynamic pump (motor) and brake. Technical report, Department of Engineering Science and Mathematics, May 2017.

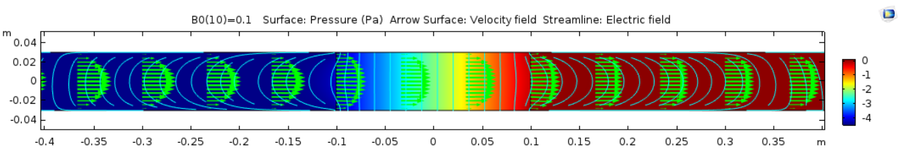
A APPENDIX

A.1 Figures with negative $V0 = -1.5$ V

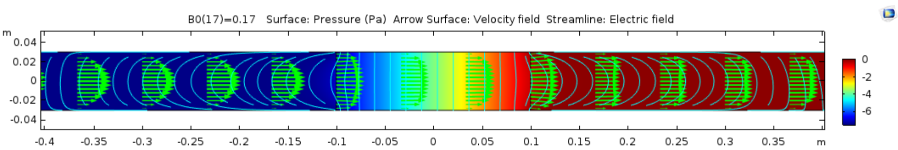
A.1.1 Simulated figures for $B0$ sweep



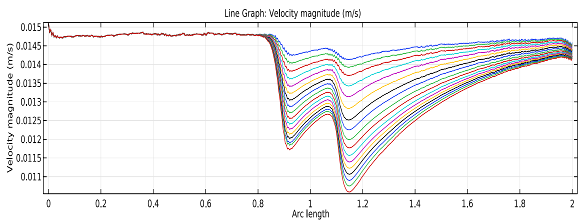
(a) Center Region field plot for: $B0 = .01$ T



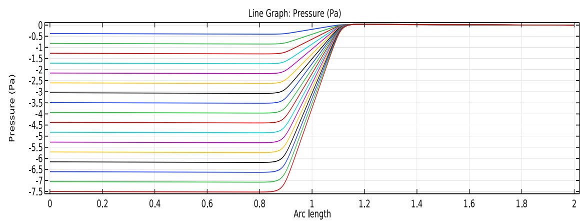
(b) Center Region field plot for: $B0 = .10$ T



(c) Center Region field plot for: $B0 = .17$ T



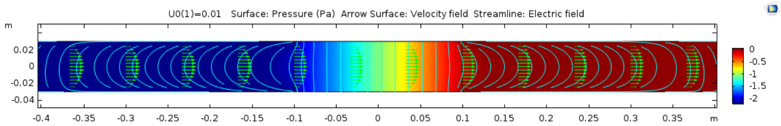
(d) Velocity line graph for entire $B0$ sweep



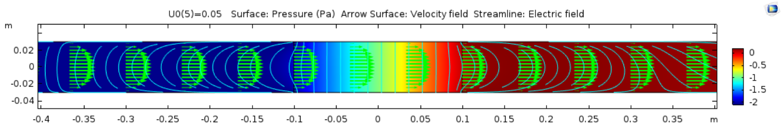
(e) Pressure line graph for entire $B0$ sweep

Figure 4: Simulated figures for magnetic field, $B0$, sweep when center region potential is negative.

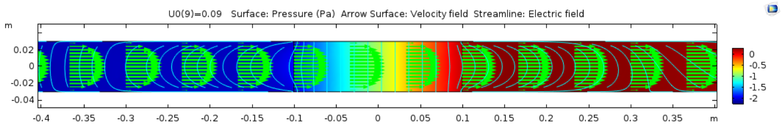
A.1.2 Simulated figures for $U0$ sweep with $V0 = -1.5$ V



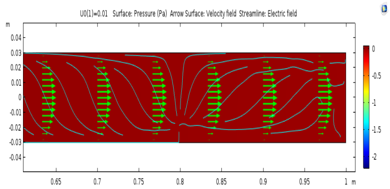
(a) Center Region field plot for: $U0 = .01\ ^m/_s$



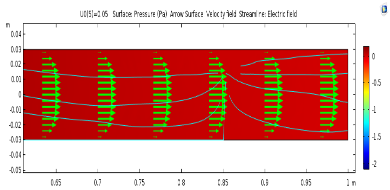
(b) Center Region field plot for: $U0 = .05\ ^m/_s$



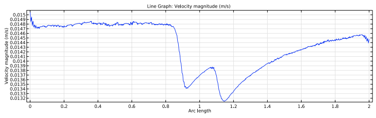
(c) Center Region field plot for: $U0 = .09\ ^m/_s$



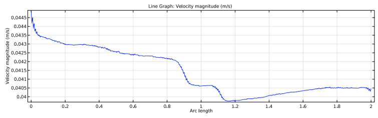
(d) Right Region field plot for: $U0 = .01\ ^m/_s$



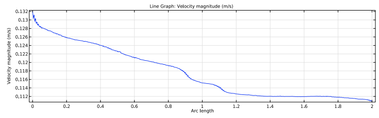
(e) Right Region field plot for: $U0 = .05\ ^m/_s$



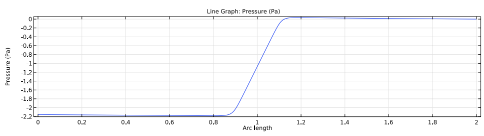
(f) Velocity line graph for: $U0 = .01\ ^m/_s$



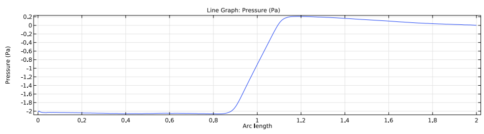
(g) Velocity line graph for: $U0 = .03\ ^m/_s$



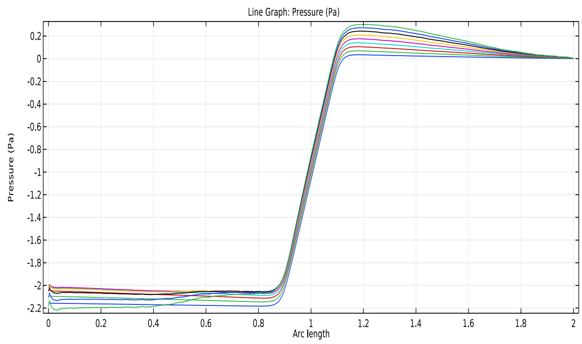
(h) Velocity line graph for: $U0 = .09\ ^m/_s$



(i) Pressure line graph for: $U0 = .01\ ^m/_s$



(j) Pressure line graph for: $U0 = .09\ ^m/_s$

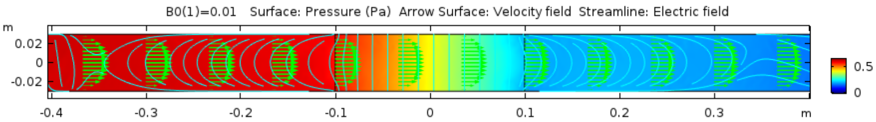


(k) Pressure line graph for entire $U0$ sweep

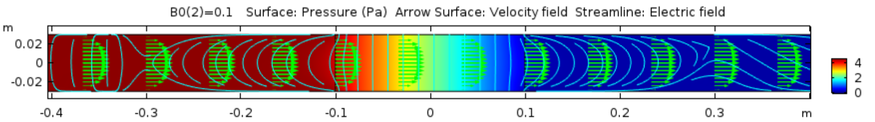
Figure 5: Simulated figures for inflow velocity, $U0$, sweep when center region potential is negative.

A.2 Figures with positive $V0 = 1.5$ V

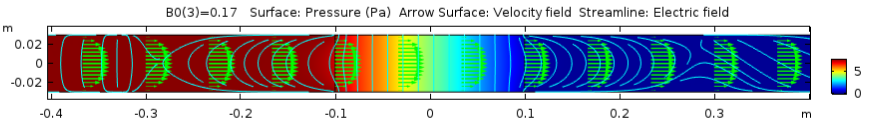
A.2.1 Simulated figures for $B0$ sweep with $V0 = 1.5$ V



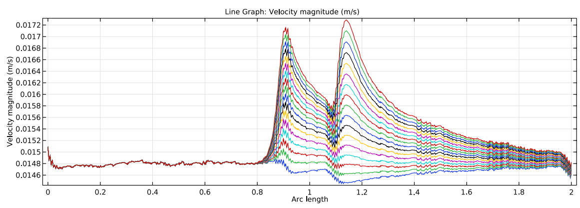
(a) Center Region field plot for: $B0 = .01$ T



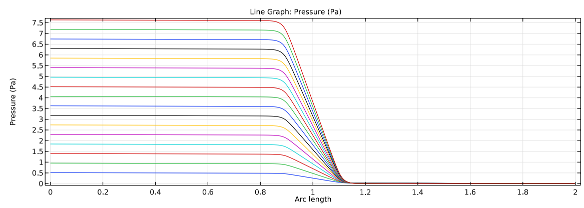
(b) Center Region field plot for: $B0 = .10$ T



(c) Center Region field plot for: $B0 = .17$ T



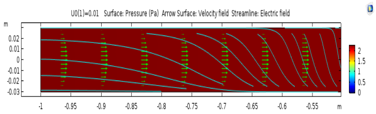
(d) Velocity line graph for entire $B0$ sweep



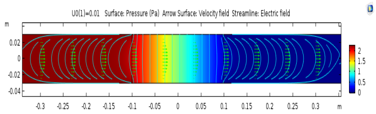
(e) Pressure line graph for entire $B0$ sweep

Figure 6: Simulated figures for magnetic field, $B0$, sweep when center region potential is positive.

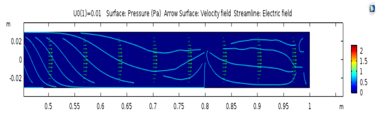
A.2.2 Simulated figures for $U0$ sweep with $V0 = -1.5$ V



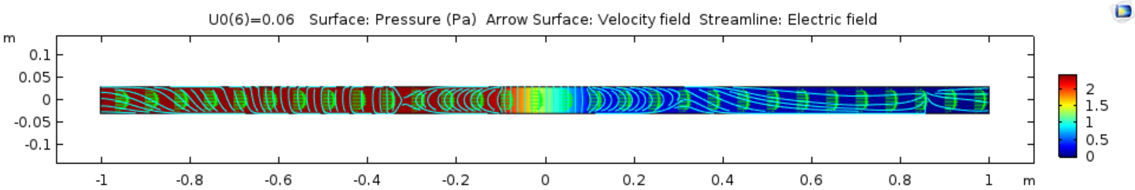
(a) Left Region field plot for: $U0 = .01$ $^m/_s$



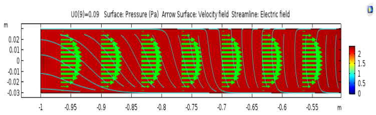
(b) Center Region field plot for: $U0 = .01$ $^m/_s$



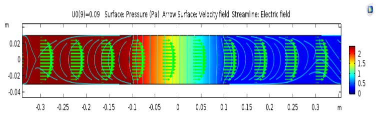
(c) Right Region field plot for: $U0 = .01$ $^m/_s$



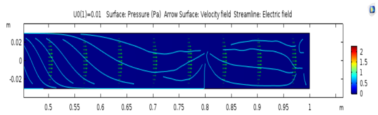
(d) Entire field plot for: $U0 = .06$ $^m/_s$



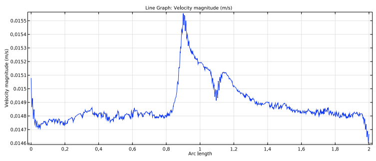
(e) Left Region field plot for: $U0 = .09$ $^m/_s$



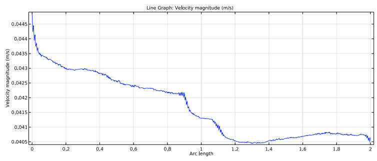
(f) Center Region field plot for: $U0 = .09$ $^m/_s$



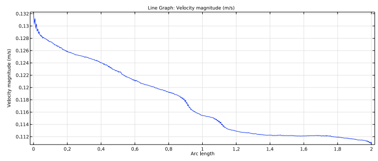
(g) Right Region field plot for: $U0 = .09$ $^m/_s$



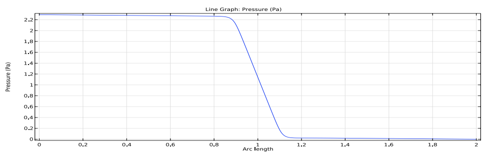
(h) Velocity line graph for: $U0 = .01$ $^m/_s$



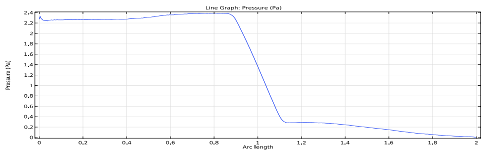
(i) Velocity line graph for: $U0 = .03$ $^m/_s$



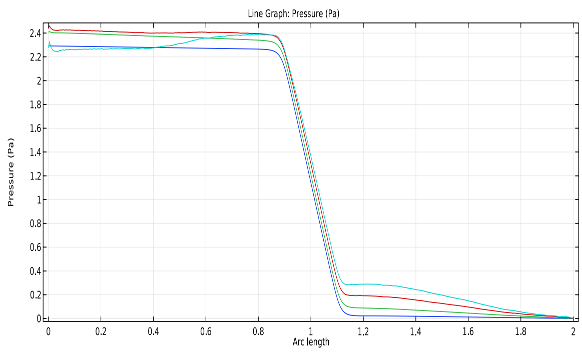
(j) Velocity line graph for: $U0 = .09$ $^m/_s$



(k) Pressure line graph for: $U0 = .01$ $^m/_s$



(l) Pressure line graph for: $U0 = .09$ $^m/_s$



(m) Pressure line graph for $U0$ sweep. Blue: $U0 = .01$; green: $U0 = .03$; red: $U0 = .06$, cyan: $U0 = .09$

Figure 7: Simulated figures for inflow velocity, $U0$, sweep when center region potential is positive.