Luleå 2017-04-05

ASSIGNMENT #1 2017
COMPUTATION OF CAPACITY USING COMSOL MULTIPHYSICS

ASSIGNMENT
The assignment is to calculate the capacity of a 2D plate capacitor using Comsol and compare the result with the approximate theoretical value given by

$$C_{approx} = \frac{\varepsilon_0 S}{d}.$$

Here *S* is the ”area” of the plates and *d* the distance between the plates.
First the electrostatic potential problem of solving the Laplace equation together with suitable boundary conditions is considered. The geometry of the problem is shown below. Two cases should be studied case 1 for 2D geometry and case2 for 3D geometry.

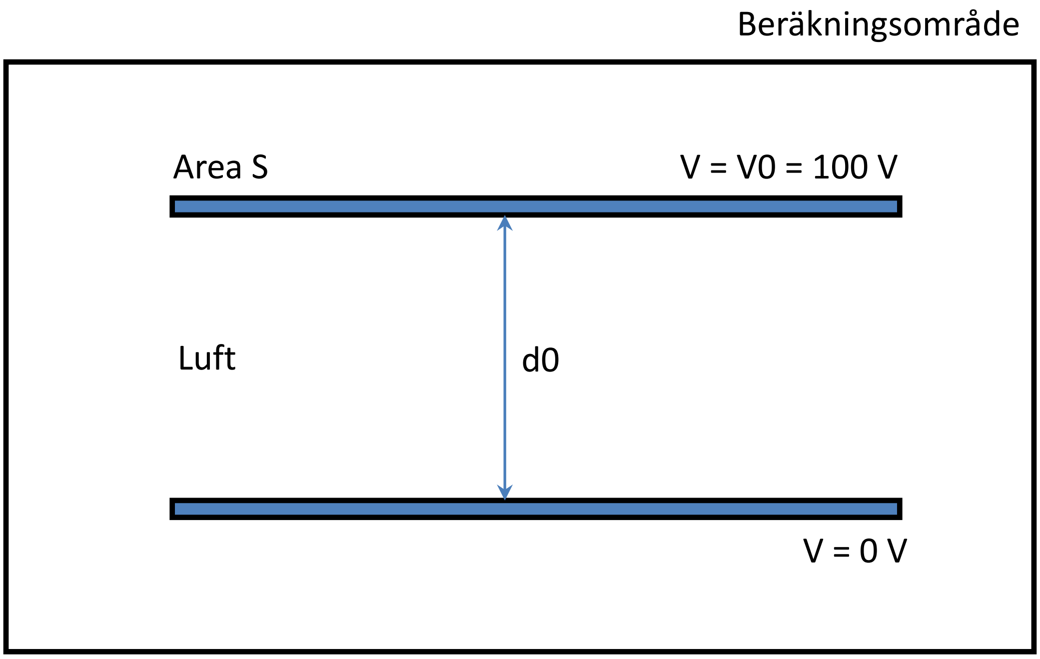

Figure 1: The geometry of the plate capacitor with its surroundings.

INSTRUCTIONS case1

1. Open COMSOL Multiphysics Model wizard and select the dimension of the problem (Here 2D). Under” AC/DC”, choose ”Electrostatics” – and press add. Click on the study button and select ”Stationary” under ”Preset studies”. Click on the done button.

2. Introduce a parameter list. Under *Model Builder* there is *Global Definitions*. Right click and choose *Parameters.* In the middle there is now a window with *Settings*. Under *Parameters* introduce the parameters defined i figure 1. The distance between the conductors may be chosen initially to d0 equal to 0.1 m. Write it as 0.1[m], where the physical dimension is in between the brackets. The “area” of the plates S is written with the dimension [m], since in 2D the default value of the depth is 1 m. Choose S as 0.5 m. Introduce the value of the potential V0 for the upper conductor. For instance V0=1[V].

3. Next step is to create the geometry. Under Component*1 (comp1)* there is *Geometry1*. Click on it. Choose a rectangle *Rectangle1(r1)* with length 1 and height 0.8, with centre at (0,0). This is now the domain of the calculation. Click Build selected. Next step is to construct the upper conductor. Choose a rectangle of length S and height 0.01. The corner should be at (-0.25,d0/2). Click Build selected.

   The lower conductor is built by mirroring the upper conductor in the plane y = 0. Right click *Geometry 1* and choose *Mirror* under *Transforms. Choose the upper conductor*(*r2*) as *Input objects.* Choose also ***Keep input objects.*** The normal of mirror line should be in the y-direction – so x:0, y:1. Click on *Build selectedl*. Choose **Form union** and click *Build all* .Now the geometry is finished.

4. Next step is to define the variables to be used, the different definitions of capacity. To do this we have to introduce an integration operator which is needed to calculate the total charge of the upper conductor. Right click *Definitions* under *Component1*. Under *Model Couplings* there is *Integration*. Choose this and click on *Integration1 (intop1).* Under *Geometric entity level,* choose the integration domain type *Boundary*. Choose the boundaries of the upper conductor and add them by selection and right clicking. The integration operator *intop1 is* now defined.

5. We continue with defining the three different expressions for how to calculate the capacity. Under the model tree (*Model Builder)* there is *Definitions* under *Model 1 (mod 1).* Right click *Variables 1.* Specify the three different expressions(see formulas below) for the capacity. ε_0 is in COMSOL denoted *epsilon0_const*. The total charge of the upper conductor q is found by integrating the surface charge over the total boundary of the conductor. In COMSOL this value is *intop1(es.nD)*. The capacity can also be written in terms of total electrostatic energy which in COMSOL is denoted *es.intWe.*

   You should by now have defined three different variables for the capacity. First the approximate theoretical value and secondly the capacity defined by the energy and finally the capacity defined by the total charge of the conductors.

$$C_{approx} = \frac{\varepsilon_0 S}{d} \quad C_1 = \frac{2 \cdot W_E}{V_0^2} \quad C_2 = \frac{q}{V_0}$$

6. To proceed we need to introduce material parameters of the different domains of the problem. We have air in the region between the conductors and for the conductors we can choose copper.
   Right click *Materials* and choose *Add Material*. Find for air (air, $\varepsilon_r = 1$). Select air for the domain between the conductors and select copper for the conductors.

7. Finally we have to consider boundary conditions and conditions of the domain. Under *Electrostatics (es)* consider *Charge Conservation* and select the complete domain. The boundary condition of the domain of computation should be chosen as *Zero Charge*. Zero charge means that no field lines can cross this boundary.
   To add the boundary conditions of the conductors right click *Electrostatics (es)* and consider electric potential. Choose the value of the potential on the conductors.

8. Next step is to choose the mesh. Choose default mesh, Physics Controlled Mesh Normal. Click on Build All to view the mesh.

9. Now we are finished with the formulation of the problem. To solve the problem right click on *Study1* and choose compute.

10. To view the solution, go under *Results* and *Electric Potential* where you find Surface plot. To find electric field lines right click on *Electric Potential* and choose Streamlines. Under Data set choose Solution1. Under streamline positioning choose uniform density. Click on the plot button.

11. To find the capacity go to *Derived values* Right-click on it and choose Global evaluation. Choose Solution 1 and enter expression for Ctheo, press evaluate. Next enter C1 and evaluate and finally C2 and evaluate.

12. Next we consider a comparison of the three different expressions for the capacity for different *d0*. One can do this calculation as above for each chosen value of *d0.* In Comsol it is however possible to directly consider a parameter sweep of for instance *d0.* Under *Study1* there is *Parametric Sweep*. Add the parameter name *d0.*
    You can then specify the values of the sweep parameter by using the command *range(startvalue,step,endvalue)*.

13. You now have a new study *Study1.* Right click and choose *Compute*. COMSOL now calculates the solution for all parameter values. To get a plot of the different capacities as a function of *d0* go to global evaluation enter the expressions for C_{approx}, C_1 and C_2.
    When compared with the more exact expressions C_1 and C_2 the error in using C_{approx} are

$$error_1 = \frac{\left|C_{approx} - C_1\right|}{C_{approx} + C_1}$$

$$error_2 = \frac{\left|C_{approx} - C_2\right|}{C_{approx} + C_2}$$

INSTRUCTIONS case2

To introduce you to 3D geometry solve the same problem but in 3D. Therefore start with a new model but select 3D from the start. Then replace the rectangular regions in case1 with cylindrical objects. Limit your calculations to the case of defining the capacitance as

$$C_2 = \frac{Q}{V}$$

and compare with the approximate theoretical result

$$C_{approx} = \frac{\varepsilon_0 S}{d_0}$$

Make a plot of the error2 as a function of d_0

$$error_2 = \frac{\left|C_{approx} - C_2\right|}{C_{approx} + C_2}$$

Write a report about your results. The report about your results should include

- A physical problem formulation together with a mathematical formulation with the equations used together with the boundary conditions.
- Some words about how the mesh size modifies the result
- Plots showing the results together with comments