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Luleå Tekniska Universitet
F7024T Multifysik, simulering och beräkning

Assignment 1: Computation of capacity using Comsol Multiphysics

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With supervisor
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Abstract

This work contains the result and analysis of the first COMSOL-laboratory exercise consisting of the electrostatic problem in which the edge effects on the capacitance of a plate capacitor is to be analyzed in two- and three dimensions.
The capacitance of the plate capacitor is calculated from a theoretical approximation, the energy and the total charge of the system. Where the two latter methods are implemented in COMSOL Multiphysics®. The resulting capacity of from the approximate solution is finally evaluated by introducing an error-equation and the obtained error is analyzed for varying distance between the conducting plates of the capacitor.
The results indicates that a slight increase in distance between the conducting plates results in significant decline in the accuracy of the approximate solution. This is due to the dismissals and assumptions of the approximation, from which, the error becomes more prominent in the result for greater distances between the conducting plates.
For sufficiently small distances the error introduced by the dismissals and assumptions of the approximation are negligible.

1 Introduction

COMSOL Multiphysics® is a general-purpose software platform based on advanced numerical methods. It is a powerful tool useful to simulation of flow, fields, force and such in models provided either by files of built directly in COMSOL. To practice rudimentary calculations and plots, COMSOL is used to solve the laboratory exercises numerically.

This report is a part of a written documentation of the laboratory exercises made in the course Multiphysics, Simulation and Computation at Luleå University of Technology. This exercise serves as practice in formulating mathematical models to describe physical and technical problems in way that is suitable for implementation of the finite element method. This work contains the result and analysis of the first COMSOL-laboratory exercise where the electrostatic problem in which the edge effects on the capacitance of a plate capacitor is to be analyzed in two- and three dimensions.

In its simplest form, a capacitor is a passive two-terminal component that stores electrical energy in an electric field when a voltage difference is applied across the two terminals. The capacitors ability to store the electric energy is the capacitance[1] The geometrical representation of the system in consideration for this laboratory exercise is presented in figure 1 and the formulae used to calculate the capacitance are given in equations (1), (2) and (3).

Equation (1) is the theoretical approximation of the capacitance. The approximate method of calculating the capacitance is directly deduced from the Gauss's flux theorem, relating the distribution of electrical charge to the resulting electric field. This theorem, however, only consider the ideal capacitors. Meaning that the overlapping area of the plates is infinitely greater than the distance between the plates, the plates are perfectly parallel and infinitely thin. Giving a perfectly uniform electric field in the capacitor. Also, Gauss's flux theorem does not take the electric field along the edges of the conductive plates into consideration. Where as equation (2) is an energy defined capacitance and for equation (3) the total charge of the plates defines the capacity.

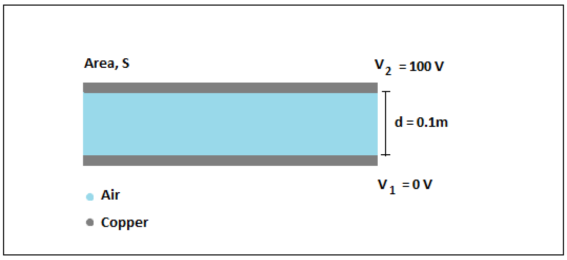

Figure 1: Illustration of the 2-dimensional geometry studied in the first case.

$$C_1 = \frac{\epsilon_0 S}{d} \tag{1}$$

$$C_2 = \frac{2W_e}{V_2^2} \tag{2}$$

$$C_3 = \frac{q}{V_2} \tag{3}$$

The objective of this laboratory exercise is to evaluate the approximate solution to the numerical solutions and determine the error induced by the effects of the assumptions made for the approximation.

2 Method

2.1 General

The exact method to calculate the coefficients and simulate the system is detailed and well explained in the instructions[2], but the general way to go about it is the same as most COMSOL projects.

1. Choose system type (fluids, laminar, $2D$).
2. Introduce global parameters.
3. Build geometry.
4. Set study specifications for your system type:
   - Set fluid parameters.
   - Set Boundary conditions.
   - Set initial conditions.
5. Build a mesh grid of your geometry.
6. Compute system.

2.2 Evaluation

When the capacities from each method is obtained the error in the approximate method is to be evaluated. For this purpose a parametric sweep is made for the distance, $d0$, between the plates. The sweep is made for an interval between $0m$ and $0.07m$. The error in the resulting approximate capacitance is then evaluated by the error-factors obtained from the formulae in equations (4) and (5). For the three-dimensional study the error of the approximate capacitance is only evaluated against the capacitance obtained from the total charge of the plates.

$$E_1 = \frac{|C_1 - C_2|}{C_1 + C_2} \tag{4}$$

$$E_2 = \frac{|C_1 - C_3|}{C_1 + C_3} \tag{5}$$

3 Results and interpretation

3.1 Case 1: 2D

The resulting plot of the initial system with the distance $d0$ between the conductive plates set to $0.1m$ is presented in figure 2. Furthermore, in figure 3 a plot with stream lines is presented of the system. In this simulation, however, the distance $d0$ between the conductive plates is set to $0.2m$.

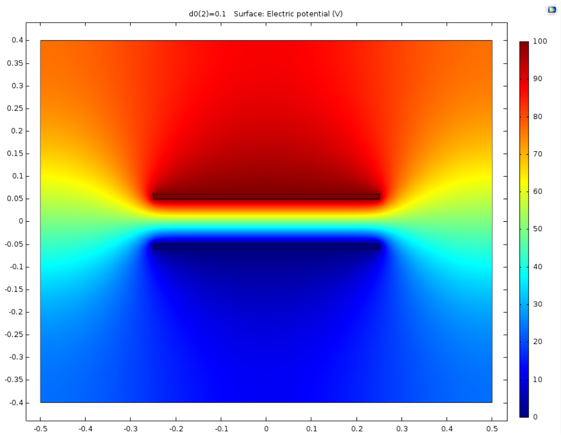

Figure 2: This figure display the resulting potential of the plate capacitor from the two-dimensional simulation in COMSOL Multiphysics®. The distance d0 between the plates is set to $0.1m$.

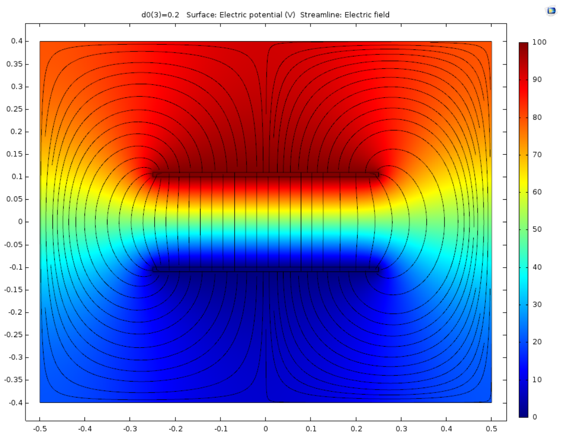

Figure 3: This figure display the resulting potential of the plate capacitor from the two-dimensional streamline simulation in COMSOL Multiphysics®. The distance d0 between the plates is set to $0.2m$.

For each method presented in equations (1), (2) and (3), one plot is presented as the result of the parametric sweep of the distance $d0$. In figure 4, 5 and 6, the obtained capacities, depending on the distance between the plates, are presented for each method. From the obtained capacities the calculated error from equations (4) and (5) are presented in figure 7.

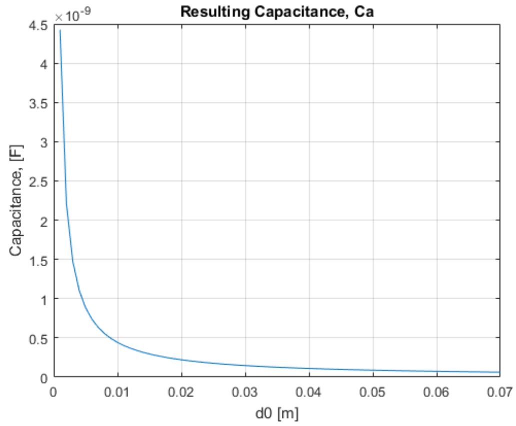

Figure 4: In this figure the capacitance obtained from the approximate method from equation (1) is illustrated. The capacitance is plotted against the distance, d0, between the conductive plates.

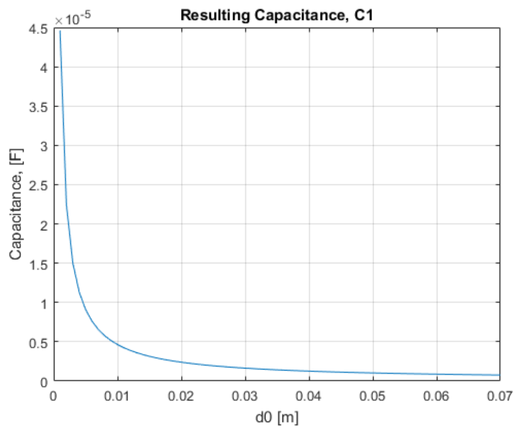

Figure 5: In this figure the capacitance is calculated from the method in equation (2). The capacitance is plotted against the varying distance, d0, between the conductive plates.

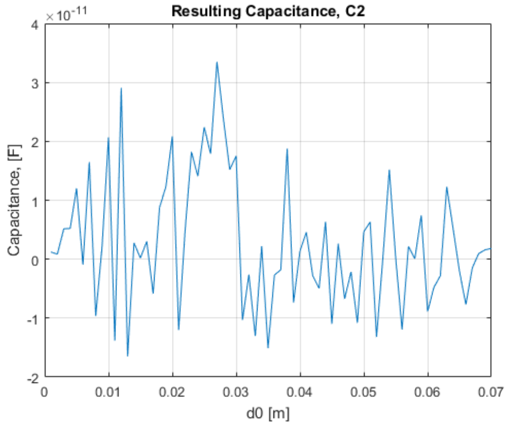

Figure 6: In this figure the capacitance is calculated from the method in equation (2). The capacitance is plotted against the varying distance, d0, between the conductive plates.

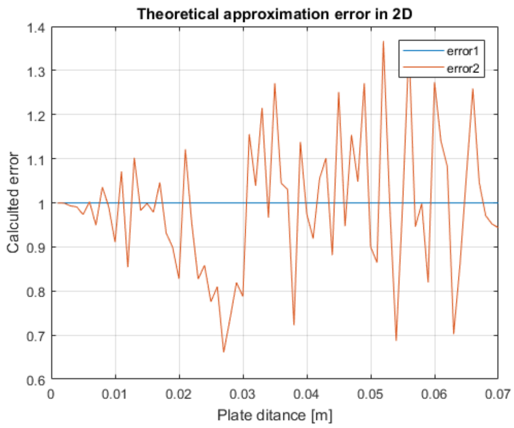

Figure 7: This plot displays the error found from the comparison of the theoretical approximation and the capacities obtained from work and the total charge. The error is plotted against the distance d0 between the conductive plates.

3.2 Case 2: 3D

As for the 3-dimensional part of this laboratory exercise, as mentioned in section 2 in the three-dimensional case, the capacities studied are the ones obtained from the approximate method presented in equation (1) and the method presented in equation (3), where the capacitance is derived from total charge. The resulting capacitance obtained for the approximate case in simulation of three dimensions is presented in figure 8. The resulting capacitance obtained from the total charge is presented in figure 6. As for the error between the two methods for the simulation in three dimensions, the result from equation 5 is plotted in figure 10.

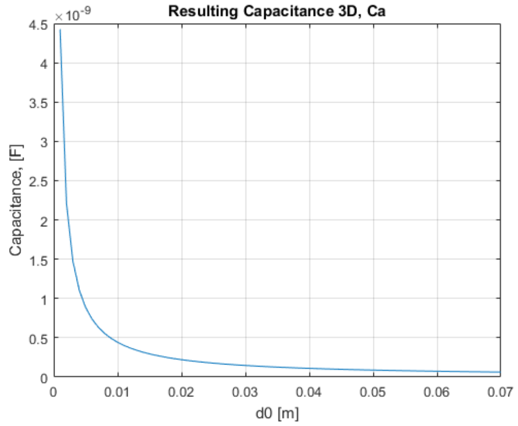

Figure 8: This figure display the three dimensional simulation result for the capacities obtained from equation (1) with a parametric sweep of the distance d0, between the conductive plates.

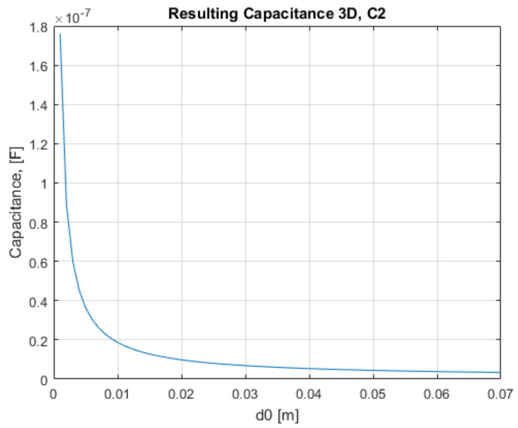

Figure 9: This figure display the three dimensional simulation result for the capacities obtained from equation (3) with a parametric sweep of the distance d0, between the conductive plates.

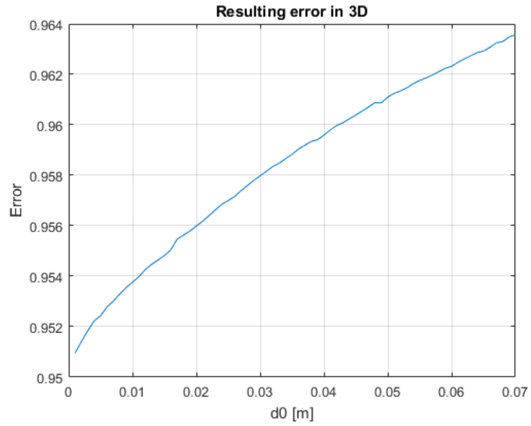

Figure 10: This figure show a plot of the error obtained from the results displayed in figure 8 and 9. The error is plotted against the different distances d0, between the conductive plates.

4 Discussion and Conclusion

4.1 Error sources

In the two cases studied in this laboratory exercise the first source of error is the limited dimensions of the plates. This error is, however, negligible for very thin plates and where the distance between the plates is very small. Furthermore, since the parallel-plate capacitor is simulated and the capacitance is obtained numerically there is some error induced by step-wise approximations in COMSOL Multiphysics®. Another source of error in the approximation is that it assumes that the insulating material perfectly fill the area between the two plates. For the setup in this exercise, however, this error is avoided by the choice of air between the plates. Finally, the assumptions and dismissals mentioned in section 1 for the approximate solution add to the resulting error.

4.2 Possible improvements

In order to improve the results and minimize the error induced by step-wise approximations, finer mesh size can be used in the simulation.

References

[1] Wikipedia. *Capacitor*. https://en.wikipedia.org/wiki/Capacitor, February, 2018.

[2] Multiphysics F7024T Hans Åkerstedt. Assignment #1, computation of capacity using comsol multiphysics. Technical report, Department of Engineering Science and Mathematics, April 2014.