



Luleå 2017-04-10

### ANALYSIS OF A COIL

This assignment is about the influence of end effects on the inductance of a coil. Use Comsol to calculate the magnetic field of a coil of length  $l$  and radius  $a$ , wound with  $n$  current turns per unit length. Find the error of using the approximate inductance for an infinitely long coil  $L = \mu_0 n^2 S l$ .

### Instructions

1. Open Comsol Multiphysics 5.2 and click on model wizard. Choose axisymmetry(2D). Under AC/DC click Magnetic fields and add. Click on Study. Choose Stationary and click on done.  
A dotted symmetry axis with  $r=0$  should appear in the geometry window.
2. Under Global Definitions choose parameters. Enter the radius of the coil  $a=0.2[m]$  and the surface current  $K=nl$  where  $n$  is the number of current turns per unit length and  $l$  the current. Choose values for  $n$  and  $l$ ,  $n=1000$  turns/m [1/m] and  $l=0.1[m]$ .
3. Right-click Geometry and choose Rectangle. Choose the width 2 and height 3. Place the left lower corner at  $r=0, z=-1.5$ . Press the buttons "Build selected". Right-click Rectangle and choose a new rectangle with width  $a$  and height  $l$ . Place the left lower corner at  $r=0, z=-l/2$ . Press "Build selected". Right-click Geometry and choose Boolean operations and compose  $r1+r2$ . Press "Build selected" and "Build all"
4. Under Magnetic fields choose Amperes law. Look at equations to see the equations to be solved. Add the rectangles  $r1$  and  $r2$ . Under relative permeability, choose user defined and put  $\mu_r=1$ . Under conduction current choose user defined and enter conductivity equal to zero. Under Electric field choose user defined and put  $\epsilon_r=1$ .
5. Right-click Magnetic fields and choose surface current. Select element 8 where the surface current flows and enter  $K$  in the phi-component of the surface current. Make sure that Axial symmetry is chosen along the  $r=0$ -axis. Right-click Magnetic fields and choose Magnetic potential. Select the boundaries 2,7,9 and as boundary condition put all components of the phi-component of the vector potential equal to zero.
6. Choose mesh and press "Build all". Later vary the mesh and see how the result is modified.
7. Right-click Study and compute. Right-click Magnetic Flux Density and choose streamline with field components  $mf.Br$  and  $mf.Bz$ . Choose uniform density. Press plot.
8. To calculate the inductance of the coil  $L$ , the total magnetic flux of the coil should be calculated. So that

$$\psi_{total} = 2\pi a A_\phi(a,0)nl = LI. \quad (1)$$

The value of  $A_{\phi}(a,0)$  may be found by plotting  $A_{\phi}(r,0)$ .

To make a plot of  $A_{\phi}(r,0)$  right-click Data sets and choose Cut line 2D.

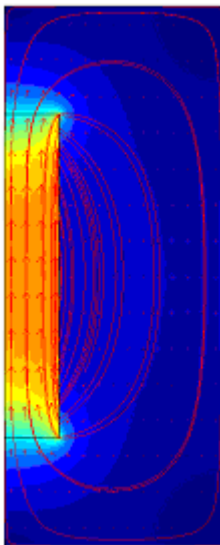
Choose point1 as (0,0) and point2 as (2,0) press plot. The plot-line should then appear in the geometry window. Right-click on Results and choose 1D plot group.

Right-click 1D Plot Group and choose Line Graph. Choose the phi-component of the vector potential. As data set choose Cut Line 2D and a line from  $(r,z)=(0,0)$  to  $(r,z)=(2,0)$ . Plot the curve.

9. To compare the result from the numerical calculation and the analytical expression introduce the dimensionless number

$$\Pi_1 = \frac{L}{\mu_0 n^2 l^3} = \frac{2\pi A_{\phi}(a,0)a}{\mu_0 I n l^2} = f\left(\frac{a}{l}\right)$$

and make a plot of this as a function of  $\alpha = a/l$ . Compare with the result using the approximate analytic expression  $L = \mu_0 n^2 S l$ . To plot the two dimensionless numbers as functions of  $\alpha = a/l$  define a point probe under definitions and choose an "expression" corresponding to the two dimensionless numbers. Solve the problem using a parameter sweep in the parameter  $\alpha$ .



Write a report about your results. The report about your results should include

- A physical problem formulation together with a mathematical formulation with the equations used together with the boundary conditions. Derive equation (1)
- Some words about how the mesh size modifies the result
- Plots showing the results together with comments

