

### Assignment #3a

#### Fluid flow past cylinder. Drag and lift forces

For fluid flow past a cylinder (see fig.) the drag force ( $F_D$ ) on the cylinder is in general very difficult to calculate analytically. With the help of Comsol it is possible to calculate this force at least for sufficiently small Reynolds numbers. To determine the drag force you first need to solve Navier-Stokes equations together with suitable boundary conditions. After the solution is found the drag force is calculated by integrating over the pressure and shear stress over the circular boundary. If the cylinder is rotating one also gets a side force (Magnus effect). An approximate result can be obtained using potential flow theory. An expression for this force is

$$F_L = \rho V_\infty \Omega 2\pi a^2 \quad (1)$$

Here  $V_\infty$  is the velocity far from the cylinder,  $a$  is the radius, and  $\Omega$  the angular velocity of the cylinder and  $\rho$  the density of the fluid. The assignment is to calculate these two forces using a numerical solution of Navier-Stokes equations together with boundary conditions.

To get the most general results it is useful to introduce the corresponding dimensionless forces, the drag-coefficient and lift-coefficient according to

$$C_D = \frac{F_D}{\frac{1}{2} \rho V_\infty^2 A_{proj}} \quad (2)$$
$$C_L = \frac{F_L}{\frac{1}{2} \rho V_\infty^2 A_{proj}}$$

According to dimensional analysis these coefficients may depend upon two dimensionless parameters, the Reynolds number

$$\text{Re} = \frac{V_\infty 2a\rho}{\mu} \quad (3)$$

where  $\mu$  is the dynamic viscosity of the fluid and a dimensionless number

$$Q = \frac{\Omega a}{V_\infty} \quad (4)$$

#### Instructions

1. Determine how the drag-coefficient depends on the Re-number. Start by considering the case with no rotation. Test for the maximum Reynolds to get a converged solution. Make a plot of  $C_D = C_D(\text{Re})$  and compare with wind-tunnel data below.
2. Introduce a rotation of the cylinder and find how the lift-coefficient depends on second dimensionless number  $Q$ . Perform the calculations with  $\text{Re}=100$ . Use  $1 < Q < 40$   
Compare your result with the theoretical result (1) Write a report about your results.

## Comsol-instructions

1. Open Comsol 5.2 and choose Fluid dynamics, laminar flow (2D) steady flow.
2. Introduce parameters for the fluid for instance air with density  $\rho=1.225 \text{ [kg/m}^3\text{]}$  and viscosity  $\mu=1.7\text{e-}5 \text{ [kg/m/s]}$  and cylinder radius  $a=0.01\text{[m]}$ .
3. Under geometry choose a rectangular domain(r1) for the fluid for instance  $[-0.1,0.4]*[-.15,15]$
4. Choose a cylinder with origin at (0,0) and radius a. (c1)
5. Choose compose r1-c1 or use diff under Booleans and Partitions so that the cylinder is cut out from the domain.
6. Under laminar flow choose fluid parameters rho och mu.
7. Introduce boundary conditions at inlet and outlet. Assume that the cylinder is placed within a wind-tunnel with a bottom and upper wall with no-slip boundary conditions. Introduce no-slip boundary conditions on the cylinder wall. Choose inlet velocity sufficiently small and so that  $Re < 1$ .
8. Mesh and compute
9. Integrate to find the forces in the x- and y-directions.



