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Last revision: March 7, 2018

Luleå Tekniska Universitet
F7024T Multifysik, simulering och beräkning

Assignment 2: Analysis of a coil

With supervisor
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Abstract

This work contains the result and analysis of the first COMSOL-laboratory exercise consisting of the magnetostatics problem in which the influence of end effects on the inductance of a long, thin coil is to be analyzed in two dimensions. The inductance of the coil is calculated from a theoretical approximation and calculated numerically in COMSOL Multiphysics®.

The resulting inductance from the approximate solution is finally evaluated with the inductance obtained numerically by introducing a dimensionless number for the inductances. This dimensionless number is then plotted as a function of the ratio between the radius and the length of the coil.

The results shows that a slight increase in the ratio between the radius and the length of the coil results in significant decline in the accuracy of the approximate solution. This is due to the dismissals and assumptions of the approximation, from which, the error becomes more prominent in the result for greater ratios between the radius and the length of the coil. For sufficiently small distances the error introduced by the dismissals and assumptions of the approximation are negligible.

1 Introduction

COMSOL Multiphysics® is a general-purpose software platform based on advanced numerical methods. It is a powerful tool useful to simulation of flow, fields, force and such in models provided either by files of built directly in COMSOL. To practice rudimentary calculations and plots, COMSOL is used to solve the laboratory exercises numerically.

This report is a part of a written documentation of the laboratory exercises made in the course Multiphysics, Simulation and Computation at Luleå University of Technology. This exercise serves as practice in formulating mathematical models to describe physical and technical problems in way that is suitable for implementation of the finite element method. This work contains the result and analysis of the first COMSOL-laboratory exercise where the magnetostatics problem in which the influence of end effects on the inductance of a coil is to be analyzed in two dimensions.

In its simplest form, an inductor consists of an insulated wire wound into a coil around a core and is a passive two-terminal component that stores electrical energy in a magnetic field when an electric current flow through it. The inductors ability to store the electric energy is the inductance [1]. The objective of this laboratory exercise is to evaluate the the approximate method for calculating the inductance for an infinitely long, thin coil (Solenoid) with the numerical solution obtained in COMSOL.

1.1 Approximate solution

A long, thin coil is also called a Solenoid and is defined as a inductor whose length is infinitely greater than its diameter[2]. By assuming a non-magnetic material is used for the coil gives a, close to, constant magnetic flux density, B. The magnetic flux is given by

$$B = \frac{\mu_0 N i}{l} \tag{1}$$

Where μ_0 is the magnetic constant, N is the number of turns, i is the current flowing through the wire and l is the length of the inductor. By the dismissal of edge effects, the total magnetic flux trough the inductor is obtained as the product of the cross-sectional area and the magnetic flux density. This giving

$$\Phi = \frac{\mu_0 N i 2\pi a^2}{l} \tag{2}$$

Finally, the definition of inductance is given by

$$L = \frac{N\Phi}{i} \tag{3}$$

By insertion of equation (2) in equation (3) and solve for the inductance L, the approximate expression for calculating the inductance for an infinitely long and thin coil is obtained. The result is presented in equation (4). Finally, when calculating the approximate inductance the number of turns N, will be given by $N = nl$, where n is the turns per unit length and l is length of the coil.

$$L_{app} = \frac{\mu_0 N^2 2\pi a^2}{l} \tag{4}$$

1.2 Numerical solution

To calculate the inductance numerically, the total magnetic flux, Ψ_{tot} of the coil needs to be derived. The system in consideration is a long, thin coil, as presented in figure 1, consisting of a radius a, with N turns and steady current i running through it.

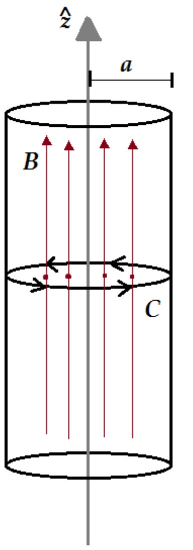

Figure 1: The system in consideration for the derivation of Ψ_{tot}. Where C is the flux- enclosing loop.

The magnetic flux is given by

$$\Psi = \int_C \bar{A}(\bar{r}) \cdot d\bar{l} \tag{5}$$

By rewriting equation (5) we get that

$$\Psi = \int_{S'} (\bar{\nabla} \times \bar{A}(\bar{r})) \bullet d\bar{a} \tag{6}$$

Since $\bar{B} = \bar{\nabla} \times \bar{A}(\bar{r})$, equation (6) can be further rewritten to the following expression

$$\Psi = \int_{S'} \bar{B} \bullet d\bar{a} \tag{7}$$

From Ampere's Circuital Law we know that the field in the coil is uniform and constant. Therefore, giving

$$\bar{B}(r < a) = \mu_0 N i \hat{z} \tag{8}$$

The total magnetic flux, Ψ_{tot}, through the coil is obtained by multiplying the flux density by the cross-section area, $2\pi a^2$.

$$\Psi_{tot} = (\mu_0 N i)(\pi a^2) = \mu_0 n i \pi a^2 \tag{9}$$

However, $\bar{A}(r = a)$ gives that

$$\Psi_{tot} = \int_C \bar{A}(r = a) \bullet d\bar{l} \tag{10}$$

Where $d\bar{l} = a d\varphi \hat{\varphi}$, giving that the

$$\Psi_{tot} = \bar{A}(r = a) 2\pi a \tag{11}$$

The magnetic vector potential for an ideal solenoid (with no pitch angle) must be parallel to $\bar{i} = i\hat{\varphi}$, giving that

$$\bar{A}(\bar{r}) = A_{\varphi}(\bar{r}) \tag{12}$$

The total magnetic flux in the coil can now be calculated by inserting equation 12 in equation 11, giving

$$\Psi tot = 2\pi a A_{\varphi}(a, 0) \tag{13}$$

From the definition of inductance, as presented in equation 3, we get that

$$\Psi_{tot} = \frac{2\pi a A'_{\phi}(a, 0) N}{l} = Li \tag{14}$$

From this the numerical calculation for the inductance is obtained as

$$L_{num} = \frac{2\pi a^2 A_{\varphi}(a, 0) N}{il} \tag{15}$$

As for the approximate solution, N is rewritten to $N = nl$.

Parameter	Name	Value	Unit
Radius	a	0.2	m
Turns/unit length	n	1000	1/m
Length	l	0.1	m
Magnetic constant	μ	$4\pi 10^{-7}$	H/m
Current	i	1.0	A
Surface Current	K	ni	A/m

Table 1: Simulation parameters

2 Method

The exact method to calculate the inductance from the approximate and numerical solutions and simulate the system is detailed and well explained in the instructions[3], the general way to go about this is as most COMSOL-projects.

1. Choose the system type (2D or 3D geometry, AC/DC, stationary conditions)
2. Introduce global variables
3. Build geometry
4. Set study specifications
5. Set study specifications of the system
   - Set conditions of the domain
   - Set boundary conditions
6. Build a mesh grid of your geometry
7. Compute the system.

The parameters defined for the simulation is presented in the following table.

2.1 Evaluation

To evaluate the approximate expression to the numerical calculations, a dimensionless number Π is introduced. Where Π is given by

$$\Pi = \frac{L}{\mu_0 n^2 l^3} \tag{16}$$

By inserting the inductance for the approximate and the numerical solutions, from equations 4 and 15, the following equations are obtained.

$$\Pi_{app} = \frac{2\pi^2 a^2}{l^2} \tag{17}$$

$$\Pi_{num} = \frac{2\pi a A_{\varphi}(a,0)}{\mu_0 n i l^2} \tag{18}$$

Since the accuracy of the approximate solution is limited by the ratio between the coil radius, a, the length of the coil, l, this ratio is assigned another dimensionless number α. As given in the following equation.

$$\alpha = \frac{a}{l} \tag{19}$$

By rewriting equations 17 and 18 so that they are expressed as functions of α the two following equations are obtained. The dimensionless numbers are then Π_{app} and Π_{num} swept for different values of α, raging between 0.001 and 0.07.

$$\Pi_{app} = 2\pi^2 \alpha^2 \tag{20}$$

$$\Pi_{num} = \frac{2\pi A_{\varphi}(a,0)}{\mu_0 n i l}\alpha \tag{21}$$

The change in numerical solution is also evaluated for two different sets for the measurement grid. The inductance is calculated with a coarse and extremely fine mesh-settings.

3 Results

In this section the results obtained from the previous section is presented. In this following figure the resulting inductances represented by the dimensionless numbers Π_{app} and Π_{num} is presented.

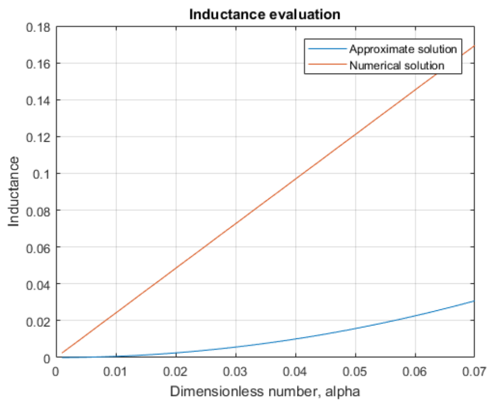

Figure 2: The resulting inductances for the approxiamte and numerical solution represented by the dimensionless numbers Π_{app} and Π_{num}.

In the two following figures the inductance of the coil is shown for the coil, simulated for when $\alpha = 0.07m$. In figure 3 the simulation is made with *coarse* grid of measurement. In figure 4 the simulation is made with *extremely fine* grid of measurement.

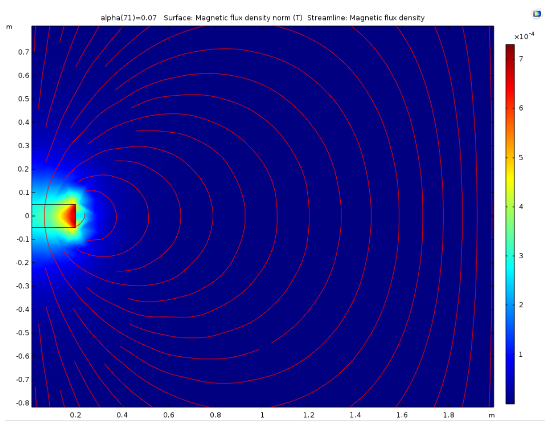

Figure 3: Simulation of the inductance of the coil made with a coarse *grid of measurement*

In this final figure the inductance, represented by the dimensionless number Π_{num}, from the two simulations with *coarse* and *extremely fine* grid of measurement is presented.

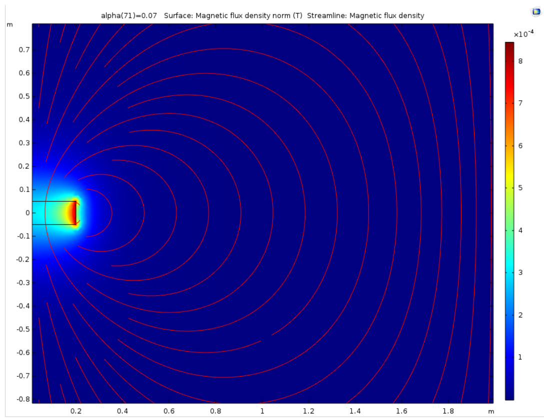

Figure 4: Simulation of the inductance of the coil made with an extremely fine *grid of measurement.*

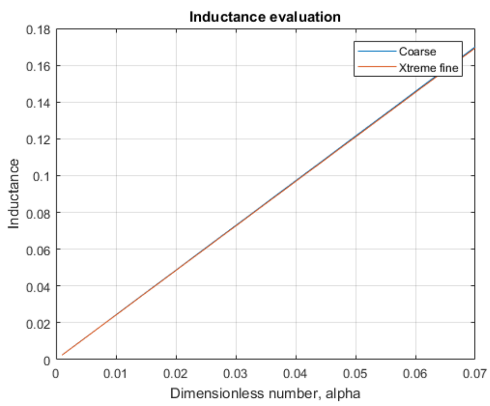

Figure 5: Evaluation of the inductance obtained from simulation with a coarse *and an* extremely fine *grid of measurement.*

4 Discussion and Conclusions

4.1 Error sources

In the two cases studied in this laboratory exercise the first source of error is the limited dimensions of the coil. This error is, however, negligible for very small values of α.

Furthermore, since the solenoid is simulated and the inductance is obtained numerically there is some error induced by step-wise approximations in COMSOL Multiphysics®.

Another source of error in the approximation is that it assumes that the insulating material perfectly fill the area between the two plates. For the setup in this exercise, however, this error is avoided by the choice of air enclosing the coil.

Finally, the assumptions and dismissals mentioned in section 1 for the approximate solution add to the resulting error.

As for the grid of measurement for the calculation of this exercise, the finer grid size does not give any significant improvements for the result. Finer grid size does, however, affect the time of computation noticeably. For this simulation the *coarse* grid size is sufficient for fast and accurate calculations.

References

[1] Wikipedia. Inductance. `https://en.wikipedia.org/wiki/Inductance`, 2018.

[2] Wikipedia. Solenoid. `https://sv.wikipedia.org/wiki/Solenoid`, 2017.

[3] Multiphysics F7024T Hans Åkerstedt. Assignment #2, analysis of a coil. Technical report, Department of Engineering Science and Mathematics, April 2017.