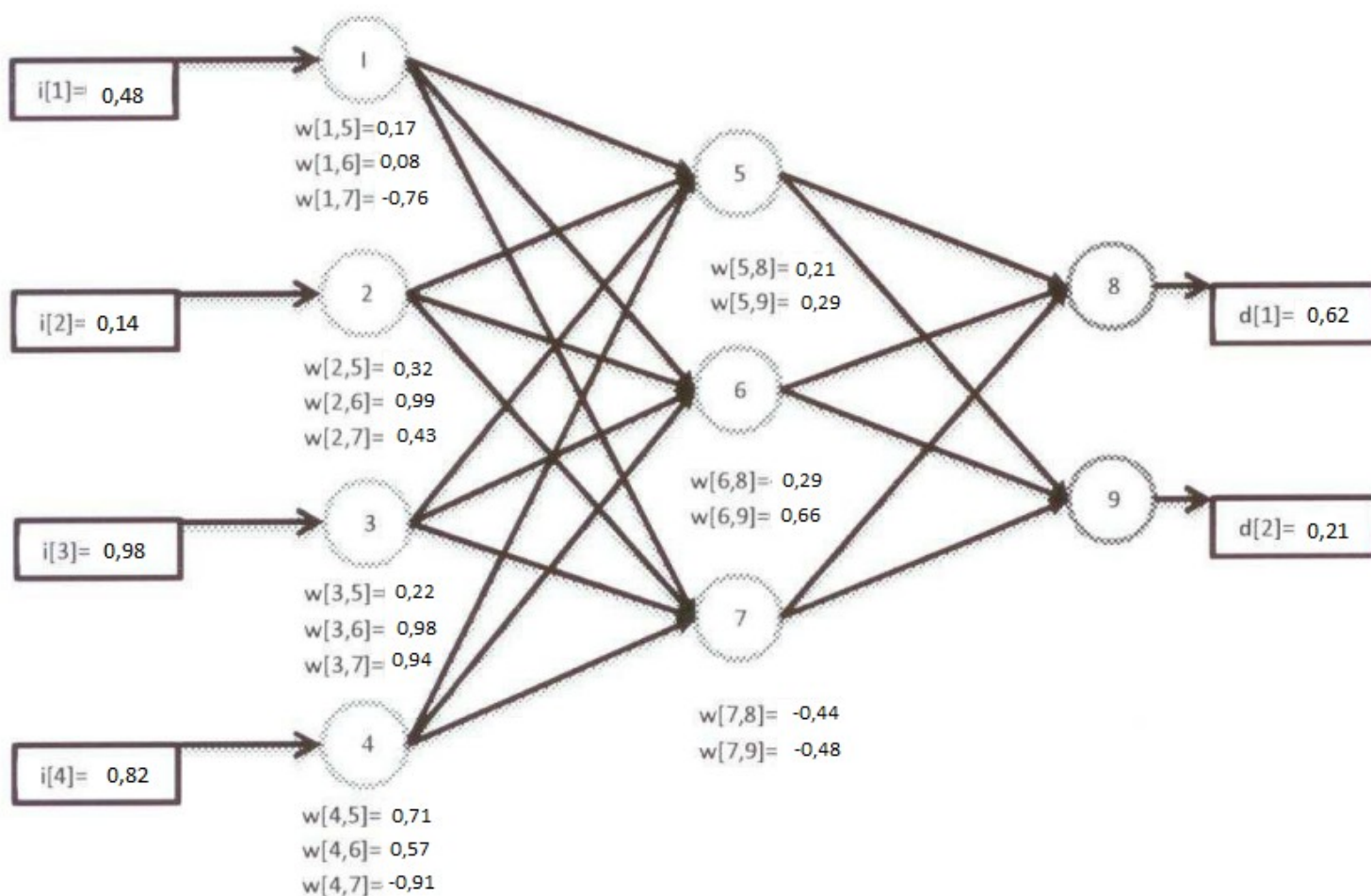


Examen la disciplina
Rețele neuronale. Aplicații.

Fie următoarea rețea neuronală MLP:



cu rata de învățare $\eta=0,71$ și eroarea maximă $E_{\max}=0,01$.

Asupra acestei rețele se aplică algoritmul Back-propagation.

Iterația 1

Propagarea înainte

Strat ascuns

$$o_5 = f(w_{15} * o_1 + w_{25} * o_2 + w_{35} * o_3 + w_{45} * o_4)$$

$$o_5 = f((0,17)(0,48) + (0,32)(0,14) + (0,22)(0,98) + (0,71)(0,82)) = f(0,9242)$$

$$o_5 = 0,7159$$

$$o_6 = f(w_{16} * o_1 + w_{26} * o_2 + w_{36} * o_3 + w_{46} * o_4)$$

$$o_6 = f((0,08)(0,48) + (0,99)(0,14) + (0,98)(0,98) + (0,57)(0,82)) = f(1,6048)$$

$$o_6 = 0,83269$$

$$o_7 = f(w_{17} * o_1 + w_{27} * o_2 + w_{37} * o_3 + w_{47} * o_4)$$

$$o_7 = f((-0,76)(0,48) + (0,43)(0,14) + (0,94)(0,98) + (-0,91)(0,82)) = f(-0,1296)$$

$$o_7 = 0,46765$$

Stratul de ieșire

$$o_8 = f(w_{58} * o_5 + w_{68} * o_6 + w_{78} * o_7)$$

$$o_8 = f((0,21)(0,716) + (0,29)(0,833) + (-0,44)(0,468)) = f(0,18605)$$

$$o_8 = 0,54638$$

$$o_9 = f(w_{59} * o_5 + w_{69} * o_6 + w_{79} * o_7)$$

$$o_9 = f((0,29)(0,716) + (0,66)(0,833) + (-0,48)(0,468)) = f(0,53271)$$

$$o_9 = 0,63012$$

Propagarea înapoi

Stratul de ieșire

$$\delta_8 = (d_8 - o_8) * o_8 * (1 - o_8)$$

$$\delta_8 = ((0,62 - 0,54638) * (0,54638) * (1 - 0,54638))$$

$$\delta_8 = 0,01825$$

$$\delta_9 = (d_9 - o_9) * o_9 * (1 - o_9)$$

$$\delta_9 = ((0,21 - 0,63012) * (0,63012) * (1 - 0,63012))$$

$$\delta_9 = -0,09792$$

Stratul ascuns

$$\delta_5 = (\delta_8 * w_{58} + \delta_9 * w_{59}) * o_5 * (1 - o_5)$$

$$\delta_5 = ((0,01825) * (0,21) + (-0,09792) * (0,29)) * (0,7159) * (1 - (0,7159))$$

$$\delta_5 = -0,005$$

$$\delta_6 = (\delta_8 * w_{68} + \delta_9 * w_{69}) * o_6 * (1 - o_6)$$

$$\delta_6 = ((0,01825) * (0,29) + (-0,09792) * (0,66)) * (0,83269) * (1 - (0,83269))$$

$$\delta_6 = -0,00827$$

$$\delta_7 = (\delta_8 * w_{78} + \delta_9 * w_{79}) * o_7 * (1 - o_7)$$

$$\delta_7 = ((0,01825) * (-0,44) + (-0,09792) * (-0,48)) * (0,46765) * (1 - (0,46765))$$

$$\delta_7 = 0,0097$$

Valorile delta:

$$\Delta w_{15} = \eta * \delta_5 * o_1 = (0,71)(-0,005)(0,48) = -0,0017$$

$$\Delta w_{16} = \eta * \delta_6 * o_1 = (0,71)(-0,00827)(0,48) = -0,00282$$

$$\Delta w_{17} = \eta * \delta_7 * o_1 = (0,71)(0,0097)(0,48) = 0,00331$$

$$\Delta w_{25} = \eta * \delta_5 * o_2 = (0,71)(-0,005)(0,14) = -0,0005$$

$$\Delta w_{26} = \eta * \delta_6 * o_2 = (0,71)(-0,00827)(0,14) = -0,00082$$

$$\Delta w_{27} = \eta * \delta_7 * o_2 = (0,71)(0,0097)(0,14) = 0,00096$$

$$\Delta w_{35} = \eta * \delta_5 * o_3 = (0,71)(-0,005)(0,98) = -0,00348$$

$$\Delta w_{36} = \eta * \delta_6 * o_3 = (0,71)(-0,00827)(0,98) = -0,00575$$

$$\Delta w_{37} = \eta * \delta_7 * o_3 = (0,71)(0,0097)(0,98) = 0,00675$$

$$\Delta w_{45} = \eta * \delta_5 * o_4 = (0,71)(-0,005)(0,82) = -0,00291$$

$$\Delta w_{46} = \eta * \delta_6 * o_4 = (0,71)(-0,00827)(0,82) = -0,00481$$

$$\Delta w_{47} = \eta * \delta_7 * o_4 = (0,71)(0,0097)(0,82) = 0,00565$$

$$\Delta w_{58} = \eta * \delta_8 * o_5 = (0,71)(0,01825)(0,7159) = 0,00927$$

$$\Delta w_{59} = \eta * \delta_9 * o_5 = (0,71)(-0,09792)(0,7159) = -0,04977$$

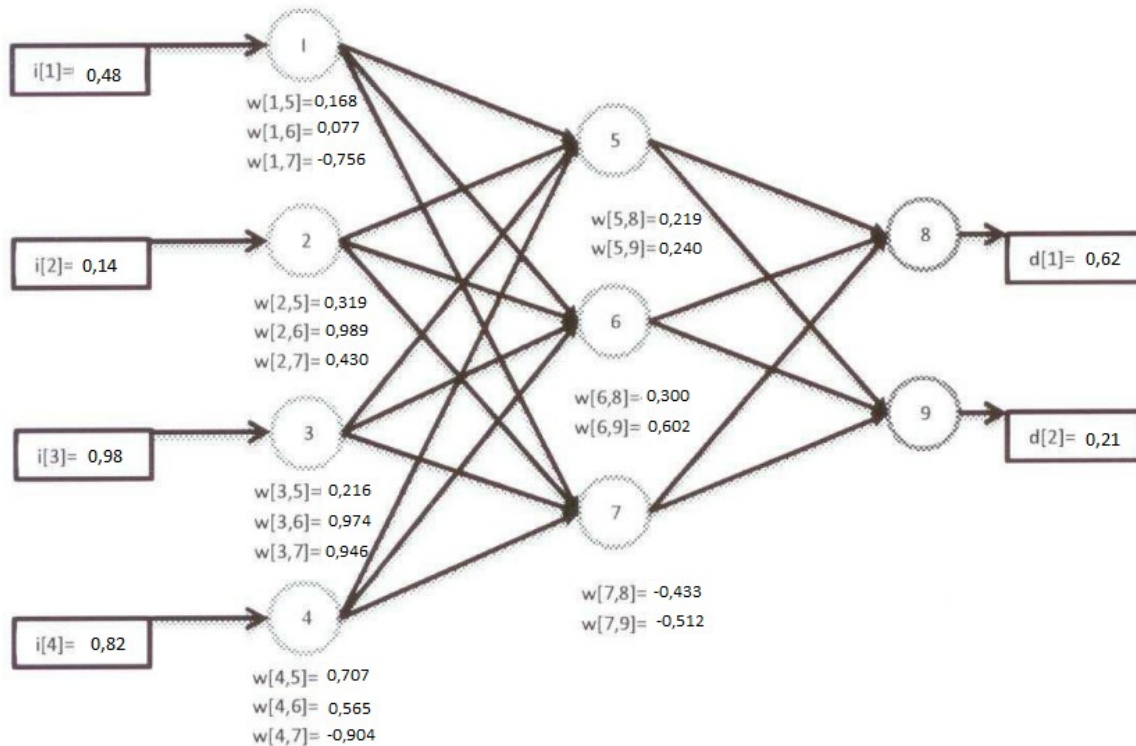
$$\Delta w_{68} = \eta * \delta_8 * o_6 = (0,71)(0,01825)(0,83269) = 0,01079$$

$$\Delta w_{69} = \eta * \delta_9 * o_6 = (0,71)(-0,09792)(0,83269) = -0,05789$$

$$\Delta w_{78} = \eta * \delta_8 * o_7 = (0,71)(0,01825)(0,46765) = 0,00606$$

$$\Delta w_{79} = \eta * \delta_9 * o_7 = (0,71)(-0,09792)(0,46765) = -0,03251$$

În funcție de aceste valori se ajustează ponderile rețelei folosite în următoarea iterație a algoritmului, adăugând valoarea fiecărui delta la ponderea corespunzătoare. Astfel, rețeaua se modifică în:



Iterația 2

Propagarea inainte

Strat ascuns

$$o_5 = f(w_{15} * o_1 + w_{25} * o_2 + w_{35} * o_3 + w_{45} * o_4)$$

$$o_5 = f((0,168)(0,48) + (0,32)(0,14) + (0,217)(0,98) + (0,707)(0,82)) = f(0,91752)$$

$$o_5 = 0,71454$$

$$o_6 = f(w_{16} * o_1 + w_{26} * o_2 + w_{36} * o_3 + w_{46} * o_4)$$

$$o_6 = f((0,077)(0,48) + (0,989)(0,14) + (0,974)(0,98) + (0,565)(0,82)) = f(1,59375)$$

$$o_6 = 0,83114$$

$$o_7 = f(w_{17} * o_1 + w_{27} * o_2 + w_{37} * o_3 + w_{47} * o_4)$$

$$o_7 = f((-0,757)(0,48) + (0,431)(0,14) + (0,947)(0,98) + (-0,904)(0,82)) = f(-0,11663)$$

$$o_7 = 0,47088$$

Stratul de iesire

$$o_8 = f(w_{58} * o_5 + w_{68} * o_6 + w_{78} * o_7)$$

$$o_8 = f((0,219)(0,715) + (0,301)(0,831) + (-0,434)(0,471)) = f(0,20234)$$

$$o_8 = 0,55041$$

$$o_9 = f(w_{59} * o_5 + w_{69} * o_6 + w_{79} * o_7)$$

$$o_9 = f((0,24)(0,715) + (0,602)(0,831) + (-0,513)(0,471)) = f(0,43076)$$

$$o_9 = 0,60606$$

Propagarea inapoi

Stratul de iesire

$$\delta_8 = (d_8 - o_8) * o_8 * (1 - o_8)$$

$$\delta_8 = ((0,62 - 0,55041) * (0,55041) * (1 - 0,55041))$$

$$\delta_8 = 0,01722$$

$$\delta_9 = (d_9 - o_9) * o_9 * (1 - o_9)$$

$$\delta_9 = ((0,21 - 0,60606) * (0,60606) * (1 - 0,60606))$$

$$\delta_9 = -0,09456$$

Stratul ascuns

$$\delta_5 = (\delta_8 * w_{58} + \delta_9 * w_{59}) * o_5 * (1 - o_5)$$

$$D_5 = ((0,01722) * (0,21927) + (-0,09456) * (0,24023)) * (0,71454) * (1 - (0,71454))$$

$$D_5 = -0,00386$$

$$\delta_6 = (\delta_8 * w_{68} + \delta_9 * w_{69}) * o_6 * (1 - o_6)$$

$$\delta_6 = ((0,01722) * (0,30079) + (-0,09456) * (0,60211)) * (0,83114) * (1 - (0,83114))$$

$$\delta_6 = -0,00726$$

$$\delta_7 = (\delta_8 * w_{78} + \delta_9 * w_{79}) * o_7 * (1 - o_7)$$

$$\delta_7 = ((0,01722) * (-0,43394) + (-0,09456) * (-0,51251)) * (0,47088) * (1 - (0,47088))$$

$$\delta_7 = 0,01021$$

Valorile delta:

$$\Delta w_{15} = \eta * \delta_5 * o_1 = (0,71) * (-0,00386) * (0,48) = -0,00132$$

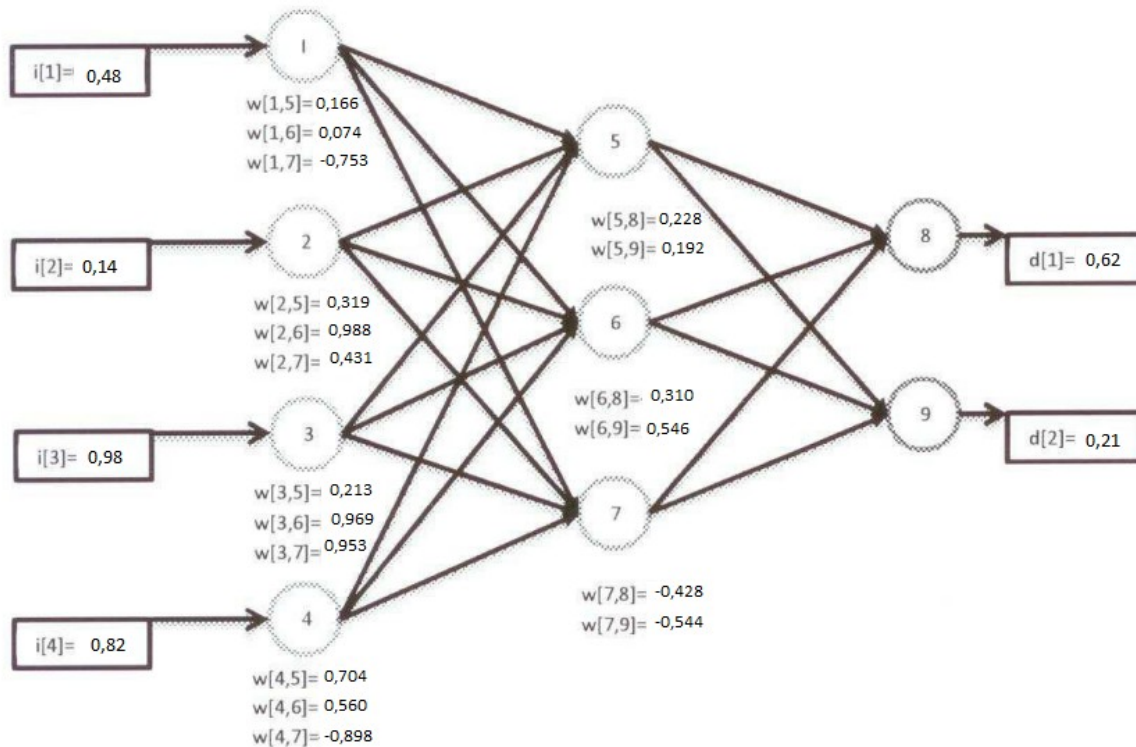
$$\Delta w_{16} = \eta * \delta_6 * o_1 = (0,71) * (-0,00726) * (0,48) = -0,00248$$

$$\Delta w_{17} = \eta * \delta_7 * o_1 = (0,71) * (0,01021) * (0,48) = 0,00348$$

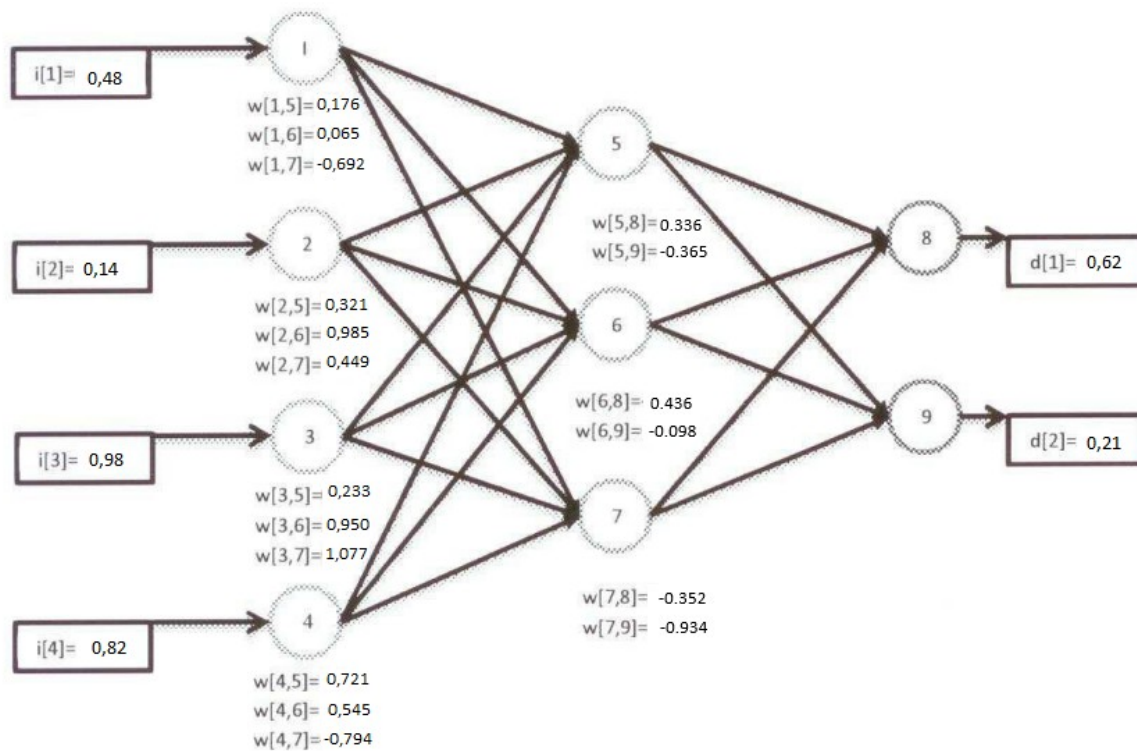
$$\begin{aligned}\Delta w_{25} &= \eta * \delta_5 * o_2 = (0,71)(-0,00386)(0,14) = -0,00038 \\ \Delta w_{26} &= \eta * \delta_6 * o_2 = (0,71)(-0,00726)(0,14) = -0,00072 \\ \Delta w_{27} &= \eta * \delta_7 * o_2 = (0,71)(0,01021)(0,14) = 0,00102 \\ \Delta w_{35} &= \eta * \delta_5 * o_3 = (0,71)(-0,00386)(0,98) = -0,00269 \\ \Delta w_{36} &= \eta * \delta_6 * o_3 = (0,71)(-0,00726)(0,98) = -0,00505 \\ \Delta w_{37} &= \eta * \delta_7 * o_3 = (0,71)(0,01021)(0,98) = 0,00711 \\ \Delta w_{45} &= \eta * \delta_5 * o_4 = (0,71)(-0,00386)(0,82) = -0,00225 \\ \Delta w_{46} &= \eta * \delta_6 * o_4 = (0,71)(-0,00726)(0,82) = -0,00423 \\ \Delta w_{47} &= \eta * \delta_7 * o_4 = (0,71)(0,01021)(0,82) = 0,00595\end{aligned}$$

$$\begin{aligned}\Delta w_{58} &= \eta * \delta_8 * o_5 = (0,71)(0,01722)(0,71454) = 0,00874 \\ \Delta w_{59} &= \eta * \delta_9 * o_5 = (0,71)(-0,09456)(0,71454) = -0,04797 \\ \Delta w_{68} &= \eta * \delta_8 * o_6 = (0,71)(0,01722)(0,83114) = 0,01016 \\ \Delta w_{69} &= \eta * \delta_9 * o_6 = (0,71)(-0,09456)(0,83114) = -0,0558 \\ \Delta w_{78} &= \eta * \delta_8 * o_7 = (0,71)(0,01722)(0,47088) = 0,00576 \\ \Delta w_{79} &= \eta * \delta_9 * o_7 = (0,71)(-0,09456)(0,47088) = -0,03161\end{aligned}$$

Ca rezultat al acestei iterații, rețeaua se modifică astfel:



Algoritmul se va executa pentru încă 23 de iterații, rețeaua finală fiind:



Pentru această rețea se obține eroarea de 0,00947, iar vectorul real de ieșire este: [0,60213; 0,30066].