




Article

Analysis of Dance Data Company Endowments, Assets, and Labor in the United States

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Abstract: To understand how we can advance equity in dance, it is crucial to understand existing financial structures and incentives that shape these companies' success. Assessment of publicly available tax documents on dance companies over the past decade can provide important insights into how different companies fared financially over this time period. Additionally, this work can provide an important basis of understanding for future analyses on the impact of the pandemic on existing trends on the financial performance. By analyzing publicly available Form 990 documents on 169 dance companies across the United States, we present information on companies' endowments, investments, compensation of employees, and sources of labor. In the process, we also bring light to issues of data quality to consider when analyzing Form 990 documents.

Keywords: Endowment; Non-Profit; Volunteer Labor; Dance.

1. Introduction

1.1. Scope

As a matter of transparency and public accountability, tax documents filed by nonprofit corporations in the United States have always theoretically been available to the public, but it wasn't until the development of electronic tax filing systems through the IRS that these tax documents became easily accessible for analysis. Online tax filing systems have been available since the late 1990's, but the 2019 Taxpayer First Act required all nonprofits to electronically file their tax returns (Ely et al., 2023). This legislation changed the landscape for public accessibility of the financial information of nonprofits as well as presented novel research opportunities for data activism. Incorrect reporting, inequitable wealth distribution, and tax fraud by nonprofit organizations has always happened, but mandatory online reporting allows data analysts to identify suspected cases and draw attention to them (Ely et al., 2023).

1.2. Inequality in the Arts

The study of women and gender has consistently drawn focus to inequitable distribution of wealth between men and women. One of the most heavily researched phenomena regarding wealth inequality is the wage gap between men and women that persists even when they have the same education, experience, skills, and job titles (Jikar, 2022). Contrary to the popular opinion that the gender wage gap is narrowing, Weichselbaumer & Winter-Ebmer (2005) showed that financial returns in relation to skills and education are increasingly higher for men than for women. Even less research has been done on the wage gap for transgender or non-binary individuals. Additionally, compounding factors

such as lack of corporate oversight and socially-imposed gender roles further entrench the gender pay gap into everyday life (Pub, 2013). Further understanding the wage gap in female-dominated fields, such as in dance, can inform future solutions to lessen the inequitable distribution of wealth between men and women.

The wage gap has long been exacerbated by disproportionate pay in the arts, and especially dance, in part because the vast majority of entry-level dancers are women and girls (Jikar, 2022). Additionally, unpaid labor in dance is much more commonplace than in other industries (Pub, 2013). Because dance is a performing art, many companies that employ dancers are entitled to government subsidies and tax breaks; however, there are no current regulations to ensure that this governmental assistance is used to lessen unpaid and underpaid labor by dancers (Fuchs, 2021). Unrestricted financial assistance may worsen existing inequalities in the field of dance by enabling company owners and executives to profit while retaining unpaid and underpaid labor, so further understanding how this assistance is used is essential in combating the problem.

1.3. *Dance and Tax Returns*

In the United States, nonprofit organizations file either a Form 900 or a Form 990 EZ with the IRS each year. These documents include, but are not limited to, information on the organization's geographic location, number of employees and volunteers, operating costs, income, compensation, and net assets. This information is more accessible than ever now that the government has required nonprofit organizations to electronically file them. Yet, much of the public does not understand the terms on publicly available tax forms nor does the average person possess the skills to readily analyze hundreds of tax forms every year (Lamonthe, 2023). Thus, data scientists can play a very important role in analyzing these forms filed by dance companies and presenting findings to the public. Further elucidating the financial practices of nonprofit organizations 1) in the field of dance, 2) in pandemic-related economic shifts and 3) through geographical analysis can help inform proper solutions to a wide range of issues, including the gender pay gap in dance.

1.4. *The Dance Data Project*

We completed this project in collaboration with our project sponsor, the Dance Data Project (also known as the DDP). The Dance Data Project is a non-profit organization dedicated to equality in the arts. Founded in 2015 by former ballet dancer and philanthropist Elizabeth Yntema, the DDP uses data and research to promote transparency, accountability, and action towards gender equality in dance.

The DDP collects, analyzes, and presents data to outline lack of leadership opportunities for female directors, choreographers, composers, set, costume, lighting designers, and back-of-house positions in dance. The organization seeks to create a world where all dancers, regardless of gender, race, ethnicity, or socioeconomic status, have equal access to opportunities and resources in the dance world. The organization also advocates for removing barriers for female employment and advancement in this industry, such as lack of parental and elder leave, day care system, and protocol for sexual harassment by presenting their findings on television news such as MSNBC and publishing articles in magazines such as *Vogue* and *People*. Through its data-driven research, advocacy, and education efforts, the DDP works to create a more equitable and inclusive dance community where all individuals have the opportunity to pursue their passion for dance.

1.5. *Scope*

This project will be a preliminary analysis of publicly available financial information on dance companies in the United States. Results will be collected for and shared with Dance Data Project for any publications or future analyses that they perform. Because little analysis has been done on financial practices of dance companies in the United States, our exploratory approach will examine broad research areas regarding how dance companies manage endowments, employment, and properties.

The bulk of our report consists of examining how companies' financial resources (particularly endowments) behave over time, in comparison to one another, and in comparison to the S&P 500. We also look into the use of volunteer labor alongside the use of paid employment labor and how geographical location impacts the use of each type of labor.

The data source for this project is a collection of IRS Form 990's and IRS Form 990 EZ's from dance companies in the United States. These publicly available tax documents are required annual filings for nonprofits¹ and include information including each the number of employees and volunteers, compensation to employees, geographical location, and annual revenue. We primarily present analysis based on Schedule D of the Form 990. Only companies meeting certain criteria regarding the possession of certain funds and assets must file Schedule D, which contains data on endowment funds as well as building values and other assets. Our analysis seeks to capture trends in how these companies have managed their assets and endowments, primarily in the time span of 2014 to 2020, with the goal of bringing transparency to the financial side of the dance industry.

2. Ethics Statement

This project included signing a non-disclosure agreement with Dance Data Project in which each group member agreed to keep specific collections of data produced by DDP confidential. Still, data that was collected during our analyses was not covered by the NDA but was still sensitive. The potential impact of this information getting into the wrong hands would be harmful to the Dance Data Project and their ability to publish the findings. Certain information could be used by dance companies to silence the DDP's message or competitors could publish and get recognition for DDP's proprietary information. Therefore, each team member was especially careful not to disclose any information that may harm DDP's work to third parties.

We also assumed that data reported on the IRS 990, 990EZ, and Schedule D forms was accurate unless identifiable discrepancies were found or amendments were reported. The social harm that could occur from this assumption is dance companies who accidentally misreport on their tax documents may be identified in our analyses as violating ethical or financial norms. In order to prevent this, our team and DDP collaborated to reach out to companies to ask for clarification of our findings before potentially publishing their name.

Finally, all of us have training in data science, but we are not dancers. Because none of us have lived experience in dance, we cannot provide personal expertise in analyzing this data. Additionally, our group all identifies as female and none of us are Black. The standpoints of dancers, men, and people of diverse backgrounds in data analyses may offer a more comprehensive understanding of the context behind our dataset. In order to reduce our bias (although it was not possible to change our social identities), we spoke with our project sponsors about how to approach this data that may impact other social groups.

3. Detailed Methodology

3.1. Data Acquisition

Data files were gathered by Andrew Hoekstra, data consultant from DDP, prior to February 8th, 2022 from publicly available IRS APIs. Form 990s were downloaded as XMLs. Our sample contained 169 US dance companies in total, with Form 990s submitted between 2015 and 2022, thus representing fiscal years 2014 through 2021. For Schedule D in particular, companies are required to report endowment totals up to 4 fiscal years prior, thus our endowment data spans 2010 to 2021.

¹ Most major dance companies are nonprofits.

3.2. Initial Wrangling & Filtering

Form 990 can be completed in three different formats: Form 990, Form 900-EZ, and Form 990-T. We excluded all Form 990-Ts because these forms do not contain relevant information on variables considered in our analyses. Additionally, when a company amended a filing, and hence had multiple filings corresponding to a single fiscal year, we took the amended filing.

3.3. Company Communication

Because companies report data across years, a useful data quality check can be if the year they reported for the previous years in a filing correspond to the values they actually reported in those years.

For example, when a company reports in their 2020 filing the beginning of year endowment balance for years 2016 through 2020, we can check whether the value they reported for 2016 in 2020 was indeed what they reported in 2016. Reporting was highly consistent across the companies considered; however, there were some notable discrepancies, particularly in earlier years. When we identified these discrepancies, we reached out to the companies to clarify which values were correct. We received prompt responses from BalletMet and Pittsburgh Ballet, who confirmed the more recently reported values were correct.

Because discrepancies were typically in earlier years, and our communications with the companies who did respond noted their most recently reported values were correct, in our analyses we took the most recent data available. For example, we took values for 2016-2020 from the 2020 filing instead of taking the current year values for the filing from each year separately.

Another result of this approach is that we can look back through 2010 in some cases, since although we only have filings that go back to 2014, each filing has data going back four years.

3.4. Standard Definitions

Term	Definition
Endowment	The form 990 uses the definition of the endowment (and types of endowments) from the Financial Accounting Standards Board , which defines an endowment fund as “An established fund of cash, securities, or other assets to provide income for the maintenance of a not-for-profit entity (NFP). The use of the assets of the fund may be with or without donor-imposed restrictions.”
Rank	
Percent Change	The percent change is $\frac{\text{End Value} - \text{Beginning Value}}{\text{Beginning Value}} \times 100$.
S&P 500	The Standard and Poor’s 500 Index is a standard index used as a benchmark to describe the behavior of the stock market overall. The index includes 500 top publicly traded companies and is weighted by the market value of stocks currently held by stakeholders, where companies with higher market value receive higher weight.
Compound Annual Growth Rate	The compound growth rate (CAGR) describes a company’s average growth over multiple years. It is useful as a smoothed metric to look over growth over a set of years and compare companies in the same time period. For a time period of t years, we compute the annual growth rate as $\text{Compound Annual Growth Rate} = \left(\frac{\text{End Value}}{\text{Beginning Value}} \right)^{\frac{1}{\text{End Year} - \text{Beginning Year}}} - 1.$

Table 2. Organizations with 100% of their Endowments in One Category for All Years on File

Organization Name	Number of Years on File
Board designated or quasi-endowment	
The Tallahassee Ballet	6
Ballet Quad Cities	2
Canyon Concert Ballet	1
Permanent endowment	
Pittsburgh Ballet Theatre	7
Dance Theatre of Harlem, Madison Ballet	6
BalletMet	5
Aspen Santa Fe Ballet, Ballet West	4
New Mexico Ballet Company	3
Oregon Ballet Theatre	2
American Repertory Ballet, Colorado Ballet, Orlando Ballet	1
Temporarily restricted endowment	
First State Ballet Theatre	6
Ballet Des Moines	2

4. Findings

4.1. Endowments

Endowments are donated funds to nonprofit organizations which are invested. Endowments are typically designed to maintain initial donations (principal funds); the investment income produced by the invested donation can be utilized for specific purposes. Depending on endowment management and policies, some organizations can remove a certain amount of principal assets from their endowment per year.

4.1.1. Types of Endowments

There are **three categories** of endowment funds as defined by the IRS.

- **Term endowments**(temporarily restricted endowments) are funds that are donor-restricted, which means they are meant to be used for a certain amount of time or until a particular event.
- **Permanent endowments** are endowment funds from donor-restricted gifts where the initial fund must be invested; however gains or losses from these investments can be used by the organization.
- **Board-designated or quasi-endowment** are funds that were internally designated for specific use but are not donor-restricted. Investment gains and losses are typically utilized; however, the principal funds can be expended at any time.

Most dance companies have their endowment funds primarily in the permanent endowment category, whereas the proportion held as a board designated/quasi-endowment is typically less than 25%, as is the proportion held as a temporarily restricted endowment (1). For the majority of companies, the proportions held in each category are fairly consistent across the years. In fact, some companies held their endowment in a single category across all years on file (Table 2).

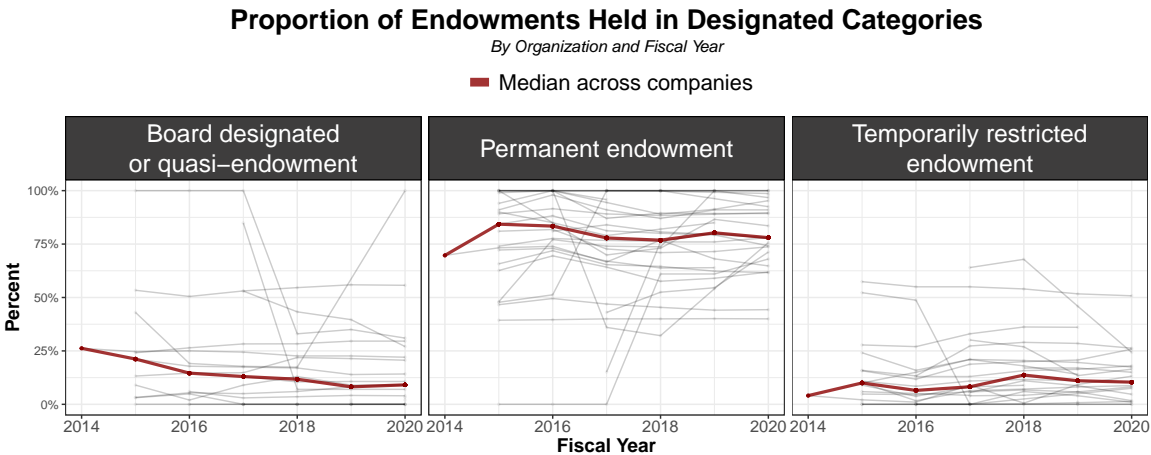


Figure 1. The percent of endowments held as a temporarily restricted endowment, permanent endowment, or board designated or quasi-endowment. The median across all companies by fiscal year is shown in red.

However, there are several notable exceptions to the general trend of consistency across years. Defining variability as the maximum standard deviation of the proportions for each category, the most variable 5 companies were Fort Wayne Ballet, San Francisco Ballet, Nashville Ballet, Atlanta Ballet, and the Washington Ballet (Figure 2).

- For Fort Wayne Ballet, most of the endowment funds (86%) were board designated/quasi-endowment in 2017, but this dropped to a mere 7% in 2018. The percentage of the endowment funds in the permanent endowment category increased accordingly.
- 100% of San Francisco Ballet’s endowment was in the board designated/quasi-endowment category up until 2018, when the percentage in board designated/quasi-endowment dropped to 33% and most (61%) was a permanent endowment.
- The trends in Nashville Ballet’s endowment went the opposite direction, with a large increase in the percentage in the board designated/quasi-endowment category (17% to 74%) and decrease in the percentage in the permanent category.
- For Atlanta Ballet, there is a dramatic shift in 2017 where the percentage held as temporarily restricted goes from 0% to 64%.
- The Washington Ballet had a high proportion of its endowment in the temporarily restricted category (52%) but by 2017 all their endowment funds were in the permanent endowment category.

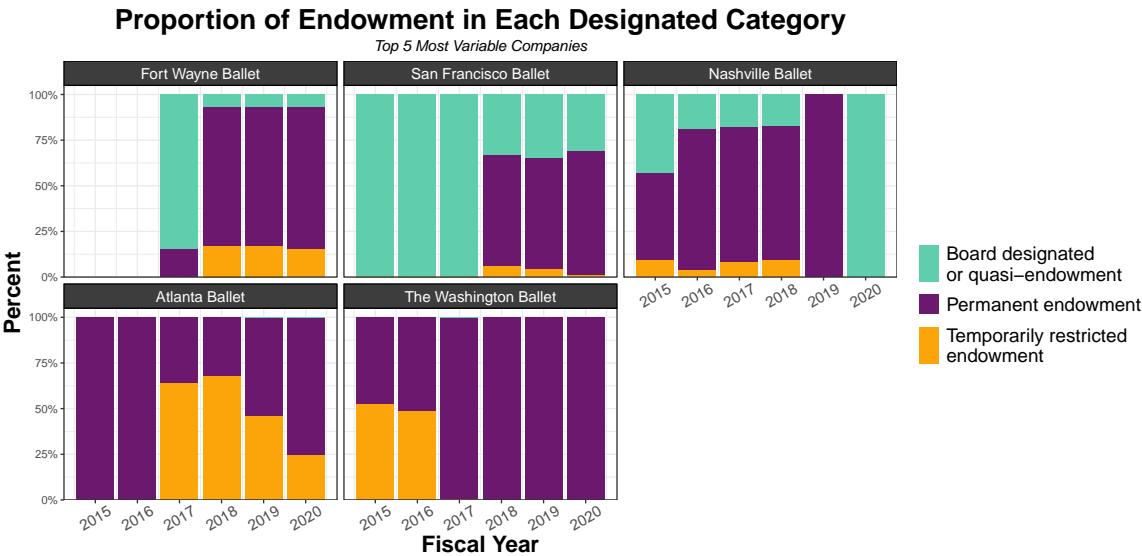


Figure 2. Proportions of endowments in each designated category over time for the 5 companies with the most variability. We defined the most variable companies by considering the maximum standard deviation in the proportion in any one category.

4.1.2. Which Companies have Endowments?

As we discuss these endowment analyses, one of the most fundamental questions about endowments is how many companies report them, and how that varies over time. To report endowments, nonprofits fill out Schedule D in Form 990. Out of 169 dance companies we investigated, 47 reported endowments at least once in Schedule D (Table ??).

Table 3. Number of Companies that Reported an Endowment

	Reported an Endowment	Did Not Report an Endowment
By Year		
2014	6	1
2015	70	35
2016	79	37
2017	83	42
2018	96	40
2019	106	40
2020	83	40
2021	21	6
Reported an Endowment at Least Once	122	47

4.1.3. Ranking Companies' Endowments

Ranking companies can be useful to see how endowments did relative to each other rather than looking at the raw values, which are on immensely different scales.

When we look at the rankings of the beginning of year balance of companies' endowments, we see immediately that the top 7 companies, New York City Ballet, San Francisco Ballet, Houston Ballet, Alvin Ailey American Dance Theater, American Ballet Theatre, Pacific Northwest Ballet, and Boston Ballet, see no changes in ranking from 2013 to 2020 (Figure 3).

Below the top 7, there are more shifts in the rankings across time, with some notable companies changing dramatically in ranking. This includes:

- A dramatic decrease in Aspen Santa Fe Ballet's ranking from 2018 through 2020
- A marked increase in:
 - Joffrey Ballet's ranking
 - Orlando Ballet's ranking
 - Fort Wayne Ballet's ranking
 - Ballet Arizona's ranking
- A decrease in Atlanta Ballet's ranking from 2013 to 2015 that then recovered.

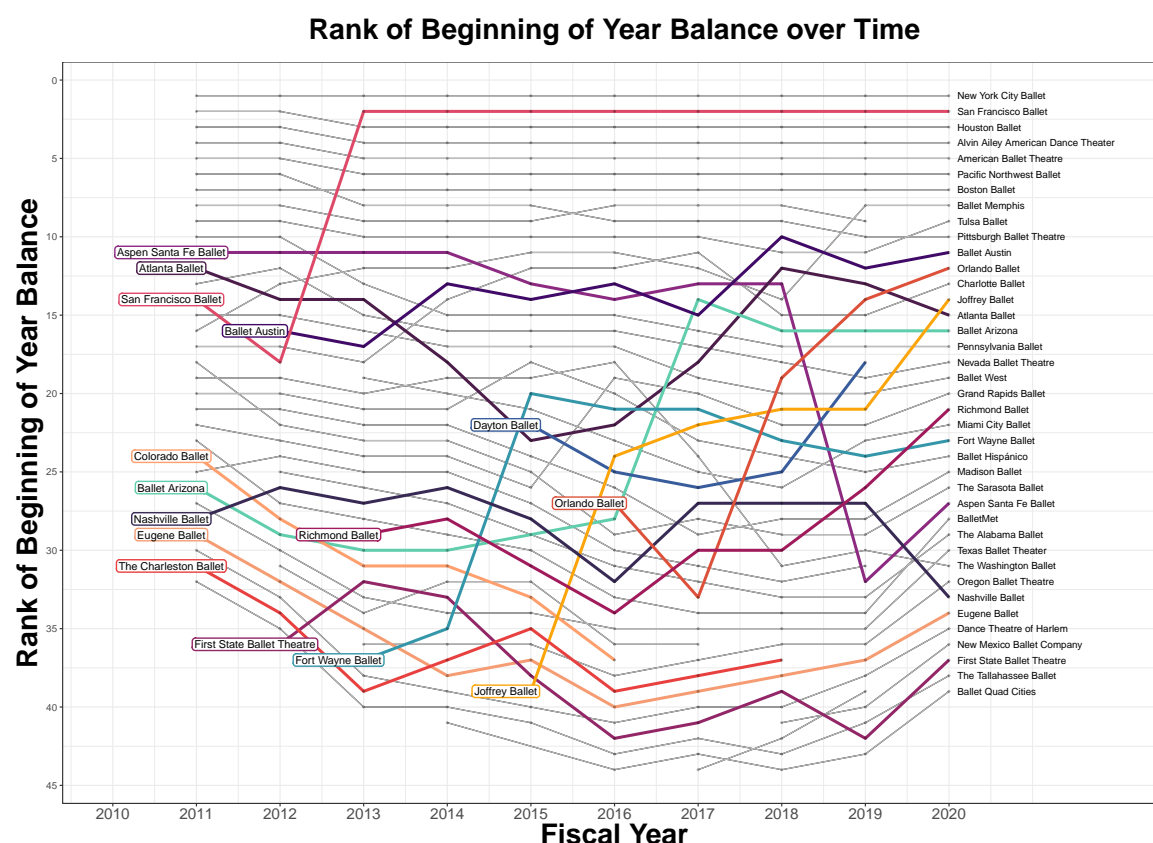


Figure 3. Rank of the endowment beginning of year balance over time. The 15 companies with the most variability in ranking, defined as the mean difference in rankings between fiscal years, are shown in color. Names of all companies are on the right.

When we add information on how these companies ranked in contributions (Figure 4), we see that although some organizations that are top ranked in endowment balance are also top ranked in mean contributions, several of the companies that experienced notable changes in their rankings also were ranked high in contributions, in particular, Orlando Ballet and Joffrey Ballet.

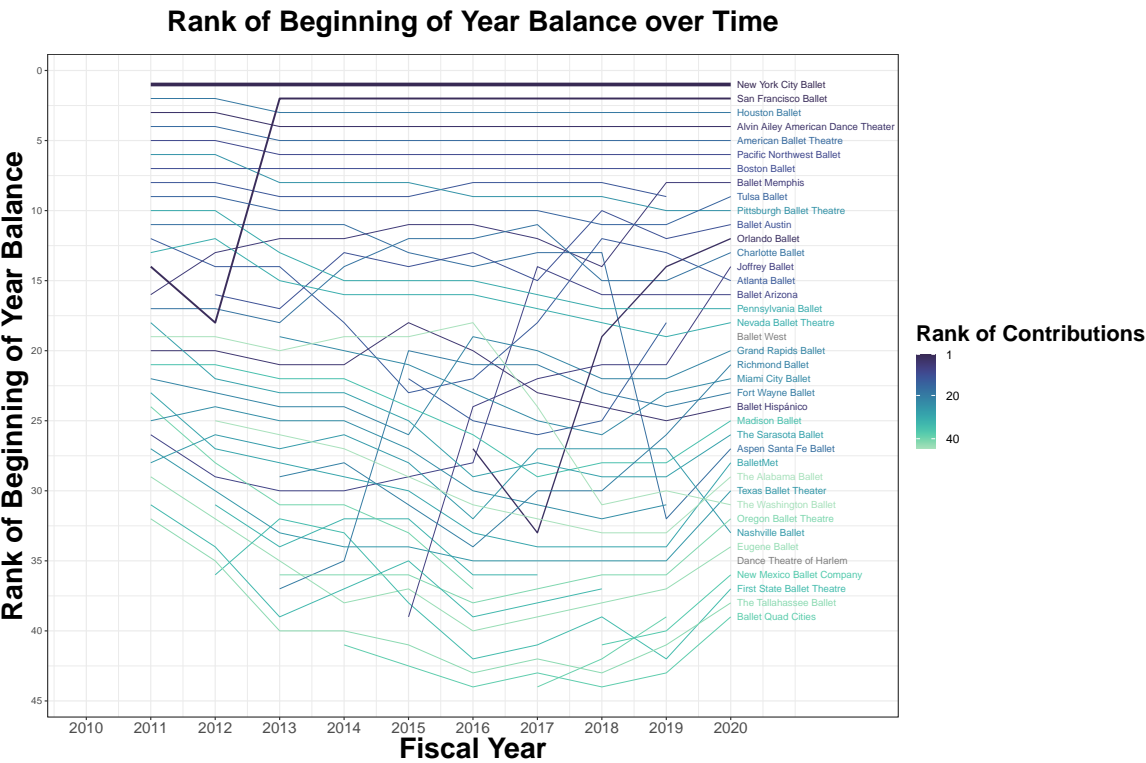


Figure 4. Rank of the endowment beginning of year balance over time, where the color indicates the ranking of the mean contributions received over all years on file for the company.

Looking more closely at the relationship between contribution rankings and beginning of year balance rankings, in Figure 5, there is a strong relationship between how a company ranks with regard to their contributions relative to the other companies and how a company ranks in the endowment beginning of year balance. That is, when companies are ranked high in the beginning of year balance, they tend to rank high in contributions as well. As we would expect, this trend holds across the full set of fiscal years considered.

However, the rankings are often not identical. If they were, all points would fall on the red line, which represents an exact correspondence between rankings. In some cases, a company consistently ranks higher in contributions relative to the beginning of year balance. We summarize whether the contributions or beginning of year balance tends to rank higher for a given company in Figure 6. For example, Ballet West ranked higher in the endowment beginning of year balance for each year available (2016-2020), while Nashville Ballet ranked higher in contributions than beginning of year balance for every year on file (2011-2022).

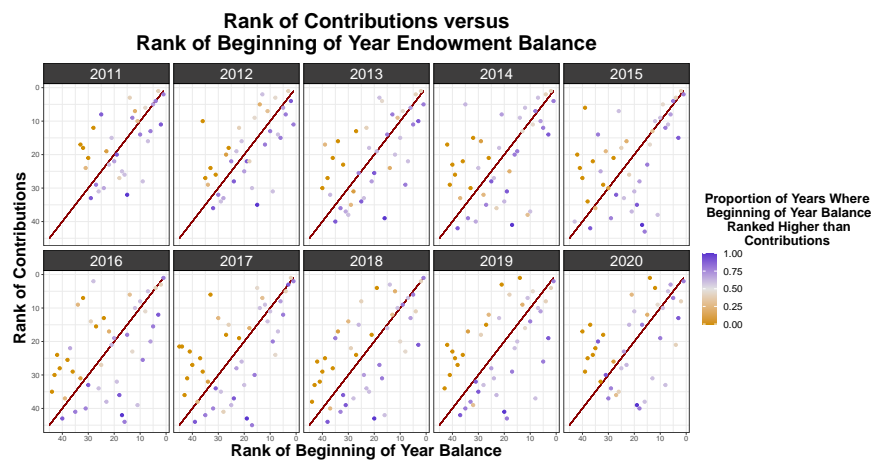


Figure 5. Comparing the rankings of beginning of year balance of the endowment to the ranking of contributions recieved.

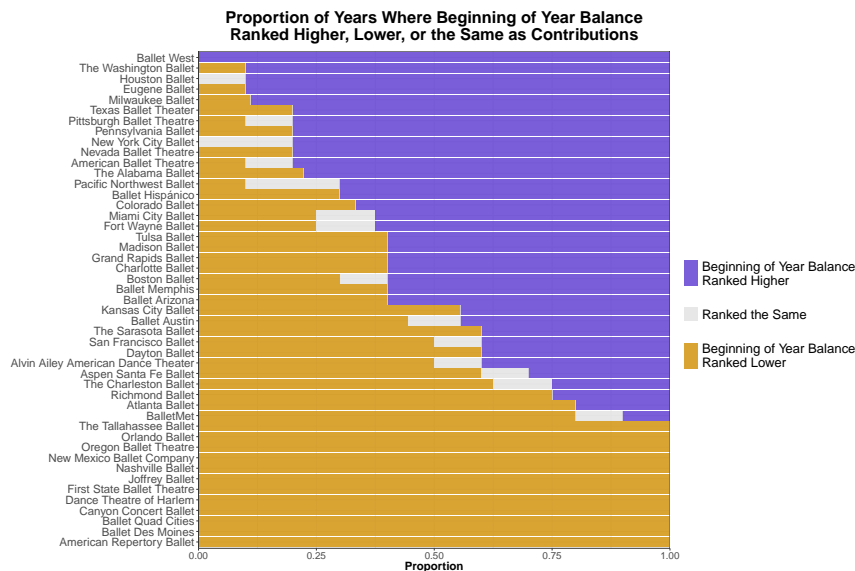


Figure 6. Comparing the proportion of years where a company ranked higher, lower, or the same in beginning of year balance compared to contributions received. A higher rank means a rank closer to 1, where 1 is the top possible rank.

220 In contrast to what we saw in the rankings of the beginning of year balance (Figures 4 and 3, we
221 see in Figure 7 that the rankings of contributions are much less consistent.

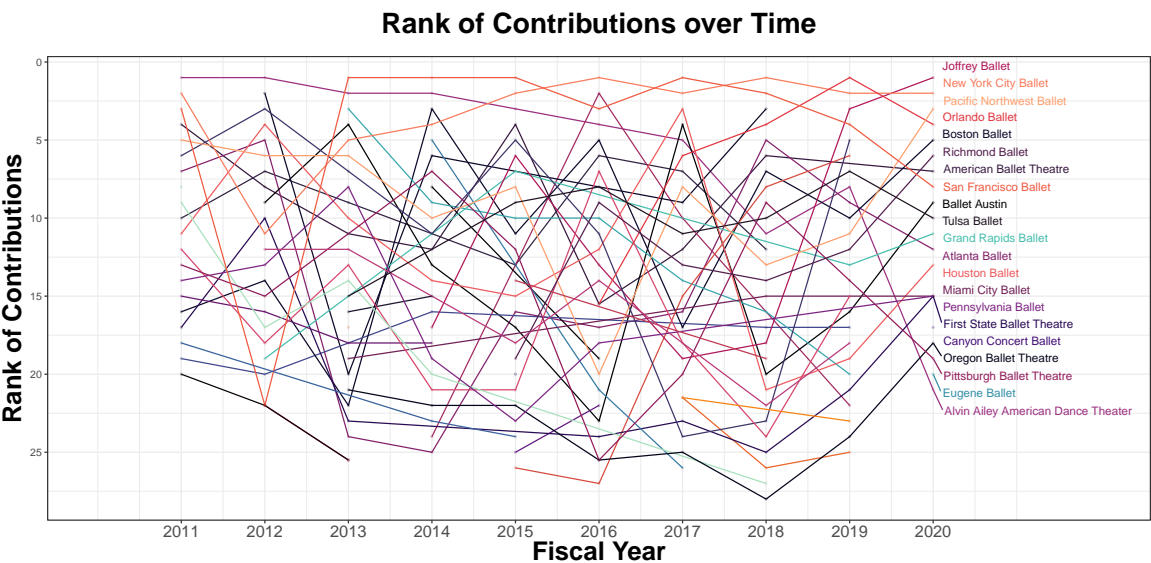


Figure 7. The rankings of contributions over time, by organization.

4.1.4. Reported Endowment Balances over time - Is the Math right?

Theoretically, one can calculate an endowment’s fiscal year end balance based on all information provided in Schedule D. We calculated the end of year balance (see the equation below), and then compared our calculated balance to the reported end of year balance. The majority of calculations are concordant; however, we noted discrepancies in thirteen companies. Discrepancies range from \$-20,000 (Orlando Ballet, 2016) to \$8,301,066 (Atlanta Ballet, 2017). Due to the scaling of the below figure, differences in reported and calculated balance below a hundred thousand dollars are difficult to see.

$$\begin{aligned} \text{Calculated Year End Balance} = & \text{Beginning Year Balance} + \text{Contributions} + \\ & \text{Investment Earnings or Losses} - |\text{Administrative Expenditures}| - \\ & |\text{Other Expenditures}| - |\text{Grants or Scholarships}| \end{aligned} \tag{1}$$

For values related to expenses (administrative expenditures, other expenditures, grants or scholarships), we took the absolute value to ensure all were positive numbers. Four companies (Ballet Hispánico, Atlanta Ballet, Miami City Ballet, and Dance Theatre of Harlem) report their other expenditures as a negative value; thus, when calculating end year balance, subtracting a negative value would result in an additive, not subtractive, effect²

We see the difference in the extent of discrepancies when we take the absolute value (Figure 8 (a)), as in equation (1), versus when we do not (Figure 8 (b)). There are less discrepancies when we do take the absolute value.

Since we compute the difference by taking Reported End Balance – Calculated End Balance, negative values indicate the calculated end balance was larger than the reported, and positive values indicate the reported end balance was larger. The calculated differences are split between being negative or positive.

Atlanta Ballet has the largest discrepancy in (a); however, we see that if we don’t take the absolute value and take the negatives as is (b), the values are concordant.

² We hope to reach out to the prior companies regarding their decision to report negative values, to ensure we do not misrepresent their decision.

Considering panel (a), most companies only miscalculate once; however, there are multiple miscalculations for Fort Wayne Ballet, Atlanta Ballet, and Ballet Arizona. Of note, some of these differences were trivial (e.g., \$10). Eugene Ballet does not report an end year balance for 2011, yet reports a beginning balance of \$45,000, hence the -\$45,000 difference we see for this year.

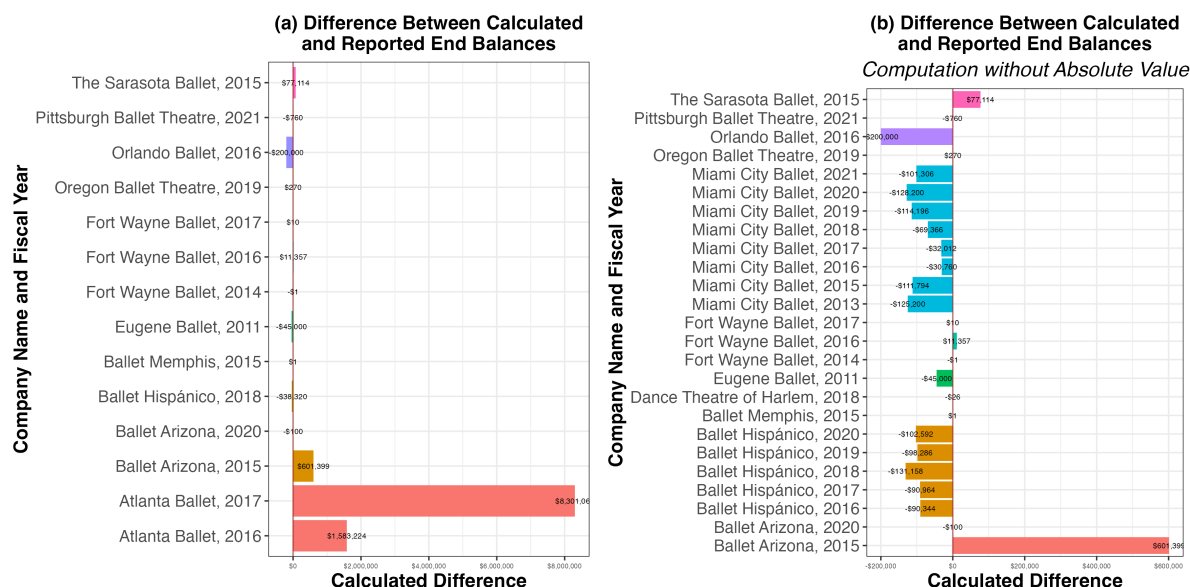


Figure 8. Comparing the reported end of year balance to that we computed based on other reported variables. Calculation is done with the absolute value in (a) and without in (b). Each organization is in a different color.

We summarize these discrepancies by company in Figure ??.

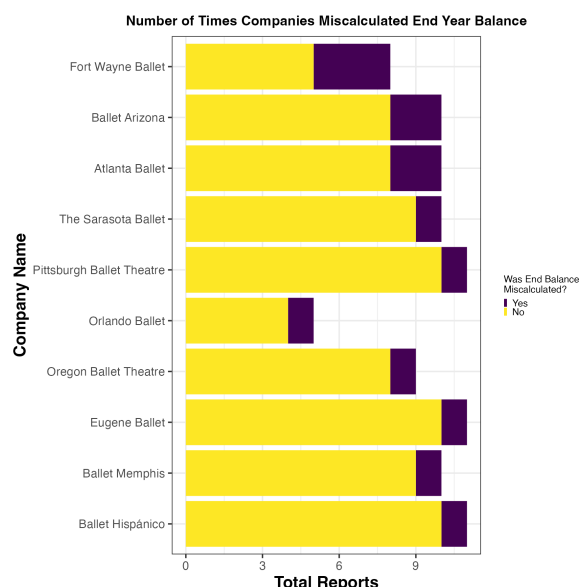


Figure 9. Number of observations where there was a discrepancy between the calculated and reported end of year balance, using the absolute value equation given in equation (1).

4.1.5. How Endowments Did Over Time

Annual percent change is the percent change from one value to the next at the end of a year-long period. With regard to endowment balances, the annual percent change in endowment balance is a comparison between the endowment at the beginning of the fiscal year and the endowment of the end

of the fiscal year, which allows us to interpret by what percent the endowment has grown or shrunk. Charting percent change over time allows us to view endowment behavior over time; further, we can compare percent changes between companies to get a sense for trends in how different companies' endowments change.

To provide some context on the interpretation of percent change, we first state a couple of standard definitions.

The relative change, which represents how much a value has changed relative to its initial value, is

$$\text{Relative change} = \frac{\text{End Value} - \text{Start Value}}{\text{Start Value}}.$$

The effect of considering relative change means that we can more easily compare companies that have enormously different beginning of year balance sizes. For a company with a small endowment, a difference of 10,000 may be substantial, while the same difference would be minimal if the company has over a million in its endowment.

The percent change, which is the relative change in percentage form, is simply $\text{Relative Change} \times 100$.

When we are considering the percent change within a fiscal year, that is, from the beginning of year balance to the end of year balance, a couple of examples of the interpretation include³:

- If the percent change is -50% , the endowment's value at the end of the fiscal year is half of what it was at the beginning.
- If the percent change is 100% , the endowment's value at the end of the fiscal year is twice that it was at the beginning.
- If the percent change is -100% , the endowment's value dropped to zero throughout the fiscal year.

We calculated each company's within-year percent change of endowment balance, as this allows us to compare the performance of different companies' endowments over time. A positive percent change indicates growth within the fiscal year; a negative percent change indicates loss.

The percent change of most companies falls between -100% and 200% (Figure 11). There are notable outliers, however, such as Joffrey Ballet in 2016 with a $\sim 3,000\%$ increase (10). By focusing on lines between -100% and 200% , we can see a trend appearing, with many companies growing and shrinking at similar rates around similar times. Thus, plotting the within-year percent change of the S&P 500, we can see that many companies' endowment balances reflect the performance of the stock market. To assess companies' "raw" performance, we adjusted all percent changes for investment earnings or losses, which flattened the stock market trend (Figure 12).

³ For a relative change of value R , it can be more intuitive to interpret it by considering the expression $\text{End Value} = (R + 1) \times \text{Start Value}$. That is, if we have a relative change, we simply add 1 to it and multiply it by the starting value to acquire the end value.

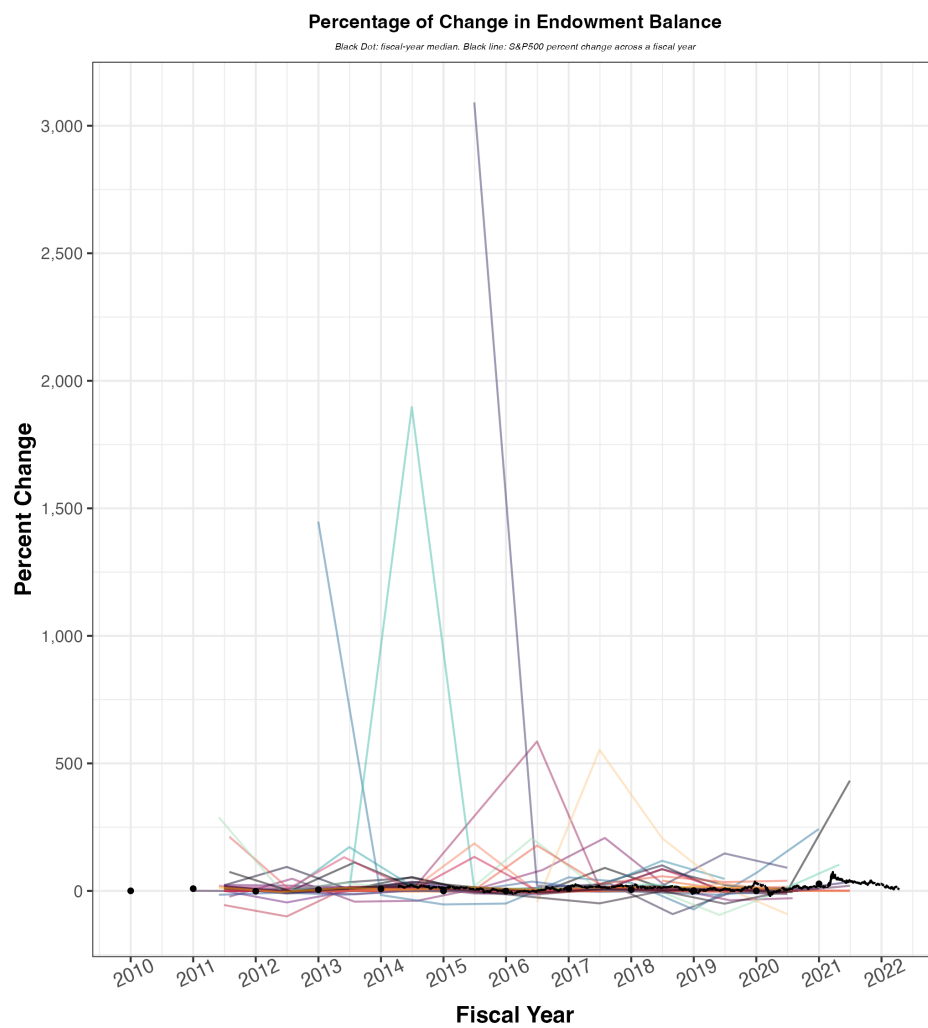


Figure 10. Percent change in endowment balance over time, including the full range of percent changes, revealing several clear outliers.

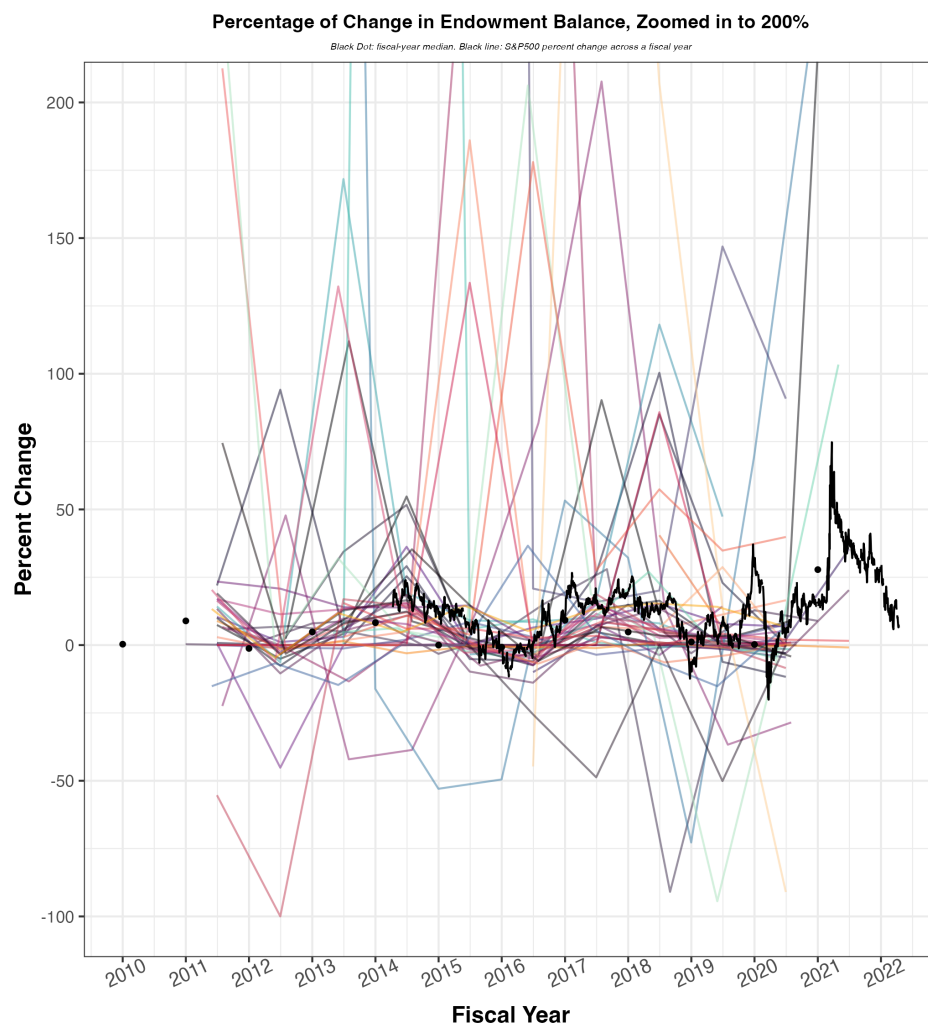


Figure 11. Percent change in endowment balance over time, restricting the range to -200 percent to 200 percent to remove outliers than reduce our ability to see trends for the majority of companies.

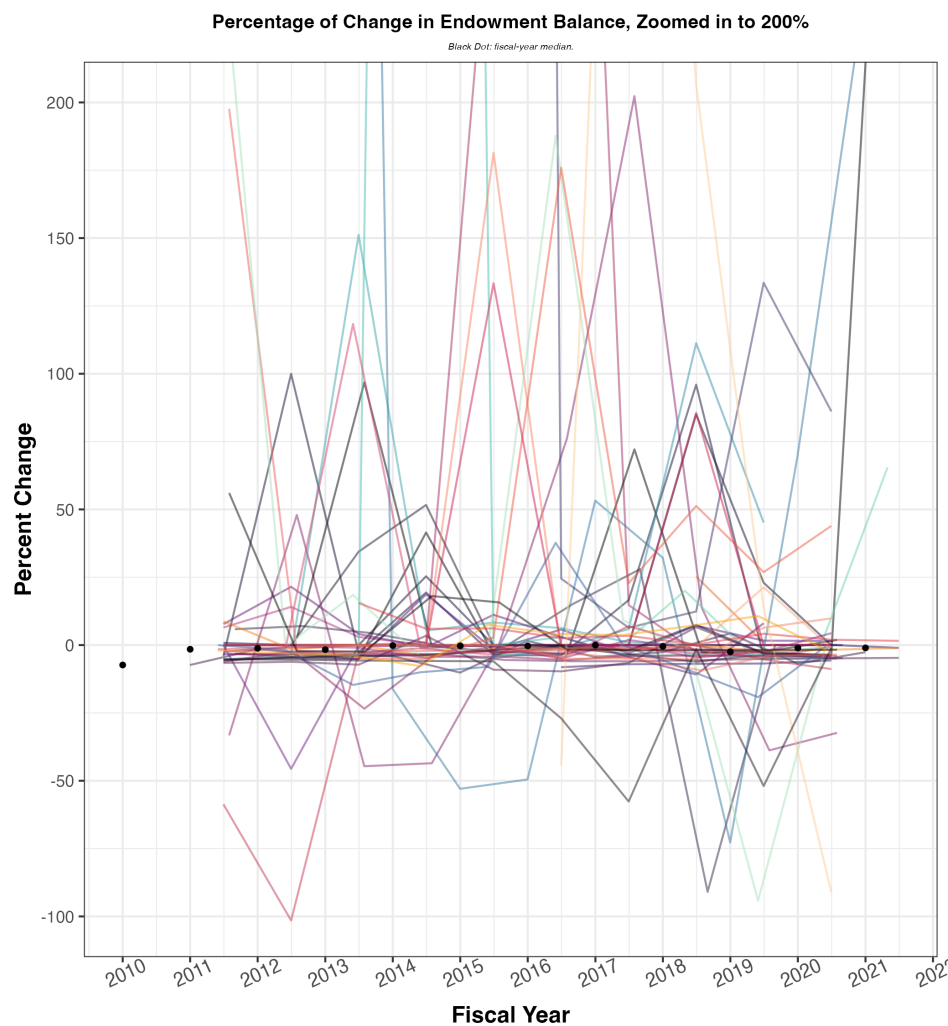


Figure 12. Adjusting for investment earnings or losses when computing the endowment percent changes flattens the trends we see when using the unadjusted endowment values.

Examining particular companies (Figure 13), we can see that eight companies reduce their endowment by over 40% (Percent Change lower than -40%) across multiple years. There are eight companies that do so: Aspen Santa Fe Ballet, Atlanta Ballet, First State Ballet Theatre, Nashville Ballet, Orlando Ballet, San Francisco Ballet, and The Washington Ballet. Some of these companies reduce their endowments by over 40% multiple times.

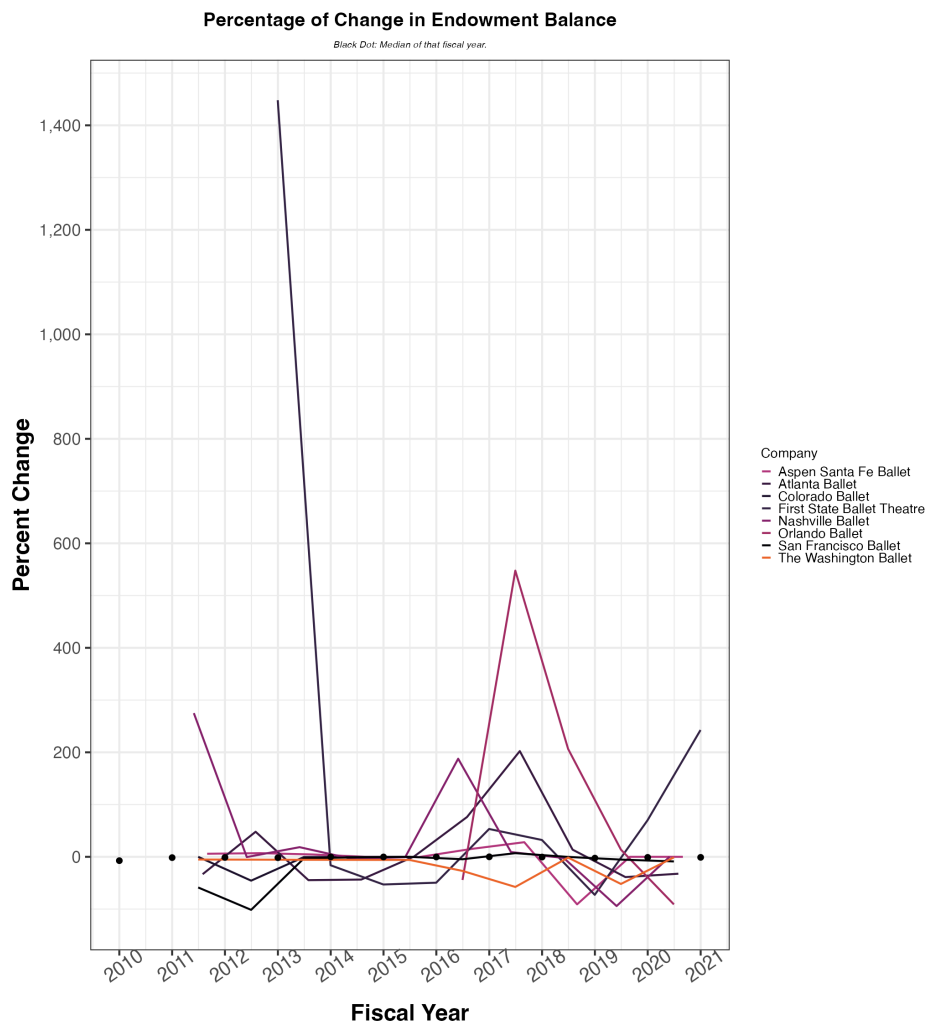


Figure 13. Companies that reduce their endowment by over 40 percent.

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`\begin{table}[!h]`
`\caption{Endowment Percent Change Dropping Below 40% Of Beginning Year balance}`

Company Name	Percent Change	Beginning Balance	End Balance	Fiscal Year
Aspen Santa Fe Ballet	-90.9	6065013	550000	2018
Atlanta Ballet	-43.5	1706513	1046921	2014
Atlanta Ballet	-44.6	2947203	1706513	2013
Colorado Ballet	-45.6	182437	100000	2012
First State Ballet Theatre	-72.7	36693	10000	2018
First State Ballet Theatre	-49.5	35876	18107	2015
First State Ballet Theatre	-53.0	76261	35876	2014
Nashville Ballet	-94.2	1095624	61350	2019
Orlando Ballet	-91.0	7732855	696082	2020
Orlando Ballet	-44.3	613186	338943	2016
San Francisco Ballet	-101.5	1035814	174	2012
San Francisco Ballet	-58.7	2318646	1035814	2011
The Washington Ballet	-51.9	621423	310000	2019
The Washington Ballet	-57.6	1212247	621423	2017

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Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

DDP	Dance Data Project
IRS	Internal Revenue Service

Appendix A

Appendix A.1

The appendix is an optional section that can contain details and data supplemental to the main text. For example, explanations of experimental details that would disrupt the flow of the main text, but nonetheless remain crucial to understanding and reproducing the research shown; figures of replicates for experiments of which representative data is shown in the main text can be added here if brief, or as Supplementary data. Mathematical proofs of results not central to the paper can be added as an appendix.

Appendix B

All appendix sections must be cited in the main text. In the appendixes, Figures, Tables, etc. should be labeled starting with 'A', e.g., Figure A1, Figure A2, etc.

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