

# DND: The Controllability of Dynamic Temporal Network in Smart Transportations

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**Abstract**—Along with the development of IoT and mobile edge computing in recent years, everything can be connected into the network at anytime, resulting in quite dynamic networks with time-varying connections. Controllability has long been recognized as one of the fundamental properties of such temporal networks, which can provide valuable insights for the construction of new infrastructures, and thus is in urgent need to be explored. In this paper, we take smart transportation as an example, first disclose the controllability problem in IoV (Internet of Vehicles), and then design an DND (Driver NoDe) algorithm based on Kalman's rank condition to analyze the controllability of dynamic temporal network and also to calculate the minimum number of driver nodes. At last, we conduct a series of experiments to analyze the controllability of IoV network, and the results show the effects from vehicle density, speed, connection radius on network controllability. These insights are critical for varieties of applications in the future smart connected living.

**Index Terms**—IoT, Dynamic Network, Internet of Vehicles, Network Controllability, Driver Nodes

## I. INTRODUCTION

In recent years, with the rapid development of Internet of Things technology, a variety of smart devices that can be connected to the network have emerged. However, these smart devices are different from the traditional ones, especially some of these kind are highly mobile, such as the rapid driving vehicles in the internet of vehicles, which led to the internet of vehicles becoming a kind of dynamic networks. In fact, dynamic networks exist in all aspects of our lives, such as delay-tolerant networks, opportunistic-mobility networks, social networks, friendship networks, etc [1], [2]. The topology of these networks is not static, but changes over time, where the connections among nodes establish or destruct irregularly. This results in rapidly changing network topology, which makes it difficult to hold controllability of the entire network. To address this challenge, it is necessary to propose a way to analyze and solve the controllability of dynamic networks.

There has been some research and progress on controllability of networks in recent years. However, these research just proved the controllability of complex networks in mathematical theory, but do not apply the theory to the controllability of mobile dynamic networks. This paper refers to the mathematical theory knowledge of these papers and applies it to the

controllability of dynamic networks. For a dynamic network, it needs several driver nodes to ensure the controllability of the entire network. By analyzing the connection state of the dynamic network during control time, the minimum number of driver nodes can be obtained.

First, this article raised some issues of controllability challenges faced by dynamic networks. Then, took the vehicle network on a road as an example with the moving speed, communication radius, average density of vehicles and the control time as different variable parameters. And designed an algorithm to calculate the optimal solution of the minimum number of driver nodes required for a dynamic network. Finally, after the experiment of simulation data, we got the influence of the vehicle's moving speed, communication radius, average density and the control time on the number of required driver nodes of a vehicle network. By deploying a minimum of driver nodes to ensure the controllability of the entire network, and maximizes resource conservation, improves efficiency, ultimately, guides the deployment of mobile networks.

The remainder of this paper is organized as follows. We give a review of related work in Section II. In Section III we introduce the background of the application scenario. Section IV describes some variables and specific formulas of our modeling. Section V presents the results and analysis. Finally, the conclusions and future work are laid out in Section VI.

## II. RELATED WORK

### A. Dynamic network

The network is composed of nodes and the relationship between nodes. The rapid development of the Internet makes human beings enter the network era, such as all kinds of social networks. The development of industrial Internet forms such as large power grid and the development of the Internet of things, which connects the whole world into a huge network. At present, network research has infiltrated into mathematics, life science, information science and many other fields, and its fundamental goal is to find effective means to control network behavior and make it serve human beings. The paper [3], for the first time to apply the classical control theory to the analysis of network control. And points out that in network science, if there is a directed edge between the node N1 and the node N2 in the directed network, the node N2 is usually said to be controlled by the node N1. In this paper, the network is abstracted into a linear time-invariant system, and the Kalman

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controllability criterion in the control theory is introduced into the research, and the sufficient and necessary condition for the network to achieve complete controllability is that the controllability matrix reaches full rank, so the controllability problem of the network is transformed into the problem of calculating the rank of the controllability matrix.

What mentioned in [4] is theoretically proved that the minimum driver nodes needed to control the entire network depends on the maximum matching in the network, in which all unmatching nodes are driver nodes of the network. Thus, the problem of network structure controllability is transformed into a classical graph theory matching problem, which reduces the time complexity of network controllability problem. However, this method can only be applied to directed networks and networks with unknown edge weights. According to Popov-Belevitch-Hautus criterion, the paper [5] is proved that the minimum number of driver nodes required by the network is equal to the maximum geometric multiplicity of all eigenvalues of the network matrix, as for undirected networks, the minimum driver nodes is the maximum algebraic multiplicity of all eigenvalues. It reduces the computational complexity of related problems greatly.

[6]–[9] presented a theoretical approach to describing the controllability of networks or proposed a way to change the state of some nodes to stabilize the network. All of the papers and methods mentioned above do not have solved the problem of controllability generated in dynamic networks well until now.

### B. Internet of Vehicles

Internet of Vehicles refers to the use of vehicle electronic sensing devices to enable two-way data exchange and sharing between vehicles, vehicles, people, vehicles and transportation facilities through wireless communication technology, car navigation systems, intelligent terminal facilities and information processing systems. It is comprehensive intelligent decision-making information system that realizes real-time monitoring, scientific dispatching and effective management of vehicles, people, objects, roads, etc., thereby improving road transportation conditions and traffic management efficiency [10].

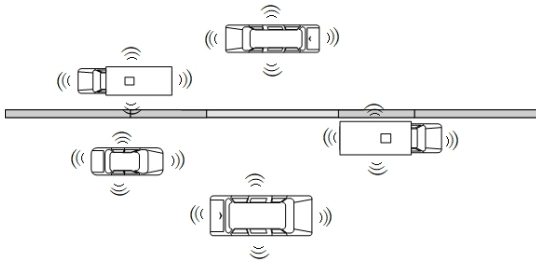


Fig. 1. Illustration of Vehicle-to-vehicle communication.

Vehicle-to-vehicle communication as shown in Fig. 1, the communication between the vehicle and the vehicle through

the vehicle-mounted terminal, and it is mainly used for two-way data transmission between vehicles. The communication technologies used include microwave, infrared technology, dedicated short-range communication, etc., featuring high safety and real-time requirements. The vehicle terminals can collect information such as the speed, position, direction and vehicle running alarm of the surrounding vehicles in real time. Those vehicles form an interactive communication platform through wireless communication technology, which can exchange pictures, text messages, videos and audio information in real time [11], [12].

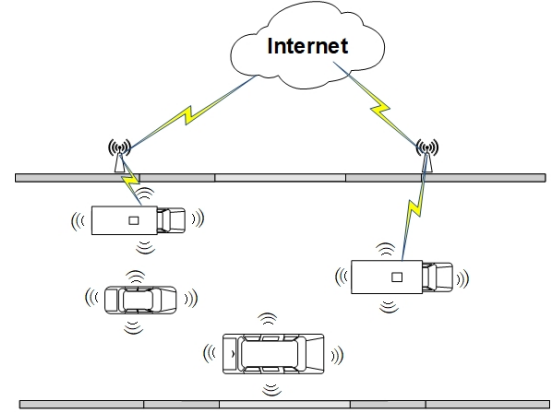


Fig. 2. Illustration of communication between the vehicle and the control center.

The communication between the vehicle and the control center means that the vehicle-mounted mobile terminal establishes interconnection with the remote traffic control center through the public access network, to complete data transmission and information exchange, and to accomplish the interaction, storage of data between the vehicle and the traffic control center. It is mainly used to vehicle navigation, vehicle remote monitoring, emergency rescue, information entertainment services and so on. Moreover, it has the characteristics of long distance and high speed movement. [10]

In a vehicular network, vehicles are virtualized as mobile network nodes, and road side units(RSUs) are virtualized as stationary network nodes. The environmental information of the road and the vehicle is collected through sensors in the vehicle and the RSU. The structure of the vehicle network presents a dynamic topology. The high-speed moving vehicle nodes make the topology of the vehicle network change rapidly, and the access status changes dynamically due to the dynamic network topology.

### III. BACKGROUND

Taking a scene in life as an example, vehicles that traveling in a section of a road exist at a certain density, and each vehicle travels along the road at a certain speed. Each vehicle has a certain communication capability to exchange the required data with other vehicles within the communication radius. If each car is regarded as a node and there is an edge between the

two cars which can exchange data to each other, then at some point in time, the nodes and the connection between them can be abstracted as an undirected graph as shown in Fig. 3 .

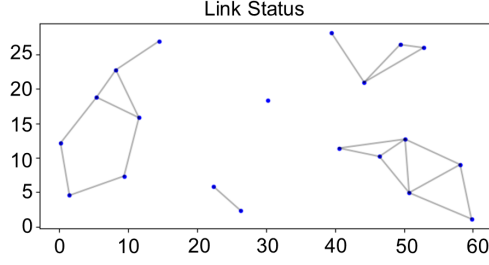


Fig. 3. The abstract graph of vehicle network.

Since the vehicle is moving constantly, the connection relationship between the vehicles is dynamic, so the connection relationship between the vehicles needs to be calculated every small period of time. The time interval for recalculating the connection relationship is called refresh time. The refresh time is usually related to the velocity of vehicles. In order to ensure the controllability of the network, the faster the vehicle speed, the higher the refresh frequency. When we set a so-called control time, it is necessary that the network of all vehicles in the entire area needs to reach a controllable state during this time period, and this time period will include at least one refresh time, usually more than one. During the control time, all the vehicle connection status in the entire area will be updated after each refresh. We not only need the connection status at the current time, but also need to retain the previous connection status. Finally, we get the connection state needed in a control time, and it can be abstracted into an undirected graph.

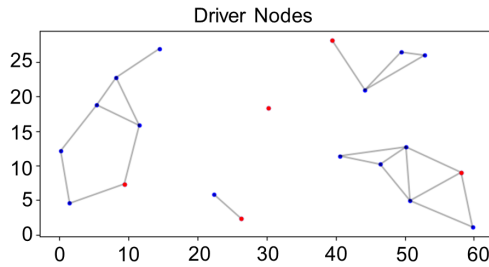


Fig. 4. The abstract graph of vehicle network with driver nodes.

According to the undirected graph generated by those parameters, under the premise of controllability of the whole network, we can calculate the required driver nodes. As shown in the Fig. 4 , where the red point is the driver node. This will be explained in detail in IV.

In the area we want to calculate, the density, the velocity, the radius of communication of the vehicle, the refresh time and control time which we set will affect the state of the entire network, and then affecting the minimal number of driver

TABLE I  
NOTATION DEFINITIONS

Notation	Meaning
X	the length of the calculated range
Y	the width of the calculated range
N	the number of nodes in the calculated range
$N_D$	number of control nodes in the calculated range
P	device density in every $Y \times Y$ area of calculated range
R	communication radius of the device
$(x_i, y_i)$	coordinate value of the device
$V_{ix}$	the speed of device i in the X direction
$V_{iy}$	the speed of device i in the Y direction
T	control time
t	refresh time
A	0-1 adjacency matrix of the graph, $A_{ij} = 1$ means that devices i and j are connected by edges, and $A_{ij} = 0$ means that there is no edge connecting between devices i and j.

nodes network. We want to find the number of minimum driver nodes and the relationship between these parameters by using the control variable method for these main parameters.

#### IV. DND: DRIVER NODE ALGORITHM

##### A. Parameters and variable declarations

In this section, we formally define the driver nodes problem in vehicle network which aims to obtain the minimum number of driver nodes of the network. Table I defines the notations to be used in the problem definition or make it easier to describe.

##### B. Modeling

Set a coordinate system for calculating range:  $X \times Y$ .

Device nodes are randomly generated in the whole region according to the normal distribution probability of  $X \sim N(P, 1)$ , where the coordinates of the nodes are:

$$x_i = \text{random}[0, X) \quad (1)$$

$$y_i = \text{random}[0, Y) \quad (2)$$

Calculate the adjacency matrix according to the generated graph:

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix}, 1 \leq i, j \leq N \quad (3)$$

and note that:

$$A_{ij} = \begin{cases} 1, & \|n_i, n_j\|^2 \leq R \\ 0, & \|n_i, n_j\|^2 > R \end{cases} \quad (4)$$

$$\|n_i, n_j\|^2 = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (5)$$

Coordinate update of node in a unit interval:

$$(x_i, y_i) = \begin{cases} x_i + V_i x, y_i + V_i y, & x_i + V_i x \leq X, y_i + V_i y \leq Y \\ X, y_i + V_i y, & x_i + V_i x > X, y_i + V_i y \leq Y \\ x_i + V_i x, Y, & x_i + V_i x \leq X, y_i + V_i y > Y \\ X, Y, & x_i + V_i x > X, y_i + V_i y > Y \end{cases} \quad (6)$$

Node velocity update:

$$V_i x = \begin{cases} -V_i x, & x_i = X \\ V_i x, & x_i < X \end{cases} \quad (7)$$

$$V_i y = \begin{cases} -V_i y, & y_i = Y \\ V_i y, & y_i < Y \end{cases} \quad (8)$$

The adjacency matrix after refresh:

$$A_{ij}^{t+1} = \begin{cases} 0, & A_{ij}^t = A_{ij} = 0 \\ 1, & A_{ij}^t \neq 0 \text{ or } A_{ij} \neq 0 \end{cases} \quad (9)$$

Where,  $A$  is the adjacency matrix calculated according to the node distance at current moment, and  $A^t$  is the state of the adjacency matrix at the previous moment.

The number of driver nodes [5]:

$$N_D = \mu(\lambda^M) \quad (10)$$

It needs to be noted that  $A$  should be connected graph. If the graph in the current state is not a connected graph, it needs to calculate its connected subgraph graph separately, and finally add the results of each connected subgraph.

### C. Abstraction of topology

In this section, we introduce how to map a network topology into a matrix briefly by using a simple example and the process of calculate the required driver nodes for the network based on this matrix.

We take a connected subgraph in the Link Status diagram above as an example. Each node in the figure is given a serial number. As shown in Fig. 5. According to the topology, an

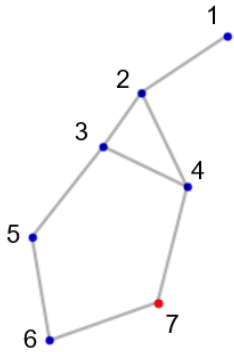


Fig. 5. Network topology.

adjacency matrix  $A$  as Fig. 6. can be obtained.

After calculation, the eigenvalue vector of the matrix  $\lambda = [3.521, 2.284, 1.272, -0.846, -0.231, -0.618, 1.618]^T$ . And each eigenvalue is different, then we select  $\lambda^M = 1.618$ , the maximum algebraic multiplicity  $\mu(\lambda^M) = 1$ .

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Fig. 6. Matrix  $A$  of network shown in Fig. 5.

Now the number of driver nodes can be obtained, and it can proceed to calculate to get which nodes are the driver nodes by follows.

Then, We get the matrix  $B = A - \lambda^M E_N$  as Fig. 7. where  $E_N$  is the identity matrix.

$$\begin{bmatrix} -0.618 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.618 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -0.618 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -0.618 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -0.618 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.618 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & -0.618 \end{bmatrix}$$

Fig. 7. Matrix  $B$ .

We perform the elementary column transformation on matrix  $B$  to get the column canonical form as Fig. 8.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.618 & -0.618 & 1 & 0 & 0 \end{bmatrix}$$

Fig. 8. Column canonical form of  $B$ .

The last row is linearly dependent on others in the column canonical form. The node corresponding to it is the driver node colored by red in Fig. 5.

## V. EVALUATION

The experiment sets a scene of a road with  $Y$  meters wide and  $X$  meters long. According to the width of the road, the entire road is divided into  $X/Y$  segments. Nodes in each segment are generated according to the density and obey the Poisson distribution. In each segment the position  $(x_i, y_i)$  of each node is random, and with a velocity  $V_i x$  in the  $X$  direction and a velocity  $V_i y$  in the  $Y$  direction according to the velocity. And note that the value of the velocity is  $Vx \pm 2$  or  $Vy \pm 2$

randomly. Both the communication radius and the control time are set according to the experimental requirements. When updating the position of each node according to its velocity, it is set that the movement in the opposite direction at the next moment when the node reaches the boundary of the region, so as to ensure the number of nodes of the entire region do not changed.

According to the model, we compare the results of experiments by assigning different values to the parameters of node speed, communication radius, density and control time. In order to eliminate the randomness of the results caused by randomly generated nodes, when the density is fixed, the same random nodes are used in calculations of different parameter assignment. Moreover, the driver nodes were calculated 100 times for each assignment and get the average value as the final result.

#### A. Communication Radius

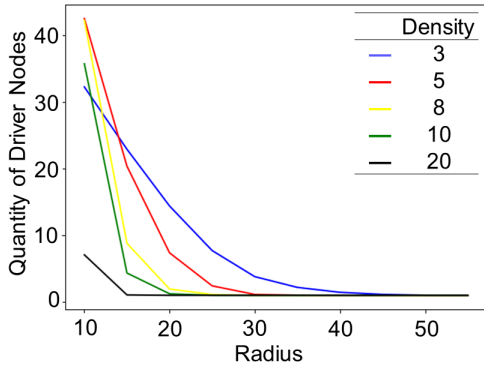


Fig. 9. Relationship between number of driver nodes and communication radius.

When only considering the state at a certain time, that is, not considering the impact of refresh and node movement, the relationship between the number of driver nodes and node communication radius is shown in Fig. 9.

The abscissa is the value of the velocity in  $m/s$  and the ordinate represents the number of required driver nodes. The density is in units of  $90m^2$ . Five curves with different colors represent different node density. It is necessary to explain that at the velocity of 10, the density of 3, the blue curve, requires less driver node than the density of 5, the red curve or 8, the yellow curve. This is because the total number of driver nodes at density of 3 is much smaller than in other cases, however, actually, the proportion of driver nodes of density of 3 is much higher than in other cases.

It can be seen from the graph that with the increasement of communication radius, the number of driver nodes needed in the whole network decreases and tends to the minimum of 1. The reason for the overall downward trend is that as the communication radius increases, each node can establish connection relations with more nodes, and the number of edges in the whole network increases. Under different density

conditions, it is that the rate of increase of the number of connections is different, so the rate of decline is different, and the higher the density, the higher the rate of increase of the number of connections.

#### B. Nodes Density

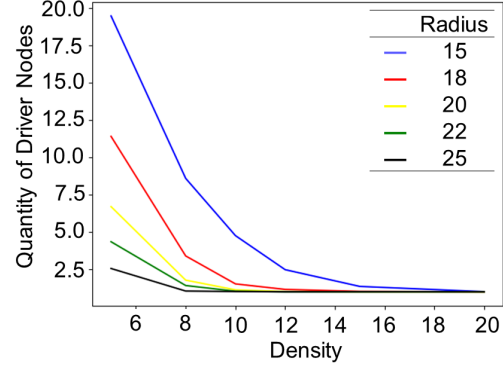


Fig. 10. Relationship between number of driver nodes and density.

Similarly, all conditions are consistent with those mentioned above, when only considering the static state of a certain time point, the relationship between the number of driver nodes and the density of nodes is shown in Fig. 10. Five curves with different colors represent different communication radius of nodes. The number of driver nodes decreases with the increase of density and tends to a minimum of 1.

Moreover, under different communication radius conditions, the decline rate of the number of driver nodes is different with the increase of node density, and the larger the communication radius is, the faster it decreases to the minimum value. This is because, when increasing the same node density, the number of new connections with others of each node is proportional to the square of the radius, so the larger the communication radius, the faster it approaches the minimum value.

#### C. Nodes Velocity

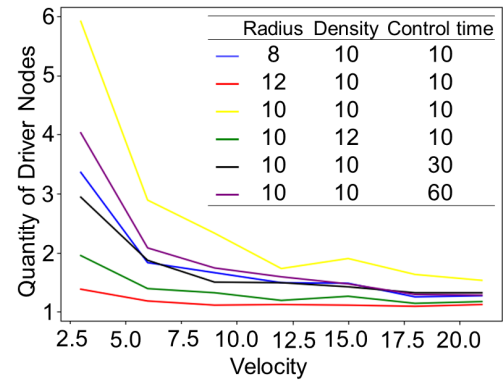


Fig. 11. Relationship between number of driver nodes and velocity.

For the node velocity as a control variable parameter, other variables are combined in different cases. The result of number of driver nodes is shown in Fig. 11. Six curves with different colors correspond to different control time, node communication radius or node density. According to the overall trend of the six color curves, with the increase of node velocity, the number of required driver nodes in the network shows a downward trend, ultimately, it tends to the minimum value of 1. This is because that with the increasement of velocity, refresh frequency will increase, then the connection degree of the whole network will increase in the same control time, longer control time is even more obvious. And the different decline rates of each curve we've explained above.

#### D. Control Time

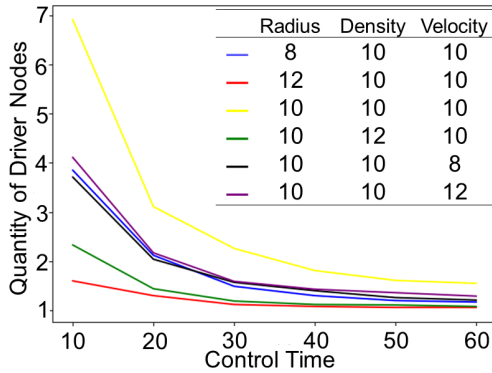


Fig. 12. Relationship between number of driver nodes and control time.

For the control time as a control variable parameter, other variables compose different situations. The results of number of driver nodes as shown in Fig. 12. Six curves with different colors correspond to different node density, node communication radius or node velocity. With the increase of control time, the total times of refresh will increase which means the connection degree of the whole network will increase, which leads to the number of required driver nodes in the network shows a downward trend, and tends to the minimum of 1 finally.

The driver nodes required in the latter two experiments are much less than those in the former two experiments. This is

Although the experimental data are simulated, but they are generated according to the actual situation. If there is a chance to obtain the actual road condition data through the relevant agencies, the real results can be obtained. However, the regular relationship between the parameters obtained through the experiment is universal.

because the influence of velocity and control time on dynamic network is considered, resulting in the connection degree of the whole graph is much higher than that of the former two experiments.

## VI. CONCLUSION

The rapid movement of nodes in dynamic network leads to the rapid change of network structure, and the controllability of network is facing enormous challenges. We apply cybernetics and system theory to real network scenarios, and accurately calculate the minimum number of driver nodes needed under the conditions of the whole network controllability under the corresponding scenarios. We get the relationship between several main parameters and the minimum number of driver nodes in the network. Applying the conclusion to the specific scenario can greatly save the deployment cost and improve the efficiency of the network. This paper only completes a part of the work in related fields. The deployment of specific access points and other application scenarios need to be further improved.

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